Int. J. Production Economics 143 (2013) 316–326

Contents lists available at SciVerse ScienceDirect

Int. J. Production Economics

journal homepage: www.elsevier.com/locate/ijpe

A fractal echelon approach for inventory management in supply chain networks

Kwangyeol Ryu [a,*], Ilkyeong Moon [a], Seungjin Oh [b], Mooyoung Jung [c]

[a] Department of Industrial Engineering, Seoul National University, Seoul 151-744, Korea
[b] Department of Industrial and Management Engineering, Pohang University of Science and Technology, San31 Hyoja-dong, Nam-gu, Pohang 790-784, South Korea
[c] School of Technology Management, Ulsan National Institute of Science and Technology, 100 Banyeon-ri, Eonyang-eup, Uljin-gun, Ulsan 689-798, South Korea

ARTICLE INFO

Available online 17 January 2012

Keywords:
Fractal-echelon
Vendor managed inventory (VMI)
Fractal-based VMI (fVMI)
Inventory management
Supply chain management (SCM)

ABSTRACT

A major issue in supply chain inventory management is the coordination of inventory policies adopted by different members in a supply chain including suppliers, manufacturers, distributors, etc. This paper presents a fractal-based approach for inventory management in order to minimize inventory costs and smooth material flows between supply chain members while responsively meeting customer demand. Within this framework, each member in the supply chain is defined as a self-similar structure, referred to as a fractal. A fractal-based echelon does not indicate a functional level or composition of supply chain members but indicates a group of multi- or hetero-functional fractals. The basic fractal unit (BFU) consists of five functional modules including an observer, an analyzer, a resolver, an organizer, and a reporter. The application of the fractal concept into inventory management makes it easy to intuitively understand and manage supply chain inventories because similar functional modules can be iteratively applied to an inventory management system. More specifically, we apply the fractal concept to a vendor managed inventory (VMI) model, referred to as fractal-based VMI (fVMI), where a vendor assumes responsibility for maintaining inventory levels and determining order quantities for his buyers. In this paper, we develop mathematical models for the analyzer and resolver to effectively manage supply chain inventories. For validating the proposed approach, a comprehensive simulation model, representing two VMI initiatives including traditional VMI and fVMI, is constructed and used for comparative analyses of case studies.

1. Introduction

A supply chain is a complex network consisting of several organizations with different objectives for the production and distribution of products according to customer demand. Especially, supply chain management (SCM) is concerned with finding the best strategy for the entire supply chain (SC) by coordinating different enterprises along the logistics network or establishing business partnerships (Simchi-Levi et al., 2003). Many researchers have studied ways to optimize SCs so that manufacturers, distributers, and suppliers can maximize their profits. In order to find the best strategy in this complex network, intensive communication, and coordination between trading partners is required so that material and information flow along the SC can be optimized (Sari, 2008).

Among various SCM issues, inventory management is to a greater extent relevant when the entire supply chain, namely a network of procurement, transformation or production, and delivery firms, is considered. According to the literature, inventory usually represents from 20% to 60% of the total assets of manufacturing firms (Giannoccaro et al., 2003). Supply chain inventory management (SCIM) is focused on end-customer demand and aims at improving customer service while lowering relevant costs (Verwijmeren et al., 1996). Inventory management policies prove critical in determining the profit of each supply chain members. The main considerations regarding SCIM policies include (i) nature of optimization (e.g., local, global), (ii) control type (e.g., centralized, distributed), (iii) nature of review of inventory levels (e.g., periodic, continuous, hybrid), (iv) type of demand function (e.g., linear, distribution), and (v) responsibility for inventory control (e.g., self-managed, vendor-managed), as illustrated in Table 1.

An inventory policy can possess local or global goals (Axsäter and Juntti, 1996). Regarding the pursuit of local goals, the SC inventory policy results from a collection of local policies in which every SC member tends to make decisions on its own inventory individually based on local performance criteria. Several effective incentive mechanisms including quantity discounts, profit sharing, buybacks,

* Corresponding author. Tel.: +82 51 510 2473; fax: +82 51 512 7603.
E-mail addresses: kyryu@pusan.ac.kr (K. Ryu), ikmoon@pusan.ac.kr (I. Moon), halmea@postech.ac.kr (S. Oh), myjung@unist.ac.kr (M. Jung).

0925-5273/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
doi:10.1016/j.ijpe.2012.01.002

etc., can be applied to align members' goals or profits in the supply chain. On the contrary, regarding the pursuit of global goals, the SC inventory policy tends to make decisions to optimize the entire inventory based on global performance criteria.

There are two different strategies for managing SC inventory including centralized and distributed (or decentralized) inventory control (Petrovic et al., 1999). Under centralized control, a central decision-maker determines the policy that minimizes the entire SC cost using a high degree of coordination and communication between the SC members. However, under distributed control, each SC member monitors the status of their own local inventory and places orders to their predecessors based on their own performance criteria. With the help of several incentive mechanisms, distributed inventory policies, which are adopted in most cases in SCs, can achieve performance levels almost as high as those realized through centralized policies.

Inventory management policies can also differ in terms of the manner of reviewing inventory levels. Under a periodic-review control policy, the inventory status is reviewed at every stage at a constant time interval. At each review, a replenishment order can be issued in order that the inventory status should meet the target level. In this case, the optimal quantity for replenishment, Q^*, has to be calculated. Under a continuous-review control policy, a replenishment order is issued when the inventory position at the considered stage falls below a predetermined level, i.e., the reorder point. In that case, a fixed quantity, Q, is ordered. A hybrid control policy may also be applied for inventory management. The most common is the (s, S) policy. Under this policy, if the inventory position falls below the reorder point s, an order is issued to raise the stock up to the target level S.

Inventory management policies are characterized based on the demand functions used. Most of the literature assumes that demand follows a certain probability distribution such as normal, Poisson, gamma, and so on. Some researchers use a linear demand function to simplify their models (Dong and Xu, 2002; Nachiappan and Jawahar, 2007). Even though demand is the most important factor for inventory control, it is very difficult to forecast the exact demand in advance. The lack of visibility of real demand can and does cause a number of problems in a SC if it is not properly designed; even then fluctuations cannot be completely eliminated. One of the easiest ways to prepare for such problems is to apply probability distributions or linear functions for demand expectation. However, progressively shorter product life cycles as well as growing innovation rates make demand extremely variable and the collection of statistics, that are required by stochastic models, less and less reliable (Blackburn, 1991).

Finally, inventory policies can be characterized based on the responsibility for inventory control. In traditional inventory control policies, each member is responsible for his own inventory control and production or distribution ordering activities. One fundamental characteristic and problem that all members in a traditional SC including retailers, distributors, and manufacturers must solve is just "how much to order the production system to make (or the suppliers to supply) to enable a SC echelon to satisfy its customers' demands." Each member strives to develop local strategies for optimizing her own organizational goals without considering the impact of her strategies on the performance of other members. Upstream members do not know actual demand information from the market place because no information is shared between members (see Fig. 1(a)). SC members use only replenishment orders placed by their immediate downstream member to create demand forecasts and inventory plans. In other words, each echelon in the SC has information only about what their immediate customers want and not on what the end customer wants. Each member of SC, therefore, replenishes her own inventory by considering her local inventory position.

In contrast to the traditional inventory control, many companies have been compelled to improve their SC operations by sharing demand and inventory information with their upstream and downstream members including customers (Disney and Towill, 2003a). Vendor managed inventory (VMI), also known as continuous replenishment, supplier-managed inventory, or consignment inventory, is a SC strategy where the vendor or supplier is given the responsibility of managing the customer's stock. VMI is one of the most widely discussed partnering initiatives for encouraging collaboration and information sharing between trading partners (Angulo et al., 2004). Under the VMI program, the retailer provides the vendor with access to its real-time inventory level through for instance POS (point of sale) data. The retailer may set a certain service level or spatial capacity for stocks, which are then taken into consideration by the vendor. Based on the information on retailers through POS systems for example, the vendor decides on the appropriate inventory level of each of the products and appropriate inventory policies to maintain those levels (see Fig. 1(b)). As a consequence, the retailer's role shifts from managing inventory to simply renting retail space (Simchi-Levi et al., 2003; Mishra and Raghunathan, 2004).

