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Strategic inventory: Manufacturer vs. retailer investment

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ABSTRACT

This paper presents a two-period supply chain model under demand induced by selling price and investment effort in the presence of strategic inventory. We compared six different scenarios to identify optimal pricing decisions. An incremental quantity discount contract was applied to verify supply chain coordination. Our findings show that manufacturer-investment efforts cannot always persuade the retailer carrying strategic inventory to maintain harmony among supply chain participants; however, retailer-investment efforts can promote harmony when strategic inventory is used. The retailer's decision to carry strategic inventory is catastrophic from the perspective of supply chain coordination, but benign for the decentralized supply chain.

1. Introduction

In multi-period supply chain environments, retailers carry additional inventories strategically to encourage the manufacturer to reduce the wholesale price in early periods. Anand et al. (2008) were the first to recognize this strategic measure of the retailer. They showed that the retailer's optimal strategy is carrying strategic inventory and that the manufacturer cannot prevent the retailer from this action. Arya and Mittendorf (2013) explored the role of consumer rebates offered by manufacturers in ameliorating the effectiveness of strategic inventories. They claimed that consumer rebates make the retailer less aggressive in carrying strategic inventories and the manufacturer less exploitative in setting wholesale prices. Moreover, the retailer, manufacturer, and consumer all benefit from manufacturer consumer rebates even when the retailer carries inventory strategically. Arya et al. (2014) further extended the role of strategic inventory to determine the trade-offs in procurement decisions in centralization versus decentralization environments. They showed that the seemingly imperfect nature of centralization may prove desirable when a retailer relies on external suppliers and strategically manages inventory. In this paper, we explore the influence of manufacturer and retailer investment efforts on strategic inventory. In an alternative approach to that of Arya and Mittendorf (2013), we sought to verify whether the manufacturer-investment effort can encourage harmony between supply chain members. To the best of our knowledge, the impact of a retailer's joint decision to carry strategic inventory and investment-effort expenditures on each supply chain member has not been discussed.

Unlike most of the literature on supply chain management for exploring a single period decision, we studied the roles of strategic inventory in a two-period setting. However, little literature shows the characteristics of the two-period supply chain model. Recently, Linh and Hong (2009) studied a two-period newsvendor model for short life cycle products in dynamic markets. They applied a revenue sharing contract for coordination. Pan et al. (2009) claimed that products such as personal computers and mobile phones are sold with multi-period pricing, and they formulated a two-period model to determine pricing and ordering decisions. Chen and Xiao (2011) applied a dynamic programming approach to find the optimal decision in a two-period supply chain model with a game-

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Table 1
Articles on investment decision under two-echelon supply chain.

Articles	Manufacturer investment–effort	Retailer investment–effort	Period	Demand	Strategic Inventory	Information
Gurnani and Erkoc (2008)	✓	✓	Single	Deterministic	×	Asymmetric
He et al. (2009)	✓	×	Single	Stochastic	×	Symmetric
Lau et al. (2010)	×	✓	Single	Deterministic	×	Symmetric
Lau et al. (2012)	×	✓	Single	Deterministic	×	Symmetric
Brunner (2013)	×	✓	Single	Stochastic	×	Symmetric
Saha (2013)	✓	×	Single	Deterministic	×	Symmetric
Wu (2013)	×	✓	Single	Deterministic	×	Symmetric
Ma et al. (2013)	✓	✓	Single	Deterministic	×	Symmetric
Zhao and Wei (2014)	×	✓	Single	Fuzzy	×	Asymmetric
Zha et al. (2015)	×	✓	Single	Stochastic	×	Asymmetric
Gao et al. (2016)	✓	✓	single	deterministic	×	symmetric
Yan and Zaric (2016)	×	✓	single	deterministic	×	symmetric
Basiri and Heydari (2017)	✓	✓	single	deterministic	×	symmetric
Ma et al. (2017)	✓	✓	single	deterministic	×	asymmetric
Present study	✓	✓	two	deterministic	✓	symmetric

theory framework model under stochastic price-dependent demand. Wang et al. (2015) developed a two-period supply chain model for short life cycle products in dynamic markets to indemnify optimal advertising policies of a retailer. Yang et al. (2016) proposed a flexible trade-credit contract mechanism for a financially constrained retailer in a two-period supply chain framework. Maiti and Giri (2017) found that the two-period supply chain is a common framework in the fashion and textile industries, and they studied the impact of dynamic pricing under price-dependent demand. The literature cited explains the importance of two-period supply chain. However, none of it addressed the inventory-carrying decision of a retailer in the presence of investment effort.

According to Taylor (2002) and Cachon and Lariviere (2005), investment effort is an integral part of retailing that stimulates demand. Although those who use it incur significant operational cost, the increased demand benefits supply chain participants. An increasing number of retailers and manufacturers are incorporating various types of investment efforts into their operations to attract customers who buy their products and to compete with their rivals. For instance, a retailer can invest in local media advertising, provide attractive shelf space, and hire well-trained sales personnel to guide consumer purchases and enhance demand. However, the manufacturer can stimulate demand through investment in global advertisement, product quality improvement, packaging, green technology innovation, and so forth. In general, the retailer or manufacturer can apply various techniques for boosting demand by investment, referred to as *investment effort* throughout this paper. We made no assumption about the specific type of investment effort used; the term refers only to the dollars expended by the supply chain member on demand-enhancing activities. A great deal of research in operation management considers the importance of joint decisions on pricing and investment effort. Table 1 presents a summary of some of the important published research on this key issue and also shows the contribution of our study.

From Table 1, one can observe that the optimal nature of the investment effort under the two-period supply chain model has not been discussed to date. In addition, no attention has been paid to strategic inventory decisions. In this study, our focus is on the behavior patterns of each supply chain member regarding investments over two periods. Some interesting research questions linked to this study include the following:

Does the manufacturer's investment effort in both periods eliminate the retailer's strategic inventory?
What is the role of strategic inventory in the retailer's own investment effort?
Are the investment patterns of the manufacturer and retailer identical when under the influence of strategic inventory?
How do equilibrium decisions in decentralized and centralized supply chains evolve with the use of strategic inventory?

To explore answers to these research questions and provide insights into the influence of a retailer's strategic inventory on the performance of supply chain members, we developed a two-period supply chain model with a manufacturer and a retailer under price-investment-induced demand. Because the supply chain performance can be enhanced if participants use contractual incentives to enhance the profit of the entire supply chain (Cachon, 2003), researchers have paid considerable attention to the designs of coordination schemes for chain members. In this study, an incremental quantity discount (IQD) contract was suggested for the supply chain. The abundant academic literature on quantity-discount contracts includes work by Sheen and Tsao (2007), Cachon and Kok (2010), and Nie and Du (2017). In addition to linear wholesale price discounts, under an IQD, the manufacturer offers an aggressive quantity-discount plan to coordinate the supply chain and improve the profit of each member while maintaining fairness for all. This type of discount model is continuous, differentiable, and concave (Cachon and Kok, 2010).

We developed the proposed supply chain model in a decentralized environment by considering the Stackelberg game structure. In a supply chain model, it is assumed that the retailer orders the exact amount of product required for each period, and the manufacturer attempts to distribute the quantity. However, the influence of ordering extra products and carrying them as a strategic inventory to fulfill the demand of the next period has not been discussed in the literature. In a common practice, the retailer orders several times through the life cycle of a product, and over the time between these orders,some products are carried from one period to another. Therefore, it is necessary to invest in optimal ordering and pricing behaviors by considering multiple time periods. In this study, six different scenarios based on the manufacturer and retailer investment-effort decisions are discussed to identify the impact of investment effort in the presence of strategic inventories. With strategic inventory, Scenario LI represents the decision model in which the investment effort is made in both periods; Scenario SI represents the decision model in which the investment effort is made only in the first period; and Scenario NI represents the benchmark model in which no investment effort is made. Similarly, without strategic inventory, Scenario L represents the decision model in which the investment effort is made in both periods; Scenario S represents the decision model in which the investment effort is made only in the first period; and Scenario N represents the benchmark model in which no investment effort is made. By comparing the investment decision in six different scenarios, we drew a number of interesting managerial implications. Our study reveals that the retailer can enhance the amount of the strategic inventory with self-investment effort, but the reverse trend follows in the face of manufacturer investment effort. The amount of strategic inventory and the investment decision largely depend on the price sensitivity of the product. Presence of strategic inventory is sometime harmful from the perspective of supply chain coordination. In addition, the manufacturer-investment effort does not always maintain harmony among supply chain participants, but the retailer-investment effort can promote cooperation.

The rest of the paper is organized as follows: Notations and assumptions are provided in Section 2. In Section 3, we formalize all models of manufacturer- and retailer-investment efforts in a decentralized environment. We also suggest managerial implications of the proposed study and offer numerical simulations in support of the analytical findings. Section 4 shows the analyses of optimal results under coordination. Finally, in Section 5, we conclude the paper with remarks and suggestions for additional research. Derivations of decentralized supply chain models and proofs of some propositions are presented in the Appendices.

2. Notations and assumptions

We present a two-echelon distribution channel consisting of a single manufacturer and a single retailer in which the manufacturer produces a product and sells it to consumers through the retailer. Pricing and investment efforts decisions are analyzed for two consecutive selling periods. The retailer orders at the beginning of each of the two periods and sets different retail prices. In addition, the retailer can carry forward excess products procured in the firstperiod as strategic inventory for the second period. Excess products and new order quantities at the beginning of the second period are available to satisfy the demand of the second period. Therefore, the retailer needs to invest for carrying additional inventories. The manufacturer can also adjust wholesale prices at each period. Market demand of the product depends on retail price and the individual investment effort of channel members. In making pricing and investment-effort decisions, the manufacturer acts as the Stackelberg leader and the retailer is the follower, and a two-stage Stackelberg game model was constructed to obtain optimal decisions in decentralized scenarios. Using the backward induction procedure, the optimal solutions were evaluated. We made the following assumptions on the basis of Arya and Mittendorf (2013):

(1) The manufacturer and retailer are both rational and risk neutral. Distribution is instantaneous;that is, the lead time at each period is zero. The capacity of the manufacturer is infinite;that is, the manufacturer is always able to fulfil all orders.
(2) The game reflects a symmetric information environment; that is, the parameters related to this study are common knowledge for the manufacturer and retailer (Hafezalkotob, 2017).
(3) The pricing and investment decisions for two consecutive periods and product characteristics remain the same between the two periods. The retailer can carry inventory between those periods with additional holding costs.
(4) Market demand is influenced by product price and individual investment efforts. The investment efforts have a positive effect on demand. The manufacturer and retailer decide the duration of investment effort (i.e., whether the investment effort is provided for single or both periods).
(5) The functions of investment-effort cost of the manufacturer and the retailer are $\frac{\alpha e_{ijk}^2}{2}$ and $\frac{\beta f_{ijk}^2}{2}$, respectively, which represent extensively accepted assumptions (Chintagunta and Jain, 1992).
(6) The marginal costs of the manufacturer and retailer are normalized to zero (Anand et al., 2008; Arya and Mittendorf, 2013). For feasibility of the optimal solution, the retail price at each period (p_i) is larger than the wholesale price(w_i);that is, $p_i > w_i$,$\forall\ i = 1,2$.

Model notation with explanation.

i	number of periods, $i = 1,2$
j	supply chain model under investment effort of the supply chain member, $j = m,r$
k	number of scenarios, $k = li,si,ni,l,s,n$
a	overall size of the potential market demand in each period
b	price sensitivity parameter of the demand in each period
α	investment cost coefficient of the manufacturer
β	investment cost coefficient of the retailer
c	sensitivity of the manufacturer investment effort
d	sensitivity of the retailer investment effort
h	holding cost of the retailer

p_{ijk}	retail price of the retailer per unit
w_{ijk}	wholesale price of the manufacturer to the retailer
e_{ijk}	level of investment effort of the manufacturer
f_{ijk}	level of investment effort of the retailer
π_{mijk}	manufacturer's profit
π_{rijk}	retailer's profit
Q_{ijk}	sales volume of the supply chain
I_{jk}	amount of strategic inventory

The following Table presents a summary of the model notation.

