Manufacturing System Analysis Experiment

EOQ, (Q, r)

1. Experiment Overview

■ Title

EOQ vs. (Q, r) model

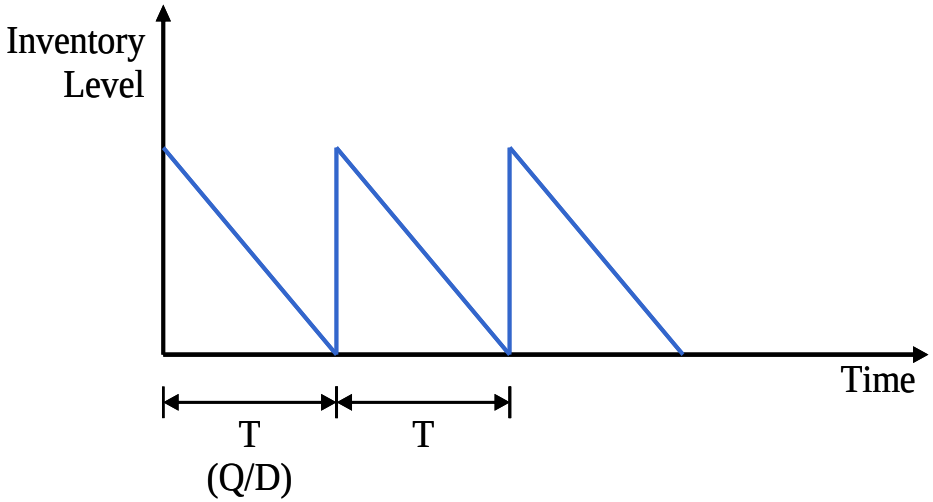
■ Objective

- Understanding the concept and principle of the EOQ and (Q, r) model.
- Considering the implications in real life inventory management issues when applying the model.

2. Theoretical Background

■ EOQ (Economic Order Quantity) Model

- Assumptions
  - Demand is constant and continuous
  - Ordering and Holding cost are constant over time
  - The whole batch quantity is delivered at the same time (zero lead time)
  - No shortages are allowed



2. Theoretical Background

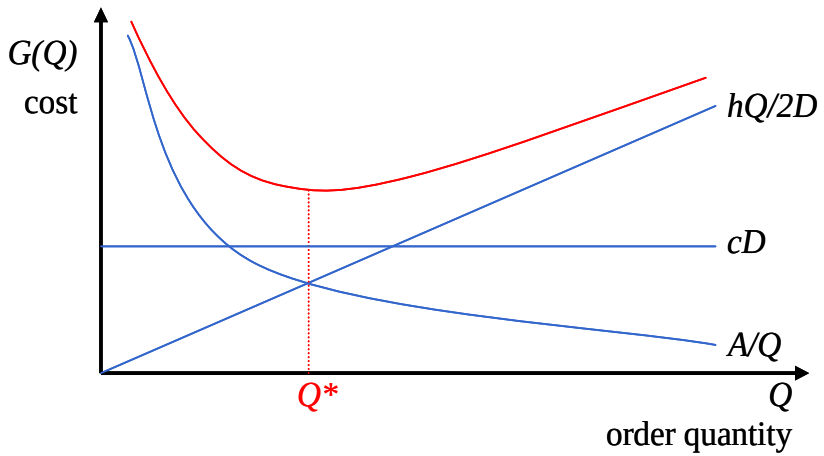
■ EOQ (Economic Order Quantity) Model (cont.)

