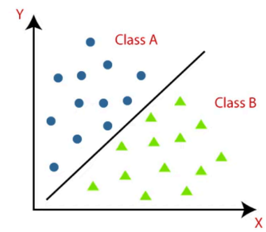
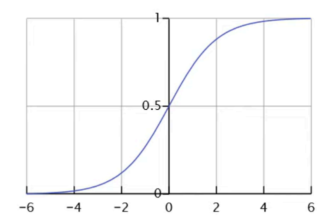
Logistic Regression

Logistic Regression

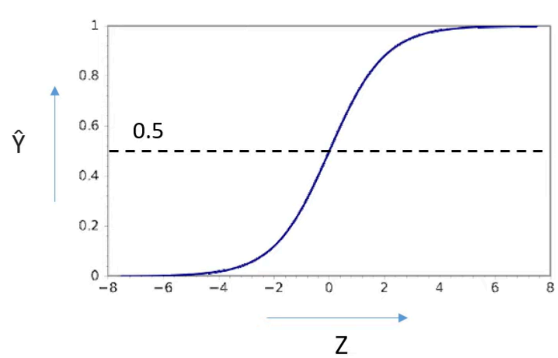
About Logistic Regression:

1. Supervised Learning Model
2. Classification model
3. Best for Binary Classification Problem
4. Uses Sigmoid function





When there are two classes then logistic regression is best suited for it. Also known as binary classification.



$$\hat{Y} = \frac{1}{1 + e^{-Z}} \qquad Z = w.X + b$$

Sigmoid Function

$\hat{Y}$ - Probability that (y = 1)

$\hat{Y}$ = P(Y=1 | X)

X - input features

Siddhar

w – weights
(number of weights is equal to the number of input features in a dataset)

b - bias

$\hat{Y} = \sigma(Z)$

Every input feature has a specific weight. so it is very important. Y cap will always be in the range of 0 and 1.

Advantages:

1. Easy to implement
2. Performs well on data with linear relationship
3. Less prone to over-fitting for low dimensional dataset

Disadvantages:

1. High dimensional dataset causes over-fitting
2. Difficult to capture complex relationships in a dataset
3. Sensitive to Outliers
4. Needs a larger dataset

dimensional represents number of input features. so, high number of input features causes over-fitting.

Logistic regression uses classification. Thus, we can use accuracy score for it.

Math Behind Logistic Regression

X	-9	-8	0	8	9
Ŷ					

$Z = 5X + 10$ $\qquad$ $\hat{Y} = \frac{1}{1+e^{-Z}}$

X = -9
$Z = 5(-9) + 10$
Z = -35
$\hat{Y} = \frac{1}{1+e^{35}}$
$\hat{Y} = 0$

X = -8
$Z = 5(-8) + 10$
Z = -30
$\hat{Y} = \frac{1}{1+e^{30}}$
$\hat{Y} = 0$

X = 0
$Z = 5(0) + 10$
Z = 10
$\hat{Y} = \frac{1}{1+e^{-10}}$
$\hat{Y} = 1$

X = 8
$Z = 5(8) + 10$
Z = 50
$\hat{Y} = \frac{1}{1+e^{-50}}$
$\hat{Y} = 1$

X = 9
$Z = 5(9) + 10$
Z = 55
$\hat{Y} = \frac{1}{1+e^{-55}}$
$\hat{Y} = 1$

the y cap values are very very close to 1 or 0

Inference:

If Z value is a large positive number,

$$\hat{Y} = \frac{1}{1+0}$$

$$\hat{Y} = 1$$

If Z value is a large negative number,

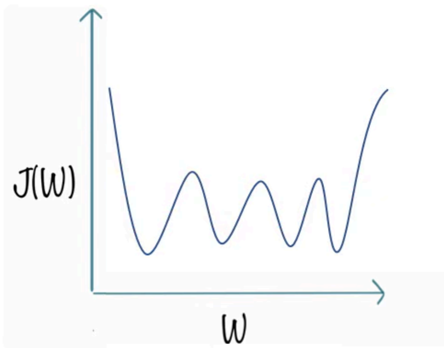
$$\hat{Y} = \frac{1}{1 + (large\ positive\ number)}$$

