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Robust multiperiod inventory model considering trade-in program and refurbishment service: Implications to emerging markets

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ABSTRACT

We propose a closed-loop supply chain system that incorporates a trade-in program and refurbishment service simultaneously. Through the trade-in program, retailers collect used old-generation products from customers and provide them with new-generation products at a discount price. It helps to acquire the additional products required for the refurbishment service. The proposed integrated system could be useful to retailers selling the smartphone which has a high potential in emerging markets. We approximate the multistage stochastic optimization model to the robust counterpart which features a second-order cone program. Computational results provide managerial insights that could be beneficial to the retailer.

1. Introduction

Since the introduction of *sustainable development* in 1987 (Brundtland et al., 1987) and the announcement of the 1992 *Rio Declaration* (Declaration, 1992) on environmental protection, various efforts have been made throughout the world to preserve the environment. One of the challenges to the manufacturing sector was the concept of *remanufacturing*. It refers to the process of collecting broken, discarded, or returned products and transforming them into “like-new” products through reconstruction, repair, cleaning, and repurposing. The performance and condition of the remanufactured products are similar to those of new products, but remanufacturing can lower the purchasing cost of raw materials and reduce adverse effects on the environment. By drawing on these strengths of remanufacturing, several companies that produce high-tech devices, such as mobile phones and personal computers, have recently launched a *trade – in program*. As an effective sales promotion strategy, companies collect used products from customers and provide them with new-generation products at discount prices. As an example of this, *Apple* launched the *iPhone Upgrade Program* in 2016, which, by collecting used iPhone 7 models from customers, allowed the company to offer iPhone 8 models to customers at discount prices. For the manufacturing industry, whose development cycle of new-generation products becomes ever shorter, the trade-in program plays a prominent role in customer retention and cutting down on incursion from competitors. Also, it mitigates the decline in sales of the new-generation product that results from the released old-generation products. Due to the trade-in program, companies can increase their customers’ repeating purchases and alleviate the regret of customers who have already bought an old-generation product (Van Ackere and Reyniers, 1993; Van Ackere and Reyniers, 1995). This successful strategy has caught the attention of researchers, including the area of management science, operations management, and supply chain

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management. Thus, various studies have been conducted to examine strategies for ongoing operations and to demonstrate the effects of the trade-in program.

Despite the successful introduction and operation of the trade-in program, little attention has been posed to the research analyzing the effect of the trade-in program on a *refurbishment service*. The refurbishment service is one of the warranty services that the customer submits the malfunctioning product for repair, and the retailer provides a *refurbished product* immediately instead of repairing that product. In general, a refurbished product refers to a product in a like-new condition, sold at a discount price. Many companies, under the refurbishment service, occasionally sell defective products, products on display, product demos, or returned products at a discount price. For most retailers nowadays and especially for larger retailers such as *Apple* or *Samsung*, the term "refurbished" refers to products that have truly been reconditioned to a like-new state, thereby sparing customers the inconvenience of waiting for repairs or other replacement hassles. In the case of the mobile phone, which customers use every day, it might be inconvenient when the repairing process takes a too long time. Through the refurbishment service, customers can enjoy the same effect as repairing the product immediately, which leads the high satisfaction to the customers.

From the retailer's perspective, however, it is challenging to prepare sufficient refurbished products since the exchange is a one-to-one arrangement. Predicting the number of products customers will return, and matching that number with a sufficient supply of refurbished products, can be difficult if the refurbished product under warranty is out of stock. Furthermore, if the corresponding product is a discontinued model, it would be cumbersome and inefficient to ask the manufacturer to produce a discontinued product. By introducing the trade-in program, retailers can acquire additional products that are in better condition than returned malfunctioning products. This suggests that research into inventory management considering the trade-in program and refurbishment service simultaneously is required.

The integrated operation of the trade-in program and refurbishment service is expected to become more and more important because of emerging markets. As the publication *Smartphones*: *Worldwide Trends and Forecasts 2019–2024* reported, smartphone penetration in emerging markets is predicted to continue to increase. That is, it is necessary to operate a trade-in program and refurbishment service systematically in order for retail shops to run efficiently in emerging markets (We refer readers to the website: www.businesswire.com/news/home/20200304005300/en/Global-Smartphones-Market-Trends-Forecasts-2019–2024– for further details). The spread of smartphones in emerging markets has been significant but did not show a large market share in the past (Kalba, 2008; DeBerry-Spence et al., 2008). In recent years, however, smartphone penetration in emerging markets has been increasing and is expected to increase. Although many existing studies considered the emerging market from the perspective of operations management or supply chain management, there was no study on the operation of the trade-in program or refurbishment service at the tactical or operational level (see review papers Zhou et al. (2016) and Tang (2018)). Most existing studies are from the perspective of medium- or long-term decision, such as the supply chain contract, tax planning, or advertising (Shou et al., 2016; Niu et al., 2019; Swami and Dutta, 2010). Research also has been conducted on the analysis of smartphone penetration in emerging markets, but not at the tactical or operational level (Steven and Britto, 2016; Han et al., 2013).

To bridge the gap between academia and industry, we started the research from the following research questions:

(i) Could the introduction of a trade-in program not only play a role in a sales promotion that increases customer demand but also ensure the stable operation of a refurbishment service?
(ii) How should an inventory model or supply chain system that incorporates the trade-in program and refurbishment service be modeled?
(iii) How should demands in this closed-loop supply chain system be modeled if they are uncertain and correlated?

To examine the refurbishment service and trade-in program in the context of the inventory management problem, we considered a supply chain system, including a manufacturer, remanufacturer, and retailer. The inclusion of a remanufacturer shifts the system from an *open − loop supply chain system* to a *closed − loop supply chain system* (Govindan et al., 2015). In the closed-loop supply chain system, there exists a reverse flow of returned products from customers to the retailer or remanufacturer. According to Souza (2008), the type of product returned can be classified into three categories in the closed-loop system; *consumer returns*, *end − of − use returns*, and *end − of − life returns*. In the case of consumer returns, the returned product is rarely used and is repurposed mostly through a refurbishment process. In the case of end-of-use returns, the product has been used sufficiently, but defects are difficult to notice, and the product can be submitted to a trade-in program. For end-of-life returns, the product has reached the end of its useful life; that is, it cannot perform its function adequately. Although it can be remanufactured, the process would require a relatively high cost and a long processing period as compared against the work required by a product from an end-of-use return. Meanwhile, the terms *remanufacturing* and *refurbishing* often have been used in remanufacturing contexts without discrimination. Throughout this study, we clearly distinguish between the two terms, using remanufacturing to refer to the process of turning end-of-life products into like-new products and using refurbishing to refer to the process of turning end-of-use products into like-new products.

Before exploring the effect of the trade-in program on the refurbishment service, the decision level from the company should be specified. According to Souza (2013), decisions made along the lines of a closed-loop system can be divided into three levels, which are *strategic*, *tactical*, and *operational* levels. Strategic-level decisions are made for long-term planning purposes, such as network-design or contracts of the supply chain. Tactical-level decisions are made for mid-term planning purposes, such as inventory policies. Operational-level decisions are made for short-term planning purposes, such as scheduling, lot-sizing, or routing in the manufacturing or remanufacturing plant. Our focus in this paper is on a company's (retailer's) acquisition of returned products, and on its return policy, which is at the tactical level. In other words, we focus on an inventory policy, which incorporates both refurbishment service and trade-in program at the time.

With the introduction of the trade-in program, the service level of the refurbishment process can be expected to increase. However, decisions on replenishment and inventory control can be complicated, and uncertainty in the system can increase. As Tang et al. (2012) pointed out, uncertainties inherent in the closed-loop system can include the proper correlation of demand and return, the quality of returned products, and the several parameters in the production planning. Since the main purpose of this study is to analyze the refurbishment service and trade-in program at the tactical level, we focus on the correlations between demand and return. We consider three types of demands, which are for new-generation products, refurbishment service, and the trade-in program. For the refurbishment service and trade-in program, the customers return the used product to the retailer. The structure of demand and return, which are correlated in this manner, implies that a new policy of inventory control is required. In this study, we consider the multiperiod inventory problem, which incorporates the three types of uncertain demands that are correlated. By adopting a factor-based demand model, the correlations of these uncertain demands can be characterized. Detailed explanations will be discussed in Section 5.1.

To sum up, we introduced the factor-based demand model in the multiperiod inventory mode which is based on the closed-loop supply chain. Due to the intractability of the model, we approximated the model by utilizing the robust optimization approach with the linear decision rule. We also conducted various experiments to get answers from research questions (i) to (iii). Thus, the main contributions of this research can be summarized as follows:

- We developed a mathematical formulation based on the closed-loop system with the refurbishment service and trade-in program.
- We approximated the multistage stochastic optimization model to the second-order cone program that is computationally tractable.
- We found managerial insights that could be beneficial to the inventory manager of the retailer.

The remainder of this paper is organized as follows: Previous relevant studies are investigated in Section 2. In Section 3, we describe the inventory model considering the refurbishment service and trade-in program (IMRSTIP). Section 4 deals with the mathematical formulation of the IMRSTIP. In Section 5, we present our computational experiments and analyses. In Section 6, we summarize the findings of this research.

2. Literature review

We investigated previous studies related to the trade-in program and inventory or lot-sizing problem in the closed-loop supply chain system. We also examined several papers that are related to emerging markets from the perspective of operations management or supply chain management. Especially, we described the studies considering the effect of the trade-in program or pricing strategy in the trade-in program in Section 2.1. In Section 2.2, the previous studies relevant to the operation at the tactical or operational level in the closed-loop system are described. In Section 2.3, we described previous research, which considered emerging markets. After explaining previous relevant studies, we summed up the distinguishable features of this study in Section 2.4.