Table 1
Main considerations affecting SCIM policies.

Considerations for SCIM	Exemplary choices
Nature of optimization	Local; global
Control type	Centralized; distributed
Nature of review of inventory levels	Periodic; continuous; hybrid
Type of demand function	Probability distribution; linear
Responsibility of inventory control	Self-managed; vendor-managed

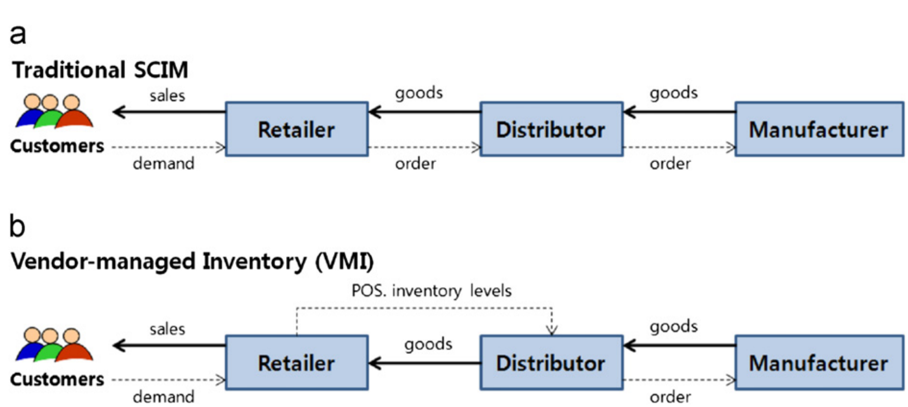

Fig. 1. SC structure under SCIM and VMI. (a) Material and information flow under traditional SCIM policy and (b) material and information flow under VMI policy.

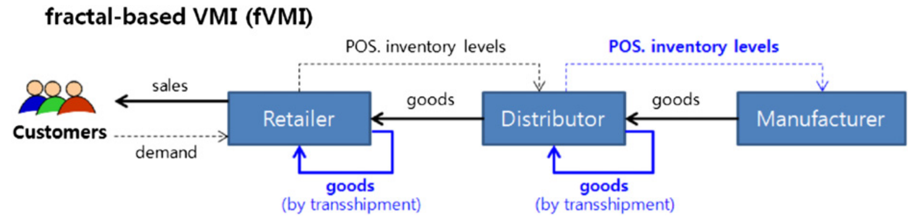

Fig. 2. SC structure under fVMI.

As illustrated in Fig. 1, the main difference between a traditional inventory control policy and a VMI program is who makes the decision to replenish retailer's inventory, i.e., whether or not the retailer places replenishment orders. When we consider a three-stage SC, which consists of three echelons including a manufacturer, a distributor, and a retailer, the distributor looks at its own inventory status and that of the retailer as well. On the other hand, all other echelons of the SC, i.e., between the distributor and manufacturer, operate in the same way as in traditional inventory management.

VMI has become more popular in the grocery sector in the last 20 years due to the success of retailers such as Wal-Mart (Andel, 1996; Waller et al., 1999). It was subsequently implemented by various leading companies in diverse industries including Electrolux (De Toni and Zamolo, 2005), Nestle and Tesco (Watson, 2005), Boeing (Micheau, 2005), etc. The popularity of VMI has led to the claim that VMI is the wave of the future and the concept will revolutionize the distribution channel (Burke, 1996; Cottrill, 1997). VMI is beneficial to both buyers and suppliers though the supplier may take a longer period of adjustment and reconfiguration before the benefits of VMI can be realized. VMI offers a competitive advantage for retailers with respect to higher product availability as well as reductions in holding costs and some operational costs plus cash flow benefits (Benefield, 1987; Achabal et al., 2000; Yao et al., 2007). On the other hand, vendors may gain opportunities to improve production and marketing efficiencies (Cottrill, 1997; Waller et al., 1999; van der Vlist et al., 2007), and synchronize replenishment planning (Waller et al., 1999; Çetinkaya and Lee, 2000) while reducing the bullwhip effect (Lee et al., 1997a, b; Disney and Towill, 2001, 2003a, b). However, a number of challenges may exist in practice that can potentially reduce the benefits gained from VMI or lead to failures in VMI programs. The readers can refer to the literature for some unsuccessful cases of VMI, such as Spartan Stores (Simchi-Levi et al., 2003) and Kmart (Fiddis, 1997).

The objective of this paper is to propose a fractal-based conceptual framework for inventory management in order to minimize inventory costs and smooth material flows between SC members while responsively meeting customer demand. In this paper, the fractal concept has been adopted because of its intrinsic benefits in dealing with a dynamically changing SC environment including customer demand. By applying the fractal concept to SCIM, each member in the SC is defined as a self-similar structure, which is referred to as a fractal. We will discuss the concept, structure, functions, etc. of a fractal in the following sections in detail. In this paper, we also consider a VMI model, which consists of a three-stage SC including a manufacturer, a distributor, and a retailer. As shown in Fig. 2, however, we extend the concept of vendor-managed control to the relationship between distributors and the manufacturer. In other words, we maintain consistency in the relationships regarding "one vendor-multiple buyers," and the vendor manages the buyer's inventories. For example, the manufacturer (vendor) manages the distributor's (buyer's) inventories, and the distributor (vendor) manages the retailer's (buyer's) inventories. We also consider transhipments between members in the same echelon to prevent backorders and ensure smooth material flow as in Fig. 2. Since we have applied the fractal concept to VMI, our model is referred to as fractal-based VMI (i.e., fVMI). According to the main considerations affecting SCIM policies as shown in Table 1, the proposed inventory policy in this paper can be classified as summarized in Table 2.

Table 2
Adopted SCIM policies.

Considerations for SCIM	Policy
Nature of optimization	Local
Control type	Distributed
Nature of review of inventory levels	Hybrid
Type of demand function	Probability distribution
Responsibility of inventory control	Vendor-managed

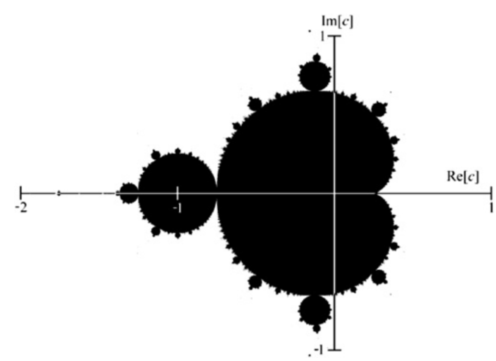

Fig. 3. Mandelbrot cell.

The remainder of this paper is organized as follows. Section 2 describes the concept, structure, and intrinsic characteristics of a fractal. A fractal-based framework for VMI is proposed in Section 3, followed by mathematical models in Section 4. A comparative study using simulation is discussed in Section 5 before concluding remarks are presented in Section 6.

2. Fundamentals of fractals

A fractal is a rough or fragmented geometric shape that can be split into parts, each of which is a reduced-size copy of the whole, a property called self-similarity (Mandelbrot, 1982). The term 'fractal' was coined by Benoit Mandelbrot in 1975 with a very simple formula such as $Z=Z^2+C$, where Z and C are complex numbers, to describe complex organisms and structures in nature using self-similar characteristics (see Fig. 3 for a Mandelbrot cell). The architectural model of a fractal represents a hierarchical structure built from elements of a basic fractal unit (BFU), and the design of a basic unit incorporates a set of pertinent attributes that can fully represent any level in the hierarchy (Tirpak et al., 1992). This means that the term 'fractal' can represent an entire organization at the highest level or an elementary unit at the bottom level. This paper basically adapts the formal definition of the fractal to solve SCIM problems, referred to as a fractal-based vendor-managed inventory (fVMI) model.

