A Paper Review on Aircraft Control using LQR and LQG Controllers with Optimal Estimation-Kalman Filter Design

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Abstract

This paper explores the development of an LQG controller for the control of a fixed wing aircraft's longitudinal and lateral flight dynamics. This uses the LQG as a robust controller while taking into account the varying dynamic parameters of the aircraft and the various disturbances of the system. The simulations have been run on MATLAB and Simulink and have been compared against the original paper, of which this is a review. This paper also briefly covers the design of the Kalman filter for achieving relatively noise free full state feedback. This allows the system to take advantage of multiple sensors to get a more accurate measurement of the state. In an actual aircraft, these could be the GPS, gyro, RADAR and IMU readings for the measurement of position and velocity. This paper also briefly covers aircraft dynamics and the various factors affecting them, such as the control surfaces.

1 Introduction

This paper is a review of the paper on Aircraft control using LQR and LQG controllers with optimal estimation-Kalman filter design [1]. Modern aircraft are complicated systems with many non-linear aspects to their dynamics. Standard controllers like the PID controller come up short when trying to control the various modes of operation. This is without considering the obvious measurement and process noise present in the aircraft. Secondly, the performance of any control system is improved through adding more sensors and actuators. Hence, the control system should be able to handle multiple sensors and actuators at once. The Linear Quadratic Gaussian controller (LQG) is one the more modern approaches to the problem. This controller is made to be optimal with respect to a quadratic cost function. This basically converts the problem into that of reducing a quadratic cost function to achieve optimal control schemes. This also takes into account the various disturbances in the system.

The LQG is based on the Linear Quadratic Regulator (LQR) and the Linear Quadratic Estimator (LQE) or the Kalman filter [2]. This has been further elaborated on in [3],[4] and [5].This allows the LQR to regulate performance, while receiving relatively noise free full state measurement from the Kalman filter. Both the LQR and LQE can be designed independently, allowing simplicity in the design. The design of the kalman filter requires the calculation of noise covariances, which is only accurate if done experimentally. Hence the controller has to be robust enough to adjust to the modeling errors in the noise.

2 Flight Equations

An aircraft typically has six degrees of freedom for motion. Three of these specify the position and the other three specify orientation. The change in orientation with respect to the passing air can be denoted by *Yaw*, *Pitch* and *Roll*. *Pitch* refers to the rotary motion of the aircraft about its lateral axis. *Yaw* refers to the rotation about the vertical axis of the aircraft, while *Roll* refers to the rotation about its longitudinal axis. Changing the angle of heading or turning the aircraft however requires it to roll first and achieve the desired bank angle and then apply a reverse roll to stabilize itself [6].

2.1 Nomenclature

Symbol	Indication
γ	Path Angle
θ	Pitch Angle
α	Angle of Attack
ϕ	Roll Angle
ψ	Yaw Angle
β	Sideslip Angle
q	Pitch Rate
u_0	Longitudinal Velocity
m	Aircraft Mass
δ_e	Elevator Deflection
δ_r	Rudder Deflection
δ_a	Aileron Deflection

2.2 Aircraft Dynamics

Aircraft dynamics can typically be described by six differential equations. There are three kinds of control surfaces on most fixed wing aircraft i.e. *Ailerons*, *Elevators* and *Rudders*. Deflection in the *Ailerons* gives rise to change in the *Roll* angle. Deflections in the elevator controls the *Pitch* of the aircraft, and the Rudders control the *Yaw*.

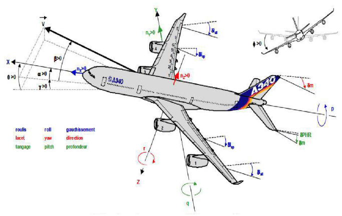


Figure 1: Aircraft dynamics nomenclature reference.

2.2.1 Velocity

The velocity can be described by the following equation:

$$\vec{v} = u\vec{i} + v\vec{j} + w\vec{k} \tag{1}$$

Where u,v and w are components of the velocity vector $\vec{v}$.

2.2.2 Forces

The forces acting on the aircraft can be described by the following equation:

$$\vec{F} = x\vec{i} + y\vec{j} + z\vec{k} \tag{2}$$

Where,
x is the Axial force
y is the transverse force
z is the Normal force

2.2.3 Moments

The moments acting on the aircraft can be described by the following equation:

$$\vec{T} = l\vec{i} + m\vec{j} + n\vec{k} \tag{3}$$

Where,
l is the Axial moment
m is the transverse moment
n is the Normal moment

2.2.4 Angular Velocities

The angular velocities can be described by the following equation:

$$\vec{\omega} = p\vec{i} + q\vec{j} + r\vec{k} \tag{4}$$

Where,
p is the Roll rate
q is the Pitch rate
r is the Yaw rate

2.3 Control and sign conventions

There are typically four basic forces acting on an aircraft, i.e. Lift, Drag, Weight and Thrust. These forces can be controlled using the control surfaces on the aircraft to produce desired changes in motion. The function of the control surfaces in a broad sense is as follows:
Ailerons control Roll rate.
Elevators control the Pitch.
Rudders control the Yaw.

2.3.1 Ailerons

Ailerons are the primary control surfaces for rolling, and can be operated differentially. The downward deflection of the aileron deflects air downward, causing lift to be generated under the wing and lifting it up. Thus, the positive aileron deflection causes a corresponding negative deflection with its orientation axis
Ailerons do not actually control roll angle. They control the roll rate.

2.3.2 Elevators

Elevators are used as control surfaces for increasing or decreasing pitch. The downward deflection of the elevator causes the airflow to be deflected downward at the very end of the aircraft, causing a moment which causes the aircraft to decrease the pitch angle. Thus positive elevator deflection causes a corresponding positive deflection with its orientation axis.

2.3.3 Rudders

Rudders are the primary control surfaces for controlling the Yaw. The counterclockwise deflection causes a flow of air towards the left at the tail end of the aircraft. This causes a moment, turning it about the z axis. Here, a positive angle of deflection gives rise to a positive deflection in the Yaw angle.

2.4 Equations of Motion

The general equations of motion can be derived from the equations of mechanics. They can be generalized as forces and moments respectively:

$$m\frac{du}{dt} = \sum \vec{F}_e \tag{5}$$

$$\frac{dC}{dt} = \sum \vec{M}_e \tag{6}$$

Thus the dynamic equations can be described as longitudinal dynamics and lateral dynamics.

3 Longitudinal Dynamics

Aircraft longitudinal dynamics assume,

$$\beta = p = r = \phi = 0 \tag{7}$$

Hence, Longitudinal equations can be written as follows [7]:

$$\dot{u} = \frac{X_u}{m}u + \frac{X_w}{m}w - \frac{g\cos\theta_0}{m}\theta + \Delta X^c \tag{8}$$

$$\dot{w} = \frac{Z_u}{m - Z_{\dot{w}}}u + \frac{Z_w}{m - Z_{\dot{w}}}w - \frac{Z_q + mU_0}{m - Z_{\dot{w}}}q - \frac{mg\sin\theta_0}{m - Z_{\dot{w}}} + \Delta Z^c \tag{9}$$

$$\dot{q} = \frac{M_u + Z_u\Gamma}{Iyy}u + \frac{M_u + Z_u\Gamma}{Iyy}w - \frac{M_q + (Z_q + mU_0)\Gamma}{Iyy} - \frac{mg\sin\theta_0\Gamma}{Iyy}\theta + \Delta M^c \tag{10}$$

$$\dot{\theta} = q \tag{11}$$

Where,

$$\Delta X^c = \frac{X_{\delta e}}{m}\delta e + \frac{X_p}{m}\delta e \tag{12}$$

$$\Delta Z^c = \frac{Z_{\delta e}}{m - Z_{\dot{w}}}\delta e + \frac{X_{\delta p}}{m - Z_{\dot{w}}}\delta e \tag{13}$$

$$\Delta M^c = \frac{M_{\delta e} + Z_{\delta e}\Gamma}{Iyy}\delta e + \frac{M_{\delta p} + Z_{\delta p}\Gamma}{Iyy}\delta p \tag{14}$$

.Now, u is assumed to be approximately zero. Hence, $\dot{u} = 0$. We can now rewrite the equations in state space form:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} \frac{Z_w}{m - Z_{\dot{w}}} & \frac{Z_q + mU_0}{m - Z_{\dot{w}}} & \frac{-mg\sin\theta_0}{m - Z_{\dot{w}}} \\ \frac{M_w + Z_w\Gamma}{Iyy} & \frac{M_q + (mU_0)\Gamma}{Iyy} & \frac{-mg\sin\theta_0}{Iyy} \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} \Delta Z^c \\ \Delta M^c \\ 0 \end{bmatrix} \delta e \tag{15}$$

This is a non-linear state space. The author of the paper has linearized the plant about an unspecified equilibrium point to achieve the following matrix:

$$\begin{bmatrix} \dot{w} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \begin{bmatrix} -0.3149 & 235.8928 & 0 \\ -0.0034 & -0.4282 & 0 \\ 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} -5.5079 \\ 0.00021 \\ 0 \end{bmatrix} \delta e \tag{16}$$

$$y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} w \\ q \\ \theta \end{bmatrix} + \begin{bmatrix} 0 \end{bmatrix} \tag{17}$$

The current variable of interest is the pitch angle and it is reflected in the values of the C matrix.