2.1. Effects of the trade-in program and strategic-level decisions for the trade-in program

Various studies have been conducted to analyze the benefits of introducing the trade-in program to the company. Rao et al. (2009) demonstrated the effects of a trade-in program and claimed that introducing a trade-in program would inevitably raise the profit. Yin et al. (2015) adopted the two-period dynamic game, including first and second generations, to examine the effectiveness of a trade-in program. By incorporating customers as forward-looking customers, they analyzed conditions that are beneficial to a company. They claimed that the durability of the product in a first generation, degree of the market heterogeneity, and uncertainty of the product in a second generation are the key determinants of the successful trade-in program. Meanwhile, Agrawal et al. (2016) studied the trade-in program that operated between an original equipment manufacturer and a third-party remanufacturer. By analyzing the effect of the trade-in program based on the game theory scheme, they derived several insights. Also, their numerical analysis revealed that the trade-in program has environmental advantages. As evidenced by previous literature, the introduction of the trade-in program not only brings additional profits to a company but also has a tremendous environmental impact. Additionally, we examined existing studies on how strategic-level decisions should be made for the medium- and long-term by introducing the trade-in program.

In addition to analyzing the effects that trade-in programs have on companies, various research has analyzed how companies might operate the trade-in program successfully, such as by establishing pricing strategies. Ray et al. (2005) focused on determining an optimal price and rebate for trade-in products. The analysis was conducted by varying the price setting and customer segment for each scenario. Accordingly, the study identified the most favorable conditions of pricing strategies for each scenario. Li et al. (2011) conducted research to find a more effective way to operate a trade-in program in a business-to-business context. They argued for the effectiveness of the trade-in program through customer segmentation and accurately predicting product returns in the trade-in program. The proposed method allows a company to design a segment-based trade-in policy. They claimed that predicting product returns, particularly for the initial return, plays a critical role in the successful operation of a trade-in program. Li and Xu (2015) found an optimal pricing strategy from the perspective of a monopolistic manufacturer, where the trade-in program and leasing option were available. They identified the differences between a trade-in program and leasing option in terms of the return time and the profitability of the new product. Furthermore, they analyzed which policies are advantageous to a company based on the given conditions. In another paper, Chen and Hsu (2015) derived the optimal price, trade-in rebate, and the strategic choice for a company by considering the deterioration rate and the recovery cost of used goods. They addressed that the degree of the trade-in rebate increases according to the deterioration rate, and decreases in the manufacturing and remanufacturing costs. Zhu et al. (2016) derived an optimal price to charge new customers and an optimal rebate to offer trade-in program customers under the duopoly situation in which one company operates the trade-in program, and another company does not operate the trade-in program. By analyzing the results from the Nash equilibrium, they identified positive impacts on the market share and the profitability of the trade-in program. Han et al. (2017) investigated conditions necessary for a successful trade-in program by determining the price and production quantity. By taking into account the factors of receptivity, durability, and subsidy, they identified insights for both companies and governments to implement a trade-in program effectively.

An *omni-channel*, which integrates an online channel with the retail service, was also considered in the trade-in program context. Cao et al. (2018) examined the trade-in program based on three types of distribution channels, including online, offline, and dual channels. They analyzed the condition for which channels are the best choice for retailers according to the shipping cost. Cao et al. (2019) considered the trade-in program with the dual-format retailing model, including the self-run store and third-party store. Differing from the traditional trade-in program, gift cards or cash coupons were provided to customers. The introduction of a trade-in program in such a system can lower the trade-in rebate but brings significant benefits to a company. Meanwhile, Sheu and Choi (2019) investigated pricing strategies under the variable of market competition. By including the concept of extended consumer responsibility in the model, they provided syncretic value-oriented prices and trade-in rebates in a trade-in program operating within a market competition setting. De Giovanni and Zaccour (2019) analyzed investment decisions for quality improvement and pricing in a trade-in program. In their work, they divided returns into two categories, including the passive return and active return, and they divided the pricing strategy into two types, which remained constant or varied over time. Ma et al. (2019) studied how the quality of returned products affected the trade-in program, unlike previous studies that only focused on how pricing affected the trade-in program under the remanufacturing environment. They identified the effects of these double references and derived the condition under which both the manufacturer's profits and customer surplus would benefit.

As can be seen from the above-mentioned literature, many researchers demonstrated the profitability of the trade-in program. If a long-term plan, such as a pricing strategy, is appropriately established, revenue would undoubtedly be increased. After that, planning from a mid-term perspective, such as order policy for inventory management, should be established. Unlike an existing inventory system, an inventory system that includes the trade-in program makes the supply chain a closed-loop system. Therefore, we investigated previous studies related to the inventory model based on the closed-loop supply chain system and identified the differences we found from this research.

2.2. Inventory or lot-sizing model in a closed-loop supply chain system

Mathematical formulations based on the closed-loop supply chain system can be categorized into two types; an *economic order quantity*, which is based on a continuous-time nonlinear program, and a mixed-integer program which is based on a discrete-time planning horizon. We mainly investigated the latter type, which is close to this study. Özceylan and Paksoy (2013) developed a mathematical formulation based on the mixed-integer program considering not only the quantities for the manufacturing and remanufacturing products but also the fixed cost for the opening decision of the plants. In the work of Li et al. (2014), they introduced two types of binary variables to incorporate the fixed costs for manufacturing and remanufacturing. Gaur et al. (2017) considered reconditioned products in a closed-loop system that could be involved in several stages, depending on the condition of the returned product. Mardan et al. (2019) developed the multi-echelon network as the closed-loop supply chain system. To overcome the complexity, they developed the Benders decomposition algorithm and validated the performance of the algorithm.

Especially in recent years, many studies in the area of the remanufacturing have incorporated an uncertain environment. Denizel and Ferguson (2009) considered the uncertain quality of the returned products in the closed-loop supply chain. They developed a multistage stochastic optimization model by generating scenarios of possible qualities of returned products. With a focus on robust optimization, various research has been conducted by optimizing the problem against the worst-case scenario. By developing robust counterparts to existing set-ups, feasible solutions have been guaranteed under all possible scenarios. For the cases of Eslamipoor et al. (2015) and Pishvaee et al. (2011), they regarded the uncertain demand set as a box shape. In addition, Eslamipoor et al. (2015) considered inventory-related costs as uncertainty. Meanwhile, the concept of the *budget of uncertainty* in demand, which was proposed initially by Bertsimas and Sim (2004), was adopted in the closed-loop supply chain system (Kim et al., 2018; Zhang et al., 2019; Zhou and Sun, 2019; Yavari and Geraeli, 2019). In the case of Hasani et al. (2012), they considered the perishable item under uncertain demand. As can be found from the above studies, the robust optimization model under the closed-loop supply chain has mainly considered the uncertain set as a box or polyhedron shape. Wei et al. (2011) regarded demand as an uncertain value that belongs to the polyhedron. To make the formulation including the uncertain parameter as a robust counterpart, they adopted the budget of uncertainty. Although the prevailing purpose of the robust optimization through the budget of uncertainty was based on a static situation, they handled the multiperiod setting by forcing the independence of parameter Γ, as Bertsimas and Thiele (2006) proposed. Talaei et al. (2016) focused on reducing the rate of carbon emission in the closed-loop system. They investigated the effects of uncertainties on the cost and demand rate by adopting the robust fuzzy programming approach. Jabbarzadeh et al. (2018) considered the disruption risk in the closed-loop system. They developed the mathematical formulation as the stochastic robust optimization model and solved the problem through the Lagrangian relaxation. Bi-level optimization was also considered in the closed-loop supply chain network by Hassanpour et al. (2019). They considered the incentive strategy for different qualities of the returned products. By considering the government as a leader and supply chain designer as a follower, the robust bi-level optimization model was developed. To solve the problem efficiently, they utilized the meta-heuristic algorithm.

As detailed in this section, plenty of studies related to the inventory or lot-sizing model under the closed-loop supply chain system were conducted. Owing to the remanufacturing process, the complexity of the model increased and tractability issue occurred. Accordingly, many studies have been conducted to improve the solution methodology in terms of efficient computation. In addition, as Tang et al. (2012) mentioned, the closed-loop supply chain can raise a variety of uncertainties. Thus, many studies considered the robust optimization approach to handle such uncertainties.