Warnecke (1993) defined a fractal as an independently acting corporate entity whose goal and performance can be precisely described. Considering an implementation methodology for fractals, Ryu and Jung (2003) defined a fractal as a set of self-similar agents whose goal can be achieved through cooperation, coordination, and negotiation with others. Each BFU provides services with an individual goal and acts autonomously and independently, and reorganize the configuration of the fractal system to a more efficient and effective one. However, for the fractal to function as a coherent whole, goal consistency is maintained though a goal-formation process that is supported by an inheritance mechanism (Tharumarajah et al., 1996). The conceptual structure of fractals in fVMI is depicted in Fig. 4; it looks like a hybrid control architecture when represented in IDEF0 notation, which is frequently cited in the manufacturing domain (Bravoco and Yadav, 1985). However, the composition of fractals shown in Fig. 4 can be restructured in order to optimize the performance of the top-level fractal by the dynamic restructuring process (DRP), which is one of the intrinsic characteristics of a fractal. The fractal has several intrinsic characteristics including self-similarity, self-organization, self-optimization, and goal-orientation. For more details on fractal-specific characteristics, see Ryu et al. (2003a) and Ryu and Jung (2003, 2004).

The characteristic of self-similarity not only refers to the structural characteristics of organizational design, but also circumscribes the manner of performing a job (service), as well as the formulation and pursuance of goals (Warnecke, 1993). For achieving the goals of the top-level fractal (e.g., a manufacturing firm), there can be a variety of possible solutions to individual problems. Even though fractals may arrange their internal structures differently, they are 'self-similar' with others if they can make the same outputs with the same inputs regardless of their structures. This characteristic can be used to organize the functional structures of members in SCs and manage SCs (Ryu et al., 2003b).

Self-organization in fVMI affects both theoretical and operational methods. On the one hand, theoretical self-organization methods referred to as self-optimization, mean the application of suitable methods for SCIM. On the other hand, operational self-organization methods, referred to as DRP, support the reconfiguration of connections between fractals and the reorganization of fractals in the system. DRP is a distinctive and unique characteristic of fractal-based systems compared with others; fractals can autonomously reconfigure and reorganize themselves. Specifically, DRP is an attractive mechanism to apply to SCM since relationships between SC members are variable.

Normally, the goals of a fractal may be different from those of other fractals. For example, a customer wants to get a quality product from the retailer and all other members in fVMI, including the manufacturers, distributors, and retailers as in Fig. 2, want to minimize their costs. To coherently achieve these goals, goal consistency should be maintained. Goal-formation is the process of generating goals by coordinating processes between participating fractals and modifying them as necessary. It is supported by an inheritance mechanism to ensure goal consistency. Warnecke (1993) pointed out that the goal-formation process is a reliable method for revealing any conflicts between competing goals. It should continue to be developed autonomously in order to harmonize fVMI by resolving conflicts. The goal in fVMI can be achieved in an iterative fashion by developing individual goals for each fractal and their respective feedbacks.

3. Fractal-based framework for vendor managed inventory (fVMI)

In this paper, we apply the fractal concept to VMI for the effective management of inventory levels and minimization of relevant costs as well as the use of the intrinsic characteristics of a fractal, as mentioned in the previous section. To do so, we first need to take into account the fractal architecture and functional modules. The BFU consists of five functional modules including an observer, an analyzer, a resolver, an organizer, and a reporter regardless of its hierarchical position (Ryu et al., 2003a). In short, all the fractals defined in the system have the same functional structure. In fVMI, the compositions of the functional modules of a fractal are the same as that defined in manufacturing domains but with slightly different functions, as illustrated in Fig. 5.

3.1. The fVMI framework

Three-stage SCs are considered for the fVMI model in this paper, including a manufacturer (a top-level fractal), distributors (middle-level fractals), and retailers (bottom-level fractals), as illustrated in Fig. 6. Consistent with the fractal concept, each member of the SC may be defined as a fractal by itself, and any combination of members can also be defined as a fractal. Combinatorial compositions of fractals are normally possible depending on the transactions between members. In this paper,

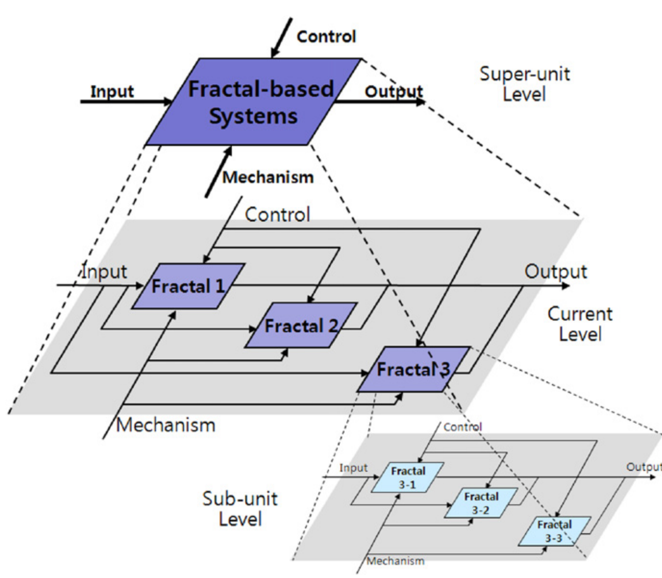

Fig. 4. Conceptual structure of fractal-based systems.

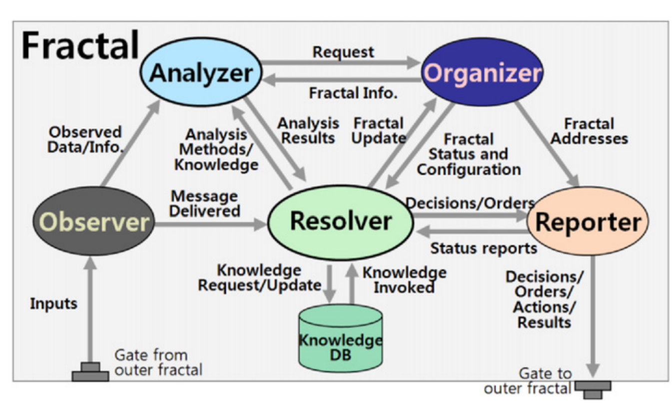

Fig. 5. Fractal architecture and functional modules.

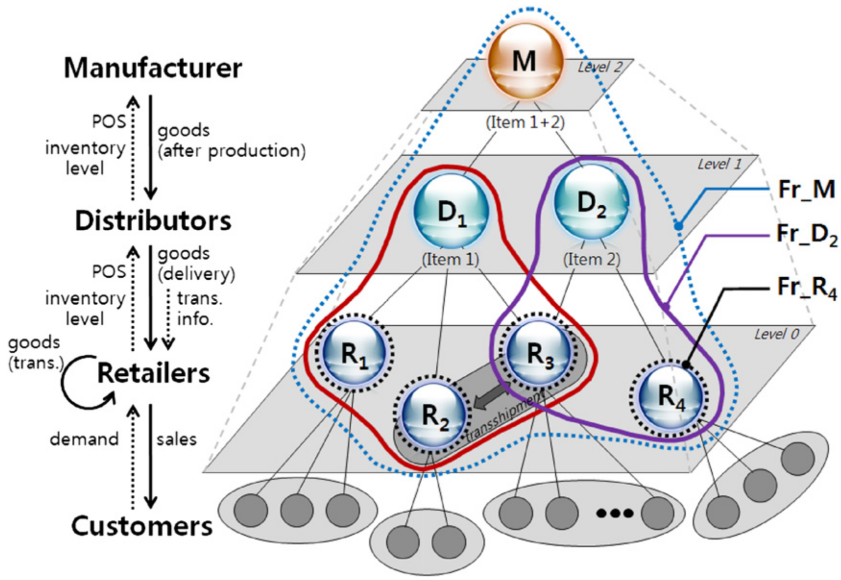

Fig. 6. Composition of fractals in fVMI.

however, we assume the following rules to define fractals to accommodate the realities of fVMI and realize model simplicity.

