# Computer simulations in room acoustics: Concepts and uncertainties[a)]

Michael Vorländer[b)]
*Institute of Technical Acoustics, RWTH Aachen University, D-52056 Aachen, Germany*

(Received 10 May 2012; revised 17 December 2012; accepted 21 December 2012)

Geometrical acoustics are used as a standard model for room acoustic design and consulting. Research on room acoustic simulation focuses on a more accurate modeling of propagation effects such as diffraction and other wave effects in rooms, and on scattering. Much progress was made in this field so that wave models also (for example, the boundary element method and the finite differences in time domain) can now be used for higher frequencies. The concepts and implementations of room simulation methods are briefly reviewed. After all, simulations in architectural acoustics are indeed powerful tools, but their reliability depends on the skills of the operator who has to create an adequate polygon model and has to choose the correct input data of boundary conditions such as absorption and scattering. Very little is known about the uncertainty of this input data. With the theory of error propagation of uncertainties it can be shown that prediction of reverberation times with accuracy better than the just noticeable difference requires input data in a quality which is not available from reverberation room measurements. 
[http://dx.doi.org/10.1121/1.4788978]

PACS number(s): 43.55.Ka, 43.55.Ev [ADP] Pages: 1203–1213

## I. INTRODUCTION

Computer simulations of room acoustics were first introduced by Schroeder[1] in 1962 and, thus, they have their 50 year anniversary. First implementations were made a little later by Krokstad *et al*.[2] The methods defined in those days seem to be well-established in research and widely used for room acoustic design. The algorithms of standard programs are based on geometrical acoustics. Interference effects are neglected in geometrical acoustics. According to the particle wave dualism, the description of sound fields is based on energy decays and the direction of particles or rays incident on the receiver. This approach is correct as long as the relevant dimensions of the room geometry are large compared with wavelengths and broadband signals are taken into account. These approximations are usually valid and with accuracy enough in large rooms intended for speech and music far above the Schroeder frequency.

For small and medium sized rooms and for studies of distinct wave effects, such as the seat dip effect, however, these geometrical methods fail. The simple reason is that the seats and the audience as such are not small compared with wavelengths at 100–200 Hz. Full wave-based models must then be chosen. The models were recently used for the first time for practical studies.[3] This is where the finite element method (FEM) and the method of finite differences in time domain (FDTD) come into play. Implementations on graphics processing units allow a rapid calculation of impulse responses even in large spaces[4–6] for frequencies up to the Schroeder frequency, and even above.

Another significant step forward was the integration of acoustic simulation methods into virtual reality systems. Real-time performance is not a huge problem anymore, so that the auralization of indoor and outdoor environments can be combined with three-dimensional stereoscopic display systems and used for research and design applications, and for games.[7,8] It is also worth mentioning that in virtual reality scenarios the listener can move unrestrictedly, turn the head, etc., and thus perceive a natural situation. This provides more realistic conditions for psychoacoustic tests.

Quantitative uncertainties in all the acoustic prediction and simulation tools mentioned above, however, have not been studied in detail yet. The users of prediction tools cannot be sure that they will obtain correct results when they use some published material data and individually created computer-aided design (CAD) models of the room shape. Nevertheless, the reliability of results is often taken for granted. Computer simulations are, however, also rejected by acousticians because they have severe doubts about their reliability.

In this contribution the latest findings in the field of indoor sound field simulation are briefly summarized. This part is intended to give a review on the state of the art, particularly concerning the methods applied in acoustic consulting and education. Furthermore, the uncertainties and the limitations of these methods are reviewed, and the reasons of the uncertainties of such computer simulations discussed and quantitatively analyzed. The sources of uncertainties are, for instance, material data, approximations in CAD models, and algorithmic details. The methods to obtain quantitative data on uncertainties can be based on intercomparisons (referred to as "round robins"). The statistical method of error propagation, however, where independent variables are analyzed with mean and variance forming a final result such as reverberation time, sound level, clarity, etc. enables us not only to

[a)]Portions of this work were presented in "Performance of computer simulations for architectural acoustics," Proceedings of the International Congress on Acoustics 2010, Sydney, Australia, August 2010.

[b)]Author to whom correspondence should be addressed. Electronic mail: mvo@akustik.rwth-aachen.de

obtain a quantitative result of the uncertainty but also to analyze the reason for the uncertainty.

## II. GEOMETRICAL ACOUSTICS: RAY TRACING AND IMAGE SOURCES

### A. Classical methods

In geometrical acoustics the two basic models of geometrical sound propagation, ray tracing and image sources, are used. Often, however, these two approaches are confused, and the physical meaning is distorted. It is therefore important to highlight the differences: Ray tracing describes a stochastic process of particle radiation and detection.[2] This concept is based on energy propagation while the phases are only included in the delay between radiation and detection. In contrast, image sources are geometrically constructed sources which correspond to specular paths of sound rays.[9,10] They can be used with complex pressure data or energy data. However, even with complex pressures the image source model is still a geometric method since it fulfills the wave equation only in special cases and then it is only an approximation.

Worth mentioning is that image sources can also be constructed by using rays, beams, or cones via a kind of "tracing." Nevertheless, these models are still "image source models." The fundamental difference between image sources and ray tracing is the way contributions in impulse responses are calculated. Ray tracing only yields impulse response low-resolution data like envelopes in spectral and time domains (Fig. 1). Image sources in the classical algorithm or constructed via tracing rays, beams, cones, etc.[11–13] may be used for an exact construction of amplitude and delay of reflections.

### B. Hybrid time domain models

Given the contradictory advantages and disadvantages of ray tracing and image sources, an attempt was made to combine their advantages to achieve high-accuracy results without spending too much algorithmic complexity, and without too high of a computational load. Either ray tracing or radiositylike algorithms were used to overcome the extremely high calculation time inherent in the image source model for simulation of the late part of the impulse response. Reverberation tails are added to the simulation of the early part of the response while hybrid ray tracing is used to detect audible image sources in a kind of "forward audibility test." The idea behind this approach is that a ray, beam, or cone detected by a receiver can be associated with an audible image source. The order, the indices, and the position of this image source can be reconstructed from the ray's history by recording the walls hit and the total free path. Hence, the total travel time, the direction, and the chain of image sources involved can be linked to the image source. Almost all other algorithms used in commercial software are some sort of dialect of the algorithms described in Sec. II A, and they differ in the way the specular is mixed with the scattered component. Which dialect is used depends on the type of results, particularly on the accuracy, spatial and temporal resolution.

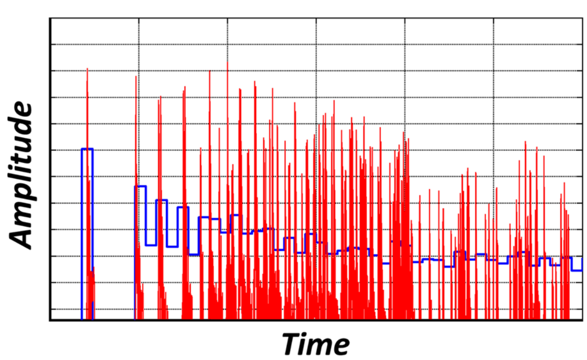



FIG. 1. (Color online) Fundamental energy impulse response computed by using image sources (detailed response) and ray tracing (histogram).

## III. WAVE MODELS

### A. Wave-based frequency domain models

In frequency domain calculations a harmonic excitation signal is used to calculate the counterpart of the impulse response: the stationary transfer function. This problem can be solved by using numerical methods and by spatial discretization by introducing a "mesh." A mesh is a discretized grid of nodes and corresponding elements formed by groups of nodes.