3. Investment effort in a decentralized environment

In this section, we relate the investigation into the equilibrium pricing and investment decisions in a decentralized environment under six different scenarios from the perspectives of the manufacturer and retailer, respectively.

3.1. Decentralized models with manufacturer investment effort

We analyzed the effect of long-term investment effort on the strategic-inventory carrying decision of the retailer. The demand function under the long-term investment effort considered in this analysis is linearly decreasing in retail price and linearly increasing in investment effort. The forms of the demand functions used for the first and second periods are $D_1(p_1,e_1) = a-bp_1 + ce_1$ and $D_2(p_2,e_2) = a-bp_2 + ce_2$, respectively. At the beginning of the first period, the manufacturer sets the unit wholesale price, w_1, and the investment effort, e_1. After the manufacturer sets the wholesale price and effort, the retailer procures $D_1(p_1,e_1) + I$ units of the product,sets the retail price, p_1, and makes decisions on how many additional units, I, to purchase as strategic inventory. Therefore, the retailer needs to invest hI as a holding cost. In the second period, the manufacturer sets the unit wholesale price, w_2, and investment effort, e_2. Finally, the retailer procures $D_2(p_2,e_2)-I$ units of the product and sets the retail price, p_2. We determined the unique equilibrium of the game by employing backward induction. The second-period optimization problem of the retailer and manufacturer are given as follows:

$$\text{Max} \quad \pi_{r2mli}(p_2) = p_2D_2(p_2)-w_2(D_2(p_2)-I) \tag{1}$$

$$\text{Max} \quad \pi_{m2mli}(w_2,e_2) = w_2(D_2(w_2,e_2)-I)-\frac{\alpha e_2^2}{2} \tag{2}$$

Similarly, the first-period optimization problem of the retailer and manufacturer are given as follows:

$$\text{Max} \quad \pi_{r1mli}(p_1,I) = p_1D_1(p_1,e_1)-w_1(D_1(p_1,e_1) + I)-hI + \pi_{r2mli}(I) \tag{3}$$

$$\text{Max} \quad \pi_{m1mli}(w_1,e_1) = w_1(D_1(w_1,e_1) + I(w_1)) + \pi_{m2mli}(w_1) \tag{4}$$

The following Lemma gives the optimal decision under Scenario LI.

Lemma 1. *Under Scenario LI, the manufacturer's optimal investment efforts and wholesale prices and the retailer's optimal retail prices are given respectively by:*

$$w_{1mli} = \frac{2\alpha}{\Delta}[c^8(a + bh) + 2bc^6(3a-5bh)\alpha-4b^2c^4(13a-7bh)\alpha^2-16ab^3c^2\alpha^3 + 32b^4(9a-2bh)\alpha^4] \tag{5}$$

$$e_{1mli} = \frac{1}{\Delta}[4b^2c^5(13a-7bh)\alpha^2 + 16ab^3c^3\alpha^3-32b^4c(9a-2bh)\alpha^4-c^9(a + bh)-2bc^7(3a-5bh)\alpha] \tag{6}$$

$$p_{1mli} = \frac{1}{2\Delta}[c^{10}h + 8c^8(a-bh)\alpha + 4bc^6(5a + 2bh)\alpha^2-56b^2c^4(5a-bh)\alpha^3-16b^3c^2(5a + 4bh)\alpha^4 + 128b^4(13a-bh)\alpha^5] \tag{7}$$

$$I_{mli} = \frac{b(20b^2\alpha^2-c^4-2bc^2\alpha)(2\alpha(2b\alpha-c^2)(a-4bh)-c^4h)}{2(c^8 + 4bc^6\alpha-44b^2c^4\alpha^2-16b^3c^2\alpha^3 + 272b^4\alpha^4)} \tag{8}$$

$$w_{2mli} = \frac{8b^2c^2(5a-7bh)\alpha^3 + 32b^3(3a + 5bh)\alpha^4-2c^6(a-bh)\alpha-4bc^4(3a + bh)\alpha^2}{c^8 + 4bc^6\alpha-44b^2c^4\alpha^2-16b^3c^2\alpha^3 + 272b^4\alpha^4} \tag{9}$$

$$e_{2mli} = \frac{4b^2c^3(5a-7bh)\alpha^2 + 16b^3c(3a + 5bh)\alpha^3-c^7(a-bh)-2bc^5(3a + bh)\alpha}{c^8 + 4bc^6\alpha-44b^2c^4\alpha^2-16b^3c^2\alpha^3 + 272b^4\alpha^4} \tag{10}$$

$$p_{2mli} = \frac{c^8h-4ac^6\alpha-4bc^4(9a + 8bh)\alpha^2 + 24b^2c^2(3a + bh)\alpha^3 + 16b^3(23a + 10bh)\alpha^4}{2(c^8 + 4bc^6\alpha-44b^2c^4\alpha^2-16b^3c^2\alpha^3 + 272b^4\alpha^4)} \tag{11}$$

where $\Delta = 1088b^5\alpha^5 + 60b^2c^6\alpha^2-160b^3c^4\alpha^3-336b^4c^2\alpha^4-c^{10} > 0$.

Proof. See Appendix A.

Using the equilibrium outcomes, the total profit of the retailer, the manufacturer, and sales volume in Scenario LI are obtained as follows:

$$\pi_{r1mli} = \frac{b\alpha \Upsilon_1}{\Delta^2} \tag{12}$$

$$\pi_{m1mli} = \frac{\alpha}{\Delta}[a^2(c^8 + 4bc^6\alpha - 40b^2c^4\alpha^2 - 32b^3c^2\alpha^3 + 288b^4\alpha^4) + b^2h^2x^4 - 4ab^2h\alpha x^2y] \tag{13}$$

$$Q_{1mli} = \frac{2b\alpha}{\Delta}[a(c^8 + 2bc^6\alpha - 28b^2c^4\alpha^2 - 56b^3c^2\alpha^3 + 304b^4\alpha^4) - bhx^3y] \tag{14}$$

where $\Upsilon_1 = 4a^2\alpha(c^{16} - 38b^2c^{12}\alpha^2 - 88b^3c^{10}\alpha^3 + 888b^4c^8\alpha^4 + 1504b^5c^6\alpha^5 - 9184b^6c^4\alpha^6 - 8960b^7c^2\alpha^7 + 39680b^8\alpha^8)$ $- bh^2x^5(3c^8 - 4bc^6\alpha - 52b^2c^4\alpha^2 + 16b^3c^2\alpha^3 + 304b^4\alpha^4) - ahx^2y(c^{12} - 10bc^{10}\alpha + 8b^2c^8\alpha^2 + 200b^3c^6\alpha^3 - 416b^4c^4\alpha^4 - 1440b^5c^2\alpha^5 + 3776b^6\alpha^6), x = 4b\alpha - c^2$, and $y = 2b\alpha - c^2$.

Using optimal decisions, one can find that the difference between manufacturer investment efforts in the first and second periods is $e_{1mli} - e_{2mli} = \frac{2bc(12b^2\alpha^2 - c^4)((4b\alpha - 2c^2)(a - 4bh)\alpha - hc^4)}{\Delta} > 0$. Similarly, the difference between wholesale prices in the first and second periods is $w_{1mli} - w_{2mli} = \frac{4b\alpha(12b^2\alpha^2 - c^4)((4b\alpha - 2c^2)(a - 4bh)\alpha - hc^4)}{\Delta} > 0$. Therefore, we propose the following proposition:

Proposition 1. *With strategic inventory, the manufacturer-investment effort and wholesale price in the first period are always greater than they are in the second period.*

Proposition 1 is partially consistent with the literature on strategic inventory. The retailer can force the manufacturer to reduce the wholesale price in the second period. In addition, our study suggests that the manufacturer needs to reduce investment effort. Therefore, strategic inventory may be harmful from the perspective of the overall supply chain profit because it reduces second-period investment efforts and hence market demand. The consumer also needs to pay more in the first period because of $p_{1mli} - p_{2mli} = \frac{(2b\alpha + c^2)(12b^2\alpha^2 - c^4)((4b\alpha - 2c^2)(a - 4bh)\alpha - hc^4)}{\Delta} > 0$. We explain the analytical implications of the proposed study using numerical illustrations. The following parameters are used for the illustration: $a = 100, b = 1, c = 0.8, d = 0.8, h = 1, \alpha = 1$, and $\beta = 1$. When the value of one parameter varied, all others remained unchanged. The detailed results of the sensitivity analysis of the manufacturer-investment model for Scenario LI are given in Appendix F. Table 4 shows the change of the investment efforts ($\Delta e_m = e_{1mli} - e_{2mli}$), wholesale prices ($\Delta w_m = w_{1mli} - w_{2mli}$), and retail prices ($\Delta p_m = p_{1mli} - p_{2mli}$) in two periods; the corresponding amounts of strategic inventory; and profits of the manufacturer and retailer in Scenario LI. The results support the presented analytical findings. In addition, Table 4 shows that the amount of strategic inventory increased as market potential (a) and price sensitivity (b) increased. As a increases, the demand also increases, and the retailer can compensate for the higher holding cost incurred as a consequence of the increasing amount of strategic inventory. If price sensitivity is high, then the retailer cannot charge a high retail price. In this scenario, the retailer should carry a large amount of inventory for reducing the wholesale price of the product in the second period. However, the retailer needs to reduce the amount of strategic inventory if consumers are sensitive to investment efforts.

To obtain more insights, we derived the simplified values of the equilibrium outcomes for the remaining five scenarios and present the results in Table 2. With strategic inventory, in Scenario SI, the manufacturer invests only in the first period, and in

Table 2
Optimal decisions in different scenarios under the manufacturer investment effort.

	SI	NI	L	S	N
w_{1mk}	$\frac{4(9a-2bh)\alpha}{(68b\alpha-9c^2)}$	$\frac{9a-2bh}{17b}$	$\frac{2a\alpha}{4b\alpha-c^2}$	$\frac{2a\alpha}{4b\alpha-c^2}$	$\frac{a}{2b}$
p_{1mk}	$\frac{a(9c^2+104b\alpha)-4bh(c^2+2b\alpha)}{2b(68b\alpha-9c^2)}$	$\frac{13a-bh}{17b}$	$\frac{3a\alpha}{4b\alpha-c^2}$	$\frac{3a\alpha}{4b\alpha-c^2}$	$\frac{3a}{4b}$
e_{1mk}	$\frac{2c(9a-2bh)}{68b\alpha-9c^2}$	–	$\frac{ac}{4b\alpha-c^2}$	$\frac{ac}{4b\alpha-c^2}$	–
w_{2mk}	$\frac{8(3a+5bh)\alpha-6c^2h}{68b\alpha-9c^2}$	$\frac{2(3a+5bh)}{17b}$	$\frac{2a\alpha}{4b\alpha-c^2}$	$\frac{a}{2b}$	$\frac{a}{2b}$
p_{2mk}	$\frac{4b(23a+10bh)\alpha-3c^2(3a+2bh)}{2b(68b\alpha-9c^2)}$	$\frac{23a+10bh}{17b}$	$\frac{3a\alpha}{4b\alpha-c^2}$	$\frac{3a}{4b}$	$\frac{3a}{4b}$
e_{2mk}	–	–	$\frac{ac}{4b\alpha-c^2}$	–	–
I_{mk}	$\frac{60b(a-4bh)\alpha-9c^2(3a-4bh)}{6(68b\alpha-9c^2)}$	$\frac{5(a-4bh)}{34}$	–	–	–
π_{r1mk}	$\frac{\Upsilon_2}{2b(68b\alpha-9c^2)^2}$	$\frac{155a^2-118abh+304b^2h^2}{34b}$	$\frac{2a^2b\alpha^2}{(4b\alpha-c^2)^2}$	$\frac{a^2(32b^2\alpha^2-8bc^2\alpha+c^4)}{16b(4b\alpha-c^2)^2}$	$\frac{a^2}{8b}$
π_{m1mk}	$\frac{2[9a^2\alpha-bh(c^2h+4(a-2bh)\alpha)]}{68b\alpha-9c^2}$	$\frac{9a^2-4abh+8b^2h^2}{34b}$	$\frac{a^2\alpha}{4b\alpha-c^2}$	$\frac{a^2(8b\alpha-c^2)}{8b(4b\alpha-c^2)}$	$\frac{a^2}{4b}$
Q_{1mk}	$\frac{b(c^2h+38a\alpha-16bh\alpha)}{68b\alpha-9c^2}$	$\frac{19a-8bh}{34}$	$\frac{2ab\alpha}{4b\alpha-c^2}$	$\frac{a(8b\alpha-c^2)}{4(4b\alpha-c^2)}$	$\frac{a}{2}$

Scenario NI, the manufacturer does not invest. Similarly, without strategic inventory, in Scenario L, the manufacturer invests in both periods, in Scenario S the manufacturer invests only in the first period, and in Scenario N the manufacturer does not invest. where, $\Upsilon_2 = c^4(81a^2-117abh + 62b^2h^2) + 8b^2c^2h(83a-94bh)\alpha + 8b^2(155a^2-118abh + 304b^2h^2)\alpha^2$.

