OMEGA Int. J. of Mgmt Sci., Vol. 21, No. 5, pp. 597–599, 1993
Printed in Great Britain.

0305-0483/93 $6.00 + 0.00

A Note of Correction on "Multi-product Scheduling on a Single Machine"

INTRODUCTION

THE ECONOMIC LOT SCHEDULING PROBLEM (ELSP) is the problem of scheduling the production of several items in a single facility so that demands are met without stockouts or backorders and the long run average inventory carrying and setup costs are minimized. This problem occurs in many production situations, for example in metal forming and plastic production lines, assembly lines, blending and mixing facilities, weaving production lines, etc. [1].

The ELSP has been widely studied over 35 years, and more than 100 papers have been published in a variety of journals. The earliest contributions to this problem include Eilon [5] and Rogers [12]. In the ELSP it is typically assumed that production and demand rates are known item-dependent constants, and that setup times and setup costs are known item-dependent, but sequence-independent constants. In addition, research in the ELSP has focused on cyclic schedules, i.e. schedules that are repeated periodically.

The ELSP is still being investigated in many different ways. The most recent contributions in this area include Dobson [4], Gallego and Moon [7], Gallego and Roundy [8], Mallya [9], Moon [10] and Moon *et al.* [11]. The most recent paper by Mallya [9] developed a scheme to schedule without the determination of the cost factors. He also provided a method to compute a lower bound using economic batch quantity (EBQ). However, we show that significant improvements on his approach can be achieved. First, we show how to find an optimal cycle time under his approach without enumeration. Second, we provide a tight lower bound than he used by computing EBQs under a constraint on the capacity of the machine. Finally, we give clarifying explanations on and relationships between the optimal total daily cost, lower bounds and upper bounds.

AN IMPROVED PROCEDURE ON MALLYA'S MODEL

We use similar notations as in Mallya [9]:

$j = 1, \ldots, m$ = the index for the products
D_j = demand rate (units per day)
P_j = production rate (units per day)
r_j = ratio of D_j/P_j
S_j = setup time of the machine (days)
A_j = machine setup cost (money units)
c_j = standard cost of the product (money units)
i = inventory holding cost factor (money units per unit per day)
Q_j = batch quantity (units)
T_j = consumption time for the batch (days)
$\kappa = 1 - \Sigma_{j=1}^{m} r_j$ = proportion of time available for setups.

If we restrict ourselves to the case that the consumption cycle times for all products must be the same, i.e. $T_1 = T_2 = \cdots = T_m = T$ as in Mallya, the problem is a simpler version of the ELSP and known as the common cycle (CC) approach [12]. The total cost per day is (note that we can ignore the $c_j D_j$ term which is always constant).

$$K(T) = \sum_{j=1}^{m} \left[\frac{A_j}{T} + \frac{ic_j T D_j}{2}(1 - r_j) \right]$$

However, the sum of production and setup times for all products must be no more than the consumption cycle T, we need the following constraint:

$$T \geqslant \sum_{j=1}^{m} (S_j + Tr_j)$$

Consequently, the CC approach is used to find a consumption cycle T which minimizes the total cost per day. The optimum consumption cycle time (under CC approach) can be easily obtained as follows since the problem is a minimization of a convex function subject to a single constraint:

$$T^* = \max \left\{ \sqrt{\frac{2 \sum_{j=1}^{m} A_j}{\sum_{j=1}^{m} ic_j D_j (1 - r_j)}}, T_{\min} \right\} \quad (1)$$

where

$$T_{\min} = \frac{\sum_{j=1}^{m} S_j}{\kappa}$$

Now we compare this approach to Mallya's [9]. Mallya's approach is finding a cycle time without using any cost factors and any idle times, which means using $T_{\min}$ as a consumption cycle time. If we decide to use $T_{\min}$ as a consumption cycle time, the batch size for each product can be easily obtained as in Mallya [9], that is:

$$Q_j = D_j \sum_{j=1}^{m} S_j/\kappa$$

When the cost factors are available, he enumerated cycle times to check the daily cost deviations of using $T_{\min}$ as a cycle time. However, we do not need to enumerate the possible cycle times to find out the maximum deviation if we use (1). That is, we only need to check

$$K(T_{\min}) - K(T^*)$$

which can be easily computed. Note that sometimes $T_{\min}$ itself can be optimal.

Table 1 shows the data used in Mallya [9]. (The data of the standard cost is different from Mallya's as there must be some data errors in Mallya.) The inventory holding cost factor i is 0.35. Using (1), we can obtain $T_{\min} = 10$ (days), $T^* = 21.07$ (days), and $K(10) - K(21.07) = 53.92 - 41.76 = 12.16$ (money units). Mallya observed that the optimum cycle time is around 20 days by enumerating possible cycle times. However, we can easily obtain the exact optimum cycle time T^*.

Since the optimal cycle times for each product for the general ELSP are not necessarily the same, we can obtain the following relationship [let T_j^* be the optimal cycle time for product j and $K(T_1^*, T_2^*, \ldots, T_m^*)$ be the optimum daily cost for the general ELSP]:

$$K(T_1^*, T_2^*, \ldots, T_m^*) \leqslant K(T^*) \leqslant K(T_{\min}) \qquad (2)$$

Note that it is impossible to find T_j^* in polynomial time since the general ELSP is a NP-complete problem. Consequently, most researchers are devoted to developing efficient heuristics. In summary, Mallya's solution provides an upper bound on the minimum total daily cost of the general ELSP, but it is less tight than using an optimum common cycle time T^*.

