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Analysis of Sabine and Eyring equations and their application to concert hall audience and chair absorption

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Historically, two equations have been used for predicting reverberation times, Sabine and Eyring. A precise means is presented for determining Eyring absorption coefficients α_{eyring} when the Sabine coefficients α_{sabine} are known, and vice versa. Thus, either formula can be used provided the absorption coefficients for the Sabine formula are allowed to exceed 1.0. The Sabine formula is not an approximation to the Eyring equation and is not a shortcoming. Given low reverberation times, the ratio of α_{sabine} to α_{eyring} may become greater than 2.0. It is vital that, for correct prediction of reverberation times, the absorption coefficients used in either formula must have been determined in spaces similar in size and shape, with similar locations of high absorption (audience) areas, and with similar reverberation times. For concert halls, it is found that, when the audience area (fully occupied) and midfrequency reverberation time are postulated, the hall volume is directly proportional to the audience absorption coefficient. Approximately 6% greater room volumes are needed when choosing nonrectangular versus classical-rectangular shaped halls and approximately 10% greater volumes when choosing heavily upholstered versus medium upholstered chairs. Determinations of audience sound absorption coefficients are presented, based on published acoustical and architectural data for 20 halls.
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I. INTRODUCTION

The prediction of reverberation times in concert halls began with the researches of Sabine (1900). His simple formula for relating reverberation time directly to the volume of a room and inversely to the absorbing power of the audience and other surfaces and objects in the room has found widespread use. With it, zero reverberation time in a room fully covered with the material requires an absorption coefficient of infinity, a fact that Sabine appeared to disregard when he stated (1900, 1906, 1915) that the absorbing power of an open window, meaning a surface with no reflected sound, is 1.000. By contrast, in a 1912 paper, he shows an absorption coefficient of 1.26 at 1024 Hz for a felt material and, in a 1915 paper, an absorption coefficient of 1.10 at 512 Hz for "upholstered settees" and 1.12 at 512 Hz for wood sheathing, 2 cm thick.

Eyring (1930), presented an alternate equation that calculates zero reverberation time for a room fully lined with a material having an absorption coefficient of 1.0. Both authors assumed in their derivation that the absorbing power is nearly uniformly distributed over all the surfaces in the room, and that the sound field is nearly diffuse so that the results are almost independent of a room's shape. For rooms where the sound field is not perfectly diffuse, a controversy has existed over which of the two reverberation equations is more accurate even for the case where the absorbing power is evenly distributed over the surfaces and the average sound absorption coefficient does not exceed 0.5.

Audience absorption has received attention in recent years (Bradley, 1992, 1996; Davis *et al.*, 1994; Kirdegaard, 1996; Beranek and Hidaka, 1998; Hidaka *et al.*, 2001; Barron and Coleman, 2001; Beranek, 1962, 1969, 1996, 2004). Two goals of this paper are to investigate where and how to use the Sabine/Eyring equations (particularly in concert halls) and to determine audience and chair absorption coefficients. The following topics are treated. (1) The Sabine and Eyring equations and a precise means for deriving Eyring audience absorption coefficients from Sabine coefficients. (2) Under what conditions will either equation calculate accurate reverberation times. (3) Choice of unit-area versus per-person method for specifying the sound absorption of the audience in a concert hall. (4) Residual (nonaudience) absorption coefficients in halls for music. (5) Audience absorption coefficients for 20 concert halls. 7. Chair absorption coefficients for 20 halls. (6) Effect of room shape and degree of upholstering on audience and chair absorptions. (7) Hall volume related to room shape and chair upholstering.

II. THE SABINE EQUATION

The Sabine equation at normal room temperature, 22 °C, is

$$T_{60} = 0.161V/(A + 4mV)\ \text{s}, \tag{1}$$

$$A = \alpha_{\text{tot}} S_{\text{tot}}\ \text{m}^2, \tag{2}$$

$$\alpha_{\text{tot}} = (\alpha_T S_T + \alpha_R S_R + \Sigma \alpha_i S_i)/S_{\text{tot}}, \tag{3}$$

and

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$$S_{tot} = S_T + S_R + \Sigma S_i \text{ m}^2. \quad (4)$$

The audience absorption coefficient is

$$\alpha_T = (\alpha_{tot} S_{tot} - \alpha_R S_R - \Sigma \alpha_i S_i)/S_T, \quad (5)$$

where V is the room volume in cubic meters; A is the total sound absorption in the room in square meters; S_T is the acoustical audience area (i.e., area beneath chairs plus areas of strips 0.5 m wide around audience blocks except for sides at balcony rails or walls, and measured on the slope) plus the area the orchestra sits over (for the orchestra no sloping and limited to a maximum of 180 m^2); S_i's are any areas of highly absorbing surfaces in the hall (sometimes introduced for echo control); and S_R (called "residual absorption" area) is the area of all other surfaces in the hall, including under-balcony areas. The α's are the Sabine sound absorption coefficients associated with their corresponding areas; and m is the energy attenuation constant for sound traveling through air in units of m^{-1}.

In this study, the absorption by objects, lighting fixtures, ventilating openings, cracks around doors in the room, thin carpeting in some of the aisles, etc., are included in the residual absorption. The absorption by the air itself is assumed here to be of importance only for the frequency bands of 2000 Hz and higher (4 m in this paper is taken to be 0.0089 m^{-1} at 2000 Hz and 0.0262 at 4000 Hz). In only three halls of this study, New York Carnegie, Sapporo Kitara, and Amsterdam Concertgebouw are there significant areas of high absorptivity besides the audience and they are accounted for in the calculations that follow.

III. THE EYRING EQUATION[1]

The Eyring equation, with the same assumptions as above, is

$$T_{60} = 0.161V/(A' + 4mV) \text{ s}, \quad (6)$$

$$A' = S_{tot}[-2.30 \log_{10}(1 - \alpha_{ey})] \text{ m}^2, \quad (7)$$

where

$$\alpha_{ey} = (\acute{\alpha}_T S_T + \acute{\alpha}_R S_R + \Sigma \acute{\alpha}_i S_i)/S_{tot}, \quad (8)$$

and the $\acute{\alpha}$'s are the Eyring sound absorption coefficients associated with their corresponding areas.

IV. DERIVING EYRING COEFFICIENTS FROM SABINE COEFFICIENTS

A simple and precise means for transfer from the Sabine sound absorbing coefficients to the Eyring ones is possible because the same procedure for obtaining the average absorption coefficients in a room is followed in both Eqs. (5) and (8). Thus

$$(\alpha_{ey}/\alpha_{tot}) = (\acute{\alpha}_T S_T + \acute{\alpha}_R S_R + \Sigma \acute{\alpha}_i S_i)/(\alpha_T S_T + \alpha_R S_R + \Sigma \alpha_i S_i) \quad (9)$$

and

$$(\acute{\alpha}_T S_T + \acute{\alpha}_R S_R + \Sigma \acute{\alpha}_i S_i) = (\alpha_{ey}/\alpha_{tot})(\alpha_T S_T + \alpha_R S_R + \Sigma \alpha_i S_i),$$

hence,

$$\acute{\alpha}_T = (\alpha_{ey}/\alpha_{tot})\alpha_T, \quad (10a)$$

$$\acute{\alpha}_R = (\alpha_{ey}/\alpha_{tot})\alpha_R, \quad (10b)$$

$$\Sigma \acute{\alpha} = (\alpha_{ey}/\alpha_{tot})\Sigma \alpha_i. \quad (10c)$$

V. ABOUT THE EQUATIONS—INTERRELATIONS AND ACCURACY

The Sabine and Eyring equations were derived under different assumptions. The Sabine equation assumes that as a sound wave travels around a room it encounters surfaces "one after another." The Eyring equation assumes that all the surfaces are simultaneously impacted by the initial sound wave, and that successive simultaneous impacts, each diminished by the average room absorption coefficient, are separated by mean free paths. Cremer and Mueller (1982) call the former "one-after-another" and the other "side-by-side." In a real room, neither condition is met. Schroeder (1973) assumed a special distribution of free paths and he got Sabine's values exactly.

