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Empty Container Repositioning with Stochastic Demand/Supply and Transportation Capacity*

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We study the empty container repositioning problem for a shipping company in this paper. We develop a mathematical model with stochastic parameters. The model seeks to minimize the total costs of repositioning, leasing, and inventory holding of empty containers, subject to meeting the requirements for empty containers. We consider that the demand/supply for empty containers consists of two parts; one is deterministic and the other is stochastic. We also consider the transportation capacity of ships used for the transportation of empty containers as a stochastic factor. In order to solve the model, Chance-Constrained Programming is applied. The developed model can be an effective decision support for a shipping company.

Keywords: Chance-Constrained Programming, Empty Container, Stochastic Parameters

1. Introduction

With the increase in economic globalization, containerized transportation has become more and more popular, especially in maritime transportation. Thus, shipping companies face a container management issue, which largely concerns empty container repositioning. It involves dispatching empty containers of various types in response to requests by export customers and repositioning other containers to storage depots or ports in anticipation of future demands (Cranic *et al*. [6]). Because of trade imbalances, some ports that import more than they export will face the systematic accumulation of empty containers and become surplus ports, while other ports that export more than they import will face shortages of containers and become shortage ports. If this situation endures, a large inventory holding cost for these empty containers will occur in the surplus ports, while because of the lack of empty containers in the shortage ports, some customers might not wait, but will change to other shipping companies. Thus, to save cost and have quick response to a customer's requests, empty container repositioning will be required among these ports. In this paper, we consider the empty container repositioning problem for a shipping company with stochastic demand/supply and transportation capacity. The problem will be discussed in detail in section 2.

Recently, there have been many studies about empty container repositioning. Cranic *et al*. [6] had introduced the basic structure and main characteristics of empty container repositioning and provided a general modeling framework for this class of problems. Choong *et al*. [5] presented a computational analysis of the effect of planning horizon length on empty container management for intermodal transportation networks. They developed an integer programming problem

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that seeks to minimize the total cost related to the repositioning of empty containers. Li *et al*. [8] studied the management of empty containers in a port considering long-run average criterion, and developed (U, D) policies, which involves importing empty containers up to U when the number of empty containers in the port is less than U, or exporting down to D when the number of empty containers is more than D, doing nothing otherwise. Li *et al*. [9] extended this one port empty container management model to a multi-port model, concerned with the allocation of empty containers from supply ports to demand ports. Shintani *et al*. [11] studied the calling port sequence with empty container repositioning, trying to minimize ship related costs, port related costs, and penalty costs of cargo traffic imbalance. Cheung and Chen [4] studied the dynamic empty container allocation problem. They formulated the problem as a two-stage stochastic network and used a quasi-gradient method and a stochastic hybrid approximation procedure to solve the problem.

Though there are many studies about empty container repositioning, studies considering stochastic demand/supply and transportation capacity for empty containers are rare. However, Cheung and Chen [4] considered these factors. Liu *et al*. [10] and Wang *et al*. [12] also studied empty container repositioning model, but for inland repositioning and only dealt with single commodity, considering both deterministic demand and stochastic demand.

Based on these studies, we also considered the deterministic part in addition to the stochastic part of demand/supply and formulated a new model for maritime empty container repositioning. Furthermore, we used a different solution method, Chance-Constrained Programming, to solve the problem. Chance-Constrained Programming was first introduced by Charnes and Cooper [1] and was further developed by the same authors [2] in 1959. In 1963, Charnes and Cooper [3] introduced the deterministic equivalents of chance constraints.

This paper is organized as follows: Section 2 presents the problem definition. Section 3 shows a mathematical model. Section 4 introduces the solution method. Section 5 presents a numerical example. Section 6 concludes this study.

2. Problem Definition

In this paper, we consider the empty container repositioning problem for a shipping company with stochastic demand/supply and transportation capacity.

The demand/supply consists of two parts: one part is deterministic and the other part is stochastic. The deterministic part is due to certain long-term contracts with customers, orders placed before the decision period, or other foreseeable factors. The stochastic part is due to unpredictable factors: for example, the demand/supply from inland depots where the shipping company has no control, some temporary orders placed during the decision periods, and so on.