The boundary element method (BEM) and the finite element method (FEM) are discussed in literature at length. Advanced techniques like fast multipole BEM are developments which allow meshes to be divided into regions of high discretization and other regions, with the effect of transfer propagation. The complex linking between mesh elements is thus re-arranged in a hierarchical way. When using FEM, the field space for the acoustic problem must be discretized into suitable volume elements.[14] Solvers can be programmed to determine the matrix' eigenvalues by using the matrix equation without further subspace conditions. When using the indirect method, the problem is projected on a modal basis onto an equivalent eigenvalue problem of orthogonal modes. The latter method has the great advantage that sources and boundary conditions can be studied in a second step. The numerical complexity is then given by the size of the modal basis and not by the finite element mesh size. It is also interesting to compute the complex eigenmodes and post-process the total field in superposition depending on source and receiver location.

### B. Wave-based time domain models

Time domain mesh models, such as the FDTD were recently introduced. These methods are similar to waveguide models [digital waveguide models (DWM)] where the sound propagation is considered as a wave transfer through delay networks.[15] If the grid is discretized into fine elements, not only the wave transfer in the d'Alembert solution but the differential equation itself (Newton's law of motion) of pressure and particle velocity can be approximated by finite differences of the spatial and temporal derivatives, $\partial r$ and $\partial t$, respectively. Originally described for electromagnetic waves it was adapted to acoustics by Botteldooren.[16]

Dispersion errors and boundary conditions might cause some problems that must be solved.[4]

Recent algorithmic implementations made use of the fact that FDTD requires updates of one mesh node at the actual time step just from the neighboring nodes at the last time step in the past. Accordingly, the processing can be to a great extent parallelized, and the advantage of parallel computing leads to acceleration of the simulation by orders of magnitude, as published by Savioja[5] and more recently by Borrel-Jensen.[6] In the latter study it was shown that it is possible to simulate a volume of approximately 15 000 $m^3$ in real time up to a frequency of 180 Hz, which is far above the Schroeder frequency of 20 Hz.

It is now the first time in history that high-speed wave model algorithms rather easily exceed the Schroeder frequency significantly! The list of wave models available and newly developed models could be extended by adding details of many other specific algorithms. This kind of review would give an impression about the variety of computational implementations and information about the computational efficiency related to prediction of the sound fields in chosen case studies. In fact, all numerical wave models have advantages and disadvantages concerning the computational efficiency and the accuracy of the results. And this strongly depends on the case study chosen. It is therefore not reasonable to create a recommendation or guideline for using wave models in room acoustic problems.

Instead, the focus of this paper is put on aspects, which are in common for all wave models. At this point, the physical relevance of wave modeling above the Schroeder frequency must be discussed. One very important task for future research is to identify a robust frequency limit between wave models and geometric models. The large room assumption and the Schroeder frequency are not sufficient to identify the crossover frequency. In case of significant wave effects, such as the seat dip effect,[3] the room size is not the only important parameter, as geometric features of the surfaces or on the floor (for example, rows of seats) and their dimensions are important as well.

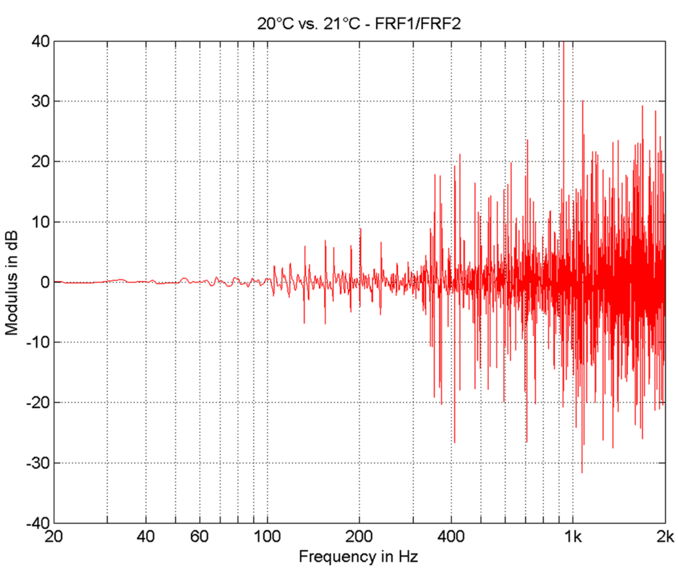
The solution, of course, could be to aim at using wave models throughout the whole frequency range. But it should be kept in mind that the room transfer function "far" above Schroeder frequency is stochastic in nature.[1] The fine structure of magnitudes and phases depends on the specific geometric input data, on certain assumptions of medium properties such as static pressure, temperature and humidity, and on the complex boundary conditions. The fact that very little is known about the exact boundary conditions in real rooms will be discussed in Sec. V. Another fact is that if small changes are introduced in a room, such as having more or less persons present in the room during a measurement (or a change in the air conditioning system) will create a completely different fine structure of the room transfer function, particularly in the phase response, while its frequency average (in 1/3 octave bands or critical loudness bands) is robust against those changes. In Fig. 2, the results are shown of the level differences between the stationary room transfer functions for room temperatures of 20 °C and 21 °C.

The listening impression is not sensitive to small changes in the fine structure of the room transfer function as the sound strength impression corresponds roughly to averages in one-third octave bands. Accordingly, the differences observed in the averages are very small.

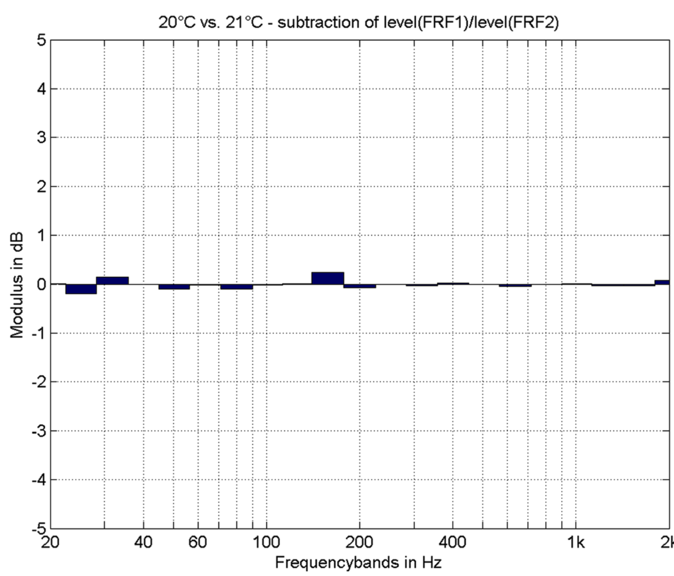
Hence, there is no point in computing a specific, but arbitrary complex transfer function which is not more accurate than the results from approximations in geometric models.

### C. Hybrid low/high frequency domain models

In the 1990s, Kleiner *et al.* combined low frequency and high frequency models.[17] At that time their combination techniques were restricted by the limited frequency range for the numerical FEM simulation. More recently, similar contributions were published.[18,19] In contrast to these earlier




(a)




(b)

FIG. 2. (Color online) (a) Level differences ($L_{20} - L_{21}$) of fine structure between the room transfer functions obtained for temperatures of 20 °C and 21 °C, and (b) differences of the energy averages ($L_{1/3,20} - L_{1/3,20}$) in one-third octave bands.

attempts, researchers nowadays can use increased computer power, and they now deal with an overlap range of the wave models and the geometric models above the Schroeder frequency. Due to well-controlled boundary conditions in case studies, audible differences are so small that test subjects cannot identify which sound example was simulated and which was recorded.[20,21] It should also be mentioned that to achieve this degree of concordance in binaural listening tests, all components of the recording and the simulation must be taken into account with high accuracy, and this does not only involve the field simulation, but also the sound source model and the binaural cues of the listener model (dummy head).[21]

One of the next tasks is to find a psychoacoustically reasonable crossover frequency for wave models and geometric models. A balance is to be settled between the computationally expensive wave models and their advantages and the computationally very inexpensive geometrical models. All in all, it is very good to have numerical methods available which allow computation of room sound fields with almost perfect accuracy provided that correct input data are available.

