대한산업공학회/한국경영과학회
2010년 춘계공동학술대회

Supply chain coordination with revenue sharing contracts

풍설호, 문일경, 류광열, 권보배

Department of Industrial Engineering
Pusan National University

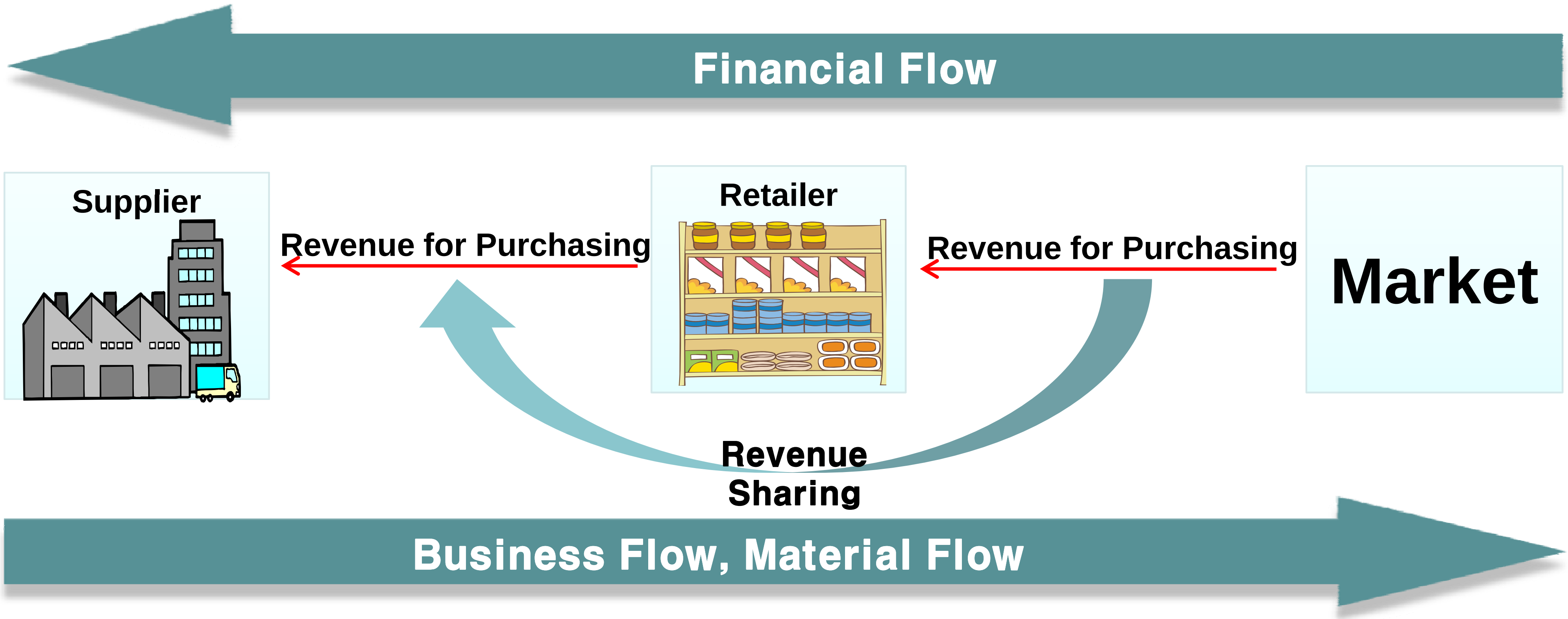
이 논문은 2009년 정부(교육과학기술부)의 재원으로 한국연구재단의 지원을 받아 수행된 연구임(2009-0087861)

Contents

1. Introduction

1.1 Definition of RS contract

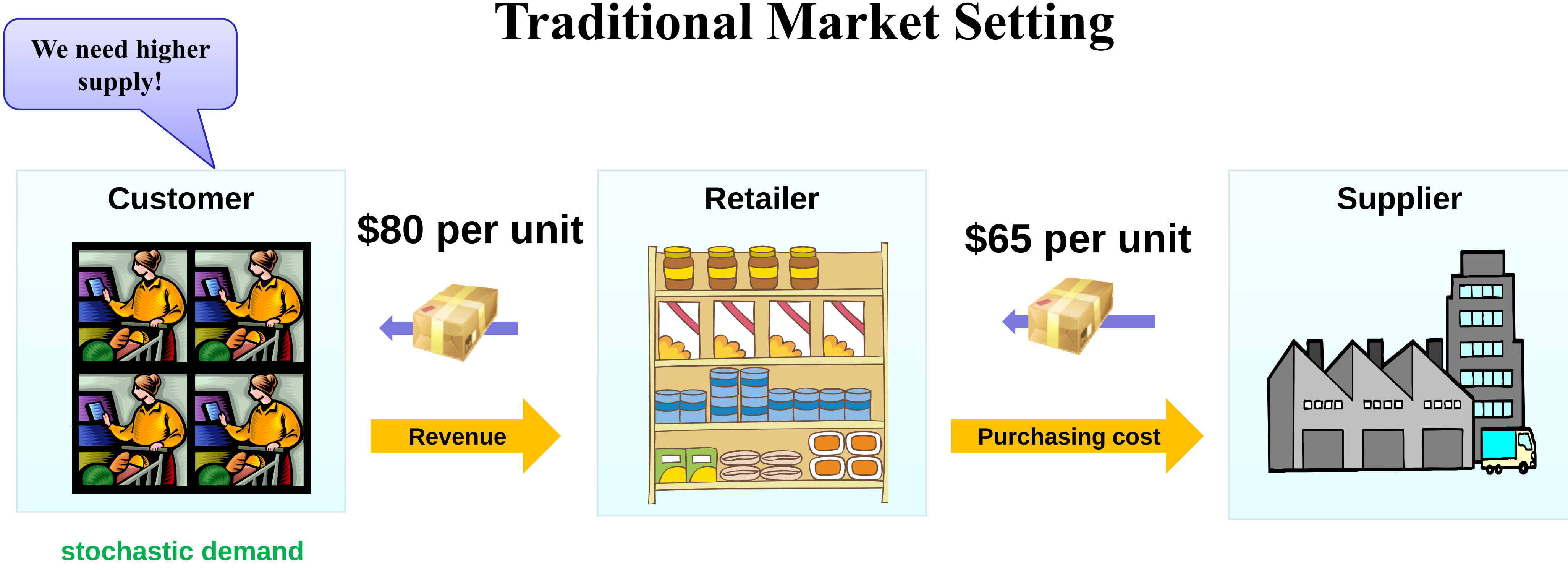
- Under a RS contract, a downstream company pays upstream company a wholesale price for each unit purchased, plus a percentage of the revenue the retailer generates.