$$Y(Q) = \frac{hQ}{2} + \frac{AD}{Q} + cD$$

$$\frac{dY(Q)}{dQ} = \frac{h}{2} - \frac{AD}{Q^2}$$

$$\frac{d^2Y(Q)}{dQ^2} = \frac{2AD}{Q^3} > 0$$

$$Q^* = \sqrt{\frac{2AD}{h}}$$



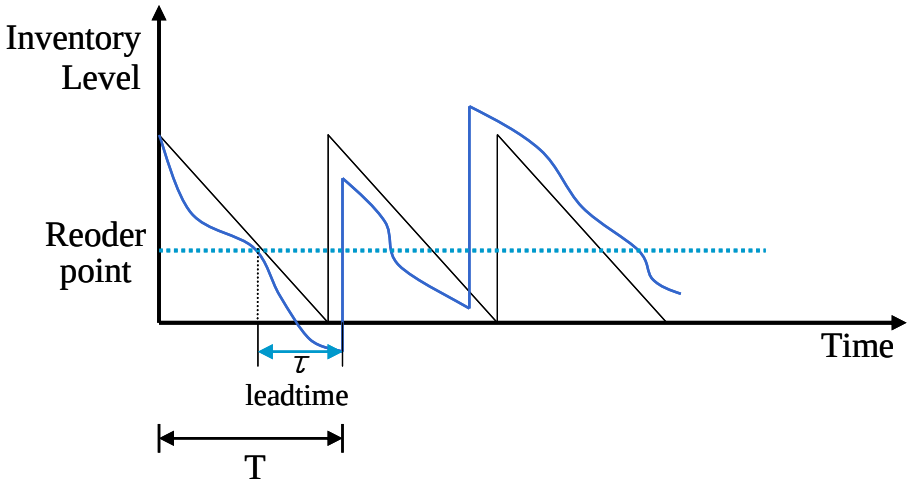
- **Total Cost**

$$Y^* = Y(Q^*) = \frac{hQ^*}{2} + \frac{AD}{Q^*} = \frac{h\sqrt{\frac{2AD}{h}}}{2} + \frac{AD}{\sqrt{\frac{2AD}{h}}} = \sqrt{2ADh}$$

2. Theoretical Background

■ (Q, r) Model

- Assumptions
  - Demand: random & stationary
  - Lead time: fixed
  - Inventory level: continuous review



2. Theoretical Background

■ (Q, r) Model (cont.)

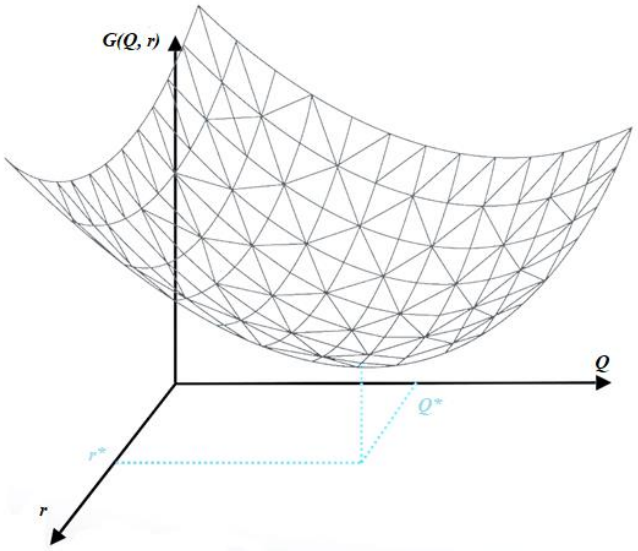
- Q

$$G(Q,r) = A\frac{D}{Q} + h\left(\frac{Q}{2} + r - \theta\right) + b\frac{D}{Q}(n(r))$$

$$\frac{dG}{dQ} = -A\frac{D}{Q^2} + \frac{h}{2} - b\frac{D}{Q^2}n(r) = 0$$

$$\frac{1}{Q^2}\{AD + bDn(r)\} = \frac{h}{2}$$

$$\therefore Q^* = \sqrt{\frac{2D\{A + bn(r)\}}{h}}$$



- r

$$G(r^*) = 1 - \frac{hQ_n}{bD}$$

3. Experiment Design

Production department at Yonsei Electronics has the basic data value as <Table 1>. They are trying to make a purchase plan based on the forecasted demand as <Table 2>. Create a minimum cost order plan using the EOQ and (Q, r) model.

T	Total planning horizon	30
D	Expected demand rate over T	360
l	Replenishment lead time	2
c	Unit cost (per unit)	50
A	Fixed ordering cost	150
h	Holding cost (per unit per week)	10
b	Backorder cost	75
	Initial inventory	30

<Table 1> Basic parameter

Week	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Demand	11	12	12	14	14	12	11	11	12	13	12	13	12	12	11
Week	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Demand	13	11	13	12	13	12	12	10	10	12	12	12	13	12	11

<Table 2> Demand data (sum = 360)