Without strategic inventory, one can observe the following in the models:

(i) In Scenario L, investment efforts of the manufacturer remained uniform.
(ii) Investment efforts in both periods benefit supply chain participants because of the profit differences of the manufacturer $\pi_{m1ml}-\pi_{m1ms} = \frac{a^2c^2}{8b(4b\alpha-c^2)} > 0$ and $\pi_{m1ms}-\pi_{m1mn} = \frac{a^2c^2}{8b(4b\alpha-c^2)} > 0$; therefore, $\pi_{m1ml} \geqslant \pi_{m1ms} \geqslant \pi_{m1mn}$. Similarly, the profit differences of the retailer are $\pi_{r1ml}-\pi_{r1ms} = \frac{a^2c^2(8b\alpha-c^2)}{16b(4b\alpha-c^2)^2} > 0$ and $\pi_{r1ms}-\pi_{r1mn} = \frac{a^2c^2(8b\alpha-c^2)}{16b(4b\alpha-c^2)^2} > 0$, therefore; $\pi_{r1ml} \geqslant \pi_{r1ms} \geqslant \pi_{r1mn}$.
(iii) In Scenario S, the retailer needs to charge a higher price in the first period than in the second period because $p_{1ms}-p_{2ms} = \frac{3ac^2}{4b(4b\alpha-c^2)} > 0$.
(iv) In Scenario S, the manufacturer needs to charge a higher wholesale price in the first period because of $w_{1ms}-w_{2ms} = \frac{ac^2}{2b(4b\alpha-c^2)} > 0$.

We made the following propositions to explore the significance of strategic inventory. The term *harmony* is used in this regard. *Economy harmony* is frequently referenced in the literature regarding the history of economic thought, and it attributes ethical virtues to the distributive results of competitive markets (Kirzner, 1987). Harmony encompasses the individual activities and approaches toward the overall system, and it is used to describe the performance of the supply chain system oriented to individuals. However, coordination cannot reflect the individual activity of each supply chain member. Supply chain coordination includes the activity of every member. In our study, we found that the retailer can force the manufacturer to reduce the wholesale price by carrying strategic inventory, but this action causes retailer conflict with the manufacturer as the retailer pursues profit maximization. In Section 4, we address the supply chain coordination issue in detail and reveal that the retailer decision to carry inventory does not create an impact on the centralized supply chain profit. Therefore, the following proposition was used to analyze the effect of the retailer's activity in a decentralized supply chain.

Proposition 2. *When the retailer carries strategic inventory, the manufacturer's decision to make a first-period investment effort can maintain harmony among supply chain participants.*

Proof. See Appendix B.

Similar to Arya and Mittendorf (2013), who suggested the implementation of consumer rebates to create harmony among supply chain participants, in this study, we found that the manufacturer's first-period investment decision can also create harmony. The term *harmony* is used because the investment decision is beneficial to both the manufacturer and retailer. The retailer's decision to carry strategic inventory does not affect the profitability of the manufacturer, and the manufacturer investment decision does not eliminate the retailer's strategic inventory. However, the difference between the amount of strategic inventory in Scenarios SI and NI is $I_{msi}-I_{mni} = \frac{-6c^2(9a-2bh)}{17(68b\alpha-9c^2)} < 0$. Therefore, to maintain harmony in these scenarios,the retailer needs to reduce the amount of strategic inventory. The consumer needs to pay more in the first period than in the second period because of $p_{1msi}-p_{2mni} = \frac{6(c^2h + 2(a-4bh)\alpha)}{68b\alpha-9c^2}$.

Proposition 3. *The manufacturer's decisions to make an investment effort in both periods can maintain harmony among supply chain participants if* $\max\{h_1,h_3\} \leqslant h \leqslant \min\{h_2,h_4\}$.

Proof. See Appendix C.

Proposition 3 indicates that manufacturer investment decisions cannot always bring harmony to the manufacturer and retailer. If the manufacturer invests in both periods, then both the retailer and manufacturer may earn less profit. In fact, manufacturer decisions may not only lead to reduced volume of strategic inventory but might also prevent the retailer from carrying any strategic inventory. Therefore, the retailer loses the power of the wholesale price negotiation and the manufacturer can use the price to stave off the retailer's power. From Eq. (8), one can observe that in Scenario LI the retailer can carry inventory when $2b\alpha > c^2$.

The graphical representation of strategic inventories in Scenarios LI, SI, and NI are shown in Fig. 1.

Fig. 1 shows that retailer strategic inventories decreased drastically in all three scenarios and each became negative if the holding cost became large. Overall, the manufacturer-investment effort always reduces the volume of the retailer's strategic inventory. The manufacturer can sometimes eliminate retailer strategic inventory without providing any benefits. The graphical representation of manufacturer and retailer profits in Scenarios LI, SI, and NI are shown in Figs. 2a and 2b.

Figs. 2a and 2b support the findings of Proposition 3. The manufacturer's decision to make investment efforts in both periods (Scenario LI) means that harmony between the manufacturer and retailer cannot be maintained. The retailer can receive a higher profit in Scenario NI than in Scenario LI. Without strategic inventory, both the manufacturer and retailer received higher profits in Scenario L. Therefore, the equilibrium outcomes of the supply chain are affected by the retailer inventory-carrying decision.

3.2. Decentralized models with retailer investment efforts

In this section, the characteristics of the retailer-investment effort are described. As explained in the previous subsection on the manufacturer's decision, the demand function is linearly decreasing in the retail price and linearly increasing in the sales efforts. The

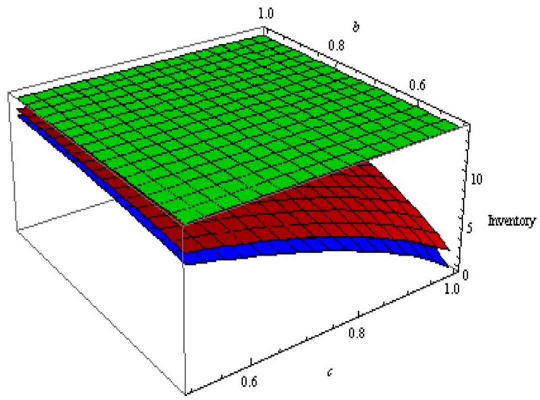

Fig. 1. Volume of the strategic inventory under the manufacturer investment effort in Scenarios LI (blue), SI (red), and NI (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

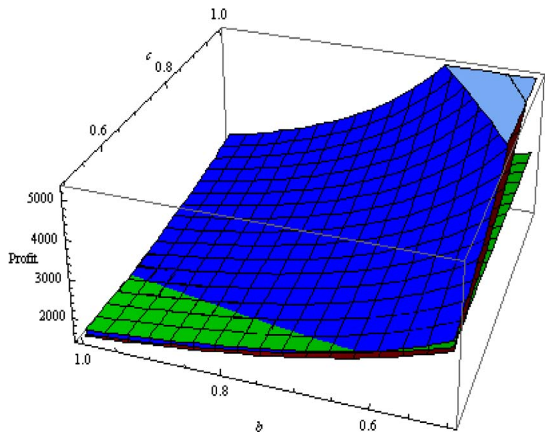

Fig. 2a. The retailer profits under the manufacturer investment effort in Scenarios LI (blue), SI (red), and NI (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

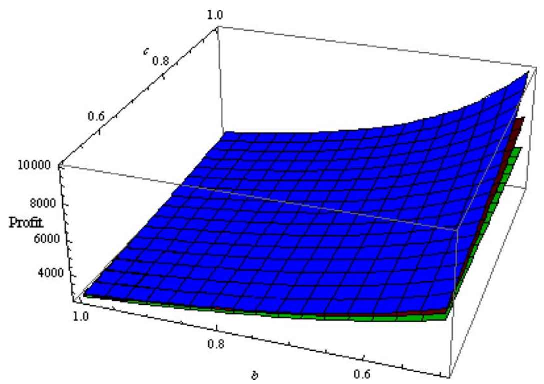

Fig. 2b. The manufacturer profits under the manufacturer investment effort in Scenarios LI (blue), SI (red), and NI (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

functional forms of the demand function in the first and second periods are $D_1(p_1,f_1) = a-bp_1 + df_1$ and $D_2(p_2,f_2) = a-bp_2 + df_2$, respectively. At the beginning of the first period, the manufacturer sets the unit wholesale price, w_1. After the manufacturer sets the wholesale price, the retailer procures $D_1(p_1,f_1) + I$ units of the product and sets the retail price, p_1, makes decisions on investment effort, f_1, and determines the amount of additional units, I, to purchase. In the second period, the retailer procures $D_2(p_2,f_2)-I$ units of the product, sets the retail price, p_2, and decides the investment effort, f_2. Finally, the manufacturer sets the unit wholesale price, w_2. Similar to the explanation in the previous subsection, the unique equilibrium of the game is determined by employing backward induction. The second-period optimization problem of the retailer and manufacturer is represented as follows:

$$\text{Max} \quad \pi_{r2rli}(p_2,f_2) = p_2 D_2(p_2,f_2) - w_2(D_2(p_2,f_2) - I) - \frac{\beta f_2^2}{2} \tag{15}$$

$$\text{Max} \quad \pi_{m2rli}(w_2) = w_2(D_2(w_2) - I) \tag{16}$$

Similarly, the first-period optimization problem of the retailer and manufacturer are given as follows:

$$\text{Max} \quad \pi_{r1rli}(p_1,f_1,I) = p_1 D_1(p_1,f_1) - w_1(D_1(p_1,f_1) + I) - hI - \frac{\beta f_1^2}{2} + \pi_{r2rli}(I) \tag{17}$$

$$\text{Max} \quad \pi_{m1rli}(w_1) = w_1(D_1(p_1,f_1) + I(w_1)) + \pi_{m2rli}(w_1) \tag{18}$$

The following Lemma gives the optimal decision under Scenario LI with retailer investment efforts.