1. Introduction

1.2 Motivation

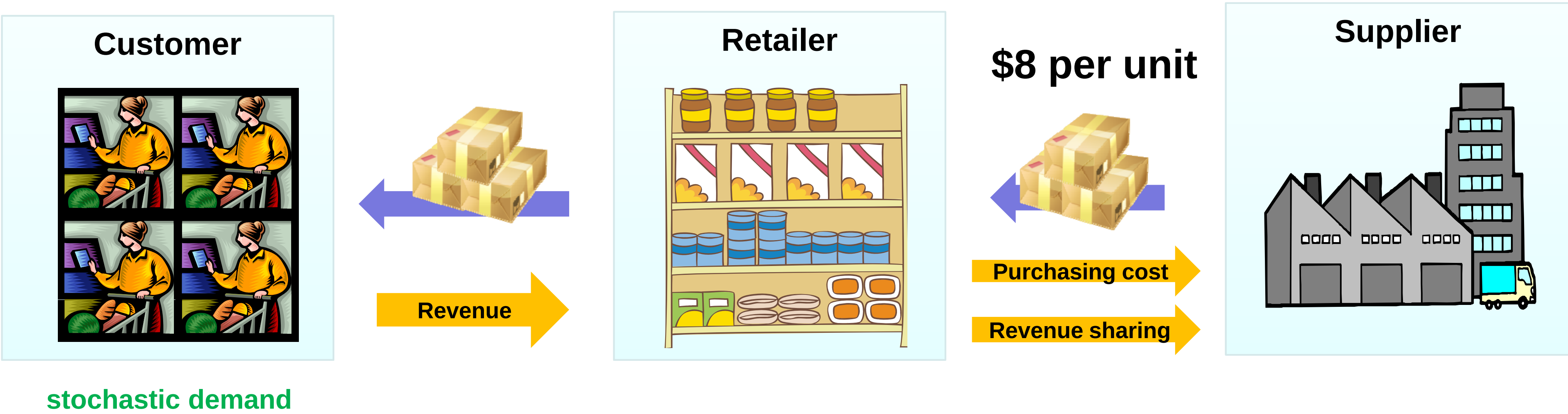
Traditional Market Setting



- We can prove that the traditional market setting can hardly maximize the profit of supply chain.

1.2 Motivation

RS Contract

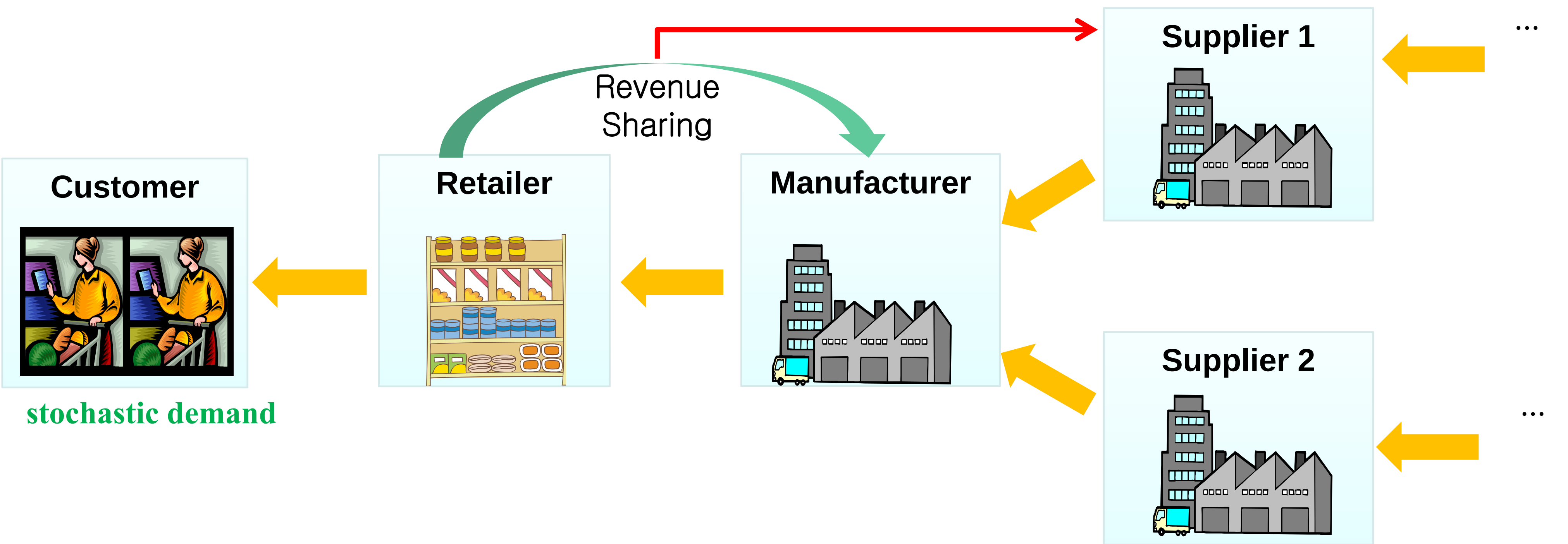


❖Under the RS contract, both the retailer and supplier can get higher profit than under the traditional market setting.