- Each bottom-level fractal (i.e., retailer) can be a fractal by itself, and deal with multiple items. For example in Fig. 6, the retailers (from R_1 to R_4) are defined as fractals (from Fr_R_1 through Fr_R_4), respectively. R_1 through R_3 sell item 1, and R_3 and R_4 deal with item 2. Note that R_3 sells items 1 and 2 together. Also note that the term 'item' here means a type of goods such as clothes, shoes, etc.
- Each middle-level fractal (i.e., distributor) can be a fractal that includes sub-fractals, and deal with its own item (i.e., different distributors deal with different items). In Fig. 6, fractals for two distributors, namely, D_1 and D_2, are defined as Fr_D_1 including Fr_R_1 through Fr_R_3 and Fr_D_2 including Fr_R_3 and Fr_R_4, respectively. D_1 and D_2 deal with item 1 and item 2, respectively.
- Multiple bottom-level fractals (i.e., retailers) can be managed as an intermediate fractal by a parent fractal (i.e., distributor) if they are located close to each other because of the transhipment issue. For example, if we assume that retailers R_2 and R_3 are located very close to each other, it is better for Fr_D_1 to manage Fr_R_2 and Fr_R_3 together by grouping them into an intermediate fractal such as Fr_R_{2+3}, for example. As a consequence, Fr_D_1 will manage two fractals, viz. Fr_R_1 and Fr_R_{2+3}. This is effective when backorders occur at either R_2 or R_3 because D_1 can flexibly control the inventories of R_2 and R_3 by letting them get goods faster via transhipment.
- A top-level fractal (i.e., manufacturer) can be a fractal including sub-fractals and deal with different items. In Fig. 6, a manufacturer is defined as a fractal named Fr_M, which includes Fr_D_1 and Fr_D_2 as sub-fractals.
- In case that information on exact transactions between customers and retailers is extremely important, fractals consisting of a retailer and a group of customers may be defined. However, if a demand function is assumed to follow a certain probability distribution such as normal, Poisson, or gamma, such fractals may not be used in this paper.
- Information (e.g., demand, POS, and inventory levels) usually flows from customers (bottom) to the manufacturer (top), and on the other hand products flow from top to bottom. In Fig. 6, information flows are represented using dotted arrows to differentiate them from the flows of physical products.
- Transhipment is permitted only between closely located fractals, if and only if they are dealing with the same item. Since we consider just two distributors who deal with different items for model simplicity, transhipment between distributors does not occur in our case, as in Fig. 6.

The proposed fVMI approach has advantageous features for managing VMI by adopting the fractal concept. Firstly, it becomes easy to understand a complicated system. The system developed under the fVMI framework can be understood intuitively owing to the self-similarity of fractals. Similar functional modules are iteratively applied to the entire system. In this way, the traditional concept of echelons used in SCIM no longer exists in the fVMI model. Hierarchical layers of fractals, referred to as fractal-echelons, are used in this research instead. Secondly, fractal-specific characteristics can be added to the system. A system developed under the fractal concept already has the intrinsic characteristics of a fractal including self-similarity, self-organization (self-optimization and DRP), goal-formation, and so on. The use of the self-organizing characteristic generates rules for optimizing the goal and structure of fractals during goal formations. The rules for optimization can be customized to use depending on the hierarchical position of each fractal, or whom it represents (e.g., retailers, distributors, the manufacturer), and so on. Thirdly, local optimization reduces the computational burden and time. fVMI tries to optimize the system through local optimization rather than global optimization. Usually, the use of local optimization techniques distributes the computational load, reduces the computing time, and encourages rapid responses of the system. Since fluctuations in customer demand make it difficult to find global optimum in SCIM problems, the aggregated decision from local optimum may give faster responsiveness to customers, and thereby increase customer satisfaction. A large SCM problem can be divided into smaller ones so that problems can be solved easier and faster. Lastly, flexible management of VMI is possible. The numerical model for fVMI can emerge from existing models. If models exist in the system for a specific member, such models can be integrated as a part of the fVMI model if goal consistency can be maintained. The allowance of

Table 3
Comparison of traditional VMI and fVMI.

Characteristics	Traditional VMI	fVMI
Representation of SC members	Echelon (a group of members with the same functions in SC)	Fractal-echelon (a group of members with multi- or hetero-functional fractals)
Range of application of VMI	In between two echelons including the bottom echelon (VMI control is restricted)	In between any vendor and any buyer in all fractals (VMI control is applied recursively)
Consideration of transhipment	Not considered normally	Considered between buyers (only between fractals that are closely located and deal with the same item)

model integration makes it possible to manage inventories with more flexibility. Furthermore, flexibility in inventory control increases in fVMI because parent fractals may promote transhipment between child fractals.

In addition to the aforementioned features, an fVMI approach is different from the traditional VMI. A comparison of Fig. 1(b) with Fig. 2 reveals differences, as described in Table 3.

3.2. Functional modules of a fractal in fVMI

The basic fractal unit (BFU) consists of five functional modules including an observer, an analyzer, a resolver, an organizer, and a reporter, as illustrated in Fig. 5. These modules conduct assigned jobs to achieve their goals through cooperation. Especially, the analyzer and the resolver play more important roles than other functional modules since they perform operations with respect to decision-makings in fVMI. The following are detailed functions of each module of a fractal.

(i) Observer: the function of the observer is to monitor the state of the fractal, receive data or messages from outer fractals, and transmit composite information to the correspondent modules in the fractal. The messages or data from outer fractals differ from each other with respect to the hierarchical position of the fractal. Common inputs to a fractal mainly include inventory levels or POS data of sub-fractals and backorder information, if it exists, as illustrated in Fig. 6. If the fractal is functioning as a retailer, the observer monitors orders and any kind of requests from customers.

(ii) Analyzer: the function of the analyzer is to analyze and forecast (expected) costs of the fractal based on information provided by the observer and the predefined criteria (e.g., maximum inventory capacity of each fractal, lead time, unit costs for holding stocks and backorders, etc.). The analyzer calculates the expected replenishment quantity of each sub-fractal using a numerical formula if it is not included in bottom-level fractals (i.e., retailers). The analyzer summarizes the analysis results and provides them to the resolver so that the resolver may take them into account while making decisions. A detailed mathematical model will be discussed in Section 4.

(iii) Resolver: the resolver plays the most important role among the functional modules of a fractal because it is in charge of decision-making processes. Based on information from the observer such as transaction records and real-time inventory levels of sub-fractals as well as the results from the analyzer, the resolver may employ a variety of numerical optimization or heuristic models to find and pursue the optimal goal of the fractal during the goal-formation processes. Specifically, the resolver makes decisions on the replenishment quantities of sub-fractals and transhipment quantities if backorders are expected to occur in any sub-fractal. Each fractal may have its own optimization model, which is appropriate for realizing its goal. In addition to making decisions, the resolver can give the fractal insight for developing future strategies. For example, the resolver of the top-level fractal, representing the manufacturer in this paper, can help a factory manager to develop a production schedule and future strategies by suggesting useful information such as modification of distributors' composition, addition or deletion of items to produce, etc. The optimization model of the resolver will be described in detail in Section 4.

(iv) Organizer: the organizer manages the statuses and addresses of fractals, namely, structural information. For DRP, the organizer may use numerical optimization methods, thereby an optimal configuration of fractals can be found. Making an efficient and effective configuration of fractals allows a system to have balanced workloads between fractals. As the workload of a fractal proportionally increases along with the increase in transactions, the organizer iteratively computes the number of transactions and provides them to the resolver. The reconfiguration of fractals is decided by the resolver, if necessary. The fractal address is used to find the physical address (e.g., *distributor_id*, *retailer_id*, etc.) of each fractal by the reporter.

