

# Toward real-time decision making for bus service reliability

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**Abstract**—This paper addresses real-time decision making for a transit system based on a Bayesian network. It addresses issues associated with real-time control of bus operations to improve headway and minimize passenger wait time. The randomness of passenger arrivals at bus stops and external factors, such as traffic congestion and weather, in high frequency transit operations often cause highly irregular headway that can result in decreased service reliability. The approach proposed in this paper, which has the capability of handling the uncertainty of transit operations, applies holding control strategies to forestall bus unreliability and, where unreliability is evident, restore reliability. A real-life bus route operating in Wollongong, Australia together with its passenger load data are used in the simulation analysis to verify and evaluate the proposed approach.

**Index Terms**—transit modeling and simulation; Bayesian network; bus control strategies; transit service reliability

## I. INTRODUCTION

Unreliability in a bus service affects passengers because it causes passengers to wait longer. Particularly, on high frequency routes, headway regularity is important to passengers because of its impact on waiting time and overcrowding. Overcrowding is key to passengers – for their comfort – and to operations – because it slows boarding and alighting. Passenger numbers are also important in planning because it is a measure of transport network efficiency. For transit services with short headways, passengers can be assumed to randomly arrive independently of the schedule. Headway variability makes passengers feel that a service is unreliable, especially when “bunching up” of buses occurs (clustering of the buses within a short distance of one another). The transit industry has lacked a measure of service reliability that is measured in terms of its impact on customers because traditional measures do not express how much reliability impacts on passengers’ perceptions. In this paper, service reliability is measured based on passenger wait time, in-vehicle wait time and even headway [4].

In order to minimize the occurrence of unreliability, it is important to identify its possible causes in bus operations. Prevention strategies focus on reducing the variability of running times and dwell times, while corrective strategies focus on reducing the negative impacts to passengers. Passenger costs, operation costs and feasibility of implementation are used to evaluate control strategies. The most common corrective

strategies are reviewed in this section: “holding”, “expressing”, “short-turning” and “deadheading” [9].

“Holding” is the control strategy of delaying a bus at some points in the network for a set amount of time. It aims to rectify a bus-running-early event or to prevent buses from forming short headways, or “bunching up” (clustering of the buses within a short distance of one another). Holding can be schedule-based to ensure on-time performance, or headway-based to maintain even headways between consecutive buses [9].

“Expressing” involves sending a bus to a stop further downstream and skipping (not servicing) some, or all, intermediate stops. The objective of this strategy may be either to increase the headway between buses (splitting bunched up buses apart) or to close a service gap further downstream, both in an attempt to balance headways and improve service past the end of the express segment [9].

“Short-turning” involves directing a bus to end its current trip before it reaches the terminus, and service the route in the opposite direction. This strategy is employed to return a late bus to schedule, or when an extra service is needed in the opposite direction, whether due to higher passenger demand or from large gaps in service [9].

“Deadheading” involves pulling a bus from service and running it empty for a segment of the route [9].

Dessouky et al. [2], Zolfaghari et al. [11] and Lo & Chang [6] propose real-time holding strategies to minimize passenger wait time at all stops in a transport route. While Delle Site and Filippi [1] present short-turn strategies to discover the effects on service patterns and the trade-offs over operation periods between user and operator costs.

Eberlein et al. [3] formulates the real-time deadheading problem, which determines at dispatch time which vehicles to deadhead and how many stations to ignore in order to minimize overall passenger cost in the system. A distributed control approach based on multi-agent negotiation is presented. Stop and bus agents communicate with others in real-time to achieve dynamic coordination of bus dispatching at various stops. A comparison between the negotiation algorithm, on-schedule and even-headway strategies is made in [10].

Previous studies do not provide methods that have the ability to handle the uncertainty in transit operations that arises from

within the transit environment and via the randomness of passenger arrivals. This paper focuses on an approach for real-time decision-making based on Bayesian networks – which have the ability to handle uncertainty – in order to find holding control strategies that maintain bus service reliability.

The remainder of the paper is organized as follows. In Section 2 we review decision-making based Bayesian networks. The proposed model for real-time decision making is presented in Section 3. The simulation and results are reported and discussed in Section 4. Finally, we conclude the paper in Section 5.

