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A Semi-empirical Determination of the Properties of Nuclear Matter

N. J. Davidson[a*,b], S. S. Hsiao[b†], J. Markram[a‡], H. G. Miller[a*,b] and Yiharn Tzeng[b]

[a] Department of Physics, University of Pretoria,
Pretoria 0002, Republic of South Africa

[b]Institute of Physics, Academia Sinica,
Taipei 11529, Taiwan, Republic of China

The temperature dependence of the coefficients in the semi-empirical mass formula has been determined from a least squares fit to the canonical ensemble average of the excitation energy for nuclei throughout the periodic table. The low temperature behavior in the leading coefficient, the bulk energy density, is in disagreement with current parameterizations used in the equations of state for symmetric nuclear matter. Peaked structure in the specific heat occurs at a temperature of approximately 1 MeV in agreement with theoretical predictions of the critical temperature of a pairing phase transition in nuclear matter.

1. Introduction

The semi-empirical mass formula [1–3] provides a simple parameterization of the binding energy for all known nuclei. The coefficients in the mass formula are determined by fitting the experimentally determined ground state energies of nuclei throughout the periodic table to a function of the mass number A and the nuclear charge Z. Aside from providing systematic information about known nuclei, the leading coefficient in the mass formula, the bulk energy per nucleon, is also the binding energy per nucleon of symmetric nuclear matter. This empirical quantity is a limiting constraint on all equations of state for nuclear matter [4–7]. For applications in areas of physics where high energies are experienced, *e.g.* astrophysics and high energy heavy ion collisions, an extension of the mass formula to non-zero excitation energies is of great interest. Since reliable information about the low-lying spectra (*i,e.* energy levels and spins) of most nuclei is available, such an extension can be obtained in a reasonably simple fashion in the canonical ensemble, where the excitation energy is given in terms of the temperature T of the system under consideration.

*permanent address
†Permanent address: Department of Physics, Soo-Chow University, Taipei 11102, Taiwan, Republic of China
‡Present address: Department of Geology, University of Pretoria, Pretoria 0002, South Africa

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2. Temperature Dependence Mass Formula

In the present work we assume that the temperature dependent binding energy [3, 8] has a similar form as the usual semi-empirical mass formula

$$
\begin{aligned}
E_b(A,Z,T) &= \alpha(T)A + \beta(T)A^{2/3} + (\gamma(T) - \frac{\eta(T)}{A^{1/3}})(\frac{4t_\zeta^2 + 4|t_\zeta|}{A}) \\
&\quad + \frac{Z^2}{R(T)A^{1/3}}(1 - \frac{0.7636}{Z^{2/3}} - \frac{2.29}{[R(T)\,A^{1/3}]^2}) + \delta(T)\frac{f(A,Z)}{A^{3/4}},
\end{aligned}
$$

where $A = N+Z$, $t_\zeta = \frac{1}{2}(Z-N)$ and $f(A,Z) = (-1,0,+1)$ for (even-even,even-odd,odd-odd) nuclei. The nucleon radius $R(T)$ is not fitted, but parameterized with a weak linear dependence on T, $R(T) = 1.07\,(1 + 0.01\,T)$[9]. Note that no attempt has been made to parameterize the surface diffuseness [10] and hence $\beta(T)$ is not simply the surface energy.

The excitation energy is given by

$$E_{ex}(A,Z,T) = E_b(A,Z,T) - E_b(A,Z,0). \tag{1}$$

At $T = 0$ the coefficients are given by $\alpha(0) = -16.11$ MeV, $\beta(0) = 20.21$ MeV, $\gamma(0) = 20.65$MeV and $\eta(0) = 48.00$MeV obtained from a fit to the experimental nuclear ground state energies of 488 odd mass nuclei [3]. The $T = 0$ coefficient for the pairing term is taken as $\delta(0) = 33.0$ MeV [8] and $\delta(T)$ is constrained to be positive definite at all temperatures.

To obtain the temperature dependence of the coefficients we have used the available experimental information about the excited states of nuclei throughout the periodic table to determine the partition function of each nucleus in the canonical ensemble

$$Z(A,Z,T) = \sum_i^n g_i\ exp(-\beta E_i) + \int_{E_n}^{E_{max}} dE\ g_{A,Z}(E)\ exp(-\beta E) \tag{2}$$

where $g_i = 2j_i + 1$ is the spin degeneracy factor and E_i the excitation energy of the ith state of the nucleus, and $\beta = 1/T$. The sum in the first term of equation (2) runs over the experimentally measured (discrete) excited states.

