

Assignment #2

1). Using the properties of a general Boolean Algebra (whose elements are not only 0 and 1), reduce the following expressions into a minimum sum of products form:

(a) $x'y'z' + x'y'z + x'yz + xy'z + xyz + xyz'$

(b) $a'b'c' + a'bc' + a'bc + ab'c + abc' + abc$

$$\begin{aligned} \text{a) } x'y'z' + x'y'z + x'yz + xy'z + xyz + xyz' &= \\ &= x'y'z' + xyz + x'y'z + xyz' + x'yz + xy'z \quad (\text{Ax. 1}) \\ &= (x'y'z + xyz) + (x'y'z + xyz') + (x'yz + xy'z) \quad (\text{Associativity}) \\ &= 1 + 1 + (x'yz + xy'z) \\ &= 1 + z(x'y + xy') \\ &= 1 + z \cdot 1 \\ &= 1 + z \\ &= 1 \end{aligned}$$

$$\begin{aligned} \text{b) } a'b'c' + a'bc' + a'bc + ab'c + abc' + abc &= \\ &= a'b'c' + abc + a'bc' + ab'c + a'bc + abc' \\ &= (a'b'c' + abc) + (a'bc' + ab'c) + (a'bc + abc') \\ &= 1 + 1 + (a'bc + abc') \\ &= 1 + b(a'c + ac') \\ &= 1 + b \cdot 1 \\ &= 1 + b \\ &= 1 \end{aligned}$$

2) Consider a Boolean algebra with $S = \{0, 1\}$. Let $f : S^3 \rightarrow S$ whose values are given as $f(i) = 0$ for $i = 1, 2, 5, 7$ and $f(i) = 1$ for $i = 0, 3, 4, 6$.

(a) Derive the Boolean function in sum of minterms canonical form.

(b) Derive the Boolean function in product of maxterms canonical form.

(c) Derive the Boolean function in Reed-Muller canonical form.

a) We will take $f(i)=1$ for sum of minterms.

$$\begin{aligned} f(x_1, x_2, x_3) &= f(0)m_0 + f(3)m_3 + f(4)m_4 + f(6)m_6 \\ &= x_1'x_2'x_3' + x_1'x_2x_3 + x_1x_2'x_3' + x_1x_2x_3' \end{aligned}$$

All of $f(0)$, $f(3)$, $f(4)$ and $f(6)$ is equal 1 cause of definition. And $(1.x) = x$

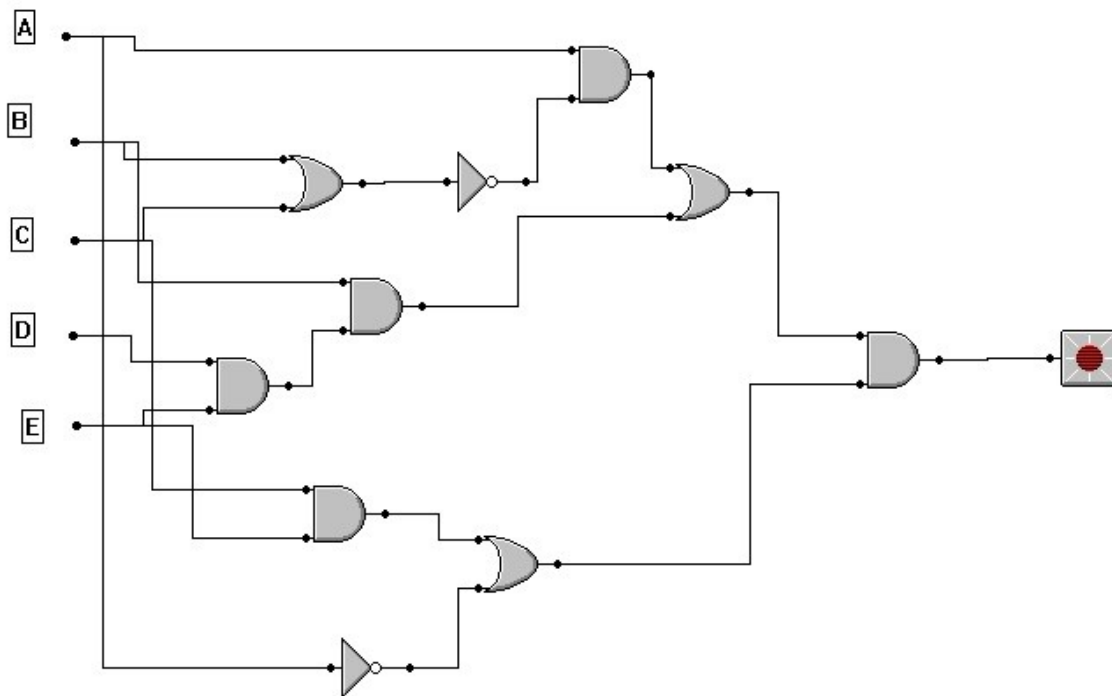
b) We will take $f(i)=0$ for product of maxterms.