## II. DECISION MAKING BASED BAYESIAN NETWORKS

Rational decision-making in the context of this paper depends upon “both the relative importance of various goals and the likelihood that, and degree to which, they will be achieved” [8]. Probability offers a means of summarizing the uncertainty that originates from “laziness” and “ignorance”. “Laziness” here means there is too much work in listing the complete set of antecedents and consequents needed to ensure an exception-less ruleset. The term “ignorance” splits in meaning between theoretical and practical. In theoretical terms “ignorance” here means there maybe no complete theory so the point at which a complete coverage of rules for the problem domain can never be adequately determined. In terms of practical “ignorance”, even though we know all the rules, we might be uncertain about specific circumstances because not all the necessary deterministic tests have been (or can be) run [8]. Decision-making Bayesian networks have the ability to handle these types of uncertainty.

In this paper, a conditional linear-quadratic Gaussian (CLQG) influence diagram is used for decision making. Influence diagrams for solving decision problems extend Bayesian networks with two additional types of nodes, namely *decision* nodes and *utility* nodes. A decision node defines the action alternatives considered by the decision maker. A utility node represents preferences that may depend on both random (or chance) variables, and decision variables. CLQG influence diagrams can be used to compute expected utilities for the various decision options given the observations (and decisions) [5].

A conditional linear-quadratic Gaussian (CLQG) influence diagram consists of a  $DAG = (\mathcal{V}, \mathcal{E})$  with nodes  $\mathcal{V}$  and directed links  $\mathcal{E}$ . Nodes  $\mathcal{V}$  represents the set of random variables,  $\mathcal{X}$ , which are partitioned into the set of continuous variables,  $\mathcal{X}_\Gamma$ , and the set of discrete variables,  $\mathcal{X}_\Delta$ . Continuous variables are assumed to follow a linear Gaussian distribution conditional on a subset of discrete variables while utility functions are assumed to be linear-quadratic in the continuous variables (and constant in the discrete). CLQG influence diagram is a compact representation of a joint expected utility function over continuous and discrete variables [5].

$$EU(\mathcal{X}_\Delta = i, \mathcal{X}_\Gamma) = \prod_{v \in \mathcal{V}_\Delta} P(i_v | i_{(pa(v))}) \times \prod_{w \in \mathcal{V}_\Gamma} p(y_w | X_{(pa(w))}) \times \sum_{z \in \mathcal{V}_u} u(X_{pa(z)}) \quad (1)$$

where  $P(i_v | i_{(pa(v))})$  is a conditional probability distribution for each discrete random variable  $i_v$  in subset  $\mathcal{V}_\Delta \in \mathcal{V}$ , in which  $pa(v)$  is the parent set of  $i_v$ .  $p(y_w | X_{(pa(w))})$  is a conditional linear Gaussian probability density for each continuous random variable  $y_w$  in subset  $\mathcal{V}_\Gamma \in \mathcal{V}$ , in which  $pa(w)$  is the parent set of  $y_w$ .  $u(X_{pa(z)})$  is a utility function for each node  $z$  in the subset  $\mathcal{V}_u \in \mathcal{V}$  of utility nodes, in which  $pa(z)$  is the parent set of utility node. The utility function  $EU$  encodes the preferences of the decision maker on a numerical scale.

A Gaussian distribution function can be specified by its mean  $\mu$  and variance  $\sigma^2$ . The probability density function of a Gaussian distributed variable,  $X$  is calculated as follows:

$$p(x; \mu, \sigma^2) = \mathcal{N}(\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - \mu)^2}{2\sigma^2}\right] \quad (2)$$

where  $p$  is the probability density function of  $X$  and  $x \in \mathcal{R}$ .

By making decisions we influence the probabilities of the configurations of the network. To identify the decision option with the highest expected utility, we compute the expected utility of each decision alternative. Inference in an influence diagram is to determine an optimal strategy  $\hat{\Delta} = \{\hat{\delta}_1, \dots, \hat{\delta}_n\}$  for the decision maker and then compute the maximum expected utility of adhering to  $\hat{\Delta}$ .