The vessel transportation capacity for empty containers is stochastic. This is because a ship usually carries both full containers and empty containers and it will not refuse to carry full containers due to the fierce competitive environment. The number of full containers is usually uncertain. Even if the number of full containers is known, the weight of the individual containers may vary substantially. A large number of heavy containers can reduce the effective capacity of a ship. So the capacity left (called residual capacity) for empty containers is stochastic. The distributions of the stochastic parameters can be obtained from historical data.

The problem can be illustrated as in Figure 1. At port i, there are deterministic and stochastic supplies of empty containers, as well as deterministic and stochastic demands for empty containers. If current inventory and supplies cannot satisfy the demands, the shipping company may lease containers from leasing sources or reposition empty containers from other ports. If current inventory and supplies of

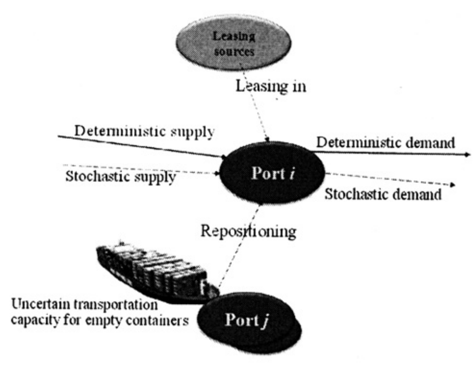

Fig 1. Illustration of the problem

containers are more than the demands, the surplus containers might just be kept in inventory or repositioned out to other ports (not shown in the figure).

The objective of this paper is to obtain decisions in terms of minimizing total costs, on the following aspects:

(1) Repositioning surplus empty containers to shortage ports in anticipation of future demand;
(2) Where, when, and how many containers the shipping company needs to lease when it experiences container shortages;
(3) How many empty containers to keep in inventory.

3. Mathematical Model

In order to formulate the mathematical model, we first define the notations in this section. The notation is as below.

Notation:

Indices:

i, j: Port index, $i,j = 1,2,...,N$.

k: Type of container, $k = 1,2,...,K$.

t: Discrete decision period (usually a week), $t = 1,2,..., T$.

Deterministic Parameters:

$C_{ij}^{k,t}$: Unit transportation cost of type k empty containers from port i to port j in period t.

$L_i^{k,t}$: Unit leasing cost of type k empty containers at port i in period t.

$H_i^{k,t}$: Unit inventory holding cost of type k empty containers at port i in period t.

$S_i^{k,t}$: Deterministic supply of type k empty containers at port i in period t.

$D_i^{k,t}$: Deterministic demand for type k empty containers at port i in period t.

M_i^k : Inventory capacity for type k empty containers at port i.

η_{ij} : Transportation time (in periods) from port i to port j.

Random Parameters:

$\beta_i^{k,t}$: Stochastic supply of type k empty containers at port i in period t.

$\gamma_i^{k,t}$: Stochastic demand for type k empty containers at port i in period t.

w_{ij}^t : Transportation capacity for empty containers from port i to port j in period t.

Decision Variables:

$X_{ij}^{k,t}$: Number of type k empty containers repositioning from port i to port j in period t.

$I_i^{k,t}$: Number of type k empty containers kept in inventory at port i at the end of period t.

$Z_i^{k,t}$: Leasing number of type k empty containers at port i in period t, which will arrive at port i for use in period t+1.

Given the previous problem statement, a mathematical model was formulated for solving the problem. The model is explained as below.

$$Min \sum_{k,t,i,j,i\neq j} C_{ij}^{k,t} X_{ij}^{k,t} + \sum_{k,t,i} L_i^{k,t} Z_i^{k,t} + \sum_{k,t,i} H_i^{k,t} I_i^{k,t} \tag{1}$$

s.t.:

$$I_i^{k,t-1} + S_i^{k,t} + \beta_i^{k,t} + \sum_{j,j\neq i} X_{ji}^{k,t-\eta_{ji}} + Z_i^{k,t-1} \geq D_i^{k,t} + \alpha_i^{k,t}, \forall i,k,t \tag{2}$$