Joyce (1978, 1980) has developed an exact equation for the geometrical-acoustics (ray-tracing) value of reverberation time (RT) in a room of arbitrary shape and arbitrary distribution of angular and spatial absorptivity and arbitrary surface irregularities. This RT formula can be solved numerically using available Fredholm solvers (Joyce, 1980). Unfortunately, the calculations are exceedingly difficult, except for the cases where the geometry is easy to express mathematically, such as a two wall "enclosure" or a spherical or rectangular enclosure. In the two-wall case, half the surface area can be made sound absorbent and the other half perfectly reflecting. But, even in the case of mathematically simple enclosures, e.g., sphere or rectangular in shape, the calculations are simple only if all surfaces have the same absorption coefficients and irregularities.

As an *illustration* of how the reverberation time RT varies as a function of a wide range of surface absorptivities and surface irregularities, Joyce presents the exact solution for a spherical enclosure with (a) the same absorptivity and roughness over the entire inner surface, (b) variation of absorptivity from 0 to 1.0, and (c) variation of reflectivity (measure of roughness) from specular to random, i.e., $s=1$ for specular and $s=0$ for random. Joyce then defines a reference sound absorption coefficient α_{ref} to be used in the reverberation equations.

The Sabine equation for the spherical example becomes

$$T_{60} = \text{constant} \times V/(S\alpha_{ref}). \quad (11)$$

The Eyring equation for the spherical example is

$$T_{60} = \text{constant} \times V/(S\alpha_K), \text{ where}, \quad (12)$$

$$\alpha_K = -2.3 \log(1 - \alpha_{ref}). \quad (13)$$

The results are given in Fig. 1 for a range of α_{ref} from 0.05 to 0.5.

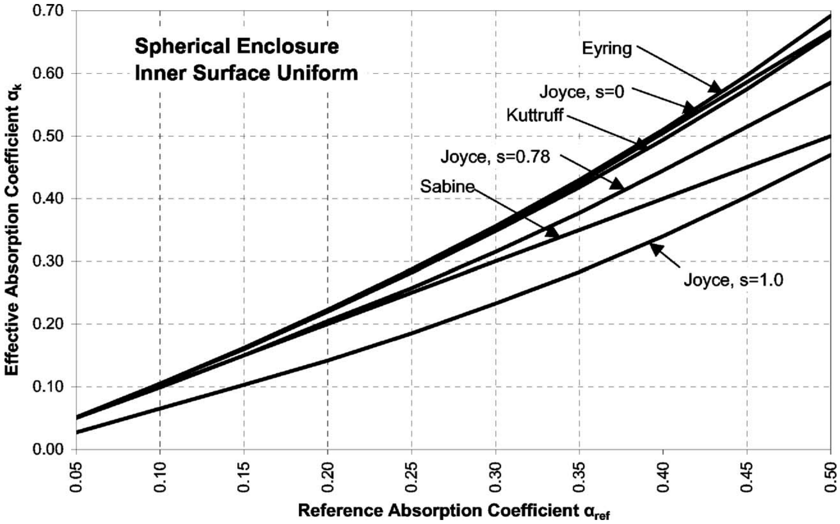

FIG. 1. Comparison of reverberation theories. Joyce labels the ordinate "Normalized decay rate." Here, it is called "Effective absorption coefficient." Joyce labels the abscissa "Surface absorptivity." Here, it is called "Reference absorption coefficient." From Joyce (1978).

Kuttruff (1973) derived a modification of the Eyring formula that assumed that the reflectivity of the surface was everywhere isotropically absorptive and also maximally diffusive

$$\alpha_{\mathrm{kut}} = [-2.3 \log(1-\alpha_{\mathrm{ref}})][1+(2.3/16)\log(1-\alpha_{\mathrm{ref}})]. \quad (14)$$

Otherwise, he made no assumption about the diffusivity of the sound field itself. Carrol and Chen (1977) obtained an analytic solution to the equation for a sphere with a spatially uniform absorptivity, i.e., Joyce's derivation with $s=0$. Kuttruff's equation is plotted in Fig. 1.

The effective absorption quotient from the Joyce theory for random reflection, $s=0$ is almost equal to the Eyring coefficient. Kuttruff's method gives a slightly lower absorption coefficient. Inspection of the Joyce curves for $s=0.78$ and 1.0, show that they lie closer to the Sabine curve than to the Eyring or Kuttruff curves. But surface considerations are not alone of importance. Joyce is careful to point out that "randomnizing" of the sound field is a function of the enclosure shape as well as the absorptivity/roughness characteristics of the material on its surfaces, and unless the sound field is nearly completely random there is a chance that the Sabine formula is even more correct than the Eyring or Kuttruff formulas. This statement is limited to values of α_{ref} that are less than about 0.4 (Joyce suggests 0.58, where the two bottom curves of Fig. 1 intersect).

There is another reason why one must beware of thinking that the sound field in a room is truly random. Joyce points out, and it is logical, that sound intensity tends to be greater for paths in the enclosure that mainly involve surfaces with low absorption coefficients. Or, as Joyce puts it, "there is a tendency of sound to accumulate on paths of longer life."

Hodgson (1993) in his Fig. 2 clearly shows that in actual room measurements, Eq. (10) is correct, but he incorrectly concludes, "At high frequencies the Sabine formula gives coefficients which are greater than 1.0—an apparently physically impossible result, indicating a short-coming of the Sabine theory," and Mange (2005) incorrectly states, "The standard Sabine equation was used…absorption coefficients greater than one are often measured. This is theoretically impossible." When using the Sabine theory, one must accept coefficients greater than one, which is neither theoretically impossible nor a shortcoming in the theory. If coefficients less than 1.0 are always desired, the Eyring alternate must be used. Hodgson gives measurements of the absorption coefficients in a test room with low reverberation time. At 8000 Hz he obtained an Eyring coefficient of 0.85 and a Sabine absorption coefficient of 1.90 and, i.e., $(\alpha_{\mathrm{ey}}/\alpha_{\mathrm{tot}})=0.45$, a legitimate result, not a shortcoming in the Sabine theory.

VI. PREMISE OF THIS PAPER

The question must be asked, "What is the measure of correctness?" Obviously, the answer ought to be that the chosen equation should predict the correct reverberation time for a room regardless of how much or where the principal absorbing material (with a constant absorption coefficient) is placed. But, is this possible? Look backwards at the problem. Start with the reverberation time and the complete physical characteristics of a rectangular enclosure. Using Joyce's exact theory, calculate the average sound absorption coefficient of the material for a particular area and location in the room. But, if this transfer function (RT to α) is now held constant, it well known that the absorption coefficient of a material will take on different values depending on where and how much of the material exists on the walls [Andre (1932)].