Lemma 2. *With the retailer Under Scenario LI, the manufacturer's optimal wholesale prices, and the retailer's optimal retail prices and investment efforts are given respectively by:*

$$w_{1rli} = \frac{9a - 2hb}{17b} \tag{19}$$

$$f_{1rli} = \frac{2d(4a + bh)}{17(2b\beta - d^2)} \tag{20}$$

$$p_{1rli} = \frac{2bd^2h + 26ab\beta - 2b^2h\beta - 9ad^2}{17b(2b\beta - d^2)} \tag{21}$$

$$I_{rli} = \frac{5b(a - 4bh)\beta}{17(2b\beta - d^2)} \tag{22}$$

$$w_{2rli} = \frac{2(3a + 5bh)}{17b} \tag{23}$$

$$f_{2rli} = \frac{d(11a - 10bh)}{17(2b\beta - d^2)} \tag{24}$$

$$p_{2rli} = \frac{a(23b\beta - 6d^2) + 10bh(b\beta - d^2)}{17b(2b\beta - d^2)} \tag{25}$$

Proof. See Appendix D.

Using the above equilibrium outcomes, the total profit of the manufacturer, the retailer, and sales volume in Scenario LI under the retailer investment are obtained as follows:

$$\pi_{r1rli} = \frac{\beta(155a^2 - 118abh + 304b^2h^2)}{578(2b\beta - d^2)} \tag{26}$$

$$\pi_{m1rli} = \frac{\beta(9a^2 - 4abh + 8b^2h^2)}{17(2b\beta - d^2)} \tag{27}$$

$$Q_{1rli} = \frac{b\beta(19a - 8bh)}{17(2b\beta - d^2)} \tag{28}$$

Using Eqs. (20) and (24), one can find that the difference in retailer investment efforts between the first and second periods is $f_{1rli} - f_{2rli} = -\frac{3d(a-4bh)}{17(2b\beta - d^2)} < 0$. Similarly, the difference between wholesale price in the first and second periods is $w_{1rli} - w_{2rli} = \frac{3(a-4bh)}{17b} > 0$. Therefore, we propose the following proposition:

Proposition 4. *When a retailer carries strategic inventory, the wholesale price in the first period is greater than it is in the second period. The retailer-investment effort in the second period is always greater than it is in the first period.*

The nature of the wholesale price in our study is consistent with that described in the existing literature. By comparing Propositions 1 and 3, one can find that the investment trends are different. Strategic inventory encourages the retailer to spend more in investment efforts in the second period. The retailer can force the manufacturer to reduce the wholesale price and utilize it to make greater investment efforts that stimulate market demand. With the manufacturer's investment, the consumer needs to pay more in the first period, but in the case of the retailer's investment, the consumer needs to pay more if $b\beta > d^2$ because of $p_{1rli} - p_{2rli} = \frac{3(a-4bh)(b\beta - d^2)}{17b(2b\beta - d^2)}$. The detailed result of the sensitivity analysis of the retailer investment effort model in Scenario LI is also shown in Table 4. It represents the change of the investment efforts ($\Delta f_r = f_{1rli} - f_{2rli}$), wholesale prices ($\Delta w_r = w_{1rli} - w_{2rli}$), retail prices ($\Delta p_r = p_{1rli} - p_{2rli}$) in two periods, the corresponding amounts of strategic inventory, and profits of the manufacturer and retailer in Scenario LI. The result supports the analytical findings described. The higher sensitivity of manufacturer-investment effort reduced

Table 3
Optimal decisions in different scenarios under the retailer investment effort.

	SI	NI	L	S	N
w_{1rk}	$\frac{4bd^2h+36ab\beta-9ad^2-8b^2h\beta}{68b^2\beta-16bd^2}$	$\frac{9a-2bh}{17b}$	$\frac{a}{2b}$	$\frac{a}{2b}$	$\frac{a}{2b}$
p_{1rk}	$\frac{d^4(9a-4bh)-bd^2(61a-12bh)\beta+8b^2(13a-bh)\beta^2}{4b(17b\beta-4d^2)(2b\beta-d^2)}$	$\frac{13a-bh}{17b}$	$\frac{a(3b\beta-d^2)}{2b(2b\beta-d^2)}$	$\frac{a(3b\beta-d^2)}{2b(2b\beta-d^2)}$	$\frac{3a}{4b}$
f_{1rk}	$\frac{bd(8b(4a+bh)\beta-d^2(7a+4bh))}{(17b\beta-4d^2)(2b\beta-d^2)}$	–	$\frac{ad}{4b\beta-2d^2}$	$\frac{ad}{2(2b\beta-d^2)}$	–
w_{2rk}	$\frac{4b(3a+5bh)\beta-d^2(3a+4bh)}{2b(17b\beta-4d^2)}$	$\frac{2(3a+5bh)}{17b}$	$\frac{a}{2b}$	$\frac{a}{2b}$	$\frac{a}{2b}$
p_{2rk}	$\frac{a(46b\beta-11d^2)-4bh(5b\beta-d^2)}{68b^2\beta-16bd^2}$	$\frac{23a+10bh}{17b}$	$\frac{a(3b\beta-d^2)}{2b(2b\beta-d^2)}$	$\frac{3a}{4b}$	$\frac{3a}{4b}$
f_{2rk}	–	–	$\frac{ad}{4b\beta-2d^2}$	–	
I_{rk}	$\frac{(a-4bh)(5b\beta-d^2)}{34b\beta-8d^2}$	$\frac{5(a-4bh)}{34}$	–	–	
π_{r1rk}	$\frac{\Upsilon_3}{32b(17b\beta-4d^2)^2(2b\beta-d^2)}$	$\frac{155a^2-118abh+304b^2h^2}{1156b}$	$\frac{a^2\beta}{4(2b\beta-d^2)}$	$\frac{a^2(4b\beta-d^2)}{16b(2b\beta-d^2)}$	$\frac{a^2}{8b}$
π_{m1rk}	$\frac{(4b\beta-d^2)(9a^2(4b\beta-d^2)+16b^2h^2M-8abhM)}{16b(17b\beta-4d^2)M}$	$\frac{9a^2-4abh+8b^2h^2}{34b}$	$\frac{a^2\beta}{2(2b\beta-d^2)}$	$\frac{a^2(4b\beta-d^2)}{8b(2b\beta-d^2)}$	$\frac{a^2}{4b}$
Q_{irk}	$\frac{(4b\beta-d^2)(4bd^2h+19ab\beta-8b^2h\beta-5ad^2)}{4(17b\beta-4d^2)(2b\beta-d^2)}$	$\frac{19a-8bh}{34}$	$\frac{ab\beta}{2b\beta-d^2}$	$\frac{a(4b\beta-d^2)}{4(2b\beta-d^2)}$	$\frac{a}{2}$

the amount of strategic inventory; however, the opposite trend is observed for the retailer investment-effort sensitivity. By providing higher investment effort in the second period to enhance demand, the retailer can carry higher inventory to fulfil the larger demand.In addition, the investment cost coefficient also plays a critical role under the retailer-investment effort model. By being more efficient,the retailer can carry higher amount of inventory.

As in the previous subsection, we present the simplified values of equilibrium outcomes for the remaining five scenarios under the retailer investment efforts presented in Table 3. With strategic inventory in Scenario SI, the retailer invests only in the first period, and in Scenario NI, the retailer does not invest. In a similar manner, without strategic inventory, in Scenario L, the retailer invests in both periods; in Scenario S, the retailer invests only in the first period; and in Scenario N, the retailer does not invest. where $\Upsilon_3 = a^2(4b\beta-d^2)(38d^4-305bd^2\beta+620b^2\beta^2)-8abh(2b\beta-d^2)(6d^4-53bd^2\beta+118b^2\beta^2)+16b^2h^2(2b\beta-d^2)(6d^4-61bd^2\beta+152b^2\beta^2)$.

Without strategic inventory, one can observe the followings from Table 3:

(i) In Scenario L, investment effort of the manufacturer remains uniform.
(ii) Investment efforts in both periods benefit both supply chain participants because of the profit difference of the manufacturer $\pi_{m1rl}-\pi_{m1rs} = \frac{a^2d^2}{8b(2b\beta-d^2)} > 0$ and $\pi_{m1rs}-\pi_{m1rn} = \frac{a^2d^2}{8b(2b\beta-d^2)} > 0$; therefore, $\pi_{m1rl} \geqslant \pi_{m1rs} \geqslant \pi_{m1rn}$. Similarly, the profit differences of the retailer are $\pi_{r1rl}-\pi_{r1rs} = \frac{a^2d^2}{16b(2b\beta-d^2)}$ and $\pi_{r1rs}-\pi_{r1rn} = \frac{a^2d^2}{16b(2b\beta-d^2)}$; therefore, $\pi_{r1rl} \geqslant \pi_{r1rs} \geqslant \pi_{r1rn}$.
(iii) In Scenario S, the retailer needs to charge a higher price in the first period because $p_{1rs}-p_{2rs} = \frac{ad^2}{4b(2b\beta-d^2)} > 0$.
(iv) In Scenario S, the wholesale price remains unaltered.

Without a retailer carrying strategic inventory, the wholesale pricing strategy of the manufacturer differs significantly more in Scenario S than it does in the other scenarios. If the manufacturer makes investment efforts, then the manufacturer needs to adjust wholesale prices. If the retailer makes investment efforts, then the manufacturer sets uniform wholesale prices. However, the retailer makes an investment effort to stimulate market demand and the retailer decision enhances profits of each supply chain member. The following two propositions were used to explore the characteristics of the supply chain when strategic inventory is used:

Proposition 5.

(i) *The retailer's decision to make an investment effort in only the first period can maintain harmony between the manufacturer and retailer.*
(ii) *The retailer's decision to make an investment effort in both periods can always maintain harmony between the manufacturer and retailer.*

Proof. See Appendix E.

The difference between the amount of the retailer strategic inventories in Scenarios SI and NI is $I_{rsi}-I_{rni} = \frac{3d^2(a-4bh)}{34(17b\beta-4d^2)} > 0$. Similarly, the difference between the amount of retailer strategic inventories in Scenarios LI and NI is $I_{rli}-I_{rni} = \frac{5d^2(a-4bh)}{34(2b\beta-d^2)} > 0$. Unlike the scenarios of manufacturer investment effort, the amount of strategic inventory increases if the retailer makes the investment effort. Therefore, the manufacturer makes the investment effort not only to stimulate demand but also to eliminate retailer strategic inventory. However, the retailer's decision to make investment efforts enhances both the amount of strategic inventory and the market demand.

The graphical representation of strategic inventories in Scenarios LI, SI, and NI when the retailer makes investment effort

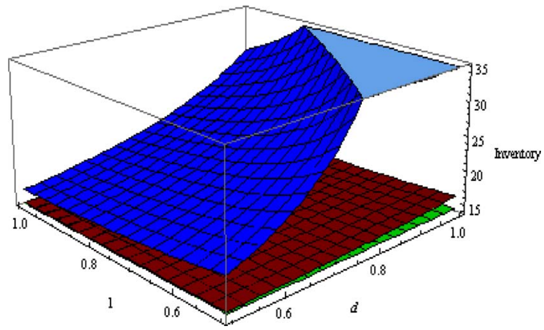

Fig. 3. Volume of the strategic inventory under the retailer investment effort in Scenarios LI (blue), SI (red), and NI (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

decisions is shown in Fig. 3.

Fig. 3 shows that the amount of strategic inventory is maximized in Scenario LI. The amount of strategic inventory decreased as did the holding cost for the retailer. The graphical representation of the manufacturer and retailer profits in Scenarios LI, SI, and NI are shown in Figs. 4a and 4b.

Figs. 4a and 4b support Propositions 5 and 6. With strategic inventory, the retailer's decision to make an investment effort always allows for harmony between the manufacturer and retailer. Therefore, the equilibrium outcomes of the supply chain under a retailer investment effort decision is unaffected by the inventory carrying decision, such as when the manufacturer makes the investment effort.