2.3. Emerging markets from the perspective of operations management or supply chain management

This study is relevant to the operation of a retail shop selling smartphones, which have a high potential for increased market share in emerging markets. The importance of operations management, supply chain management, and inventory management in emerging markets should not be overlooked (Han et al., 2013; Fleury et al., 2015; Wang et al., 2016). We investigated prior relevant studies that studied emerging markets from the perspective of operations management or supply chain management to clarify the distinction between our study and existing studies. Steven and Britto (2016) analyzed the effects that entering emerging markets had on inventory performance and product recall. By establishing several hypotheses and verifying them, it is identified that inventory performance plays a vital role in the performance of companies in emerging markets. In the meantime, Boulaksil and Van Wijk (2018) studied the inventory management of *nanostores* (small retailers), which are numerous in developing countries. By establishing a multistage stochastic inventory model based on uncertain supplier credits, they conducted computational experiments. From the results of these experiments, several managerial insights were provided. Although this study considered the problem on a tactical or operational level, which is the same decision level considered by this study, they mainly focused on the credit of the supplier in the inventory model, rather than on the service structure, such as a trade-in program or refurbishment service. Palsule-Desai (2015) proposed an analytical model based on a game theory approach, in order to examine the roles of the coordinator in a two-tier network, including small and marginal producer farmers. This system easily can be seen in emerging markets, such as in the fruit and vegetable industry of India. By deriving the equilibrium from the non-cooperative game-theoretic model, the roles of the coordinator and profit-sharing mechanism are identified. Choi and Luo (2019) argued that data quality affects supply chain management with regard to profits and welfare in emerging markets. They also found a sufficient condition of positive effect with the introduction of the blockchain. Through their analytical model, they compared a decentralized system with a centralized system in terms of data volatility from poor data quality. Meanwhile, Boulaksil and Belkora (2017) compared two distribution systems, *van sales* and *pre-sales* strategies, based on the nanostores in emerging markets. The former is a system where a van is loaded with goods, traverses the nanostores, and sells the goods to each nanostore if the store has a payment capability.The latter, pre-sales strategy, refers to a system where products are provided after being ordered in advance. The researchers of this study also conducted a case study with a mathematical model. The results of this case study showed that the pre-sales strategy outperformed the van sales strategy. Although more workloads are required for the pre-sales strategy, it is expected to be an outstanding strategy to use in the future, in emerging markets. Jerath et al. (2016) analyzed how an organized retailer affects an unorganized retailer. With their theoretical model, they provided several managerial insights which could be intuitive for retailing in emerging markets. Especially, they identified that an organized retailer could lower the number of unorganized retailers under the equilibrium model. Lorentz et al. (2013) conducted case studies to characterize the emerging market within the sector of international food manufacturers. The analysis showed that network strategy and position, boundaries of the company, product mobility, and geographic configuration are key components of the adjustment pattern with regard to the supply chain network. In the case of the Huq et al. (2016), a case study was conducted based on the apparel industry of Bangladesh. They mainly focused on social management capabilities and social issues, including health, safety, quality of life, and worker right. Their results claimed that stakeholder pressure and regulatory mechanisms affect a supplier's social issue. As can be seen from the literature reviewed above, studies considering emerging markets from the perspective of operations management or supply chain management have received attention from academia. However, research on the trade-in program or refurbishment service within a closed-loop supply chain system was conducted insufficiently. Although Batista et al. (2019) considered the emerging market in the closed-loop supply chain system, their research was not based on a mathematical model for optimization.

2.4. Distinctive features of this research

As evident from the above literature, no study has yet considered the trade-in program and refurbishment service simultaneously from a tactical or operational level. Bian et al. (2019) integrated the model based on the trade-in program with warranty service, but they focused on pricing rather than inventory control. In the case of Huang (2018), the trade-in program was incorporated in the closed-loop system structure, but the refurbishment service was not considered. Meanwhile, Hong et al. (2019) investigated the role of the value-added service in the closed-loop supply chain, which has a similar property to the trade-in program. They examined the effects of the value-added service on a retailer and manufacturer by analyzing the model and conducting numerical experiments. They also designed a supply chain contract, based on the service cost-sharing mechanism, to improve the profitability of the whole system. However, this study was also oriented to make the decision based on the strategical level.

In this study, we focused on a multiperiod inventory model that took into account both a trade-in program and refurbishment service at the same time. To address uncertain demands in the multistage decision process, we considered an adjustable robust optimization approach. This robust optimization approach, which is based on the *wait and see* decision model, is more natural than the *here and now* decision model (Ben-Tal et al., 2004). In this scheme, the decision can be delayed by observing the part of uncertain factors that are realized within the course of the period. Also, no study has considered the adjustable optimization approach to the closed-loop supply chain system. To develop the closed-loop supply chain model based on the adjustable robust optimization

Table 1
Comparisons of this study and previous research relevant to the closed-loop system.

Authors (year)	Mathematical formulation	Uncertainty	Refurbishment service or Trade-in program	Solution methodology
Teunter et al. (2006)	Mixed-integer program			Dynamic programming
Özceylan and Paksoy (2013)	Mixed-integer program			Commercial solver
Li et al. (2014)	Mixed-integer program			Meta-heuristic
Gaur et al. (2017)	Mixed-integer nonlinear program			Heuristic
Huang (2018)	Stackelberg game		✓	Closed-form solution
Mardan et al. (2019)	Mixed-integer program			Benders decomposition
Denizel and Ferguson (2009)	Linear program	✓		Stochastic program
Wei et al. (2011)	Linear program	✓		Budget of uncertainty
Pishvaee et al. (2011)	Mixed-integer program	✓		Robust counterpart from box uncertainty set
Hasani et al. (2012)	Mixed-integer nonlinear program	✓		Linearization
Kim et al. (2018)	Mixed-integer program	✓		Budget of uncertainty
Eslamipoor et al. (2015)	Mixed-integer program	✓		Robust counterpart from box uncertainty set
Talaei et al. (2016)	Mixed-integer program	✓		Chance constrained fuzzy programming
Jabbarzadeh et al. (2018)	Mixed-integer program	✓		Robust counterpart and Lagrangian relaxation
Zhang et al. (2019)	Mixed-integer program	✓		Fuzzy theory and Robust optimization
Zhou and Sun (2019)	Linear program	✓		Budget of uncertainty
Yavari and Geraeli (2019)	Mixed-integer program	✓		Budget of uncertainty and Heuristic
Hassanpour et al. (2019)	Bi-level mixed-integer program	✓		Robust counterpart and Meta-heuristic
This study	Linear program	✓	✓	Adjustable robust optimization approach

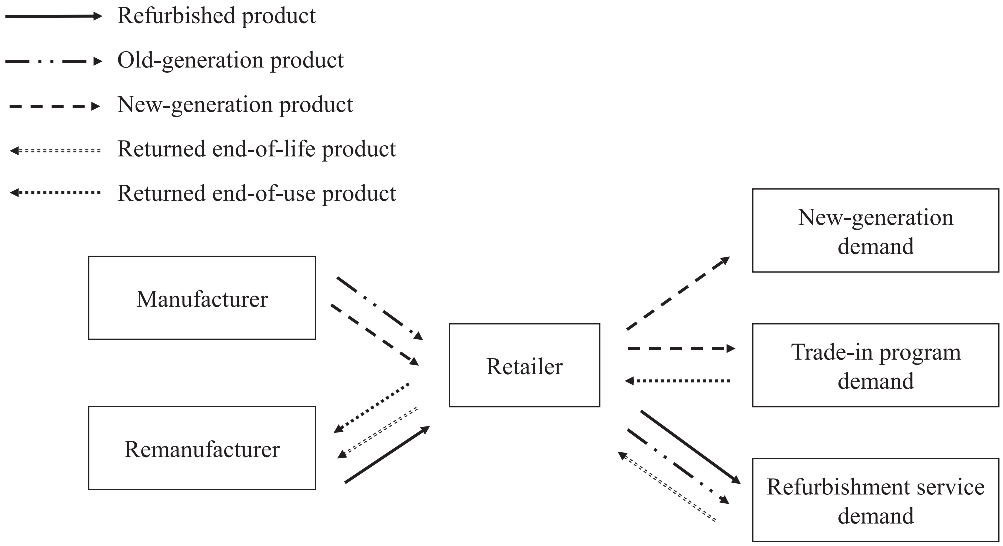

Fig. 1. Flow of products within the supply chain.

approach, we refer to the multiperiod linear program model. We mainly referred to the models of Teunter et al. (2006) and Wei et al. (2011) and modified those models to fit the circumstance of this study. To emphasize the characteristics of this study from the previous studies related to the closed-loop system, we summarize the relevant literature in Table 1.

3. Problem description

We consider a multiperiod inventory model based on the discrete-time planning horizon $t \in \{1, ..., T\}$. Let $\widetilde{d}_t$, $\widetilde{\phi}_t$, and $\widetilde{\xi}_t$ denote the uncertain demands of the new-generation product, trade-in program, and old-generation product, respectively, at period $t \in \{1, ..., T-1\}$. The retailer provides new-generation products to customers of the new-generation demands and trade-in program demands. Customers who purchase the new-generation product through the trade-in program return end-of-use products, which are old-generation products, to the retailer. Then, the retailer determines how many products to send to the remanufacturer and how many to keep in inventory. If the retailer sends the end-of-use product to the remanufacturer, the retailer receives the like-new conditioned product after a lead time L_r with paying the refurbishing cost $c_{r,t}$ per unit at period t. Otherwise, when the retailer holds the product in inventory, the inventory holding cost $h_{w,t}$ per unit occurs at period t. In the case of customers who arrive at the store to get refurbishment service, they return the end-of-life product to the retailer and receive the like-new conditioned product. The retailer also determines how many products to send for remanufacturing and how many to keep in inventory. In this case, lead time L_m takes and remanufacturing cost $c_{m,t}$ occurs per unit at period t. Otherwise, the holding cost $h_{I,t}$ occurs per unit at period t. Meanwhile, the manufacturer produces the new-generation product with unit-cost $c_{n,t}$ by taking the lead time L_n. The manufacturer produces the old-generation product only when the product required for refurbishment service is out of stock. Afterward, the lead time L_o and relatively higher cost (including opportunity cost) $c_{o,t}$ occurs per unit at period t. These forward and reverse flows in the closed-loop system are illustrated in Fig. 1. When excess inventories of new-generation and old-generation products occur, we assume that the unit inventory holding costs $h_{u,t}$ and $h_{v,t}$, respectively, impose themselves at the end of period t by carrying over to the next period $t+1$. In the case of the unsatisfied demand, the backlog is assumed. We feel that the unit backlog cost of customers who want refurbishment service, p_t, is higher than that of customers who want to buy the new-generation product or join a trade-in program, b_t. Therefore, we assume that the unit penalty cost p_t is relatively larger than b_t in period t for the unsatisfied demands. A supplementary explanation is added to Appendix A to highlight the difference between when the trade-in program was introduced and when it was not. Before developing the mathematical formulation, we made the following assumptions:

Assumption 1. Customers who want a refurbishment service return end-of-life products to a retailer.