## IV. VALIDATION TESTS

The computational performance and the accuracy of computer simulations can only be validated when existing rooms are modeled and the results are compared with measurement results. Auralization, of course, can also be validated by listening tests with recordings in the original room. This procedure was used by the author for the first intercomparison in a lecture hall in Braunschweig, Germany, in 1993 and 1994. The initial results were partly disappointing.[22] Data were obtained from 17 participants in computer simulations, and from 7 participants in measurements.[23] One result is shown in Fig. 3. It shows the predicted reverberation time based on a visual inspection of the test room and individually selected absorption coefficients. The results of this phase showed a surprisingly large scatter with a strong tendency to underestimate the absorption coefficients and thus to overestimate the reverberation time.

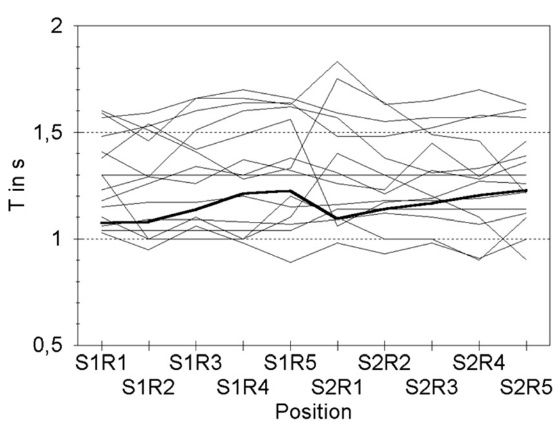



FIG. 3. Results from the first round robin on room acoustical computer simulations (Ref. 21). Plot of reverberation times $T$ predicted for the 1 kHz octave band in an auditorium. Bold line: average measurement result which has an uncertainty of 5% ($\pm$0.05 s; Ref. 22).

Even rather robust quantities such as the sound level (strength) were predicted with up to $\pm$5 dB maximum deviation. In an overall accuracy rating, some programs were deemed reliable. Moreover it was important that algorithms with purely specular reflection modeling are not sufficient which was supported by the results of the second phase where the input data were fixed for all participants. Still the programs which only used specular reflections overestimated the reverberation time systematically. Today it is common knowledge, of course, that in typical rooms after reflection order three or four, the main energy propagation goes through diffuse (scattered) sound. In the following years two more round robins were created.[24,25]

## V. SOURCES AND CONSEQUENCES OF UNCERTAINTIES

In this section the sources of uncertainties and their impact on the results are studied. Uncertainties must be treated as research objects. The author is of the opinion that it is not adequate to "calibrate" a computer model by modification of input data so that, for instance, reverberation times or other damping effects match measurement results. Computer simulations should be independent of calibration factors. They should be solely based on physical data and databases containing input data (usually material properties).

If correct input data are used, the question remains which geometric model and method are suitable to solve the acoustic problem. The latter aspect depends on the expertise and experience of the operator.

For the analysis of uncertainties, a very powerful tool can be used which is based on analysis of uncertainties of measurements. The principles suggested in this "ISO guide to the expression of uncertainty in measurement" (ISO GUM[26]) have not yet been widely considered for acoustics. Computational acoustics mostly assumes perfect input data and compares results with analytic results based on the same input data. There is hardly a systematic approach to tackle the problem of uncertainties with an adequate knowledge which is available for experimental work in acoustics.

### A. Systematic uncertainties

Deviations of simulations and measurements are caused by shortcomings of their algorithms and modeling approach. As described in Sec. II, ray tracing (or similar approaches) and the image model provide the basis for all simulations.

#### *1. Level of detail in the room model*

A proper polygon model of the room is essential for room acoustic simulations with geometrical acoustics. The surface elements, usually polygons, must be large compared with wavelengths in three decades in order to cover the audio frequency range. This is practically impossible. For engineering applications, compromise solutions are used that lack any scientific basis. The results may be wrong due to a physically unfortunate choice of the level of detail. A high level of detail that is too high will lead to unnecessary long computation times. Accordingly, a large potential is identified in the acceleration of algorithms at low frequencies at low spatial resolution in the polygon model, and at late times in the impulse response, where the late decay is built by scattering rather than by deterministic specular reflections in a detailed polygon model. An ongoing project is currently trying to establish which criteria can be used to choose an appropriate level of detail for polygon model[27] (see Fig. 4). These findings will also be relevant for simulations of large volumes such as cathedrals, stadiums, airports, and trains stations, at reasonable computation times.

### 2. Curved surfaces

To the author's knowledge, none of the simulations packages available allows the modeling of curved surfaces. Usually curved surfaces are approximated by a number of planes. Curved surfaces produce very special features like focal points or caustics. The questions are (i) whether an approximation by planes produces a focus as well and (ii) whether the sound level in the focal region and the size of the focal region are correct.[28]

A study recently published by Vercammen[29,30] sheds more light on the problem of focusing. He provided mathematical formulas for sound reflections from concave spherical surfaces. Calculations of the sound pressure are stipulated, particularly for the region near the focus. For a hemisphere, the energy is distributed over a circular area with a width of $\lambda/2$. For small wavelengths the focusing effect is therefore quite strong. Generally, a reduction of the extremely high pressure by absorbers or diffusers in the curved boundary is not sufficient to eliminate the focusing effect. Outside the focal, a strong interfering sound field is observed. Vercammen concludes that within reasonable accuracy the sound field outside the focal point can be calculated with geometrical acoustics. Computer models based on image source methods are not capable of describing the focal pressure, and wave models are the only solution to the problem.

### 3. Diffraction

Diffraction in room acoustics mainly occurs for two reasons: There are obstacles in the room (e.g., stage reflectors) or there are edges at surroundings of finite room boundaries. In the latter case, either the boundary forms an obstacle, such as columns or the edge of an orchestra pit, or the boundary forms the edge between different materials with different impedances (and absorption). Since diffraction is a typical wave phenomenon, it is not accounted for by the basic simulation algorithms listed above. In the past, some ideas were put forward to integrate diffraction as a statistical feature into ray models. But the success was quite limited because the increase in calculation time was a severe problem. In optics and radiowave physics, ray tracing models were generalized into so-called uniform geometrical diffraction theory as presented by Svensson.[31] They are very useful for the determination of first-order diffractions. All methods of geometrical diffraction are, however, very time consuming for simulations of a multiple-order diffraction and corresponding reverberation.