(v) Reporter: the reporter conveys the results acquired from all processes of a fractal to other fractals. Under the fVMI approach, the message sent by the reporter to a sub-fractal may include the transhipment instruction, which informs the certain sub-fractal how to get additional goods, and the expected delivery time or delivery status regarding the replenishment of the sub-fractal's inventory. Regarding the response to the parent fractal, the reporter should inform the inventory level or status in real time according to the request from the parent fractal.

3.3. SCIM policy in fVMI

As summarized in Table 2, the type of review of inventory levels is hybrid, and the type of demand function is stochastically distributed in fVMI. In this paper, therefore, we adopt the (s, S) inventory control policy for SCIM. In Fig. 7, S indicates the order-up-to level, and s indicates the reorder point. Generally, the determination of s and S is important to minimize the cost for managing inventory under an (s, S) policy, and many researchers have studied ways to optimize s and S under various demand functions (Archibald and Silver, 1978; Kelle and Milne, 1999; Ching, 2001; Christina and Christina, 2005). In this paper, we focus on the determination of the replenishment quantity, q or q' in Fig. 7, for predetermined s and S. In fVMI, s and S are predetermined before inventory is operated, and are not changed during operation. Instead of modifying s and S, the replenishment quantity is optimized to reflect changes in the demand patterns at time T_r.

Most studies assumed that the backorder cost is significantly larger than the other cost factors such as the inventory holding cost, ordering cost, and transportation cost (Herer et al., 2002). In fVMI, transhipments are also considered to minimize the backorder cost. At time T_r, the expected backorder quantity (viz. s—expected demand during L) is forecasted based on the previous demand patterns, and immediately procured from closely located fractals.

4. Mathematical fVMI Model

Among five functional modules, the analyzer and the resolver are associated with decision-making in fVMI. The analyzer

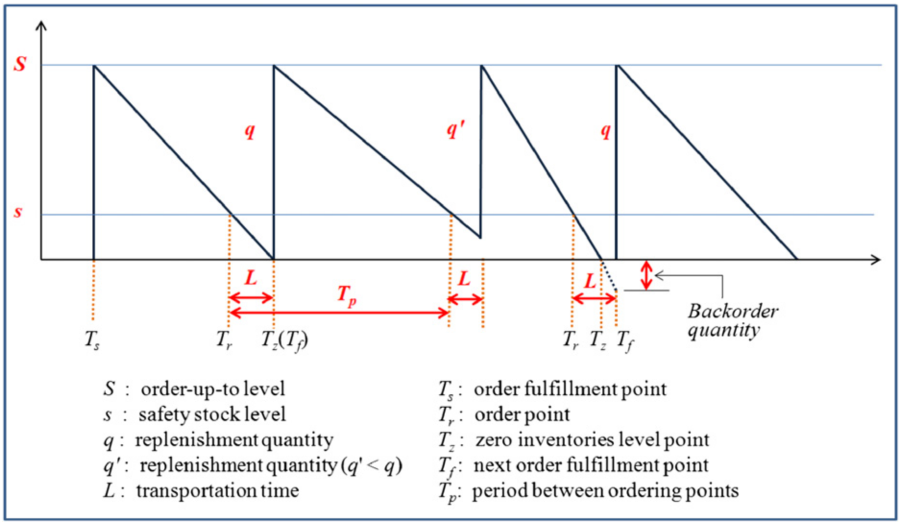

Fig. 7. (s, S) Policy.

performs cost analysis, and the resolver makes decisions with necessary information. The observer monitors the inventory statuses of the child fractals, and delivers the status information to the analyzer and resolver. The formulas for the analyzer and the resolver are different depending on the levels and concerned issues. Each formula is to be customized with respect to the situation and goals. Numerical models for fractals in fVMI are based on only the cost of fractals at each level since intangible factors are difficult to express numerically. Therefore, in order to achieve the goals, the goal of each fractal is regarded as the minimization of its cost. The computations are performed iteratively from bottom-level fractals to the top-level fractal.

4.1. Conceptual model and computational procedure

The analyzer of a fractal performs cost analysis. While analyzing costs, the analyzer calculates the expected inventory costs of the sub-fractals, the expected backorder costs of the sub-fractals, the transportation cost, and the ordering cost for the replenishment quantity q. If a fractal contains n sub-fractals, the cost of the fractal with replenishment quantity q, C_q, can be calculated as follows:

$$C_q = \sum_{i=1}^{n}\{E(C_{Invi})+E(C_{Bci})\}+C^f+\sum_{i=1}^{n}C_{qi} \tag{1}$$

where $E(C_{Invi})$ is the expected inventory cost of child fractal i ($i=1, \ldots, n$), $E(C_{Bci})$ is expected backorder cost of child fractal i ($i=1, \ldots, n$), C^f is additional cost of fractal f including the transportation and ordering costs, and C_{qi} is cost of child fractal i ($i=1, \ldots, n$).

The cost of lower level fractals (C_{qi}) can be calculated in a similar manner until the calculation of the costs of all fractals is completed. Additional cost is to be customized depending on the situation of the fractal. The results of the analyzer will be used by the resolver to optimize the goal of each fractal. The optimization process is performed locally at each sub-fractal first; then, the supervisory fractal integrates the sub-fractals. In case additional factors are optimized, the fractal also integrates them. The goal of the fractal (g_f) can be optimized as follows:

$$g_f = F_f \oplus g_1 \oplus g_2 \oplus \cdots \oplus g_n \tag{2}$$

where F_f is the numerical expression for the additional factors of fractal f, and g_i is the goal of child fractal i ($i=1, \ldots, n$).

The sign '$\oplus$' refers to the integration of models. Depending on the goal of the fractal, the model can be maximized or minimized during optimization processes. However, the consistency of goals should be maintained to facilitate optimization of the goals. For example, if the model contains the cost of the fractal, the cost should be minimized. If once the fractal decides to minimize its goal, other goals also should be minimized. Integration of numerical models can be applied iteratively to any level of fractals in a manner similar to that shown in Eq. (2).

Complex and integrated numerical models of the top-level fractal can be built from a simple model of bottom-level fractals by iteratively adapting simple models. The results of adaptation at each level are used to not only control intra-fractal processes but also give information to upper-level fractals. Fig. 8 illustrates the procedure when the numerical models are applied to the problems.

4.2. Numerical model for the analyzer

The analyser performs cost analysis of the fractal at time T_r, as shown in Fig. 7. To define and customize numerical models for the fractal in fVMI, the following notation is used in this paper:

i	set of sub-fractals ($i=1, \ldots, N$)
j	set of products ($j=1, \ldots, M$)
hR_{ij}	inventory holding cost of sub-fractal i for product j per unit time
cD_{ij}	ordering cost of sub-fractal i for product j per unit time
tc_{ij}	transportation cost of sub-fractal i for product j per unit time
$tct_{jii'}$	transportation cost of product j between sub-fractals i and i'
cB_{ij}	backorder cost of sub-fractal i for product j per unit time
C_i	expected cost of sub-fractal i per unit time
C_f	expected cost of fractal f with the replenishment quantity q_{ij} per unit time
x_{ij}	demand for product j in sub-fractal i during time period L (probability function$=f(x_{ij})$)
L	transportation time
S_{ij}	order-up-to level
s_{ij}	inventory level at the ordering point of sub-fractal i for product j
T_r	ordering point
T_p	expected period between ordering points