3.1 Stability and Controllability

The Eigen values for the longitudinal dynamic equations are as follows:

$$0.0000 + 0.0000i$$

$$-0.3715 + 0.8938i$$

$$-0.3715 - 0.8938i$$

Thus, based on the Eigen values, it can be concluded that the system is marginally stable. The controllability of the system can be deduced by the rank of the controllability matrix:

$$\begin{bmatrix} B & AB & ... & A^{n-1}B \end{bmatrix}$$

The controllability matrix for the longitudinal dynamics is as follows:

$$\begin{bmatrix} -5.5079 & 2.2298 & 3.5032 \\ 0.0021 & 0.0178 & -0.0152 \\ 0 & 0.0021 & 0.0178 \end{bmatrix}$$

Which has rank 3, i.e. full rank. Thus it can be concluded that the longitudinal dynamics of the aircraft are controllable. A similar test can be run for observability, to verify if full state feedback is possible. The observability matrix is as follows:

$$\begin{bmatrix} 0 & 0 & -0.0034 \\ 0 & 1.0000 & -0.4282 \\ 1.0000 & 0 & 0 \end{bmatrix}$$

The rank for this matrix is 3, i.e. full rank. Thus full state feedback is possible for the longitudinal dynamic states. The input to the system is the elevator deflection impulse of $0.2rad$ or $11\,\deg$.

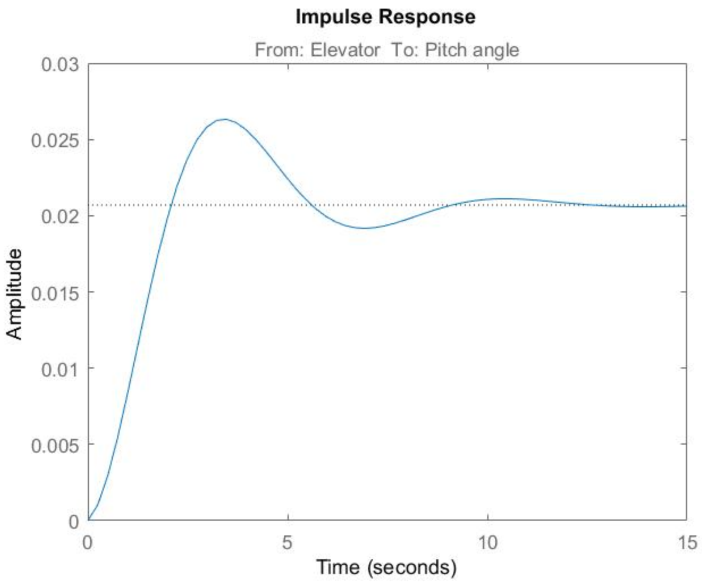


Figure 2: Open loop impulse response (Pitch Angle).

The output is in the form of the Pitch Angle θ.

4 Lateral Dynamics

Based on the equations in [], and using a similar method to the longitudinal dynamic equations, the state equations for the aircraft's lateral dynamics can be deduced. The state variables are chosen as Sideslip Angle(β), Roll rate(p), Yaw rate (r) and Roll angle(ϕ). The inputs to the system are aileron and rudder (δ_a and δ_r) deflection. These can be represented as a state space model, as follows [7]:

$$\dot{x} = Ax + Bu$$

$$y = Cx + Du$$

$$\begin{bmatrix} \dot{\beta} \\ \dot{p} \\ \dot{r} \\ \dot{\phi} \end{bmatrix} = \begin{bmatrix} \frac{Y_\beta}{U_0} & \frac{Y_p}{U_0} & \frac{Y_r}{U_0} & \frac{g\cos\theta}{U_0} \\ L_\beta & L_p & L_r & 0 \\ N_\beta & N_p & N_r & 0 \\ 0 & 1 & \tan\theta_0 & 0 \end{bmatrix} \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix} + \begin{bmatrix} \frac{Y_{\delta_r}}{U_0} & \frac{Y_{\delta_a}}{U_0} \\ L_{\delta_r} & L_{\delta_a} \\ N_{\delta_r} & N_{\delta_a} \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{18}$$

This is a nonlinear state space equation. The author of the paper as linearized it about a non-specific equilibrium point. The author also hasn't mentioned the values of the various constants used in the state space models used to achieve the following result:

$$\begin{bmatrix}\dot{\beta}\\ \dot{p}\\ \dot{r}\\ \dot{\phi}\end{bmatrix} = \begin{bmatrix}-0.0558 & -0.9968 & 0.0802 & 0.0415\\ 0.5980 & -0.1150 & -0.0318 & 0\\ -3.0500 & 0.3880 & -0.4650 & 0\\ 0 & 0.0805 & 1.0000 & 0\end{bmatrix}\begin{bmatrix}\beta\\ p\\ r\\ \phi\end{bmatrix} + \begin{bmatrix}0.0729 & 0\\ -4.7500 & 0.0077\\ 0.1530 & 0.1430\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix} \tag{19}$$

$$y = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}\begin{bmatrix}\beta\\ p\\ r\\ \phi\end{bmatrix} + \begin{bmatrix}0 & 0\\ 0 & 0\end{bmatrix}\begin{bmatrix}\delta_a\\ \delta_r\end{bmatrix} \tag{20}$$

It can be seen that the outputs of concern are the Sideslip angle β, and the Roll angle ϕ.

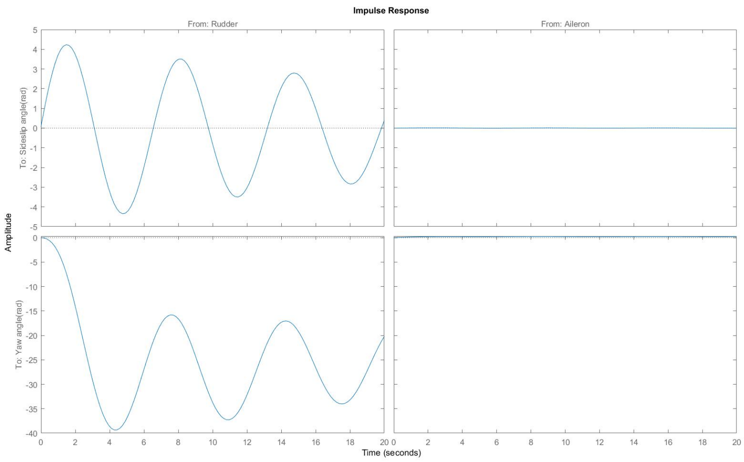


Figure 3: Open loop impulse response (Sideslip Angle and Roll Angle).

4.1 Stability and Controllability

The Eigen values for the lateral dynamic equations are as follows:

$$-0.0329 + 0.9467i - 0.0329 - 0.9467i - 0.5627 + 0.0000i - 0.0073 + 0.0000i$$

Thus, since all the real parts of the EIgen values are negative, the system is stable. The controllability of the system can be deduced by the rank of the controllability matrix:

$$\begin{bmatrix}B & AB & ... & A^{n-1}B\end{bmatrix}$$

The controllability matrix for the lateral dynamics is as follows:

$$\begin{bmatrix}0.0729 & 0 & 4.7430 & 0.0038 & -1.0286 & 0.0061 & -3.9195 & -0.0066\\ -4.7500 & 0.0077 & 0.5850 & -0.0054 & 2.8370 & 0.0049 & -0.5202 & 0.0026\\ 0.1530 & 0.1430 & -2.1365 & -0.0635 & -13.2457 & 0.0159 & 10.3973 & -0.0240\\ 0 & 0 & -0.2294 & 0.1436 & -2.0894 & -0.0639 & -13.0173 & 0.0162\end{bmatrix}$$

The rank for this matrix is 4, i.e. the matrix is half rank. This means that not all poles can be controlled. However, the matrix is stabilizable due to its inherent stability.

Here, the impulse response for the roll angle is different from the output the author observed in the paper. The paper has an output which rises up to a value of $0.18rad$, before showing an under-damped oscillation. The simulations run during the review of the paper show that the roll angle drops below zero to a value of $-40rad$ followed by under-damped oscillations similar to those seen in the paper. The reason for this discrepancy has not been fully explored yet, but preliminary examining reveals no differences in the state space or the input to the state space from the paper.

5 LQG Controller

The Linear Quadratic Gaussian (LQG) controller is one of the most prominent optimal controllers . It can be used to control linear system with additive white Gaussian noise by reducing the state space error to a quadratic cost function which can be then minimized. The solution to this minimization is unique and is computed with relative ease. The LQG controller is one of the most popular controllers for the optimal control of nonlinear systems, after they have been linearized about an equilibrium point. It has previously been used for motion prediction [8]
LQG control allows regulation of the trade off between performance and control effort, while taking into account the noise from the system and its measurements. The LQG is simply an extended version of the Linear Quadratic Regulator (LQR). However, since the controller takes into account that all the states may not be available and the measurements may be noisy, it requires a Kalman state estimator for full state feedback.

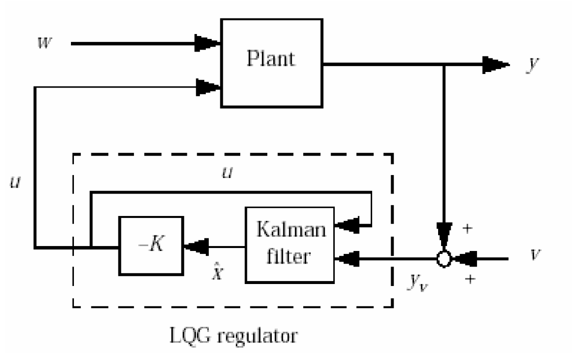


Figure 4: Block representation of the LQG controller.

State space equations for the regulator can be given as:

$$\frac{d}{dt}\hat{x} = [A - LC - (B - LD)K]\hat{x} + Ly_v \tag{21}$$

$$u = -Kx \tag{22}$$

Regulating the output y to near 0 is the goal. The plant has disturbances incorporated which are given by $y_v = y + v$. These disturbances are accounted for by the controller while generating the control signals. The state equations can be given by the following:

$$\dot{x} = Ax + Bu + Gw \tag{23}$$

$$y_v = Cx + Du + Hw + v \tag{24}$$

Here, v and w are modeled as white noise. As mentioned above, the LQG consists of both optimal state feedback and a Kalman filter. For the purposes of the LQG, the Kalman filter and the Optimal state feedback can be designed independently.