Assumption 2. Customers who buy the new-generation product through the trade-in program return the end-of-use products to a retailer.

Assumption 3. A remanufacturer receives two types of products from the retailer, which are end-of-use products and end-of-life products.

Assumption 4. A manufacturer produces only new-generation products, except when the refurbished product is out of stock.

Assumption 5. Lead times and costs for producing the new-generation and old-generation products, and remanufacturing and refurbishing processes are different.

The quality of the returned product can be classified into several levels that could variegate the lead times and remanufacturing costs. However, we assumed returned products as two levels, including end-of-use and end-of-life products, to focus on the operation of the closed-loop supply chain under the correlated uncertain demands. In the cases of Assumptions 1 and 2, two customer types, who receive a refurbishment service by returning end-of-use products and who participate in the trade-in program by returning end-of-life products, could be considered further. In the former case, however, the retailer will not offer the warranty service to customers who return the products that have been used for a long time but still function properly. Also, if customers are reasonable, they will buy the new-generation product through a trade-in program and not receive the refurbishment service when an additional cost occurs. Therefore, we ruled out the former case in this study. In the latter case, the submission of an end-of-life product to the trade-in program was also ruled out because the retailer would either refuse to honor the return or offer a minimal benefit for it. From a retailer's perspective, even if customers participate in a trade-in program for a small benefit, such a program will not differ much, in terms of profit, from selling new-generation products to those customers. Thus, the latter case was also excluded in this study.

3.1. Demand modeling

We assumed that three types of uncertain demands, which are for the new-generation product, refurbishment service, and trade-in program, are correlated. When the release of the new-generation product is successful, customers who use the old-generation product will typically prefer the new-generation product without using the refurbishment service. When the trade-in program is introduced, it can encroach on the refurbishment service. That is, the correlation between the demands of new-generation and refurbishment service shows negative. Also, the correlation between the demands of the trade-in program and refurbishment service features negative. Let $\rho_{n,r}$ and $\rho_{r,t}$ represent the correlations between demands of new-generation product and refurbishment service, and refurbishment service and trade-in program, respectively. Then, the correlation between trade-in program and refurbishment service, $\rho_{t,n}$, can be derived by the *Cauchy − Schwarz inequality* as follows (Olkin, 1981):

$$\rho_{t,n} \leqslant \rho_{n,r}\rho_{r,t} + \sqrt{(1-\rho_{n,r}^2)(1-\rho_{r,t}^2)}$$
$$\rho_{t,n} \geqslant \rho_{n,r}\rho_{r,t} - \sqrt{(1-\rho_{n,r}^2)(1-\rho_{r,t}^2)}$$

From the above inequalities, the range of the correlation between the demands of the trade-in program and new-generation product can be identified. If the inequality $\rho_{n,r}^2 + \rho_{r,t}^2 \geqslant 1$ holds true, the correlation between the demands of the trade-in program and new-generation product features positive. The three types of demands are correlated with both positive and negative, which makes the demand model complicated in an uncertain environment. That is, the demand modeling, which can capture the correlations, is required. Accordingly, we adopted a factor-based demand model as follows:

$$\begin{aligned}
&\widetilde{d}_t(\widetilde{\mathbf{z}}_t) \triangleq d_t^0 + \sum_{k=1}^{N} d_t^k \widetilde{z}_k \\
&\widetilde{\phi}_t(\widetilde{\mathbf{z}}_t) \triangleq \phi_t^0 + \sum_{k=1}^{N} \phi_t^k \widetilde{z}_k \\
&\widetilde{\xi}_t(\widetilde{\mathbf{z}}_t) \triangleq \xi_t^0 + \sum_{k=1}^{N} \xi_t^k \widetilde{z}_k
\end{aligned} \tag{1}$$

where $1 \leqslant N_1 \leqslant N_2 \leqslant \cdots \leqslant N_{T-1} = N$ and $\widetilde{\mathbf{z}}_k$ are unfolded until $k = 1, \cdots, N_t$.

The factor-based demand model is the stochastic demand model that is affinely dependent on the predefined uncertain factors. Each demand model shares some uncertain factors and thus captures the correlations. Another advantage of the factor-based demand model is that it does not require full information about demand distribution. It can be characterized by only the mean, support, and covariance of the uncertain factor. We describe in detail how the correlation among the three types of demands is incorporated with mean, covariance, and support of the uncertain factor in Section 5.1. To sum up, the correlations among the three types of uncertain demands can be captured by adopting the factor-based demand model without full information about the demand distributions.

3.2. Decision of the inventory manager

We assume that the inventory manager of the retailer makes the four types of decisions at the beginning of each period, simultaneously. These decisions are (i) the order quantity of the new-generation product, (ii) the order quantity of the old-generation product, (iii) the refurbishing quantity with the end-of-use product, and (iv) the remanufacturing quantity with the end-of-life product. Decision (i) is made to respond to future demands of the new-generation product and trade-in program from the manufacturer. On the other hand, decision (ii) is made to attain the old-generation product from the manufacturer to respond to the refurbishment service when the refurbished product is out of stock. Decisions (iii) and (iv) are made to attain the like-new conditioned product for the refurbishment service from the remanufacturer. The sequence of these decisions by the inventory manager is illustrated in Fig. 2.

To distinguish the flow of each product from the retailer, we partitioned the product flow with four types; (i) new-generation product, (ii) refurbishment service (old-generation and refurbished products), (iii) refurbishing process (end-of-use product), and (iv)

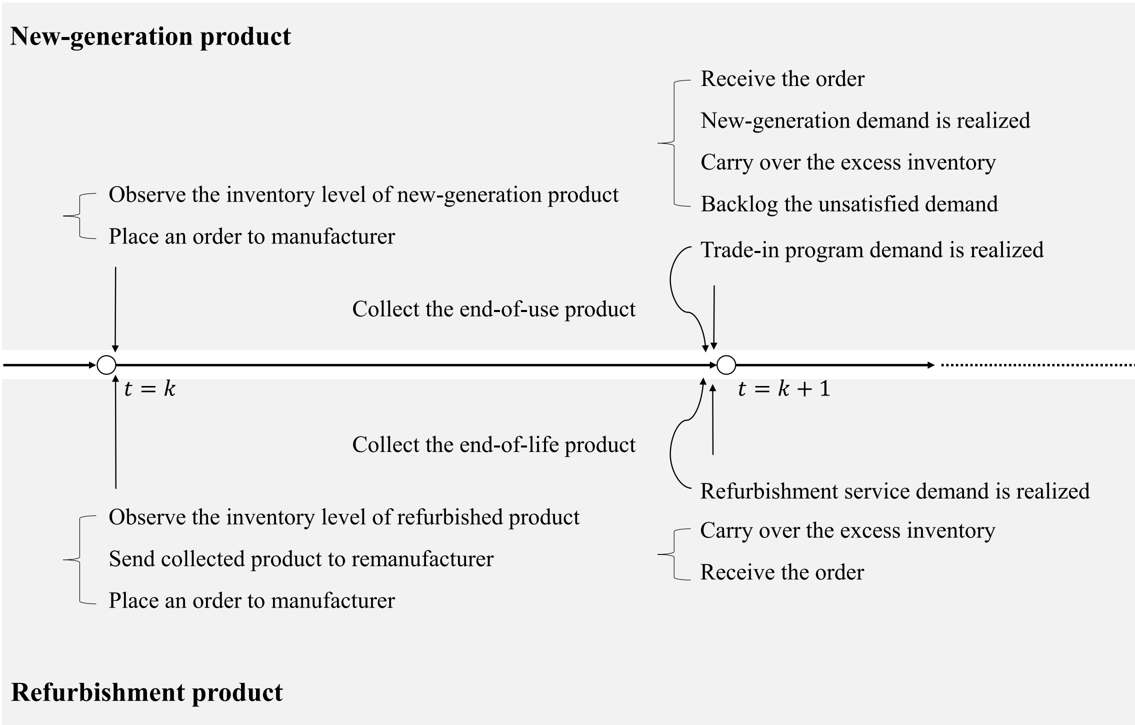

Fig. 2. Sequence of the decisions made by the inventory manager.

remanufacturing process (end-of-life product). Their flows can be merged, but we partitioned the flow to incorporate the different costs of the product from process to process. The retailer receives the new-generation product from the manufacturer to respond to the demands of the trade-in program and new-generation product. To handle the refurbishment service demand, the retailer acquires like-new conditioned products from a remanufacturer or old-generation products from a manufacturer. The like-new conditioned product should be regarded as two types which are from end-of-use and end-of-life products because different costs are imposed on them. For the refurbishing process, which transforms end-of-use products to like-new conditioned products, the retailer receives the same quantity of end-of-use products with the demand generated by the trade-in program. Then, the retailer determines how many products will be sent to the remanufacturer or kept in inventory. In the case of the remanufacturing process, which transforms end-of-life products to like new conditioned products, the retailer receives the same quantity of end-of-life products with the demand generated by refurbishment service. In the same manner as used in the refurbishing process, the retailer determines how many products will be sent to the remanufacturer or kept in inventory. For the mathematical formulation, flow conservation of each inventory process is illustrated in Fig. 3 and related decision variables are summarized as follows:

x_t: Order quantity of new − generation products from a manufacturer at period t
y_t: Order quantity of old − generation products from a manufacturer at period t
q_t: Quantity of end − of − use products sent to a remanufacturer for refurbishing process at period t
m_t: Quantity of end − of − life products sent to a remanufacturer for remanufacturing process at period t
u_t: Inventory level related to new − generation products at period t
v_t: Inventory level related to refurbishment service at period t
w_t: Inventory level related to the refurbishing process at period t
I_t: Inventory level related to the remanufacturing process at period t