Since the experimentally known spectrum is most cases only sufficient to allow the accurate determination of Z for very low temperatures ($T \ll 1$ MeV), it is necessary to supplement the experimentally known spectrum with an appropriate approximation to the continuum $g_{A,Z}(E)$. For this purpose, we make use of the fits obtained in [11]. For sufficiently large energies, the usual Fermi gas expression for the total density of states (*i.e.* including the spin degeneracy) is used:

$$g_{A,Z}(E) = \frac{\sqrt{\pi}}{12}\frac{\exp[2\sqrt{a_{A,Z}U}]}{a_{A,Z}^{1/4}U^{5/4}}. \tag{3}$$

Here $a_{A,Z}$ is the level density parameter and $U = E - P(N) - P(Z)$, where $P(N)$ and $P(Z)$ are the pairing corrections for neutron number N and proton number Z respectively [11]. The form for this parameterization is obtained from a saddle point approximation to a Fermi gas, a procedure which is only valid up to $T \approx 7$ MeV. However, since we are mainly interested in the region up $T \approx 4$ MeV, this parameterization is probably quite acceptable.

The fits given in [11] assume that $a_{A,Z}$ is independent of the excitation energy. Since $a_{A,Z}$ is closely correlated to the shell corrections, this means that shell effects in the excitation energy increase with increasing temperature. However, such effects are expected to decrease with increasing temperature [12], implying the change of the level density parameter with energy. Recent experiments also indicate an energy dependence in $a_{A,Z}$ [13–18]. Calculations using a simple realistic model [19] indicate that $a_{A,Z}$ is reasonably constant for the lowest few MeV of temperature. Since we are interested in the low temperature behaviour, a constant level density parameter should not be too bad an approximation. However, the parameterization (3) with constant $a_{A,Z}$ is probably not valid for $T \geq 4$ MeV.

At lower energies, a suitable empirical fit to the nuclear energy level density can be obtained to the form

$$g_{A,Z}(E) = \frac{\sqrt{2\pi}\sigma}{\tau} \exp{(E - E_0)/\tau}, \tag{4}$$

with σ the spin-dependence parameter. Values for the parameters $a_{A,Z}$, τ, E_0 and σ, as well as the respective regions where (3) and (4) should be used, can be found in [11] for a large number of nuclei.

The nuclei used to determine the aforementioned coefficients can therefore be divided into three groups:

1. Nuclei where sufficient discrete states are known to allow the use of the discrete spectrum at low energies and the Fermi gas expression (3) at higher energies.
2. Nuclei where the discrete spectrum does not extend high enough for (3) to be valid. For these nuclei, the discrete spectrum is used for low excitation energies, followed by the exponential form (4) for intermediate energies and finally the Fermi gas (3) at high energies.
3. Nuclei where very little of the discrete spectrum is known. In these cases, the (4) is used for the low- and intermediate- excitation portions of the spectra and (3) for the highly excited part.

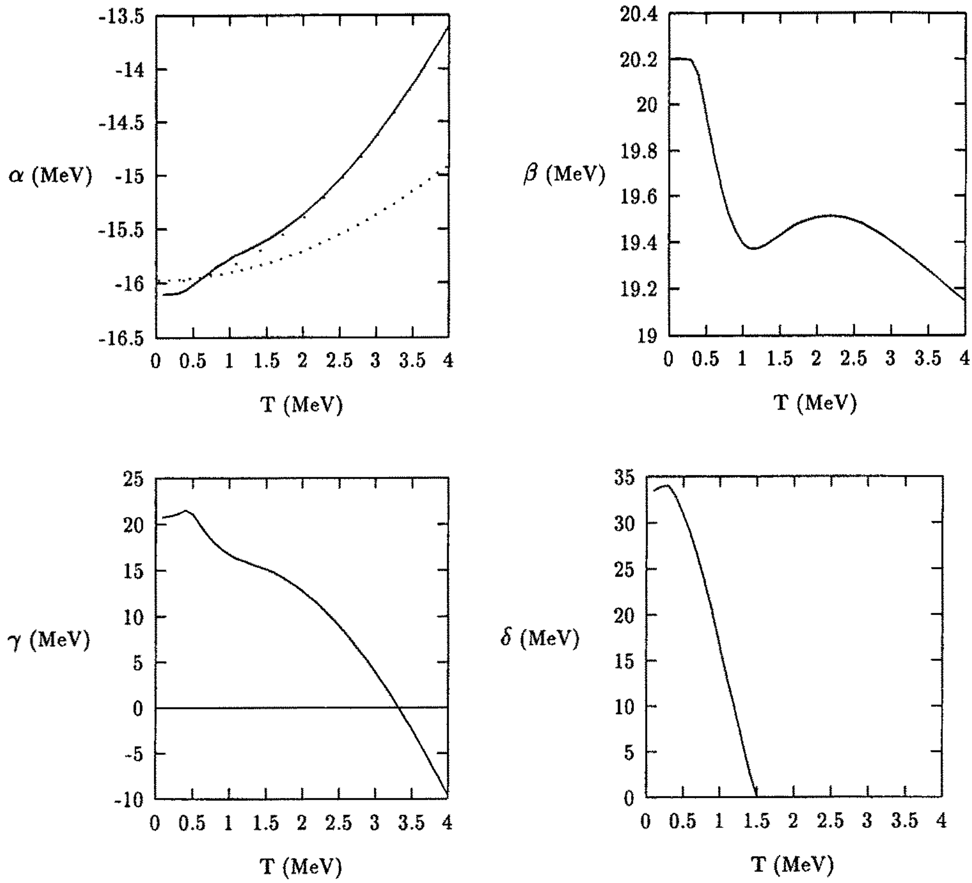
All three groups are spread across the whole periodic table.