The premise of this paper is that if a particular reverberation equation (transfer function) is used and the reverberation time is held constant, the calculated absorption coefficient for the absorbing material will vary as it is moved around or made larger or smaller. Alternatively, if one requires that the absorption coefficient must remain the same regardless of its position or size, there is no *single* equation (transfer function) that will calculate the correct reverberation times. To explore this premise, this paper concentrates on the absorption of sound by a seated audience in concert halls.

Using an appropriate CAD program, a computer model of a room can be used to determine the reverberation time from a given audience absorption coefficient. The transfer function (α_T to RT) will be different for every room shape and absorption configuration and may be near to or differ greatly from the Sabine or Eyring equation. Hidaka (2006) found that ta change in the mean-free-path length would enable the use of the Sabine equation for calculation of the RT's for non-shoebox type concert halls. The α_T's used therein were determined in shoebox type concert hall where he found MFP$=4V/S_{tot}$.

As will be demonstrated, the sound absorption by an audience depends on the shape of the enclosure, on the location of the audience in the enclosure, and on the nature of the sound field in the enclosure. Hence, for calculation of room reverberation times using a particular formula the audience absorption coefficients should have been determined from measurements in approximately the same shapes and sizes of enclosure, on the same position of the audience area in the enclosure, and with approximately the same enclosure reverberation times.

From studies made in classrooms, Bistafa and Bradley (2000) write, "The Sabine/Eyring formula is concluded to be a reasonable choice among the analytical expressions compared here." They found computer simulations of that year to give no better results. They also conclude, as does the above premise, that the absorption coefficients used in predicting reverberation times in classrooms should be determined in rooms that simulate the actual classroom sizes and shapes and locations of acoustical materials.

VII. CHOICE OF UNIT-AREA OR PER-PERSON AUDIENCE ABSORPTION IN CONCERT HALLS

Beranek (1960, 1962, 1969) has demonstrated that in concert halls the audience absorption is proportional to the area (with edge corrections—see S_T in Sec. II) over which the audience sits and not to the number of seats. The impetus for that series of studies followed the failure of preconstruction computations of reverberation times to predict those actually measured. Notable were the differences in Boston Symphony Hall (calculated 2.3 s, measured 1.9 s), Royal Festival Hall (1.8 s vs 1.5 s), Mann Auditorium (1.9 s vs 1.5 s); and Edmonton Jubilee Auditorium (1.8 s vs 1.4 s). In those four cases, the audience absorption used in the design stage was proportional to the seat count and the value chosen was nearly the same as Sabine's original value of 0.44 m^2 per person (Sabine, 1900), a number that is at least 20% too low for modern seat and row-to-row spacing (Nishihara *et al.*, 2001). Sabine published audience absorptions at 512 Hz both on "per person" (0.44) and "per unit area" (0.96) bases. If Sabine had used the unit-area number instead of 0.44 per person in calculating the reverberation time of Boston Symphony Hall, his calculated RTs would have been much closer to those measured. [The number of seats per 100 square meters varies from 244 (Amsterdam, Concertgebouw) to 172 (Munich, Philharmonie).]

Barron and Coleman (2001) have reinforced the "audience area" concept, reporting on measured data in seven British concert halls, saying "there is a very good correlation coefficient ($r=0.98$) with the [occupied] seating area, and a non-significant [very low] correlation with the number of seats… . This…supports Beranek's proposition that seat absorption should be treated by area." It must be noted, that in a reverberation chamber where the average room sound absorption coefficient is very low and the sample size is small, the absorption coefficient for an audience usually measures somewhere in-between the per person and the unit area values.

Clearly, which method is used would make no difference if all halls had the same row-to-row and seat-to-seat spacing, i.e., the same area per person. But, another consideration forces the choice of the unit-area method. The sound absorption is greater for an audience seating area that is steeply sloped (raked) because more of the human body is directly exposed to the sound (Nishihara *et al.*, 2001). If the per person assumption is made, the predicted total absorption in a room will not change with audience sloping. The sloped areas in a hall may be as much as 15% greater than the projected areas.

In the basic data for concert halls that follow, the per-area method is used. It must be noted that the acoustic audience areas S_T given here may differ significantly from the S_T's in Beranek (2004) for the same halls because the areas here are measured on the slopes, that is, they are not projected areas.

VIII. REMARKS ON THE PREMISE OF SECTION VI

Some believe that if the sound absorption by the seats in a hall is very large, the reverberation times will be independent of whether or not they are occupied. But, as is investigated here, even when the seats are occupied, the audience sound absorption is dependent on the seat design and on different degrees of upholstering. Furthermore, the sound absorption of an audience depends on whether a hall has upper areas where reverberation can develop, which is usual in older rectangular halls, or on whether the seats extend upward in front of one or more walls thus largely preventing the development of overhead reverberation.

In summary, the audience absorption coefficients shown in this paper for use in the Sabine or Eyring equations are probably only meaningful for halls with reverberation times in the range of 1.6–2.0 s, cubic volumes in the range of 10 000–30 000 m^3, and one of two shapes, i.e., (1) rectangular with large open wall areas above the highest balconies and (2) nonrectangular or near-rectangular with audience areas so steep that they mask or cover one or more side or end walls.

Following the belief that sound tends to accumulate on paths of longer life, investigation of the 85 concert halls presented in Beranek (2004), revealed that of the many rectangular halls in the book, a sample of nine would be an adequate representation. They evidenced upper interiors where the sound could reverberate without fully involving the audience below. Of the nonrectangular halls, a group of 11 halls was selected in all of which the seating sloped upward so steep that the rear or one or more side walls or both were masked [In Beranek (2004), more were not possible because all of the data needed were not available]. For all 20 halls, available were (1) the architectural plans, (2) a descrip-

TABLE I. Basic data for the halls of this study. The residual absorption coefficients used throughout this study are shown for use in the Sabine equation. Eyring residual absorption coefficients=Sabine coefficients times (α_{ey}/α_{tot}).