$$\hat{Y} = 0$$

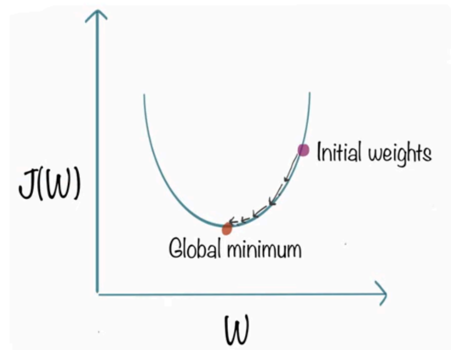
Loss and cost func

Loss function measures how far an estimated value is from its true value.

$$\textbf{Loss} = \frac{1}{n}\sum_{i=1}^{n}(Y_i - \hat{Y}_i)^2$$



Gradient Descent
With Local minima



Gradient Descent
With Global minima

there are many local minima and it causes a wiggly structure. Thus, we have to take log of this loss function to get one single global minima.

Binary Cross Entropy Loss Function (or) Log Loss :

$$L(y, \hat{y}) = -(y \log \hat{y} + (1 - y) \log(1 - \hat{y}))$$

When $y = 1$, $\Rightarrow$ $L(1, \hat{y}) = -(1 \log \hat{y} + (1 - 1) \log(1 - \hat{y}))$ $\Rightarrow$ $L(1, \hat{y}) = -\log \hat{y}$

We always want a smaller Loss Function value, hence, ŷ should be very large,
so that (– log ŷ) will be a large negative number.

When $y = 0$, $\Rightarrow$ $L(0, \hat{y}) = -(0 \log \hat{y} + (1 - 0) \log(1 - \hat{y}))$ $\Rightarrow$ $L(0, \hat{y}) = -\log(1 - \hat{y})$

We always want a smaller Loss Function value, hence, ŷ should be very small,
so that – log (1 - ŷ) will be a large negative number.

we want loss function to be minimum.

Cost Function for Logistic Regression

Loss function (L) mainly applies for a single training set as compared to the cost function (J) which deals with a penalty for a number of training sets or the complete batch.

$$L(y, \hat{y}) = -(y \log \hat{y} + (1 - y) \log(1 - \hat{y}))$$

Sid

$$J(w, b) = \frac{1}{m} \sum_{i=1}^{m} (L(y^{(i)}, \hat{y}^{(i)})) = -\frac{1}{m} \sum_{i=1}^{m} (y^{(i)} \log \hat{y}^{(i)} + (1 - y^{(i)}) \log(1 - \hat{y}^{(i)}))$$

('m' denotes the number of data points in the training set)

Gradient descent for logistic regression

About Logistic Regression:

1. Supervised Learning Model

2. Classification model

3. Best for Binary Classification Problem

4. Uses Sigmoid function

5. Binary Cross Entropy Loss Function (or) Log Loss

Gradient Descent is an optimization algorithm used for minimizing the cost function in various machine learning algorithms. It is used for updating the parameters of the learning model.

$$w_2 = w_1 - L*dw$$

$$b_2 = b_1 - L*db$$

w --> weight
b --> bias
L --> Learning Rate

dw --> Partial Derivative of cost function with respect to w

db --> Partial Derivative of cost function with respect to b

$$dw = \frac{1}{m} * (\hat{Y} - Y).X$$

$$db = \frac{1}{m} * (\hat{Y} - Y)$$

The concepts of gradient descent are same except that the cost function is different here. so, the partial derivative of cost function will be different.

ALL EQUATIONS REQUIRED :

Logistic Regression model:

- ❖ Sigmoid Function

$$\hat{Y} = \frac{1}{1 + e^{-Z}} \qquad Z = w.X + b$$

Sid

- ❖ Updating weights through Gradient Descent

$$w_2 = w_1 - L*dw$$

$$b_2 = b_1 - L*db$$

- ❖ Derivatives

$$dw = \frac{1}{m} * (\hat{Y} - Y).X$$

$$db = \frac{1}{m} * (\hat{Y} - Y)$$