1. RS model in N-stage Supply Chain

1.2 Motivation

RS Contract in N-stage Supply Chain



- In some supply chains, there are more than two stages and some members face more than one upstream members.

2. Two objectives of RS model

Feasible RS Model

Effective and Desirable

All members of SC choose
the same quantity that maximizes
the total profit of SC

All members can get higher profit
under the RS contract
than the traditional market setting

3. RS model in N-stage Supply Chain

3.1 Notation

Notation

Parameters

- q : Order quantity
- p : Selling price per unit
- s : Salvage value
- $R(q)$: Total revenue of retailer
- N : Number of members in the supply chain
- $C_i(q)$: Production cost of member i $\quad i=1,2\ldots N$
- $C(q)$: Total cost of supply chain
- $\pi_i(q)$: Profit of member i $\quad i=1,2\ldots N$
- $\pi(q)$: Profit of total supply chain

Decision variables

- ω_i : Wholesale price that member i charges $\quad i=2,3\ldots N$
- Φ_i : Percentage of retailer's revenue that member i keeps $\quad i=1,2\ldots N$

3. RS model

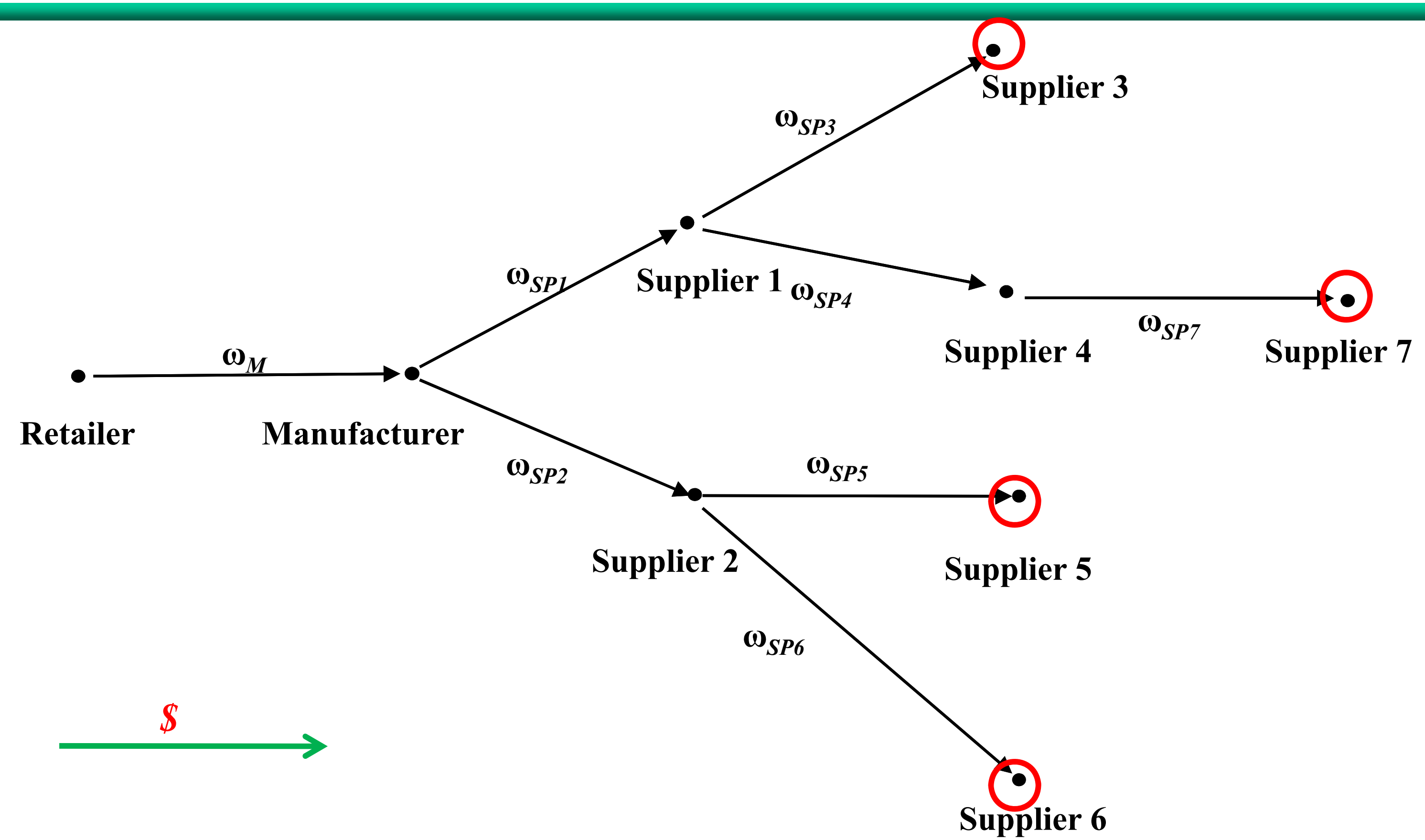
3.2 Assumptions

- Demand from market is stochastic.

- Sale price and wholesale price are fixed by the market in one selling season.

