

The University of Derby
Faculty of Arts, Design and Technology

Efficient Acoustic Modelling of Large Spaces using Time Domain Methods

Analysis of Time Domain Numerical Methods
for Acoustic Modelling of Large Spaces

Simon Durbridge

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Submitted for in part-fulfilment of the requirements for the
MSc in Audio Engineering.

for Bethany

Acknowledgements

I would like to dedicate this work to anyone of remote importance.

Contents

1	Introduction	1
1.1	Context	1
1.2	Problem Definition	1
2	Loudspeaker Systems & Large Room Acoustics	3
2.1	The Acoustic Wave Equation	3
2.1.1	The Wave Equations	3
2.2	Loudspeaker Systems	6
2.2.1	Subsection Heading	6
2.3	Section Heading	7
2.3.1	Subsection Heading	8
	Appendix	9
	Problems	9

Acronyms

Use the template *acronym.tex* together with the Springer document class `SVMono` (monograph-type books) or `SVMult` (edited books) to style your list(s) of abbreviations or symbols in the Springer layout.

Lists of abbreviations, symbols and the like are easily formatted with the help of the Springer-enhanced `description` environment.

ABC	Spelled-out abbreviation and definition
BABI	Spelled-out abbreviation and definition
CABR	Spelled-out abbreviation and definition

Chapter 1

Introduction

The intro Text

1.1 Context

1.2 Problem Definition

Real time acoustic modelling could be of significant benefit to many applications; Engineers could make design changes and see results 'on the fly', and entertainment users could have more realistic experiences. These benefits should be possible for an arbitrary number of sources and receivers, in proportionally large environments with high quality results. Is it possible to further reduce computation time for simulations of large acoustic problems, to provide results in real time for the full human audio frequency range? There are two 'branches' of computation solution that should be considered: the direct solution i.e. direct outputs or audio samples from the simulation, and indirect solutions i.e. a system impulse response that may be convolved with mixed source signals in order to create an auralization of the system.//

Fig. 1.1 A visualisation of a 2D explicit FDTD simulation [?]

Chapter 2

Loudspeaker Systems & Large Room Acoustics

Abstract Acoustics is a branch of physics that aims to characterise Newton's law of motion applied to wave propagation, while obeying the physical conservation law and often focussing on propagation in an audible spectrum. This characterisation of sound propagation is intrinsically linked to many other branches of physics, as well as psychoacoustics and perception. Many aspects of acoustic modelling may be of interest when considering the design and application of loudspeaker systems. Both small and large scale simulations may allow a user to make informed decisions about the design and placement of a loudspeaker system, so that the performance of the system may be validated and optimised before application. In this chapter we will evaluate the lossless acoustic wave equation for gasses, and consider the application of the wave equation in bounded space. We will then consider some specific use cases for applying such an equation for modelling loudspeaker system performance.

2.1 The Acoustic Wave Equation

In the McGraw-Hill Electronic and Electrical Engineering Series of books, the late L. Beranek authored the Acoustics volume. This volume contains an elegant summary of the wave equation, that will be the subject of paraphrase in the following section.

2.1.1 The Wave Equations

Acoustic waves are classified as fluctuations of pressure in a given medium. In room acoustics and loudspeaker system engineering, these fluctuations are often cyclical in nature around an ambient pressure, as opposed to the jets described in aeroacoustic study. Similar to the behaviour of heat convection or fluid diffusion, these cyclical fluctuations propagate and spread through the medium of interest. As these fluctuations of pressure propagate energy is often lost, and eventually the medium will often come to a state of relative rest where the energy of the propagating waves have been almost entirely dissipated. It is possible to calculate an approximate solution the the propagation of pressure through a space, by solving a system of second

order partial differential equations that can be collectively lumped into a 'Wave Equation'. Below, we will introduce the three building blocks of the wave equation in both one dimension, and three dimensions (based on vector notation). These building blocks are Newton's Second Law of Motion, the gas law, and the laws of conservation of mass.

