Traveling Salesman Problem With a Drone Station

Sungwoo Kim and Ilkyeong Moon

***Abstract*—The importance of drone delivery services is increasing. However, the operational aspects of drone delivery services have not been studied extensively. Specifically, with respect to truck-drone systems, researchers have not given sufficient attention to drone facilities because of the limited drone flight range around a distribution center. In this paper, we propose a truck-drone system to overcome the flight-range limitation. We define a drone station as the facility where drones and charging devices are stored, usually far away from the package distribution center. The traveling salesman problem with a drone station (TSP-DS) is developed based on mixed integer programming. Fundamental features of the TSP-DS are analyzed and route distortion is defined. We show that the model can be divided into independent traveling salesman and parallel identical machine scheduling problems for which we derive two solution approaches. Computational experiments with randomly generated instances show the characteristics of the TSP-DS and suggest that our decomposition approaches effectively deal with TSP-DS complexity problems.**

***Index Terms*—Drone delivery, drone station, mixed integer programming, truck-drone service.**

I. Introduction

GROWING e-commerce and m-commerce increases the importance of efficient logistics. In 2013, Amazon announced drone technology as a future logistic innovation, and many companies have invested into drone research. For example, Amazon unveiled *Amazon Prime Air*, and Google announced *Project Wing* [1], [2]. Drones have many advantages over the typical truck delivery system [3], [4]. As drones operate independently, they are free from operating labor costs and have relatively unlimited working time. Further, they move through the air and thus avoid the traffic congestion problems of ground transportation. These advantages lead to the highly energy-efficient use of drones. Moreover, the transportation cost per kilometer is much lower than that of other means. However, because of technological limitations, a drone can carry only one parcel of limited weight and volume, and it can deliver to a single customer within a short flight range. To overcome these limitations, drone and truck delivery services can be used such that the characteristics of one complement the other. Table I summarizes comparison of trucks and drones [3], [4]. To demonstrate the combined means of delivery, the HorseFly team at the University of Cincinnati developed a system in which a drone can attach to and launch from a truck [4].

TABLE I
Comparison of Trucks and Drones

Transportation	delivery space	delivery speed	parcel weight	parcel capacity	delivery range
drone	**air**	**fast**	light	one	short
truck	ground	slow	**heavy**	**many**	**long**

Drones seem a good logistic alternative for industries, but the technology needs further development to overcome some realistic problems. Aside from controlling a certain type of drones [5] or motion sensing issues [6], battery capacity is a main concern for drone utilization. As many distribution centers with drone facilities are located far from central cities, relatively few customers are serviceable by drones. For this reason, large retail companies such as Amazon strive to build more distribution centers near major cities, but the expenses of constructing distribution centers are still a huge obstacle to completion. To deal with this logistical problem, a different concept of drone facilities is proposed. Roblin [7] introduced *Pylons Dronairports*, which contain drone recharge and shelter devices. Designed by Bruni and Sardo, these compact devices can be easily installed any place. In addition, Amazon plans to use street lights and church steeples as drone docking stations [8]. Another problem is that the weight and volume capacities of drones are not enough to accommodate commercial delivery services [9]. Drone security is also affected by issues with GPS and sensor accuracies [10].

Because many researchers and companies have tried to overcome these problems, some companies have been able to utilize drones for commercial purposes. For instance, DHL express launched the first commercial delivery drone, called *Parcelcopter*, in 2014 [11], [12], and the plan for building the first airport for drones is ongoing in Rwanda [13]. In contrast, research on the operational aspects of drone delivery has been neglected, and only a handful of papers in drone-truck systems have been presented. One of the initial papers about the traveling salesman problem (TSP) in tandem with drones was conducted by Murray and Chu [14], who described two different models. The flying sidekick TSP (FSTSP) describes the way a single drone is used with a truck. A drone is attached onto the truck, and a truck driver launches the drone and also

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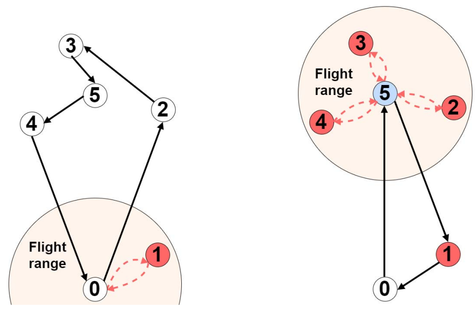

Fig. 1. Comparison of the (a) PDSTSP and (b) TSP-DS (red circle: drone-serviceable customer, white circle: truck-only customer, and blue circle: a drone station).

retrieves it. The other model is the parallel drone scheduling TSP (PDSTSP) and is the key reference for this paper. Unlike the FSTSP, the PDSTSP can utilize a sufficiently large number of drones. However, drones deliver parcels only within the flight range of the distribution center such that problems arise when the distribution center is far away from a majority of customers.

To overcome the limitations of the PDSTSP, we developed the TSP with a drone station (TSP-DS), through which we exploit a drone station, defined as a facility that stores drone and charging devices. The station is ready to launch drones that is, it is "activated" after a truck supplies parcels for drone delivery. We assume that the station can furnish a sufficiently large number of drones and that the location of the station does not depend on that of the distribution center. Specifically, the drone station is located near customer areas and away from the distribution center. The facility can deliver parcels using drones after a truck supplies the deliverables to the drone station, and a truck and a drone station operate independent of the distribution center after the truck supplies parcels for drone delivery. Fig. 1 depicts the difference between the PDSTSP and the TSP-DS.

We first analyze the fundamental features of the TSP-DS. We define route distortion, and the lower bound of the number of drones to eliminate route distortion was presented. By applying the assumptions of the sufficient number of drones and by considering the distance between the distribution center and a drone station is far enough, we show that the TSP-DS can be divided into the TSP and the parallel identical machine scheduling problem (PMS). Through this approach, we successfully reduce the complexity of the problem, and obtain the exact solution. In addition, we explain the tradeoff between the TSP-DS and PDSTSP.

The remainder of this paper is composed as follows. Section II introduces previous research related to truck-drone systems. Section III describes the TSP-DS. Fundamental features of the TSP-DS is presented in Section IV. Section V shows the analyses of computational results and discussion of several issues, and Section VI presents conclusions.

II. Literature Review

The TSP-DS is one variation of the TSP and the vehicle routing problem. A recent review of the TSP was offered by Applegate *et al.* [15] and a review of multiple TSP (MTSP) problems was written by Bektas [16]. Other excellent overviews of the vehicle routing problem were provided by Golden *et al.* [17] and Toth and Vigo [18]. The proposed model is also related to the PMS. Allahverdi *et al.* [19], Ruiz and Vázquez-Rodríguez [20], and Baker and Trietsch [21] summarized studies of the PMS. As traditional studies do not exploit drones, we concentrate on the drone-truck models to which they are directly related.