3. Experiment Design

1. Create an order plan using EOQ and (Q, r) model, and compare the results to see which model is better.

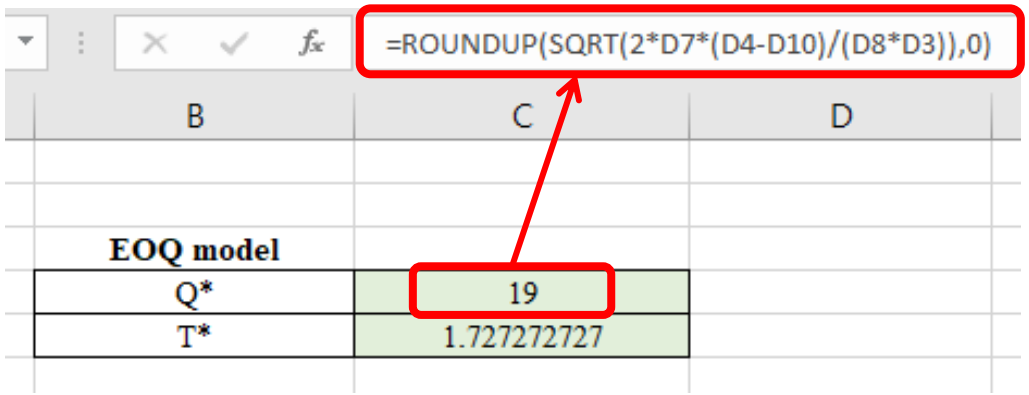
2. Conduct an experiment with different demand data, and discuss the applicability to the reality through the result analysis.

4. Experimental Procedure

■ Step 1. EOQ Model

1) Q* calculation

- $Q^* = \sqrt{\frac{2AD}{h}}$, (A = Fixed ordering cost, D = Expected demand rate over T – initial inventory, h = Holding cost * Total planning horizon T)
- Order quantity must be an integer (Roundup function)

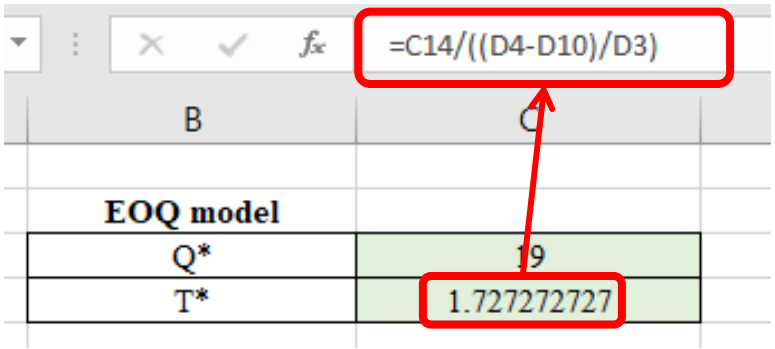


4. Experimental Procedure

- **Step 1. EOQ Model**

2) T* calculation

- $T^* = \frac{Q^*}{D}$ (Q*=Q*, D = Expected demand rate over T – initial inventory / total planning horizon)