From the analysis, one can conclude that (i) a retailer's decision to maintain investment efforts in two periods always ensures harmony between the manufacturer and retailer, but when the manufacturer makes the investment, harmony is not ensured; (ii) the amount of strategic inventory always decreases if the manufacturer makes an investment effort in two periods, but it reaches a maximum under the retailer investment decision; and (iii) investment effort in two periods benefits the manufacturer and retailer under the decentralized environment. In the next section, we discuss the supply chain coordination issue when the retailer makes the investment effort in both periods, and we address the impact of the retailer's decision to carry strategic inventory.

4. Supply chain coordination

In this section, we first address the benchmark centralized model under the two-period retailer investment effort; that is, we look at the simultaneous decisions of all the channel members in a vertically integrated firm on the retail price and investment effort that maximize supply chain profit in both periods. The benchmark model is controlled by a central planner, and the wholesale prices and strategic inventory are insignificant. The profit functions in the second period(π_{2crli}) and first period (π_{1crli}) under the centralized retailer investment model are as follows:

$$\pi_{2crli} = p_2(a - bp_2 + df_2) - \frac{\beta f_2^2}{2} \tag{29}$$

$$\pi_{1crli} = p_1(a - bp_1 + df_1) - \frac{\beta f_1^2}{2} + \pi_{2crli} \tag{30}$$

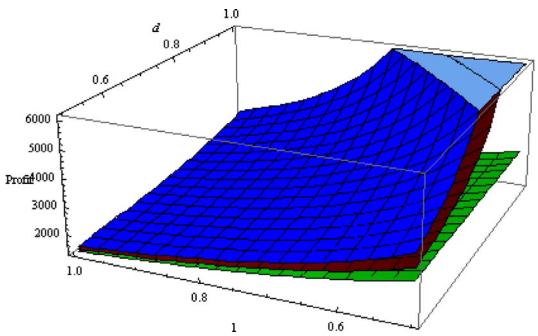

Fig. 4a. The retailer profits under the retailer investment effort in Scenarios LI (blue), SI (red), and NI (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

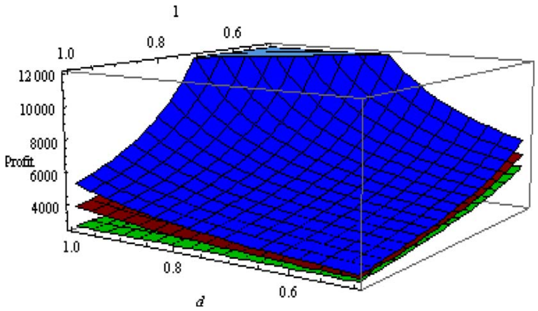

Fig. 4b. The manufacturer profits under the retailer investment effort in Scenarios LI (blue), SI (red), and NI (green). (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

One can verify that the equilibrium decision for the above two-period optimization problem is as follows:

$$p_{1crli} = p_{2crli} = \frac{a\beta}{2b\beta - d^2} \tag{31}$$

$$f_{1crli} = f_{2crli} = \frac{ad}{2b\beta - d^2} \tag{32}$$

Using the equilibrium solution, the total profit of the centralized supply chain and the corresponding sales volume in two periods are obtained as follows:

$$\pi_{1crli} = \frac{a^2\beta}{2b\beta - d^2} \tag{33}$$

$$Q_{crli} = \frac{2ab\beta}{2b\beta - d^2} \tag{34}$$

Using Eqs. (26), (27) and (33), we determine the difference between the centralized and the decentralized supply chain profit with strategic inventory as given bellow:

$$\pi_{1crli} - (\pi_{r1rli} + \pi_{m1rli}) = \frac{(13a - 18bh)(9a + 32bh)\beta}{578(2b\beta - d^2)} > 0$$

Therefore, the supply chain profit is maximum in the centralized environment. Similarly, the difference between the centralized and decentralized supply chain profit (Eq. (33) and results given in Table 3) without the retailer strategic inventory, that is in Scenario L, is obtained as follows:

$$\pi_{1crli} - (\pi_{r1rl} + \pi_{m1rl}) = \frac{a^2\beta}{4(2b\beta - d^2)} > 0$$

Therefore, the profit of the supply chain always greater in the centralized scenario than those in the decentralized scenarios. The result is well established and consistent with the existing literature. To explore the influence of the retailer strategic inventory in decentralized environment, we compute the difference between decentralized supply chain profits as follows:

$$(\pi_{r1rli} + \pi_{m1rli}) - (\pi_{r1rl} + \pi_{m1rl}) = \frac{(55a - 288bh)(a - 4bh)\beta}{1156(2b\beta - d^2)}$$

From the analysis, the following proposition is proposed:

Proposition 6. *The retailer decision to carry inventory enhances the total profit of the decentralized supply chain if* $a > 5.24bh$.

Proposition 6 suggests that if the holding cost for the retailer is not too high, then the retailer's decision to carry strategic inventory benefits the entire decentralized supply chain. By combining Propositions 6 and 7, one can conclude that the joint impact of retailer investment efforts in both periods and strategic inventory always enhance the performance of the decentralized supply chain. At this point, we introduce the IQD contract to remove channel inefficiency and subsequently verify efficiency from the perspective of supply chain coordination.

The IQD contract can be described by two parameters: wholesale price, w, and quantity discount factor, ϵ. To entice the retailer to order more products, and make investments that benefit the supply chain, the manufacturer provides incremental discounts on the basis of order quantity. The wholesale-price contract is a subset of the IQD contract, which is equivalent to the wholesale-price contract if $\epsilon = 0$. Under this contract, the profit functions of the retailer (π_{r2qdli}) and manufacturer (π_{m2qdli}) in the second period are as follows:

$$\pi_{r2qdli} = p_2(a-bp_2 + df_2)-(w_2-\epsilon(a-bp_2 + df_2-I))(a-bp_2 + df_2-I)-\frac{\beta f_2^2}{2} \tag{35}$$

$$\pi_{m2qdli} = (w_2-\epsilon(a-bp_2 + df_2-I))(a-bp_2 + df_2-I) \tag{36}$$

To verify whether the IQD contract can coordinate the supply chain, it is necessary to determine response of the retailer in the second period by solving $\frac{\partial \pi_{r2qdli}}{\partial p_2} = 0$ and $\frac{\partial \pi_{r2qdli}}{\partial f_2} = 0$. After simplification, the retail price of the product and investment effort of the retailer are obtained as follows:

$$p_2 = \frac{a\beta + b(2(a-I)\epsilon-w_2)\beta-d^2(w_2 + 2I\epsilon)}{2b(1-b\epsilon)\beta-d^2} \tag{37}$$

$$f_2 = \frac{d(a-b(w_2 + 2I\epsilon))}{2b(1-b\epsilon)\beta-d^2} \tag{38}$$

Therefore, the manufacturer has two alternatives: (i) the manufacturer can maximize profit by maximizing π_{m2qdli} with respect to w_2, or (ii) the manufacturer can coordinate with the retailer to enhance the supply chain profit. If the manufacturer coordinates with the retailer, then by equating the value of the retailer investment obtained in Eq. (38) and price of product obtained in Eq. (37) with the respective centralized values in Eqs. (32) and (31), the manufacturer wholesale price is obtained as follows:

$$w_2 = \frac{2\epsilon(b(a-2I)\beta + d^2I)}{2b\beta-d^2} \tag{39}$$

Therefore, first-period optimization problems of the retailer and manufacturer become

$$\pi_{r1qdli} = p_1(a-bp_1 + df_1)-(w_1-\epsilon(a-bp_1 + df_1 + I))(a-bp_1 + df_1 + I) + \pi_{r2qdli}-\frac{\beta f^2}{2}-hI \tag{40}$$

$$\pi_{m1qdli} = (w_1-\epsilon(a-bp_1 + df_1 + I))(a-bp_1 + df_1 + I) + \pi_{m2qdli} \tag{41}$$

To verify whether the IQD contract can coordinate the supply chain, it is necessary to determine the response of the retailer in the first period by solving $\frac{\partial \pi_{r1qdli}}{\partial p_1} = 0, \frac{\partial \pi_{r1qdli}}{\partial I} = 0$, and $\frac{\partial \pi_{r1qdli}}{\partial f_1} = 0$. After simplification, the amount of strategic inventory, the retail price, and investment effort of the retailer are obtained as follows:

$$I = \frac{1}{4}\left(\frac{4ab\beta}{2b\beta-d^2}-\frac{2h}{\epsilon} + \frac{2(h + w_1-2a\epsilon)}{b\epsilon^2}-\frac{d^2(h + w_1)}{b^2\epsilon^2\beta}\right) \tag{42}$$

$$p_1 = \frac{a\beta + b(2(a-I)\epsilon-w_2)\beta-d^2(w_2 + 2I\epsilon)}{2b(1-b\epsilon)\beta-d^2} \tag{43}$$

$$f_1 = \frac{d(a-b(w_2 + 2I\epsilon))}{2b(1-b\epsilon)\beta-d^2} \tag{44}$$

Also, in the first period, the manufacturer has two alternatives: (i) the manufacturer can maximize profit by maximizing π_{m1qd} with respect to w_2, or (ii) the manufacturer can coordinate with the retailer to enhance supply chain profit. If the manufacturer coordinates the retailer, then by equating the value of the retailer investment obtained in Eq. (44) and price of the product obtained in Eq. (43) with the respective centralized values in Eqs. (32) and (31), the manufacturer wholesale price is obtained as follows:

$$w_1 = \frac{4ab\beta\epsilon}{2b\beta-d^2}-h \tag{45}$$

Using the above decision, the profit of the retailer, the manufacturer, and total profit of the supply chain are obtained as follows:

$$\pi_{r1qdli} = \frac{a\beta[2b(a + bh-2abx)\beta-d^2(a + bh)]}{(2b\beta-d^2)^2} \tag{46}$$

$$\pi_{m1qdli} = \frac{d^4h^2 + 4bd^2h(ax-h)\beta + 4b^2(h^2-2ahx + 2a^2x^2)\beta^2}{4x(2b\beta-d^2)^2} \tag{47}$$

$$\pi_{c1qdli} = \pi_{r1qdli} + \pi_{m1qdli} = \frac{2a^2\epsilon\beta + 2bh^2\beta-d^2h^2 + 2abh\epsilon\beta}{2\epsilon(2b\beta-d^2)} \tag{48}$$

Now, the win‾win outcomes of the supply chain in two periods will be achieved only when all the members of the supply chain achieve a higher profit than what they achieve without coordination. For the win‾win outcome of all the channel members, we must have $\pi_{r1qd} \geqslant \pi_{r1rli}$ and $\pi_{m1qd} \geqslant \pi_{m1rli}$. Simplifying the first inequalities, one can find $\pi_{m1qd}-\pi_{r1mli} = \frac{\beta[a^2(2b(423-1156b\epsilon)\beta-423d^2) + 304b^2h^2(2b\beta-d^2) + 696abh(2b\beta-d^2)]}{578(2b\beta-d^2)^2}$. Therefore, $\pi_{r1qd} \geqslant \pi_{r1mli}$ holds if $\epsilon \leqslant \epsilon_{Uli} = \frac{(423a^2 + 696abh-304b^2h^2)(2b\beta-d^2)}{2312a^2b^2\beta}$.

Similarly, the difference between the manufacturer profits in IQD contract and decentralized scenario is obtained as follows: $\pi_{r1qd}-\pi_{r1rli} = -\frac{17d^4h^2+4d^2(9a^2\epsilon+bh(15a\epsilon-h(17-4b\epsilon)))\beta-4b(9a^2\epsilon+b(30ah\epsilon-h^2(17-8b\epsilon)-34a^2\epsilon^2))\beta^2}{34\epsilon(2b\beta-d^2)^2}$. Therefore, $\pi_{r1qd} \geqslant \pi_{r1rli}$ holds if. $\epsilon \geqslant \epsilon_{Lli} = \frac{(2b\beta-d^2)(9a^2+30abh+8b^2h^2+\sqrt{81a^4+540a^3bh-1268a^2b^2h^2+480ab^3h^3+64b^4h^4})}{136a^2b^2\beta}$.

