Warehouse capacity sharing via transshipment for an integrated two-echelon supply chain

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A R T I C L E   I N F O

A B S T R A C T

Warehouse capacity constraint has been one obstacle to achieving the channel-wide optimal decision in inventory management. We studied an integrated inventory model consisting of a single vendor and multiple buyers with warehouse capacity sharing via transshipment. We proposed an optimal transshipment policy by developing nonlinear programming models and genetic algorithms as well as obtaining Karush-Kuhn-Tucker points. This inventory policy can significantly reduce the channel-wide cost and the performance is influenced by the consideration of fixed transshipment costs. Sensitivity analyses show that parameters have different impacts on the channel-wide cost and the performances of the algorithms.

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1. Introduction

Warehouse capacity has been an important consideration that managers need to take into account to make optimal decisions in many industries. On one hand, extending warehouse capacity usually involves a long-run investment that is a strategic decision. Such investment would require a certain or predictable increment in demand for a long period. On the other hand, many supply chains have some certain peak time of demand in a short period due to short-term sales promotions or emergencies. Therefore, it would not be economical for those managers to invest for higher warehouse capacities which cannot be fully utilized in the long run. Moreover, it is usually not easy to rent warehouses for a short time, such as one month. There exist many studies on inventory system with tight warehouse capacity (refer to Wang et al., 2012; Fiestras-Janeiro et al., 2015), and delayed replenishment is considered as a solution to the single buyer case (refer to Lee and Wang, 2008; Yi and Sarker, 2013). For multi-buyer cases, we show that transshipment between warehouses of buyers could be an efficient approach to improving the supply chain performance. The following examples in China motivate our research.

Chinese on-line business companies conduct sales promotions under several periods in each year, such as the first week in October, the second week in November, and the month corresponding to spring festival, which are related to new or conventional festivals in China. Because demands during those periods are much higher than those at other time, warehouse capacities at many retailers are not sufficient. However, it is not easy for them to expand their warehouse capacities. First, building new warehouses may fail to be beneficial because each peak time may last for two or three weeks. Second, warehouse owners prefer long-term contracts (longer than one year) that make leasing warehouse capacity for a short time usually unavailable in

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China. Although some retailers may sell multiple types of products, it is still difficult for them to smooth out the utilization of the warehouses by scheduling the arrival of products because the demands are much higher than the warehouse capacities. In this case, the companies replenish products to retailers simultaneously in every cycle, and transship products between them to fully utilize warehouse capacities during the demand peaks which can reduce the ordering and replenishment costs.

The second example is from the medicine supply chains during the Severe Acute Respiratory Syndromes (SARS) disaster in China in 2003. The SARS lasted for about half a year when citizens generated high and specific demand for some Chinese traditional medicines. Because this demand was caused by emergency, supply chains could not extend warehouse capacities of retailers to satisfy the high demand. Therefore, transshipment between retailers was implemented to increase the utilization ratio of warehouse capacities of supply chains. In addition, the suppliers replenished retailers simultaneously in every cycle, to satisfy the constraints for long-distance transportation, including limited number of trucks and drivers.

In an integrated inventory system, the vendor can arbitrarily move the products from one buyer to another, which is known as transshipment between buyers. Transshipment can also be achieved under the vendor managed inventory (VMI) system, in which the vendor has the authority to conduct transshipment between buyers. The advantage of transshipment is twofold. First, under uncertain demands, transshipment between buyers can improve the customer service level, and it is popular in many industries, such as automobile and consumer goods (refer to Anupindi et al., 2001). In this case, transshipments transpire between a buyer with excess inventory and another with excess demand. To the best of our knowledge, most of existing studies on transshipment are motivated by this consideration. Second, under the transshipment policy, the vendor may improve the utilization ratio of the buyers' warehouse capacities by keeping products that exceed one buyer's warehouse capacity at other buyers' warehouses for a time. In this way, the vendor can move those products to that buyer at appropriate time points and benefit from this type of warehouse capacity sharing strategy. The above two examples reveal that this strategy has been implemented in the real world to improve the supply chain performance under some short-term selling seasons.

In this paper, we study a continuous review inventory model under integrated control and transshipment between buyers who have limited warehouse capacity. The vendor simultaneously replenishes buyer inventories by using ordinary deliveries with relatively long lead times; however, in a relatively short time, inventories can be moved from one buyer to another when inventory levels prompt buyers to reorder. Fig. 1 illustrates a supply chain model with transshipment between buyers.

In this paper, we propose a nonlinear programming model for a supply chain with warehouse capacity constraints under transshipment policies. We attempt to address the following three questions:

(a) Can warehouse capacity sharing with lateral transshipment (TRAN) lead to a lower channel-wide average cost than the integrated inventory model without transshipment (INT)?
(b) If it can, what is the reason for this advantage and how do channel parameters affect it?
(c) How do the fix transshipment costs affect the performance of the TRAN model?

Because the three research questions have been addressed rarely, we explain the contribution of our study to the existing literature by revealing the advantages of the TRAN model and the necessity for considering fixed transshipment costs during decision making. In addition, with respect to methodology, we develop an efficient algorithm to find the channel-wide optimal solution under any assignment of buyers for transshipment. For the algorithm, we propose an optimal transshipment
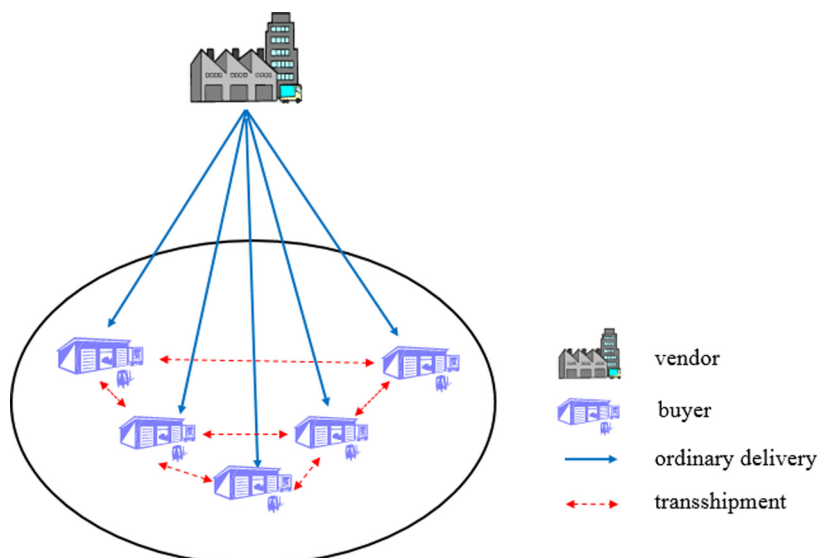

Fig. 1. Supply chain model with transshipment.

policy on the basis of which we can obtain the channel-wide optimal order quantity and the transshipment quantities between buyers by using the Karush-Kuhn-Tucker (KKT) condition.

This paper is organized as follows. We review the related literature in Section 2 and propose a TRAN model in Section 3. Section 4 describes an algorithm that can be used to obtain the optimal solution when the assignment of buyers for the transshipment is given. The GA that accounts for the fixed transshipment cost is proposed in Section 5. A numerical example is discussed in Section 6, and we provide concluding remarks in Section 7.

2. Literature review

A large body of literature is devoted to integrated inventory models with a single vendor and multiple buyers. Woo et al. (2001) studied an integrated inventory model for a single vendor and multiple buyers based on ordering cost reduction. In that model, the vendor orders raw materials and produces identical products for one cycle. Ordering costs can be reduced along with expenditure cost, and all of the buyers use the same replenishment cycle length. Zhang et al. (2007) extended the above model by considering that buyers can have different cycle lengths. Zavanella and Zanoni (2009) studied a VMI model by considering that buyers may receive the orders in different shipments. Darwish and Odah (2010) studied a VMI model with a single vendor and multiple buyers. In their model, the vendor orders and obtains the products at the beginning of each cycle, which is divided into several identical sub-cycles. All of the buyers share the same sub-cycle, and in each sub-cycle, the buyers obtain the amount of product to satisfy their demand. In that paper, they also considered a penalty cost for stock that exceeds buyers' warehouse capacity. Hariga et al. (2014) extended the above VMI model by considering a variable replenishment cycle. Sadeghi et al. (2014) studied a bi-objective inventory model for a three-stage supply chain with multiple retailers. In that model, the vendor has one replenishment frequency for all of the retailers. Mateen et al. (2015) analyzed a single vendor and multi-buyer inventory model and each buyer has an upper limit stock. Moreover, replenishment exceeding the limit generates a penalty cost in that model.

By considering delivery lead time, Prasertwattana and Chiadamrong (2004) analyzed the optimal purchasing and inventory policy in a supply chain with a single manufacturer and multiple buyers under periodic review. They developed a genetic algorithm (GA) to find the appropriate ordering and inventory levels of the manufacturer and the buyers. Hsu and Lee (2009) studied an integrated inventory system with a single manufacturer and multiple buyers in which each buyer has the same lead time, which can be reduced with a crashing cost. Taleizadeh et al. (2012) proposed a multiple products, single vendor, and multiple buyers' inventory problem. In each cycle, the vendor sends products to the buyers in n shipments, and lead time is assumed to vary linearly with respect to the lot size. Moreover, the vendor's warehouse capacity and buyers' budgets are considered in that model. Jha and Shanker (2013) studied a system wherein the buyers have the same length of sub-cycle but different lead times. In that model, the length of lead time can also be reduced with a crashing cost. All of the cited studies did not offer an account of transshipment between buyers.