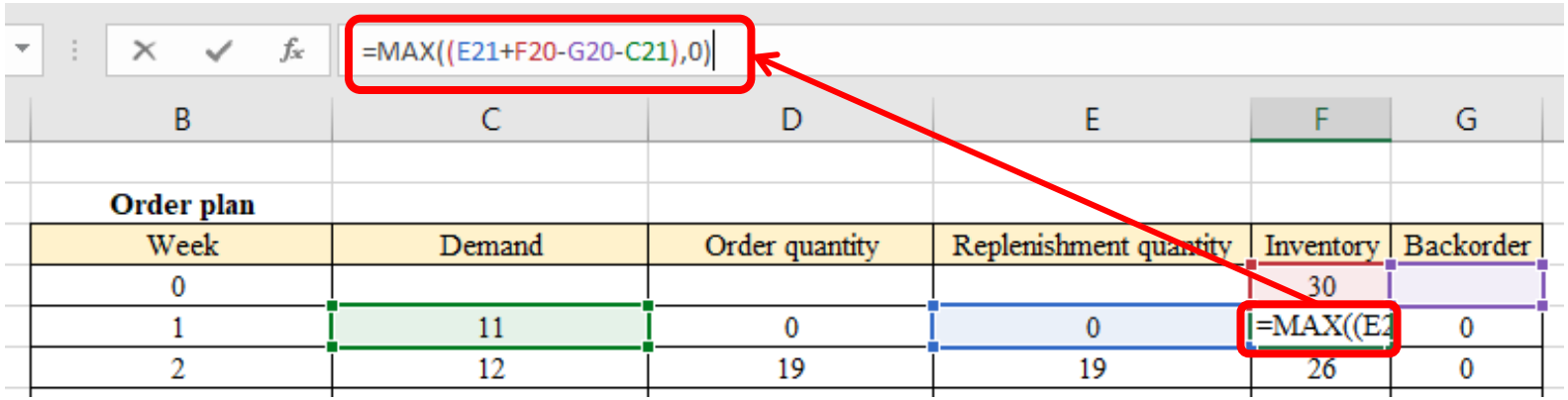


4. Experimental Procedure

■ Step 1. EOQ Model

3) Inventory calculation

- [Inventory of t] = [Inventory of t – 1] + [Replenishment quantity of t] – [Demand of t] – [Backorder of t – 1]
- Inventory must be a positive value (MAX function)

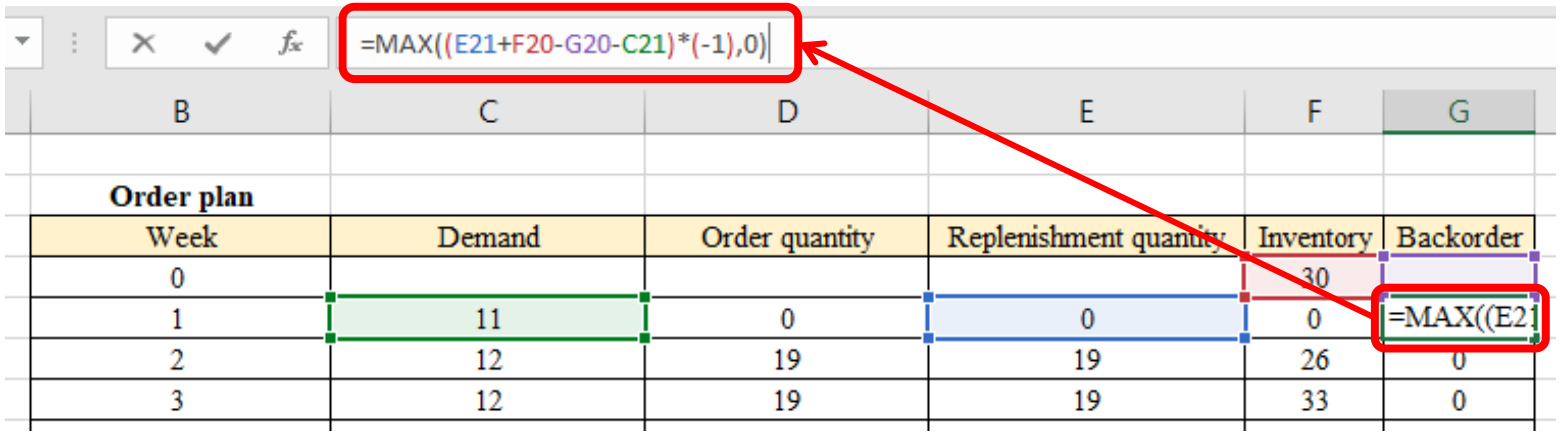


4. Experimental Procedure

■ Step 1. EOQ Model

4) Backorder calculation

- Backorder occurs when demand is not satisfied each period
- [Backorder of t] = [Inventory of t – 1] + [Replenishment quantity of t] – [Demand of t] – [Backorder of t – 1]
- Backorder must be a positive value (*(-1), MAX function)