4. Mathematical formulation

4.1. Mathematical formulation of the IMRSTIP under the deterministic demand model

This subsection presents the mathematical formulation under the deterministic demand model. In this model, the inventory manager regards all demands across the entire planning horizon as deterministic values. The notations, d_t, ξ_t, and ϕ_t, are denoted to distinguish the deterministic demands from the uncertain demands, $\widetilde{d}_t$, $\widetilde{\xi}_t$, and $\widetilde{\phi}_t$, respectively. The objective of the inventory manager is to minimize the total costs within the entire planning horizon, including purchasing, excess inventory holding, and

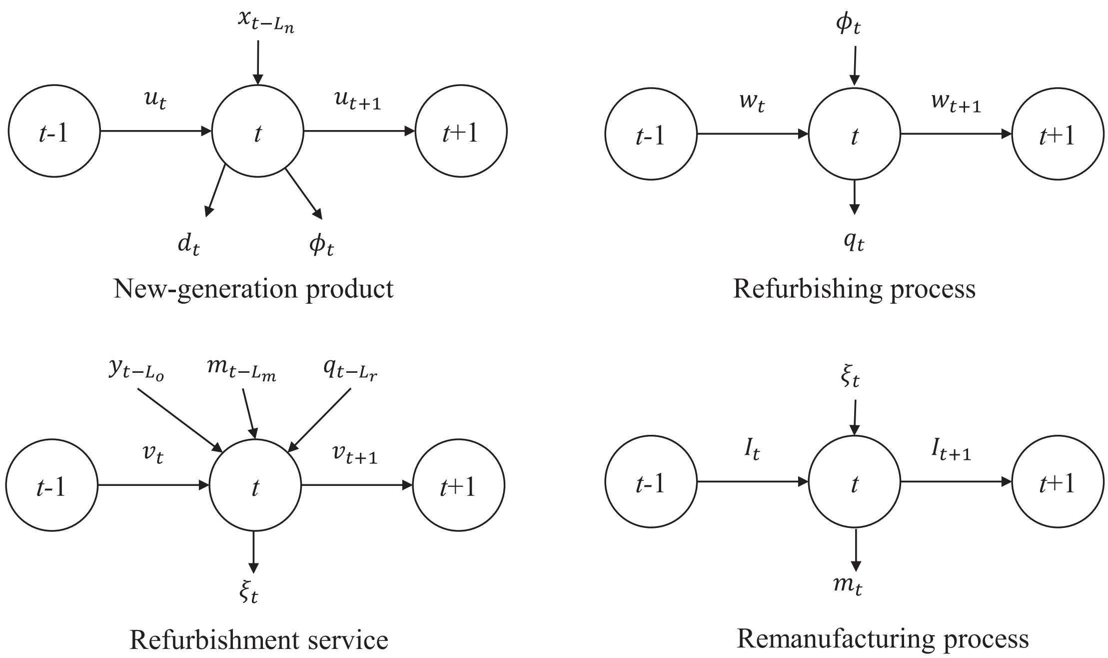

Fig. 3. Balance equations for the four types of inventories.

backlog costs. Under the deterministic demand model, the total costs of the entire planning horizon are represented as follows:

$$\begin{aligned}
&\text{Total purchasing costs (TPC)} = \sum_{t\in\mathfrak{T}} [c_{n,t}x_t + c_{o,t}y_t + c_{r,t}q_t + c_{m,t}m_t]\\
&\text{Total holding costs (THC)} = \sum_{t\in\mathfrak{T}} [h_{u,t}(u_{t+1})^+ + h_{v,t}(v_{t+1})^+ + h_{w,t}(w_{t+1})^+ + h_{I,t}(I_{t+1})^+]\\
&\text{Total backlog costs (TBC)} = \sum_{t\in\mathfrak{T}} [b_t(u_{t+1})^- + p_t(v_{t+1})^-]\\
&\text{where } (\mathbf{x})^+ = \max(\mathbf{x},0),\ (\mathbf{x})^- = -\min(\mathbf{x},0),\ \text{and } \mathfrak{T} \triangleq \{1,\ldots,T-1\}
\end{aligned} \tag{2}$$

Remark 1. The backlogged inventories related to **w** and **I** are not featured in this model. Since the quantities of remanufacturing and refurbishing cannot exceed the returned products through the refurbishment service and trade-in program, backlogged inventories are not presented in TBC. We describe them in detail in Remark 2.

The inventory manager has to consider the capacities of the manufacturer and remanufacturer. That is, we assume that order quantities of new-generation and old-generation products are limited to the upper bound, C_t, at period t. We also assume that the upper bound, U_t, is limited for remanufacturing and refurbishing processes at period t. In addition, remanufacturing quantity cannot exceed the available end-of-use products. In a similar manner, refurbishing quantity cannot exceed the available end-of-life products. By taking account of the related capacity constraints and flow conservation constraints, which are illustrated in Fig. 3, we developed the linear program (3) as follows:

$$\begin{aligned}
&\min\quad \text{TPC} + \text{THC} + \text{TBC}\\
&\text{s. t.}\quad u_{t+1} = u_t + x_{t-L_n} - d_t - \phi_t \quad t\in\mathfrak{T};\\
&v_{t+1} = v_t + y_{t-L_o} + m_{t-L_m} + q_{t-L_r} - \xi_t \quad t\in\mathfrak{T};\\
&w_{t+1} = w_t + \phi_t - q_t \quad t\in\mathfrak{T};\\
&I_{t+1} = I_t + \xi_t - m_t \quad t\in\mathfrak{T};\\
&x_t + y_t \leqslant C_t \quad t\in\mathfrak{T};\\
&q_t + m_t \leqslant U_t \quad t\in\mathfrak{T};\\
&q_t \leqslant w_t + \phi_t \quad t\in\mathfrak{T};\\
&m_t \leqslant I_t + \xi_t \quad t\in\mathfrak{T};\\
&x_t,\ y_t,\ q_t,\ \text{and } m_t \geqslant 0 \quad t\in\mathfrak{T};
\end{aligned} \tag{3}$$

Remark 2. Constraints related to the quantities of refurbishing and remanufacturing can be reformulated as the non-negative constraints of inventory levels w_{t+1} and I_{t+1}, respectively, as follows:

$$q_t \leqslant w_t + \phi_t \Leftrightarrow w_t + \phi_t - q_t \geqslant 0 \Leftrightarrow w_{t+1} \geqslant 0 \quad t \in \mathfrak{T}; \tag{4}$$

$$m_t \leqslant I_t + \xi_t \Leftrightarrow I_t + \xi_t - m_t \geqslant 0 \Leftrightarrow I_{t+1} \geqslant 0 \quad t \in \mathfrak{T};$$

We do not include the fixed cost for purchasing the product from the manufacturer or remanufacturer. Also, we relax the decision variables as the real variables, which could be considered as integer variables. Although the fixed cost or integer variable can be considered, we formulated a mathematical model as a linear program to retain tractability for the robust optimization approach, as several previous studies did (See and Sim, 2010; Ang et al., 2012; Shin et al., 2019; Lee and Moon, 2020).

4.2. Mathematical formulation of the IMRSTIP under the stochastic demand model

For the inventory manager, it is challenging to predict future demand exactly as a nominal value. When the realized demand differs from the predicted nominal value, this can occur at an enormous cost. To incorporate the uncertain demand while capturing the correlation among three types of demands, we adopt the factor-based demand model. Due to uncertain factors, the objective function is expressed as an expectation form $\mathbb{E}(\cdot)$. Accordingly, the objective function is to minimize the expectation of total costs during the entire planning horizon. The expectation of total purchasing, inventory holding, and backlog costs under the stochastic demand, TPC-S, THC-S, TBC-S, respectively, can be represented as follows:

$$\begin{aligned}
&\mathrm{TPC-S} = \mathbb{E}\left[\sum_{t\in\mathfrak{T}} (c_{n,t}x_t(\tilde{\mathbf{z}}_{t-1}) + c_{o,t}y_t(\tilde{\mathbf{z}}_{t-1}) + c_{r,t}q_t(\tilde{\mathbf{z}}_{t-1}) + c_{m,t}m_t(\tilde{\mathbf{z}}_{t-1}))\right]\\
&\mathrm{THC-S} = \mathbb{E}\left[\sum_{t\in\mathfrak{T}} (h_{u,t}(u_{t+1}(\tilde{\mathbf{z}}_t))^+ + h_{v,t}(v_{t+1}(\tilde{\mathbf{z}}_t))^+ + h_{w,t}(w_{t+1}(\tilde{\mathbf{z}}_t))^+ + h_{I,t}(I_{t+1}(\tilde{\mathbf{z}}_t))^+)\right]\\
&\mathrm{TBC-S} = \mathbb{E}\left[\sum_{t\in\mathfrak{T}} (b_t(u_{t+1}(\tilde{\mathbf{z}}_t))^- + p_t(v_{t+1}(\tilde{\mathbf{z}}_t))^-)\right]
\end{aligned}\tag{5}$$