Many studies have focused on the lateral transshipment between buyers. One motivation of considering transshipment is to improve the channel performance under demand uncertainty (e.g., Lee, 1987; Axsäter, 1990; Herer and Rashit, 1999; Rudi et al., 2001; Xu et al., 2003). Shao et al. (2011) studied the incentives for transshipment in a supply chain with decentralized buyers. They showed that the manufacturer prefers a high transshipment price while the buyers prefer a low one. Chen et al. (2012) studied transshipment in the newsvendor problem wherein each selling season is divided into two phases: two retailers order at the beginning of the first phase, and one can obtain transshipped products from the other retailer at the beginning of the second phase. Noham and Tzur (2014) discussed a multi-item transshipment problem by considering the fixed cost of transshipment. Dan et al. (2016) proposed a two-period model with two retailers and a single supplier. The retailers can implement preventive transshipment at the beginning of the second period to rebalance their inventories. Lee and Park (2016) analyzed the inventory and transshipment policy for a supply chain with two retailers and a single supplier who has a random supply capacity. Zhao et al. (2016) studied the transshipment problem in an online-to-offline supply chain. In that dual channel, transshipment is conducted between the retailer and the manufacturer's direct channel to pool the inventory risk. These studies focused on the transshipment in the newsboy problem. In addition, Torabi et al. (2015) considered an inventory transshipment problem of an e-retailing channel in which products can be transshipped among retailers after the uncertain demand is revealed.

There exist several studies on transshipment by considering warehouse or production capacities. Kawamura et al. (2006) analyzed the case of a Brazilian sugar supply chain which consists of multiple products, production plants, and capacitated transshipment points. A linear programming model is developed for the problem and it is solved by a software package. Drechsel and Kimms (2011) discussed the cooperative lot sizing problem with transshipment among multiple independent players. Moreover, the cost sharing among those players is considered from the game theoretical aspect. Shen et al. (2011) developed a Lagrangian relaxation approach for crude oil transportation network by considering multi-mode inventory routing problem incorporated with transshipments. Coelho et al. (2012) combined the inventory-routing problem with transshipment and considered the inventory capacities of retailers. Ahmadi et al. (2016) analyzed the location and inventory problem in a three-stage supply chain with transshipment. Refer to Paterson et al. (2011) for a detailed survey of the studies on lateral transshipments.

We can see that the continuous review inventory model combining the warehouse capacity constraints and transshipment policies has rarely been studied in previous research. Table 1 shows a comparison of our paper to other relevant papers which considered either transshipment or upper stock limit.

Table 1
Comparison of this study and some relevant studies.

Authors (Year)	Covered cont.	In lead time	Study trans.	Fixed cost of trans.	Upper stock limit	Demand uncertainty
Ahmadi et al. (2016)			✓	✓	✓	
Anupindi et al. (2001)			✓			✓
Axsäter (1990)			✓			✓
Chen et al. (2012)			✓			✓
Coelho et al. (2012)			✓		✓	
Dan et al. (2016)			✓			✓
Darwish and Odah (2010)	✓				✓	
Drechsel and Kimms (2011)			✓			✓
Hariga et al. (2014)	✓				✓	
Herer and Rashit (1999)			✓			✓
Kawamura et al. (2006)			✓		✓	
Lee (1987)			✓			✓
Lee and Park (2016)			✓			✓
Mateen et al. (2015)		✓			✓	
Noham and Tzur (2014)			✓	✓		✓
Rudi et al. (2001)			✓			✓
Shao et al. (2011)			✓			✓
Shen et al. (2011)			✓		✓	
Taleizadeh et al. (2012)	✓	✓			✓	
Torabi et al. (2015)			✓			✓
Xu et al. (2003)	✓	✓	✓			✓
Yi and Sarker (2013)		✓			✓	
Zhao et al. (2016)			✓			✓
This study	✓	✓	✓	✓	✓	

Note: *cont.* and *trans.* represent continuous review and transshipment, respectively.

3. Integrated inventory control and transshipment model

3.1. Notation

The following notation is used to develop the TRAN model:

m	number of buyers
D_i	product demand (consumption) rate of buyer i
W_i	warehouse capacity of buyer i
A_v	order cost of the vendor
h_v	unit holding cost of the vendor
A_i	fixed cost of sending products to buyer i
h_b	unit holding cost of each buyer (we assume that the buyers have the same unit holding cost)
c_b	unit holding cost of the products delivered from the vendor to each buyer during the lead time (we assume that the buyers have the same unit holding cost during the lead time)
f_i	fixed cost of transshipping products from buyer i to other buyers in one sub-cycle
L_i	lead time of buyer i
E	set of buyers whose order quantity is greater than the warehouse capacity
u	upper bound on the number of buyers in set E
TC_v	total cost per cycle for the vendor
AC_v	average cost for the vendor
TC_i	total cost per cycle for buyer i
AC_i	average cost for buyer i
AC^{nt}	average channel-wide cost without transshipment
AC^{nf}	average channel-wide cost obtained from Algorithm 2
AC^f	average channel-wide cost obtained from the GA
r_i	reorder point of buyer i without transshipment
s_{ij}	transshipment cost from buyer i to buyer j per unit product $i \neq j$
T_i	cycle time of buyer i
M	set of buyers, $M = \{1, 2, \ldots, m\}$
H_i	set of buyers receiving transshipped products from buyer $i = 1, 2, \ldots, m$
P_i	set of buyers transshipping products to buyer $i = 1, 2, \ldots, m$
Z	big number

Decision variables:

Q	order quantity of the vendor
n	number of shipments from the vendor to a buyer in one cycle
T	cycle time of the vendor
q_i	number of products sent to buyer i in each shipment without transshipment option
r'_i	reorder point of buyer i under transshipment
Δq_{ij}	amount of products delivered from buyer i to buyer j in one transshipment
x_i	1, if f_i occurs; 0, otherwise

3.2. Model description

Consider an integrated supply chain with a single vendor and multiple buyers under lateral transshipment strategy. The vendor is geographically far away from the buyers who are relatively geographically close to each other. The vendor orders and obtains the products at the beginning of each cycle, which is divided into several identical sub-cycles. The replenishment quantity of buyer i is equal to the demand during the sub-cycle (i.e., T_i), and $T_1 = T_2 =, \cdots, = T_m$. After receiving products from the supplier, the vendor sends the products to the buyers simultaneously with lead time L_i for buyer i in each sub-cycle. We consider L_i values to be exogenously specified parameters. Because we focus on the lateral transshipment under warehouse capacity constraints, we assume that each buyer has a certain product demand (consumption) rate which is a usual assumption in inventory literature (refer to Darwish and Odah, 2010; Hariga et al., 2014; Mateen et al., 2015). The order-quantity, reorder-point (Q, r) continuous review ordering policies are used at the buyers, each of whom has an inventory capacity. Preventive transshipment between buyers under centralized control occurs when the inventory levels reach the reorder points. Because the buyers are located in one region, we assume that the lead times of transshipments are negligible. We also assume that $s_{ij} = s_{ji}$ and $s_{ij} < s_{ik} + s_{kj}$, $i, j, k = 1, 2, \ldots, m$ and $i \neq j \neq k$.

Because $T_1 = T_2 =, \ldots, = T_m$, we know that $q_1/D_1 = q_2/D_2 =, \ldots, = q_m/D_m$. Without transshipment, the quantity of products sent to buyer i in each shipment satisfies

$$q_i = D_i q_1/D_1 \quad i = 2, 3, \ldots, m \tag{1}$$

The vendor's order quantity is determined by

$$Q = Dnq_1/D_1 \tag{2}$$

where $D = \sum_{i=1}^{m} D_i$.

Let R_i be the quantity of products delivered to buyer i with ordinary delivery in each sub-cycle when lateral transshipments between buyers are allowed. Then, we have

$$R_i = q_i + \sum_{j=1, j\neq i}^{m} \Delta q_{ij} - \sum_{k=1, k\neq i}^{m} \Delta q_{ki} \tag{3}$$

In addition, $r_i = D_i L_i$. Therefore, the reorder point under transshipment is $r'_i = r_i + \sum_{j=1, j\neq i}^{m} \Delta q_{ij} - \sum_{k=1, k\neq i}^{m} \Delta q_{ki}$, $i = 1, 2, \ldots, m$. Fig. 2 shows the inventory levels of a single vendor and three buyers, in which Buyer 1 transships Δq_{13} to Buyer 3 and Buyer 2 is not involved in transshipment in any sub-cycle. I_v represents the inventory level of the vendor and I_i represents the inventory level of buyer i, $i = 1, 2, 3$. Compared with the inventory level without transshipment, Buyer 1 receives more products in an ordinary delivery while Buyer 3 receives fewer products. Moreover, upon transshipment, the inventory level of Buyer 1 immediately decreases, as the red lines show.

From Eq. (1), we see that the total quantity of products sent to buyers satisfies $\sum_{i=1}^{m} q_i = Dq_1/D_1$. The length of each sub-cycle at each buyer is equal to q_1/D_1. The total cost per cycle of the vendor is

$$
\begin{aligned}
TC_v &= A_v + n\sum_{i=1}^{m} A_i + h_v\left((n-1)D\left(\frac{q_1}{D_1}\right)^2 + (n-2)D\left(\frac{q_1}{D_1}\right)^2 + \ldots + D\left(\frac{q_1}{D_1}\right)^2\right) \\
&= A_v + n\sum_{i=1}^{m} A_i + \frac{h_v n(n-1)D}{2}\left(\frac{q_1}{D_1}\right)^2
\end{aligned} \tag{4}
$$