A drone station can operate drones after a truck arrives and supplies parcels. This characteristic is closely related to the PMS with precedence constraints. Tanaka and Sato [22] studied a single machine scheduling problem with precedence constraints. The objective was to minimize total job completion time, and job idle time was not permitted. A successive sublimation dynamic programming method was applied to find the exact solution. Bilyk *et al.* [23] defined a batch scheduling problem with precedence constraints. Identical machines were assumed, and ready time for each job was considered. A variable neighborhood search and a greedy randomized adaptive search procedure were applied to solve the problem. Davari *et al.* [24] solved a single machine scheduling problem with time windows and precedence constraints. A branch-and-bound algorithm was proposed to solve the problem. Hassan *et al.* [25] studied a PMS with precedence constraints to minimize the makespan. Three valid inequalities were proposed, and their strengths were checked by computational experiments. Nicosia and Pacifici [26] addressed a multiple machine scheduling problem with precedence constraints. A heuristic method related to the bin packing problem was developed, and a lower bound was proposed. Because traditional studies did not exploit drones, we concentrate on the drone-truck models in this paper.

Murray and Chu [14] offered one of the earliest studies of truck-drone delivery problems and introduced two fundamental models. First, the PDSTSP describes a drone facility within a distribution center. To our knowledge, it is the only model in which a drone facility is considered in truck-drone problems. A sufficiently large number of drones can be utilized at the distribution center, but the limited flight range creates practical issues. To alleviate this problem, the FSTSP was developed to describe a truck driver launching and retrieving a drone. This model overcomes the flight range limitation from the distribution center of the PDSTSP, but it only applies to a single drone. Our research is directly related to the PDSTSP and serves as a complementary model applicable to a drone facility separated from the distribution center. To solve the PDSTSP, Murray and Chu [14] developed a heuristic method based on decomposition of the model into the TSP and PMS. We also used the similar decomposition approach; however, our approach focused on the conditions on the decomposition which guarantees the optimal solution.

Although we take into account a drone facility problem with a truck TSP, a majority of research has concentrated on truck-launch delivery problems, which are intricately related to the FSTSP. In related studies, Agatz *et al.* [3] assumed that drones and a truck share the same road network, which allowed them to find the worst-case approximation ratios for the heuristics.

However, the assumption fails to take advantage of the drones capacity to freely move off truck paths and remain unaffected by road conditions.

Ha *et al.* [27] introduced the TSP with a drone. They assumed that launching and retrieving a drone is impossible at the same customer node. The mathematical formulation and two heuristic algorithms were developed. Mathew *et al.* [28] described the heterogeneous delivery problem by considering a team using a truck and drones with complementary capabilities based on the assumption that drone-serviceable customers can only receive deliveries by drones. The problem can be reduced to the generalized traveling problem, which can be solved with many heuristics methods. In addition, they defined the multiple warehouse delivery problem by showing a special case of the heterogeneous delivery problem and developing two heuristic approaches. Ferrandez *et al.* [29] compared the overall travel times and energy consumption of truck-only and truck-drone tandem deliveries. They proposed a clustering-first and routing-second approach. *K*-means algorithm, used to find an efficient launch location of drones, and genetic algorithms were applied to solve a truck-routing problem.

We introduce several studies not directly related to a truck-drone delivery service; however, these works show solutions to drone problems. Boone *et al.* [30] introduced the MTSP which can be applied to the drone swarm route plan. They divided the MTSP into two components: 1) clustering and 2) TSP problems. The *K*-means clustering method was applied to divide cities into multiple clusters, and each drone was allocated to each cluster. A constructive heuristic approach, called 2-Opt, was applied to solve the TSP in each cluster. This approach helps to reduce significant computation time. Dorling *et al.* [31] developed the vehicle routing problem for drone delivery services by deriving an approximated and linearized cost function that accounts for the energy consumption model of multiple drones and by developing mixed integer-based programming for the problem. Further, Dorling *et al.* [31] built a string-based simulated annealing heuristic. A drone system in an indoor environment was introduced by Khosiawan and Nielsen [32]. The system focused on a scheduling issue, and a system architecture for drone applications in an indoor environment was developed. Furthermore, a framework of scheduler component was presented.

III. Traveling Salesman Problem With Drone Station

The TSP-DS is an extension of the PDSTSP, with the major difference in the location of off-duty drones. In the TSP-DS, drones are stored in and launched from a drone station, not the package distribution center. In the previous research conducted by Murray and Chu [14], it was assumed drones can only be used at a distribution center and the number of drones are infinite. However, in many cases, distribution centers are located far from a city center where a majority of customers are located. Constructing additional distribution centers might be a solution to handle this problem, but it is hard to be realized because of enormous costs. The motivation of this paper was to relax the first assumption, and the concept of a drone station which is relatively cheap and easy to install was defined. We need to emphasize that our study is focused on a more general work of the PDSTSP model which overcomes the limited usage of drones; TSP-DS can be reduced to the PDSTSP when the locations of the distribution center and drone station are the same.

A drone station can store and utilize a sufficiently large number of drones that deliver drone-fitting parcels with a limited flight range. A large number of drones seems to be vague, therefore, we present the lower bound of the number of drones which guarantees the minimum makespan of the total delivery time in a latter section. After a truck arrives at the station, drone-fitting parcels are processed for drone delivery and the station is said to be activated. We assume that the location of the station is relatively far from the distribution center; a drone station is farther than the maximum flight distance of a drone launched from the distribution center. If a drone station is located to near a distribution center, there are not meaningful differences between the TSP-DS and PDSTSP. In this case, a decision maker does not operate a drone station in that operating an independent facility needs additional costs. If a drone station is far away from a city center, the decision maker also does not operate the station because of the same reason. That is why we assumed that the location of a drone station is far away from the distribution center but near the city center. Although the decision where to build a drone station can be an important issue, the location of the drone station is assumed to be given. The reason for this assumption is that the location problem should be solved based on the long-term perspective while our topic mainly focuses on the daily delivery service.

Because of safety and weight issues, a single drone cannot carry multiple parcels. Therefore, a drone visits only one customer per sortie while a truck can visit multiple customers in one trip. In addition, some customers order products that exceed the volume and weight capacities of drones. The limited flight range is due to the capacity of drone batteries. We assume that the travel time of vehicles are proportional to distances and drones are faster than a truck because the drones cross air space and the truck must follow ground routes. Because charged batteries are supplied from a drone station, battery charging times for returned drones are not considered. A truck or a drone delivers an order only once to a customer.