Nonrectangular halls	Volume (m^3)	Occupied areas S_T (m^2)	Residual areas S_R (m^2)	Combined areas S_{tot} (m^2)	RT occupied halls (sec) frequency (Hz) 125	250	500	1000	2000	4000
Sapporo, Kitara Concert Hall	28800	1786	6582	8368	2.05	1.95	1.90	1.90	1.75	1.50
Munich, Philharmonie am Gasteig	29700	2000	6134	8134	1.95	2.00	1.90	1.95	1.75	1.50
Rotterdam, DeDoelen Concert Hall	24070	1730	4098	5828	2.30	2.00	1.90	1.90	1.85	1.65
Berlin, Philharmonie	21000	1620	3784	5404	2.10	1.85	1.85	1.95	1.80	1.60
New York, Avery Fisher Hall	20400	1660	3936	5596	1.60	1.76	1.78	1.74	1.55	1.46
Cleveland, Severance Hall	16290	1486	3502	4988	1.75	1.70	1.65	1.55	1.45	1.30
Baltimore, Meyerhoff Hall	21530	1700	4154	5854	2.30	2.10	2.00	2.00	1.65	1.35
Manchester, Bridgewater Hall	25000	1850	5126	6976	2.30	2.12	2.00	2.00	1.80	1.65
New York, Carnegie	24270	1874	4553	6427	2.12	2.00	1.83	1.75	1.57	1.40
Tokyo, Suntory	21000	1578	4548	6126	2.14	2.08	1.95	2.00	1.90	1.75
Buffalo, Kleinhans	18280	2200	3626	5826	2.15	1.65	1.60	1.40	1.27	1.23
Rectangular halls										
Boston Symphony Hall	18750	1522	4150	5672	1.95	1.90	1.90	1.90	1.59	1.43
Berlin, Konzerthaus	15000	1101	3717	4818	2.20	2.10	2.00	2.00	1.80	1.60
Vienna, Musikvereinssaal	15000	1118	3147	4265	2.25	2.18	2.04	1.96	1.80	1.62
Lenox, Ozawa Hall	11610	919	2978	3897	2.00	1.90	1.80	1.75	1.70	1.40
Seattle, Benaroya Hall	19263	1668	4335	6003	2.15	1.90	1.80	1.75	1.65	1.55
Kyoto, Concert Hall	17800	1342	4571	5913	2.16	2.06	1.99	1.98	1.82	1.59
Amsterdam, Concertgebouw	18780	1285	3706	4991	2.20	2.15	2.05	1.95	1.80	1.55
Lucerne, Concert Hall	17823	1465	4018	5483	2.10	2.00	1.90	1.80	1.65	1.50
Tokyo, Tokyo Opera City Concert Hall	15300	1220	4791	6011	2.07	2.03	1.99	1.93	1.84	1.66
Sabine equation: Residual absorption coefficients used in calculations, all halls					0.14	0.12	0.10	0.09	0.08	0.07
Except: Boston					0.17	0.14	0.11	0.09	0.09	0.08
Tokyo, TOC					0.14	0.115	0.089	0.079	0.075	0.059
Berlin, Konzerthaus					0.15	0.12	0.10	0.09	0.08	0.07
New York, Avery Fisher					0.28	0.18	0.13	0.11	0.10	0.10
Berlin, Philharmonie					0.19	0.20	0.15	0.10	0.09	0.08
Eyring equation: Residual absorption coefficients must be determined from Sabine coefficients times (α_{ey}/α_{tot}).										

tion of the interior surfaces, (3) the type of audience chair, and (4) measured reverberation times of sufficient accuracy.

No halls were selected that have steeply, inwardly sloped upper side walls in which an unusually high percentage of the early sound energy is directed to the audience areas (e.g., Christchurch Town Hall). This style should be studied separately, but for the three in Beranek (2004), the necessary accurate basic data are available on only one.

The chosen nine rectangular halls and eleven nonrectangular halls are listed in Table I along with basic information on each. They have midfrequency reverberation times in the range of 1.6–2.0 s. Comparative data on all 20 halls, with seats fully occupied, using the Sabine equation are presented in Table II.

IX. RESIDUAL ABSORPTION COEFFICIENTS

Necessary to this study are residual absorption coefficients α_R's, that is, the average of the sound absorption coefficients of the surfaces in a hall other than those covered by the audience, orchestra, and heavily absorbing materials. Measurements have been reported of α_R's in ten fully completed halls with no seats present (Beranek and Hidaka, 1998). Of those, Boston Symphony Hall and Tokyo Opera City (TOC) Concert Hall are included here. For 15 of the halls the values for α_R has been estimated from the data for similar halls in the 1998 paper. Even if the assumed residual coefficients for the 16 halls are different by small amounts (as indicated by comparison with the residual coefficients for the Boston, Tokyo, and Berlin Konzerthaus Halls, at the bottom of Table I) the effect on the computed reverberation times will not be great because 60%–80% of the total room absorption comes from (occupied) audience areas. All residual absorptions are shown at the bottom of Table I.

For two of the halls, where the walls have unusual construction details, New York Avery Fisher and Berlin Philharmonie, the values of α_R were determined by assuming that the *audience absorption coefficients were the same* as the average of those in the Baltimore, Manchester, and Tokyo-Suntory Halls and calculating backwards. The residual values derived this way are plotted in Fig. 2. In the New York Avery Fisher Hall the walls are wooden and the seats are mounted on a wooden floor with airspace beneath. As expected, the wooden surfaces absorb more sound in the lowest two frequency bands than that for "most halls." In the Berlin

TABLE II. Comparative reverberation, physical, and audience absorption data for all halls in this study. Sabine equation used in calculating absorption coefficients.

Nonrectangular halls	Reverberation times (sec)			Total hall absorption coefficient $(0.161V/T_{60}S_{tot})$-$4mV/S_{tot}$			V/S_T (m)	Audience absorption coefficient			% of absorption in occupied areas			Resid. over other S_R/S_T	Area per Seat S_a/N
	125	500	2000	125	500	2000		125	500	2000	125	500	2000	–	(m^2)
Sapporo, Kitara	2.05	1.90	1.75	0.27	0.29	0.29	16.1	0.70	0.93	0.97	57	72	77	3.7	0.54
Munich, Philharmonie	1.95	1.90	1.75	0.30	0.31	0.33	14.9	0.80	0.95	0.99	65	76	80	3.1	0.58
Rotterdam, DeDoelen	2.30	1.90	1.85	0.29	0.35	0.35	13.9	0.64	0.94	0.90	66	80	83	2.4	0.55
Berlin, Philharmonie	2.10	1.85	1.80	0.30	0.34	0.34	13.0	[a]	[a]	[a]	56	69	80	2.3	0.50
New York, Avery Fisher	1.60	1.78	1.55	0.37	0.33	0.37	12.3	[a]	[a]	[a]	46	72	79	2.4	0.43
Cleveland, Severance	1.75	1.65	1.45	0.30	0.32	0.36	11.0	0.68	0.83	0.93	67	78	83	2.4	0.47
Baltimore, Meyerhoff	2.30	2.00	1.65	0.26	0.30	0.35	12.7	0.54	0.78	0.93	61	76	83	2.4	0.49
Manchester, Bridgewater	2.30	2.00	1.80	0.25	0.29	0.32	13.5	0.56	0.81	0.87	59	75	80	2.8	0.60
New York, Carnegie	2.12	1.83	1.57	0.29	0.33	0.38	13.0	0.63	0.86	0.93	65	78	83	2.4	0.43
Tokyo, Suntory	2.14	1.95	1.90	0.26	0.28	0.29	13.3	0.60	0.81	0.78	65	77	80	2.9	0.54
Buffalo, Kleinhans	2.15	1.60	1.27	0.24	32	0.39	8.3	0.39	0.67	0.85	63	80	87	1.6	0.58
										Avg	61	77	82	2.6	0.52
Rectangular halls															
Boston Symphony Hall	1.95	1.90	1.59	0.27	0.28	0.31	12.3	0.55	0.74	0.89	56	72	79	2.7	0.40
Berlin, Konzerthaus	2.20	2.00	1.80	0.23	0.25	0.25	13.6	0.49	0.76	0.83	50	70	76	3.4	0.50
Vienna, Musikvereinssaal	2.25	2.04	1.80	0.25	0.28	0.28	13.4	0.59	0.78	0.86	57	72	78	2.8	0.41
Lenox, Ozawa Hall	2.00	1.80	1.70	0.24	0.27	0.26	12.6	0.56	0.81	0.82	56	72	77	3.2	0.42
Seattle, Benaroya Hall	2.15	1.80	1.65	0.24	0.29	0.28	11.5	0.50	0.77	0.82	58	75	80	2.6	0.47
Kyoto, Concert Hall	2.20	2.00	1.80	0.25	0.27	0.27	14.9	0.63	0.86	0.91	59	74	79	3.4	0.48
Amsterdam, Concert Hall	2.20	2.05	1.80	0.28	0.30	0.30	14.6	0.64	0.82	0.90	62	74	80	2.9	0.41
Lucerne, Concert Hall	2.10	1.90	1.65	0.25	0.28	0.29	12.2	0.55	0.76	0.86	59	73	80	2.7	0.46
Tokyo, Tokyo Opera City	2.07	1.99	1.84	0.20	0.21	0.20	12.5	0.43	0.67	0.69	44	66	70	3.9	0.48
										Avg	56	72	78	3.1	0.45

[a]Assumed about same as average Baltimore & Manchester.