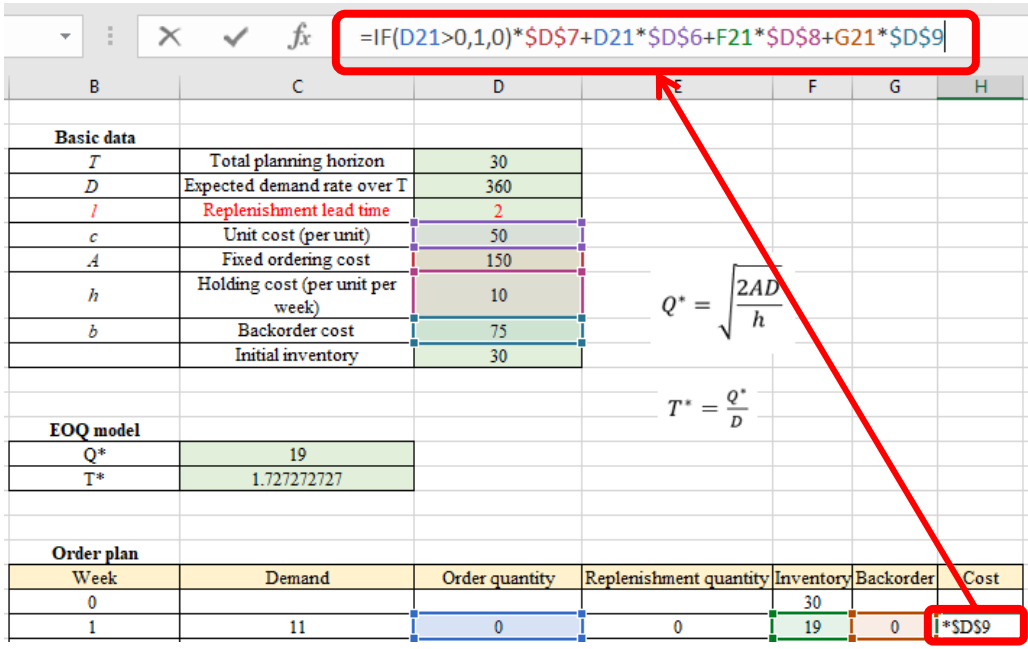


4. Experimental Procedure

■ Step 1. EOQ Model

5) Total cost calculation

- Cost of the current period = (Replenishment quantity * Expected demand rate) + (Inventory amount * Holding cost) + (Backorder amount * Backorder cost) + (Ordering cost)
- Use IF function to see if any ordering occur, (IF(replenishment is 0, if true = value is 0, if false = value is 1) * ordering cost)