Most recently, Schröder *et al.*[32] and Antani *et al.*[33] integrated diffraction modules into simulation software for both stochastic ray tracing and deterministic image sources. Tests and comparisons with experiments are subject to ongoing research.[34]

### 4. Spherical wave impedances

Scattering, diffraction, etc. are examples of wave phenomena which are not covered by geometrical acoustics. Nevertheless it is sometimes assumed that image source algorithms including of complex wall reflections factors can yield "correct" modal sound fields in rooms. This assumption is, however, wrong. The image source model is a correct solution of the wave equation for rigid boundaries. For the well-known problem of a point source above an impedance plane as one non-rigid boundary, it can be shown that the

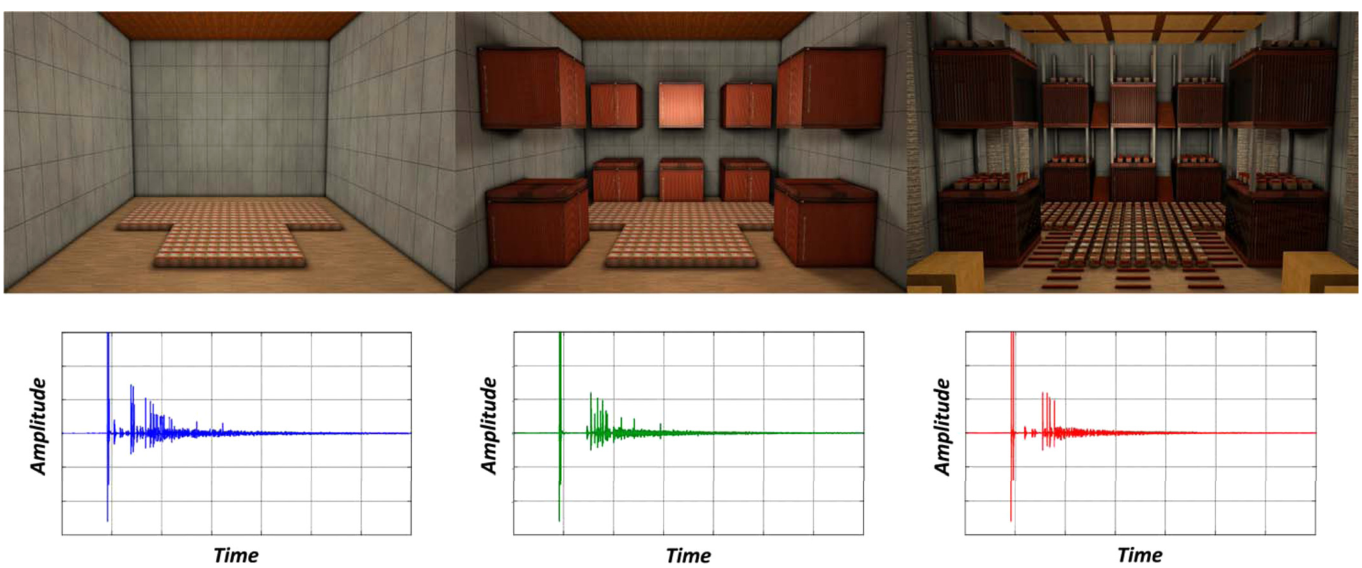



FIG. 4. (Color online) Level of detail in a CAD model of a hall illustrated for (top), left to right for low, mid, and high details, and corresponding impulse responses (bottom).

spherical wave solution based on complex impedance is a good approximation, if the position of source and receiver are not located too close (>one wavelength) to the wall.[35] This is basically the content of the large room assumption. The relevant dimension compared with the wavelength is then not the room volume but the smallest dimension, height or width. In flat rooms or in long rooms like corridors, grazing incidence angles may occur quite often so that the plane wave reflection is too rough an approximation at low frequencies.

Therefore, in all cases where room modes are to be calculated, for instance, small studio rooms, living rooms, etc., only wave-based models can be used, such as BEM or FEM or similar approaches.

## B. Stochastic uncertainties

Stochastic uncertainties in simulations with geometrical acoustics are usually caused by uncertain input data, namely by the choice of the number of rays, and by the boundary conditions of absorption and scattering. These data are often obtained from databases or textbooks, or they are integrated databases in software. Software parameters such as the number of rays or particles used influence the computation time and are thus of interest if compromises between accuracy and effort must be found.

### 1. The number of rays

The number of rays must be discussed independently for stochastic ray tracing and for deterministic ray tracing, which is basically a dialect of image source algorithms, as described in Sec. II B. To find images using ray (beam, cone,…) tracing, a sufficient number of rays must be used so that no reflection (image source) is missed. Accordingly, the minimum number of rays, $N_{\min}$, that is needed to resolve the cone of visibility for a spherical receiver with radius $r_d$ up to time $t$, is[12]

$$N_{\min} = \frac{4(ct)^2}{r_d^2}. \quad (1)$$

In contrast, for stochastic ray tracing or radiositylike algorithms which are used for estimation of reverberation tails, a diffuse field of rays (particles) could be assumed. The result of stochastic ray tracing is collected in time intervals ("bins") and/or surface patches. The rays coincide in temporal bins and surface patches, depending on the specific algorithms and parameters chosen in the program for detection and absorption. As a rule of thumb,[36] the total energy in the impulse response and, correspondingly, the sound strength, $G$, and clarity, $C_{80}$, are uncertain with a standard deviation of

$$\sigma_G = 4.34\sqrt{\frac{A}{8\pi N r_d^2}}, \quad (2)$$

with $N$ the number of rays used and $A$ the equivalent absorption area so that a sufficient repeatability of the test runs of the software can be ensured. This does not mean, however, that the result is free of systematic uncertainties, as other sources of uncertainties are introduced by the choice of input data.

### 2. Absorption coefficients

Stochastic uncertainties are caused by the operator who is responsible for choosing the data, and by uncertainties of material properties, that means deviations of the product specification from standard measurements or by manufacturing variations of the products. The geometrical model, the "polygon model," is for the time being considered perfect for our purpose. In Sec. V C these uncertainties are neglected. Also neglected are uncertainties caused by an unsuitable computation time due to an insufficient number of rays, low reflection order, etc. Only material input data are studied and their variation in tables that are published in textbooks and standard measurements.

For geometrical acoustics there are a few preliminary studies of the influence of material data on the prediction results. Tables of absorption coefficients are widely available in textbooks and online. The first question concerning the simulation software focuses on the implementation of absorption. Should $\alpha$ be modeled angle-dependent or just be constant (for random incidence)? Of course this depends on the sound field incident on the boundary. Angle-dependent data would, in general, be better, but very few data of this kind are available.

At least, ISO 354[37] provides a standard method for measuring random-incidence absorption coefficients in reverberation rooms. The uncertainty inherent in the method can be expressed as follows:

The question is whether these uncertainties are sufficient to obtain acceptable results of room acoustic simulations. This question will be discussed in Sec. V C.

### 3. Scattering coefficients

Surface scattering occurs when wall surfaces are corrugated. The specific reflection pattern depends, to a great extent, on the frequency. However, under diffuse field conditions and the corresponding uniform sound incidence, the detailed reflection characteristic is not needed, but a random-incidence scattering coefficient, $s$, which is defined as the ratio between the scattered sound energy and the totally reflected sound energy.[38] There are no detailed tables available, except the ones published in Ref. 39. It is also not clear whether scattering should be implemented in the software with angle dependence or just for the random-incidence average? Comparisons using different software implementations show that the results reflect the same trends when the scattering coefficients are changed, but the absolute results differ although the same input data are used.[40]

The question of angle dependence cannot be settled easily. If the sound field provides a good mixing and, thus, a good diffuse field approximation, the random-incidence data are certainly sufficient. In non-mixing geometries such as corridors or flat halls, this effect may not be taken for granted, and instead of the average, specific angles of incidence dominate the losses. For scattering walls it can be expected that differences are noticeable for vertical or horizontal orientation of 1D structures. Accordingly, Dalenbäck implemented an orientation-dependent ray scattering for 1D diffusors.[41] Even for random incidence and 2D diffusers it must be accepted at the moment that the measurement uncertainty in ISO 17497-1 measurements is rather large compared with the uncertainty in measurements of absorption coefficients.[41]

Now, in Sec. V C, a first attempt is made to predict the uncertainty of room acoustic simulation, if the input data of absorption coefficients have typical uncertainties.

### C. Propagation of uncertainty of absorption coefficients

When performing measurements in physics, the uncertainties can be calculated or predicted according to the fundamental concept of error propagation,[42] which is also the basis for the ISO guide.[23] Usually the results (measurement, simulation) are based on one or more input parameters, and each of them can be characterized by their specific uncertainties. The question is how these input uncertainties affect the uncertainty of the result in the end.