From here, one can infer that the IQD contract can upsurge the profit of the supply chain members for any arbitrary values of discount factor $\epsilon \in [\epsilon_{Lli},\epsilon_{Uli}]$. Using Eq. (45), the simplified value of the amount of strategic inventory is $I_{qd} = \frac{2ab\epsilon\beta-h(2b\beta-d^2)}{2\epsilon(2b\beta-d^2)}$. Therefore, the retailer can able to carry inventory in IQD contract if $\epsilon \geqslant \epsilon_{L1} = \frac{h(2b\beta-d^2)}{2ab\epsilon}$, that is if $\epsilon = \epsilon_{L1}$ then the amount of the strategic inventories becomes zero. Substituting, $\epsilon = \epsilon_{L1}$ in Eq. (48), then the profit of the supply chain becomes $\frac{a^2\beta}{2(2b\beta-d^2)}$. Because $\frac{\partial I_{qd}}{\partial\epsilon} = -\frac{h^2}{2\epsilon^2}$, the centralized supply chain profit cannot be achieved with strategic inventory.

Proceeding in a similar way, one can verify that the IQD contract can coordinate the supply chain perfectly without strategic inventory and allocate the profit surplus among supply chain members. Without strategic inventory, the profit functions of the retailer (π_{r2qd}) and manufacturer (π_{m2qd}) in each period are as follows:

$$\pi_{r2qdl} = p_2(a-bp_2+df_2)-(w_2-\epsilon(a-bp_2+df_2))(a-bp_2+df_2)-\frac{\beta f_2^2}{2} \tag{49}$$

$$\pi_{m2qdl} = (w_2-\epsilon(a-bp_2+df_2))(a-bp_2+df_2) \tag{50}$$

To verify whether the IQD contract can coordinate the supply chain without strategic inventory, it is necessary to determine the response of the retailer in each period by solving $\frac{\partial\pi_{r1qdl}}{\partial p_2} = 0$ and $\frac{\partial\pi_{r1qdl}}{\partial f_2} = 0$. After simplification, the retail price and investment effort in each period of the retailer are obtained as follows:

$$p_2 = \frac{(a+bw_2-2ab\epsilon)\beta-d^2w_2}{2b(1-b\epsilon)\beta-d^2}$$

$$f_2 = \frac{ad-bdw_2}{2b(1-b\epsilon)\beta-d^2}$$

If the manufacturer coordinates with the retailer decision and inclined to imply centralized decision to maximize supply chain profit, then the manufacturer sets the following wholesale price in each period:

$$w_2 = \frac{2ab\epsilon\beta}{2b\beta-d^2}$$

The corresponding profits of the retailer and the manufacturer in two periods are obtained as follows:

$$\pi_{r1qdl} = \frac{a^2\beta(2b(1-b\epsilon)\beta-d^2)}{(2b\beta-d^2)^2} \tag{51}$$

$$\pi_{m1qdl} = \frac{2a^2b^2\epsilon\beta^2}{(2b\beta-d^2)^2} \tag{52}$$

From Eqs. (51) and (52), one can verify that $\pi_{r1qdl}+\pi_{r1qdl} = \pi_{1cr}$, that is the supply chain becomes coordinated. Now, the win¯win outcomes of the system will be achieved only when all the members of the supply chain achieve a higher profit than what they achieve in the decentralized scenario. For the win¯win outcome of all the channel members, we must have $\pi_{r1qdl} \geqslant \pi_{rl}$ and $pi_{m1qdl} \geqslant \pi_{ml}$. Simplifying the inequalities, we have obtained the following range $\epsilon \in [\epsilon_{Ll},\epsilon_{Ul}]$, where

$$\epsilon_{Ll} = \frac{2b\beta-d^2}{4b^2\beta} \quad \text{and} \quad \epsilon_{Ul} = \frac{3(2b\beta-d^2)}{8b^2\beta}$$

From here, one can infer that the IQD contract always coordinates the supply chain without strategic inventories. Note that the profit of the manufacturer will be maximum or the profit of the retailer will be minimum at the upper bound of the interval, i.e. at $\epsilon = \epsilon_{Ul}$. From the above discussion, we propose the following proposition:

Proposition 7.

(i) *With strategic inventory, any arbitrary values of discount factor, $\epsilon \in [\epsilon_{Lli},\epsilon_{Uli}]$, lead to acceptable outcomes for all the supply chain members, but the supply chain profit remains suboptimal.*
(ii) *Without strategic inventory, any arbitrary values of discount factor, $\epsilon \in [\epsilon_{Ll},\epsilon_{Ul}]$, coordinate the system perfectly and lead to acceptable outcomes for all supply chain members.*

The graphical representation of the profits of the manufacturer and retailer under the coordination in Scenario L are shown in Figs. 5a and 5b.

The discount factor for the parameter values comes from $\epsilon \in [0.34,0.51]$. Figs. 5a and 5b indicate that the profit of the manufacturer increased while the profit of the retailer decreased, which is also consistent with the profits of the retailer and manufacturer

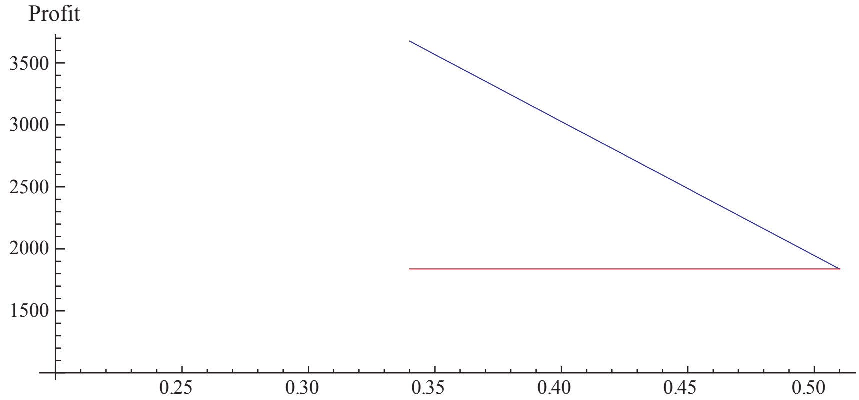

Fig. 5a. The retailer profits in decentralized (Red) and IQD contract (Blue) in Scenario L. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

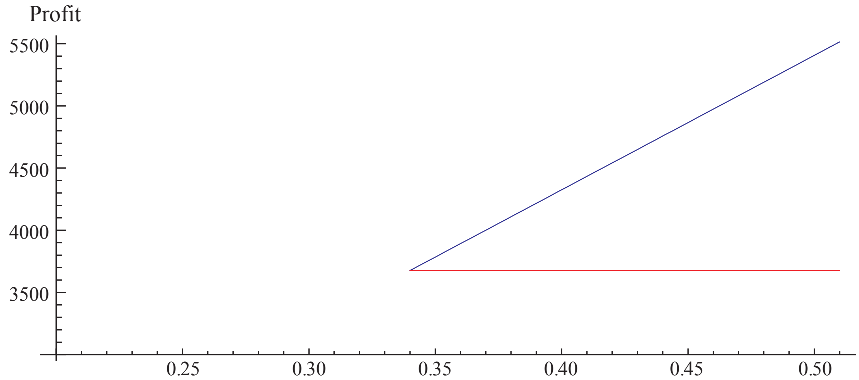

Fig. 5b. The manufacturer profits in decentralized (Red) and IQD contract (Blue) in Scenario L. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

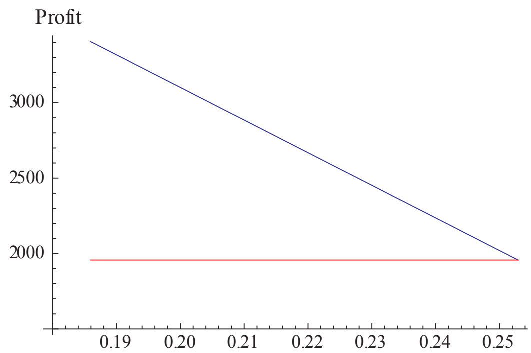

Fig. 6a. The retailer profits in decentralized (Red) and IQD contract (Blue) in Scenario LI. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

obtained from Eqs. (51) and (52).

The graphical representation of the profits of the manufacturer, retailer, and the total supply chain under coordination in Scenario LI are shown in Figs. 6a, 6b and 6c.

The range of discount factors is $\epsilon \in [0.1859,0.2529]$. The retailer can carry strategic inventory if $\epsilon \geqslant 0.0068$. Similar to Scenario L, in Scenario LI, the profit of the manufacturer increased while the profit of the retailer decreased; however, Fig. 6c shows that the optimal supply chain profit cannot be achieved with strategic inventory. The detailed result of the sensitivity analysis for coordination in Scenario LI is shown in Table 5Appendix F). It represents the amount of strategic inventory at lower bound ($I_{rc\epsilon_{Li}}$) and upper bound ($I_{rc\epsilon_{Uli}}$) of a feasible discount factor; the deviations of the centralized profits ($\Delta\pi_{c\epsilon_{Li}} = \pi_{crli} - \pi_{c1qdli}|_{\epsilon_{Li}}$) and ($\Delta\pi_{c\epsilon_{Uli}} = \pi_{icrli} - \pi_{c1qdli}|_{\epsilon_{Uli}}$). The results indicate that as the market potential (a) and investment effort (d) increased, the amount of strategic inventory increased and the corresponding profit deviation was also increased. However, the opposite trend was observed

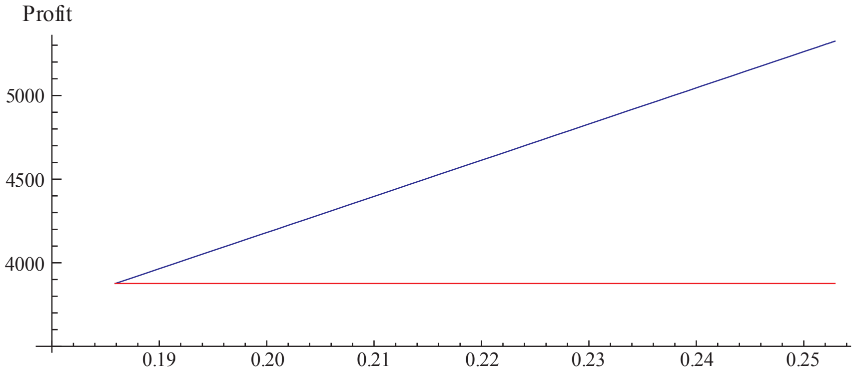

Fig. 6b. The manufacturer profits in decentralized (Red) and IQD contract (Blue) in Scenario LI. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

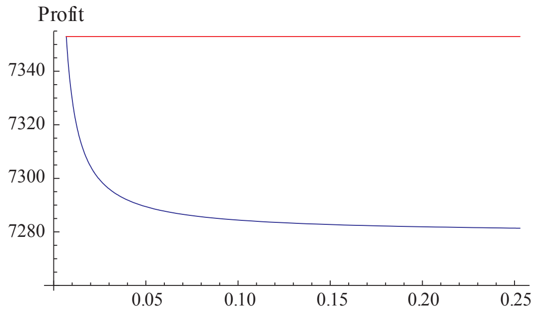

Fig. 6c. Total profit of the centralized supply chain (Red) and IQD contract (Blue) in Scenario LI. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

for the price and efficiency of investment-effort-sensitive parameters. The results are consistent with the decentralized model featuring retailer-investment effort. It is expected that the amount of strategic inventory decreases as the holding cost increases, and the results confirm the accuracy of this intuition. Overall, the presence of strategic inventory is an obstacle for achieving optimal profit for a coordinated supply chain.