The total cost per cycle of buyer i satisfies:

$$
\begin{aligned}
TC_i/n &= h_b\frac{q_i}{2}\frac{q_i}{D_i} + c_b L_i q_i + \sum_{j=1, j\neq i}^{m} s_{ij}\Delta q_{ij} + \left(\frac{h_b(q_i - D_i L_i)}{D_i} + c_b L_i\right)\left(\sum_{j=1, j\neq i}^{m} \Delta q_{ij} - \sum_{k=1, k\neq i}^{m} \Delta q_{ki}\right) + x_i f_i \\
&= h_b\frac{q_1 D_i}{2D_1}\frac{q_1}{D_1} + c_b L_i\frac{q_1 D_i}{D_1} + \sum_{j=1, j\neq i}^{m} s_{ij}\Delta q_{ij} + \left(h_b\left(\frac{q_1}{D_1} - L_i\right) + c_b L_i\right)\left(\sum_{j=1, j\neq i}^{m} \Delta q_{ij} - \sum_{k=1, k\neq i}^{m} \Delta q_{ki}\right) + x_i f_i
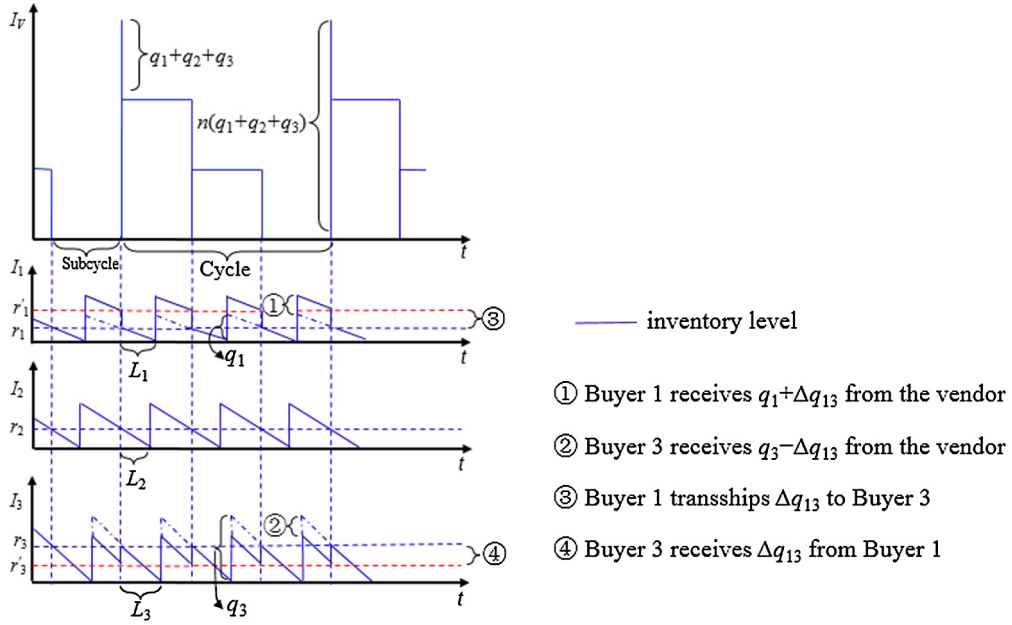\end{aligned} \tag{5}
$$

Fig. 2. Inventory levels of a single vendor and three buyers.

From Eqs. (4) and (5), we can obtain the average cost per unit time of the vendor and buyer i as follows:

$$AC_v = \frac{A_v D_1}{n q_1} + \frac{D_1}{q_1} \sum_{i=1}^{m} A_i + \frac{h_v(n-1)Dq_1}{2D_1} \tag{6}$$

$$AC_i = h_b \frac{q_1 D_i}{2D_1} + c_b D_i L_i + \frac{D_1}{q_1} \sum_{j=1,j\neq i}^{m} s_{ij} \Delta q_{ij} + \frac{D_1}{q_1} x_i f_i + \left(\frac{h_b(q_1 - D_1 L_i) + c_b D_1 L_i}{q_1} \right) \left(\sum_{j=1,j\neq i}^{m} \Delta q_{ij} - \sum_{k=1,k\neq i}^{m} \Delta q_{ki} \right) \tag{7}$$

The decision framework of the supply chain within the TRAN model (P1) is

$$\min \quad AC = AC_v + \sum_{i=1}^{m} AC_i \tag{8}$$

$$\text{subject to} \quad q_i + \sum_{j=1,j\neq i}^{m} \Delta q_{ij} - \sum_{k=1,k\neq i}^{m} \Delta q_{ki} \leqslant W_i \quad i = 1, 2, \ldots, m \tag{9}$$

$$r_i - \sum_{k=1,k\neq i}^{m} \Delta q_{ki} + \sum_{j=1,j\neq i}^{m} \Delta q_{ij} \geqslant 0 \quad i = 1, 2, \ldots, m \tag{10}$$

$$q_i > 0 \quad i = 1, 2, \ldots, m \tag{11}$$

$$\Delta q_{ij} \geqslant 0 \quad i, j = 1, 2, \ldots, m \quad i \neq j \tag{12}$$

$$\sum_{j=1,j\neq i}^{m} \Delta q_{ij} \leqslant Z x_i \quad i = 1, 2, \ldots, m \tag{13}$$

$$n > 0, \text{ integer} \tag{14}$$

$$x_i = 0, 1 \quad i = 1, 2, \ldots, m \tag{15}$$

This problem is a mixed 0–1 nonlinear programming problem with $m(m-1) + 2$ decision variables. The objective function, i.e., Eq. (8), is used to minimize the channel-wide average cost per unit time. Constraints (9) guarantee that the inventory level of each buyer cannot exceed the warehouse capacity before the transshipment. Constraints (10) illustrate that the inventory levels cannot be negative. Constraints (11) guarantee that the number of products received by each buyer during each sub-cycle is greater than the reorder point. Constraints (12) ensure that the number of products delivered in transshipment cannot be a negative value. Constraints (13) present the condition stating that if buyer i transships its inventory to others, then x_i = 1, i = 1, 2, ..., m. It means that x_i = 1 is a necessary condition for $\sum_{j=1,j\neq i}^{m} \Delta q_{ij} > 0$.

Under the TRAN model, transshipment between buyers is implemented allowing each buyer's order to arrive at multiple time points. As a consequence, the total quantity of the products that some buyers can obtain during one sub-cycle can be greater than their warehouse capacities. In addition, the utilization ratio of the warehouse capacities of some buyers is increased. Because warehouse capacity can be shared, the channel-wide optimal replenishment solution can be achieved.

4. Optimal solution algorithm

P1 is a nonlinear programming model that is difficult to solve. In this section, we discuss the optimal transshipment policy and propose an algorithm to simplify P1. Let Δq^*_{ij} be the optimal Δq_{ij} in P1, $i, j = 1, 2, \ldots, m$.

Theorem 1. *For any pair of $\{q_1, n\}$, the optimal transshipment solution of P1 should satisfy*

$$\Delta q^*_{ij} \times \Delta q^*_{jk} = 0 \quad \forall i, j, k \in M \tag{16}$$

Proof. For a pair of $\{q_1, n\}$, consider a feasible transshipment solution of P1 with $0 < \Delta q_{ij} < \Delta q_{jk}$ and $x_i = x_j = 1$. Consider a transshipment solution with $\Delta q'_{ij}$, $i, j = 1, 2, \ldots, m$, $i \neq j$. Let $\Delta q'_{ij} = 0$, $\Delta q'_{ik} = \Delta q_{ik} + \Delta q_{ij}$, $\Delta q'_{jk} = \Delta q_{jk} - \Delta q_{ij}$, and the transshipment quantities between other buyers remain as the original solution (see Fig. 3).

In this case, only AC_i, AC_j, and AC_k are influenced and the difference between the average costs under these two transshipment solutions is $D_1(s_{ij} + s_{jk} - s_{ik})\Delta q_{ij}/q_1$. Under $\Delta q'_{ij}$, $\Delta q'_{ik}$, and $\Delta q'_{jk}$, the inventory levels of each buyer when the products from the vendor arrive and when the transshipments occur are, respectively, the same as the levels under Δq_{ij}, Δq_{ik}, and Δq_{jk}. These conditions mean that the new solution satisfies Constraints (9)–(15) and is feasible for P1. Because $s_{ij} + s_{jk} - s_{ik} > 0$, the average cost of the supply chain per unit time under $\Delta q'_{ij}$, $\Delta q'_{ik}$, and $\Delta q'_{jk}$ is lower than the cost under Δq_{ij}, Δq_{ik}, and Δq_{jk}. Using the same approach, we can obtain the same results in the case in which $0 < \Delta q_{kj} < \Delta q_{ij}$. Consequently, the optimal transshipment solution should satisfy Eq. (16). □

Theorem 1 illustrates that, there exists an optimal transshipment solution in which the vendor cannot simultaneously transship products to one buyer and move the recipient buyer's inventory to other buyers. Therefore, under the optimal transshipment solution, H_i and P_i should not be nonempty at the same time. Theorem 1 also indicates that $\Delta q^*_{ij} \times \Delta q^*_{ji} = 0$, $i, j = 1, 2, \ldots, m$, and $i \neq j$. If we assume that $s_{ij} \leq s_{ik} + s_{kj}$, $i, j, k = 1, 2, \ldots, m$ and $i \neq j \neq k$, then three buyers may be located on the same line. In these cases, it is possible to obtain optimal solutions that satisfies $\Delta q^*_{ij} \times \Delta q^*_{jk} \neq 0$ and Theorem 1 only captures a specific optimal solution.