#### 1. Concepts of error propagation

Let us assume a function $f(x,y)$ which describes how the two input data $x$ and $y$ affect the final result, $f$. The variance of $f$ is[42]

$$\sigma_f^2 \approx \left(\frac{\partial f}{\partial x}\right)^2 \sigma_x^2 + \left(\frac{\partial f}{\partial y}\right)^2 \sigma_y^2, \tag{3}$$

which depends on the variances $\sigma_x^2$ and $\sigma_y^2$, of the variables $x$ and $y$, respectively. The same can be expressed by the relative uncertainty

$$\frac{\sigma_f}{f} \approx \frac{\sqrt{\left(\frac{\partial f}{\partial x}\sigma_x\right)^2 + \left(\frac{\partial f}{\partial y}\sigma_y\right)^2}}{f}. \tag{4}$$

As an example the sound power, $W$, is calculated by measuring the spatial average root-mean-square sound pressure, $p$, and the reverberation time, $T$, in a reverberation chamber. In this case, the function has the mathematical structure $W = \text{const} \cdot p^2/T$. Needless to say that in measurements, the spatial average of the sound pressure suffers from uncertainties, as does the reverberation time, and they are characterized by the standard deviations, $\sigma_p$ and $\sigma_T$, respectively. Now, what is the uncertainty of the final result, the sound power? The result of the error propagation according to Eq. (4) is

$$\frac{\sigma_W}{W} \approx \frac{\sqrt{\left(\frac{\partial W}{\partial p}\sigma_p\right)^2 + \left(\frac{\partial W}{\partial T}\sigma_T\right)^2}}{W} = \sqrt{\left(2\frac{\sigma_p}{p}\right)^2 + \left(\frac{\sigma_T}{T}\right)^2}. \tag{5}$$

The relative uncertainty of the sound pressure is, thus, more relevant as the relative uncertainty of the reverberation time, as its term contributes more to the uncertainty of the sound power.

If this concept is applied to room acoustical simulation, equations to estimate the final result based on the input data with uncertainties are required. The latter are uncertainties of absorption coefficients, and these are known from the information about reverberation room measurements according to ISO 354.[37] The procedure of uncertainty propagation is now illustrated by three examples.

#### 2. Application to uncertainty of room acoustic data: Reverberation time

For this approach an exponential decay is expected. The early reflections are therefore treated as a part of the exponential envelope without any specific local structure. Thus, the uncertainties discussed are global estimates and expectation values and not specific for a specific location in the room.

According to Sabine's equation, the reverberation time in a diffuse sound field in a room volume $V$ and a number of $i$ surfaces can be calculated.

With

$$A = \sum_i S_i \alpha_i, \quad T = 0.16\frac{V}{A} \tag{6}$$

yields the error propagation with independent (uncorrelated) uncertainties of the absorption coefficients. The standard deviation of the absorption coefficient $\alpha_i$ is denoted $\sigma_{\alpha_i}$. According to Eq. (3) the error propagation into the uncertainty of the equivalent absorption area, $A$, is

$$\sigma_A^2 \approx \left(\frac{\partial A}{\partial \alpha_1}\sigma_{\alpha_1}\right)^2 + \left(\frac{\partial A}{\partial \alpha_2}\sigma_{\alpha_2}\right)^2 + \cdots \tag{7}$$

which yields

$$\sigma_A^2 = \sum_i \left(S_i \sigma_{\alpha_i}\right)^2 \tag{8}$$

and

$$\frac{\sigma_A}{A} = \frac{\sqrt{\sum_i \left(S_i \sigma_{\alpha_i}\right)^2}}{\sum_i S_i \alpha_i} = \frac{\sigma_T}{T} = \frac{\sigma_{\langle\alpha\rangle}}{\langle\alpha\rangle}, \tag{9}$$

with the average absorption coefficient $\langle\alpha\rangle$, the reverberation time $T$, and the surface areas of the room boundaries, $S_i$. It is worth mentioning that the relative standard deviations of $T$ and $\alpha$, $\sigma_{\langle T\rangle}/\langle T\rangle$ and $\sigma_{\langle\alpha\rangle}/\langle\alpha\rangle$, respectively, are equal to the relative standard deviation of the equivalent absorption area, $\sigma_{\langle A\rangle}/\langle A\rangle$, because $T$, $\alpha$, and $A$ are connected with each other by linear equations. This is a direct consequence of Eq. (3).

If Eq. (9) is applied to an example of a hall with a volume of 11 000 $\mathrm{m}^3$, the result as shown in Fig. 5 is obtained. Two boundary materials are analyzed, one absorbing with $\alpha_1 = 0.7$ ("absorber" corresponding to the "audience," $S_1 = 800\ \mathrm{m}^2$) and one reflecting with $\alpha_2 = 0.03$ ("hard"

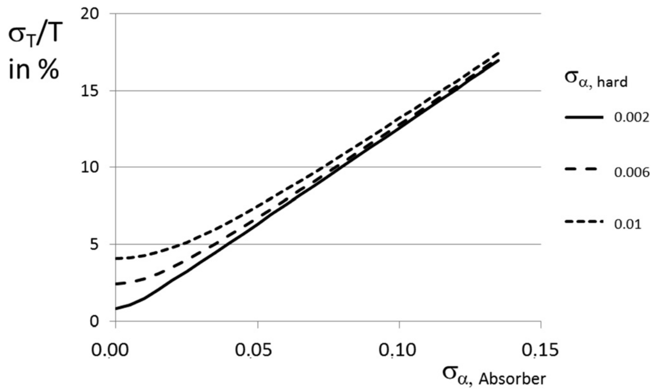



FIG. 5. Result of Eq. (7) of the estimated relative uncertainty of the reverberation time as function of the standard deviation, $\sigma_\alpha$, of an absorber with $\alpha = 0.7$. Curve parameter: standard deviation, $\sigma_\alpha$, of the absorption coefficient of $\alpha = 0.03$ for the reflecting walls.

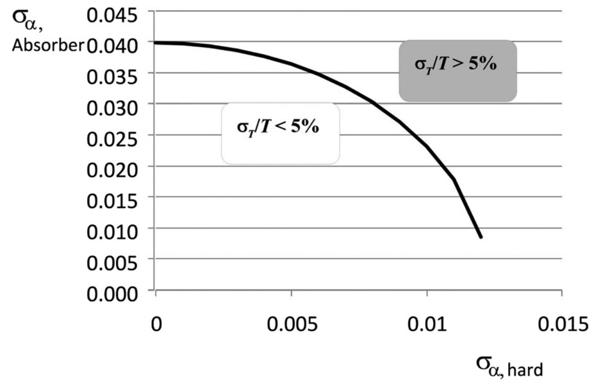



FIG. 6. Limen of maximum standard deviations, $\sigma_\alpha$, to achieve a relative standard deviation of the reverberation time of 5%.

represented by the walls and the ceiling, $S_2 = 2500\,\mathrm{m}^2$). The curves are displayed with the standard deviation of the absorption coefficient of the hard surface as parameter. The abscissa is the standard deviation of the absorption coefficient of the absorber.

The standard deviation of the reflecting surface has only little influence. The main trend of the uncertainty is the increase with the standard deviation of the absorber. As the main absorbers in this auditorium are the seats and the audience, the absorption coefficient of them may in practice easily vary by $\pm 0.1$. In this case the uncertainty of the reverberation time already exceeds 10%.