$$I_i^{k,t} = I_i^{k,t-1} + S_i^{k,t} + \beta_i^{k,t} + \sum_{j,j\neq i} X_{ji}^{k,t-\eta_{ji}} + Z_i^{k,t-1} - \sum_{j,j\neq i} X_{ij}^{k,t} - D_i^{k,t} - \alpha_i^{k,t}, \forall i,k,t \tag{3}$$

$$I_i^{k,t} \leq M_i^k, \forall i,k,t \tag{4}$$

$$\sum_k X_{ij}^{k,t} \leq w_{ij}^t, \ \forall i,j,j\neq i,t \tag{5}$$

$$X_{ij}^{k,t},\ Z_i^{k,t},\ I_s^{k,t} \geq 0, \text{integer} \tag{6}$$

Formula (1) is the objective function of the problem. It seeks to minimize the summation of repositioning cost, leasing cost, and inventory holding cost. Constraint (2) states that all the containers that come to port i should be more than or equal to the demands at port i. Constraint (3) indicates the inventory level of type k empty containers at port i at the end of period t after the repositioning decision is made. Constraint (4) means that the inventory cannot exceed the inventory capacity. Constraint (5) indicates that the amount of repositioning for empty containers should not exceed the transportation capacity for empty containers.

4. Solution Method

Since there are stochastic parameters in the model, we apply the Chance-Constrained Programming (CCP) method to solve the problem. Chance-Constrained Programming is a kind of stochastic optimization approach. It is suitable for solving optimization problems with random variables included in constraints and sometimes in the objective function as well. The constraints are guaranteed to be satisfied with a specified probability or confidence level when the optimal solution is found. Its general form is as the following formulas:

$$Min\ \ z = \sum_{j=1}^{n} c_j x_j \tag{7}$$

s.t.:

$$\Pr\{\sum_{j=1}^{n} a_{ij}x_j \ge b_i\} \ge 1-\alpha_i, i=1,2,\dots,m; x_j \ge 0, \forall j. \tag{8}$$

where a_{ij}, bi are random parameters, c_j is a deterministic coefficient, x_j is the decision variable, and α_i is the confidence level of constraint i. (9)

The way to solve the problem is to convert the stochastic constraints into their respective deterministic equivalents with respect to the pre-specified confidence level. The following theorem can be applied (Liu *et al*. [7]):

Theorem: Assume Y is a random variable with distribution function of F. α is the confidence level. Constraint $\Pr\{h(y) \le Y\} \ge \alpha\cdot$ holds only if $h(y) \le K_\alpha$, $K_{\alpha\cdot}=\sup\{K| K_{\alpha\cdot}=F^{-1}(1-\alpha\cdot)\}$. $\Pr\{\cdot\}$means the probability of event $\{\cdot\}$ while $\sup\{\cdot\}$means the upper bound of variable in $\{\cdot\}$

Here we introduce in detail a case where b_i is normal with mean $E\{b_i\}$ and variance $\mathrm{var}\{b_i\}$.

Let $K_{\alpha i}$ be the standard normal value such that

$$F(K_{\alpha i}) = 1-\alpha_i \tag{10}$$

Constraint (8) is equivalent to:

$$\Pr\{\frac{b_i - E\{b_i\}}{\sqrt{\mathrm{var}(b_i)}} \le \frac{\sum_{j=1}^{n} a_{ij}x_j - E\{b_i\}}{\sqrt{\mathrm{var}(b_i)}}\} \ge \alpha_i \tag{11}$$

Formula (11) holds only if

$$\frac{\sum_{j=1}^{n} a_{ij}x_j - E\{b_i\}}{\sqrt{\mathrm{var}(b_i)}} \ge K_{\alpha_i} \tag{12}$$

Then the stochastic constraint is equivalent to the deterministic linear constraint:

$$\sum_{j=1}^{n} a_{ij}x_j \ge E\{b_i\} + K_{\alpha_i}\sqrt{\mathrm{var}\{b_i\}}. \tag{13}$$

Using Chance-Constrained Programming to solve the model in this paper, the specific steps are as follows:

Step 1: Convert the model into Chance-Constrained Programming:

$$\begin{aligned} Min \sum_{k,t,i,j,i\ne j} C_{ij}^{k,t} X_{ij}^{k,t} + \sum_{k,t,i} L_i^{k,t} Z_i^{k,t} \\ + \sum_{k,t,i} H_i^{k,t} I_i^{k,t} \end{aligned} \tag{14}$$

s.t.:

$$\begin{aligned} \Pr\{ I_i^{k,t-1} + S_i^{k,t} + \sum_{j,j\ne i} X_{ji}^{k,t-\eta_{ji}} + Z_i^{k,t-1} - D_i^{k,t} \\ \ge \gamma_i^{k,t} - \beta_i^{k,t}\} \ge \lambda_i^{k,t}, \forall i,k,t \end{aligned} \tag{15}$$

$$\begin{aligned} \Pr\{ I_i^{k,t-1} + S_i^{k,t} + \sum_{j,j\ne i} X_{ji}^{k,t-\eta_{ji}} + Z_i^{k,t-1} \\ - \sum_{j,j\ne i} X_{ij}^{k,t} - D_i^{k,t} - M_i^k \\ \le \gamma_i^{k,t} - \beta_i^{k,t}\} \ge \delta_i^{k,t},\ \forall i,k,t \end{aligned} \tag{16}$$

$$\Pr\{\sum_k X_{ij}^{k,t} \le w_{ij}^t\} \ge \mu_{ij}^t,\ \forall i,j,j\ne i,t \tag{17}$$

$$\Pr\{X_{ij}^{k,t},\ Z_i^{k,t},\ I_s^{k,t} \ge 0, \text{integer}\}=1 \tag{18}$$

$$\lambda_i^{k,t}, \delta_i^{k,t}, \mu_{ij}^t \in [0,1]. \tag{19}$$

$\Pr\{\cdot\}$ is an operator of probability computation. $\lambda_i^{k,t}, \delta_i^{k,t}$, μ_{ij}^t are the confidence levels defined by the user. If a larger confidence level is defined, the resulting decision will be more conservative, and the reverse is true if a smaller one is chosen. If $\lambda_i^{k,t}$ is defined, constraint (15) means that it is permissible to be violated with, at most, probability $(1-\lambda_i^{k,t})$ in any admissible choice of $X_{ij}^{k,t}$, $Z_i^{k,t}$, $I_s^{k,t}$ values. Similarly, constraints (16) and (17) are permissibly violated with, at most, probability $(1-\delta_i^{k,t})$, $(1-\mu_{ij}^{t})$ respectively in any admissible choice of $X_{ij}^{k,t}$, $Z_i^{k,t}$, $I_s^{k,t}$ values. The confidence level in formula (18) is one. This means that constraint (18) cannot be violated.

Step 2: Let $\Delta_i^{k,t} = \gamma_i^{k,t} - \beta_i^{k,t}$, Then obtain the distribution function of $\Delta_i^{k,t}$, w_{ij}^{t} from historical data, denoted as F_i^k , F_{ij}. Here we assume that the distribution of $\Delta_i^{k,t}$, w_{ij}^{t} are not changed in different periods and we already have their distributions. Thus, the index *t* could be omitted from $\Delta_i^{k,t}$, w_{ij}^{t}.

Step 3: Set the confidence levels of $\lambda_i^{k,t}$, $\delta_i^{k,t}$, μ_{ij}^{t}. They are usually prescribed at high levels, such as 0.90. Especially, $\lambda_i^{k,t}$ can be prescribed by the company's customer service level such as fill rate. $\delta_i^{k,t}$ is usually set as 1, because the storage limits cannot be violated. But if the company has some flexible storage facilities, $\delta_i^{k,t}$ can be less than 1.

Step 4: Convert the stochastic constraints into their respective deterministic equivalents. In this paper, we assume all the random variables are normally distributed. Therefore, the model can be converted as follows:

$$Min \sum_{k,t,i,j,i\neq j} C_{ij}^{k,t} X_{ij}^{k,t} + \sum_{k,t,i} L_i^{k,t} Z_i^{k,t} + \sum_{k,t,i} H_i^{k,t} I_i^{k,t} \tag{20}$$

s.t.:

$$I_i^{k,t-1} + S_i^{k,t} + \sum_{j,j\neq i} X_{ji}^{k,t-\eta_{ji}} + Z_i^{k,t-1} - D_i^{k,t} \geq E\{\gamma_i^{k,t} - \beta_i^{k,t}\} + K_{\lambda_i^{k,t}} \sqrt{Var(\gamma_i^{k,t} - \beta_i^{k,t})}, \forall i,k,t \tag{21}$$

$$I_i^{k,t-1} + S_i^{k,t} + \sum_{j,j\neq i} X_{ji}^{k,t-\eta_{ji}} + Z_i^{k,t-1} - \sum_{j,j\neq i} X_{ij}^{k,t} - D_i^{k,t} - M_i^k \leq E\{\gamma_i^{k,t} - \beta_i^{k,t}\} + K_{\delta_i^{k,t}} \sqrt{Var(\gamma_i^{k,t} - \beta_i^{k,t})},\ \forall i,k,t \tag{22}$$

$$\sum_k X_{ij}^{k,t} \leq E\{w_{ij}^t\} + K_{\mu_{ij}^t} \sqrt{Var(w_{ij}^t)},\ \forall i,j,j\neq i,t \tag{23}$$

$$X_{ij}^{k,t},\ Z_i^{k,t},\ I_s^{k,t} \geq 0, \text{integer} \tag{24}$$

After converting, the model involves the optimization of a linear objective function, subject to linear inequality constraints and the unknown variables are all required to be integers. Therefore, the model becomes an integer programming (IP) problem and can be solved by some optimization software.

5. Numerical Example

In this example, we consider 2 ports, 2 types of containers, and 3 decision periods and there is no repositioning or leasing in period 0. The input data is as shown in the following tables.

Table 1. Repositioning Costs

From\To		Port1	Port2
Port1	Container Type1	—	250
	Container Type2	—	300
Port2	Container Type1	245	—
	Container Type2	290	—

Table 2. Leasing Costs

	Port1	Port2
Container Type1	180	200
Container Type2	300	320

Table 3. Inventory Holding Costs

	Port1	Port2
Container Type1	250	260
Container Type2	280	295

Table 4. Deterministic Demands

From\To		Period1	Period2	Period3
Port1	Container Type1	90	20	60
	Container Type2	30	80	70
Port2	Container Type1	60	10	20
	Container Type2	60	90	110

Table 5. Deterministic Supplies

From\To		Period1	Period2	Period3
Port1	Container Type1	135	140	10
	Container Type2	110	130	50
Port2	Container Type1	20	60	120
	Container Type2	60	130	20

Table 6. Distributions of Δ_i^k

		Mean	Variance
i=1	k=1	40	16
	k=2	35	36
i=2	k=1	40	36
	k=2	20	25

Table 7. Transportation Time (in period)

From\To	Port1	Port2
Port1	—	1
Port2	1	—

Table 8. Distributions of Transportation Capacity

From\To	Port1	Port2
Port1	—	Normal(120,25)
Port2	Normal(120,25)	—

Table 9. Initial Inventory

From\To	Port1	Port2
Container Type1	110	78
Container Type2	130	40

Table 10. Inventory Capacity

From\To	Port1	Port2
Container Type1	150	150
Container Type2	140	120

We set al $\lambda_i^{k,t}$ as 0.95, all $\delta_i^{k,t}$ as 0.90, and all μ_{ij}^t as 0.90.

Then we use the solution method in section 4 to solve this example. The result shows that the optimal objective value is 134,015. The decisions are: in period 1, lease 167 type 1 containers from leasing sources and keep 95 type 2 containers at port 1, lease 100 type 1 containers from leasing sources at port 2; in period 2, keep 25 type 2 containers at port 1 and lease 150 type 1 containers from leasing sources at port 2.

6. Conclusions

In this paper, we have considered the empty container repositioning problem with stochastic demand/supply and transportation capacity. After a description of the problem, we proposed a mathematical model with stochastic parameters for this problem, in terms of minimization total repositioning cost, leasing cost, and inventory holding cost of empty containers. In order to solve the mathematical model, we first briefly introduced Chance-Constrained Programming and then introduced the specific solution method for this model. Afterwards, we conducted a numerical example. In this paper, we only considered the empty container flows. We could consider both full container flows and empty container flows together in a further study.

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