Let $\Delta AC_{ij}(\Delta q_{ij}|q_1)$ be the part of $AC_i + AC_j$, which is influenced by Δq_{ij}. From Eq. (7) and Theorem 1, we obtain

$$\Delta AC_{ij}(\Delta q^*_{ij}|q_1) = \frac{D_1(s_{ij} - h_b L_i + c_b L_i + h_b L_j - c_b L_j)}{q_1}\Delta q^*_{ij} \tag{17}$$

Theorem 2. *For any pair of $\{q_1, n\}$, in the optimal transshipment solution of P1, $\Delta q^*_{ij} = 0$ when $0 \leq q_i \leq W_i$ and $0 \leq q_j \leq W_j$.*

Proof. For a pair of $\{q_1, n\}$, without loss of generality, suppose an original transshipment solution in which $\Delta q_{12} > 0$, $\Delta AC_{12}(\Delta q_{12}|q_1) > 0$, $x_1 = 1$, $0 \leq q_1 \leq W_1$, and $0 \leq q_2 \leq W_2$. From Theorem 1, we can infer that $\Delta q_{2i} = 0$ and $\Delta q_{j1} = 0$, for all $i, j = 3, 4, \ldots, m$. The relevant constraints of P1 can be written as

$$q_1 + \sum_{i \in H_1, i \neq 2} \Delta q_{1i} + \Delta q_{12} \leqslant W_1 \tag{18}$$

$$q_2 - \sum_{j \in P_2, j \neq 1} \Delta q_{j2} - \Delta q_{12} \leqslant W_2 \tag{19}$$

$$r_1 + \sum_{i \in H_1, i \neq 2} \Delta q_{1i} + \Delta q_{12} > 0 \tag{20}$$

$$r_2 - \sum_{j \in P_2, j \neq 1} \Delta q_{j2} - \Delta q_{12} > 0 \tag{21}$$

$$\sum_{j=2}^{m} \Delta q_{1j} \leqslant Zx_1 \tag{22}$$

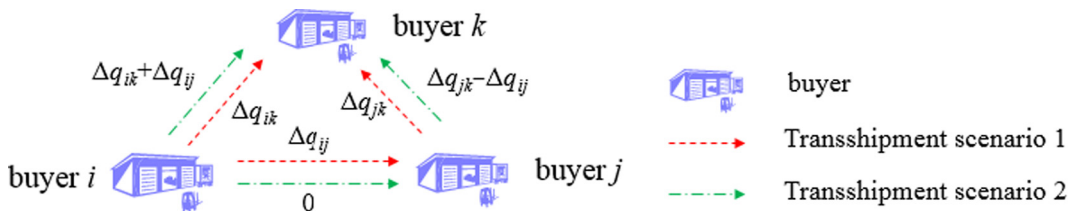$$\Delta q_{j2}, \Delta q_{1i} > 0 \quad \forall i \in H_1, j \in P_2 \tag{23}$$

Fig. 3. The transshipment quantities among buyers i, j, and k.

Consider a transshipment solution with $\Delta q'_{ij}$, $i, j = 1, 2, \ldots, m$, $i \neq j$. Let $\Delta q'_{12} = 0$ and other transshipment quantities remain as the original solution. Because $0 \leq q_1 \leq W_1$ and $0 \leq q_2 \leq W_2$, Constraints (18)–(23) can be transformed to Constraints (24)–(29) under the solution with $\Delta q'_{12} = 0$.

$$q_1 + \sum_{i \in H_1, i \neq 2} \Delta q_{1i} \leqslant W_1 \tag{24}$$

$$q_2 - \sum_{j \in P_2, j \neq 1} \Delta q_{j2} \leqslant W_2 \tag{25}$$

$$r_1 + \sum_{i \in H_1, i \neq 2} \Delta q_{1i} > 0 \tag{26}$$

$$r_2 - \sum_{j \in P_2, j \neq 1} \Delta q_{j2} > 0 \tag{27}$$

$$\sum_{j=3}^{m} \Delta q_{1j} \leqslant Z x_1 \tag{28}$$

$$\Delta q_{j2}, \Delta q_{1i} > 0 \quad \forall i \in H_1, i \neq 2, j \in P_2, j \neq 1 \tag{29}$$

Therefore, by setting $\Delta q'_{12} = 0$, we obtain a new solution feasible for P1 from the original solution. The difference between the average costs under these two transshipment solutions is determined by

$$\sum_{i=1}^{m} AC_i - \sum_{i=1}^{m} AC'_i = \Delta AC_{12}(\Delta q_{12}|q_1) \tag{30}$$

Because $\Delta AC_{12}(\Delta q_{12}|q_1) > 0$, the solution with $\Delta q'_{12} = 0$ leads to a lower average cost. Therefore, we can always find a feasible transshipment solution with $\Delta q'_{12} = 0$, which leads to a lower average cost than $\Delta q_{12} > 0$. □

From Theorem 2 we can infer that inventories should not be transshipped from buyer i to buyer j when q_i and q_j are no greater than W_i and W_j, respectively, for all $i, j = 1, 2, \ldots, m$, and $i \neq j$.

To satisfy the warehouse capacity constraint for buyer k with $q_k > W_k$, we have $\sum_{i \in M} \Delta q_{ik} > 0$ and $\sum_{j \in M} \Delta q_{kj} = 0$. It means that if $q_k > W_k$, buyer k must obtain transshipped inventory from others. Based on Theorems 1 and 2, we propose an approach to obtain the optimal transshipment solution. For a pair of $\{q_1, n\}$, let E denote the set of all buyers with $q_k > W_k$. Without loss of generality, we assume that

$$\frac{W_1}{D_1} \leqslant \frac{W_2}{D_2} \leqslant \ldots \leqslant \frac{W_m}{D_m} \tag{31}$$

If $q_{i+1} > W_{i+1}$, then $q_i > W_i$ because $q_i/D_i = q_{i+1}/D_{i+1}$. Obviously, q_m should not be greater than W_m, and Buyers 1 to $i + 1$ need to receive products transshipped from other buyers when $q_{i+1} > W_{i+1}$, $i = 1, 2, \ldots, m - 2$. If $x_2 = x_3 = \ldots = x_m = 0$, then Buyer 1 cannot obtain any transshipped inventory from other buyers and q_1 should be no greater than W_1. Moreover, we have $q_i \leq W_i$, for all $i = 1, 2, \ldots, m$. In this case, E is empty and there is no transshipment.

Lemma 1. *Under $\{x_1, x_2, \ldots, x_m\}$ with $x_2 + x_3 + \ldots + x_m \neq 0$, there exists an upper bound, u, on the number of buyers in E and u is equal to the largest k that satisfies*

$$\frac{W_k}{D_k} \leqslant \min_{i=1,2,\ldots,k} \left(\frac{W_i + r_i}{D_i} \right) \tag{32}$$

$$\sum_{i=1}^{k} \frac{W_k D_i}{D_k} + \sum_{i=k+1}^{m} \frac{x_i W_k D_i}{D_k} \leqslant \sum_{i=1}^{k} W_i + \sum_{i=k+1}^{m} x_i W_i \tag{33}$$

Moreover, the minimum value of u is 1.

Proof. From Constraints (9) and (10), we obtain $q_i \leq W_i + r_i$, $i = 1, 2, \ldots, m$. From Inequality (33), suppose that buyer $u + 1$ has

$$\frac{W_{u+1}}{D_{u+1}} > \min_{i=1,2,\ldots,u+1} \left(\frac{W_i + r_i}{D_i} \right) = \frac{W_j + r_j}{D_j} \tag{34}$$

If buyer $u + 1$ is in E, then we have $q_{u+1} \geq W_{u+1}$. Moreover, we can obtain $q_j/D_j \geq W_{u+1}/D_{u+1}$ because $q_j = D_j q_{u+1}/D_{u+1}$. Considering Inequality (34), we can obtain $q_j > W_j + r_j$, which does not satisfy $q_j \leq W_j + r_j$. In this case, there is no feasible solution.

When the number of buyers in E is equal to u, the minimum q_1 is $D_1 W_u/D_u$. The minimum total order quantity of buyers in E and other buyers (e.g., buyer i) with $x_i = 1$ can be obtained as $\sum_{i=1}^{u} \frac{W_u D_i}{D_u} + \sum_{i=u+1}^{m} \frac{x_i W_u D_i}{D_u}$. Inequality (33) guarantees that this minimum value cannot exceed the total warehouse capacity of those buyers. For any buyer in E, in which the number of buyers is 0, 1, 2, ..., or u, a feasible solution that satisfies Constraints (9)–(15) can be found if Inequalities (32) and (34) are maintained. Suppose that buyer $u + 1$ has $\sum_{i=1}^{u+1} \frac{W_{u+1} D_i}{D_{u+1}} + \sum_{i=u+2}^{m} \frac{x_i W_{u+1} D_i}{D_{u+1}} > \sum_{i=1}^{u+1} W_i + \sum_{i=u+2}^{m} x_i W_i$.

If buyer $u + 1$ is in E, then the minimum total order quantity of buyers in E and other buyers (e.g., buyer i) with $x_i = 1$ exceeds the total warehouse capacity of those buyers, which cannot lead to any feasible solution. Therefore, there exists an upper bound equal to the largest k that satisfies Inequalities (32) and (33).

Because $W_1/D_1 < (W_1 + r_1)/D_1$ and $x_2 + x_3 + \ldots + x_m \neq 0$, the minimum value of u is 1. $\square$

If the number of buyers in E is no greater than u, we define E as feasible; therefore, all of feasible Es are $\varnothing$, $\{1\}$, $\{1, 2\}$, ..., $\{1, 2, \ldots, u\}$. It is easy to show that the number of feasible Es does not exceed m; i.e., $u \leq m - 1$. Buyers in a feasible E obtain transshipped products that do not exceed their warehouse capacities. From Theorem 2, we can see that the value of u is influenced by $\{x_1, x_2, \ldots, x_m\}$. Theorem 3 explains the range of q_1 under $\{x_1, x_2, \ldots, x_m\}$ and a feasible E.

Theorem 3. *For $\{x_1, x_2, \ldots, x_m\}$ and a feasible E*:

(i) *If $E = \{1, 2, \ldots, u\}$, q_1 should satisfy*

$$\frac{W_u D_1}{D_u} < q_1 \leqslant \min\left(\min_{i=1,2,\ldots,u}\left(\frac{W_i + r_i}{D_i}\right)D_1, \frac{D_1(\sum_{i=1}^{u} W_i + \sum_{i=u+1}^{m} x_i W_i)}{\sum_{i=1}^{u} D_i + \sum_{i=u+1}^{m} x_i D_i}, \frac{W_{u+1}D_1}{D_{u+1}} \right) \tag{35}$$

(ii) *If E is $\varnothing$, q_1 should satisfy*

$$0 < q_1 \leqslant W_1 \tag{36}$$

(iii) *If $E = \{1, 2, \ldots, j\}$ with $j < u$, q_1 should satisfy*

$$\frac{W_j D_1}{D_j} < q_1 \leqslant \frac{W_{j+1}D_1}{D_{j+1}} \tag{37}$$

Proof.