Based on the expected total costs presented in (5), the multistage stochastic optimization model can be developed as follows:

$$\min \quad \mathrm{TPC-S} + \mathrm{THC-S} + \mathrm{TBC-S} \tag{6}$$

$$\begin{aligned}
\text{s. t.} \quad & u_{t+1}(\tilde{\mathbf{z}}_t) = u_t(\tilde{\mathbf{z}}_{t-1}) + x_{t-L_n}(\tilde{\mathbf{z}}_{t-L_n-1}) - \tilde{d}_t(\tilde{\mathbf{z}}_t) - \tilde{\phi}_t(\tilde{\mathbf{z}}_t) && t \in \mathfrak{T};\\
& v_{t+1}(\tilde{\mathbf{z}}_t) = v_t(\tilde{\mathbf{z}}_{t-1}) + y_{t-L_o}(\tilde{\mathbf{z}}_{t-L_o-1}) + m_{t-L_m}(\tilde{\mathbf{z}}_{t-L_m-1}) + q_{t-L_r}(\tilde{\mathbf{z}}_{t-L_r-1}) - \tilde{\xi}_t(\tilde{\mathbf{z}}_t) && t \in \mathfrak{T};\\
& w_{t+1}(\tilde{\mathbf{z}}_t) = w_t(\tilde{\mathbf{z}}_{t-1}) + \tilde{\phi}_t(\tilde{\mathbf{z}}_t) - q_t(\tilde{\mathbf{z}}_{t-1}) && t \in \mathfrak{T};\\
& I_{t+1}(\tilde{\mathbf{z}}_t) = I_t(\tilde{\mathbf{z}}_{t-1}) + \tilde{\xi}_t(\tilde{\mathbf{z}}_t) - m_t(\tilde{\mathbf{z}}_{t-1}) && t \in \mathfrak{T};\\
& x_t(\tilde{\mathbf{z}}_{t-1}) + y_t(\tilde{\mathbf{z}}_{t-1}) \leqslant C_t && t \in \mathfrak{T};\\
& q_t(\tilde{\mathbf{z}}_{t-1}) + m_t(\tilde{\mathbf{z}}_{t-1}) \leqslant U_t && t \in \mathfrak{T};\\
& q_t(\tilde{\mathbf{z}}_{t-1}) \leqslant w_t(\tilde{\mathbf{z}}_t) + \tilde{\phi}_t(\tilde{\mathbf{z}}_t) && t \in \mathfrak{T};\\
& m_t(\tilde{\mathbf{z}}_{t-1}) \leqslant I_t(\tilde{\mathbf{z}}_t) + \tilde{\xi}_t(\tilde{\mathbf{z}}_t) && t \in \mathfrak{T};\\
& x_t(\tilde{\mathbf{z}}_{t-1}), y_t(\tilde{\mathbf{z}}_{t-1}), q_t(\tilde{\mathbf{z}}_{t-1}), \text{ and } m_t(\tilde{\mathbf{z}}_{t-1}) \geqslant 0 && t \in \mathfrak{T};
\end{aligned}$$

4.3. Robust optimization approach

In general, probability distributions of the random variables are assumed to be known or can be estimated in the stochastic optimization model (Shapiro and Philpott, 2007). In other words, the inventory manager should decide on the order quantity based on the stochastic demand with full information. In practice, however, obtaining full information about uncertain factors is difficult. Even if the distribution is estimated exactly, in general, evaluating the expected value in the multistage decision process is computationally intractable (Shapiro, 2003; Shapiro and Nemirovski, 2005; Shapiro, 2008). Instead, we assumed that the first and second moments and support of the uncertain factors are available for the inventory manager. By approximating the upper bound of the multistage stochastic optimization model (6), we could derive a robust counterpart that is tractable. That is, minimizing the upper bound was considered rather than directly minimizing the expected value. To approximate the multistage stochastic optimization model to the tractable deterministic model, the following assumption is required for the uncertain factors $\tilde{\mathbf{z}}$.

Assumption 1. (Chen and Sim, 2009) Uncertain factors $\tilde{\mathbf{z}}$ are zero mean random variables, $\mathbb{E}[\tilde{z}_k] = 0, \forall\, k \in 1, ..., N$, with the positive definite covariance matrix Σ. The uncertain factors are defined on the support set $\boldsymbol{W}$ which is a second-order conic representable set, such as intervals, polyhedrons, or ellipsoids.

4.3.1. Linear decision rule

We adopted the *linear decision rule* (LDR) to handle the multistage stochastic optimization model. By restricting decision variables as affinely dependent on the uncertain factors, the decision can be delayed by observing the realization of part of the uncertain factors. Related decision variables with the LDR are expressed as follows:

$$x_t^{\mathrm{LDR}}(\tilde{\mathbf{z}}) = x_t^0 + \sum_{k=1}^{N} x_t^k \tilde{z}_k$$
$$y_t^{\mathrm{LDR}}(\tilde{\mathbf{z}}) = y_t^0 + \sum_{k=1}^{N} y_t^k \tilde{z}_k$$
$$q_t^{\mathrm{LDR}}(\tilde{\mathbf{z}}) = q_t^0 + \sum_{k=1}^{N} q_t^k \tilde{z}_k$$
$$m_t^{\mathrm{LDR}}(\tilde{\mathbf{z}}) = m_t^0 + \sum_{k=1}^{N} m_t^k \tilde{z}_k \tag{7}$$

Because the decision is based on the realized uncertain factors, which is referred to as the *non anticipative* property, uncertain factors are restricted that are unavailable from period t. This property can be incorporated by adding non-anticipative constraints as follows:

$$x_t^k = 0, \quad \forall\, k \geqslant N_{t-1} + 1$$
$$y_t^k = 0, \quad \forall\, k \geqslant N_{t-1} + 1$$
$$q_t^k = 0, \quad \forall\, k \geqslant N_{t-1} + 1$$
$$m_t^k = 0, \quad \forall\, k \geqslant N_{t-1} + 1 \tag{8}$$

Hence, the order decision is based on the observed information available at the beginning of each period t.

Remark 3. The decision variables relevant to inventory levels, u_{t+1}, v_{t+1}, w_{t+1}, and I_{t+1}, are also affinely dependent on uncertain factors, $\tilde{\mathbf{z}}$, as follows:

$$u_t(\tilde{\mathbf{z}}) = u_t^0 + \sum_{k=1}^{N} u_t^k \tilde{z}_k$$
$$v_t(\tilde{\mathbf{z}}) = v_t^0 + \sum_{k=1}^{N} v_t^k \tilde{z}_k$$
$$w_t(\tilde{\mathbf{z}}) = w_t^0 + \sum_{k=1}^{N} w_t^k \tilde{z}_k$$
$$I_t(\tilde{\mathbf{z}}) = I_t^0 + \sum_{k=1}^{N} I_t^k \tilde{z}_k$$

By deriving the balance equation for the inventory level with the closed-form expression, it can be easily discerned that decision variables indicating inventory levels are also affine function of the uncertain factors:

$$\begin{aligned} u_{t+1}(\tilde{\mathbf{z}}) &= u_1^0 + \sum_{i=1}^{t} x_i(\tilde{\mathbf{z}}) - \sum_{i=1}^{t} d_i(\tilde{\mathbf{z}}) \\ &= u_1^0 + \sum_{i=1}^{t} \left((x_i^0 - d_i^0) + \sum_{k=1}^{N} (x_i^k - d_i^k)\tilde{z}_k \right) \\ &= u_t^0 + \sum_{k=1}^{N} u_t^k \tilde{z}_k \\ &= u_t^0 + \sum_{k=1}^{N} u_t^k \tilde{z}_k \end{aligned} \tag{9}$$

The remaining three inventory levels can also be derived in the same manner. From the non-anticipative constraints in (8), inventory levels also feature the non-anticipative property seen in the second equality in (9). As a result, decision variables related to the inventory level also feature the non-anticipative property as follows:

$$u_{t+1}^k = 0, \quad \forall\, k \geqslant N_t + 1$$
$$v_{t+1}^k = 0, \quad \forall\, k \geqslant N_t + 1$$
$$w_{t+1}^k = 0, \quad \forall\, k \geqslant N_t + 1$$
$$I_{t+1}^k = 0, \quad \forall\, k \geqslant N_t + 1 \tag{10}$$

4.3.2. Upper bound of the expected positive parts

Instead of directly minimizing the expectation of the multistage stochastic optimization model, we primarily focused on minimizing the approximated upper bound of the expected value. By adopting the LDR, the upper bound of the expected value can be estimated without considering the expected cost function. The expected purchasing cost can be obtained directly but expected holding and backlog costs are approximated to the upper bounds. For the purchasing cost of new-generation products, the expected value can be obtained by the following process:

$$\begin{aligned}
&\mathbb{E}\left(c_{n,t}(x_t^0 + \sum_{k=1}^{N} x_t^k \tilde{z}_k)\right)\\
&= c_{n,t}\mathbb{E}(x_t^0) + c_{n,t}\mathbb{E}\left(\sum_{k=1}^{N} x_t^k \tilde{z}_k\right)\\
&= c_{n,t}x_t^0\\
&\because\ \mathbb{E}[\tilde{z}_k] = 0
\end{aligned} \tag{11}$$

In the same manner, total purchasing costs based on the LDR can be expressed as follows:

$$\mathrm{TPC} - \mathrm{R} = \sum_{t \in \mathfrak{T}} (c_{n,t}x_t^0 + c_{o,t}y_t^0 + c_{r,t}q_t^0 + c_{m,t}m_t^0)$$