The results for a coordinated supply chain are consistent with the findings of Anand et al. (2008), who claimed that "first-best is never achieved in the dynamic (two-period) setting." One can verify that a commonly used coordination contract mechanism, such as wholesale price discounts, revenue sharing, or quantity-discount contracts, cannot provide win–win outcomes in Scenario LI. Through the IQD contract, the manufacturer always wants to apply the centralized decision to achieve maximum supply chain profit. However, if the retailer wants to carry strategic inventory, then the maximum supply chain profit is never obtained. One can use bargaining theory to determine the amount of strategic inventory that supports coordination.

5. Summary and concluding remarks

After the characteristics of strategic inventories were described by Anand et al. (2008) in a multiple-period supply chain, the concept enticed researchers to study the emerging implications. Anand et al. (2008) described the strategic inventory as a hazard and claimed that "the equilibrium solution as well as supplier, buyer, and channel profits are affected by the threat of strategic inventories." In this study, we expanded the research by analyzing the effects of investment efforts of independent supply chain participants on strategic inventory. We found that manufacturer first-period investment effort can maintain harmony when the retailer uses strategic inventory, but it reduces the sales volume. To stave off retailer exploitation, the manufacturer must make investment efforts in both periods to eliminate retailer strategic inventory. However, the investment effort in both periods sometimes provides the opportunity for every supply chain member to earn extra profits. In a surprising finding, a retailer-investment effort maintains harmony between the manufacturer and retailer despite the retailer using investment efforts in self-interest to stimulate demand. In response to the retailer-investment effort over two periods, the volume of the strategic inventory increases, but the supply chain integrity is safeguarded. The manufacturer can also receive a benefit. The total profit of the supply chain is greater in a decentralized environment when the retailer caries more strategic inventory. We employed an IQD contract to coordinate the supply chain and found that it can provide win–win outcomes with the use of strategic inventory. Therefore, the retailer strategic inventory is not always a threat but can also prove advantageous. The retailer can always make sure the manufacturer's sacrifice does not come to naught.

The present analysis can be extended to include several important features. One immediate and uncomplicated extension of this study involves exploring the characteristics of joint investment efforts. In this study, we assumed that both the manufacturer and retailer can apply dynamic pricing,but an interesting extension of the model calls for analysis of the impact of strategic inventory when uniform wholesale or retail prices are used in both periods. In addition, the demand function of the proposed study is deterministic and steady. Incorporating demand uncertainty or seasonal variation (e.g., fashion goods or holiday merchandise) can extend the generalized aspects of the present analysis because in the uncertain scenarios the retailer does not necessarily carry inventory from one period to the next.

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Appendix A. Proof of Lemma 1

The optimal solution for the retailer second-period optimization problem presented in Eq. (1) is obtained by solving $\frac{d\pi_{r2mli}}{dp_2} = 0$. On simplification, we have $p_2 = \frac{a+ce_2+bw_2}{2b}$. The profit function of the retailer in the second-period is concave because $\frac{d^2\pi_{r2mli}}{dp_2^2} = -2b < 0$.

The optimal solution for the manufacturer second-period optimization problem presented in Eq. (2) is obtained by solving $\frac{\partial\pi^2_{m2mli}}{\partial w_2} = 0$ and $\frac{\partial\pi^2_{m2mli}}{\partial e_2} = 0$. On simplification, one can obtain $w_2 = \frac{2(a-2I)\alpha}{4b\alpha - c^2}$ and $e_2 = \frac{c(a-2I)}{4b\alpha - c^2}$. We check the conditions of optimality as $\frac{\partial^2\pi_{m2mli}}{\partial w_2^2} = -b < 0$and $\frac{\partial^2\pi_{m2mli}}{\partial w_2^2}\frac{\partial^2\pi_{m2mli}}{\partial e_2^2} - \left(\frac{\partial^2\pi_{m2mli}}{\partial w_2\partial e_2}\right)^2 = \frac{4b\alpha - c^2}{4}$. Therefore, the profit function of the manufacturer is concave if $4b\alpha > c^2$.

Substituting the optimal response obtained in second-period into Eq. (3), the profit function for the retailer in first-period is obtained as follows:

$$\pi_{r1mli}(p_1,I) = (p_1 - w_1)(a + ce_1 - bp_1) - (w_1 + h)I + \frac{c^4I^2 - 4abc^2I\alpha + b^2(a^2 + 12aI - 12I^2)\alpha^2}{b(4b\alpha - c^2)^2}$$

The optimal solution for the retailer in first-period is obtained by solving $\frac{\partial\pi_{r1mli}}{\partial p_1} = 0$ and $\frac{\partial\pi_{r1mli}}{\partial I} = 0$. On simplification, the following solutions are obtained:

$$p_1 = \frac{a+ce_1+bw_1}{2b} \quad \text{and} \quad I = \frac{b[4b(3a - 4b(h+w_1))\alpha^2 - c^4(h+w_1) - 4c^2(a - 2b(h+w_1))\alpha]}{2(12b^2\alpha^2 - c^4)}$$

Because $\frac{\partial^2\pi_{r1mli}}{\partial p_1^2} = -2b < 0$and $\frac{\partial^2\pi_{r1mli}}{\partial p_1^2}\frac{\partial^2\pi_{r1mli}}{\partial I^2} - \left(\frac{\partial^2\pi_{r1mli}}{\partial I\partial p_1}\right)^2 = \frac{4(12b^2\alpha^2 - c^4)}{4b\alpha - c^2}$, therefore, the profit function of the retailer is concave if $12b^2\alpha^2 > c^4$.

Finally, the manufacturer sets its wholesale price and investment effort in the first period by solving the optimization problem presented in Eq. (4). Substituting p_1 and I in Eq. (4) one can obtain $\pi_{m1mli}(w_1,e_1) = \frac{\alpha(4b\alpha - c^2)(ac^2 + b(h+w_1)(4b\alpha - c^2))^2}{2(12b^2\alpha^2 - c^4)^2} - \frac{\alpha e_1^2}{2} + \frac{w_1}{2}\left(a + ce_1 - bw_1 - \frac{b(c^4(h+w_1) + 4c^2(a - 2b(h+w_1))\alpha - 4b(3a - 4b(h+w_1))\alpha^2)}{12b^2\alpha^2 - c^4}\right)$.

The optimal decision for the manufacturer in first period is obtained by solving $\frac{\partial\pi_{m1mli}}{\partial w_1} = 0$ and $\frac{\partial\pi_{m1mli}}{\partial e_1} = 0$. Collectively, by using back substitution, one can obtain all the decision variables given in Lemma 1. Note that, $\frac{\partial\pi_{m1mli}}{\partial e_1^2} = -\alpha < 0$ and $\frac{\partial\pi_{m1mli}}{\partial e_1^2}\frac{\partial\pi_{m1mli}}{\partial w_1^2} - \left(\frac{\partial\pi_{m1mli}}{\partial e_1\partial w_1}\right)^2 = \frac{\Delta}{4(12b^2\alpha^2 - c^4)^2} > 0$, that is the profit function of the manufacturer in the first period is also concave if $\Delta = 1088b^5\alpha^5 + 60b^2c^6\alpha^2 - 160b^3c^4\alpha^3 - 336b^4c^2\alpha^4 - c^{10} > 0$.

Appendix B. Proof of Proposition 2

The difference between the manufacturer profits in Scenario SI and NI is

$$\pi_{m1msi} - \pi_{m1mni} = \frac{c^2(9a - 2bh)^2}{34b(68b\alpha - 9c^2)} > 0$$

Similarly, the difference between retailer profits in Scenario SI and NI is

$$\pi_{r1msi} - \pi_{r1mni} = \frac{c^2(9a - 2bh)(c^2(3807a - 5606bh) + 680b(31a + 46bh)\alpha)}{1156b(9c^2 - 68b\alpha)^2} > 0$$

Therefore, the manufacturer's first-period investment decision can maintain harmony between the manufacturer and retailer.

Appendix C. Proof of Proposition 3

Proof. The difference between the manufacturer profits in Scenarios LI and NI is

$$\pi_{m1mli} - \pi_{m1mni} = \frac{1}{34b\Delta}[c^{10}(9a^2 - 4abh + 8b^2h^2) + 34bc^8(a^2 + b^2h^2)\alpha - 4b^2c^6(101a^2 - 94abh + 256b^2h^2)\alpha^2 + 16b^3c^4(5a^2 - 125abh + 284b^2h^2)\alpha^3 + 16b^4c^2(121a^2 + 188abh - 376b^2h^2)\alpha^4]$$

Therefore, $\pi_{m1mli} \geqslant \pi_{m1mni}$ holds if the holding cost of the retailer satisfies the following relation:

$$h_1 = \frac{2a\psi_1 - ab\sqrt{34\psi_2}}{2b^2(4b\alpha - c^2)\psi_3} \leqslant h \leqslant \frac{2a\psi_1 + ab\sqrt{34\psi_2}}{2b^2(4b\alpha - c^2)\psi_3} = h_2$$

where $\psi_1 = 752b^5\alpha^4 - bc^8 + 94b^3c^4\alpha^2 - 500b^4c^2\alpha^3$; $\psi_2 = (4b\alpha - c^2)(2c^6 + 17bc^4\alpha - 190b^2c^2\alpha^2 + 376b^3\alpha^3)(c^8 + 4bc^6\alpha - 44b^2c^4\alpha^2 - 16b^3c^2\alpha^3 + 272b^4\alpha^4)$, and $\psi_3 = 4c^6 + 33bc^4\alpha - 380b^2c^2\alpha^2 + 752b^3\alpha^3$.

Similarly, the difference between retailer profits in Scenarios LI and NI satisfies $\pi_{r1mli} \geqslant \pi_{r1mni}$, if

$$h_3 = \frac{ab(4b\alpha - c^2)^2\psi_4 - 17ab\sqrt{\psi_5}}{4b^2(4b\alpha - c^2)^2\psi_6} \leqslant h \leqslant \frac{ab(4b\alpha - c^2)^2\psi_4 + 17ab\sqrt{\psi_5}}{4b^2(4b\alpha - c^2)^2\psi_6} = h_4$$

where $\psi_4 = 59c^{14} + 1050bc^{12}\alpha - 11184b^2c^{10}\alpha^2 - 6472b^3c^8\alpha^3 + 245120b^4c^6\alpha^4 - 260192b^5c^4\alpha^5 - 1748544b^6c^2\alpha^6 + 3333632b^7\alpha^7$, $\psi_5 = (c^{10} - 60b^2c^6\alpha^2 + 160b^3c^4\alpha^3 + 336b^4c^2\alpha^4 - 1088b^5\alpha^5)^2(52158464b^8\alpha^8 - c^{16} - 1624bc^{14}\alpha + 59100b^2c^{12}\alpha^2 - 320992b^3c^{10}\alpha^3 - 211120b^4c^8\alpha^4 + 4583104b^5c^6\alpha^5 - 2296832b^6c^4\alpha^6 - 32869376b^7c^2\alpha^7)$, and $\psi_6 = 76c^{14} + 1475bc^{12}\alpha - 17032b^2c^{10}\alpha^2 + 11276b^3c^8\alpha^3 + 252736b^4c^6\alpha^4 - 342608b^5c^4\alpha^5 - 1670208b^6c^2\alpha^6 + 3259648b^7\alpha^7$.

By combining the two inequalities, one can conclude Proposition 3.