3. RS model in N-stage Supply Chain

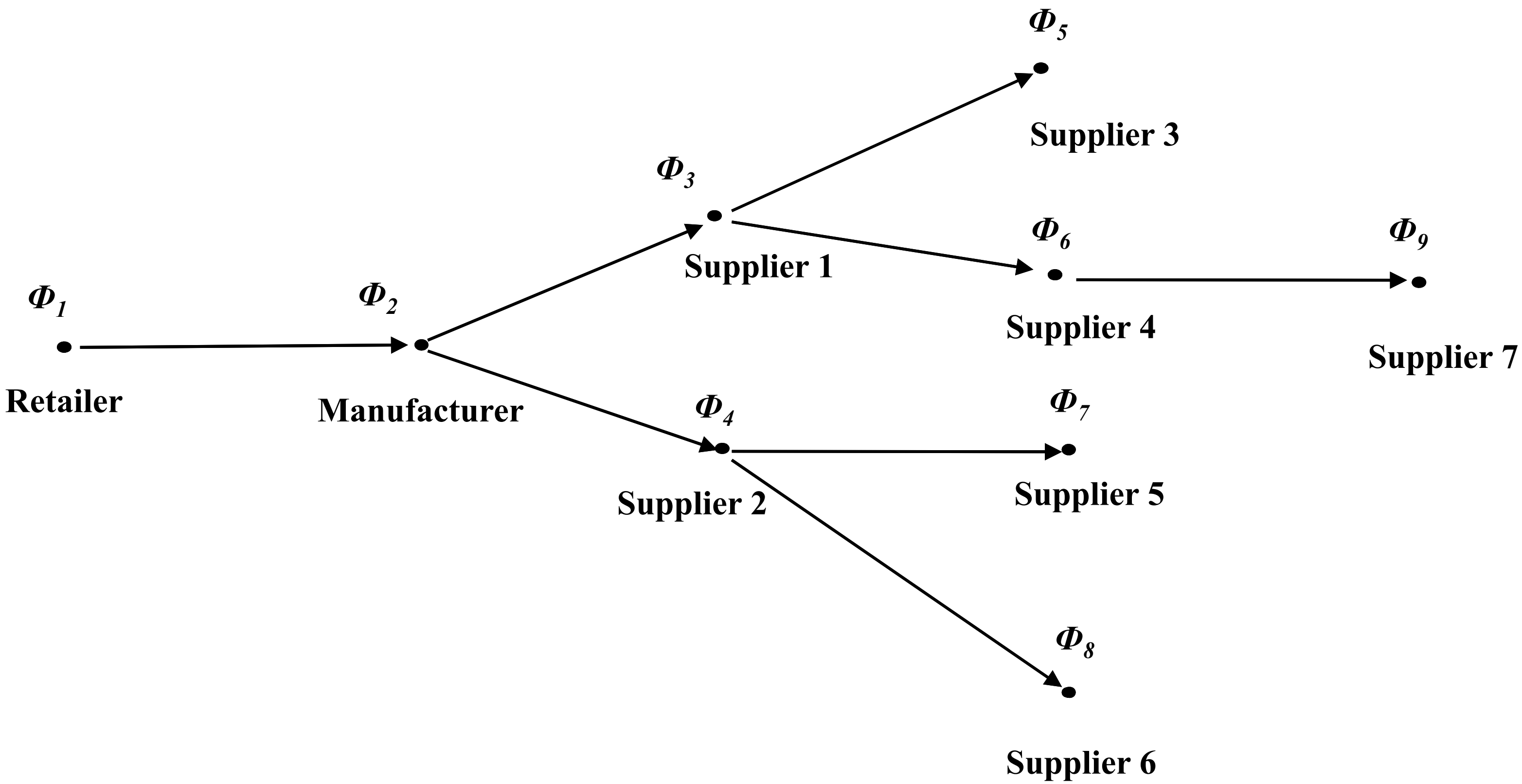
3.3 RS model in N-stage supply chain



- The left point means the downstream member in the supply chain, such as retailer.
- No point has more than one direct predecessor and the points without any successor are called terminal point.
- The value of each line means the wholesale price charged by the points on the right side.

3. RS model in N-stage Supply Chain

3.3 RS model in N-stage supply chain



- Let set S_i be the set containing all the successors of member i and herself.