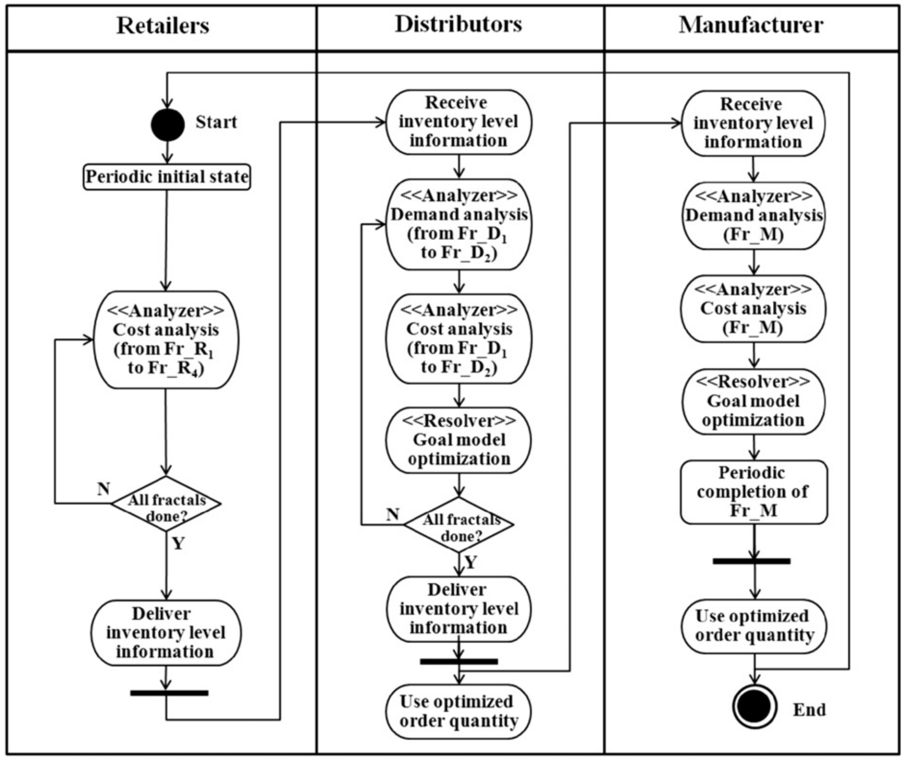

Fig. 8. Procedure for adapting the numerical model to fVMI.

$qt_{jii'}$ transhipment quantity for product j between sub-fractals i and i'
q_{ij} replenishment quantity for product j for sub-fractal i

$$C_f = \sum_{i=1}^{N}\sum_{j=1}^{M}\left[\frac{C(T_p)}{T_p}\right] + \sum_{i=1}^{N} C_i = \sum_{i=1}^{N}\sum_{j=1}^{M}\left[\frac{C(L)+C(T_p-L)}{T_p}\right] + \sum_{i=1}^{N} C_i \tag{3}$$

The expected cost during L, $C(L)$ is given as follows:

$$\begin{aligned} C(L) = {} & \int_0^{s_{ij}} \left[\left\{\frac{hR_{ij}}{2} \times x_{ij} + hR_{ij} \times (s_{ij}-x_{ij})\right\} \times L\right] f(x_{ij})dx_{ij} \\ & + \int_{s_{ij}}^{\infty} \left\{\frac{hR_{ij}}{2} \times \frac{L \times s_{ij}^{2}}{x} + (x_{ij}-s_{ij}-qt_{jii'}) \times cB_{ij}\right\} f(x_{ij})dx_{ij} \\ & + tc_{ij} \times q_{ij} + tct_{jii'} \times qt_{jii'} + cD_{ij} \end{aligned} \tag{4}$$

The expected cost during (T_p-L), $C(T_p-L)$ is given as follows:

$$C(T_p-L) = \frac{hR_{ij}}{2} \times (q_{ij}-s_{ij}) \times (T_p-L) + hR_{ij} \times s_{ij} \times (T_p-L) \tag{5}$$

where

$$T_p = \frac{(q_{ij}-s_{ij}) \times L}{\left[\int_{-\infty}^{\infty} x_{ij} f(x_{ij})dx_{ij}\right]} + L \tag{6}$$

In Eq. (4), the inventory holding, backorder, transportation, and ordering costs are considered. The first part of Eq. (4) formulates the situation where the demand (x_{ij}) is smaller than the current inventory level. If demand is larger than the current inventory level, the backorder cost should be considered in the numerical model, as shown in the second part of Eq. (4). Here, $(x_{ij}-s)$ refers to the backorder quantity. In this model, the transportation cost is imposed on a unit of product, and the ordering cost is imposed on a unit of order. Abstract constraints such as the service level of the company are ignored in the numerical model due to the complexity of formulation. Without modifying the model, the proposed cost model can be used at all levels of the fractal in fVMI. Of course, the sets of sub-fractals and products vary depending on the level of the fractal. The total cost related to the inventory management of an entire SC can be calculated by the summation of the costs of all the sub-fractals in the supply chain. The results of profit analysis will be used by the resolver to optimize the goal of each fractal.

4.3. Numerical model for the resolver

The resolver generates important information for making decisions regarding replenishment quantity, transhipment, and delivering point. Based on the results from the analyzer as well as data about the current situation and the fractal's goals, the resolver solves the objective function of the fractal in order to retrieve crucial information so that it can be used to make strategic decisions for the fractal. Assume that the goal of each fractal is to minimize cost with respect to inventory management. Because fractals are self-similar, the goal model of the fractal has the same form in all levels of the fractal. The goal model for a resolver in fVMI is formulated with two decision variables, as indicated below:

q_{ij} replenishment quantity for product j for sub-fractal i
$qt_{jii'}$ transhipment quantity for product j between sub-fractals i and i'

The goal model of the resolver in fractal f, i.e., g_f, at time T_r is as follows:

$$g_f = min\ C_f(q_{ij}, qt_{jii'}) \oplus g_1 \oplus g_2 \oplus \cdots \oplus g_N \tag{7}$$

subject to

$$\sum_{j=1}^{M} q_{ij} \le caR_i \quad for\ all\ i \tag{8}$$

$$\sum_{i=1}^{N}\sum_{j=1}^{M} q_{ij} \le caD \tag{9}$$

$$\sum_{i=1}^{N} qt_{jii'} \le s_{i'j} \quad for\ all\ i',\ j \tag{10}$$

$$q_{ij} \ge 0 \quad for\ all\ i,\ j\ (integer\ value) \tag{11}$$

$$qt_{jii'} \ge 0 \quad for\ all\ i,\ i',\ j\ (integer\ value) \tag{12}$$

where caR_i is the inventory capacity of sub-fractal *i, and caD* is the inventory capacity of fractal *f*.

Eq. (7) is the objective function of the goal model. The main objective of the goal model in fVMI is to minimize the inventory cost of the fractal. The proposed objective function includes the costs of backorder, holding inventory, ordering, transportation, and transhipment. Eq. (8) implies that the replenishment quantity for sub-fractal *i* should be smaller than the inventory capacity of sub-fractal *i*. Eq. (9) is the capacity constraint of fractal *f*. Eq. (10) means that the transhipment quantities from sub-fractal i' to all sub-fractals excluding i' should be smaller than the inventory level at the reorder point of sub-fractal i' ($s_{i'j}$).

With given *S*, *s*, and the constraints, the replenishment quantity (q) can be obtained using the following equation:

$$q_{ij} = \begin{cases} S_{ij} - s_{ij} + \frac{S_{ij}-s_{ij}}{T_r - T_s} \times L, & \text{if } \frac{S_{ij}-s_{ij}}{T_r-T_s} \times L \le s_{ij} \\ S_{ij} & \text{otherwse} \end{cases} \tag{13}$$

Transhipment is triggered when $(S_{ij} - s_{ij}) \times L/(T_r - T_s) > s_{ij}$, and the transhipment quantity ($qt_{jii'}$) can be obtained using the following objective function:

$$Minimize \sum_{i'=1}^{N}\sum_{j=1}^{M} qt_{jii'}\, tct_{jii'} + \sum_{i'=1}^{N}\sum_{j=1}^{M} \left(\frac{S_{ij}-s_{ij}}{T_r - T_s} \times L - s_{ij} - qt_{jii'} \right) \times cB_{ij} \tag{14}$$

subject to

$$qt_{jii'} \le s_{i'j} \quad for\ all\ i',\ j. \tag{15}$$

5. Comparative study based on simulation

To examine the operability and the efficiency of the fVMI framework, a comparative study has been conducted based on simulations. We have applied three SCIM initiatives to an illustrative supply chain and used various demand patterns as input parameters for the simulations. The total cost of the SCIM is used as an evaluation criterion.