Philharmonie Hall the ceiling is partly covered with, "136 pyramidal-shaped, combination sound-diffusing, low-frequency Helmholtz-resonator-type absorbing boxes" (Beranek, 2004). Because the absorption of the Helmholtz resonators is mainly at low frequencies, the residual absorption coefficients in the 125–1000 Hz bands are also high, as expected.

X. AUDIENCE ABSORPTION COEFFICIENTS IN NINE RECTANGULAR HALLS

Using the Sabine equation and the basic data from Table I, the audience absorption coefficients for nine rectangular (shoebox) halls are given in Table III and are plotted in Fig. 3. Curve A is the average for the Kyoto and Amsterdam Halls which have heavily upholstered chairs. Curve B shows

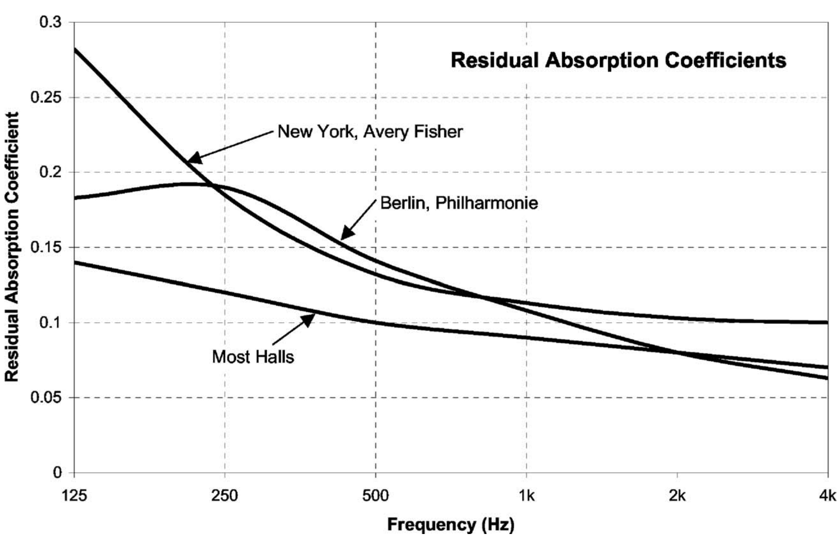

FIG. 2. Residual absorption coefficients using Sabine equation for Berlin Philharmonie and New York Avery Fisher Halls, shown along with the residual coefficients assigned to most of the halls.

TABLE III. Audience absorption coefficients for the rectangular halls in this study. The Sabine equation was used to determine the coefficients in the upper table. Eyring coefficients can be determined by multiplying the Sabine coefficients by the ratios in the lower table.

	Octave-band midfrequencies (Hz)					
Sabine equation	125	250	500	1000	2000	4000
Kyoto, Concert Hall	0.63	0.76	0.87	0.91	0.91	0.88
Amsterdam, Concertgebouw	0.64	0.71	0.82	0.90	0.90	0.89
Boston Symphony Hall	0.55	0.66	0.74	0.80	0.89	0.85
Berlin, Konzerthaus	0.49	0.64	0.76	0.79	0.83	0.78
Vienna, Musikvereinssaal	0.57	0.65	0.78	0.85	0.86	0.78
Lenox, Ozawa Hall	0.56	0.68	0.81	0.87	0.82	0.90
Seattle, Benaroya Hall	0.50	0.67	0.77	0.83	0.82	0.72
Lucerne, Concert Hall	0.55	0.65	0.76	0.84	0.86	0.80
Tokyo, Tokyo Opera City Hall	0.43	0.54	0.67	0.74	0.69	0.66
Average Sabine Audience Abs. Coef.	0.56	0.68	0.79	0.85	0.86	0.82

Eyring equation	The Eyring audience absorption coefficients are equal to the above coefficients multiplied by the ratio “α_{ey}/α_{tot}” below Ratio “α_{ey}/α_{tot}”					
Kyoto, Concert Hall	0.88	0.88	0.88	0.88	0.88	0.88
Amsterdam, Concertgebouw	0.88	0.87	0.87	0.86	0.86	0.87
Boston Symphony Hall	0.88	0.80	0.87	0.87	0.86	0.87
Berlin, Konzerthaus	0.90	0.89	0.89	0.89	0.89	0.89
Vienna, Musikvereinssaal	0.89	0.88	0.88	0.87	0.87	0.88
Lenox, Ozawa Hall	0.89	0.88	0.88	0.88	0.88	0.88
Seattle, Benaroya Hall	0.89	0.88	0.87	0.87	0.87	0.89
Lucerne, Concert Hall	0.89	0.88	0.87	0.87	0.87	0.88
Tokyo, Tokyo Opera City Concert Hall	0.91	0.90	0.90	0.89	0.90	0.92
Ratio Average	0.89	0.87	0.88	0.88	0.88	0.89
Average Eyring Audience Abs. Coef.	0.50	0.59	0.69	0.74	0.75	0.72

the average of the coefficients for the Boston, Berlin Konzerthaus, Vienna, Lenox, Seattle, and Lucerne Halls, which have light to medium upholstered chairs (Beranek, 2004). Curve C is for the TOC Hall. The average Sabine coefficients for all rectangular halls are given in the middle of Table III.

The audience absorption coefficients for TOC (curve C) are lower at all frequencies than those for the six halls of curve B. Three reasons probably account for this difference. First, the sound waves will involve the larger area of the residual surfaces more of the time than in other rectangular halls, because, from Table II, it is seen that for TOC S_R/S_T is the highest, 3.9, compared to 3.0 for the others. Second, it is possible that the unique pyramidal ceiling creates a somewhat different sound field at the surface of the audience area. Third, as will be shown later, the unoccupied chair absorption for TOC is lower.