## III. MODEL

### A. Notation list

The following variables and parameters are used in the proposed formulations:

- $n$ : number of vehicles
- $m$ : number of bus stops
- $i$ : index of vehicles,  $i = 1, \dots, n$
- $k$ : index of stops,  $k = 1, \dots, m$
- $A_{i,k}$ : number of passengers alighting vehicle  $i$  at stop  $k$
- $B_{i,k}$ : number of passengers boarding vehicle  $i$  at stop  $k$
- $L_{i,k}$ : number of on-board passengers of vehicle  $i$  when it departs stop  $k$
- $D_{i,k}$ : dwell time for vehicle  $i$  serving the passengers boarding and alighting at stop  $k$
- $\alpha$ : average alighting time for each passenger
- $\beta$ : average boarding time for each passenger
- $\lambda_k$ : passenger arrival rate (number of persons per minute) at stop  $k$
- $\rho_k$ : passenger alighting fraction of the on-board passenger at stop  $k$
- $H_{i,k}$ : leading headway of vehicle  $i$  departing from stop  $k$
- $R_{i,k}$ : Running time of vehicle  $i$  from stop  $k - 1$  to stop  $k$ , including time spent accelerating from stop  $k - 1$  and decelerating to stop  $k$
- $SH$ : scheduled headway
- $\varepsilon$ : standard headway deviation
- $AA_{i,k}$ : actual arrival time of vehicle  $i$  at stop  $k$
- $AD_{i,k}$ : actual departure time of vehicle  $i$  at stop  $k$
- $FA_{i,k}$ : forecast/estimate arrival time of vehicle  $i$  at stop  $k$  from stop  $k - 1$

$FD_{i,k}$ : forecast/estimate departure time of vehicle  $i$  at stop  $k$  from stop  $k - 1$

$HT$ : holding time

$TT$ : vehicle travel time

$VT$ : in-vehicle wait time

$W$ : total passenger wait time

$E[W]$ : expected wait time

### B. Model Formulation

This subsection presents fundamental equations that are used in the proposed method. Figure 1 presents a time space representation of a bus's operation.

The headway  $H_{i,k}$  depends on the previous headways, running time differences and dwelling differences.

$$H_{i,k} = H_{i,k-1} + \Delta R_{i,k} + \Delta D_{i,k} \quad (3)$$

$\Delta R_{i,k}$  is the difference in running time between bus  $i$  and its predecessor  $i - 1$  when they arrive at stop  $k$  and  $\Delta D_{i,k}$  is the difference in dwell time between bus  $i$  and its predecessor  $i - 1$  when they dwell at stop  $k$ .

$$\Delta R_{i,k} = R_{i,k} - R_{i-1,k} \quad (4)$$

$$\Delta D_{i,k} = D_{i,k} - D_{i-1,k} \quad (5)$$

It is assumed that boarding and alighting do not occur simultaneously.

$$D_{i,k} = \alpha A_{i,k} + \beta B_{i,k} \quad (6)$$

The number of passengers alighting and boarding is calculated by the following equations:

$$A_{i,k} = \rho_k L_{i,k} \quad (7)$$

$$B_{i,k} = \lambda_k H_{i,k} \quad (8)$$

### C. Model Solution

A decision network represents information about the vehicle's current state, its possible actions, the state that will result from the vehicle action and the utility of that state. Figure 2 shows a decision network for the holding control strategy problem. It presents the three types of nodes used:

- Chance nodes (ovals) represent random continuous variables. There are three chance variables:  $\mathcal{X}_r = \{\text{dwell time, running time, headway adherence}\}$ . Dwell time and running time influence headway adherence. Each chance node has associated with it a distribution that is indexed by the state of the parent nodes. The transit network can express uncertainty about dwell time, running time, headway adherence, and passenger wait time.
- Decision nodes (rectangles) represent points where the decision-maker has a choice of actions. A decision variable, e.g. *Action*, with states  $\mathcal{X}_D = \{\text{no action, holding}\}$ . The choice of action influences the utility that will result.

- Utility nodes (diamonds) represent utility functions. The utility node has as parents variables of headway adherence, and action, describing the outcomes that directly affect utility. The utility node represents the expected utility associated with each action.

The algorithm for evaluating decision networks is the following:

Step 1: Set evidence variables for the current state.  
Step 2: For each possible value of the decision node;  
Step 2.1: Set the decision node to the value of evidence variables.  
Step 2.2: Calculate the posterior probabilities for the parent nodes of the utility node. In this case calculating the normal distribution of headway adherence.  
Step 2.3: Calculate the expected utility for the action: holding action and no action.  
Step 3: Return the action with the highest utility.