In Fig. 6, the corresponding result of Eq. (9) is shown for a maximum uncertainty of 5% [just noticeable difference (JND) of reverberation time]. In order to keep the uncertainty below 5%, only combinations of the standard deviations on the left side of the limen are allowed. It is crucial to recognize in Fig. 5 that a reverberation time uncertainty below 5% can only be reached if $\sigma_{\alpha,\mathrm{Absorber}}$ is below 0.04 (which means $\alpha_{\mathrm{Absorber}} = 0.7 \pm 0.04$). Such a small uncertainty, however, cannot be obtained from ISO 354 measurements.

The prediction of uncertainty using Eq. (9) is confirmed by a Monte Carlo simulation with normally distributed variation of input data; see Figs. 7 and 8. The Monte Carlo experiment was performed with a hybrid image source/ray tracing

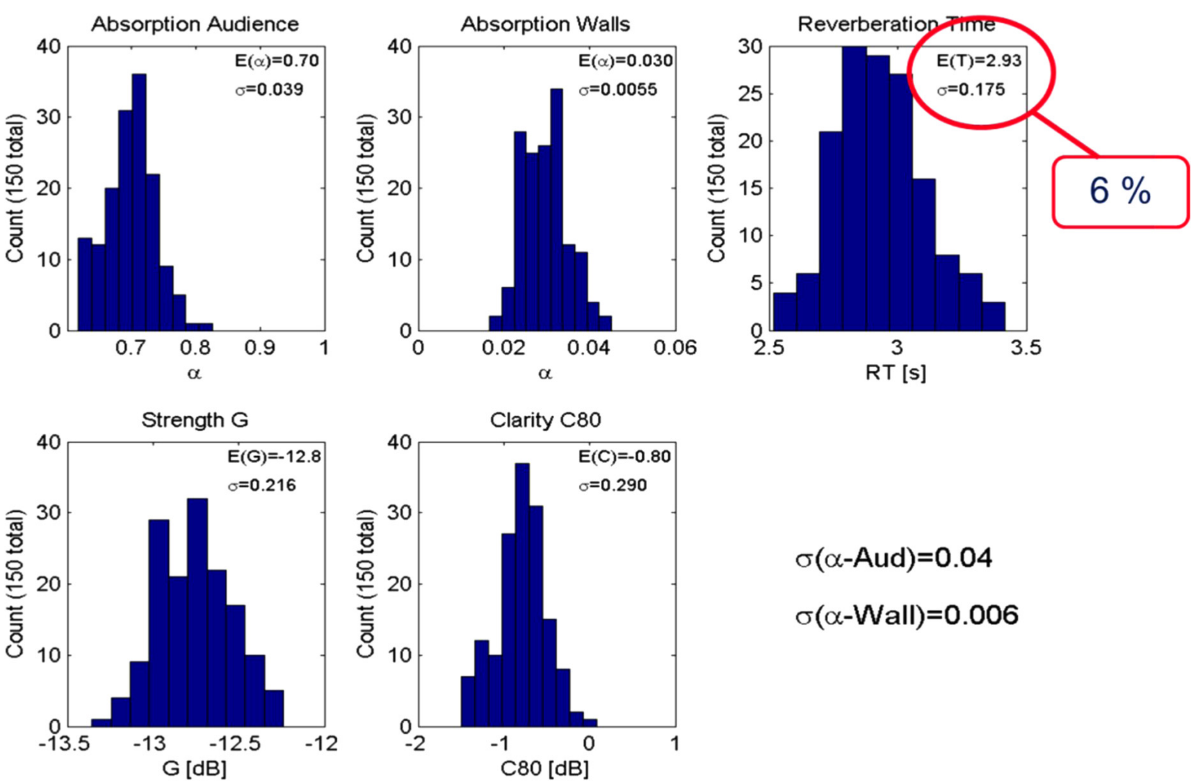



FIG. 7. (Color online) Monte Carlo experiment with a distribution of input data for a simulation of an auditorium with an absorber (audience) of $\alpha = 0.7$ (upper left) and reflecting walls and ceiling with $\alpha = 0.03$ (upper middle). Statistical analysis of the results for reverberation time (upper right), strength (lower left), and clarity (lower right). In this example the standard deviation of the distribution of input data for the absorbing and the reflecting walls is $\sigma = 0.04$ and 0.006, respectively.

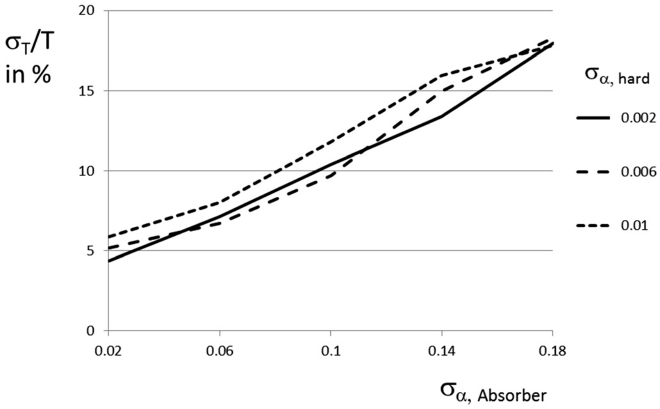



FIG. 8. Result of a Monte Carlo simulation of the relative uncertainty of the reverberation time as function of the standard deviation, $\sigma_\alpha$, of an absorber with $\alpha = 0.7$. Curve parameter: standard deviation, $\sigma_\alpha$, of the absorption coefficient of $\alpha = 0.03$ for the reflecting walls. See also Fig. 5.

algorithm.[43] Twenty test runs for each combination of standard deviations were chosen; see example in Fig. 7.

### 3. Application to uncertainty of room acoustic data: Strength

For the sound level, or "strength," the uncertainty can be derived as well. The total level in a room or strength, $G$, in the diffuse field in a room with equivalent absorption area, $A$ in m$^2$, is given by

$$G = 37 - 10 \log A. \tag{10}$$

The standard deviation, $\sigma_A$, of $A$ is known already by Eq. (9), so the propagation of this uncertainty into the standard deviation, $\sigma_G$, of the strength, $G$, by using the concept of Eq. (3) with $G = G(A)$ gives

$$\sigma_G \approx \frac{\partial G}{\partial A} \sigma_A = \frac{10}{\ln 10} \frac{1}{A} \sigma_A, \tag{11}$$

which is hence

$$\sigma_G = 4.34 \frac{\sigma_A}{A} \text{ dB} = 4.34 \frac{\sigma_T}{T} \text{ dB}. \tag{12}$$

Further calculation shows that to obtain a maximum level deviation of 1 dB, which is considered as JND for sound level, the total uncertainty of the absorption area or the absorption coefficient can be 23%, which is rather large. Uncertainties in $\alpha_{\text{Absorber}}$ between 0.15 and 0.18, are typical values of uncertainties (Table I) and are indeed no problem.