Using the Eyring equation, the ratios α_{ey}/α_{tot} are plotted in the lower half of Table III, to show the range. This ratio only varies in these halls from 0.87 to 0.90—averaging 0.875. The Eyring coefficients can be found from the prod-

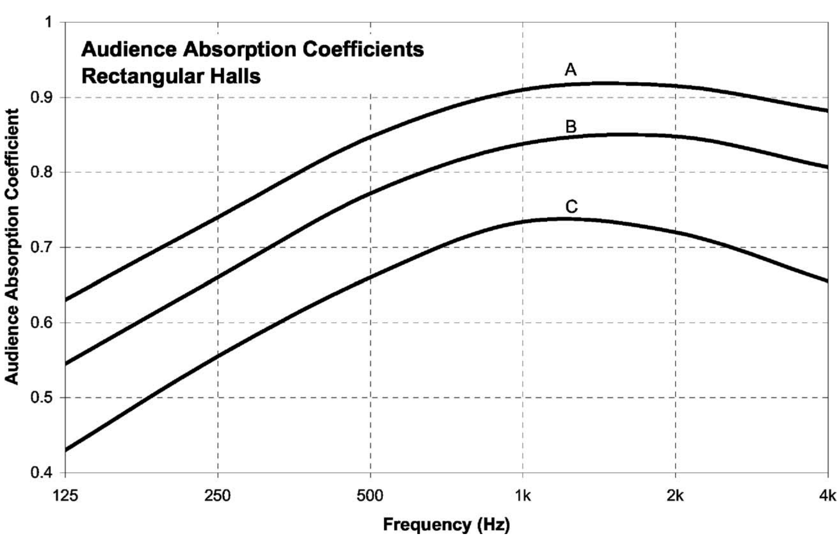

FIG. 3. Audience absorption coefficients for rectangular halls using Sabine equation. (A) Average of Kyoto and Amsterdam Halls. (B) Average of Boston, Berlin Konzerthaus, Vienna, Lenox, Seattle, and Lucerne Halls. (C) Tokyo Opera City Hall.

TABLE IV. Audience absorption coefficients for the nonrectangular halls in this study. Sabine coefficients are shown in the upper table and Eyring coefficients in the lower.

	Octave-band midfrequencies (Hz)					
Sabine equation	125	250	500	1000	2000	4000
Sapporo, Kitara Concert Hall	0.696	0.825	0.926	0.958	0.966	0.972
Munich, Philharmonie am Gasteig	0.797	0.827	0.952	0.950	0.989	0.990
Rotterdam, DeDoelen Concert Hall	0.642	0.836	0.942	0.966	0.898	0.827
New York, Carnegie	0.634	0.729	0.857	0.910	0.932	0.893
Cleveland, Severance Hall	0.679	0.755	0.834	0.927	0.931	0.905
Baltimore, Meyerhoff Hall	0.544	0.678	0.775	0.800	0.928	1.008
Manchester, Bridgewater Hall	0.558	0.694	0.811	0.838	0.867	0.771
Tokyo, Suntory	0.598	0.684	0.811	0.812	0.779	0.674
Buffalo, Kleinhans	0.391	0.613	0.671	0.807	0.848	0.755
Berlin, Philharmonie (see note)	0.557	0.673	0.775	0.827	0.801	0.778
New York, Avery Fisher Hall (note)	0.573	0.700	0.801	0.869	0.923	0.796
Av. Sabine Audience Absorp. Coef.	0.628	0.740	0.848	0.886	0.901	0.861
	Octave-band midfrequencies (Hz)					
Eyring equation	125	250	500	1000	2000	4000
Sapporo, Kitara Concert Hall	0.610	0.720	0.806	0.834	0.841	0.849
Munich, Philharmonie am Gasteig	0.687	0.718	0.818	0.819	0.854	0.856
Rotterdam, DeDoelen Concert Hall	0.558	0.711	0.797	0.817	0.768	0.718
New York, Carnegie	0.553	0.628	0.730	0.770	0.785	0.759
Cleveland, Severance Hall	0.587	0.650	0.717	0.787	0.810	0.792
Baltimore, Meyerhoff Hall	0.480	0.591	0.670	0.691	0.791	0.854
Manchester, Bridgewater Hall	0.494	0.607	0.703	0.727	0.754	0.681
Tokyo, Suntory	0.529	0.601	0.708	0.712	0.686	0.604
Buffalo, Kleinhans	0.350	0.529	0.576	0.678	0.710	0.643
Berlin, Philharmonie	0.482	0.571	0.658	0.709	0.689	0.675
New York, Avery Fisher Hall	0.481	0.596	0.683	0.737	0.781	0.686
Av. Eyring Audience Absorp. Coef.	0.546	0.639	0.729	0.760	0.776	0.747

ucts of the Sabine coefficients by these ratios. The average Eyring coefficients for the rectangular halls are given in the bottom line of Table III.

XI. AUDIENCE ABSORPTION COEFFICIENTS IN NONRECTANGULAR HALLS

The audience absorption coefficients for nonrectangular halls were calculated using the parameters in Table I. The results, for both formulas, are presented in Table IV and are plotted in Fig. 4. Curve A is for three halls, Sapporo, Munich, and Rotterdam, which have chairs with thicker upholstery than for the other halls. Curve B is for Carnegie and Cleveland Halls. Curve C is for the Baltimore, Manchester, and Tokyo-Suntory Halls. Curve D is for the Buffalo Hall which has chairs with minimum upholstering. In a section following, it will be seen that the unoccupied chairs in Carnegie Hall absorb more sound than the chairs in Baltimore and Manchester Halls.

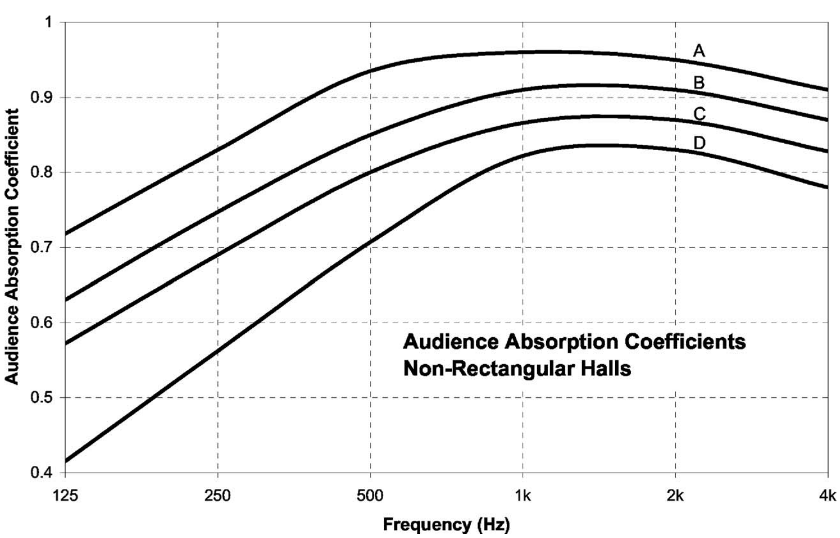

FIG. 4. Audience absorption coefficients for nonrectangular halls using Sabine equation. (A) Average of Sapporo, Munich, and Rotterdam halls. (B) New York, Carnegie, and Cleveland Halls. (C) Average of Baltimore, Manchester, and Tokyo Suntory Halls. (D) Buffalo Hall.