#### Step 1. Set the evidence variables for the current state

$H_{i,k}$  and  $H'_{i,k}$  are headway before and after applying holding action respectively. Headway depends on actual arrival time, departure time of the current bus and the actual departure time of preceding bus.

$$H_{i,k} = FD_{i,k} - AD_{i+1,k} = AA_{i,k} + D_{i,k} - AD_{i+1,k} \quad (9)$$

$$H'_{i,k} = H_{i,k} + HT = AA_{i,k} + D_{i,k} - AD_{i+1,k} + HT \quad (10)$$

#### Step 2.1. Set the decision node to the value of evidence variables

According to Abkowitz et al. (cited in [7]), the relationship between the expected wait time of passengers and headway is calculated by the following equation:

$$E[W] = \frac{E[H]}{2} + \frac{V[H]}{2E[H]} \quad (11)$$

where  $E[W]$  is the expected wait time,  $V[H]$  is the headway variation, and  $E[H]$  is the expected headway. From this equation, one can see that minimizing the headway variation will cause a reduction in passenger waiting time. The utility function  $U$  specifies the cost of passenger wait time. Utility of no action and holding action are calculated by applying (11).

$$U(\text{no action}) = \frac{H_{i,k}}{2} + \frac{|H_{i,k} - SH|}{2H_{i,k}} \quad (12)$$

$$U(\text{holding, headway adherence}) = \frac{H'_{i,k}}{2} + \frac{|H'_{i,k} - SH|}{2H'_{i,k}} \quad (13)$$

#### Step 2.2. Calculate the posterior probabilities for the parent nodes of the utility node

The current distribution of headway adherence and the distribution with applying holding action are calculated by applying (2) as follows:

$$\begin{aligned} p(H_{i,k}; SH, \sigma^2) &= \mathcal{N}(SH, \varepsilon^2) \\ &= \frac{1}{\sqrt{2\pi\varepsilon^2}} \exp\left[-\frac{(H_{i,k} - SH)^2}{2\varepsilon^2}\right] \end{aligned} \quad (14)$$

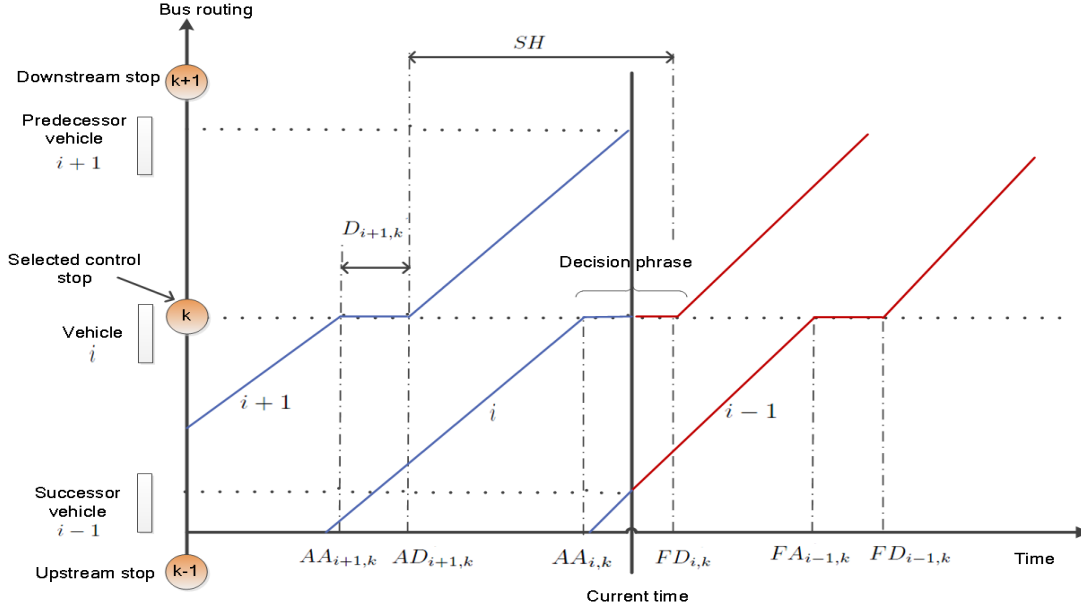


Fig. 1. Time and journey-based diagrammatic view of bus operations

$$p(H'_{i,k}; SH, \varepsilon^2) = \mathcal{N}(SH, \varepsilon^2) = \frac{1}{\sqrt{2\pi\varepsilon^2}} \exp\left[-\frac{(H'_{i,k} - SH)^2}{2\varepsilon^2}\right] \quad (15)$$