$$\begin{aligned} F(x_1, x_2, x_3) &= [f(1) + M_1][f(2) + M_2][f(5) + M_5][f(7) + M_7] \\ &= [x_1 + x_2 + x_3'][x_1 + x_2' + x_3][x_1'x_2'x_3][x_1'x_2'x_3'] \end{aligned}$$

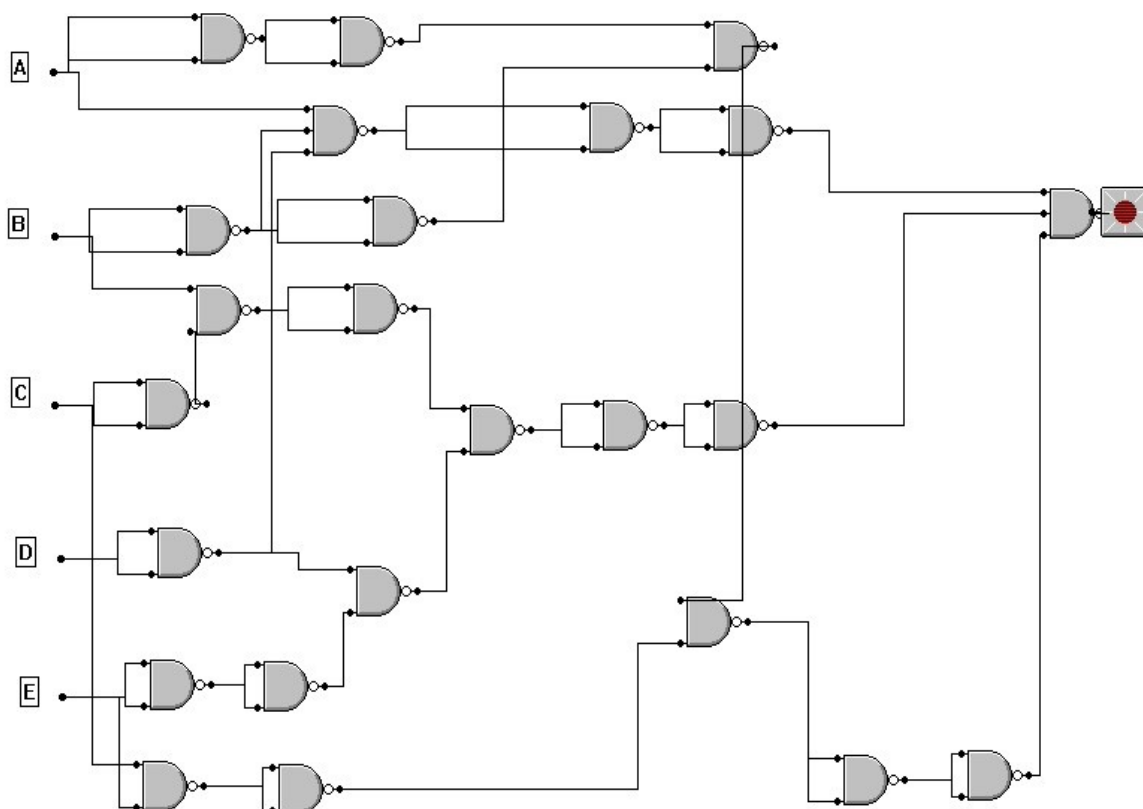
All of $f(1)$, $f(2)$, $f(5)$ and $f(7)$ is equal 0 cause of definition. And $(0 + x) = x$