(i) $E = \{1, 2, \ldots, u\}$. Because $q_u > W_u$ and $q_u = D_u q_1/D_1$, we get $q_1 > W_u D_1/D_u$. To satisfy the total inventory capacity constraint of the buyers involved in transshipment, we obtain

$$\sum_{i=1}^{u} q_i + \sum_{i=u+1}^{m} x_i q_i \leqslant \sum_{i=1}^{u} W_i + \sum_{i=u+1}^{m} x_i W_i \tag{38}$$

Then, we can infer that

$$q_1 \sum_{i=1}^{u} \frac{D_i}{D_1} + q_1 sum_{i=u+1}^{m} x_i \frac{D_i}{D_1} \leqslant \sum_{i=1}^{u} W_i + \sum_{i=u+1}^{m} x_i W_i \tag{39}$$

Because u is the upper bound, $q_{u+1} \leq W_{u+1}$. We obtain $q_1 D_{u+1}/D_1 \leq W_{u+1}$. From the proof of Lemma 1, we get $q_i \leq W_i + r_i$, $i = 1, 2, \ldots, u$. Therefore, $q_1 \leq D_1 \min_{i=1,2,\ldots,u}[(W_i + r_i)/D_i]$. Then, we can show that q_1 should satisfy Inequality (35).

(ii) E is $\varnothing$. In this case, $q_i \leq W_i$, $i = 1, 2, \ldots, m$. Therefore, $q_1 \leq D_1 \min_{i=1,2,\ldots,m}(W_i/D_i) = W_1$.

(iii) $E = \{1, 2, \ldots, j\}$ with $j < u$. Because $q_j > W_j$ and $q_j = D_j q_1/D_1$, we get $q_1 > W_j D_1/D_j$. Because $q_{j+1} \leq W_{j+1}$ and $q_{j+1} = D_{j+1} q_1/D_1$, we obtain $q_1 \leq W_{j+1}D_1/D_{j+1}$. $\square$

Theorem 3 shows that upper and lower bounds on q_1 characterize a feasible E. Let $g_{ij}(q_1)$ represent the unit cost of transshipment that is equal to $(s_{ij} - h_b L_i + c_b L_i + h_b L_j - c_b L_j)D_1/q_1$.

Theorem 4. *To determine the optimal transshipment solution for buyers not in E, the vendor should not transship products from buyer i to buyer j if buyer k with $g_{kj}(q_1) < g_{ij}(q_1)$ has unused warehouse capacity.*

Proof. Without loss of generality, suppose that Buyer 1 is in E, Buyers 2 and 3 are not in E, and $g_{21}(q_1) < g_{31}(q_1)$. Consider a transshipment solution in which $\Delta q_{21} > 0$, $\Delta q_{31} > 0$, and $W_2 - q_2 - \Delta q_{21} > 0$. Let $\Delta q'_{21} = \Delta q_{21} + \min(W_2 - q_2 - \Delta q_{21}, \Delta q_{31})$ and $\Delta q'_{31} = \Delta q_{31} - \min(W_2 - q_2 - \Delta q_{21}, \Delta q_{31})$. It follows that

$$\sum_{i=1}^{m} AC_i - \sum_{i=1}^{m} AC_i' = [g_{31}(q_1) - g_{21}(q_1)] \min(W_2 - q_2 - \Delta q_{21}, \Delta q_{31}) \qquad (40)$$

Because $g_{21}(q_1) < g_{31}(q_1)$, we can infer that the average cost under $\{\Delta q'_{21}, \Delta q'_{31}\}$ is less than the cost under $\{\Delta q_{21}, \Delta q_{31}\}$. It can be shown that Constraints (18)–(22) can be satisfied under $\{\Delta q'_{21}, \Delta q'_{31}\}$. □

For a feasible E, we can simplify P1 with Theorems 1 through 4. Because the numbers of feasible E are limited, we can obtain the optimal solution of P1 by testing all cases of a feasible E. Each E tells which buyers need to receive products from transshipment. However, which buyers transship products to those buyers in the E is still unknown. Based on the above theorems, we develop an algorithm to obtain the optimal solution of P1 for $\{x_1, x_2, \ldots, x_m\}$. Let W_i^{AL} and r_i^{AL} be artificial variables in Algorithm 1, $i = 1, 2, \ldots, m$. Let TO be the list of buyers who is able to transship products to others and let TI be the list of buyers who need to receive products via transshipments. Denote the list containing all pairs (i, j) with $i \in TO$ and $j \in TI$ as A, and denote the list containing all ranges of q_1 as B.

Algorithm 1 (*The optimal solution can be implemented as follows:*).

Step 1. Calculate the cases of feasible E based on Constraints (32)–(34) and set $\Delta q_{ij} = 0$ for all $i, j = 1, 2, \ldots, m$. Mark E that are feasible so they remain to be untested.
Step 2. Make Lists TO, TI, A, and B empty. For the untested E containing the smallest number of buyers, assign buyers (e.g., buyer i) with $x_i = 1$ not in E to List TO and assign buyers in the same untested E to List TI.
Step 3. Put all of the pairs (i, j) with $i \in TO$ and $j \in TI$ in List A. Sort the pairs of buyers in List A in ascending order according to $g_{ij}(q_1)$.
Step 4. Obtain the range of q_1 based on Theorem 3 and put the resulting range in List B.
Step 5. Set $W_i^{AL} = W_i$ and $r_i^{AL} = r_i$, $i = 1, 2, \ldots, m$.
Step 6. Consider the next pair of buyers in List A. Suppose that the buyer from List TO in this pair is buyer f, and the other buyer in the same pair is buyer e.
Step 6.1. Consider the next range of q_1 in List B. If $W_f^{AL} = -1$ or $W_e^{AL} = -1$, go to Step 6.3. Otherwise, determine the value of Δq_{fe}:

$$\Delta q_{fe} = \min\left(W_f^{AL} - \frac{D_f}{D_1} q_1, \frac{D_e}{D_1} q_1 - W_e^{AL} \right) \qquad (41)$$

Step 6.2. If $\Delta q_{fe} = W_f^{AL} - D_f q_1 / D_1$ in the current range of q_1, update W_e^{AL} by adding Δq_{fe} and set $W_f^{AL} = -1$ for this range. If $\Delta q_{fe} = D_e q_1 / D_1 - W_e^{AL}$ in the current range of q_1, update W_f^{AL} by subtracting Δq_{fe} and set $W_e^{AL} = -1$ for this range. Otherwise, divide this range into two ranges by inserting q_1 that satisfies

$$W_f^{AL} - \frac{D_f}{D_1} q_1 = \frac{D_e}{D_1} q_1 - W_e^{AL}, \qquad (42)$$

and put these two ranges as untested in List B.

Step 6.3. If all of ranges in List B have been tested, go to Step 6.4. Otherwise, go to Step 6.1.
Step 6.4. If all of the pairs in List A have been calculated, go to Step 7. Otherwise, go to Step 6.
Step 7. Obtain the optimal q_1 under the current E by using the KKT conditions. Mark this E as "tested".
Step 8. If all cases of feasible E are tested, return the optimal q_1 among all the q_1 obtained under different E and the algorithm stops. Otherwise, return to Step 2.

For a feasible E, $\{1, 2, \ldots, p\}$, we can obtain a list of ranges of q_1 (List B). For each range in List B, we obtain Δq_{ij} as a function of q_1, i.e., $\Delta q_{ij}(q_1)$ from the algorithm, which is a continuous function of q_1 in the range. Consider $\{x_1, x_2, \ldots, x_m\}$ and a range, $[q_1^l, q_1^u]$, in List B. We can obtain

$$
\begin{aligned}
\min AC &= AC_v + \sum_{i=1}^{m} AC_i \\
&= AC_v + \sum_{i=1}^{m} \left(h_b \frac{q_1 D_i}{2D_1} + c_b D_i L_i \right) + TR(q_1) + \frac{D_1}{q_1} \sum_{i=1}^{m} f_i x_i
\end{aligned}
\qquad (43)
$$

where

$$TR(q_1) = \sum_{i \notin E} \sum_{\substack{j=1 \\ j \neq i}}^{m} g_{ij}(q_1) \Delta q_{ij}(q_1) \tag{44}$$

subject to

$$\begin{cases} q_1^l \leqslant q_1 \leqslant q_1^u \\ \text{Constraint (14)} \end{cases} \tag{45}$$

It is clear that q_1 and $\Delta q_{ij}(q_1)$ obtained from the algorithm satisfy Constraints (9)–(14). We can get the KKT conditions for the range, $[q_1{}^l, q_1{}^u]$, as follows:

$$-\frac{(A_v + n\sum_{i=1}^{m} A_i)D_1}{nq_1^2} + \frac{(n-1)h_v \sum_{i=1}^{m} D_i}{2D_1} + \frac{h_b \sum_{i=1}^{m} D_i}{2D_1} + TR'(q_1) - \frac{D_1}{q_1^2} \sum_{i=1}^{m} f_i x_i = \lambda_1 - \lambda_2 \tag{46}$$

$$-\frac{A_v D_1}{n^2 q_1} + \frac{h_v q_1 \sum_{i=1}^{m} D_i}{2D_1} = 0 \tag{47}$$

$$\lambda_1(q_1 - q_1^l) = 0 \tag{48}$$

$$\lambda_2(q_1^u - q_1) = 0 \tag{49}$$

$$\lambda_1, \lambda_2 \geqslant 0 \tag{50}$$

From Eq. (47), we get

$$n = \frac{D_1}{q_1} \sqrt{\frac{2A_v}{h_v \sum_{i=1}^{m} D_i}} \tag{51}$$

Case 1. $\lambda_1 \neq 0$ and $\lambda_2 \neq 0$. From Eqs. (48) and (49), we see that $q_1 = q_1{}^l$ and $q_1 = q_1{}^u$, which cannot hold.
Case 2. $\lambda_1 \neq 0$ and $\lambda_2 = 0$. From Eq. (48), we get $q_1 = q_1{}^l$. Based on Eqs. (46) and (51), it follows that

$$\lambda_1 = -\frac{D_1 \sum_{i=1}^{m} A_i}{(q_1^l)^2} - \frac{h_v \sum_{i=1}^{m} D_i}{2D_1} + \frac{h_b \sum_{i=1}^{m} D_i}{2D_1} + TR'(q_1^l) - \frac{D_1}{(q_1^l)^2} \sum_{i=1}^{m} f_i x_i \tag{52}$$