5.1 Optimal State feedback gain

The LQG reduces the state error to a quadratic function which can be minimized. This quadratic performance criterion can be given by:

$$J(u) = \int_0^\infty [x^T Qx + 2x^T Nx + u^T Ru] \tag{25}$$

The regulator is an infinite horizon regulator. Hence the limits to the integration are from 0 to ∞. The function $J(u)$ is to be minimized by the regulator and regulated to near 0. The matrices Q,N and R are user defined, and are used to specify the trade off between control effort and regulation performance. This optimal feedback gain is called the LQ optimal gain. This is basically the optimal full state feedback gain K which minimized the error function. It is multiplied with the feedback and subtracted from the reference signal.

5.2 Kalman State Estimator

The control scheme $u = -kx$ is not possible to implement without full state feedback x. The states may not always be directly measurable. Hence, we derive the states from the output of the system and the state transition matrix of the system, while taking into account the noise from the process and the measurements. This is the basic function of the Kalman filter or the Linear Quadratic Estimator (LQE). The Kalman filter generates the state estimate $\hat{x}$ which remains optimal for the output-feedback problem. The state equation is given by:

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu + L(y_v - C\hat{x} - Du) \tag{26}$$

Where L is the observer gain.
For the controller and estimator, there are two separate sets of Riccati equations to be solved. This allows their design to be independent of each other.
For the control signal u, and the measurement y_v, noise covariance data is as follows:

$$E(ww^T) = Q_n \tag{27}$$

$$E(vv^T) = R_n \tag{28}$$

$$E(wv^T) = Nn \tag{29}$$

These covariances are calculated as follows:
$E(x)$ = Expected value of x
$E(y)$ = Expected value of y

$$E(x, y) = E[x - E(x)][y - E(y)] = E[x] - E[x]E[y]$$

The Kalman gain L can be determined through the algebraic Riccati equation. Kalman filters are the optimal estimators when dealing with white Gaussian noise. The minimization of estimation error is given by:

$$\lim_{t\to\infty} E((x - \hat{x})(x - \hat{x})^T) \tag{30}$$

For the regulation of the plant to near zero, the input disturbance at low frequency with Power Spectral Density(PSD) below $10 rad/sec.$ was chosen.

6 Linear Quadratic Regulator

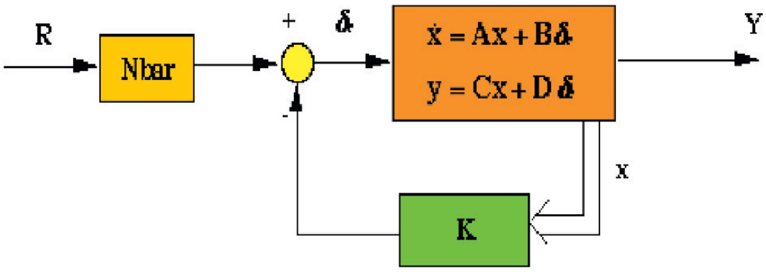


Figure 5: Block representation of the LQ Regulator.

The LQR assumes an absence of disturbance in the system. It requires the state space approach to analyze a system. This makes it easy to work with multi-output system. The LQR relies on full state feedback. Thus if the pair (A, B_k) is stabilizable, then we look for a feedback gain k which minimizes the following quadratic cost function [9]:

$$J(k, \vec{x}(0)) = \int_0^\infty \vec{x}(t)^T Q\vec{x}(t) + \vec{u_k}(t)^T R\vec{u_k}(t)dt \tag{31}$$

Here, Q and R are positive definite. The optimal solution is given by the following controller:

$$k = -R^{-1}{B_k}^T P \tag{32}$$

Here, P is a positive symmetric solution of the following stationary Riccati equation:

$$A^T P + PA - PBR^{-1}B^T P = -Q \tag{33}$$

Now, the cost function $J = (k, \vec{x}(0))$ can be expressed as :

$$J(k, \vec{x}(0)) = \int_0^\infty \vec{x}(t)^T Q\vec{x}(t) + [x^T(t)k^T]R[kx(t)]dt \tag{34}$$

$$(u = kx) \tag{35}$$

$$J(k, \vec{x}(0)) = \int_0^\infty x^T(Q + k^T Rk)xdt \tag{36}$$

Substituting value of u in the state space equations:

$$\dot{x} = Ax + B_k u = Ax + B_k kx = (A + B_k)x \tag{37}$$

$$x(t) = e^{(A+B_k K)t}x(0) \tag{38}$$

$$J(k, \vec{x}(0)) = x(0)\int_0^\infty e^{(A+B_k K)^T t}(Q + k^T Rk)e^{(A+B_k K)t}dtx(0) \tag{39}$$

$$= x^T(0)Px(0) \tag{40}$$

Where P is the symmetric positive definite solution to:

$$(A + B_k k)^T P + P(A + B_k k) = -(Q + k^T Rk) \tag{41}$$

Using completion of squares, we can rewrite the equation as:

$$A^T P + PA = -k^T Rk - k^T B_k P - Q + PB_k R^{-1}{B_k}^T P \tag{42}$$

$$A^T P + PA - PB_k R^{-1}{B_k}^T P = -Q \tag{43}$$

This is the Riccati equation.

The lqr function in MATLAB can be used to design the optimal feedback gains for any system. This paper attempts to design the feedback using the same. This is done by inputting the state matrices (A, B), and the weight matrices Q and R. The Q matrix can be calculated as $Q = C^T * C$. The paper assumes the following values for the matrices in the longitudinal dynamic model:

$$R_{longitudinal} = 1, Q_{longitudinal} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{44}$$

The matrices for the lateral model have not been mentioned in the paper, but they can be calculated as follows:

$$R_{lateral} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} Q_{lateral} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{45}$$

However, the response achieved using the values from the paper result in a different rise time for the response in each case. Different values of scalar multipliers for the R matrix were tried and the values for which the response matched the response in the paper are:

$$R_{longitudinal} = 5 * 10^-4, R_{lateral} = 0.02 \tag{46}$$

The k matrices for both the models are as follows:

$$K_{longitudinal} = \begin{bmatrix} -0.1391 & 31.5864 & 44.7213 \end{bmatrix}, K_{lateral} = \begin{bmatrix} 7.9929 & -2.0883 & -4.6226 & -6.5483 \\ -0.4177 & 0.1924 & 1.7464 & 2.6065 \end{bmatrix} \tag{47}$$

The response for these gains is give in figure 6.

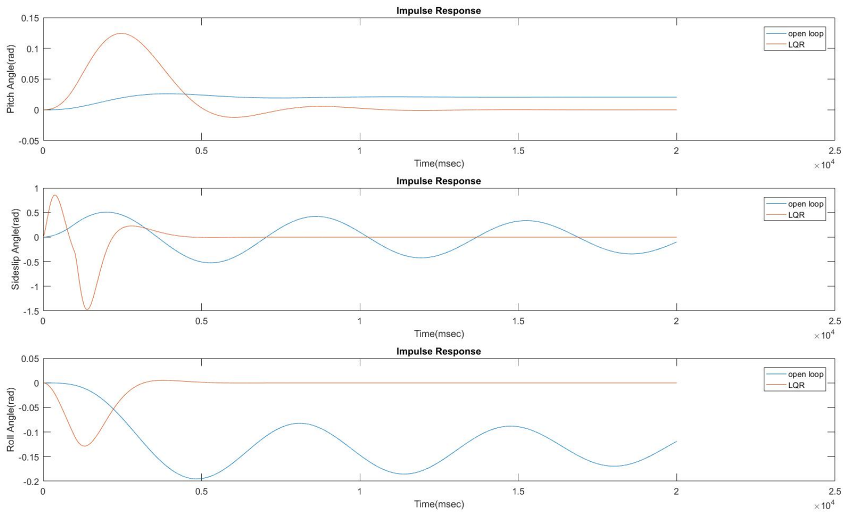


Figure 6: .(a) Comparison of Open-loop and Closed-Loop Impulse Response for the LQR (Pitch angle),(b) Comparison of Open-loop and Closed-Loop Impulse Response for the LQR (Sideslip angle,(c) Comparison of Open-loop and Closed-Loop Impulse Response for the LQR (Roll angle)

7 Kalman Filtering

For the purpose of designing the Kalman filter, we can assume the discrete plant to be represented as:

$$x(n+1) = Ax(n) + B(u(n) + w(n)) \tag{48}$$

$$y(n) = Cx(n) \tag{49}$$

$w(n)$ is the additive noise to the input. The Kalman filter should be able to estimate $y(n)$, given $u(n)$ despite $v(n)$ added to the output measurements.

$$y_v(n) = Cx(n) + v(n) \tag{50}$$

$v(n)$ is modeled as Gaussian white noise.

7.1 Discrete Kalman Filter

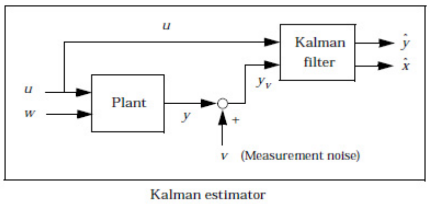


Figure 7: Block representation of the Kalman estimator.

The discrete Kalman filter has two modes i.e. Time update and Measurement update. The time update mode is responsible for the calculation of the projected value of the state based on the previous values of the state.
The measurement update is responsible for the update to the time update, so it can recalculate the projected values based on the current measurement.

The equations for the discrete steady state Kalman filter are as follows:
Measurement update:

$$\hat{x}(n/n) = \hat{x}(n/n-1) + M(y_v(n) - C\hat{x}(n/n-1)) \tag{51}$$

Time update:

$$\hat{x}(n+1/n) = A\hat{x}(n) + Bu(n) \tag{52}$$

Here,
$\hat{x}(n+1/n)$ is the estimate of $x(n)$ given past measurements up to $y_v(n-1)$.
$\hat{x}(n/n)$ is the updated measurement given the last estimate $y_v(n)$.