Travel times between nodes were assumed to be symmetric. The truck departs from the distribution center and returns to it after packages are delivered. Likewise, drones return to the station after delivering parcels. The delivery service is considered ended when a truck returns to the distribution center and all drones return to their drone station. We defined the last delivery time as the time to finish the total delivery service. The objective of the TSP-DS is to minimize the last delivery time.

A. Notation

We regard each customer as a single node and make a network with $N = \{1, \ldots, c\}$ as a node set of customers and $s \in N$ as a drone station node index. In a customer network, we add the distribution center node. We define 0 as the index of the distribution center, and to avoid symmetric problems, we define $c+1$ as the index of the distribution center node for returns. We also define origin set $N_0 = \{0, 1, \ldots, c\}$ and destination set $N_1 = \{1, 2, \ldots, c+1\}$. Multiple drones are located in a drone station, and a set of drones is defined as $V = \{1, 2, \ldots, v\}$.

Customers are sorted by their package information. Weights, volumes, and distances from the drone station are considered to distinguish drone-serviceable customers. We define D as a set of drone-serviceable customers, which is a subset of N. The travel time of a truck between a pair of nodes (i, j) ($\forall i \in N_0, j \in N_1$) is defined as $\tau_{i,j}$ and that of the drones is defined as $\tau_{i,j}^d$ ($\forall i \in N_0, j \in N_1$). The binary decision variable $x_{i,j}$ equals 1 if the truck travels from node $i \in N_0$ to node $j \in \{N_1 : j \neq i\}$; it is 0 otherwise. Similarly, the decision variable $x_{i,j}^s$ is defined for the route of a truck until it arrives at a drone station. The binary decision variable $y_{i,v}$ is 1 if customer $i \in D$ is served by drone $v \in V$ launched from a drone station. Variable z refers to the last possible delivery time of a truck and drones. u_i indicates the position of node $i \in N_1$ in the truck's path.

B. Mathematical Formulation

We can formulate the TSP-DS as follows:

$$\text{Minimize} \quad z \tag{1}$$

$$\text{subject to} \quad \sum_{\substack{i \in N_0 \\ i \neq j}} x_{i,j} + \sum_{\substack{v \in V \\ j \in D}} y_{j,v} = 1 \;\; \forall j \in N \tag{2}$$

$$x_{i,j}^s \leq x_{i,j} \quad \forall i \in N_0, j \in N_1. \tag{3}$$

The objective function (1) minimizes the delivery time of a truck and drones. Constraint (2) suggests that neither a truck nor a drone can deliver the parcel to a customer more than once. Constraint (3) ensures that $x_{i,j}^s$ follows the path of $x_{i,j}$:

$$\sum_{j \in N_2} x_{i,j}^s - \sum_{j \in N_2} x_{j,i}^s = \begin{cases} 1, & \text{if } i = 0 \\ -1, & \text{if } i = s \\ 0, & \text{otherwise.} \end{cases} \quad \forall i \in N_0 \cup \{c+1\}. \tag{4}$$

Constraint (4) restricts the route of a truck until it arrives at a drone station

$$z \geq \sum_{i \in N_0} \sum_{j \in N_1} \tau_{i,j} \cdot x_{i,j}^s + \sum_{i \in D} \left(\tau_{s,i}^d + \tau_{i,s}^d \right) \cdot y_{i,v} \;\; \forall v \in V \tag{5}$$

$$z \geq \sum_{i \in N_0} \sum_{\substack{j \in N_1 \\ i \neq j}} \tau_{i,j} \cdot x_{i,j}. \tag{6}$$

Constraint (5) imposes the criterion that z is greater than or equal to the last delivery time of drone $v \in V$ launched from a drone station. Constraint (6) restricts that z should not less than the last delivery time of a truck

$$\sum_{j \in N_1} x_{0,j} = 1 \tag{7}$$

$$\sum_{i \in N_0} x_{i,c+1} = 1 \tag{8}$$

$$\sum_{\substack{i \in N_0 \\ i \neq j}} x_{i,j} = \sum_{\substack{k \in N_1 \\ k \neq j}} x_{j,k} \quad \forall j \in N. \tag{9}$$

Constraints (7)–(9) specify the flow of the truck. Constraint (7) means that a single truck leaves the distribution center and Constraint (8) means that the truck must return to the distribution center. Constraint (9) ensures that the truck leaves customer $j \in N$ to deliver parcels after it arrives to customer node $j \in N$ from customer node $i \in N_0$

$$u_i - u_j + 1 \leq (c+2) \cdot \left(1 - x_{i,j}\right) \quad \forall i \in N, \quad j \in \{N_1 : j \neq i\} \tag{10}$$

$$1 \leq u_i \leq c+2 \quad \forall i \in N_1. \tag{11}$$

Subtours of the truck are eliminated by Constraints (10) and (11)

$$x_{i,j} \in \{0, 1\} \quad \forall i \in N_0, j \in \{N_1 : j \neq i\} \tag{12}$$

$$x_{i,j}^s \in \mathbb{R}_+ \quad \forall i \in N_0, j \in \{N_1 : j \neq i\} \tag{13}$$

$$y_{i,v} \in \{0, 1\} \quad \forall i \in D, v \in V \tag{14}$$

$$u_i \in \mathbb{R}_+ \quad \forall i \in N_1. \tag{15}$$

Constraints (12)–(15) define the decision variables.

Although the assumptions on the number of drones and location of the station are relaxed, the proposed mathematical formulation still works to the relaxed problem. In a latter section, we address that this formulation is decomposed to reduce the complexity by using the assumptions.

IV. Fundamental Features of TSP-DS

In the TSP-DS, a loaded truck reaches a drone station, and activates the drone delivery process. This station activation condition has important features, and we demonstrate the main characteristics of the TSP-DS in this section.

Proposition 1: The activation time of a drone station is always less than or equal to $z/2$.

Proof: The travel time matrix of a truck is symmetric, and the total distance of a truck does not change when the travel direction of the truck is reversed on the route. For this reason, when the activation time of a drone station is greater than $z/2$, a truck can be chosen to the same travel route with the reverse direction which activates the drone station before $z/2$. ∎

A. Routes Distortion

Generally, a drone station is used to maximize the use of drones, and a truck is used on the shortest routes. However, in some cases, a truck driver takes a longer route to activate a drone station earlier which results in the overall reduction in the objective function. Because the proposed model searches the optimal schedule of the global truck-drone system, in which a drone station and a truck interact, we define this case as *route distortion*. In analyzing the fundamental features of the drone-truck system, we do not take into consideration two assumptions: the sufficiently large number of drones in a drone station and the minimum distance between the distribution center and the drone station. However, we assume that the number of drones are less than that of customers.