This solution is a KKT point only when $\lambda_1 \geq 0$.
Case 3. $\lambda_1 = 0$ and $\lambda_2 \neq 0$. In this case, $q_1 = q_1{}^u$ leads to

$$\lambda_2 = \frac{D_1 \sum_{i=1}^{m} A_i}{(q_1^u)^2} + \frac{h_v \sum_{i=1}^{m} D_i}{2D_1} - \frac{h_b \sum_{i=1}^{m} D_i}{2D_1} - TR'(q_1^u) + \frac{D_1}{(q_1^u)^2} \sum_{i=1}^{m} f_i x_i \tag{53}$$

This solution is a KKT point only when $\lambda_2 \geq 0$.
Case 4. $\lambda_1 = 0$ and $\lambda_2 = 0$. In this case, q_1 should satisfy

$$\frac{D_1 \sum_{i=1}^{m} A_i}{q_1^2} + \frac{h_v \sum_{i=1}^{m} D_i}{2D_1} - \frac{h_b \sum_{i=1}^{m} D_i}{2D_1} - TR'(q_1) + \frac{D_1}{q_1^2} \sum_{i=1}^{m} f_i x_i = 0 \tag{54}$$

This solution is a KKT point only when q_1 satisfies Constraint (37). As the algorithm shows, for $\{x_1, x_2, \ldots, x_m\}$, we can obtain the optimal solution for E by comparing the objective functions under the ranges in List B. Then, we can obtain the optimal solution of P1 by comparing the objective functions of every feasible E. Note that the value of n obtained from Eq. (51) may not be an integer. In this case, we check the largest integer that is less than n and the smallest integer that is greater than n. When the fixed cost of transshipment is negligible, the above Theorems and algorithm can be simplified by setting $x_1 = x_2 =, \ldots, x_m = 1$. Obviously, this is a special case of the TRAN model.

5. The genetic algorithm

As discussed in Section 4, Algorithm 1 can only obtain the optimal solution of P1 for a given $\{x_1, x_2, \ldots, x_m\}$. In this section, we propose a GA for this problem. In the GA, each chromosome determines a $\{x_1, x_2, \ldots, x_m\}$. Then, we use Algorithm 1 to obtain the optimal solution under the $\{x_1, x_2, \ldots, x_m\}$. Fig. 4 illustrates the proposed GA procedure.

5.1. Chromosome representation

To execute the GA, a chromosome is coded with m genes. The value of x_i is considered as the "allele" to recall the specific language involved in the GA. The ith gene of the chromosome can be either 0 or 1; i.e., the value of x_i. Fig. 5 is an example that illustrates seven buyers. Only Buyers 3, 5, and 6 can transship products to other buyers. This representation of a chromosome is not a complete solution. We can obtain the optimal solution under a $\{x_1, x_2, \ldots, x_m\}$ by using the proposed algorithm, so we obtain a complete solution through a specific chromosome developed in the GA. In this paper, we feature an input minimiza-
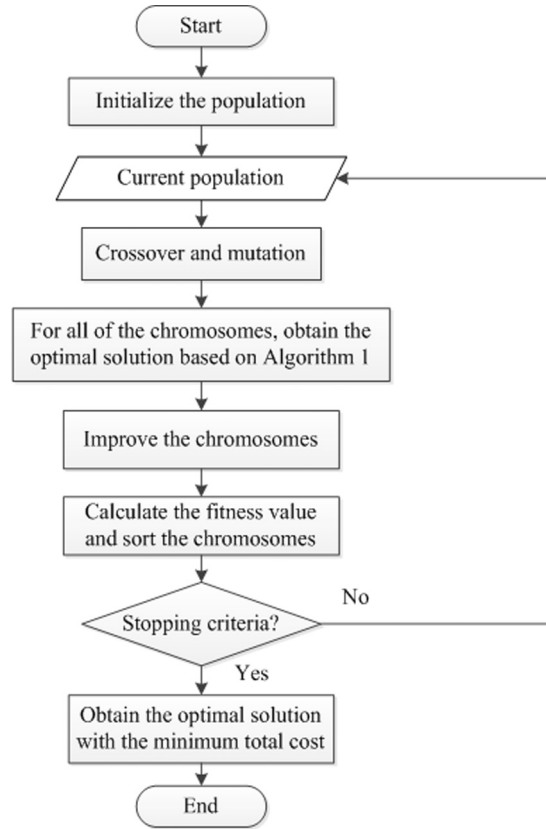
Fig. 4. The flowchart of the proposed GA.

0	0	1	0	1	1	0

Fig. 5. The chromosome used in the GA.

tion problem and use Eq. (55) to measure the fitness value of chromosome w, $w = 1, 2, \ldots, P$, in which P is the population size of the GA.

$$Fitness_w = the\ channel - wide\ cost\ in\ chromosome\ w \tag{55}$$

5.2. Crossover, mutation, and updating the solution

Eq. (55) shows that the fitness value of one chromosome is the channel-wide cost determined by it. Therefore, the best chromosome has the smallest fitness value. We select chromosomes for crossover with the popular roulette wheel approach (Goldberg, 1989), in which the selection probability of each chromosome is positively associated with its fitness value. For two chromosomes selected from the population ($F1$ and $F2$), we generate two new chromosomes ($O1$ and $O2$) by using crossover and mutation mechanisms. The two-point crossover with possibility P_c which has been widely used in GAs is used in this paper. Then, we implement a two-step mutation mechanism for each new chromosome obtained from the crossover. We omit the details of crossover and mutation; readers can refer to Feng et al. (2015) for details. Fig. 6 shows an example of crossover and mutation for the case with 10 buyers.

5.3. Solution improvement and stopping criterion

For a chromosome, we can obtain the optimal solution of P1. If $x_i = 1$ in the chromosome and $\Delta q_{ij} = 0$ in the optimal solution for all j in the corresponding E, then we can improve the chromosome by setting $x_i = 0$. Using this strategy, we find that the optimal solution remains and the fitness value is decreased by $D_1 f_i / q_1$.
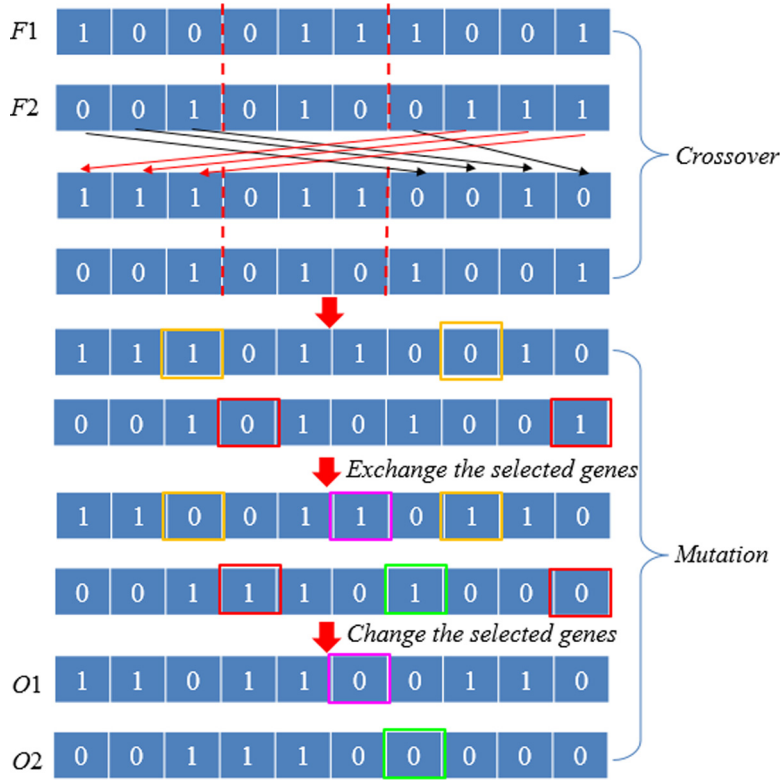
Fig. 6. An example of the crossover and mutation operators.

If the maximum number of generations is reached, then the stopping criterion is invoked. Otherwise, we create a new generation.

5.4. Impact of fixed transshipment cost consideration in the algorithm

As shown in Section 1, some of researchers who have studied transshipment did not consider the fixed transshipment cost; that is, they assumed that $f_1 = f_2 =, \ldots, = f_m = 0$. In this case, the algorithm may not obtain the optimal solution if the fixed transshipment costs exist. Algorithm 2 is proposed to get the solution when existing fixed transshipment costs are ignored in the decision making. We can reveal the impact of fixed transshipment cost consideration by comparing the results obtained from Algorithm 2 and the GA.