To consider the wave equation, we should use the analogy of a small¹ volume of gas, within a larger homogeneous medium. The faces of the volume are frictionless, and only the pressure at any face impacts on the gas inside the volume.

¹ rectilinear

One Dimension	Three Dimensions
Sound pressure p propagates across the medium like a plane wave, from one side to the other in the x direction at a rate equal to the change in space $\frac{\delta p}{\delta x}$	Sound pressure p propagates across the medium like a spherical wave, from one side to the other at a rate of grad $p = \mathbf{i} \frac{\delta p}{\delta x} + \mathbf{j} \frac{\delta p}{\delta y} + \mathbf{k} \frac{\delta p}{\delta z}$ where \mathbf{i} , \mathbf{j} and \mathbf{k} are unit vectors in the directions x , y and z .
Force acting on the volume in the positive x direction can thus be described as $-(\frac{\delta p}{\delta x} \Delta x) \Delta y \Delta z$	Force acting on the volume in the positive x direction can thus be described as $-[i(\frac{\delta p}{\delta x} \Delta x) \Delta y \Delta z] + j(\frac{\delta p}{\delta y} \Delta y) \Delta x \Delta z + k(\frac{\delta p}{\delta z} \Delta z) \Delta x \Delta y]$
A positive gradient causes acceleration in the $-x$ direction	←
Force per unit volume is given by dividing both sides of the previous equation by the volume V , $\frac{f}{V} = -\frac{\delta p}{\delta x}$	Force per unit volume is given by dividing both sides of the previous equation by the volume V , $\frac{f}{V} = -\mathbf{grad} p$
Newton's second law of motion dictates that the rate of change of momentum in the volume must balance with force per unit volume, and we can assume the mass of gas in the volume is constant.	←
The force mass balance can be described as $\frac{f}{V} = -\frac{\delta p}{\delta x} = \frac{M}{M} \frac{\delta u}{\delta t} = \rho' \frac{\delta u}{\delta t}$	The force mass balance can be described as $\frac{f}{V} = -\mathbf{grad} p = \frac{M}{M} \frac{Dq}{Dt} = \rho' \frac{Dq}{Dt}$
u is the velocity of gas in the volume, ρ' is the density of the gas, and $M = \rho' V$ is the mass of gas in the volume.	where q is the vector velocity, ρ' is the density of gas in the volume, $M = \rho' V$ is the total mass of gas in the volume. $\frac{D}{Dt}$ represents the total rate of change of velocity of a section of gas in the volume, and can be composed as $\frac{Dq}{Dt} = \frac{\delta q}{\delta t} + q_x \frac{\delta q}{\delta x} + q_y \frac{\delta q}{\delta y} + q_z \frac{\delta q}{\delta z}$ where q_x , q_y and q_z are the components of the particle velocity q in each direction. As this is a linear wave equation approximation, these velocity components have no cross terms.
If the change in density of gas in the volume is sufficiently small, the ρ' will be approximately equal to the average density ρ_0 , thus simplifying the equations above to $-\frac{\delta p}{\delta x} = \rho_0 \frac{\delta u}{\delta t}$	If the particle velocity vector is sufficiently small, the change of momentum of the gas is approximately the same as the momentum of the volume at any arbitrary point, and the density of gas within the volume ρ' will be approximately equal to the average density ρ_0 . Thus the above can be written as $-\mathbf{grad} p = \rho_0 \frac{\delta p}{\delta t}$
cell5	cell6

2.2 Loudspeaker Systems

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furtheron please use the \LaTeX automatism for all your cross-references and citations.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Use the standard `equation` environment to typeset your equations, e.g.

$$a \times b = c, \quad (2.1)$$

however, for multiline equations we recommend to use the `eqnarray` environment².

$$\begin{aligned} a \times b &= c \\ \mathbf{a} \cdot \mathbf{b} &= \mathbf{c} \end{aligned} \quad (2.2)$$

2.2.1 Subsection Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furtheron please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.2.