There are two types of the route distortions. In one, a truck delivers parcels to customers who can be serviced by drones. This happens when the last delivery time of a drone is later than that of a truck. In this case, use of

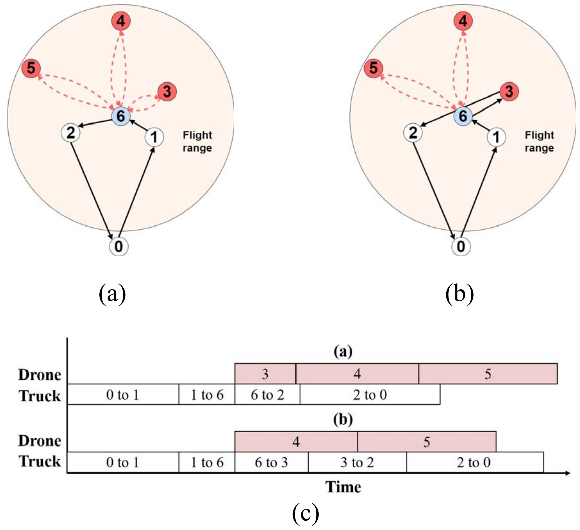

Fig. 2. Example of disadvantage of using drones to deliver parcels to all drone-available customers. (a) Drone station delivering parcels to all drone-serviceable customers. (b) Optimal schedule of a truck and a drone. (c) Comparison of schedules between (a) and (b).

a truck to deliver to drone-serviceable customers is more efficient (Fig. 2). Fig. 2(a) shows a drone delivering parcels to serviceable Customers 3–5. However, the last delivery time of a drone delivery is later than that of the truck delivery. In contrast, Fig. 2(b) shows that a truck delivers parcels to Customer 3 and alleviates the burden on the drone station which results in an earlier delivery service time.

In the other route distortion case, a truck uses a long delivery route to arrive at a drone station early. A driver would make this decision because drones are only able to deliver to customers after a truck supplies parcels to the station, and thus, an early activation time means an early delivery time by drones (Fig. 3). For example, in the case of Fig. 3(a), the shortest route of a truck is 0-1-6-2-0. However, as shown in Fig. 3(b), a truck using longer route 0-6-1-2-0 can activate a drone station so that the drones can start delivering earlier. The duration of delivery times from a drone station is independent of activation time, but the sooner the station is activated by the truck, the earlier the drones can finish deliveries. The last delivery shown in Fig. 3(b) is earlier than that shown in Fig. 3(a).

When N_0 is given, three factors affect route distortions. The main factors correspond to the number of drones in a station as well as the velocities and flight ranges of drones (the number of drone-serviceable customers) (Fig. 4). When the number of drones increases, a truck takes the shorter routes. In the case of Fig. 4(a), the last delivery time from a drone station is earlier than that of the shortest truck routes; further, early activation of a drone station is unnecessary when many drones are available. Likewise, faster drones affect the best route choices for a truck, as shown in Fig. 4(b). Decreasing the flight range or the number of potential drone-serviceable customers also offers the same result that a drone station needs not to be activated in the early stage of delivery service. In the case of Fig. 4(c), the flight range is decreased and Customer 5 is not considered

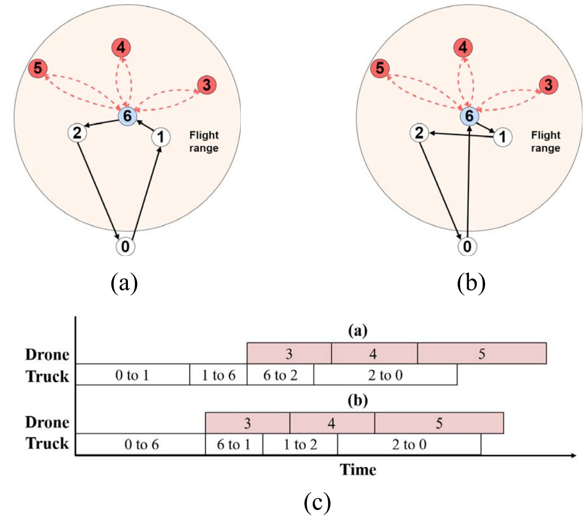

Fig. 3. Routes of a truck can be influenced by a drone station. (a) Optimal schedule of a truck. (b) Optimal schedule of a truck and a drone. (c) Comparison of schedules between (a) and (b).

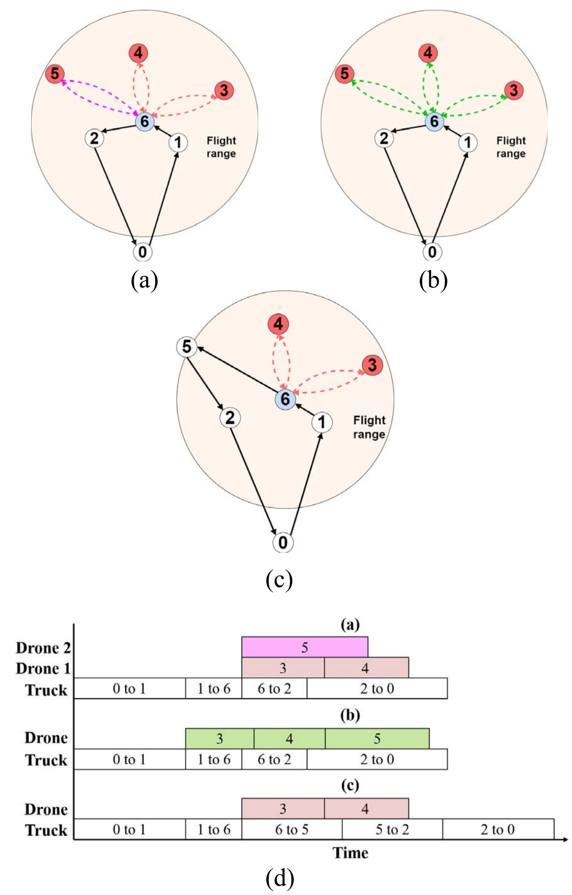

Fig. 4. Three factors can affect the routes. (a) Optimal schedule of a truck and two drones. (b) Optimal schedule of a truck and a high-speed drone. (c) Optimal schedule of a truck and a drone with a smaller flight range. (d) Comparison of schedules between (a), (b), and (c).

a drone-serviceable customer. As a result, the last delivery time from a drone station is earlier than that of a truck from a distribution center. These examples show that the factors related to the drone station workload affect the truck route.