For $[q_1{}^l, q_1{}^u]$, we rewrite Eq. (43) as

$$
\min AC = AC_v + \sum_{i=1}^{m} AC_i
$$
$$
= AC_v + \sum_{i=1}^{m} \left(h_b \frac{q_1 D_i}{2D_1} + c_b D_i L_i \right) + TR(q_1)
$$
(56)

Then, Eqs. (52)–(54) can be rewritten as

$$
\lambda_1 = -\frac{D_1 \sum_{i=1}^{m} A_i}{\left(q_1^l\right)^2} - \frac{h_v \sum_{i=1}^{m} D_i}{2D_1} + \frac{h_b \sum_{i=1}^{m} D_i}{2D_1} + TR'(q_1^l)
$$
(57)

$$
\lambda_2 = \frac{D_1 \sum_{i=1}^{m} A_i}{\left(q_1^u\right)^2} + \frac{h_v \sum_{i=1}^{m} D_i}{2D_1} - \frac{h_b \sum_{i=1}^{m} D_i}{2D_1} - TR'(q_1^u)
$$
(58)

$$
\frac{D_1 \sum_{i=1}^{m} A_i}{q_1^2} + \frac{h_v \sum_{i=1}^{m} D_i}{2D_1} - \frac{h_b \sum_{i=1}^{m} D_i}{2D_1} - TR'(q_1) = 0
$$
(59)

However, the real objective value should include the total fixed transshipment cost under the solution. We may obtain this final objective value by adding $D_1 f_i / q_1$ if $\sum_{j=1, j \neq i}^{m} \Delta q_{ij}(q_1) \neq 0$ for all $i = 1, 2, \ldots, m$.

Algorithm 2.

Step 1. Set $x_1 = x_1 = x_2 =, \ldots, = x_m = 1$.

Step 2. Obtain the solution based on Algorithm 1 by replacing Eqs. (52)–(54) for the KKT conditions with Eqs. (57)–(59), respectively. Calculate the inventory and variable transshipment cost.

Step 3. Calculate the total fixed cost of transshipment at buyer i, $i = 1, 2, \ldots, m$: the cost is equal to $D_1 f_i / q_1$ if $\sum_{j=1, j \neq i}^{m} \Delta q_{ij}(q_1) \neq 0$; the cost is 0, otherwise.

Step 4. Obtain the total cost by adding the total fixed cost of transshipment to the cost obtained at Step 2 of this algorithm.

6. Computational experiments

6.1. Performance of the algorithms

In this experiment, we considered 10 buyers. A_v = \$500; h_v = \$1.6; and h_b = \$12.3. For simplicity, we assumed $c_b = h_b$. Because $r_i = D_i L_i$, we randomly generated r_i instead of L_i. We also randomly generated five instances. In each instance, other parameters followed uniform distributions as presented in Table 2.

The TRAN model was coded in LINGO version 16.0, and the GA was coded in JAVA. The experiments were conducted on a PC with Intel Core 2.3 GHz and 4 GB RAM. We obtained the appropriate settings for the GA parameters through pilot experiments. The population size, maximum generation number, Pc, $Pm1$, and $Pm2$ are set as 100, 100, 0.7, 0.20, and 0.05, respectively. Table 3 illustrates the comparison of the solutions obtained from LINGO and the GA. For most instances, the GA can obtain optimal solutions. In the two instances in which the GA solutions were worse than those of LINGO, the optimality gaps were small. Moreover, we found that the LINGO computation time increased significantly with problem size. When m was >40, LINGO required >24 h to solve the problem, but the GA obtained solutions within acceptable timeframes.

To show the impact of transshipment policy on the channel-wide cost, we compare the INT model and the TRAN model. The INT model is a special of the TRAN model with $x_1 = x_2 =, \ldots, = x_m = 0$. We can obtain the optimal solution of the INT model with Algorithm 1 by setting a single feasible E which is empty. In addition, because the fixed transshipment cost has rarely been considered in existing studies, we compare our GA with Algorithm 2. In Algorithm 2, all of the buyers who are not in E can transship products to the buyers in E. For a solution obtained from this algorithm, only the fixed transshipment costs of the buyers who conduct transshipment are added to the channel-wide cost; this approach is based on the chromosome improvement mechanism in the GA. Let Gap I = $(AC^{nt} - AC^{nf})/AC^{nt} \times 100\%$, Gap II = $(AC^{nt} - AC^{f})/AC^{nt} \times 100\%$, and Gap III = $(AC^{nf} - AC^{f})/AC^{nf} \times 100\%$. The comparisons are summarized in Table 4. For each problem size, we ran Algorithm 2 and the GA on 10 instances.

From Table 4, we can see that the TRAN model can lead to a lower channel-wide cost than the INT model. Because Gap II is greater than Gap I, we can infer that this cost saving can be reduced if the fixed transshipment cost is not considered in the optimal solution algorithm. The influence of fixed transshipment cost is created by two circumstances. First, if we do not consider this fixed cost, then we may obtain a solution in which buyer i transships few products to buyer j while other buyers (e.g., buyer k) not in E also still transship products to buyer j. Although the unit transshipment cost, s_{ij}, is lower than s_{kj}, transshipping a small number of products cannot compensate for the fixed cost of involving three buyers in the transshipment. Second, there exist cases in which s_{ij} is lower than s_{kj} while f_i is greater than f_k. Under these conditions the transshipment cost may not be minimized if we decide the transshipment solution based on s_{ij} and s_{kj} without considering f_i and f_k. Therefore, we find a trade-off exists between the fixed cost and the transportation cost in the transshipment. Moreover, the gaps are larger for large problems because more buyers could be involved in transshipment, which increases Gaps I and II. Because many buyers characterize large problems, a wide solution space is created that affects the optimal GA solution, which increases Gap III.

6.2. Sensitivity analysis

To analyze the impact of parameters on the performances of these models and algorithms, we conducted sensitivity analysis of h_v, h_b, f_i, and W_i for the cases with $m = 10$. For each value of h_v and h_b, we randomly generated 30 instances by using the parameter distributions in Table 2. Tables 5 and 6 show the comparison of the INT model, Algorithm 2, and the GA under different h_b and h_v values, respectively. Gaps I to III decrease when h_b is increased from \$8 to \$16, respectively. When h_v is

Table 2
Distributions of stochastic parameters.

W_i	D_i	A_i	r_i	f_i	s_{ij}
[50, 300]	[500, 3500]	[20, 60]	[20, 40]	[20, 50]	[1, 10]

Table 3
Comparison of the results obtained from LINGO and the GA.

Problem size	Instance	LINGO		GA	
		Objective value ($)	Computation time (s)	Objective value ($)	Computation time (s)
$m = 10$	1	38,786	395	38,786	42
	2	32,141	267	32,141	33
	3	30,413	327	30,413	41
	4	44,398	654	44,398	48
	5	32,269	486	32,269	45
$m = 20$	1	59,645	1723	59,645	110
	2	69,056	2183	69,056	170
	3	59,040	1714	59,040	155
	4	65,712	1908	65,712	170
	5	72,643	2665	72,643	212
$m = 30$	1	96,580	11,914	96,580	281
	2	96,542	15,551	96,542	317
	3	106,456	18,019	106,456	387
	4	98,246	13,500	98,246	385
	5	101,235	15,179	101,235	543
$m = 40$	**1**	**124,414**	**>50,000**	**124,518**	**601**
	2	150,211	>50,000	150,211	889
	3	131,051	>50,000	131,051	626
	4	**128,477**	**>50,000**	**128,531**	**612**
	5	134,306	>50,000	134,306	770

Note: The instances wherein the GA fails to obtain the optimal solution appear in bold.

Table 4
The comparison of three scenarios.

m	INT ($)	TRAN ($)		Gap (%)		
		Algorithm 2	GA	I	II	III
10	38,740	35,273	34,652	8.95	10.55	1.76
20	74,588	64,770	63,419	13.47	15.28	2.09
30	115,396	96,326	93,917	16.53	18.61	2.50
40	167,967	134,839	130,786	19.72	22.13	3.01
50	210,437	165,838	159,805	21.19	24.06	3.64

Table 5
The comparison of the INT model, Algorithm 2, and the GA under different h_b.

h_b ($)	INT			TRAN						Gap (%)		
				Algorithm 2			GA					
	Average	Min	Max	Average	Min	Max	Average	Min	Max	I	II	III
8	36,796	21,939	50,999	32,642	21,574	44,100	32,019	21,501	43,001	11.289	12.982	1.909
10	37,867	23,305	51,024	33,874	23,063	45,984	33,251	22,990	44,222	10.545	12.190	1.839
12	38,136	24,670	52,601	34,456	24,552	46,808	33,875	24,479	45,209	9.650	11.173	1.686
14	40,007	26,035	53,110	36,338	26,035	48,019	35,748	25,967	46,221	9.171	10.646	1.624
16	41,076	27,400	54,001	37,571	27,400	49,912	36,979	27,347	47,816	8.533	9.974	1.576

Table 6
The comparison of the INT model, Algorithm 2, and the GA under different h_v.

h_v ($)	INT			TRAN						Gap (%)		
				Algorithm 2			GA					
	Average	Min	Max	Average	Min	Max	Average	Min	Max	I	II	III
1	37,261	23,870	47,683	33,862	23,854	42,782	33,295	23,780	44,690	9.122	10.644	1.674
2	38,843	25,339	48,522	35,316	25,287	43,853	34,702	25,202	45,861	9.080	10.661	1.739
3	39,987	26,407	49,759	36,331	26,352	44,924	35,715	26,259	47,032	9.143	10.684	1.696
4	41,529	27,325	51,298	37,690	27,120	46,495	37,071	27,024	48,203	9.244	10.735	1.642
5	42,642	27,994	52,436	38,664	27,782	47,966	38,044	27,681	49,173	9.329	10.783	1.604

increased from \$1 to \$5, however, the variation tendencies of Gaps I to III are more complex. As Tables 7 and 8 show, we investigate into an instance as an example to discuss the reason for this difference.