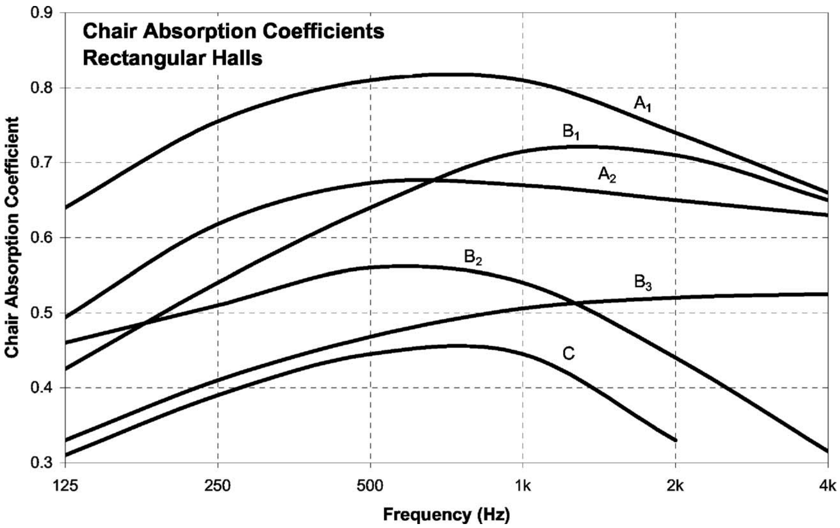

FIG. 5. Chair absorption coefficients for rectangular halls using Sabine equation. (A_1) Kyoto Hall. (A_2) Amsterdam Hall. (B_1) Average of Berlin Konzerthaus, Lenox, Seattle, and Lucerne Halls. (B_2) Boston Hall. (B_3) Vienna Hall. (C) Tokyo Opera City Hall.

XII. CHAIR ABSORPTION COEFFICIENTS IN RECTANGULAR HALLS

The absorption coefficients for unoccupied chairs in rectangular halls are given in Fig. 5. Six curves appear here as compared to three for Fig. 3 because A_1-Kyoto and A_2-Amsterdam have nearly the same absorptions when occupied, and B_1-BerlinK, Lenox, Seattle, and Lucerne, B_2-Boston, and B_3-Vienna are nearly alike when occupied. The only explanation as to why A_2 here is much lower than A_1, and B_2 and B_3 are much lower than B_1 is that those halls have the lowest areas per seat, 0.41 compared to 0.48 for A_1-Kyoto and 0.47 for B_1-Berlin, Lenox, Seattle and Lucerne (see Table II). If valid, this means that crowded people better shield the chairs, making the audience absorption less dependent on the kind of chairs on which they are seated.

For B_2-Boston the high absorptions at low frequencies are explained as follows: The seats are mounted on a layer of plywood with large airspace beneath. The high unoccupied chair absorption at 125 Hz is caused by the undamped plywood, but when occupied, it is significantly damped and assumes a greater mass due to peoples weight. Also, the seats are upholstered with leather over a 2 cm layer of felt so that there is no porosity, which means low absorption at higher frequencies. In Vienna, the rear balcony seats are bare wood, and all seats are very lightly upholstered.

The lowest curve of Fig. 5, C, is the chair absorption for the TOC. The rapid decrease in sound absorption at high frequencies for TOC and several of the other halls needs explanation. In an upholstered seat, the sound waves must penetrate the upholstery covering to get to the layer of foam or other material beneath. Several constructions can inhibit this penetration: (1) A thin plastic sheet that exists between the upholstery covering and the cushion, thus preventing the free flow of air. Such a sheet has a mass, which usually is light enough to vibrate freely for the lowest two bands, but that vibrates less and less freely as the frequency gets higher. (2) The upholstery covering has high flow resistance. This means that the sound wave is reflected off of the upholstery covering. But upholstering covering is a mass, and may vibrate freely at low frequencies. (3) The upholstery covering may have been back sprayed, which means it is nonporous—equivalent to very high flow resistance. (4) Upholstery covering is nonporous and heavy.

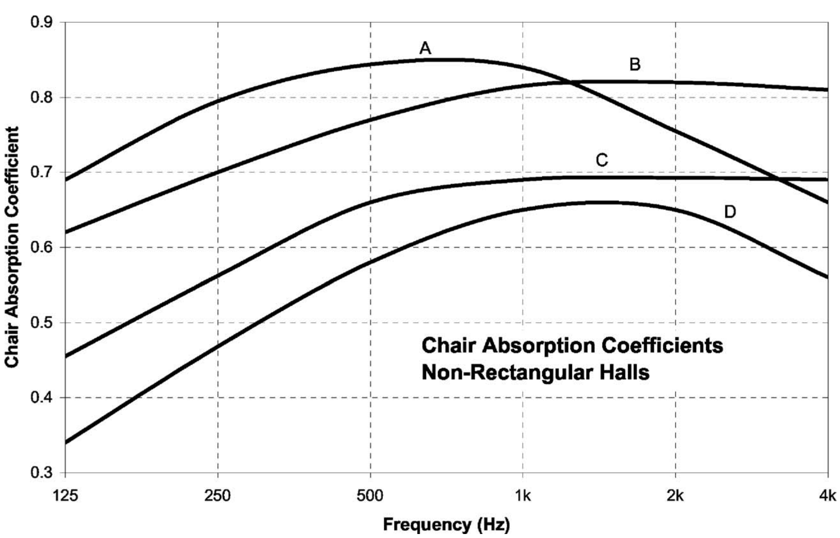

FIG. 6. Chair absorption coefficients for nonrectangular halls using Sabine equation. (A) Average of Sapporo, Munich, and Rotterdam Halls. (B) New York Carnegie Hall. (C) Average of Baltimore and Manchester Halls. (D) Buffalo Hall.

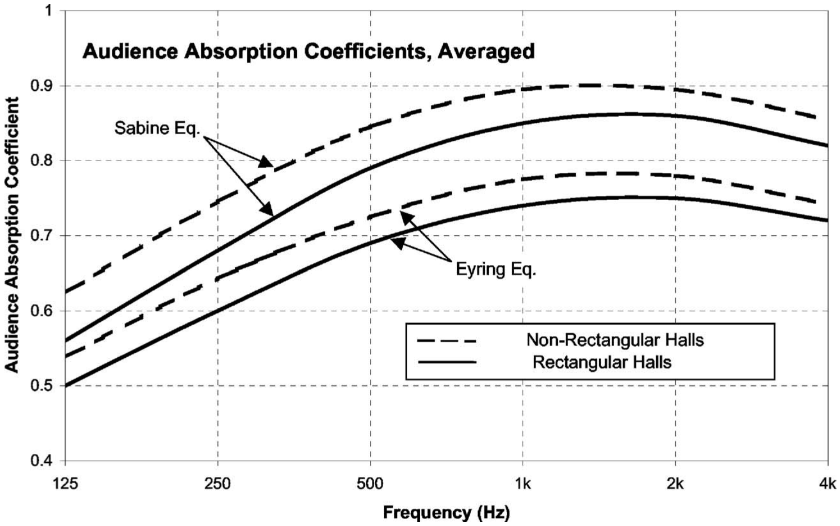

FIG. 7. Audience absorption coefficients: Average of all halls. Upper curves: Sabine equation; Lower curves; Eyring equation. Dashed lines: Nonrectangular halls. Solid lines: Rectangular halls. Data are from Tables III and IV.