5.1. Illustrative supply chain

In the simulation, three-stage supply chains are assumed, including one manufacturer (M), two distributors (D_1 and D_2), and four retailers (R_1 to R_4), as illustrated in Fig. 6. The manufacturer produces two types of items (p_1, p_2). D_1 distributes p_1 to R_1, R_2, and R_3. D_2 distributes p_2 to R_3 and R_4. Transhipment is permitted only between R_2 and R_3 for p_1 (tct_{123}=tct_{132}=\$2). The transportation lead time between echelons is one day, and the transportation lead time within an echelon (transhipment lead time) is zero day. We assume that the demand for each product follows a normal distribution ($N(\mu, \sigma^2)$). Basic information of the illustrative SC is summarized in Table 4. Basic information including the inventory holding, ordering, transportation, and backorder cost of p_1 are the same as those of p_2 in the simulation.

In the simulation, three SCIM initiatives, traditional SCIM (see Fig. 1(a)), traditional VMI (see Fig. 1(b)), and fVMI (see Fig. 2), are compared. In all three supply initiatives, the (s, S) inventory control policy is used for replenishment decisions, and a stochastic demand function is assumed for customers. In the simulation, s and S are predetermined before operating the inventory, and are not changed during operation. To determine s and S, we adopt Archibald and Silver's (1978) model.

In fVMI, a fractal determines the replenishment quantities of the sub-fractals. Because the fractal knows the demand functions of the sub-fractals, s and S can be calculated using Archibald and Silver's (1978) model. The replenishment quantities of fractals are determined based on the previous demand patterns through the upper-level fractal (parent fractal). In fVMI, transhipments are also considered to minimize the backorder cost.

Under traditional SCIM, the replenishment quantity of each member is determined by herself. The replenishment quantity equals S. At the echelon of retailers, s and S can be simply calculated. However, at the n echelons of distributors and manufacturers, distributors and manufacturers do not know the demand function of customers for each product. Therefore, they cannot use Archibald and Silver's (1978) model to obtain the optimized S. We used the s and S determined in fVMI, because the main purpose of the simulation is to formulate the effect of the replenishment and transhipment quantities under the same values of s and S.

Under traditional VMI, the replenishment quantities of retailers and distributors are determined by the distributors, and the distributor may know the demand function of customers. Values of S and s in traditional VMI are equal to the values in fVMI. The replenishment quantities of retailers are determined based on the previous demand patterns. The replenishment quantity of a distributor is equal to S for the distributor. The replenishment and transhipment quantities according to the type of SCIM are summarized in Table 5.

5.2. Simulation setup

The simulation considered two independent factors including the type of SCIM and variability in customer demand. The number of levels of these factors and their values are listed in Table 6. Customer demand for the product in the retailer is summarized in Table 7.

To evaluate the effects of independent factors, the total cost for the entire supply chain (TSC) is used. TSC includes the inventory holding cost, ordering cost, transportation cost, backorder cost, and transshipment cost. The simulation is carried out for 48

Table 4
Basic information of the illustrative supply chain.

	M	D_1	D_2	R_1	R_2	R_3	R_4
Inventory holding cost (\$/week)	0.25	0.50	0.50	0.75	0.75	0.75	0.75
Ordering cost (\$/order)	2.5	2.5	2.5	2.5	2.5	2.5	2.5
Transportation cost (\$/unit)	0.1	0.2	0.2	None	None	None	None
Backorder cost (\$/unit)	5	10	10	15	15	15	15
Inventory capacity (unit)	600	200	200	100	100	100	100

Table 5
Replenishment and transhipment quantities.

	Traditional VMI	fVMI
Retailers	q_R = Eq. (13)	q_R = Equation (13) qt_R = Equation (14)
Distributors	$q_D = S_D$	q_D = Equation (13)
Manufacturers	$q_M = S_M$	q_M = Equation (13)

q: replenishment quantity, qt: transhipment quantity.

Table 6
Independent factors of the simulation.

Independent factors	Levels		
	1	2	3
Type of SCIM	Traditional VMI	fVMI	
Variability in customer demand (α)	1	1.5	2

Table 7
Customer demand for the product at the retailer (unit/week).

	R_1	R_2	R_3	R_4
p_1	$N(150, 15\alpha)$	$N(100, 10\alpha)$	$N(50, 5\alpha)$	$N(0, 0)$
p_2	$N(0, 0)$	$N(0, 0)$	$N(150, 15\alpha)$	$N(250, 25\alpha)$

Table 8
Simulation results according to the independent factors.

	TSC ($)	
	Traditional VMI	fVMI
α=1	26,041	25,285
α=1.5	31,930	27,691
α=2	38,981	34,631
Average	32,317	29,202
Reduction rate (%)		9.64

weeks after some warm-up period using the commercial simulation software AutoMod 12.2 of Brooks Automation, Inc.

5.3. Simulation results

The simulation results, i.e., TSC, according to the independent factors are summarized in Table 8. An examination of Table 8 reveals that the reduction in TSC derived from fVMI is significantly higher than the reduction derived from traditional VMI. For example, while fVMI yields cost savings of 23.3%, traditional VMI yields 15.2% on the average. In addition to this, Table 8 also shows that the TSC under fVMI is smaller than that under traditional VMI for all levels of demand variability.

Fig. 9 shows that the enlargement of TSC under fVMI is smaller than under traditional VMI when the variability of customer demand changes from a low level (α=1) to a high level (α=2). For example, while fVMI yields a cost enlargement of 37.0%, traditional VMI yields 49.7% at the high level of variability. This means that fVMI is more robust than traditional VMI with regard to demand variability.

An examination of Table 9 reveals that significant cost savings are derived from the retailer level echelon. Table 9 also shows that the cost savings derived from fVMI are larger than under traditional VMI in the case of retailers. This means that transhipment between retailers has positive effects on cost savings.

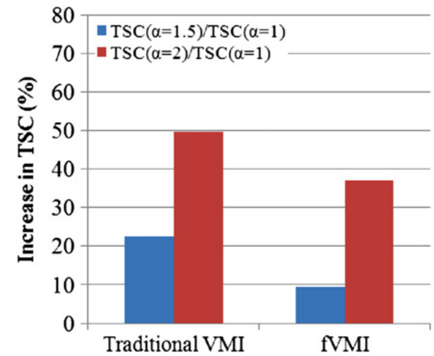

Fig. 9. Increase in TSC (%) according to the independent factor α.

Table 9
Cost at each echelon under a high level of demand variability (α=2).

	Cost in echelon ($)	
	Traditional VMI	fVMI
Manufacturers	8103	7730
Distributors	19,490	17,640
Retailers	11,388	9261

6. Conclusion

In this paper, we have proposed an fVMI framework for inventory management in order to effectively manage inventory levels and minimize relevant costs. The proposed fVMI approach has advantageous features for managing VMI by adopting the fractal concept as described in Section 3.1 in detail. In the proposed framework, each member of the SC is defined as a fractal by itself, and any combination of members can also be defined as a fractal. The SC structure for fVMI is based on the structure for traditional VMI. However, two main differences exist between the two structures. Firstly, fVMI extends the concept of vender-managed control to the entire SC structure, while traditional VMI considers only the relationship between retailers and their immediate upstream members. Secondly, transhipment is permitted between members in the same echelon in fVMI. In fVMI, an (s, S) policy is used as a basis for inventory control. Under the (s, S) policy, a fractal optimizes the replenishment quantities between fractal-echelons, and the transhipment quantities within a fractal-echelon.

To examine the efficiency of the fVMI framework, a comparative study based on simulations was conducted. In the simulation, two SCIM initiatives, including traditional SCIM, traditional VMI, and fVMI, were applied to an illustrative SC under three demand variations. The simulation results indicated that the cost savings derived from fVMI are significantly higher than those under traditional VMI for all levels of demand variability. The results also revealed that transhipment between retailers has positive effects on cost savings.