By adopting the work of Chen and Sim (2009), we derived three upper bounds of expected positive parts related to excess inventories. The first bound is presented in (12) as follows:

$$\begin{aligned}
&\mathbb{E}\left((u_{t+1}^0 + \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k)^+\right)\\
&\leqslant \left(u_{t+1}^0 + \max_{\tilde{\mathbf{z}} \in \mathbf{W}} \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k\right)^+\\
&= \pi^1(u_{t+1}^0, \mathbf{u}_{t+1})
\end{aligned} \tag{12}$$

The second bound can be derived by using the equality $a^+ = a + (-a)^+$

$$\begin{aligned}
&\mathbb{E}\left((u_{t+1}^0 + \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k)^+\right)\\
&= u_{t+1}^0 + \mathbb{E}\left((-u_{t+1}^0 - \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k)^+\right)\\
&\leqslant u_{t+1}^0 + \left(-u_{t+1}^0 + \max_{\tilde{\mathbf{z}} \in \mathbf{W}} \sum_{k=1}^{N} (-u_{t+1}^k)\tilde{z}_k\right)^+\\
&= \pi^2(u_{t+1}^0, \mathbf{u}_{t+1})
\end{aligned} \tag{13}$$

The third bound can be derived by using the equality $a^+ = (a + |a|)/2$ and Jensen's inequality as follows:

$$\begin{aligned}
&\mathbb{E}\left((u_{t+1}^0 + \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k)^+\right)\\
&= \tfrac{1}{2}\mathbb{E}\left(u_{t+1}^0 + \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k\right) + \tfrac{1}{2}\mathbb{E}\left|u_{t+1}^0 + \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k\right|\\
&= \tfrac{1}{2}u_{t+1}^0 + \tfrac{1}{2}\mathbb{E}\left|u_{t+1}^0 + \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k\right|\\
&\leqslant \tfrac{1}{2}u_{t+1}^0 + \tfrac{1}{2}\sqrt{\mathbb{E}[((u_{t+1}^0 + \sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k)^+)^2]} \quad \text{(by Jensen's inequality)}\\
&= \tfrac{1}{2}u_{t+1}^0 + \tfrac{1}{2}\sqrt{(u_{t+1}^0)^2 + \mathbb{E}[\left(\sum_{k=1}^{N} u_{t+1}^k \tilde{z}_k\right)^2]}\\
&= \pi^3(u_{t+1}^0, \mathbf{u}_{t+1})
\end{aligned} \tag{14}$$

Theorem 1. *The upper bound of the expected positive part* $\mathbb{E}((u_{t+1}^0 + \mathbf{u}_{t+1}'\tilde{\mathbf{z}})^+)$*, which is represented by* $\pi(u_{t+1}^0, \mathbf{u}_{t+1})$*, can be obtained by minimizing the three bounds,* $\pi^1(u_{t+1}^0, \mathbf{u}_{t+1})$*,* $\pi^2(u_{t+1}^0, \mathbf{u}_{t+1})$*, and* $\pi^3(u_{t+1}^0, \mathbf{u}_{t+1})$ *as follows:*

$$\begin{aligned}&\pi(u^0_{t+1},\mathbf{u}_{t+1})\triangleq\ \min\sum_{i=1}^{3}\pi^i(u^0_{i,t+1},\mathbf{u}_{i,t+1})\\&\text{s. t.}\quad\sum_{i=1}^{3}u^0_{i,t+1}=u^0_{t+1}\\&\sum_{i=1}^{3}\mathbf{u}_{i,t+1}=\mathbf{u}_{t+1}\end{aligned}\tag{15}$$

The optimization problem (15) for every t-th period ($t\in\mathfrak{T}$) can be expressed as the epigraph form by combining the inequalities (12)–(14) as follows (Chen and Sim, 2009):

$$\begin{aligned}&\pi(u^0_{t+1},\mathbf{u}_{t+1})=\min\quad r_{1,t+1}+r_{2,t+1}+r_{3,t+1}\\&\text{s. t.}\quad u^0_{1,t+1}+\max_{\tilde{\mathbf{z}}\in\mathbf{W}}\tilde{\mathbf{z}}'\mathbf{u}_{1,t+1}\leqslant r_{1,t+1}\\&r_{1,t+1}\geqslant 0\\&\max_{\tilde{\mathbf{z}}\in\mathbf{W}}\tilde{\mathbf{z}}'(-\mathbf{u}_{2,t+1})\leqslant r_{2,t+1}\\&u^0_{2,t+1}\leqslant r_{2,t+1}\\&\tfrac{1}{2}u^0_{3,t+1}+\tfrac{1}{2}\ |u^0_{3,t+1},\Sigma^{1/2}\mathbf{u}_{3,t+1}|_2\leqslant r_{3,t+1}\\&u^0_{1,t+1}+u^0_{2,t+1}+u^0_{3,t+1}=u^0_{t+1}\\&\mathbf{u}_{1,t+1}+\mathbf{u}_{2,t+1}+\mathbf{u}_{3,t+1}=\mathbf{u}_{t+1}\\&r_{i,t+1},u^0_{i,t+1}\in\mathcal{R},\quad\mathbf{u}_{i,t+1}\in\mathcal{R}^N,\quad i=1,\dots,3\end{aligned}\tag{16}$$

Remark 4. Due to the Assumption 1, the optimization problem (16) is tractable if the robust counterparts of the inner optimizations in the first and third constraints are appropriately defined. If the uncertain factors are not defined on the support set $\boldsymbol{W}$, the optimization problem (16) becomes intractable which leads to a robust optimization model also being intractable.

To sum up, we approximated the objective function of the multistage stochastic optimization model which was expressed as the expectation form. The approximated upper bound of the objective function features the second-order cone program because the quadratic constraint remains on the third upper bound of the expected positive part (14). According to the Ben-Tal and Nemirovski (1998), the robust counterpart retains tractability if the ellipsoidal uncertainty set is considered in a linear program or second-order cone program (which is also known as a conic quadratic program). For the cases of the interval or polyhedron sets, these sets do not affect the complexity of the problem. In this study, under Assumption 1, the optimization model (16) can retain the property of the second-order cone program if the robust counterpart is well defined by handling the remaining uncertain factors properly. Thus, the problem can be solved by the interior point method with the commercial optimization solver. If the support set $\boldsymbol{W}$ is not a second-order conic representable set, such as an intersection of ellipsoids, the robust counterpart of the problem becomes NP-hard which leads to the robust optimization model being intractable (Ben-Tal and Nemirovski, 1998).

By solving the optimization problem (16), a tighter upper bound can be achieved than can be achieved for each bound as presented in inequality (17).

$$\mathbb{E}\left((u^0_{t+1}+\sum_{k=1}^{N}u^k_{t+1}\tilde{z}_k)^+\right)\leqslant\pi(u^0_{t+1},\mathbf{u}_{t+1})\leqslant\min_{i=1,2,3}\pi^i(u^0_{t+1},\mathbf{u}_{t+1})\tag{17}$$

Remark 5. A reasonable upper bound can be approximated without utilizing directional deviations, which are the forward and backward deviations (See and Sim, 2010).

Chen and Sim (2009) and See and Sim (2010) used fourth and fifth bounds which are derived from the forward and backward deviations, respectively, in the optimization model (16). However, a close bound can be achieved even if the directional deviations are set to ∞. If the information of the deviations is unavailable, it could be omitted by forcing the fourth and fifth bounds to be redundant.

Using the same process, we can derive the upper bound of the total inventory holding costs for the entire planning horizon as follows:

$$\text{THC}-\text{R}=\sum_{t\in\mathfrak{T}}(h_{u,t}\pi(u^0_{t+1},\mathbf{u}_{t+1})+h_{v,t}\pi(v^0_{t+1},\mathbf{v}_{t+1})+h_{w,t}\pi(w^0_{t+1},\mathbf{w}_{t+1})+h_{I,t}\pi(I^0_{t+1},\mathbf{I}_{t+1}))$$

The upper bound of expected backlogged inventory can also be derived as follows:

$$\mathbb{E}\left((u^0_{t+1}+\sum_{k=1}^{N}u^k_{t+1}\tilde{z}_k)^-\right)\leqslant\pi(-u^0_{t+1},-\mathbf{u}_{t+1})\leqslant\min_{i=1,2,3}\pi^i(-u^0_{t+1},-\mathbf{u}_{t+1})\tag{18}$$

The total backlog cost can be approximated based on (18) as follows:

$$\mathrm{TBC}-\mathrm{R}=\sum_{t\in\mathfrak{T}}(b_t\pi(-u_{t+1}^0,-\mathbf{u}_{t+1})+p_t\pi(-v_{t+1}^0,-\mathbf{v}_{t+1}))$$

Consequently, the upper bound of the total costs on the entire planning horizon, including TPC-R, THC-R, and TBC-R, are derived based on the LDR.