B. Conditions for Elimination of Route Distortion

Based on the assumption that a sufficient number of drones is available in a station, we can draw the inequality that eliminates the route distortion.

Proposition 2: Let $c_{\max}$ be the farthest drone-serviceable customer from the drone station s and α be the travel rate of the drone speed to the truck speed. If the number of drones is sufficient and the problem satisfies $\tau_{s,0} \geq 2 \cdot \tau_{s,c_{\max}}/\alpha$, drones can finish parcel deliveries to all drone-serviceable customers before the truck returns to the distribution center.

Proof: When the number of drones in a station is sufficient, each drone can deliver a parcel to a single customer. In this case, the upper bound for the flight time of a drone from the station (UB_d) is the delivery time of a drone to $c_{\max}$. As the travel time matrix of a truck is symmetric, $\tau_{c_{\max},s} = \tau_{s,c_{\max}}$ and $\text{UB}_d = \{\tau_{c_{\max},s} + \tau_{s,c_{\max}}\}/\alpha = 2 \cdot \tau_{s,c_{\max}}/\alpha$. The lower bound of the truck travel time LB_t to return to the distribution center after leaving a drone station s is $\tau_{s,0}$. Therefore, if UB_d is less than LB_t, the last delivery time from a drone station s can be earlier than or the same as the delivery time of the truck. ■

In the real world, a sufficiently large number of drones is not needed, and the number of customers is the logical upper bound for drone inventory. However, when many drones are needed, and although we cannot find the minimum number before solving the problem, we can find the bound that likely allows for a sufficient number of drones for delivery services.

Proposition 3: If the number of drones is $\lceil |N|/\lfloor[(\alpha \cdot \tau_{0,s})/(2 \cdot \tau_{c_{\max},s})]\rfloor\rceil$, additional drones are not necessary to shorten the schedule.

Proof: In Proposition 2, $\text{UB}_d = 2 \cdot \tau_{s,c_{\max}}/\alpha$, and LB_t is $\tau_{s,0}$. Therefore, the lower bound of the maximum number of customers to which a drone can deliver before a truck returns to the distribution center is $\lfloor[(\alpha \cdot \tau_{0,s})/(2 \cdot \tau_{c_{\max},s})]\rfloor$. The number of customers is $|N|$, and thus, the required number of drones is $\lceil |N|/\lfloor[(\alpha \cdot \tau_{0,s})/(2 \cdot \tau_{c_{\max},s})]\rfloor\rceil$. ■

Combining Propositions 2 and 3, we can define the following general condition.

Corollary 1: If the number of drones is more than $\lceil |N|/\lfloor[(\alpha \cdot \tau_{0,s})/(2 \cdot \tau_{c_{\max},s})]\rfloor\rceil$ and the problem satisfies $\tau_{s,0} \geq 2 \cdot \tau_{s,c_{\max}}/\alpha$, then the route distortion is eliminated.

C. Decomposition of TSP-DS

The TSP-DS is an NP-hard problem, and the typical mathematical formulation can be solved very limited size of instances; it was hard to solve problems with more than 11 nodes of instances. One of our motivation is to reduce the complexity. By analyzing the mathematical structure of the TSP-DS, we found that there are special characteristics of the mathematical formulation and exploited them to derive decomposition methods which guarantee optimal solutions.

For our problem, we address the situation in which the majority of customers are located far from the distribution center and the maximum flight distance of a drone from the distribution center is less than the distance between the drone station and the distribution center. It means $\tau_{0,s} \geq R$ (R is the diameter of the flight range). As $c_{\max}$ does not exceed the radius of the flight range, the following inequality holds: $\tau_{0,s} \geq R \geq 2 \cdot \tau_{s,c_{\max}}$. As the drone velocity is the same or exceeds the speed of a truck, our problem always satisfies Proposition 2. we also assume that a drone station can utilize a sufficiently large number of drones, and this assumption satisfies Proposition 3. Therefore, our problem fulfils the elimination condition of route distortion (Corollary 1).

When the problem satisfies conditions for Corollary 1, a drone station can successfully initiate delivery of all drone-compatible parcels, and a truck does not need to deliver parcels to any customer serviceable by drones. Because the route distortion was eliminated, the model can be divided into two independent problems. The first problem is the TSP through which one finds the shortest truck routes by considering only customers who cannot be serviced by drones. The second problem finds the drone station schedule that minimizes the last delivery time using drones. Because the objective value of the second problem is always less than or equal to the objective value of the first problem (Corollary 1), these two independent problems successfully solve the TSP-DS. We define these two problems as an independent traveling salesman and parallel machine scheduling problem (TSPMS).

However, in terms of a drone station schedule, the PMS model can suggest an overuse of drones because the model is not designed to minimize them. Furthermore, it does not exploit the information from the solution of the TSP which provides the arrival time of a truck at the drone station. For this reason, a two-stage traveling salesman and modified parallel machine scheduling problem (TSMPMS) is developed to find a schedule that minimizes the number of drones used at a station by exploiting the solution of the TSP to set the drone station schedule. The first stage is the same as the ordinary TSP. After the TSP is solved, the activation time of a drone station a_s and the last delivery time of a truck z_t are known. As the problem satisfies Corollary 1, $z = z_t$ and the last delivery time of a drone station can be earlier or the same as z. This finding means the upper bound of the drone flight time UB_f is $z - a_s$. Reflecting this information, a modified PMS problem is solved to minimize the number of drones used under the upper bound of the flight time. The process to calculate UB_f is described in Algorithm 1.

The start node (start_node) is initialized as 0 node. The activation time of a drone at a station (active_time) and the upper bound of the flight time (UB_f) are set as 0. The algorithm finds the next node from the start node. When the next node j is found, the activation time and the new start node is updated. The algorithm repeats until the new start node is c. After the activation time is fully updated, the upper bound of the flight time is calculated. Because the problem satisfies Proposition 1, UB_f can be always greater than or equal to $z/2$.