Please do not use quotation marks when quoting texts! Simply use the `quotation` environment – it will automatically render Springer’s preferred layout.

2.2.1.1 Subsubsection Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furtheron please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.2.1, see also Fig. ??³

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Paragraph Heading Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furtheron please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.2.

² In physics texts please activate the class option `vecphys` to depict your vectors in *boldface-italic* type - as is customary for a wide range of physical subjects.

³ If you copy text passages, figures, or tables from other works, you must obtain *permission* from the copyright holder (usually the original publisher). Please enclose the signed permission with the manuscript. The sources must be acknowledged either in the captions, as footnotes or in a separate section of the book.

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1. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
 - a. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
 - b. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
2. Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.

Subparagraph Heading In order to avoid simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.2, see also Fig. ??.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

For unnumbered list we recommend to use the `itemize` environment – it will automatically render Springer’s preferred layout.

- Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development, cf. Table 2.1.
 - Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
 - Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.
- Livelihood and survival mobility are oftentimes coutcomes of uneven socioeconomic development.

Run-in Heading Boldface Version Use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.2.

Run-in Heading Italic Version Use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.2.

2.3 Section Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furtheron please use the \LaTeX automatism for all your cross-references and citations as has already been described in Sect. 2.2.

Table 2.1 Please write your table caption here

Classes	Subclass	Length	Action Mechanism
Translation	mRNA ^a	22 (19–25)	Translation repression, mRNA cleavage
Translation	mRNA cleavage	21	mRNA cleavage
Translation	mRNA	21–22	mRNA cleavage
Translation	mRNA	24–26	Histone and DNA Modification

^a Table foot note (with superscript)

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

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- Type 1 That addresses central themes pertaining to migration, health, and disease. In Sect. 2.1, Wilson discusses the role of human migration in infectious disease distributions and patterns.
- Type 2 That addresses central themes pertaining to migration, health, and disease. In Sect. 2.2.1, Wilson discusses the role of human migration in infectious disease distributions and patterns.

2.3.1 Subsection Heading

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Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

If you want to emphasize complete paragraphs of texts we recommend to use the newly defined Springer class option `graybox` and the newly defined environment `svgraybox`. This will produce a 15 percent screened box ‘behind’ your text.

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2.3.1.1 Subsubsection Heading

Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furtheron please use the

L^AT_EX automatism for all your cross-references and citations as has already been described in Sect. 2.2.

Please note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Theorem 2.1. *Theorem text goes here.*

Definition 2.1. Definition text goes here.

Proof. Proof text goes here. \square

Paragraph Heading Instead of simply listing headings of different levels we recommend to let every heading be followed by at least a short passage of text. Furthermore please use the L^AT_EX automatism for all your cross-references and citations as has already been described in Sect. 2.2.

Note that the first line of text that follows a heading is not indented, whereas the first lines of all subsequent paragraphs are.

Theorem 2.2. *Theorem text goes here.*

Definition 2.2. Definition text goes here.

Proof. Proof text goes here. \square

Acknowledgements If you want to include acknowledgments of assistance and the like at the end of an individual chapter please use the `acknowledgement` environment – it will automatically render Springer’s preferred layout.

Appendix

When placed at the end of a chapter or contribution (as opposed to at the end of the book), the numbering of tables, figures, and equations in the appendix section continues on from that in the main text. Hence please *do not* use the `appendix` command when writing an appendix at the end of your chapter or contribution. If there is only one the appendix is designated “Appendix”, or “Appendix 1”, or “Appendix 2”, etc. if there is more than one.

$$a \times b = c \tag{2.3}$$

Problems

2.1. A given problem or Exercise is described here. The problem is described here. The problem is described here.

2.2. Problem Heading

(a) The first part of the problem is described here.

(b) The second part of the problem is described here.

Methods

Difference Time Domain Method

Finite Difference Time Domain Method

Time Domain Method

Modelling Strategies

Work