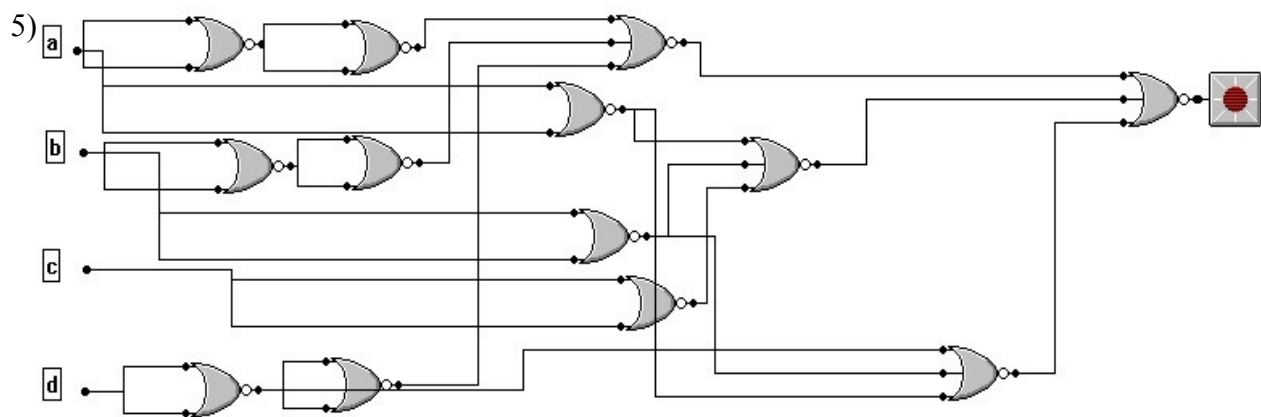
c) redd muller unutmussun

3)



4)





- 6) A minority function has an output value of 1 if there are less 1's than 0's on its inputs. The output is 0 otherwise. Consider the design of the four input minority function.

a)

X	Y	Z	T	output
0	0	0	0	1
0	0	0	1	1
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	0
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

b) $f(i) = 1$, for all $i=0,1,2,4,8$

$$SOM = f(0) \cdot (x'y'z't') + f(1) \cdot (x'y'z't) + f(2) \cdot (x' y' z t') + f(4) \cdot (x' y z' t') + f(8) \cdot (x y' z' t')$$

$f(i)=0$, for all $i=3,5,6,7,9,10,11,12,13,14,15$

$$\begin{aligned} POS = & [f(3) + (x + y + z' + t')] \\ & * [f(5) + (x + y' + z + t')] \\ & * [f(6) + (x + y' + z' + t')] \\ & * [f(9) + (x' + y + z + t')] \\ & * [f(10) + (x' + y + z' + t)] \\ & * [f(11) + (x' + y' + z + t')] \\ & * [f(12) + (x' + y' + z + t)] \\ & * [f(13) + (x' + y' + z + t')] \\ & * [f(14) + (x' + y' + z' + t)] \\ & * [f(15) + (x' + y' + z' + t')] \end{aligned}$$

$$\begin{aligned} \text{c) } SOM &= x'y'z't' + x'y'z't + x'y'z' t' + x'yz' t' + x y'z't' \\ &= x'y'z' (t' + t) + x't' (y'z + yz') + xy'z't' \\ &= x'y'z' \cdot 1 + x't' \cdot 1 + xy'z't' \\ &= x'y'z' + x't' + xy'z' t' \end{aligned}$$