After UB_f is calculated, we can find the schedule of a drone station that utilizes the minimum number of drones without changing the last delivery time. We define a new binary variable d_v; it is 1 if drone $v \in V$ is used for the delivery and 0 otherwise. The mathematical formulation of the modified PMS

Algorithm 1 Algorithm for UB_f

```
Initialization : start_node, arrival_station, UB_f = 0;
While{i ∈ N_0 − N_d}
{
  While{j ∈ N_0 − N_d}
  {
    if(x_{start_node,j} = 1) then
      arrival_station += τ_{start_node,j}
      start_node = j
      break
    end-if
  }
if(start_node = c) then break;
}
if(arrival_station > z/2) then
  UB_f = arrival_station
else
  UB_f = z - arrival_station
end-if
Output(UB_f)
```

is as follows:

$$\text{Minimize} \sum_{v\in V} d_v \tag{16}$$

$$\text{subject to} \sum_{i\in D}\left(\tau^d_{s,i}+\tau^d_{i,s}\right)\cdot y_{i,v} \le \mathrm{UB}_f \cdot d_v \quad \forall v \in V. \tag{17}$$

The objective function (16) minimizes the required number of drones for delivery service. Constraint (17) suggests that d_v is 1 when drone v is used and the flight time of a drone so it does not exceed the upper bound of the flight time

$$\sum_{v\in V} y_{j,v} = 1 \quad \forall j \in D \tag{18}$$

$$y_{j,v} \in \{0, 1\} \quad \forall j \in D, \forall v \in V \tag{19}$$

$$d_v \in \{0, 1\} \quad \forall v \in V. \tag{20}$$

Constraint (18) shows that each customer serviceable by drones should receive deliveries by a drone. Constraints (19) and (20) define decision variables.

D. Tradeoff Between TSP-DS and PDSTSP

In the proposed model, the activation time of a drone station depends on the arrival time of a parcel-laden truck. However, in the case of the PDSTSP, a drone station is located at the distribution center, so drones can be used at time 0. In this regard, even when the last delivery time and the sum of flight times of the TSP-DS is the same as those of the PDSTSP, the required number of drones for the delivery service might differ (Fig. 5). For instance, according to Fig. 5(a), a drone starts the delivery service from a drone station after a truck arrives. Therefore, an additional drone is needed in the TSP-DS so the last delivery time is no later than the time when a truck returns to the distribution center. In contrast, Fig. 5(b) shows that when all drones are activated at time 0, they can deliver the parcels to drone-serviceable customers before a truck finishes its delivery route.

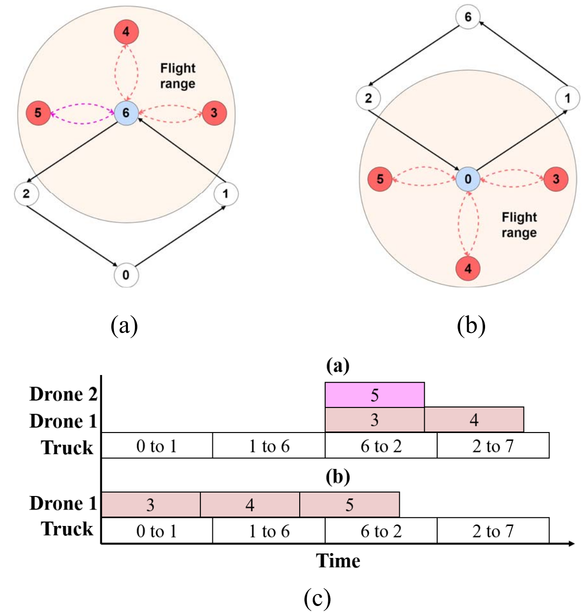

Fig. 5. Difference between the proposed model and PDSTSP. (a) Optimal schedule of the TSP-DS. (b) Optimal schedule of the PDSTSP. (c) Comparison of schedules between (a) and (b).

Proposition 4: If the problem satisfies Corollary 1, the minimum number of drones in a drone station is, at most, three times larger than the number needed to satisfy the PDSTSP when the sum of flight distances from a drone station and the last delivery time of the TSP-DS are the same as those of the PDSTSP.

Proof: Let z_s be the last delivery time of the TSP-DS and z_d be that of the PDSTSP. The lower bound of the last delivery time of the TSP-DS is z. When the problem satisfies Corollary 1, the route distortion does not exist. In addition, we assume that the sum of flight distances from a drone station and the last delivery time are the same as those from the distribution center. Therefore, $z_s = z_d = z$ and z is the same as the last delivery time of a truck. Because a drone station needs to finish the delivery service before a truck returns to the distribution center, the lower bound of the delivery time from a drone station is $z/2$. In other words, the truck can arrive the drone station before or equal to z as mentioned in Proposition 1.

Let A be a drone in the PDSTSP, and list the delivery jobs of Drone A with the shortest process time order. Consider two drones in the TSP-DS. Assign the odd index jobs of Drone A to a drone in the TSP-DS and the even index jobs to the other drone in the TSP-DS with the same order. The sum of flight distances of Drone A cannot exceed z; therefore, at most one drone's last delivery time in the TSP-DS, defined as Drone B, can exceed over $z/2$. Let $c'_{\max}$ is the last delivery job of Drone B, then $z/2 <$ the last delivery time of Drone B $< z/2+2\cdot\tau_{s,c'_{\max}}$. If the last delivery job of Drone B is allocated to a new drone, then the last delivery time of Drone $B - 2\cdot\tau_{s,c'_{\max}} < z/2$ holds. It shows that jobs in Drone A can be divided into at most three segments in which each of them is less than or equal to $z/2$. Therefore, three drones are needed at most. ■

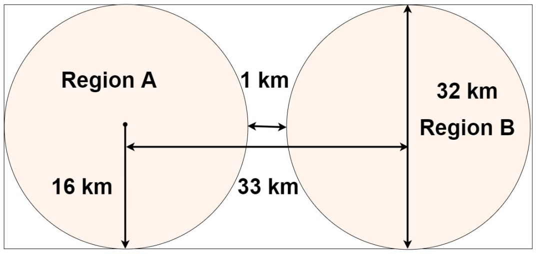

Fig. 6. Experimental design.

V. Computational Experiments

Results of computational experiments and the insight of the developed model are presented in this section. The models were built in XPRESS-IVE 7.9 with the XPRESS-MP mathematical programming solver. Experiments were conducted with an Intel Core i5-3570 CPU 3.4 GHz with 8.00 GB of RAM in Windows 10.

According to Murray and Chu [14], the flight range of a commercial drone is approximately 16 km ($\approx$10 miles). Therefore, we assumed that a circle with 16 km radius is a feasible flight region. To compare the PDSTSP and the TSP-DS, we set two different flight areas. The feasible flight area from drone station s is defined as Region A while that from the distribution center is defined Region B. To avoid overlapping feasible flight regions, we made a gap between them. As a result, the experiments were conducted in a square region of 32 km $\cdot$ 65 km (Fig. 6).