A TIGHT LOWER BOUND

Mallya [9] determined a lower bound for the minimum total daily cost by taking each product in isolation and calculating EBQs. This approach is known as independent solution (IS) since it ignores the capacity issue of the sharing of a machine by several products [3, 6, 10]. Let LB(IS) be a lower bound obtained by using IS. A tight lower bound has been implicitly suggested by Bomberger [2], and rediscovered by several researchers [3, 7, 10]. The idea is to compute EBQs under a constraint on the capacity of the machine. The capacity constraint is that enough time must be made available for setups. Since the long-run average proportion of time spent on setups is $\Sigma_j S_j/T_j$, and the proportion of time available for setups is κ, the capacity constraint is as in (3). However, the synchronization constraint, stating that no two items can be scheduled to produce at the same time, is ignored. Consequently, the value of the following non-linear programming results in a lower bound on the total daily cost for the general ELSP.

$$\min_{T_1, T_2, \ldots, T_m} \sum_{j=1}^{m} \left[\frac{A_j}{T_j} + \frac{ic_j D_j T_j}{2}(1 - r_j) \right]$$

$$\text{subject to } \sum_{j=1}^{m} \frac{S_j}{T_j} \leqslant \kappa \qquad (3)$$

Obviously, the addition of a constraint results in an objective value at least as high as that obtained without a constraint since the problem is a minimization problem. The above non-linear program can be easily solved via a line search [10]. Let the lower bound obtained using the above method be LB(capacity). Then we can get the following relationship:

$$\text{LB(IS)} \leqslant \text{LB(capacity)} \leqslant K(T_1^*, T_2^*, \ldots, T_m^*) \quad (4)$$

The lower bound using this approach results in 39.31 (money units) for Mallya's example which is exactly the same as that of IS. In other words, the above capacity constraint was not active in this example since the demand rates are moderate compared to the production rates (note that $\kappa = 0.11$ for this case). However, if we consider a highly utilized situation, i.e. if κ is close to 0, this lower bound produces a higher value than that of

Table 1. Data for Mallya's example

Product	Production rate (P_j) (units/day)	Demand rate (D_j) (units/day)	Setup time (S_j) (days)	Setup cost (A_j) (money unit)	Standard cost (c_j) (money unit)
1	1800	431	0.20	80	0.00379
2	2500	375	0.35	140	0.00252
3	4000	480	0.15	60	0.00391
4	3200	895	0.25	100	0.00282
5	1500	151	0.15	60	0.00108

IS. To see this, if we increase the demand rates by 10% without changing any other data, i.e. if daily demand rates become (474, 413, 528, 985, 166), then LB(IS) = 40.70 (money units) and LB(capacity) = 57.73 (money units), which shows the poor quality of IS as a lower bound.

By combining (2) and (4), we obtain the following overall relationship.

$$\mathrm{LB(IS)} \leqslant \mathrm{LB(capacity)}$$
$$\leqslant K(T_1^*, T_2^*, \ldots, T_m^*) \leqslant K(T^*) \leqslant K(T_{\min})$$

In conclusion, Mallya provided LB(IS) and $K(T_{\min})$ which are loose lower and upper bounds. We show that a tight lower bound LB(capacity) and a tight upper bound $K(T^*)$ can be easily obtained by utilizing well-known techniques in the ELSP literature.

CONCLUDING REMARKS

The ELSP is one of the classical production scheduling problems which has been widely studied over 35 years. More than 100 papers have been published (most of them related to developing efficient heuristics since the problem is NP-hard) on the problem, and several more have appeared recently which shows the problem's popularity. However, there are some redundancies and inefficiencies due to unawareness of the well-known techniques in the literature of the ELSP. We have provided a short note on the basic relationships between lower bounds, the optimum total daily cost and upper bounds.

REFERENCES

1. Boctor F (1987) The G-group heuristic for single machine lot scheduling. *Int. J. Prodn Res.* **25**(3), 363–379.
2. Bomberger E (1966) A dynamic programming approach to a lot size scheduling problem. *Mgmt Sci.* **12**(11), 778–784.
3. Dobson G (1987) The economic lot-scheduling problem: achieving feasibility using time-varying lot sizes. *Opns Res.* **35**(5), 764–771.
4. Dobson G (1992) The cyclic lot scheduling problem with sequence-dependent setups. *Opns Res.* **40**(4), 736–749.
5. Eilon S (1957) Scheduling for batch production. *Inst. Prodn Engng J.* **36**(9), 549–579.
6. Elmaghraby S (1978) The economic lot scheduling problem (ELSP): review and extensions. *Mgmt Sci.* **24**(6), 587–598.
7. Gallego G and Moon I (1992) The effect of externalizing setups in the economic lot scheduling problem. *Opns Res.* **40**(3), 614–619.
8. Gallego G and Roundy R (1992) The economic lot scheduling problem with finite backorder costs. *Nav. Res. Log.* **39**(4), 729–739.
9. Mallya R (1992) Multi-product scheduling on a single machine: a case study. *Omega* **20**(4), 529–534.
10. Moon I (1991) Some issues in the economic lot scheduling problem. Ph.D. thesis, Columbia University, New York.
11. Moon I, Gallego G and Simchi-Levi D (1991) Controllable production rates in a family production context. *Int. J. Prodn Res.* **29**(12), 2459–2470.
12. Rogers J (1958) A computational approach to the economic lot scheduling problem. *Mgmt Sci.* **4**(3), 264–291.

I Móon

(June 1993)

Department of Industrial Engineering
College of Engineering
Pusan National University
Pusan 609-735
Korea