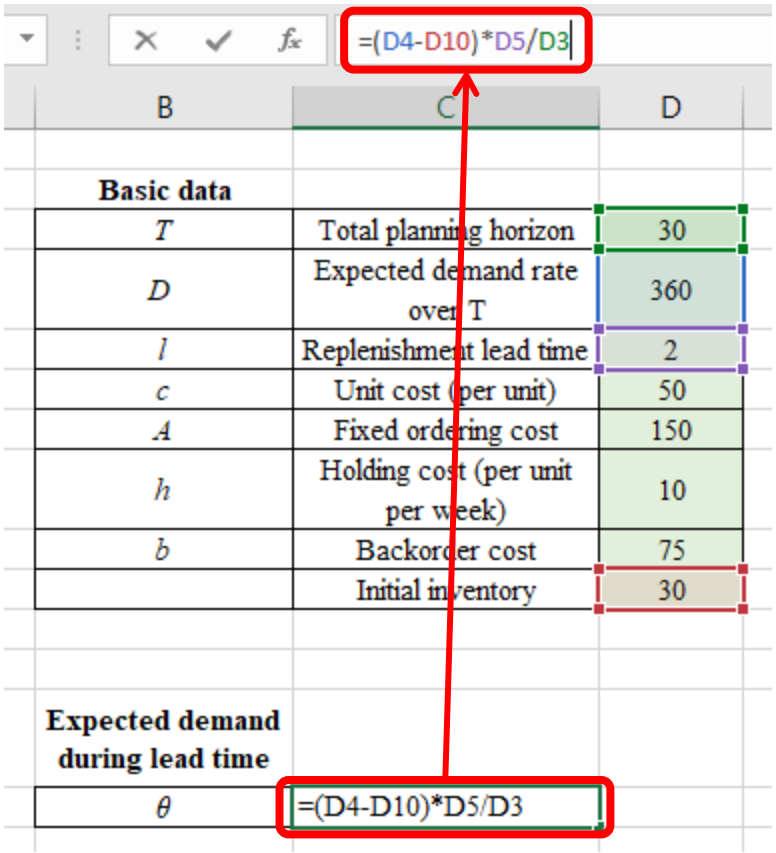
4. Experimental Procedure

- **Step 2. (Q, r) Model**

  1) θ calculation

  θ = expected demand during replenishment lead time

$$\theta = \frac{D * l}{T}$$



4. Experimental Procedure

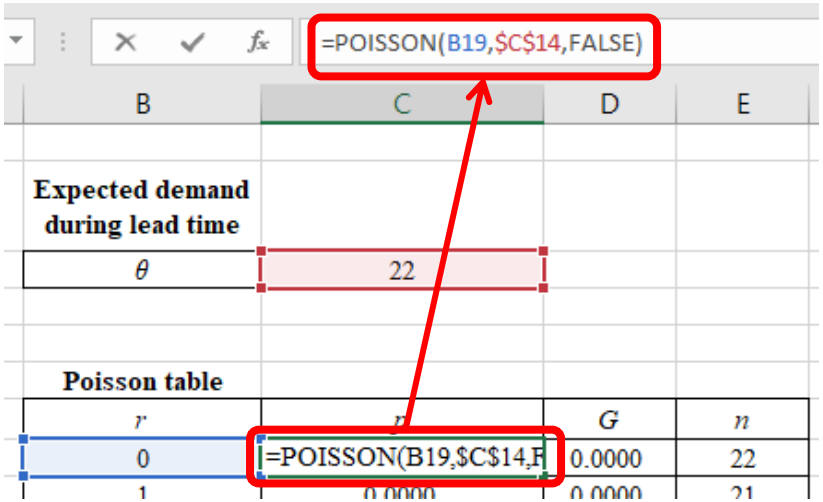
- **Step 2. (Q, r) Model**

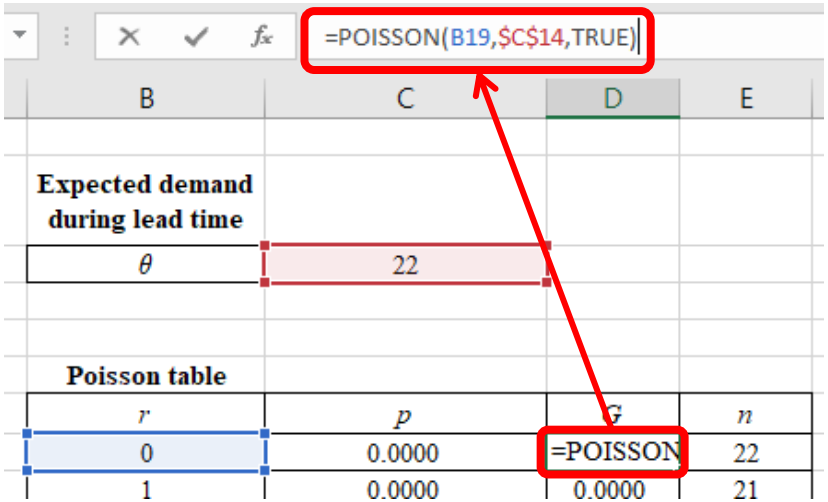
2) $p(r)$, $G(r)$calculation

$p(r)$ = density function of demand during lead time

$G(r)$ = cumulative distribution function of demand during lead time

(Round up nearest to 4 decimals because initial values are very small)





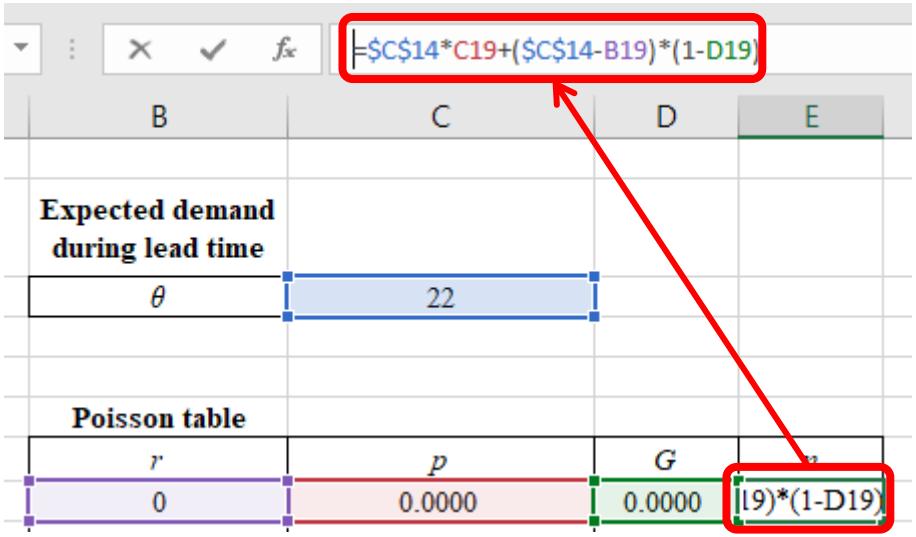
4. Experimental Procedure

- **Step 2. (Q, r) Model**

3) $n(r)$ calculation

$n(r)$ = expected quantity by which lead-time demand exceeds the base stock level