Due to the probabilistic nature of parcel ordering, customers were assigned randomly to specific locations. Furthermore, to concentrate on the effect of a drone station on delivery, we only consider small and light parcels that can be delivered by drones. For this reason, if customers are located in the flight-feasible region, they are assumed to be drone-serviceable customers. When we solve the TSP-DS, customers located in Region A are classified as drone-serviceable customers but others are considered truck-only customers. However, in the PDSTSP, customers in Region B can be serviced by drones while those in Region A cannot be serviced by drones. In addition to this, customer locations are restricted to Regions A and B.

The number of drones in a station was calculated using the bound of drones needed to satisfy Proposition 3. In detail, the radius of Region A is 16 km and the distance between the drone station and the distribution center is 33 km. When the travel rate α is set as 2, a drone can deliver parcels to at least two customers before a truck at the drone station returns to the distribution center. Therefore, the minimum number of drones to satisfy the condition for Proposition 3 is no more than $\lceil |N|/2 \rceil$.

A. Computation Times

Two data sets are generated to evaluate the computation times of the models. A small data set is used to compare the performance between the TSP-DS and other models. Due to the complexity characteristic of the TSP-DS, the number of customers is increased from 7 to 11. A large data set is generated to evaluate the performances of the other models, and the number of customers is increased from 20 to 80. In each customer set, ten random instances were generated. The travel rate α was fixed at 2. We stopped the experiment of each model when it took over 18 000 s. The detailed information of the experiments and results are shown in Table II. Fig. 7 illustrates computation times for each data set with respect to different models.

TABLE II
Average Computation Times (Seconds) of the TSP-DS, TSPMS, and TSMPMS With Respect to the Number of Customers in the Experiment Region

Data type	Number of customers	TSP-DS	TSPMS	TSMPMS
Small set	7	0.255	0.018	0.007
	8	0.647	0.032	0.019
	9	1.598	0.029	0.015
	10	5.360	0.040	0.016
	11	14.021	0.042	0.020
Large set	20	-	0.220	0.083
	30	-	0.592	0.397
	40	-	-	0.730
	50	-	-	0.910
	60	-	-	1.978
	70	-	-	9.046
	80	-	-	25.587

Although three models give the same objective value, the computation times are distinct between them. With fewer customers, the gap is small. However, the computation times of the TSP-DS are much greater for the data set with more customers. Although the computation time increases are relatively small, the TSPMS and TSMPMS models are also not free from increased computation times. The computation time difference between the TSPMS and the TSMPMS was negligible in the small problems. However, in the large problems, the TSMPMS was much faster than the TSPMS. The gap between the computations are increased according to the size of the problem because the second-stage problem of the TSMPMS uses bounds derived from the first-stage model while the TSPMS solves two problems independently.

B. Comparison Between PDSTSP and TSP-DS

As we assume that the TSP-DS is justifiable when customers live in city centers far away from the distribution center, we consider the case in which more than one-half of customers are near drone station s. To analyze the characteristics mentioned in Section IV-A, we conduct experiments by varying the number of customers in Region A, the number of drones, and travel rate α. The total number of customers is set at 10. In each case, ten experiments are conducted, and the savings between the objective values of the TSP-DS (PDSTSP) and the ordinary TSP is found. Each saving is calculated as [the objective value of the TSP—that of the TSP-DS (PDSTSP)]/the objective value of the TSP. The detailed environment setting and results are shown in Table III. The results show that

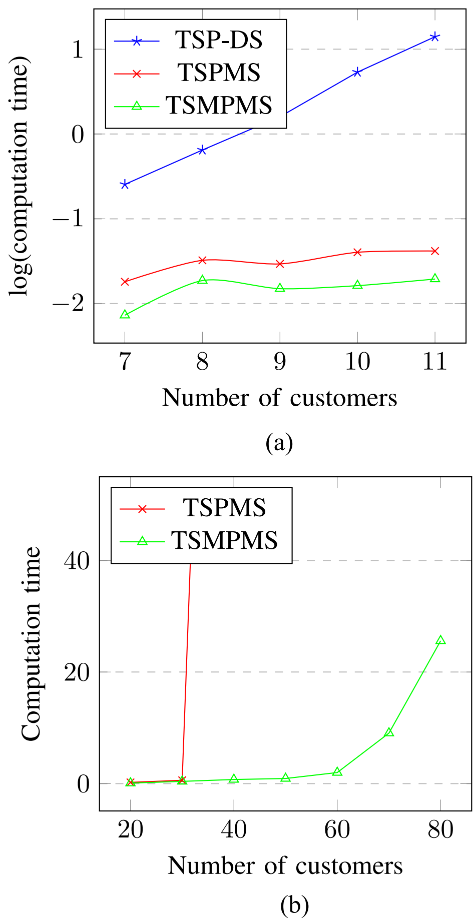

Fig. 7. Comparison of the computation times of three models. (a) Computation times of a small set. (b) Computation times of a large set.

TABLE III
AVERAGE SAVINGS (%) OF THE OPTIMAL VALUE BETWEEN THE TSP-DS (PDSTSP) AND THE TSP WITH RESPECT TO THE NUMBER OF CUSTOMERS IN REGION A, THE NUMBER OF DRONES, AND TRAVEL RATES

Number of customers	Number of drones	Travel rate α 1.5	2.0	2.5
6	1	14.25(9.44)	15.48(10.80)	15.54(12.10)
	2	15.54(12.48)	15.54(12.75)	15.54(12.75)
	3	15.54(12.75)	15.54(12.75)	15.54(12.75)
7	1	15.38(7.95)	16.54(10.13)	17.25(11.17)
	2	17.66(11.27)	17.83(11.66)	17.83(11.66)
	3	17.83(11.65)	17.83(11.66)	17.83(11.66)
8	1	19.26(6.35)	22.43(6.71)	25.66(7.01)
	2	27.13(7.01)	28.13(7.01)	28.15(7.01)
	3	28.13(7.01)	28.15(7.01)	28.15(7.01)
9	1	22.85(5.43)	27.21(5.53)	29.69(5.53)
	2	31.02(5.53)	32.33(5.53)	33.28(5.53)
	3	32.81(5.53)	33.34(5.53)	33.34(5.53)

when the number of drones is increased from 1 to 3, the delivery rates are not appreciably changed and the route distortions do not happen.