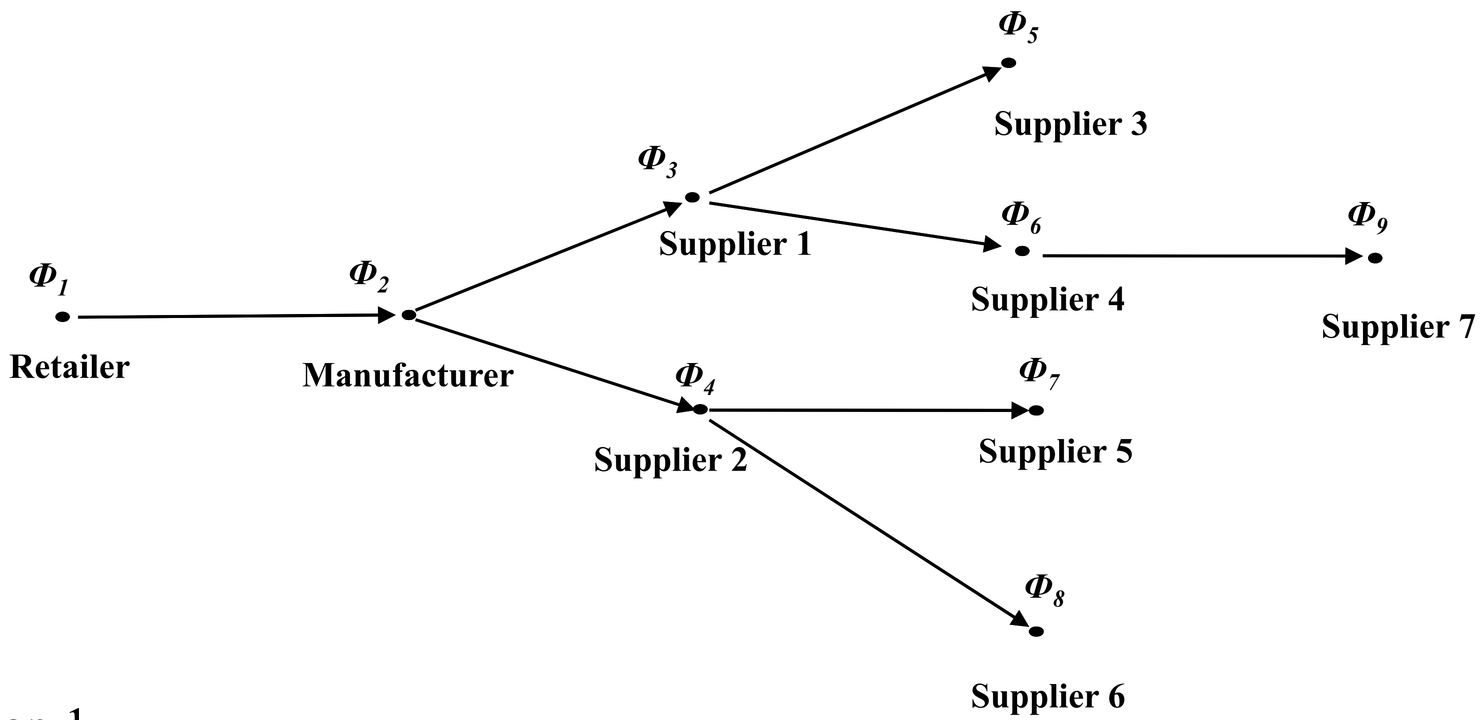
  $S_2 = \{ 2,3,4,5,6,7,8,9\}$

  $S_4 = \{ 4,7, 8\}$

  $S_5 = \{5\}$

3. RS model in N-stage Supply Chain

3.3 RS model in N-stage supply chain



- Step 1
  Determine the proportion (Φ_i) of each member in the total profit under RS contract
- Step 2
  Find a terminal point or a point whose all right lines have a known value. Then calculate the wholesale price charged by the point and mark it as the value of the left line.
- Step 3
  Repeat step 2 until all the values of the lines are known.

3. RS model in N-stage Supply Chain

3.3 RS model in N-stage supply chain

Theorem 1

Consider the set of RS contracts with $\omega_i = \dfrac{\sum_{j \in S_i} C_j(q) - \sum_{j \in S_i} \Phi_j C(q)}{q}$, $i = 2,3 \ldots N$

With those contracts, the profit function of member *i* is: $\pi_i = \Phi_i \pi$

Furthermore: $q_i^* = q_0^* = q_{i+1}^*$, $i = 1,2 \ldots N\text{-}1$

That means all of members should take the wholesale price given a set of Φ :

$$\omega_i = \frac{\sum_{j \in S_i} C_j(q) - \sum_{j \in S_i} \Phi_j C(q)}{q}$$

Then, member *i* can get Φ_i of the total profit of supply chain. All of the members will choose the quantity that maximizes the total profit of supply chain.

3. RS model in N-stage Supply Chain

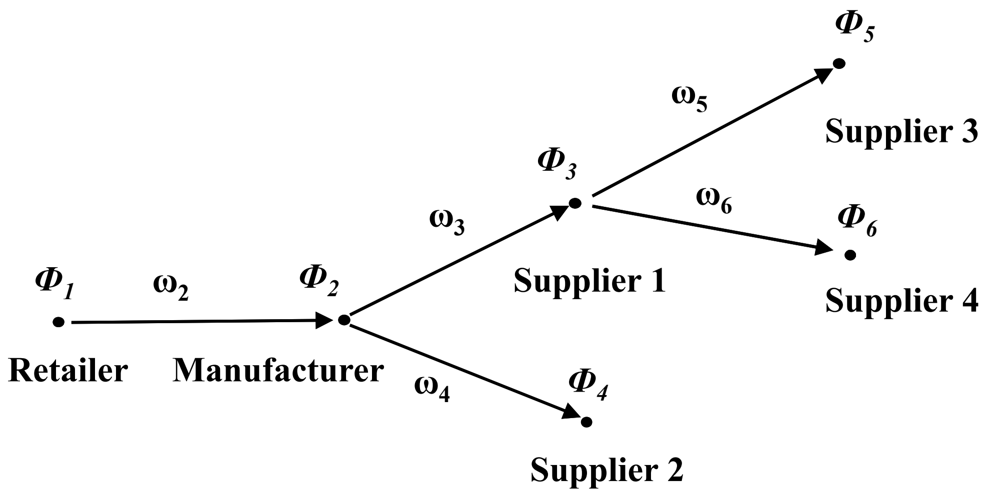
3.3 Basic RS model in 2-stage supply chain

Theorem 2

- Using the proportion of retailer's profit in the total profit of supply chain under the traditional market setting is a feasible method to determine Φ_i.

3. RS model in N-stage Supply Chain

3.4 Example



- Suppose the demand is normally distributed with average value *100* and standard deviation *30*.
  The price is $*30*, salvage value is $*1*.