The lower bound E_n on the integral in (2) is taken to be the energy at which (3) should become valid (from [11]) for case (1) above, 80% of the largest discrete energy level for case (2) above, and zero for case (3). Since we are only interested in temperatures up to $T \approx 4$ MeV, we have taken the upper bound $E_{max} \approx 3$ GeV. Increasing E_{max} has no effect on the numerical results in this temperature range.

The coefficients in the mass formula have been determined by a least squares fit of (1) to the ensemble average of the excitation energy

$$E_{ex}(A, Z, T) = -\frac{\partial}{\partial \beta} \ln \mathrm{Z}(A, Z, T) \tag{5}$$

determined from a total of 313 nuclei in the mass region $22 \leq A \leq 250$ for temperatures $T \leq 4$ MeV. The temperature dependence of the five coefficients is given in Figure 1.



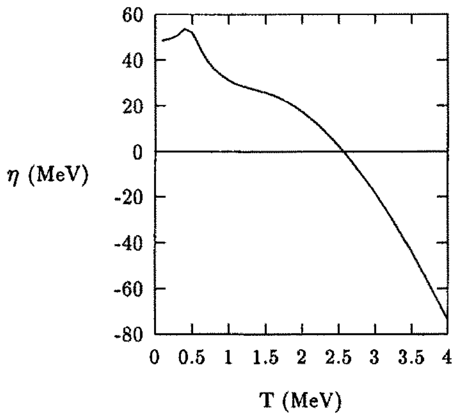


Figure 1. The coefficients of the semi-empirical mass formula (in MeV) as a function of the temperature (in MeV). In the graph for α, the solid line is the fit to the excitation energies, the dashed line is the bulk energy density for a Fermi gas (8), and the dotted line is the bulk energy density from [6].

The most interesting coefficient is $\alpha(T)$, the bulk energy density of symmetric nuclear matter. The simplest form for the bulk energy density used in the equation of state for nuclear matter [4, 6] is obtained from a Fermi gas model and is given by

$$\alpha_{FG}(T) = \alpha(0) + a_V T^2 \tag{6}$$

where the bulk level density parameter

$$a_V \approx \frac{1}{15} m^*/m \ \ \mathrm{MeV}^{-1} \tag{7}$$

is given in terms of the effective mass, m^*, of the nucleon. The value of m^*/m ranges from 0.7 to 1.2, depending on the manner in which it has been determined [20–23], but is quite often simply taken to be 1. At lower temperatures the simple quadratic parameterization (6) is clearly in disagreement with the empirical results. For $T \geq 2.7$ MeV (see figure 1) we obtain a good fit to the present semi-empirical determination with

$$\alpha(T) = -16 + 0.15T^2. \tag{8}$$

The value of a_V obtained in this manner ($\approx 1/6.7$) is slightly larger than the value obtained from extrapolation of fitted values of $a_{A,Z}$ of finite nuclei ($\approx 1/8$ [25]). This is because the value extracted from (8) has been corrected for finite size effects [24] in the sense that $\alpha(T)$ fitted from the canonical calculations is the bulk term.