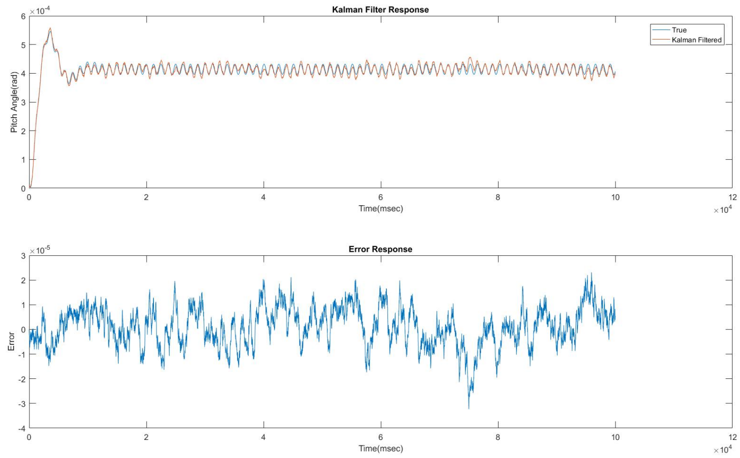


Figure 8: Kalman filter response for pitch angle θ.

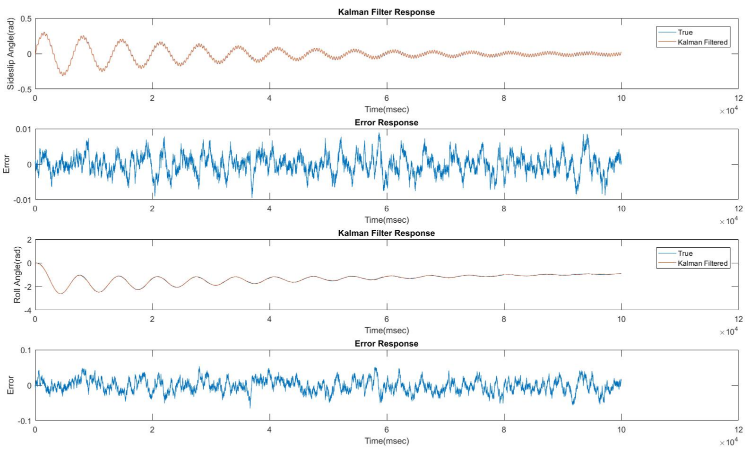


Figure 9: (a) Kalman Filter Response for the sideslip angle, β (b) Kalman Filter Response for the roll angle ϕ.

Given the current estimate $\hat{x}(n/n)$, the time update predicts the state value at the next sample $n+1$ (one-step-ahead predictor). The measurement update then adjusts this prediction based on the new measurement $y_v(n-1)$. The correction term is a function of the innovation, that is, the discrepancy between the measured and predicted values of $y(n+1)$.

$$y_v(n-1) - C\hat{x}(n/n-1) = C(x(n+1) - \hat{x}(n+1/n)) \tag{53}$$

The innovation gain M is chosen to minimize the steady-state covariance of the estimation error given the noise covariances.

$$E(w(n)w(n)^T) = Q \tag{54}$$

$$E(v(n)v(n)^T) = R \tag{55}$$

Thus the time and update equations can be bundled into one state space model, i.e. the Kalman filter:

$$\hat{x}(n+1/n) = A(I - MC)\hat{x}(n/n-1) + \begin{bmatrix} B & AM \end{bmatrix} \begin{bmatrix} u(n) \\ y_v(n) \end{bmatrix} \quad (56)$$

$$\hat{y}(n/n) = C(I_M C)\hat{x}(n/n-1) + CMy_v(n) \quad (57)$$

This generates the optimal estimate $\hat{y}(n/n)$ of $y(n)$.
State of the filter is $\hat{x}(n/n-1)$. For the design of the Kalman filter block in Simulink, the gain matrices Q, N and R had to be chosen. The matrices for the lateral and longitudinal model are as follows:

$$\hat{R}_l ongitudinal = [1], \hat{Q}_l ongitudinal = [1], \hat{N}_l ongitudinal = 0 \quad (58)$$

$$\hat{R}_l ateral = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \hat{Q}_l ateral = [1], \hat{N}_l ateral = 0 \quad (59)$$

These values or the method to obtain them is not discussed in the paper. The simulation results for the Kalman filter output for a sine wave input for each of the models has been described in the figures 8 and 9.

8 Conclusion

The control scheme developed by the end of the paper should serve as a robust controller for the regulation of the Pitch angle, Sideslip angle and the Roll angle. Compared to a more primitive controller such as the PID, the LQR and the LQG are more optimal. This controller takes into account the various process disturbances by implementing the Kalman filter for a an accurate state measurement.
Instead of tuning gains to match performance parameters, the optimal gains can be calculated by solving the Riccati equations for the system. This is a more powerful control scheme, as the system specific features in the response are accounted for instead of manually tuning the gains.

References

[1] Labane Chrif, Zemalache Meguenni Kadda, `Aircraft Control System Using LQG and LQR Controller with Optimal Estimation-Kalman Filter Design`. 3rd International Symposium on Aircraft Airworthiness, ISAA 2013

[2] Robert C. Nelson.1998. *Flight Stability and Automatic Control*. University of Notre Dame.

[3] Anonymous. *uPSD3400 PWM API with R/C Servomotors Control Example (PCM),*. AN2283 Application note, June, 2006 http://www.st.com.

[4] Wong, K.C, 2006, `UAV Design Activities in a University Environment.`. University of Sydney.

[5] Aly El-Osery.2004. MATLAB Tutorial. October. http://www.ee.nmt.edu

[6] Michael, T. Rusnell. 2004. *Morphing UAV Pareto Curve Shift for Enhanced Performance.*. University of Notre Dame.

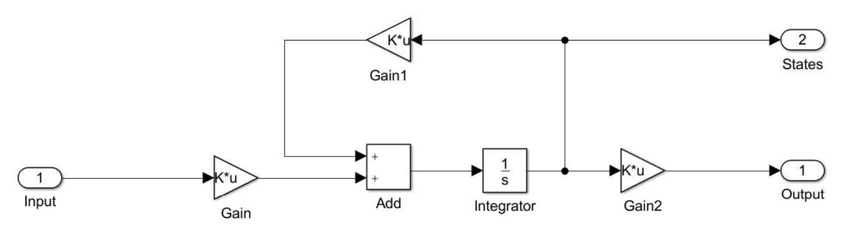
[7] Reed Siefert Christiansen (2004). `Design of an Autopilot for Small Unmanned Aerial Vehicle` Brigham Young University.

[8] Tom Bateman, Barry Blakeley, Adam Dean, Jake Hanft and Mike Sheek (2004). *UAV Stability Augmentation System (USAS),*. University of Colorado.

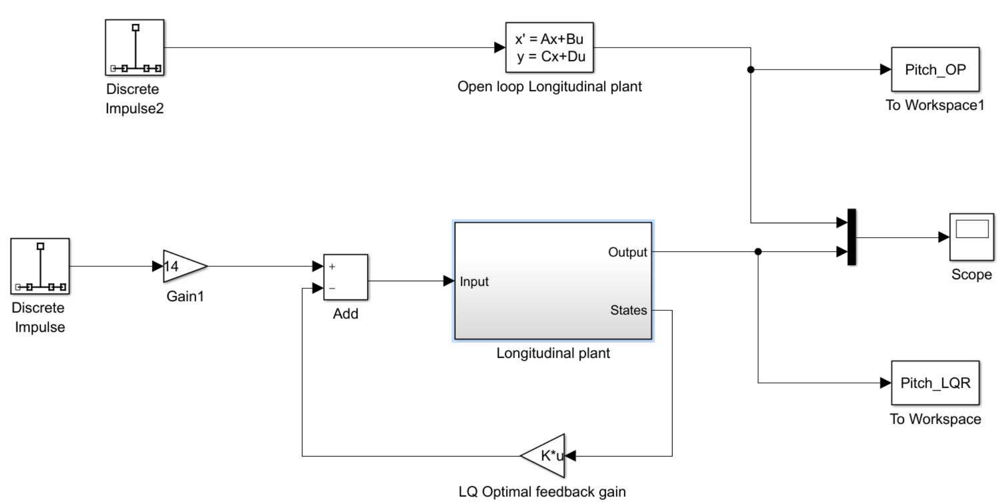
[9] R. M. Murray `Control and Dynamical Systems`. California Institute of Technology, 11 January 2006

APPENDIX:

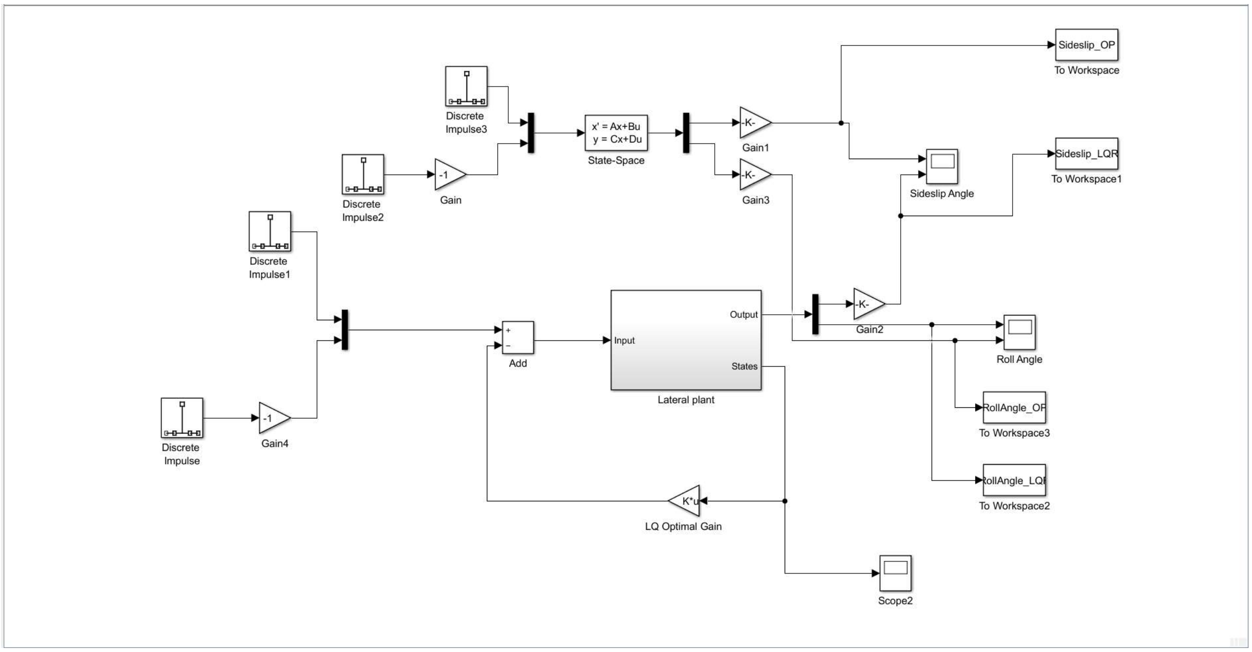
Open Loop model:



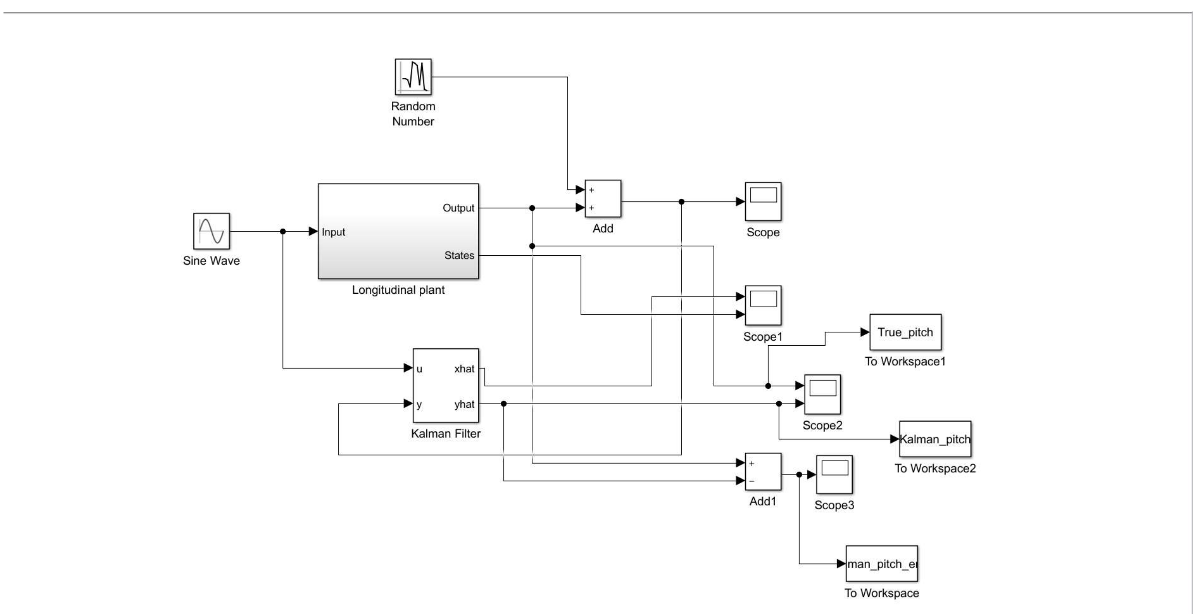
Longitudinal model LQR control:



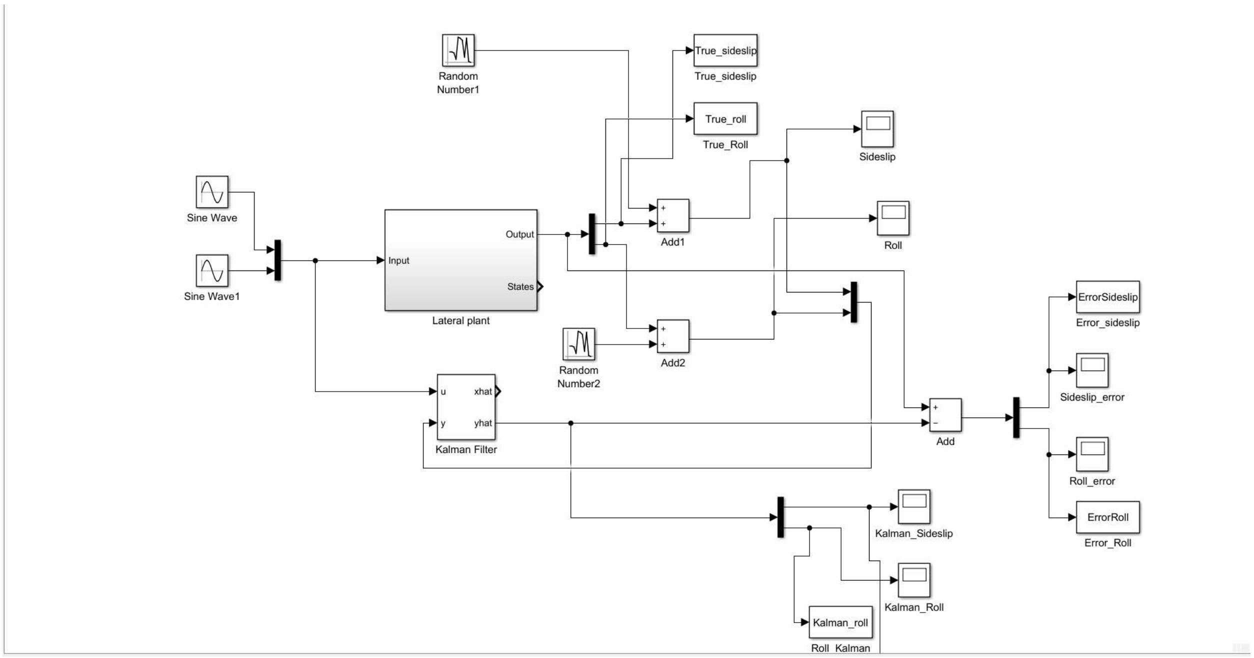
Lateral model LQR control:



Longitudinal model Kalman Filtering:



Lateral model Kalman Filtering:



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Aircraft Control System Using LQG and LQR Controller with Optimal Estimation-Kalman Filter Design

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Abstract

This paper, describes a LQG and LQR robust controller for the lateral and longitudinal flight dynamics of an aircraft control system. The controller is used in order to achieve robust stability and good dynamic performance against the variation of aircraft parameters. The application of the proposed LQG and LQR robust control scheme is implemented through the simulation. The proposed robust controller for aircraft stability is designed using Matlab/Simulink program. Simulation results confirm the performance of the proposed controller for aircraft control system. Since the time of its introduction, the Kalman filter has been the subject of extensive research and application, particularly in the area of autonomous or assisted navigation. For example, to determine the velocity of an aircraft or sideslip angle, one could use a Doppler radar, the velocity indications of an inertial navigation system, or the relative wind information in the air data system. Rather than ignore any of these outputs, a Kalman filter could be built to combine all of this data and knowledge of the various systems dynamics to generate an overall best estimate of pitch, roll and sideslip angle.

Selection and peer-review under responsibility of Airworthiness Technologies Research Center, Beihang University/NLAA.

Keywords: Aircraft motion; LQG control; LQR control; lateral stability; longitudinal stability; State estimator Kalman filter.

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Nomenclature

γ	Path angle
θ	Pitch angle
α	Angle of attack
ϕ	Roll angle
β	Sideslip angle
q	Pitch rate
U_0	Longitudinal velocity
m	Aircraft mass
δ_e	Elevator deflection
δ_r	Rudder deflection
δ_a	Aileron deflection

1. Introduction

The Feedback control systems are widely used in manufacturing, mining, automobile and military hardware applications. In response to demands for increased efficiency and reliability, these control systems are being required to deliver more accurate and better overall performance in the face of difficult and changing operating conditions. In order to design control systems to meet the demands of improved performance and robustness when controlling complicated processes, control engineers will require new design tools and better underlying theory. In particular, a standard method of improving the performance of a control system is to add extra sensors and actuators. This necessarily leads to a multi-input multi-output control system. Thus, it is a requirement for any modern feedback control system design methodology that it be able to handle the case of multiple actuators and sensors. Linear Quadratic Gaussian optimal control theory (LQG) is one of the major achievements of the modern control area. This controller design methodology enables a controller to be synthesized which is optimal with respect to a specified quadratic performance index. Furthermore, this theory takes into account the presence of Gaussian white noise disturbances acting on the system. Indeed, in many practical control problems, it is straightforward to translate the required performance objective into a problem of minimizing a quadratic cost functional. Also, in many practical control problems, the system is subject to disturbances and measurement noise which are most naturally modeled as stochastic white noise processes.

The LQG controller design methodology based on the Kalman filter who in 1960 published his famous paper describing a recursive solution to the discrete-data linear filtering problem. A more complete introductory discussion can be found in [1] which also contains some interesting historical narrative. More extensive references include [2], [3] and [4]. It has also been used for motion prediction [7] and it is used for multi-sensor . In practice, although it is possible to obtain process models either from first principles or from experimental measurements, these models will always be subject to errors. Thus, the control system needs to be designed to be robust against these modeling errors.