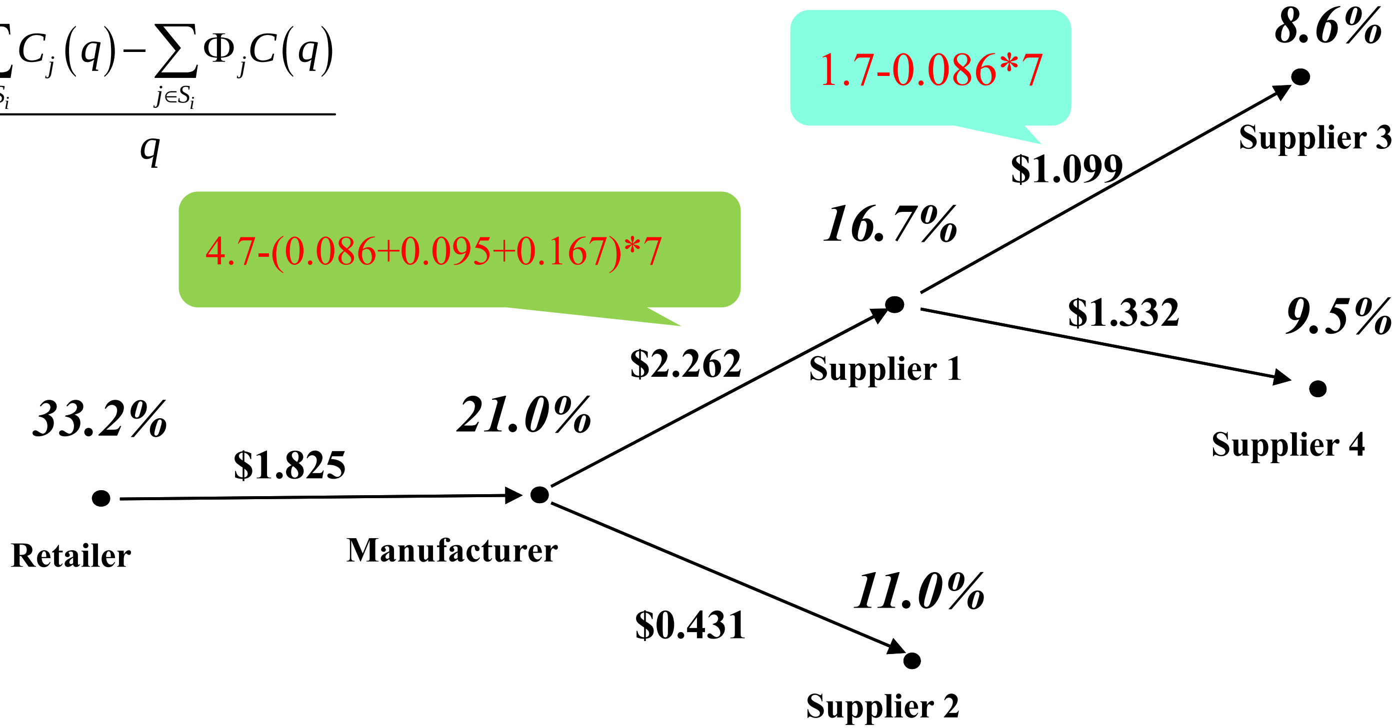
Number	Cost function	Price under traditional market setting($)
1	0.5q	
2	0.6q	20.5
3	1q	12
4	1.2q	3.5
5	1.7q	3.5
6	2q	4

3. RS model in N-stage Supply Chain

3.4 Example

Profit and cost under traditional market setting ($)							
	Retailer	Manu.	Sup.1	Sup.2	Sup.3	Sup.4	Total profit
Profit	592.9	374.7	298.0	195.9	153.3	170.3	1785.1
Percentage	33.2%	21%	16.7%	11%	8.6%	9.5%	
Production Cost	$0.5q$	$0.6q$	$1q$	$1.2q$	$1.7q$	$2q$	

$$\omega_i = \frac{\sum_{j \in S_i} C_j(q) - \sum_{j \in S_i} \Phi_j C(q)}{q}$$



3. RS model in N-stage Supply Chain

3.4 Example

Mean=100
Price=$30

c_1 =0.5q
ω_2=1.82

Capital=289.46

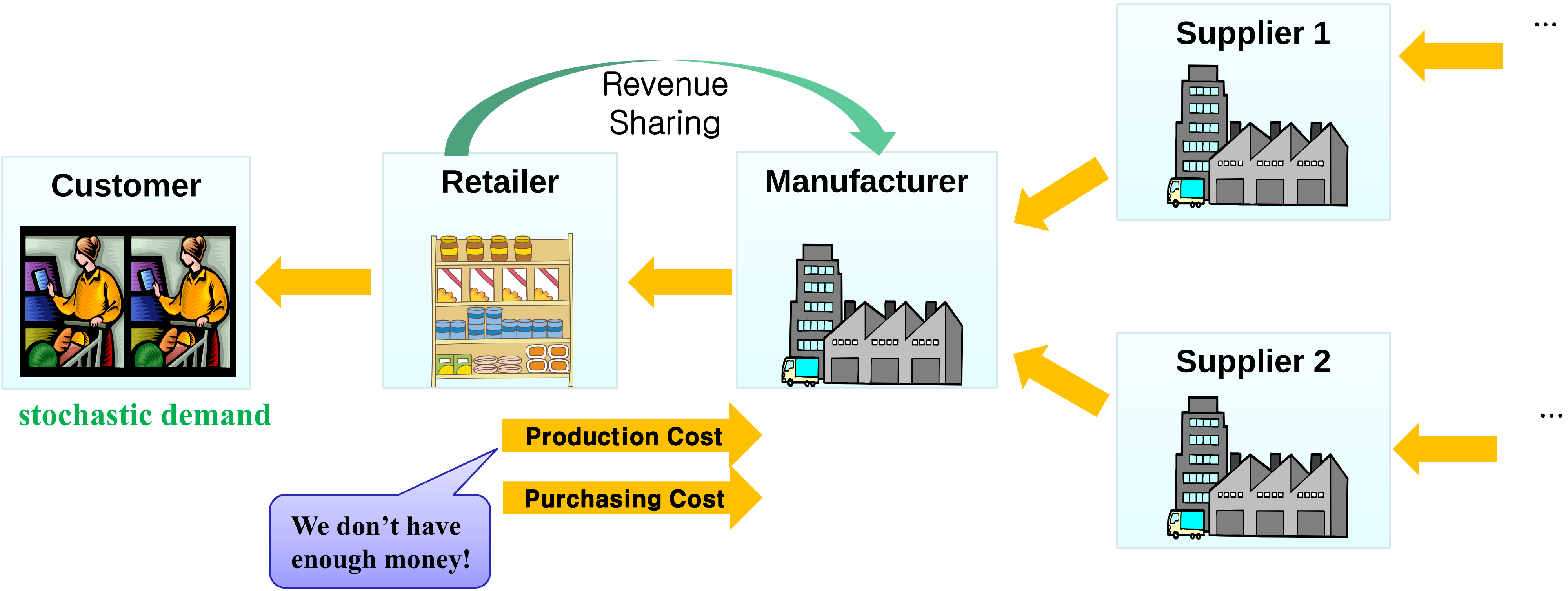
		Profit of Each Member ($)								
S.D		Retailer	Manu.	Sup.1	Sup.2	Sup.3	Sup.4	Total profit	Profit increasing	Optimal quantity
30	Market setting	592.9	374.7	298.0	195.9	153.3	170.3	1785.1		85
	RS contract	681.4	430.6	342.5	225.1	176.2	195.7	2051.5	15%	124
20	Market setting	[illegible]	[illegible]5	315.4	207.2	162.2	180.2	1956.8		90
	RS contract	758.4	432.4	343.9	226.1	176.9	196.6	2134.3	9%	116
5.47	Market setting	844.0	428.1	340.5	223.8	175.1	194.6	2206.1		98
	RS contract	962.6	437.5	348.0	228.7	178.9	198.9	2254.6	2%	104

❖ The RS contract is more effective under the demand with a lager variability.