More consistent determinations of $\alpha(T)$ in which the bulk energy for nucleons and nuclei are determined from the same function of density and temperature has been obtained from a momentum-dependent Skyrme-type parameterization [26] and a simpler momentum dependent potential [6]. In the latter case a parameterization of the bulk energy density is obtained in which the entire temperature dependence of the nuclear force is implicitly included in the kinetic energy densities. For symmetric nuclear matter at a density, $n_s = 0.155$ fm^{-3}, with bulk symmetry energy, $S_V = 29.3$ MeV, bulk incompressibility parameter, $K_s = 375$ Mev, and $\alpha(0) = -16$ MeV the bulk energy density is also shown in Figure 1. It is clear that this expression for $\alpha(T)$ behaves essentially as a Fermi gas with a smaller value of a_V, and is in poor agreement with the curve obtained from the present canonical ensemble calculation. Similar discrepancies have also been observed in finite nuclei [27].

One of the universal features of finite nuclei is an abrupt change in the density of states at excitation energies of 10 MeV or below [11]. At lower excitation energies the spectrum of most nuclei is sparse and dominated by collective states. With increasing excitation energy, the independent particle degrees of freedom predominate and the density of states grows exponentially. Calculations of the specific heat in both the finite temperature mean field approximation and in the canonical ensemble in which the partition function has been determined from the experimental spins and excitation energies display a peaked structure in the specific heat at approximately the same critical temperature [28]. Since the nuclear force is short-ranged and saturates rather quickly one would expect that such a change in the many body level density might also occur in nuclear matter.

The specific heat of symmetric nuclear matter, determined from the bulk energy density by means of a numerical derivative, is shown in Figure 2. Of major interest is the large peak in C at $T \approx 0.9$ MeV. Such a peak in C arises from of an abrupt change in the

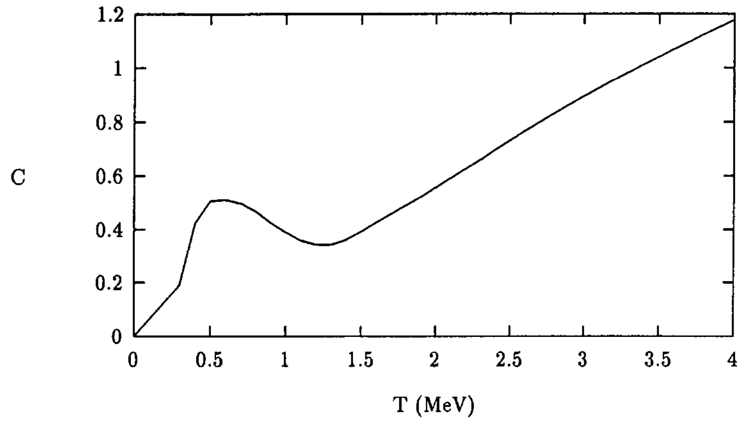


Figure 2. The specific heat of nuclear matter C as a function of temperature obtained from $\alpha(T)$ in Figure 1.

many body density of states. In analogy to the situation in finite nuclei, we suggest that the structure in C is indicative of a phase transition in nuclear matter.

The calculation in the present work contains no dynamics, and, as such, is incapable of explaining the origin of the observed phase transition. However, it has been pointed out [29] that nuclear matter or even better neutron matter could exist in a paired state. Recently it has been shown for realistic nucleon-nucleon potentials finite temperature BCS calculations exhibit a second order phase transition in symmetric nuclear matter at critical temperatures (at normal nuclear density) of 0.4 - 0.8 MeV [29]. The existence of this phase transition has important implications for nuclear astrophysics, where the commonly used equations of state for nuclear matter are based on Fermi gas models, and, as such, do not allow for any such transition [4, 6]. For an infinite system such as nuclear matter, it might be argued that such a phase transition should be signaled by a singularity in C rather than just a peak. The method by which we have obtained our fit to α, however, precludes the possibility of obtaining such a singularity. Since we are fitting to data from finite nuclei, where canonical ensemble calculations do not display singularities, there is no way that our extrapolation (which is still within the canonical ensemble) to nuclear matter can either.

3. Concluding Remarks

In conclusion, we have determined the temperature dependence of the coefficients in the semi-empirical mass formula by fitting to the canonical ensemble average of the excitation energy of over 300 nuclei for temperatures $T \leq 4$ MeV. Although the parameterization considered is simple, and neglects shell effects, there are strong indications that the low temperature behaviour of the binding energy per nucleon in symmetric nuclear matter is significantly different from that of a Fermi gas. This has important implications for the nuclear equation of state in the temperature range $T < 3$ MeV. Furthermore we have shown that there is empirical evidence for a phase transition in nuclear matter at a temperature in the order of 1 MeV. While the present calculations cannot determine the nature of the transition, the similarity in critical temperature with those obtained for the pairing phase transition from finite temperature mean field calculations makes such an identification tempting. Modification of the nuclear equation of state for applications in astrophysics and heavy ion collisions to include this transition appear to be necessary.

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