1.1 Aircraft control and movement

There are three primary ways for an aircraft to change its orientation relative to the passing air. *Pitch* (movement of the nose up or down), *Roll* (rotation around the longitudinal axis, that is, the axis which runs along the length of the aircraft) and *Yaw* (movement of the nose to left or right.) Turning the aircraft (change of heading) requires the aircraft firstly to roll to achieve an angle of bank; when the desired change of heading has been accomplished the aircraft must again be rolled in the opposite direction to reduce the angle of bank to zero. [5]

2. Aircraft longitudinal dynamics

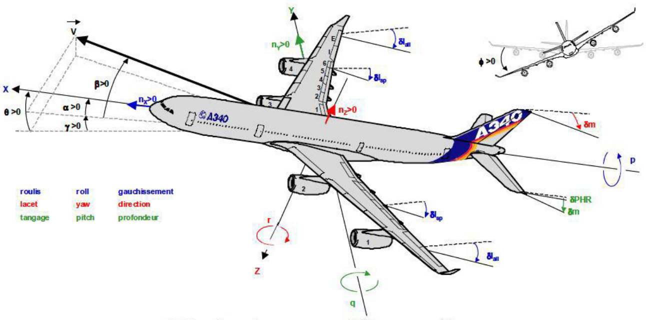

Fig.1. Aerodynamic reference

1.2. Equations of movements:

The general equations of the movement are governed by the equations of mechanics

$$\begin{cases} m\dfrac{\overrightarrow{du}}{dt} = \sum \overrightarrow{F_e} \\ \dfrac{d\vec{C}}{dt} = \sum \overrightarrow{M_e} \end{cases} \tag{1}$$

1.2.1. Equation of longitudinal motion:

$$\beta = p = r = \Phi = 0 \tag{2}$$

Longitudinal equations can be rewritten as:

$$\begin{cases} \dot{u} = \dfrac{X_u}{m}u + \dfrac{X_w}{m}w - \dfrac{g\cos\Theta_0}{m}\theta + \Delta X^c \\ \dot{w} = \dfrac{Z_u}{m-Z_{\dot{w}}}u + \dfrac{Z_w}{m-Z_{\dot{w}}}w + \dfrac{Z_q+mU_0}{m-Z_{\dot{w}}}q - \dfrac{mg\sin\Theta_0}{m-Z_{\dot{w}}}\theta + \Delta Z^c \\ \dot{q} = \dfrac{[M_u+Z_u\Gamma]}{I_{yy}}u + \dfrac{[M_u+Z_u\Gamma]}{I_{yy}}w + \dfrac{[M_q+(Z_q+mU_0)\Gamma]}{I_{yy}} \\ \quad -\dfrac{mg\sin\Theta_0\,\Gamma}{I_{yy}}\theta + \Delta M^c \\ \dot{\theta} = q \end{cases} \tag{3}$$

With:

$$\Delta X^c = \frac{X_{\delta_e}}{m}\delta_e + \frac{X_p}{m}\ \delta_p$$
$$\Delta Z^c = \frac{Z_{\delta_e}}{m - Z_{\dot\omega}}\delta_e + \frac{Z_{\delta_P}}{m - Z_{\dot\omega}}\delta_p$$
$$\Delta M^c = \frac{M_{\delta_e} + Z_{\delta_e}\Gamma}{I_{yy}}\delta_e + \frac{M_{\delta_p} + Z_{\delta_p}\Gamma}{I_{yy}}\delta_p \tag{4}$$

Rewrite in state space form as:

-Since $u \approx 0$ in this mode, then $\dot u \approx 0$ and can eliminate the X force equation:

$$\begin{bmatrix}\dot w\\ \dot q\\ \dot\theta\end{bmatrix} = \begin{bmatrix} \frac{Z_w}{m - Z_{\dot w}} & \frac{Z_q + mU_0}{m - Z_{\dot w}} & \frac{-mgsin\,\Theta_0}{m - Z_{\dot w}} \\ \frac{[M_w + Z_w\Gamma]}{I_{yy}} & \frac{[M_q + (Z_q + mU_0)\Gamma]}{I_{yy}} & \frac{-mgsin\,\Theta_0}{I_{yy}} \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}\Delta Z^c\\ \Delta \mathrm{M}^c\\ 0\end{bmatrix} \tag{5}$$

$$\begin{bmatrix}\dot w\\ \dot q\\ \dot\theta\end{bmatrix} = \begin{bmatrix} \frac{Z_w}{m} & U_0 & -gsin\Theta_0 \\ \frac{\left[M_w + Z_w\frac{\mathrm{M}_{\dot w}}{\mathrm{m}}\right]}{I_{yy}} & \frac{\left[M_q + (mU_0)\frac{\mathrm{M}_{\dot w}}{\mathrm{m}}\right]}{I_{yy}} & \frac{-mgsin\,\Theta_0}{I_{yy}}\frac{M_{\dot w}}{m} \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}\Delta Z^c\\ \Delta \mathrm{M}^c\\ 0\end{bmatrix} \tag{6}$$

The transfer function can be represented in state-space form and output equation as state by equation

$$\begin{bmatrix}\dot w\\ \dot q\\ \dot\theta\end{bmatrix} = \begin{bmatrix} -0.3149 & 235.8928 & 0 \\ -0.0034 & -0.4282 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + \begin{bmatrix}-5.5079\\ 0.0021\\ 0\end{bmatrix}\delta_e \tag{7}$$

$$y = [\,0 \;\; 0\; 1]\begin{bmatrix}w\\ q\\ \theta\end{bmatrix} + [0] \tag{8}$$

This work presents investigation into the development of pitch control schemes for pitch angle and pitch rate of an aircraft systems. Pitch control systems with full state feedback controller are investigated. A modern controller (LQG) control the pitch of an aircraft system. Performance of one control strategy with respect to the pitch. Simulation results are shown in Fig. 2

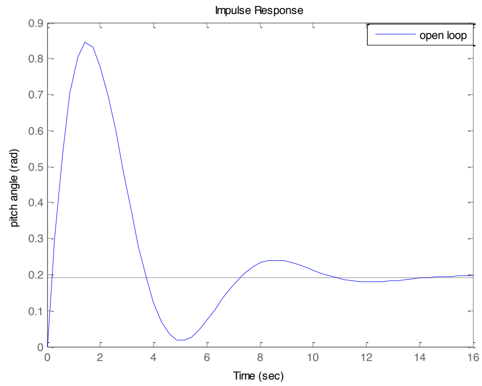

Fig.2. Open loop Impulse Response (Pitch angle)

$X = [u, \omega, q, \gamma]^T$ and $\gamma = \theta - \alpha$ represent flight path angle, with $\alpha = \omega$, $u = \begin{bmatrix} \delta_e \\ \delta_p \end{bmatrix}$

The input (elevator deflection angle, δ_e) will be 0.2 rad (11 degrees), and the output is the pitch angle (theta).

$X = [u, \omega, q, \gamma]^T$ and $\gamma = \theta - \alpha$ represent flight path angle, with $\alpha = \omega$, $u = \begin{bmatrix} \delta_e \\ \delta_p \end{bmatrix}$

The input (elevator deflection angle, δ_e) will be 0.2 rad (11 degrees), and the output is the pitch angle (theta).
There are three types of possible lateral-directional dynamic motion: roll subsidence mode, Dutch roll mode, and spiral mode.

3. Aircraft lateral dynamics

Using a procedure similar to the longitudinal mode, we can develop the equation of motion for the lateral dynamics.

$$\dot{x} = Ax + Bu, \qquad x = \begin{bmatrix} \beta \\ p \\ r \\ \phi \end{bmatrix}, \quad u = \begin{bmatrix} \delta_a \\ \delta_r \end{bmatrix} \tag{9}$$

$x^T = [\beta \; p \; r \; \phi]^T$: state vector
$u^T = [\delta_a \; \delta_r]^T$: control vector
δ_a, δ_r: aileron and rudder deflection
β, ϕ: sideslip and roll angle
p, r: roll and yaw rate

$$A=\begin{bmatrix}\frac{Y_\beta}{U_0} & \frac{Y_p}{U_0} & \frac{Y_r}{U_0} & \frac{g\cos\,\theta_0}{U_0}\\ L_\beta & L_p & L_r & 0\\ N_\beta & N_p & N_r & 0\\ 0 & 1 & \tan\theta_0 & 0\end{bmatrix},\quad B=\begin{bmatrix}\frac{Y_{\delta_r}}{U_0} & \frac{Y_{\delta_a}}{U_0}\\ L_{\delta_r} & L_{\delta_a}\\ N_{\delta_r} & N_{\delta_a}\\ 0 & 0\end{bmatrix},\qquad C=\begin{bmatrix}1\ 0\ 0\ 0\\ 0\ 0\ 0\ 1\end{bmatrix},\quad D=\begin{bmatrix}0\ 0\\ 0\ 0\end{bmatrix} \tag{10}$$

If we assume that the measurable outputs are the sideslip angle β and roll angle ϕ , the matrixes A, B and C are:

$$A=\begin{bmatrix}-0.0558 & -0.9968 & 0.0802 & 0.0415\\ 0.5980 & -0.1150 & -0.0318 & 0\\ -3.0500 & 0.3880 & -0.4650 & 0\\ 0 & 0.0805 & 1.000 & 0\end{bmatrix}, B=\begin{bmatrix}0.0729 & 0.000\\ -4.7500 & 0.00775\\ 0.15300 & 0.1430\\ 0 & 0\end{bmatrix},\quad C=\begin{bmatrix}1\ 0\ 0\ 0\\ 0\ 0\ 0\ 1\end{bmatrix} \tag{11}$$

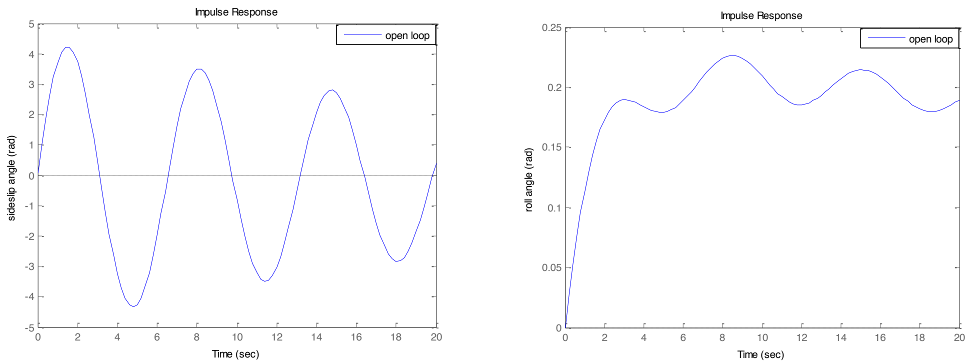

Fig.3. (a) Open loop Impulse Response (Sideslip angle); (b) Open loop Impulse Response (Roll angle)