4. RS model Considering Capital Constraint

4.1 Motivation

RS Contract in N-stage Supply Chain



❖ In the RS model, the expenditure of member *i* depends on the cost and the percentage of retailer's revenue that member *i* keeps .

4.2 Notation

Notation

Parameters

- q : Order quantity
- p : Selling price per unit
- s : Salvage value
- $R(q)$: Total revenue of retailer
- N : Number of members in the supply chain
- $C_i(q)$: Producing cost of member i $\quad i=1,2\ldots N$
- $C(q)$: Total cost of supply chain
- $\pi_i(q)$: Profit of member i $\quad i=1,2\ldots N$
- $\pi(q)$: Profit of total supply chain
- Y : Budget of retailer

Decision variables

- ω_i : Wholesale price that member i charges $\quad i=2,3\ldots N$
- Φ_i' : Percent of retailer's revenue that member i keeps $\quad i=1,2\ldots N$

4. RS model Considering Capital Constraint

4.3 Assumptions

- Demand from market is stochastic.
- Sale price and wholesale price are fixed by the market in one selling season
- There is a limit on the purchasing (Capital Constraint).

4. RS model Considering Capital Constraint

4.4 Model

Under the RS model, $\omega_2 = \dfrac{\sum_{j \in S_2} C_j(q) - \sum_{j \in S_2} \Phi_j C(q)}{q} = \dfrac{\Phi_1 C(q) - C_1(q)}{q}$

and the retailer's capital need is $\Phi_1 C(q) = \omega_2 q + C_1(q)$

Budget of retailer: Y

$$\Phi_1 \le \frac{Y}{C(q)} \qquad (1)$$

Profit of retailer under the traditional market setting

Profit of retailer: $\Phi_1 \left[R(q) - C(q) \right] \ge \pi_{Mr}$ (2)

From (1) and (2): $\dfrac{\pi_{Mr}}{\left[R(q) - C(q) \right]} \le \Phi_1 \le \dfrac{Y}{C(q)}$

4.4 Model

When the budget (Y) decreases to Y' :

$$\frac{\pi_{Mr}}{\left[R\left(q_0^*\right)-C\left(q_0^*\right)\right]} \le \Phi_1 \le \frac{Y'}{C\left(q_0^*\right)}$$

Method 1

If $\Phi_1{}'$ is chosen, which is smaller than Φ_1 :

$$\pi = \pi\left(q_0^*\right) \qquad \textbf{remain}$$

$$\pi_1 = \Phi_1'\pi\left(q_0^*\right) = \left(\Phi_1 - \Delta\Phi_1\right)\pi\left(q_0^*\right)$$

Method 2

If $q_0{}'$ is chosen, which is smaller than $q_0{}^*$:

$$\pi = \pi\left(q_0'\right)$$

$$\pi_1 = \Phi_1\pi\left(q_0'\right)$$

When $\Phi_1'\pi\left(q_0^*\right) < \Phi_1\pi\left(q_0'\right)$, retailer will choose method 2.

4. RS model Considering Capital Constraint

4.4 Model

If we let the wholesale price, $\omega_2 = \dfrac{(\Phi_1 - \Delta\Phi_1)C(q) - C_1(q)}{q}$

Then the retailer's capital need is : $(\Phi_1 - \Delta\Phi_1)C(q_0^*) = Y'$

Add penalty factors $\triangle\Phi_{p1}$ and $\triangle\Phi_{p2}$, and $\triangle\Phi_{p1}$ - $\triangle\Phi_{p2} = \triangle\Phi_1$

Let the retailer's share be : $\Phi_1 - \Delta\Phi_{p1} + \Delta\Phi_{p2}\dfrac{C(q)}{R(q)}$

and the manufacturer's share be : $\Phi_2 + \Delta\Phi_{p1} - \Delta\Phi_{p2}\dfrac{C(q)}{R(q)}$

Then the profit of retailer will be : $(\Phi_1 - \Delta\Phi_{p1})\pi(q)$

and the profit of manufacturer will be : $(\Phi_2 + \Delta\Phi_{p1})\pi(q)$

4. RS model Considering Capital Constraint

4.4 Model

Extreme case 1: $\triangle\Phi_{p1} = 0$, $\triangle\Phi_{p2} = -\triangle\Phi_1$

The profit of retailer will be : $\Phi_1 \pi(q)$

The profit of manufacturer will be : $\Phi_2 \pi(q)$

The capital constraint will not affect the profit of retailer.

Extreme case 2: $\triangle\Phi_{p1} = \triangle\Phi_1$, $\triangle\Phi_{p2} = 0$

The profit of retailer will be : $(\Phi_1 - \Delta\Phi_1)\pi(q)$

The profit of manufacturer will be : $(\Phi_2 + \Delta\Phi_1)\pi(q)$

The profit of retailer is minimum.

4. RS model Considering Capital Constraint

4.4 Model

The profit of retailer is $\left(\Phi_1 - \Delta\Phi_{p1}\right)\pi(q)$

When q is fixed, a lager $\triangle\Phi_{p1}$ can cause a lower profit.