In this study, we considered only the cost factors in SCIM. For further research, more factors such as the customer service levels of retailers and the bullwhip effects under fVMI can be considered and investigated to extend the proposed framework. Additional simulations also need to be conducted to compare the proposed framework with various optimization techniques that have not been considered in this paper. The adoption of the other characteristics of a fractal (i.e., self-organization including DRP, goal-orientation, etc.) into SCIM can also be considered as future research topics. For example, self-reconfiguration of supply networks can be dealt with to facilitate DRP under the fVMI framework, and autonomous goal generation of each fractal can be considered as another research issue with respect to the fVMI framework.

Acknowledgement

This research was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2009-0087861). The authors would like to express their gratitude for the support.

References

Achabal, D.D., McIntyre, S.H., Smith, S.A., Kalyanam, K., 2000. A decision support system for vendor managed inventory. Journal of Retailing 76 (4), 430–454.
Andel, T., 1996. Manage inventory, own information. Transport and Distribution 37 (5), 54–58.
Angulo, A., Nachtmann, H., Waller, M.A., 2004. Supply chain information sharing in a vendor managed inventory partnership. Journal of Business Logistics 25 (1), 101–120.
Archibald, B.C., Silver, E.A., 1978. (s, S) policies under continuous review and discrete compound poisson demand. Management Science 24 (9), 899–909.
Axsäter, S., Juntti, L., 1996. Comparison of echelon stock and installation stock polices for two-level inventory systems. International Journal of Production Economics 45 (1–3), 303–310.
Benefield, D., 1987. Consignment: what the hospital CFO should know. Hospital Material Management Quarterly 8 (4), 29–37.
Blackburn, J.D., 1991. Time-based Competition: The Next Battleground in American Manufacturing. Irwin, Homewood, IL.
Bravoco, R.R., Yadav, S.B., 1985. A methodology to model the functional structure of an organization. Computers in Industry 6 (5), 345–361.
Burke, M., 1996. It's time for vendor managed inventory. Industrial Distribution 85 (2), 90.
Çetinkaya, S., Lee, C.Y., 2000. Stock replenishment and shipment for vendor-managed inventory systems. Management Science 46 (2), 217–232.
Ching, W.K., 2001. Optimal (s, S) policies with delivery time guarantees for manufacturing systems with early set-up. Computers and Industrial Engineering 40 (3), 249–257.
Christina, N., Christina, L., 2005. An analytical study of the $Q(s, S)$ policy applied to the joint replenishment problem. European Journal of Operational Research 163 (3), 721–732.
Cottrill, K., 1997. Reforging the supply chain. Journal of Business Strategy 18 (6), 35–39.
De Toni, A.F., Zamolo, E., 2005. From a traditional replenishment system to vendor-managed inventory: a case study from the household electrical appliances sector. International Journal of Production Economics 96 (1), 63–79.
Disney, S.M., Towill, D.R., 2001. The effect of VMI on the bullwhip effect. In: Proceedings of the International Conference on Production Research, Prague, Czech Republic, July 30–August 2.
Disney, S.M., Towill, D.R., 2003a. The effect of vendor managed inventory (VMI) dynamics on the bullwhip effect in supply chains. International Journal of Production Economics 85 (2), 199–215.
Disney, S.M., Towill, D.R., 2003b. Vendor-managed inventory (VMI) and bullwhip reduction in a two level supply chain. International Journal of Operations and Production Management 23 (6), 625–651.
Dong, Y., Xu, K., 2002. A supply chain model of vendor managed inventory. Transportation Research Part E 38 (2), 75–95.
Fiddis, C., 1997. Manufacturer-Retailer Relationships in the Food and Drink Industry: Strategies and Tactics in the Battle for Power. Financial Times Retail & Customer Publications, London, UK.
Giannoccaro, I., Pontrandolfo, P., Scozzi, B., 2003. A fuzzy echelon approach for inventory management in supply chains. European Journal of Operational Research 149 (1), 185–196.
Herer, Y.T., Tzur, M., Yücesan, E., 2002. Transhipments: an emerging inventory recourse to achieve supply chain leagility. International Journal of Production Economics 80 (3), 201–212.
Kelle, P., Milne, A., 1999. The effect of (s, S) ordering policy on the supply chain. International Journal of Production Economics 59 (1–3), 113–122.
Lee, H.L., Padmanabhan, P., Whang, S., 1997a. Information distortion in a supply chain: the bullwhip effect. Management Science 43 (4), 543–558.
Lee, H.L., Padmanabhan, P., Whang, S., 1997b. The bullwhip effect in supply chains. Sloan Management Review 38 (3), 93–102.
Mandelbrot, B.B., 1982. The Fractal Geometry of Nature. W.H. Freeman and Company.
Micheau, V.A., 2005. How Boeing and Alcoa implemented a successful vendor managed inventory program. Journal of Business Forecasting 24 (1), 17–19.
Mishra, B.K., Raghunathan, S., 2004. Retailer- vs. vendor-managed inventory and brand competition. Management Science 50 (4), 445–457.
Nachiappan, S.P., Jawahar, N., 2007. A genetic algorithm for optimal operating parameters of VMI system in a two-echelon supply chain. European Journal of Operational Research 182 (3), 1433–1452.
Petrovic, D., Roy, R., Petrovic, R., 1999. Supply chain modelling using fuzzy sets. International Journal of Production Economics 59 (1-3), 443–453.
Ryu, K., Jung, M., 2003. Agent-based fractal architecture and modelling for developing distributed manufacturing systems. International Journal of Production Research 41 (17), 4233–4255.
Ryu, K., Jung, M., 2004. Goal-orientation mechanism in the fractal manufacturing system. International Journal of Production Research 42 (11), 2207–2225.
Ryu, K., Son, Y., Jung, M., 2003a. Modeling and specifications of dynamic agents in fractal manufacturing systems. Computers in Industry 52 (2), 161–182.
Ryu, K., Son, Y., Jung, M., 2003b. Framework for fractal-based supply chain management of e-Biz companies. Production Planning & Control 14 (8), 720–733.
Sari, K., 2008. On the benefits of CPFR and VMI: a comparative simulation study. International Journal of Production Economics 113 (2), 575–586.
Simchi-Levi, D., Kaminsky, P., Simchi-Levi, E., 2003. Designing and Managing the Supply Chain, second ed. McGraw-Hill, IL.
Tharumarajah, A., Wells, A.J., Nemes, L., 1996. Comparison of the bionic, fractal and holonic manufacturing system concepts. International Journal of Computer Integrated Manufacturing 9 (3), 217–226.
Tirpak, T.M., Daniel, S.M., LaLonde, J.D., Davis, W.J., 1992. A note on a fractal architecture for modelling and controlling flexible manufacturing systems. IEEE Transactions on Systems, Man, and Cybernetics 22 (3), 564–567.
van der Vlist, P., Kuik, R., Verheijen, B., 2007. Note on supply chain integration in vendor-managed inventory. Decision Support Systems 44 (1), 360–365.
Verwijmeren, M., van der Vlist, P., van Donselaar, K., 1996. Networked inventory management information systems: materializing supply chain management. International Journal of Physical Distribution and Logistics Management 26 (6), 16–31.
Waller, M.A., Johnson, M.E., Davis, T., 1999. Vendor-managed inventory in the retail supply chain. Journal of Business Logistics 20 (1), 183–203.
Warnecke, H.J., 1993. The Fractal Company: a Revolution in Corporate Culture. Springer-Verlag, Berlin.
Watson, E., 2005. Nestle switches to vendor managed inventory with Tesco. Food Manufacture 80 (8), 20.
Yao, Y., Evers, P.T., Dresner, M.E., 2007. Supply chain integration in vendor-managed inventory. Decision Support Systems 43 (2), 663–674.