4. Linear Quadratic Gaussian Controller

Linear Quadratic Gaussian (LQG) control is a modern state space technique for designing optimal dynamic regulators. It enables you to trade off regulation performance and control effort, and to take into account process and measurement noise. Like pole placement, LQG design requires a state-space model of the plant. This section focuses on the discrete-time case. To form the LQG regulator, simply connect the Kalman filter and LQ-optimal gain *K* as shown below:

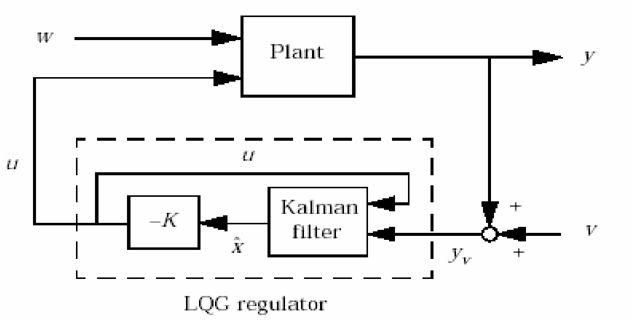

Fig.4. Block diagram of LQG Controller

This regulator has state-space equations

$$\frac{d}{dt}\hat{x} = [A - LC - (B - LD)K]\hat{x} + Ly_v \quad (12)$$
$$u = -Kx$$

The goal is to regulate the output y around zero. The plant is subject to disturbances and is driven by controls. The regulator relies on the noisy measurements $y_v = y + v$ to generate these controls. The plant state and measurement equations are of the form

$$\dot{x} = Ax + Bu + Gw$$
$$y_v = Cx + Du + Hw + v \quad (13)$$

and both w and v are modeled as white noise.
The LQG regulator consists of an optimal state-feedback gain and a Kalman state estimator. You can design these two components independently as shown next.

4.1. Optimal State-Feedback Gain

In LQG control, the regulation performance is measured by a quadratic performance criterion of the form

$$J(u) = \int_0^\infty \{x^T Qx + 2x^T Nx + u^T Ru\} \quad (14)$$

The weighting matrices Q, N and R are user specified and define the trade-off between regulation performance (how fast goes to zero) and control effort. The first design step seeks a state feedback law that minimizes the cost function. This gain is called the *LQ-optimal* gain.

4.2. Kalman State Estimator

As for pole placement, the LQ-optimal state feedback $u = -kx$ is not implementable without full state measurement. However, we can derive a state estimate $\hat{x}$ such that $u = -k\hat{x}$ remains optimal for the output- feedback problem. This state estimate is generated by the Kalman filter.

$$\frac{d}{dt}\hat{x} = A\hat{x} + Bu + L(y_v - C\hat{x} - Du) \tag{15}$$

With inputs u (controls) and y_v (measurements).The noise covariance data

$$E(ww^T) = Q_n\ , \qquad E(vv^T) = R_n\ , \qquad E(wv^T) = N_n \tag{16}$$

Determines the Kalman gain L through an algebraic Riccati equation.
The Kalman filter is an optimal estimator when dealing with Gaussian white noise. Specifically, it minimizes the asymptotic covariance of the estimation error $x - \hat{x}$.

$$\lim_{t\to\infty} E((x - \hat{x})(x - \hat{x})^T) \tag{17}$$

The goal is to regulate the plant output y around zero. The input disturbance d is low frequency with power spectral density (PSD) concentrated below 10 rad/sec. For LQG design purposes, it is modeled as white noise driving a low-pass filter with a cutoff at 10 rad/sec, as this picture shows.(fig 05-06)
There is some measurement noise n , with noise intensity given by

$$E(n^2) = 0.01 \tag{18}$$

Use the cost function

$$J(u) = \int_0^{\infty} (10y^2 + u^2)dt \tag{19}$$

to specify the trade-off between regulation performance and cost of control. Note that an open-loop state-space model is:

$$\begin{aligned} \dot{x} &= Ax + Bu + Bd \quad \text{(state equations)} \\ y_v &= Cx + n \quad \text{(measurements)} \end{aligned} \tag{20}$$

Simulation results are shown in Fig. 5-6

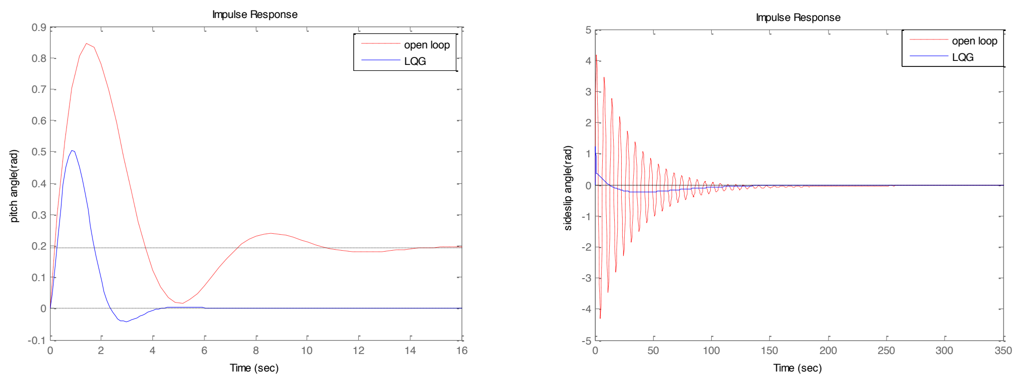

Fig.5. (a) Comparison of Open-loop and Closed-Loop Impulse Response for the LQG (Pitch angle), (b) Comparison of Open-loop and Closed-Loop Impulse Response for the LQG (Sideslip angle)

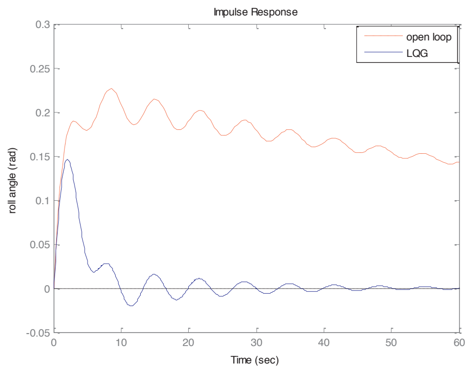

Fig.6. Comparison of Open- and Closed-Loop Impulse Response for the LQG Example (Roll angle)

5. Linear Quadratic Regulator Controller

Modern control theory has made a significant impact on the aircraft industry in recent years [10]. LQR is a method in modern control theory that used state-space approach to analyze such a system. Using state space methods it is relatively simple to work with a multi-output system. The system can be stabilized using full-state feedback system. The configuration of this control system is shown in Figure 08-09.

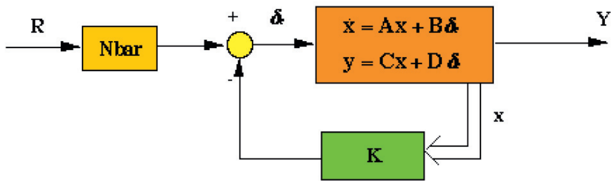

Fig.7. Full-state feedback controller with reference input

In designing LQR controller, *lqr* function in Matlab can be used to determine the value of the vector K which determined the feedback control law. This is done by choosing two parameter values, input $R = 1$ and $Q = C^T * C$ where C^T is the matrix transpose of C from state equation (6) and (11). The controller can be tuned by changing the nonzero elements in q matrix which is done in m-file code as obtained.

$$R = 1;$$
$$Q = [0\ 0\ 0; 0\ 0\ 0; 0\ 0\ x]; \tag{21}$$
$$K = lqr\ [A, B, Q, R];$$

Consequently, by tuning the value of $x = 500$, the following values of matrix K are obtained. If x is increased even higher, improvement to the response should be obtained even more. But for this case, the values of $x = 500$ is chosen because it satisfied the design requirements while keep x as small as possible.

In order to reduce steady state error of the system output, a value of constant gain *Nbar* should be added after the reference. With a full-state feedback controller all the states are feedback. The steady-state value of the states should be computed, multiply that by the chosen gain K, and used a new value as the reference for computing the input. *Nbar* can be found using the user-defined function which can be used in m-file code. The method used in simulation work is done by exported both value of matrix K and constant gain. For this controller design, the value of constant gain, *Nbar* are found to be, *Nbar = 100.*

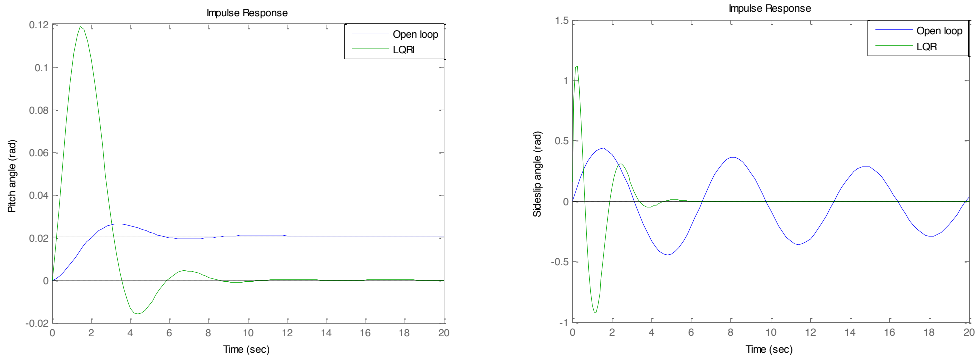

Fig.8. (a) Comparison of Open-loop and Closed-Loop Impulse Response for the LQR (Pitch angle), (b) Comparison of Open-loop and Closed-Loop Impulse Response for the LQR (Sideslip angle)

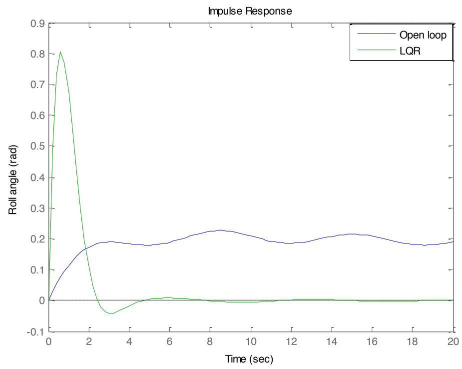

Fig.9. Comparison of Open- and Closed-Loop Impulse Response for the LQR Example (roll angle)

6. Kalman Filtering

Consider the discrete plant

$$x(n+1) = Ax(n) + B\big(u(n) + w(n)\big)$$
$$y(n) = Cx(n) \tag{22}$$

with additive Gaussian noise $w(n)$on the input $u(n)$ and data.
Our goal is to design a Kalman filter that estimates the output $y(n)$ given the inputs $u(n)$ and the noisy output measurements

$$y_v(n) = Cx(n) + v(n) \tag{23}$$

where $v(n)$ is some Gaussian white noise.