TABLE IV
AVERAGE GAPS (%) OF THE OPTIMAL VALUE BETWEEN THE LAST DELIVERY TIME OF THE TSP-T AND THAT OF THE TSP-TC WITH RESPECT TO THE NUMBER OF CUSTOMERS IN REGION A, THE NUMBER OF DRONES, AND TRAVEL RATES

Number of customers	Number of drones	Travel rate α 1.5	2.0	2.5
6	1	1.64	0.09	0.00
	2	0.00	0.00	0.00
	3	0.00	0.00	0.00
7	1	3.14	1.79	0.87
	2	0.28	0.00	0.00
	3	0.00	0.00	0.00
8	1	11.27	7.66	3.60
	2	1.55	0.03	0.00
	3	0.03	0.00	0.00
9	1	13.75	8.69	5.44
	2	3.51	1.62	0.10
	3	0.88	0.00	0.00

The objective value of the TSP was much later than those of the TSP-DS and PSDTSP, which justifies use of the truck-drone system. Moreover, the objective value of the TSP-DS is lower than that of the PDSTSP, and the maximum saving of the optimal value is increased according to the number of customers in Region A. It strengthens our argument that utilizing a drone station helps to make the last delivery time earlier when the distribution center is far away from a majority of customers. Likewise, the increasing number of drones or increased travel rate α enlarge the saving because releasing the burden of the drone alleviates the burden created by inefficient truck routes.

To analyze the impacts of the main factors, such as the number of drones, customers serviceable by drones, and travel rate α, on route distortion, the differences between the last delivery time of the truck side in the TSP-DS (TSP-T) and the last delivery time of the TSP with truck only customers (TSP-TC) are considered. The gap is calculated as (the objective value of the TSP-T—that of the TSP-TC)/(the objective value of the TSP-T), and detailed information is given in Table IV. Obviously, the last delivery time of the TSP-T is larger than or equal to that of the TSP-TC when the number of drones was not sufficient because of route distortion.

If the same number of drones is used, the gap is smaller when few customers are in Region A. When 6 customers are in Region A and travel rate α is 1.5, two drones are sufficient to avoid route distortion. However, more than three drones are needed at the same travel rate to serve 9 customers to avoid route distortion. It can be observed that the number of drones in a station has significant impact on the truck route. Because the generated examples satisfy the distance condition of Proposition 3, increasing the number of drones corresponds to the shortened truck route.

TABLE V
AVERAGE AND MAXIMUM NUMBER OF DRONES USED IN A STATION, AND RATIOS β AND γ

Number of customers	Average	Maximum	Upper bound	β	γ
10	1.95	3	5	0.20	0.30
20	2.95	6	10	0.15	0.30
30	3.93	7	15	0.13	0.23
40	5.10	10	20	0.13	0.25
50	5.55	12	25	0.11	0.24

C. Minimum Number of Drones Used in Drone Station

To analyze the relationship between the number of customers and number of drones used in the station without route distortion, we calculate the minimum number of drones in the TSMPMS. We vary the number of customers from 10 to 50 to check the trend. Because the ratio of the number of customers serviceable by drones/the total number of customers can affect the required number of drones, we vary this ratio from 0.6 to 0.9, and 10 experiments are performed for each ratio. Therefore, 40 experiments are conducted for each number of customer group. The average and maximum number of drones for each customer group are derived. Moreover, ratio β shows the average number of drones used / total number of customers. Similarly, γ (the maximum number of drones used/total number of customers) is defined to check the upper bound. The details of the experiments and results are shown in Table V.

The average number of drones used is much less than the upper bound derived from Proposition 3. The maximum number of drones used is also smaller than the upper bound. The maximum number of drones is approximately twice the average. Ratios β and γ decreased for increasing number of customers. The decreasing rate of β is higher than γ.

D. Discussion

We analyzed the flight range (the number of customers serviceable by drones), the velocity of drones, and the number of drones as main factors affecting the route distortion. However, in a realistic-world problem, the drone range and velocity are difficult to control because of safety issues and limited technologies. Fortunately, increasing the number of drones is relatively easy because the sufficient number of drones can be utilized at a drone station which leads to elimination of route distortion.

The tradeoff between the number of customers and the required number of drones for the delivery service provides insights into practical application. If the distribution center is far away from a majority of drone-serviceable customers, constructing a drone station will lead to better delivery outcomes. However, when the customers are distributed more evenly, the tradeoff for building a drone center or using trucks out of a distribution center should be considered. Although at most three times of drones compared to that for the PDSTSP is required, more customers could be efficiently serviced by drones.

Algorithm 2 Mosel Code of Algorithm 1

```
st_node := 0
arrival_station := 0
flight_time := 0
forall(i in N_tsp) do
   forall(j in N_tsp|getsol(x(st_node,j))>0.99) do
      arrival_station := arrival_station
            + tau_original(st_node,j)
      st_node := j
      break
   end-do
   if(st_node = s) then
      break
   end-if
end-do
if(arrival_station > getobjval/2) then
   flight_limit := arrival_station
else
   flight_limit := getobjval − arrival_station
end-if
```

Another interesting point is that the required number of drones to eliminate route distortion is relatively small. The required number of drones are less than one-third of customers. Moreover, ratio β is negatively affected by the number of customers because increasing the number of customers leads drones to offer more options to deliver parcels to customers in the drone-service area. Therefore, drones deliver more parcels in a given time period if the number of customers increases.

VI. CONCLUSION

We define a new drone and truck-drone TSP by exploring use of a drone station with three features: 1) it can utilize many drones; 2) it is located far away from the distribution center; and 3) it is activated for delivery after a truck arrives with parcels. The TSP-DS was formulated based on mixed integer programming and we analyzed characteristics of the TSP-DS. We proved that the mathematical model can be divided into two different mathematical models, and derived the TSPMS and the TSMPMS to give the exact solution of the TSP-DS. Computational experiments showed that the fundamental characteristics of the TSP-DS and the TSMPMS could effectively reduce the complexity problem. Another experiments revealed that the TSP-DS is more effective than the PDSTSP when a majority of customers are located far from the distribution center. We also showed that route distortion can be eliminated with relatively small number of drones. We expect our model can be used as a means to overcome the limits of drone facility problems, and it can be used to establish drone-truck delivery systems in the near future.

In this problem, we assumed that the locations of customers, a drone station, and the distribution center are given, and the results show that the distance between a drone station and the distribution center is an important factor. Therefore, the location problem of a drone station is an extended topic of our problem. Consideration of multiple drone stations may also inform future research. When some of the flight ranges of each drone station overlap, drones could freely move to each station, which would improve the utilization rates of drones.

Appendix

See Algorithm 2.

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