TABLE I. Uncertainty of absorption coefficients (Ref. 37).

| Absorption coefficient | Uncertainty |
|---|---|
| Low $\alpha$ ($\approx$0.1) | 0.1 |
| Mid $\alpha$ ($\approx$0.4) | 0.1 |
| High $\alpha$ ($\approx$0.9) | 0.2 |

### 4. Application to uncertainty of room acoustic data: Clarity

Finally the error propagation for the parameter clarity, $C_{80}$, is calculated. $C_{80}$ is based on the ratio between early and late reflections (ISO 3382) and can be derived from Barron's statistical reverberation theory[44])

$$C_{80} \approx 10 \log \Gamma, \tag{13}$$

with

$$\Gamma = e^{1.104 S/T} \left( 1 + \frac{13.8 V}{4\pi c r^2 T} \right) - 1. \tag{14}$$

$r$ denotes the source-receiver distance. The detailed calculation of the standard deviation of the clarity following Eq. (13) is similar to Eqs. (10) and (11)

$$\sigma_{C_{80}} = \frac{\partial C_{80}}{\partial \Gamma} \sigma_\Gamma = \frac{10}{\ln 10} \frac{\sigma_\Gamma}{\Gamma} = 4.34 \frac{\sigma_\Gamma}{\Gamma}. \tag{15}$$

Hence it is required to calculate the relative standard deviation of $\Gamma$ due to uncertainties in the absorption coefficients. $\Gamma$ is a function of $T$. With the standard deviation of the reverberation time, $\sigma_{\langle T \rangle}$, and its dependence on the relative standard deviation of the absorption, $\sigma_{\langle \alpha \rangle}/\langle \alpha \rangle$, it follows

$$\sigma_\Gamma = \frac{\partial \Gamma}{\partial T} \sigma_T = \frac{\partial \Gamma}{\partial T} \frac{\sigma_{\langle \alpha \rangle}}{\langle \alpha \rangle} T = \Delta \frac{\sigma_{\langle \alpha \rangle}}{\langle \alpha \rangle}. \tag{16}$$

Further calculation yields

$$\begin{aligned} \Delta &= \frac{\partial \Gamma}{\partial T} T \\ &= -e^{1.104 S/T} \left( \frac{1.104 s + \frac{13.8 V}{4\pi c r^2}}{T} + \frac{1.104 s \cdot \frac{13.8 V}{4\pi c r^2}}{T^2} \right). \end{aligned} \tag{17}$$

Finally yields

$$\sigma_{C_{80}} = \frac{4.34 \Delta}{\Gamma} \cdot \frac{\sigma_{\langle \alpha \rangle}}{\langle \alpha \rangle} = B \cdot \frac{\sigma_{\langle \alpha \rangle}}{\langle \alpha \rangle} = B \cdot \frac{\sigma_T}{T}, \tag{18}$$

with a dimensionless room constant, $B$, with

$$B = \frac{4.34 \Delta}{\Gamma} \approx 6. \tag{19}$$

Fortunately, $B$ is only slightly dependent of the term $V/r^2$ in Eq. (17). Therefore the variation of $B$ for a classroom ($B \approx 5$) with small average $r$ and a church ($B \approx 7$) with large average $r$ is small. The prediction from error propagation Eq. (18) and the results from repeated simulations with statistically varied input data fit nicely (Fig. 9).

In the end, a simple rule of thumb can be stipulated: the uncertainty of clarity, in dB, is six times the relative uncertainty of reverberation time or of the average absorption coefficient. This uncertainty is usually rather small compared

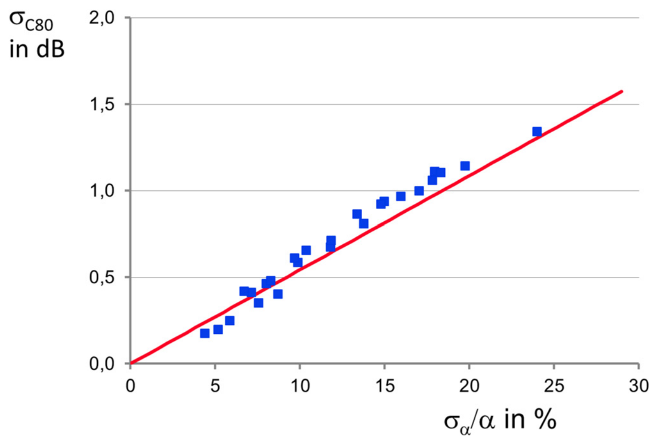



FIG. 9. (Color online) Standard deviation, $\sigma_{C80}$, of clarity $C_{80}$ as a function of the relative standard deviation of the average absorption coefficient, $\sigma_\alpha/\alpha$. Prediction (—) and results from the Monte-Carlo experiment (■).

with the JND for clarity of about 1 dB. Thus, it can be concluded that it is relatively easy to predict the parameter clarity.

## VI. CONCLUSION AND FURTHER WORK

Geometric acoustic models for room acoustics are going to be extended in order to include wave models for the low frequency range. Guidelines for a well-chosen crossover frequency, however, are not yet known. It is clear, though, that wave models can produce more reliable results due to better inclusion of diffraction and scattering, as demonstrated by Lokki *et al.*[3] This depends crucially on the input data for the boundary conditions.

From a certain frequency, the stationary transfer function becomes purely stochastic, particularly in the phase response. Whether any wave models yield better results than geometric models depends not only on the exactness of input data but also on the precision of environmental conditions (temperature, pressure, humidity) and on the relevance of those exact conditions. As long as differences caused by small changes in temperature and humidity cannot be heard, why should these subtle effects be simulated in order to achieve the "exact" complex transfer function? It will be interesting to observe research activities to find reasonable frequency limits for wave models for room acoustics.

Another very interesting field of research will be investigation on accuracy and robustness to uncertain input data. Uncertainties are an important factor as well as in experiments as in computer simulations. Particularly when performing computer simulations, more emphasis should be put on the analyses of such uncertainties because it is then possible to interpret the results with regard to the major influences of input data such as CAD models and absorption coefficients from databases. One very interesting approach of using fuzzy input data is available in the field of vibroacoustics and statistical energy analysis.[45]

Given the data available in textbooks which stem mostly from measurements using ISO 354, input data of absorption coefficients are not accurate enough to obtain simulation results with an uncertainty below the JND of reverberation time. The parameters clarity and strength are more robust in this respect.

More research is required that focuses on the sources of uncertainties and their relevance for the results of the room acoustic quantities. Furthermore, research is needed to obtain information about the JNDs of these effects. In particular, only little information about perception of scattering is available. It may be expected that the rather large uncertainties in measured random incidence scattering coefficients hardly play a role in the overall acoustic impression, but this assumption has not yet been validated.

## ACKNOWLEDGMENTS


The author is grateful for discussions with Marc Aretz and Sönke Pelzer, doctoral candidates at the Institute of Technical Acoustics, RWTH Aachen University.