Compared with the profit under method 1 and 2:

$$\left(\Phi_1 - \Delta\Phi_{p1}\right)\pi\left(q_0^*\right) \geq \max\left(\left(\Phi_1 - \Delta\Phi_1\right)\pi\left(q_0^*\right), \Phi_1\pi\left(q_0^{'}\right)\right)$$

$$\Delta\Phi_{p1} \leq \min\left(\Delta\Phi_1, \Phi_1\left(1 - \frac{\pi\left(q_0^{'}\right)}{\pi\left(q_0^*\right)}\right)\right)$$

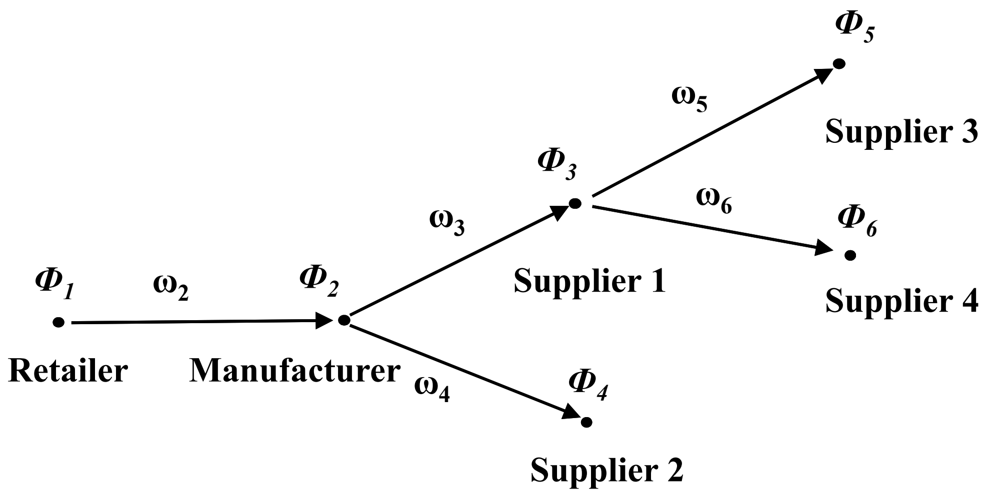
Upper bound on penalty factor $(\mathbf{U}_{pf})$

As the profit of manufacturer should not decrease : $\Delta\Phi_{p1} \geq 0$

$$0 \leq \Delta\Phi_{p1} \leq \Phi_1\left(1 - \frac{\pi\left(q_0^{'}\right)}{\pi\left(q_0^*\right)}\right)$$

4. RS model Considering Capital Constraint

4.5 Example



- Suppose the demand is normally distributed with average value *100* and standard deviation *30*.
  The price is $*30*, salvage value is $*1*.

Number	Cost function	Price under traditional market setting($)
1	0.5q	
2	0.6q	20.5
3	1q	12
4	1.2q	3.5
5	1.7q	3.5
6	2q	4

4. RS model Considering Capital Constraint

4.5 Example

Profit under the RS contract ($)								
Retailer	Manu.	Sup.1	Sup.2	Sup.3	Sup.4	Total profit	Total cost	Optimal quantity
681.4	430.6	342.5	225.1	176.2	195.7	2051.5	871.5	124

Profit under the RS contract with capital constraint($)

Budget	Φ_1'		Wholesale price		Order Quantity		Profit of retailer		Profit of supply chain	
	M1	M2	M1	M2	M1	M2	M1	M2	M1	M2
289.45	33.2%	33.2%	1.825	1.825	124.5	124.5	681.36	681.38	2051.5	2051.5
250	28.7%	33.2%	1.509	[illegible]	[illegible]	[illegible]	[illegible]88.78	666.24	2051.5	2005.9
200	22.9%	33.2%	1.103	1.825	124.5	86.0	469.79	596.79	2051.5	1796.8
150	17.2%	[illegible]	[illegible]	1.825	124.5	64.5	352.85	475.92	2051.5	1432.9
100	11.5%	33.2%	0.305	1.825	124.5	43.0	235.92	325.26	2051.5	979.3

q_{M2} = 200/(1.825+0.5)

Φ_1 =200/871.5

4. RS model Considering Capital Constraint

4.5 Example

Profit under the RS contract ($)					
Total Revenue	**Total profit**	**C/R**	**Total cost**	Φ_1	**Optimal quantity**
2923	2051.5	0.298	871.5	33.2%	124

Profit under the RS contract with capital constraint($)							
Budget	$\triangle\Phi_1$	U_{pf}	$\triangle\Phi_{p1}$	**Profit of retailer**		**Profit of Manu. (RS model)**	**Profit of SC**
				M2	**RS model**		
289.45	0	0	0	681.38	681.38	430.62	2051.5
250	4.5%	0.007	0.0035	666.24	673.81	438.19	2051.5
200	10.3%	0.041	0.0205	596.79	639.08	472.92	2051.5
150	16%	0.100	0.0500	475.92	578.65	533.35	2051.5
100	21.7%	0.173	0.0865	325.26	503.32	608.68	2051.5

5. Conclusion

- Supply chain can hardly get the maximum profit under the traditional market setting.

- When we set $\omega_i = \dfrac{\sum_{j \in S_i} C_j(q) - \sum_{j \in S_i} \Phi_j C(q)}{q}$, member *i* can get Φ_i of total profit of supply chain.

- Under the capital constraint, if we let the wholesale price, $\omega_2 = \dfrac{(\Phi_1 - \Delta\Phi_1) C(q) - C_1(q)}{q}$

  Then let the retailer's and manufacturer's shares be $\Phi_1 - \Delta\Phi_{p1} + \Delta\Phi_{p2} \dfrac{C(q)}{R(q)}$ and $\Phi_2 + \Delta\Phi_{p1} - \Delta\Phi_{p2} \dfrac{C(q)}{R(q)}$, the profit of supply chain and all members can be maximized.

감사합니다.

풍설호, 문일경, 류광열, 권보배