The GA can yield good transshipment solutions under a q_1 that represents a relatively low transshipment cost (including transportation and fixed costs). Therefore, the GA may lead to solutions with a larger optimal q_1 than Algorithm 2 (see Tables 7 and 8). When h_b is increasing, a smaller q_1 may be preferable when each buyer's order quantity in one sub-cycle is decreasing. In this case, the optimal n is an affine transformation of q_1. Hence, the optimal n is increasing with h_b. Although the channel-wide costs under the models and algorithms are all increasing with h_b, Gap III is decreasing. In addition, the decreasing rates of Gap III when h_b is increased from \$12 to \$16 are greater than the rates when h_b is increased from \$8 to \$10. In the latter cases, the optimal q_1 values from Algorithm 2 and from the GA are both 90. When h_b is increased from \$8 to \$10, the absolute values of $AC^{nf} - AC^f$ are the same. Gap III is decreased only because AC^{nf} is increasing when h_b is increased from \$8 to \$10. When h_b is equal to \$12, \$14, and \$16, the optimal q_1 obtained from Algorithm 2 is decreased to 75 while the optimal q_1 from the GA is still 90. In these cases, the increment of AC^f is higher than the increment of AC^{nf} when h_b is increased. Therefore, the absolute values of $AC^{nf} - AC^f$ are decreasing. Considering that AC^{nf} is increasing with h_b, the decreasing rates of Gap III when h_b is increased from \$12 to \$16 could be greater than the one when h_b is increased from \$8 to \$10.

Eq. (51) explains that the optimal n is decreasing with h_v and q_1. When h_v is increasing, a larger q_1 and smaller n may be preferable when the vendor's total order quantity in one cycle (i.e., nDq_1/D_1) is also decreasing. For the same q_1 and different h_v, the optimal n may vary. When h_v is sufficiently large, however, Eqs. (57) and (58) can also be used to obtain the optimal q_1. Therefore, Algorithm 2 obtained the same q_1 with the GA when h_v = \$2, \$3, \$4, and \$5. In these cases, the value for $AC^{nf} - AC^f$ remains unchanged while AC^{nf} is increasing. In this case, Gap III decreases when h_v is increased from \$2 to \$5. Moreover, q_1 = 89, which is greater than the value of q_1 obtained from Algorithm 2 when h_v = \$1. Because a larger q_1 implies more transshipments, the value of $AC^{nf} - AC^f$ when h_v = \$2 is greater than when h_v = \$1. As a consequence, Gap III may increase when h_v increases from \$1 to \$2.

Let F be the uniform distribution of f_i, i = 1 ,2, 3, . . . ,10. We conducted experiments under different mean values and standard deviations of F to see the impact of the fixed transshipment cost. For each type of F, we randomly generated 30 instances for which other stochastic parameters follow the uniform distributions presented in Table 2. Table 9 shows the average channel-wide cost of the 30 instances under F with different mean values and with the same standard deviation. We can see that when the mean value of F is increased, the fixed cost of transshipment is higher, and the cost saving from transshipment is lower; thus Gaps I and II are decreased respectively. However, Gap III is larger in those cases. These findings mean that when the fixed cost of transshipment is higher, the assignment of buyers for transshipment plays a more important role in reducing the channel-wide cost.

Table 10 shows the average channel-wide cost of the 30 instances under different standard deviations of F with the same mean value. We can see that when the standard deviation F is increased, the improvements of the two algorithms are more significant, because more buyers have lower fixed transshipment costs. The channel-wide cost can be reduced more significantly if the buyers with a lower fixed cost are involved in the transshipment. Because the GA can attempt more assignments of buyers for the transshipment, Gap III is larger in those cases.

Table 7
The solutions under different h_b for an instance.

h_b (\$)	INT			TRAN						Gap (%)		
				Algorithm 2			GA					
	AC^{nt} (\$)	q_1	n	AC^{nf} (\$)	q_1	n	AC^f (\$)	q_1	n	I	II	III
8	33,633	56	8	31,151	90	5	30,577	90	5	7.380	9.086	1.843
10	34,641	56	8	32,419	90	5	31,844	90	5	6.414	8.074	1.774
12	35,649	56	8	33,678	75	6	33,112	90	5	5.529	7.117	1.681
14	36,656	56	8	34,835	75	6	34,379	90	5	4.968	6.212	1.309
16	37,664	56	8	35,992	75	6	35,646	90	5	4.493	5.536	0.961

Table 8
The solutions under different h_v for an instance.

h_v (\$)	INT			TRAN						Gap (%)		
				Algorithm 2			GA					
	AC^{nt} (\$)	q_1	n	AC^{nf} (\$)	q_1	n	AC^f (\$)	q_1	n	I	II	III
1	34,686	56	10	32,769	75	8	32,251	89	7	5.527	7.020	1.581
2	36,429	56	7	34,427	89	5	33,852	89	5	5.496	7.074	1.670
3	37,684	56	6	35,571	89	4	34,996	89	4	5.607	7.133	1.617
4	38,753	56	5	36,557	89	3	35,982	89	3	5.667	7.150	1.573
5	39,820	56	4	37,247	89	3	36,672	89	3	6.461	7.906	1.544

Table 9
The comparison of the INT model, Algorithm 2, and the GA under F with different mean values.

F	INT ($)			TRAN ($)						Gap (%)		
				Algorithm 2			GA					
	Average	Min	Max	Average	Min	Max	Average	Min	Max	I	II	III
[10, 40]	39,096	24,874	53,893	34,834	24,365	45,441	34,274	24,241	44,618	10.901	12.334	1.608
[15, 45]	39,096	24,874	53,893	35,216	24,519	45,876	34,624	24,381	44,776	9.924	11.439	1.681
[20, 50]	39,096	24,874	53,893	35,870	24,621	47,605	35,243	24,547	45,365	8.252	9.855	1.748
[25, 55]	39,096	24,874	53,893	36,325	24,775	47,913	35,659	24,059	46,912	7.088	8.791	1.833
[30, 60]	39,096	24,874	53,893	36,615	24,874	48,121	35,915	23,765	47,430	6.346	8.136	1.912

Table 10
The comparison of the INT model, Algorithm 2, and the GA under F with different standard deviations.

F	INT ($)			TRAN ($)						Gap (%)		
				Algorithm 2			GA					
	Average	Min	Max	Average	Min	Max	Average	Min	Max	I	II	III
[10, 60]	39,096	24,874	53,893	35,045	24,570	46,218	34,382	24,307	44457	10.362	12.068	1.892
[15, 55]	39,096	24,874	53,893	35,603	24,596	47,499	34,974	24,418	45530	8.934	10.543	1.767
[20, 50]	39,096	24,874	53,893	35,870	24,621	47,605	35,243	24,547	45365	8.252	9.855	1.748
[25, 45]	39,096	24,874	53,893	36,004	24,874	47,821	35,408	24,709	46107	7.909	9.433	1.655
[30, 40]	39,096	24,874	53,893	36,254	24,874	47,963	35,780	24,813	46747	7.269	8.482	1.307

Table 11
The comparison of the INT model, Algorithm 2, and the GA under Y with different mean values.

Y	INT ($)			TRAN ($)						Gap (%)		
				Algorithm 2			GA					
	Average	Min	Max	Average	Min	Max	Average	Min	Max	I	II	III
[60, 310]	35,857	24,133	46,421	33,618	24,049	42,732	33,300	23,998	42,513	6.244	7.131	0.946
[70, 320]	33,393	23,692	41,871	31,816	23,692	38,886	31,536	23,521	38,745	4.723	5.561	0.880
[80, 330]	31,645	23,209	38,313	30,517	23,209	37,125	30,313	23,106	37,050	3.565	4.209	0.669
[90, 340]	30,138	22,856	36,394	29,422	22,856	36,354	29,320	22,710	36,304	2.376	2.714	0.347
[100, 350]	29,302	22,807	34,792	28,840	22,807	34,511	28,757	22,793	34,487	1.577	1.860	0.288

Let Y be the uniform distribution of W_i, i = 1, 2, 3, ..., 10. We conducted experiments under different mean values of Y to see the impact of the warehouse capacities. For each type of Y, we randomly generated 30 instances for which other stochastic parameters follow the uniform distributions in Table 2. Table 11 shows the average channel-wide cost of 30 instances under different mean values and with the same standard deviation. We can see that when the mean value is increased, the improvements of the two algorithms are less significant, because the buyers can obtain more products from ordinary deliver, which mitigates the essentiality of lateral transshipment.

7. Conclusions and future works

We developed several theorems to indicate the optimal transshipment solution and found that for $\{q_1, n\}$:

(i) Buyers should not simultaneously transship products to others and receive transshipped products from others.
(ii) Buyer i should not transship products to buyer j if $0 \leq q_i \leq W_i$ and $0 \leq q_j \leq W_j$.
(iii) Buyer i should not transship products to buyer j if buyer k with $g_{kj}(q_1) < g_{ij}(q_1)$ has unused warehouse capacity.

Based on the above findings, an optimal solution algorithm that alleviate the complexity of the TRAN model is be developed. From this study, we can see that the average channel-wide cost can be saved by using the TRAN model under which the transshipment allows the order to arrive at various times. As a consequence, some buyers can obtain more products than their warehouses can accommodate. In addition, the utilization ratio of the warehouse capacities of some buyers is increased. When warehouse capacity is shared among buyers, the channel-wide optimal replenishment solution can be achieved. The numerical experiment shows the advantage of the TRAN model. The advantage can be more significant when the number of buyers is increased.

Moreover, when existing fixed transshipment costs are ignored during decision making, the solution can deviate from the optimal solution, and this outcome has a negative impact on the performance of lateral transshipments. The experimental

results demonstrate that the fixed transshipment costs should be considered in the model and algorithm design. Sensitivity analyses show how this impact is influenced by other parameters. From the results, we can see that h_v has a positive impact on the advantage of the TRAN model, while h_b has a negative impact on it. This advantage is negatively associated with the warehouse capacities of buyers.

In this article, we developed our model based on several assumptions that can be reconsidered in future work. First, we assumed that product consumption rates of buyers are known, and we focused on how supply chains benefit from transshipment as result of increased warehouse capacity. Considering the advantage of transshipment in increasing customer service level when the consumption rates are uncertain can be a fruitful direction. Second, we considered absolute constraints on the warehouse capacities of buyers. This model can be extended by considering penalty costs for exceeding upper bounds of inventory levels under our tight capacity constraints, which would make our model more applicable in general. Third, we studied an integrated channel and focused solely on reducing the channel-wide cost. It would be interesting to study the performances of members in a decentralized channel under a warehouse-sharing policy.

Acknowledgement

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