[1]M. R. Schroeder, B. S. Atal, and C. Bird, "Digital computers in room acoustics," in *Proceedings of the 4th International Congress on Acoustics*, ICA, Copenhagen (1962), p. M21.
[2]A. Krokstad, S. Strøm, and S. Sørsdal, "Calculating the acoustical room response by the use of a ray tracing technique," J. Sound Vib. **8**, 118 (1968).
[3]T. Lokki, A. Southern, and L. Savioja, "Studies on seat dip effect with 3D FDTD modeling," in *Proceedings of FORUM ACUSTICUM 2011*, Aalborg Denmark (2011), pp. 1517–1522.
[4]K. Kowalczyk and M. van Walstijn, "Room acoustics simulation using 3D compact explicit FDTD schemes," IEEE Trans. Audio, Speech, Lang. Process. **19**(3), 34 (2011).
[5]L. Savioja, "Real-time 3D finite-difference time-domain simulation of mid-frequency room acoustics," in *Proceedings of the 13th International Conference on Digital Audio Effects*, DAFx Graz, Austria (2010), p. 43.
[6]N. Borrel-Jensen, "Real-time auralisation of the lower frequency sound field using numerical methods on the GPU," Master thesis, Institute of Technical Acoustics, RWTH Aachen University, and Department of Computer Science, University of Copenhagen, 2012.
[7]D. Schröder, F. Wefers, S. Pelzer, D. Rausch, M. Vorländer, and T. Kuhlen, "Virtual reality system at RWTH Aachen University," in *Proceedings of the International Symposium on Room Acoustics (ISRA)*, Melbourne, Australia (29 August 2010).
[8]A. Chandak, L. Antani, M. Taylor, and D. Manocha, "Fast and accurate geometric sound propagation using visibility computations," in *Proceedings of the International Symposium on Room Acoustics (ISRA),* Melbourne, Australia (29 August 2010)
[9]J. B. Allen and D. A. Berkley, "Image method for efficiently simulating small room acoustics," J. Acoust. Soc. Am. **65**, 943–950 (1979).
[10]F. P. Mechel, "Improved mirror source method in room acoustics," J. Sound Vib. **256**(5), 873 (2002).
[11]J.-P. Vian and D. van Maerke, "Calculation of the room impulse response using a ray tracing method," in *Proceedings of the Symposium on Acoustics and Theatre Planning*, Vancouver (1986), pp. 74–78.
[12]M. Vorländer, "Simulation of the transient and steady state sound propagation in rooms using a new combined sound particle-image source algorithm," J. Acoust. Soc. Am. **86**, 172–178 (1989).
[13]T.A. Funkhouser, I. Carlbom, G. Elko, G. Pingali, M. Sondhi, and J. West, "A beam tracing approach to acoustic modelling for interactive virtual environments," in *Proceedings of the 25th Annual Conference on Computer Graphics, SIGGRAPH'98* (1998), p. 21.
[14]O. C. Zienkiewicz and R. L. Taylor, *The Finite Element Method*, 6th ed. (Butterworth Heinemann, Oxford, UK, 2005), pp. 1–752.
[15]M. Karjalainen and C. Erkut, "Digital waveguides versus finite difference structures," EURASIP J. Appl. Signal Process. **1**, 978 (2004).
[16]D. Botteldooren, "Finite-difference time-domain simulation of low-frequency room acoustic problems," J. Acoustic. Soc. Am. **98**(6), 3302–3308 (1995).
[17]M. E. Kleiner, E. Granier, and P. Svensson, "Coupling of low and high frequency models in auralization," in *Proceedings of the 15th International Congress on Acoustics*, ICA, Trondheim, Norway (1995), pp. 533–536.
[18]M. Bansal, S. Feistel, and W. Ahnert. "First approach to combine particle model algorithms with modal analysis using FEM," in *Proceedings of the*

*118th Convention of the Audio Engineering Society*, Barcelona, Spain (2005), p. 6392.
[19] J. Summers, K. Takahashi, Y. Shimizu, and T. Yamakawa, "Assessing the accuracy of auralizations computed using a hybrid geometrical-acoustics and wave-acoustics method," J. Acoust. Soc. Am. **115**(5), 2514 (2004).
[20] M. Aretz and M. Vorländer, "Efficient modelling of absorbing boundaries in room acoustic FE simulations," Acta Acust. Acust. **96**(6), 1042 (2010).
[21] M. Aretz, "Combined wave and ray based room acoustic simulations in small rooms," Ph.D. dissertation, Institute of Technical Acoustics, RWTH Aachen University, Germany, 2012.
[22] M. Vorländer, "International round robin on room acoustical computer simulations," in *Proceedings of the 15th International Congress on Acoustics*, ICA, Trondheim, Norway (1995), pp. 689–692.
[23] A. Lundeby, M. Vorländer, T. E. Vigran, and H. Bietz, "Uncertainties of measurements in room acoustics," Acustica **81**, 344 (1995).
[24] I. Bork, "A comparison of room simulation software—The 2nd round robin on room acoustical computer simulation," Acta Acust. Acust. **86**(5), 943 (2000).
[25] I. Bork, "Report on the 3rd round robin on room acoustical computer simulation—Part II: Calculations," Acta Acust. Acust. **91**(4), 753 (2005).
[26] ISO/IEC Guide 98, Guide to the expression of uncertainty in measurement (GUM) (International Organization for Standardization, Geneva, Switzerland, 1993).
[27] S. Pelzer, M. Vorländer, and H.-J. Maempel,"Room modeling for acoustic simulation and auralization tasks: Resolution of structural detail," in *Proceedings of the 36th Deutsche Arbeitsgemeinschaft für Akustik*, DAGA Berlin (2010), available from the Deutsche Gesellschaft für Akustik e.V. (DEGA), Berlin, pp. 709–710.
[28] E. Mommertz, "Investigation of acoustic wall properties and modelling of sound reflections in binaural room simulation," Ph.D. dissertation, Institute of Technical Acoustics, RWTH Aachen University, Germany, 1996.
[29] M. Vercammen, "Sound reflections from concave spherical surfaces. Part I: Wave field approximation," Acta Acust. Acust. **96**, 82 (2010).
[30] M. Vercammen, "Sound reflections from concave spherical surfaces. Part II: Geometrical acoustics and engineering approach," Acta Acust. Acust. **96**, 92 (2010).
[31] U. P. Svensson, R. I. Fred, and J. Vanderkooy, "An analytic secondary source model of edge diffraction impulse responses," J. Acoust. Soc. Am. **106**, 2331–2344 (1999).
[32] D. Schröder and A. Pohl, "Real-time hybrid simulation method including edge diffraction," in *Proceedings of the EAA Symposium on Auralization*, Espoo, Finland (2009).
[33] L. Antani, A. Chandak, M. Taylor, and D. Manocha, "Fast geometric sound propagation with finite edge diffraction," Technical Report No. TR10-011, UNC Chapel Hill (2010)
[34] D. Schröder, M. Vorländer, and P. U. Svensson, "Open acoustic measurements for validating edge diffraction simulation methods," in *Proceedings of the Baltic-Nordic Acoustic Meeting*, BNAM Bergen, Norway (2010).
[35] J. S. Suh and P. A. Nelson, "Measurement of transient response of rooms and comparison with geometrical acoustic models," J. Acoust. Soc. Am. **105**, 2304–2317 (1999).
[36] M. Vorländer, "Die Genauigkeit von Berechnungen mit dem raumakustischen Schallteilchenmodell und ihre Abhängigkeit von der Rechenzeit (The accuracy of calculations with the room acoustic ray tracing model and its dependence on the calculation time)," Acustica **66**, 90 (1985).
[37] ISO 354:2003, Measurement of sound absorption in a reverberation room (International Organization for Standardization, Geneva, Switzerland, 2003).
[38] E. Mommertz and M. Vorländer, "Definition and measurement of random-incidence scattering coefficients," Appl. Acoust. **60**, 187 (2000).
[39] M. Vorländer, *Auralization: Fundamentals of Acoustics, Modelling, Simulation, Algorithms, and Acoustic Virtual Reality* (Springer, Berlin, 2008), pp. 311–315.
[40] L. Shtrepi, S. Pelzer, M. Rychtáriková, A. Astolfi, and M. Vorländer, "Influence of scattering coefficient on the prediction of room acoustic parameters in a virtual concert hall through three different algorithms," in *Proceedings of EURONOISE*, Prague, (2012).
[41] B.-I. Dalenbäck, "Modeling 1D-diffusers-the missing link," in *Proceedings of the Ecophon International Acoustic Symposium*, EIAS Båstad, Sweden (2011).
[42] Y. Beers, *Introduction to the Theory of Error* (Addison-Wesley, Reading, MA, 1953), pp. 1–65.
[43] D. Schröder and M. Vorländer, "RAVEN: A real-time framework for the auralization of interactive virtual environments," in *Proceedings of FORUM ACUSTICUM 2011*, Aalborg, Denmark (2011), pp. 1541–1546.
[44] M. Barron, *Auditorium Acoustics and Architectural Design* (Taylor and Francis, London, 1993), pp. 1–418.
[45] J. Cordioli, "Limits of the applicability of random matrix theory to the dynamics of structures with uncertainties," Ph.D. dissertation, Universidade Federal de Santa Catarina, Brazil, 2006.