Curve B_2 of Fig. 5 is for Boston Symphony Hall where the covering is leather, i.e., nonporous and heavy. For the TOC Hall, my notes as acoustical consultant say that there is a vinyl sheet underneath the upholstery covering (not planned). The chairs of Vienna and TOC were tested in a reverberation chamber and their unoccupied absorptions were alike at 500–4000 Hz, but the TOC chairs in that test did not have a vinyl sheet. The difference at low frequencies between the chair absorptions shown by curve B_3-Vienna and curve C-TOC is caused by the larger ratio S_R/S_T (See Table II). The chairs in the Kyoto Hall are upholstered on the rear of the seat back, which is not true for any of the other nineteen halls in this study and from curve A_1 of Fig. 5 the chair absorption for Kyoto is the highest of all halls.

XIII. CHAIR ABSORPTION COEFFICIENTS IN NONRECTANGULAR HALLS

The absorption coefficients for unoccupied chairs in nonrectangular halls are given in Fig. 6. Curve A is the average of coefficients for the Sapporo, Munich, and Rotterdam Halls, all of which have heavy upholstering (but not on rear of seatback). Curve B is for New York Carnegie Hall. Curve C is the average for Baltimore and Manchester Halls. Curve D is for Buffalo Hall where the chair upholstering is minimum. Except for variability at the high frequencies, the shapes of the curves resemble those of Fig. 4. The letterings for the curves are consistent in Figs. 4 and 6.

XIV. THEORETICAL RELATION OF HALL VOLUME TO AUDIENCE ABSORPTION

From Table II for nonrectangular halls it is seen that the percentage of absorption in the seating areas S_T to that in the residual areas S_R, $\alpha_T S_T/(\alpha_T S_T+\alpha_R S_R)$, is 77%. Assuming this percentage, on average, Eq. (1) becomes

$$V \approx 8.1(T_{60}S_T)\alpha_T \quad (\text{nonrectangular halls, 500 Hz}). \tag{15}$$

For rectangular halls where the percentage average is 72%

$$V \approx 8.5(T_{60}S_T)\alpha_T \quad (\text{rectangular halls, 500 Hz}). \tag{16}$$

Thus, for both types, the volume is directly related to the audience absorption coefficient.

XV. HALL VOLUME RELATED TO ROOM SHAPE AND SEAT UPHOLSTERING

A. Room shape

Audience absorption coefficients for the nonrectangular-shaped halls are greater than those for the other halls as seen in the plots of Fig. 7, taken from Tables III and IV. At 500 Hz, the ratio of the Sabine coefficients is $0.84/0.79=1.06$, and at 1000 Hz $0.9/0.85=1.06$. From this ratio and Eqs. (15) and (16), the cubic volume of a nonrectangular hall must be about 6% greater than that for a rectangular one, assuming the same audience size, reverberation time, and type of upholstering.

TABLE V. Audience absorption coefficients, Sabine equation, related to the degree of chair upholstering.

	Audience absorption coefficients					
Rectangular halls	125	250	500	1000	2000	4000
Heavily upholstered seats	0.63	0.75	0.85	0.91	0.91	0.88
Medium upholstered seats	0.55	0.67	0.79	0.84	0.85	0.83
Nonrectangular halls						
Heavily upholstered seats	0.73	0.82	0.90	0.95	0.95	0.92
Medium upholstered seats	0.63	0.75	0.84	0.90	0.90	0.88

TABLE VI. Upholstery details on seats in 20 concert halls.

Amount of upholstering	Front side of seat back	Rear side of seat back	Top of seat bottom	Arm rests
Heavily upholstered	7.5 cm	Sometimes[a]	10	2 cm
Medium upholstered	2.5 cm	0	5	Solid
Lightly upholstered	1.5 cm	0	2.5	Solid
Tokyo TOC[b]	65%, 2 cm	0	5	Solid

[a]Only in Kyoto in this paper.
[b]See text for details of upholstering.

B. Seat upholstering

A primary consideration in design of a concert hall is how much will the cubic volume be affected by the amount of seat upholstering if the reverberation time T_{60} and the audience size S_T are chosen. For rectangular halls, Fig. 3, the ratio between the α_T's at 500 Hz, curves A and B, is 1.1, and at 1000 Hz, 1.09 indicating a need for 10% greater cubic volume when the seats are heavily upholstered. For nonrectangular halls, Fig. 4, from curves A and B/C, the ratio of the α_T's at 500 Hz, is 1.13 and at 100 Hz, 1.05, indicating a 9% greater cubic volume. Thus, a 9%–10% increase in cubic volume is needed for a hall in which the seats are planned to be heavily upholstered as compared to light to medium upholstering.

In both rectangular and nonrectangular halls the percent increase of audience absorption with the amount of upholstering is highest in the lowest frequencies, but the increase even occurs at the highest frequencies where one might expect that the listeners' bodies would cover the absorbing surfaces (Table V). Higher audience absorption means lower sound levels in the room. At 63 Hz, the *loudness* halves for 5 dB drop in sound level. At 125 Hz, the loudness halves for 8 dB drop in sound level, while at higher frequencies the loudness halves for 10 dB drop. This means that the *loudness* in sones at low frequencies will decrease more rapidly with a change in absorption than at high frequencies, i.e., at 63 Hz a 10 dB drop will decrease the loudness by a factor of 4, while at 500 Hz and above a drop of 10 dB decreases the loudness by a factor of 2. Sounds in the 67–125 Hz bands frequency range are produced by the lower-pitched instruments in an orchestra.

Several years ago a questionnaire survey was sent to managers of concert halls asking them to describe the seats in their halls. The responses revealed that seats today are generally of shaped plywood, with no upholstering on the rear of the seat back or on the bottom of the seat. But, different degrees of upholstering were reported on other surfaces as given in Table VI.

XVI. CONCLUSIONS

The Sabine equation can be used for calculating reverberation times for a room if the Sabine sound absorption coefficients that are employed were previously determined in a site similar to the room in consideration. Sabine absorption coefficients must be allowed to take on all values from zero to infinity—false is the concept that a Sabine coefficient of 1.0 equates to complete sound absorption by a surface (as stated by Sabine, but contradicted by his own data). The Sabine absorption coefficients are easily derived and the Eyring absorption coefficients can be directly derived from Sabine coefficients because they are rigidly linked together by the logarithm. In concert halls the ratio of Eyring to Sabine coefficients is about 0.85, but in rooms with short reverberation times, the ratio may even go lower than 0.5.

Sound absorption by a seated audience is found to be higher at all frequencies if the sound absorption of the chairs on which they are seated is higher. The volume of a concert hall must be increased by approximately 10% if heavily upholstered seats are employed instead of light to medium upholstered seats. The sound absorption by an audience in halls where clear upper spaces for reverberation do not exist (usually in nonrectangular halls) is higher than that in halls were unimpeded surfaces above the top balcony are present (usually in rectangular halls). The volume of a concert hall must be about six percent greater if a design is selected that does not embody a large reverberant space above the top balcony.

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[1]The Eyring equation was derived by Pat Norris [presented as an oral footnote to a paper by Norris and Andree at an Acoustical Society of America meeting in Spring (1929)], several months before Eyring's presentation. It was first published in an appendix to Knudsen's book (1932), obviously submitted to Knudsen earlier. Knudsen, wrote to Beranek "I recognized that he…was the first to derive this equation among American acousticians, and therefore I included his derivation in my *Architectural Acoustics*." The exact date of derivation of this equation by Schuster and Waetzmann, is not known, but it was published in 1929. Cremer and Mueller (1982) wrote, "This substitution $[-\ln(1-\alpha)]$ was first introduced by Fokker" (Fokker, 1924).

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