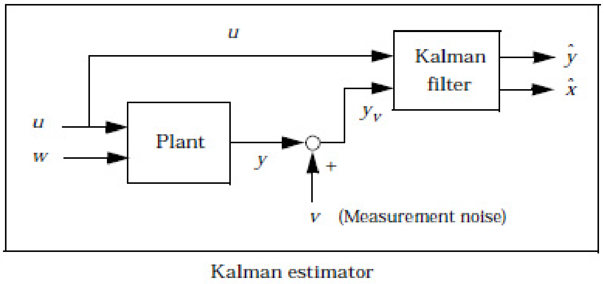

Fig.10. Kalman estimator

6.1. Discrete Kalman Filter

The equations of the steady-state Kalman filter for this problem are given as follows.
Measurement update

$$\hat{x}(n/n) = \hat{x}(n/n-1) + M(y_v(n) - C\hat{x}\,(n/n-1)) \quad (24)$$

Time update

$$\hat{x}(n+1/n) = A\hat{x}(n) + Bu(n) \quad (25)$$

In these equations:

- $\hat{x}(n/n-1)$ is the estimate of $x(n)$ given past measurements up to $y_v(n-1)$
- $\hat{x}(n/n)$ is the updated estimate based on the last measurement $y_v(n)$

Given the current estimate $\hat{x}(n/n)$, the time update predicts the state value at the next sample $n+1$ (one-step-ahead predictor). The measurement update then adjusts this prediction based on the new measurement $y_v(n-1)$. The correction term is a function of the *innovation*, that is, the discrepancy.

$$y_v(n-1) - C\hat{x}(n/n-1) = C(x(n+1) - \hat{x}(n+1/n)) \quad (26)$$

between the measured and predicted values of $y(n+1)$. The innovation gain M is chosen to minimize the steady-state covariance of the estimation error given the noise covariances.

$$E(w(n)w(n)^T) = Q, \;\; E(v(n)v(n)^T) = R, \quad (27)$$

You can combine the time and measurement update equations into one state-space model (the Kalman filter).

$$\hat{x}(n+1/n) = A(I-MC)\hat{x}(n/n-1) + [B\ AM]\begin{bmatrix} u(n) \\ y_v(n) \end{bmatrix}$$
$$\hat{y}(n/n) = C(I-MC)\hat{x}(n/n-1) + CMy_v(n) \quad (28)$$

This filter generates an optimal estimate $\hat{y}(n/n)$ of $y(n)$.
That the filter state is $\hat{x}(n/n-1)$
Simulation results are shown in Fig. 11-12

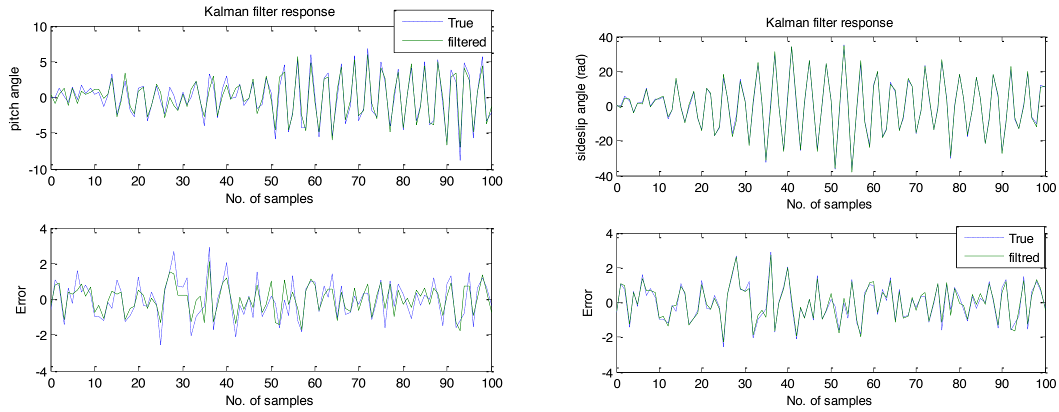

Fig.11. (a) Kalman Filter Response for the pitch angle θ, (b) Kalman Filter Response for the sideslip angle β

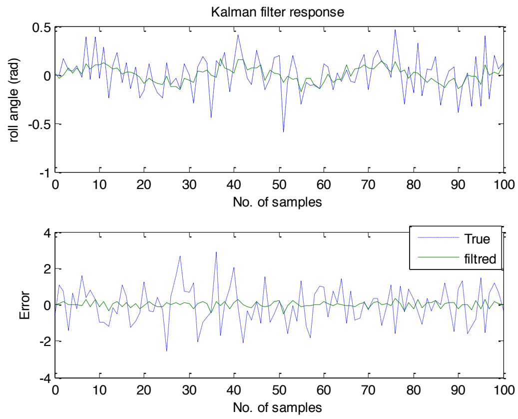

Fig.12. Kalman Filter Response for the roll angle ϕ

The first plot shows the true response y (dashed line) for the pitch angle θ and the filtered output y_e (solid line). The second plot compares the measurement error (dash-dot) with the estimation error (solid). This plot shows that the noise level has been significantly reduced. This is confirmed by the following error covariance computations.

7. Conclusion

The validated model of pitch, roll and sideslip control of an aircraft is very helpful in developing the control strategy for actual system. Pitch, roll and sideslip control of an aircraft is a system which requires a pitch, roll and sideslip controller to maintain the angle at it desired value. This can be achieved by reducing the error signal which is the difference between the output angle the desired angle. The control approach of LQR is capable on controlling the pitch angle, roll angle and sideslip angle of the aircraft system for value of 0.2 radian (11.5 degree). Simulation and analysis results show that, LQR controller relatively give the better performance. For advanced work, effort can be devoted in developing more robustness control techniques, following by implement the proposed control algorithm to real plant for validating of the theoretical result.

Finally, the LQG gives a very good following to the outputs of plant with a steady shift error limited and the Kalman filter is an optimal estimator when dealing with Gaussian white noise. Optimal estimation provides an alternative rationale for the choice of observer gains in the current estimator which is based on observer performance in the presence of process noise and measurement errors.

The Kalman filter estimates a process by using a form of feedback control: the filter estimates the process state at some time and then obtains feedback in the form of (noisy) measurements. As such, the equations for the Kalman filter fall into two groups: *time update* equations and *measurement update* equations. The time update equations are responsible for projecting forward (in time) the current state and error covariance estimates to obtain the a priori estimates for the next time step. The measurement update equations are responsible for the feedback i.e. for incorporating a new measurement into the a priori estimate to obtain an improved a posteriori estimate.

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References

[1] Robert C.Nelson.1998. Flight Stability and Automatic Control. University of Notre Dame.
[2] Anonymous. 2006: *uPSD3400 PWM API with R/C Servomotors Control Example (PCM)*, AN2283 Application note, June, http://www.st.com.
[3] Wong, K.C .2006. UAV Design Activities in a University Environment. University of Sydney.
[4] Aly El-Osery.2004. MATLAB Tutorial, October. . http://www.ee.nmt.edu
[5] Michael, T. Rusnell. 2004. Morphing UAV Pareto Curve Shift for Enhanced Performance. University of Notre Dame.
[6] Reed Siefert Christiansen (2004). Design of an Autopilot for Small Unmanned Aerial Vehicle. Brigham Young University.
[7] Tom Bateman, Barry Blakeley, Adam Dean, Jake Hanft and Mike Sheek (2004). UAV Stability Augmentation System (USAS), University of Colorado.
[8] Bahadir Karuv (2003). Building Blocks, Bergen County Academies Invention Lab.
[9] A. Maddi, "Modélisation et contrôle du vol latéral d'un avion", Magister Thesis, Electronics Department, University of Blida, Algeia, 1997.
[10] M. Grewal and A. Andrews, "Kalman Filtering Theory and Practice Using MATLAB", (Second ed.), New York, NY USA: John Wiley & Sons, Inc., 2001.
[11] P.C. Murphy and V. Klein, "Estimation of Aircraft Unsteady Aerodynamic Parameters From Dynamic Wind Tunnel Testing" , AIAA 2001- 4016, August 2001
[12] G. Bishop, G. Welch, "An Introduction to the Kalman Filter", University of North Carolina at Chapel Hill, Department of Computer Science, SIGGRAPH 2001.
[13] D. Simon, "Optimal state estimation: Kalman, H-infinity, and nonlinear approaches", John Wiley & Sons, 2006.
[14] P. S. Maybeck, "Applied Optimal Estimation—Kalman Filter Design and Implementation", notes for a continuing education course offered by the Air Force Institute of Technology, Wright-Patterson AFB, Ohio, semiannually since December 1974.