**Step 2.3. Calculate the expected utility for the action: holding action and no action**

The influence diagram is a compact representation of a joint expected utility function due to the chain rule. Expected utility of no action and holding action are calculated by applying (1), (14) and (1), (15).

$$EU(\text{no action}) = p(H_{i,k}; SH, \sigma^2) \times U(\text{no action}) \quad (16)$$

$$EU(\text{holding, headway adherence}) = p(H'_{i,k}; SH, \varepsilon^2) \times U(\text{holding, headway adherence})$$

If holding action is selected, then the holding policy  $\delta_{\text{holding}}$  is applied.

$$\delta_{\text{holding}} = \begin{cases} \text{all-stop control,} & \text{if most vehicles are bunched.} \\ \text{two-stop control,} & \text{if there is some bunching.} \\ \text{one-stop control,} & \text{if vehicles are slightly off headway.} \end{cases} \quad (17)$$

**Step 3. Return the action with the highest utility**

For simplicity, in this paper there are only two actions, *holding* and *no action* from which to choose. Their current expected utilities are  $U(\text{holding})$  and  $U(\text{noaction})$ . The information headway adherence will yield some new expected utilities  $EU$ . The idea of this decision theory is that it chooses the action that yields the highest expected utility, averaged over all the possible outcomes of the action.

#### IV. SIMULATION AND RESULTS

##### A. Simulation

A case study of bus operations on the Gwynneville-Keiraville bus route in the central region of the regional city

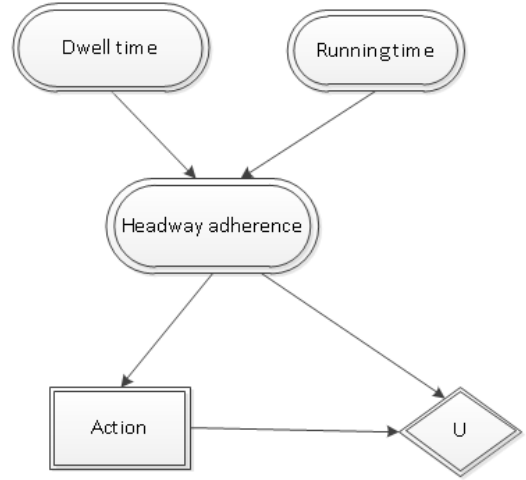


Fig. 2. Holding decision making based Bayesian network

of Wollongong, Australia whose population is about 300,000 is used to demonstrate and test the simulator. The simulator only focuses on one time period namely the peak period from 16:34 to 22:32 on weekdays.

1) *Route Characteristics:* During the evening peak, the route runs from the Eastern Entrance of the University of Wollongong and makes 11 stops on its circular route around various parts of central and inner suburban Wollongong before returning to begin its route again. The total route is about 8 kms. The scheduled headway is 15 minutes ( $SH = 15$ ). There are 3 buses running in the evening peak with start times: 16:34pm, 16:49pm, and 19:34pm. The capacity of a bus is assumed to be 70 – including passengers both seated and standing. The simulation involves generating random variables from known probability distributions of variables such as running times and passenger arrival rates. Passenger arrivals

to the system are assumed to be distributed according to a Poisson random variable with known mean. Figure 3 shows the simulation environment, which has been developed in Java.

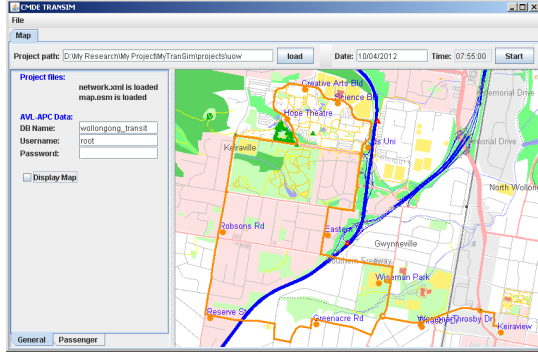


Fig. 3. The programmed Simulation environment

2) *Passenger demand*: The parameters representing passenger demand at each time point are statistically drawn from historical data from the period February 28 to September 16, 2011. Figure 4 shows passenger boarding and alighting per stop for a day. The boarding, the alighting time per passenger and passenger alighting fraction are assumed to be 5 seconds, 5 seconds and 0.3 respectively, which means  $\alpha = 5$ ,  $\beta = 5$ ,  $\rho_k = 0.3$ . Passenger arrival rate is modelled as follows:

- Calculate the distribution of passenger boarding for a week and weekdays per bus stop
- Calculate passenger boarding for 15 minute time period per day and bus stop
- Calculate passenger boarding rate per minute at each stop

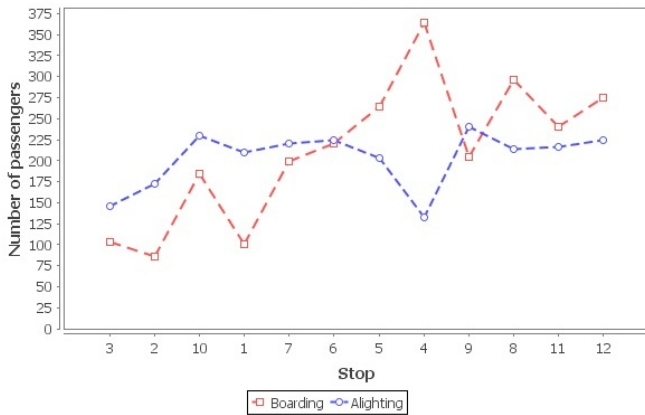


Fig. 4. Passenger boarding and alighting at each bus stop

3) *Control point*: In the holding control strategy, where to set the control point is important. In this paper, with one stop control, stop number 1 is selected as the control point. With two stop control points, stop sequence numbers 1 and 6 are the selected as the control points.

## B. Results

For short-headway services, the variability of headways is the main measure for evaluating transit reliability. An effective holding strategy improves service reliability by reducing

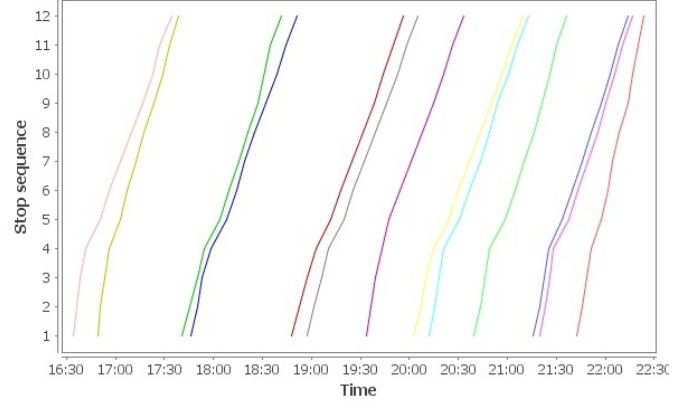


Fig. 5. Headway adherence without control

headway variability which in turn results in shorter passenger waiting times. Figures 5 and 6 present space-time headway adherence before and after applying holding control strategies for the peak hour (16:30- 23:00). There is more bunching in Figure 5 while Figure 6 shows more even headway.

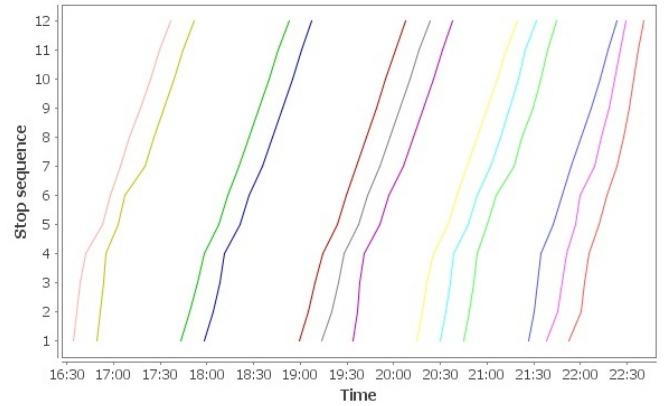


Fig. 6. Headway adherence with holding control

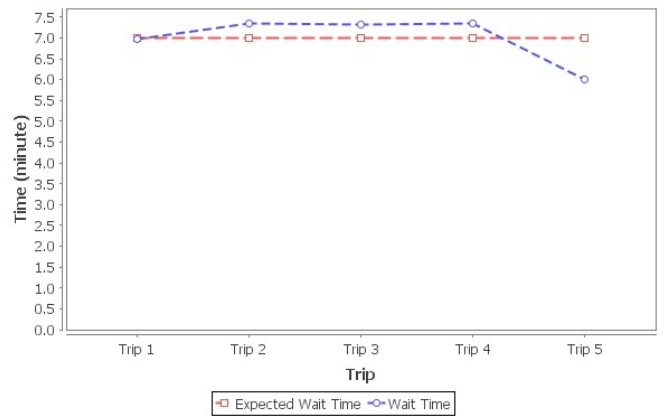


Fig. 7. Passenger wait time

Another performance measure is passenger wait time at bus stops. Figure 7 shows passenger wait time compared with expected wait time. The equations for passenger wait and expected wait time used in Figure 7 are as follows:

$$W = \frac{(\sum_{i=1}^n \sum_{k=1}^m (\frac{AD_{i,k} - AD_{i+1,k}}{2} + \frac{\varepsilon}{2(AD_{i,k} - AD_{i+1,k})}))}{K} \quad (18)$$

$$E[W] = \frac{SH}{2} + \frac{\varepsilon}{2SH} \quad (19)$$

where  $K$  is the number of runs in a trip. The results of passenger wait time analysis indicate that with holding control strategy, the level of passenger wait time is kept to expected levels.

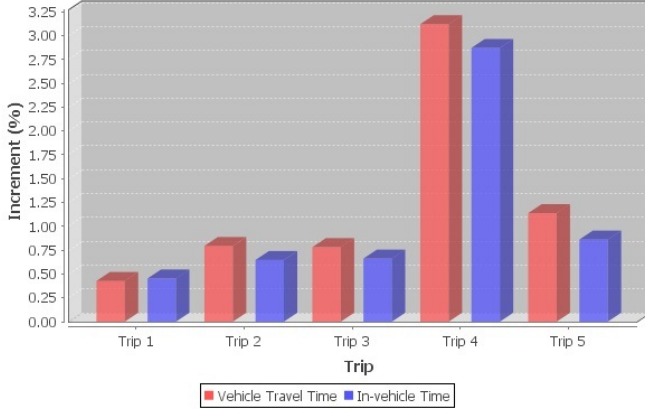


Fig. 8. Travel and in-vehicle times.

Figure 8 shows the effect of holding control strategies on travel and in-vehicle times, which are calculated by the following equations:

$$TT = (1.0 - \frac{\sum_{i=1}^n \sum_{k=1}^m (FD_{i,k} - AD_{i+1,k})}{\sum_{i=1}^n \sum_{k=1}^m (AD_{i,k} - AD_{i+1,k})}) \times 100 \quad (20)$$

$$VT = (1.0 - \frac{\sum_{i=1}^n \sum_{k=1}^m (FD_{i,k} - AD_{i+1,k}) \times L_{i,k}}{\sum_{i=1}^n \sum_{k=1}^m (AD_{i,k} - AD_{i+1,k}) \times L_{i,k}}) \times 100 \quad (21)$$

Holding strategies may cause delays to on-board passengers and longer travel times that may require higher fleet costs. However, improved regularity of headways can reduce the in-vehicle time of the passengers at the following stops. In addition, passenger waiting time at bus stops can in practice be considered more important than passenger in-vehicle time.

## V. CONCLUSION

Aiming at decreasing average passenger wait times at stops, this paper uses headway adherence and passenger wait time to measure service reliability of a high frequency bus route. In recent years, many studies focused on developing real-time control strategies. However, none of the available methods are adequately robust for dynamic decision-making under environmental uncertainty.

Our holding control-based Bayesian networks approach is notable for its ability to control real-time information and handle uncertainty. It is shown that it can cope with variables by dynamically making changes which result in greater utility. Another advantage of the approach is that it considers headway

adherence, running time, dwell time and decision-making as continuous values. The effect is that the algorithm is more flexible in making decisions compared to the existing transit control methods. A simulation-based evaluation enabled verification of the efficiency of this approach. The simulation for bus performance and level-of-service was carried out by capturing the interactions between transit operations and passenger demand. Route and stop level analysis for transit service reliability improved passenger decision-making processes and enhanced daily route service management by the transit agents.

Future work will focus on multi-criteria decision making that involves the selection of the best actions from a set of alternatives, each of which is evaluated against multiple, and often conflicting, criteria. In other words, holding, expressing, short-turning and deadheading control strategies can be used at the same time. We will also extend our approach to minimize overcrowding, which is considered among the major issues in modern public transport management.

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