$$n(r) = \theta p(r) + (\theta - r)(1 - G(r))$$



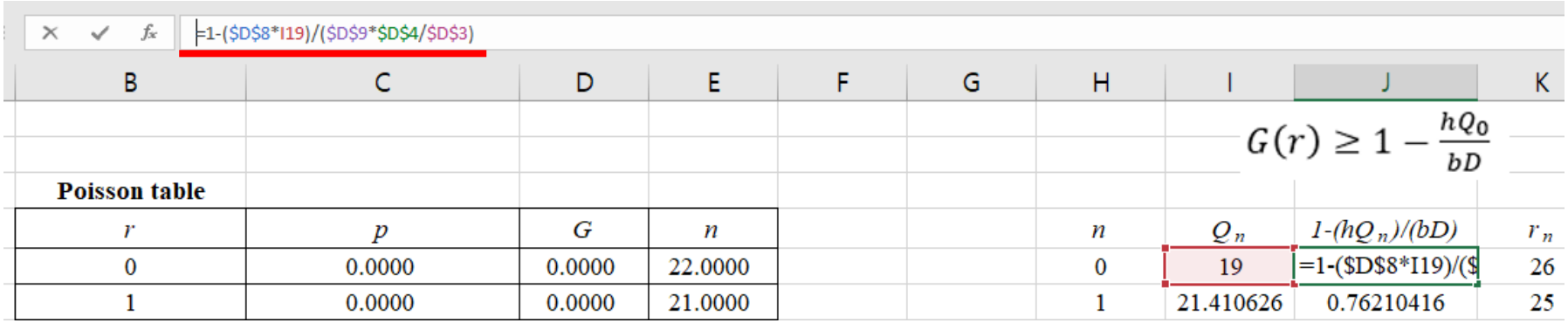
4. Experimental Procedure

- **Step 2. (Q, r) Model**

  4) Q_0, r_0 calculation

  - Use EOQ to find Q_0
  - Calculate $G(r)$ using $G(r) = 1 - \frac{hQ_n}{bD}$ and use Poisson table to find r_0

  r_0 = <u>minimum r value</u> which satisfies $G(r) \geq 1 - \frac{hQ_0}{bD}$



4. Experimental Procedure

- **Step 2. (Q, r) Model**

5) (Q, r) calculation

- Calculate Q_n & $G(r)$

$$Q_n = \sqrt{\frac{2D\left(A + bn(r_{n-1})\right)}{h}}, \qquad G(r) = 1 - \frac{hQ_n}{bD}$$

- Find r_n same way as the r_0

r_n = minimum r value which satisfies $G(r) \geq 1 - \frac{hQ_n}{bD}$

4. Experimental Procedure

- **Step 2. (Q, r) Model**

  6) (Q*, r*) calculation

  - If $|Q_n - Q_{n-1}| < 1$ and $|r_n - r_{n-1}| < 1$, $set\ Q^* = Q_n, r^* = r_n$ (according to (2))
  - Else, set $t = t + 1$ and go to (1)

4. Experimental Procedure

■ Step 2. (Q, r) Model

7) Order quantity & replenishment amount calculation

- Orders occur when (inventory at current period) + (scheduled replenishment) fell below reorder point (r^*)
- Scheduled replenishment means ordered, but not yet received quantity
- Considering (inventory at current period) + (scheduled replenishment) allow us to avoid the unnecessary extra orders
- Only orders from 2 weeks before is considered due to the lead time. Which means, replenishment quantity is received 2 weeks after the order
- Order quantity is calculated using current inventory, scheduled replenishment, safety stock, r^*, and Q^*

4. Experimental Procedure

- **Step 2. (Q, r) Model**

  8) Inventory, backorder, and cost can be calculated same as the EOQ model.

4. Experimental Procedure

■ Step 3. EOQ model result & cost calculation

1) Total order quantity, total inventory, and total backorder calculated be by summing up values from each period.
2) Total order frequency = total order quantity / Q^*
3) Total cost calculation
   - Total purchase cost = total order quantity x unit cost
   - Total ordering cost = total order frequency x ordering cost
   - Total inventory cost = total inventory x holding cost
   - Total backorder cost = total backorder x backorder cost

• Total cost = sum of each element

4. Experimental Procedure

- **Step 4. (Q, r) model result & cost calculation**
  - Same as EOQ model cost calculation
- **Step 5. Result comparison of EOQ & (Q, r)**
- **Step 6. Repeat experiments and derive the results by controlling variability when demands occur**

Q & A