Appendix D. Proof of Lemma 2

To find the optimal solution for the second-period optimization problem of the retailer, one needs to find the solution of the first order conditions $\frac{\partial\pi_{r2rli}}{\partial p_2} = 0$ and $\frac{\partial\pi_{r2rli}}{\partial f_2} = 0$. Note that, $\frac{\partial^2\pi_{r2rli}}{\partial p_2^2} = -2b < 0$and $\frac{\partial^2\pi_{r2rli}}{\partial p_2^2}\frac{\partial^2\pi_{r2rli}}{\partial f_2^2} - \left(\frac{\partial^2\pi_{r2rli}}{\partial p_2\partial f_2}\right)^2 = 2d\beta - d^2$, therefore, the optimal solution of the above problem exists if $2b\beta - d^2 > 0$. The corresponding optimal solution is $p_2 = \frac{a\beta + (b\beta - d^2)w_2}{2b\beta - d^2}$ and $f_2 = \frac{d(a - dw_2)}{2b\beta - d^2}$.

Substituting p_2 and f_2 in Eq. (16) the second-period optimization problem for the manufacturer is obtained as $\pi_{m2rli}(w_2) = \frac{w2(d^2I + b(a - 2I - bw_2)\beta)}{2b\gamma - d^2}$. Optimizing the manufacturer second period optimization problem yields $w_2 = \frac{\beta ab - I(2b\beta - d^2)}{2b^2\beta}$ and the optimal solution always exists because $\frac{d^2\pi_{m2rli}}{dw_2^2} = \frac{-2b^2\beta}{2b\beta - d^2} < 0$. One can observe that, the wholesale price in the second period w_2 decreases as the strategic inventory I increases, therefore the retailer investment effort in the second period increases. Using equilibrium values, the retailer's first-period optimization problem is obtained as follows: $\pi_{r1rli}(p_1,f_1,I) = (p_1 - w_1)(a + df_1 - bp_1) - (w_1 + h)I - \frac{f_1^2\beta}{2} + \frac{b^2(a^2 + 12aI - 12I^2)\beta^2 - 3d^4I^2 - 6bd^2(a - 2I)I\beta}{8b^2\beta(2b\beta - d^2)}$. Because the first-period profit function of the retailer is a function of three variables, we compute the Hessian matrix(H) as follows:

$$H = \begin{pmatrix} \frac{\partial^2\pi_{r1rli}}{\partial p_1^2} & \frac{\partial^2\pi_{r1rli}}{\partial p_1\partial I} & \frac{\partial^2\pi_{r1rli}}{\partial p_1\partial f_1} \\ \frac{\partial^2\pi_{r1rli}}{\partial p_1\partial I} & \frac{\partial^2\pi_{r1rli}}{\partial I^2} & \frac{\partial^2\pi_{r1rli}}{\partial I\partial f_1} \\ \frac{\partial^2\pi_{r1rli}}{\partial p_1\partial f_1} & \frac{\partial^2\pi_{r1rli}}{\partial I\partial f_1} & \frac{\partial^2\pi_{r1rli}}{\partial f_1^2} \end{pmatrix} = \begin{pmatrix} -2b & 0 & d \\ 0 & -\frac{3(2d\beta - d^2)}{4b^2\beta} & 0 \\ d & 0 & -\beta \end{pmatrix}$$

The values of the leading principal minors of the Hessian matrix are $m_1 = -2b < 0, m_2 = \frac{3(2d\beta - d^2)}{2b\beta} > 0$, and $m_3 = -\frac{3(2d\beta - d^2)^2}{4b^2\beta} < 0$; that is, the first-period profit function of the retailer is also concave. By solving the first-order conditions for optimization $\frac{\partial\pi_{r1rli}}{\partial p_1} = 0, \frac{\partial\pi_{r1rli}}{\partial f_1} = 0$, and $\frac{\partial\pi_{r1rli}}{\partial I} = 0$, the following optimal values are obtained:

$f_1 = \frac{d(a - bw_1)}{2b\beta - d^2}$, $p_1 = \frac{(a + bw_1)\beta - d^2w_1}{2b\beta - d^2}$, and $I = \frac{b\beta[3a - 4b(h + w_1)]}{3(2b\beta - d^2)}$ Substituting the retailer's response in Eq. (18), the first-period optimization problem of the manufacturer is obtained as $\pi_{m1rli}(w_1) = \frac{b(18aw_1 - b(4hw_1 + 17w_1^2 - 4h^2))\beta}{9(2b\beta - d^2)}$. One can verify that the optimal solution for the manufacturer first-period optimization problem ($w_1 = w_{1rli}$) is $w_{1rli} = \frac{9a - 2hb}{17b}$. The profit function of the manufacturer in first-period is also concave because $\frac{\partial^2\pi_{r1rli}}{\partial p_1^2} = \frac{-34b^2\beta}{9(2d\beta - d^2)}$. Using backward substitution, the equilibrium solution in Scenario LI under the retailer investment effort is presented in Lemma 2.

Appendix E. Proof of Proposition 5

Proof. The difference between retailer profits in Scenario SI and NI is

$$\pi_{r1rsi} - \pi_{r1rni} = \frac{d^2}{9248b(2b\beta - d^2)(17b\beta - 4d^2)^2}[8858(2b\beta - d^2)^2 + b(16bh^2(3043b\beta - 698d^2)(2b\beta - d^2) + 8ah(731b\beta - 154d^2)(2b\beta - d^2) + 9a^2\beta(14272b\beta - 4535d^2))] > 0$$

Similarly, the difference between manufacturer profits in Scenario SI and NI is

$$\pi_{m1rsi} - \pi_{m1rni} = \frac{d^2(135a^2 + 8abh - 16b^2h^2)(2b\beta - d^2) + 306a^2b\beta}{272b(2b\beta - d^2)(17b\beta - 4d^2)} > 0$$

Therefore, the retailer decisions to offer first-period investment effort always maintain harmony between the manufacturer and retailer.

Similarly, the difference between retailer profits in Scenario LI and NI is

$$\pi_{r1rli} - \pi_{r1rni} = \frac{d^2(9a^2 - 4abh + 8b^2h^2)}{34b(2b\beta - d^2))^2} > 0$$

Similarly, the difference between manufacturer profits in Scenario LI and NI

$$\pi_{m1rsi} - \pi_{m1rni} = \frac{d^2(155a^2 - 118abh + 304b^2h^2}{1156b(2b\beta - d^2)} > 0$$

Therefore, the manufacturer decisions to offer investment effort in both periods can also maintain harmony.

Appendix F. Sensitivity analysis

Tables 4 and 5.

Table 4
Sensitivity analysis of decentralized models in Scenario LI under the manufacturer and retailer investment efforts.

	Manufacturer investment						Retailer investment						
Para.	% Change	Δe_m	Δw_m	Δp_m	I_{mli}	π_{r1mli}	π_{m1mli}	Δf_r	Δw_r	Δp_r	I_{rli}	π_{r1rli}	π_{m1rli}
a	40	8.33	20.81	13.73	13.81	3546.34	5999.23	−14.12	24.01	6.35	29.41	3844.12	7605.88
	20	7.09	17.75	11.71	11.78	2604.46	4406.36	−12.04	20.47	5.42	25.08	2821.78	5585.12
	0	5.87	14.68	9.69	9.74	1807.67	3058.77	−9.96	16.94	4.48	20.76	1957.19	3875.78
	−20	4.65	11.62	7.67	7.71	1155.99	1956.48	-7.88	13.41	3.55	16.44	1250.34	2477.85
	−40	3.42	8.56	5.64	5.68	649.39	1099.47	-5.81	9.88	2.62	12.11	701.23	1391.35
b	40	4.32	10.82	6.64	10.73	1169.42	2082.84	-6.16	11.89	4.18	17.99	1228.75	2436.16
	20	4.99	12.48	7.91	10.33	1420.75	2478.16	-7.63	14.02	4.45	19.09	1510.18	2992.36
	0	5.87	14.68	9.69	9.74	1807.67	3058.77	-9.96	16.94	4.48	20.76	1957.19	3875.78
	−20	7.05	17.62	12.33	8.82	2478.11	3996.14	-14.23	21.35	3.55	23.72	2776.73	5495.41
	−40	8.49	21.23	16.28	7.19	3905.97	5770.52	-24.61	28.71	2.05	30.75	4767.14	9428.87
h	40	5.77	14.43	9.52	9.57	1805.37	3055.94	-9.79	16.65	4.41	20.41	1951.55	3869.19
	20	5.82	14.55	9.61	9.65	1806.52	3057.35	-9.88	16.81	4.44	20.58	1954.35	3872.47
	0	5.87	14.68	9.69	9.74	1807.67	3058.77	-9.96	16.94	4.48	20.76	1957.19	3875.78
	−20	5.92	14.81	9.78	9.83	1808.85	3060.21	-10.04	17.08	4.52	20.93	1960.05	3879.11
	−40	5.97	14.94	9.86	9.91	1810.04	3061.66	-10.13	17.22	4.56	21.11	1962.94	3882.48
c	40	5.64	9.41	8.08	4.56	3058.46	3931.22						
	20	6.41	13.35	9.75	7.97	2133.6	3310.18						
	0	5.87	14.68	9.69	9.74	1807.67	3058.77						
	−20	4.98	15.57	9.37	11.23	1600.91	2885.61						
	−40	3.88	16.18	9.02	12.44	1469.26	2767.88						
d	40							-25.45	16.94	1.23	37.86	3569.97	7069.55
	20							−15.08	16.94	2.36	26.18	2468.26	4887.85
	0							−9.96	16.94	4.48	20.76	1957.19	3875.78
	−20							−6.82	16.94	6.28	17.75	1673.65	3314.3
	−40							−4.59	16.94	7.36	15.96	1504.17	2978.67
α	40	4.39	15.39	9.46	10.92	1639.86	2919.23						
	20	5.03	15.11	9.57	10.42	1706.16	2975.33						
	0	5.87	14.68	9.69	9.74	1807.67	3058.77						
	−20	6.98	13.97	9.78	8.73	1981.47	3195.65						
	−40	8.33	12.49	9.58	7.05	2341.99	3460.31						
β	40							-6.27	16.94	5.96	18.31	1725.22	3416.43
	20							−7.71	16.94	5.39	19.25	1814.84	3593.9
	0							-9.96	16.94	4.48	20.76	1957.19	3875.78
	−20							-14.12	16.94	2.82	23.52	2218.14	4392.55
	−40							-24.21	16.94	1.81	30.25	2851.9	5647.56

Table 5
Sensitivity analysis of centralized models in Scenario LI.

Para.	% Change	$I_{rc\epsilon_L li}$	$\Delta\pi_{c\epsilon_L li}$	$I_{rc\epsilon_U li}$	$\Delta\pi_{c\epsilon_L}$
a	40	77.10	154.20	78.44	156.89
	20	65.38	130.76	66.70	133.40
	0	53.67	107.35	54.96	109.92
	−20	42.00	84.00	43.23	86.46
	−40	30.37	60.75	31.52	63.05
b	40	45.67	91.34	47.15	94.31
	20	48.89	97.79	50.28	100.56
	0	53.67	107.35	54.96	109.92
	−20	61.93	123.86	63.13	126.27
	−40	81.08	162.16	82.28	164.56
d	40	97.91	195.82	100.25	200.50
	20	67.69	135.39	69.31	138.63
	0	53.67	107.35	54.96	109.92
	−20	45.90	91.80	46.99	93.99
	−40	41.25	82.51	42.24	84.48
β	40	47.31	94.63	48.44	96.89
	20	49.77	99.54	50.96	101.92
	0	53.67	107.35	54.96	109.92
	−20	60.83	121.67	62.29	124.58
	−40	78.21	156.43	80.08	160.17
h	40	51.81	145.08	53.50	149.80
	20	52.73	126.56	54.22	130.14
	0	53.67	107.35	54.96	109.92
	−20	54.65	87.44	55.71	89.14
	−40	55.64	66.77	56.47	67.76

Appendix G. Supplementary material

Supplementary data associated with this article can be found, in the online version, at http://dx.doi.org/10.1016/j.tre.2017.10.005.

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