4.3.3. Robust counterpart

Based on the LDR formulations represented in (7), the approximated upper bound of the multistage stochastic optimization model (6) are derived as follows:

$$\begin{array}{lll}
\min & \mathrm{TPC}-\mathrm{R}+\mathrm{THC}-\mathrm{R}+\mathrm{TBC}-\mathrm{R} & \\
\text{s. t.} & u_{t+1}^k=u_t^k+x_{t-L_n}^k-d_t^k-\phi_t^k & t\in\mathfrak{T};k\in 0\ldots N\\
& v_{t+1}^k=v_t^k+y_{t-L_o}^k+m_{t-L_m}^k+q_{t-L_r}^k-\xi_t^k & t\in\mathfrak{T};k\in 0\ldots N\\
& w_{t+1}^k=w_t^k+\phi_t^k-q_t^k & t\in\mathfrak{T};k\in 0\ldots N\\
& I_{t+1}^k=I_t^k+\xi_t^k-m_t^k & t\in\mathfrak{T};k\in 0\ldots N\\
& x_t^k,\ y_t^k,\ q_t^k,\ m_t^k=0 & k\geqslant N_{t-1}+1;\ t\in\mathfrak{T};\\
& u_{t+1}^k,\ v_{t+1}^k,\ w_{t+1}^k,\ I_{t+1}^k=0 & k\geqslant N_t+1;\ t\in\mathfrak{T};\\
& x_t^0+\mathbf{x}_t'\tilde{\mathbf{z}}+y_t^0+\mathbf{y}_t'\tilde{\mathbf{z}}\leqslant C_t & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};\\
& q_t^0+\mathbf{q}_t'\tilde{\mathbf{z}}+m_t^0+\mathbf{m}_t'\tilde{\mathbf{z}}\leqslant U_t & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};\\
& q_t^0+\mathbf{q}_t'\tilde{\mathbf{z}}\leqslant w_t^0+\mathbf{w}_t'\tilde{\mathbf{z}}+\phi_t^0+\boldsymbol{\phi}_t'\tilde{\mathbf{z}} & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};\\
& m_t^0+\mathbf{m}_t'\tilde{\mathbf{z}}\leqslant I_t^0+\mathbf{I}_t'\tilde{\mathbf{z}}+\xi_t^0+\boldsymbol{\xi}_t'\tilde{\mathbf{z}} & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};\\
& x_t^0+\mathbf{x}_t'\tilde{\mathbf{z}}\geqslant 0 & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};\\
& y_t^0+\mathbf{y}_t'\tilde{\mathbf{z}}\geqslant 0 & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};\\
& q_t^0+\mathbf{q}_t'\tilde{\mathbf{z}}\geqslant 0 & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};\\
& m_t^0+\mathbf{m}_t'\tilde{\mathbf{z}}\geqslant 0 & \tilde{\mathbf{z}}\in\boldsymbol{W},\ \ t\in\mathfrak{T};
\end{array}\tag{19}$$

Constraints related to the inventory balance equation are derived to the linear equations. However, the constraint related to the order capacity for the manufacturer, C_t, remains the uncertain factors $\tilde{\mathbf{z}}\in\boldsymbol{W}$. Since the uncertain factors are assumed to be bounded and defined on the support set $\boldsymbol{W}$, the constraints can be reformulated to the robust counterpart by defining the inner optimization as follows:

$$\begin{aligned}
& x_t^0+\mathbf{x}_t'\tilde{\mathbf{z}}+y_t^0+\mathbf{y}_t'\tilde{\mathbf{z}}\leqslant C_t,\quad \tilde{\mathbf{z}}\in\boldsymbol{W}\\
&\Leftrightarrow x_t^0+y_t^0+\max_{\tilde{\mathbf{z}}\in\boldsymbol{W}}\sum_{k=1}^{N}(x_t^k+y_t^k)\tilde{z}_k\leqslant C_t
\end{aligned}\tag{20}$$

In the case of the capacity constraints for the remanufacturer, U_t, we followed the same procedure as shown in (20) and obtained the robust counterpart as follows:

$$\begin{aligned}
& q_t^0+\mathbf{q}_t'\tilde{\mathbf{z}}+m_t^0+\mathbf{m}_t'\tilde{\mathbf{z}}\leqslant U_t,\quad \tilde{\mathbf{z}}\in\boldsymbol{W}\\
&\Leftrightarrow q_t^0+m_t^0+\max_{\tilde{\mathbf{z}}\in\boldsymbol{W}}\sum_{k=1}^{N}(q_t^k+m_t^k)\tilde{z}_k\leqslant U_t
\end{aligned}\tag{21}$$

Recall that the constraints related to the refurbishing and remanufacturing quantities can be reformulated as (4). Consequently, we derived the robust counterpart of the constraints related to the refurbishing quantity for every period $t\in\mathfrak{T}$ as follows:

$$\begin{aligned}
& q_t(\tilde{z}_{t-1})\leqslant w_t(\tilde{z}_t)+\phi_t(\tilde{z}_t)\Leftrightarrow w_{t+1}(\tilde{z}_t)\geqslant 0\\
&\Leftrightarrow w_{t+1}^0-\max_{\tilde{\mathbf{z}}\in\boldsymbol{W}}\sum_{k=1}^{N}w_{t+1}^k\tilde{z}_k\geqslant 0
\end{aligned}\tag{22}$$

Constraints representing the capacity of the remanufacturing quantity can be reformulated as a robust counterpart with the same process used in (22) as follows:

$$\begin{aligned}
& m_t(\tilde{z}_{t-1})\leqslant I_t(\tilde{z}_t)+\xi_t(\tilde{z}_t)\Leftrightarrow I_{t+1}(\tilde{z}_t)\geqslant 0\\
&\Leftrightarrow I_{t+1}^0-\max_{\tilde{\mathbf{z}}\in\boldsymbol{W}}\sum_{k=1}^{N}I_{t+1}^k\tilde{z}_k\geqslant 0
\end{aligned}\tag{23}$$

We also developed robust counterparts for decision variables $x_t(\tilde{z}_{t-1})$, $y_t(\tilde{z}_{t-1})$, $q_t(\tilde{z}_{t-1})$, and $m_t(\tilde{z}_{t-1})$ which have non-negative conditions for every period $t\in\mathfrak{T}$.

$$\begin{cases} x_t^0 + \mathbf{x_t}'\widetilde{\mathbf{z}} \geqslant 0 & \widetilde{\mathbf{z}} \in \boldsymbol{W}; \\ y_t^0 + \mathbf{y_t}'\widetilde{\mathbf{z}} \geqslant 0 & \widetilde{\mathbf{z}} \in \boldsymbol{W}; \\ q_t^0 + \mathbf{q_t}'\widetilde{\mathbf{z}} \geqslant 0 & \widetilde{\mathbf{z}} \in \boldsymbol{W}; \\ m_t^0 + \mathbf{m_t}'\widetilde{\mathbf{z}} \geqslant 0 & \widetilde{\mathbf{z}} \in \boldsymbol{W}; \end{cases} \Leftrightarrow \begin{cases} x_t^0 - \max\limits_{\widetilde{\mathbf{z}} \in \boldsymbol{W}} \sum\limits_{k=1}^{N} x_t^k \widetilde{z}_k \geqslant 0 \\ y_t^0 - \max\limits_{\widetilde{\mathbf{z}} \in \boldsymbol{W}} \sum\limits_{k=1}^{N} y_t^k \widetilde{z}_k \geqslant 0 \\ q_t^0 - \max\limits_{\widetilde{\mathbf{z}} \in \boldsymbol{W}} \sum\limits_{k=1}^{N} q_t^k \widetilde{z}_k \geqslant 0 \\ m_t^0 - \max\limits_{\widetilde{\mathbf{z}} \in \boldsymbol{W}} \sum\limits_{k=1}^{N} m_t^k \widetilde{z}_k \geqslant 0 \end{cases} \tag{24}$$

Under Assumption 1, Constraints (20)–(24) can be transformed to a tractable robust counterpart by reformulating the inner optimization problem properly. Tractability of the inner optimization problem could be handled in the same manner as in Remark 4. In the case of a robust counterpart, it depends on what types of support sets $\boldsymbol{W}$ are defined. For example, if uncertain factors $\widetilde{z}_k$ are distributed on the symmetric interval set $\boldsymbol{W} = [-z, z]$ in the constraint $x_t^0 - \max\limits_{\widetilde{\mathbf{z}} \in \boldsymbol{W}} \sum_{k=1}^{N} x_t^k \widetilde{z}_k \geqslant 0$, this could be handled by introducing the absolute value as $x_t^0 - \sum_{k=1}^{N} |x_t^k| z \geqslant 0$. In this manner, all uncertain factors remaining in the optimization model (19) could be handled. Finally, the robust counterpart of the IMRSTIP (RIMRSTIP) becomes the deterministic second-order cone optimization model.

Theorem 2. *The objective value of the robust counterpart of the optimization model (19) is greater than or equal to that of the multistage stochastic optimization model (6).*

Theorem 2 can be easily derived by combining the inequalities in (11), (17), and (18). Although the robust counterpart of (19) provides the objective value that is worse than that obtained from the stochastic optimization problem (5), it features the deterministic second-order cone optimization problem that has the virtue of computational tractability. Performance gaps with the multistage stochastic optimization model are provided in Section 5.

5. Computational experiments

We conducted the computational experiments to validate the performance of the RIMRSTP. Also, we performed experiments to derive managerial insights. In addition to the research questions (1), (2), and (3) in Section 1, research questions related to the model validation and insights are raised as follows:

- Does the robust model retain tractability until a modest data size?
- What is the loss of the objective value during approximation to the robust counterpart?
- To what extent does the robust model protect against the realized uncertain factors?
- What is the aspect of the inventory operation when the correlations among the three types of demands exist or do not?

All computational experiments were performed by the optimization solver FICO XPRESS-IVE version 7.2 with an Intel® Core™ i5–7400 CPU @ 3.0 GHz. Before reporting the results of the experiments, we provide a small example of the demand process and computational results in Section 5.1.

5.1. Demand process

We begin by explaining the demand generation. The factor-based demand model, which we adopted from See and Sim (2010), involves the constant term and coefficient of the uncertain factors. The factor-based demand model is modeled as the affine function of the uncertain factor by estimating the related parameters. We utilized the time series analysis method to predict future demand and induced it as a factor-based demand model.

5.1.1. ARMA model

The *autoregressive moving* − *average* (ARMA) model is one of the stochastic processes that predict future points in the time series. It is a generalization model of the autoregressive and moving average models, which are expressed with previous observations of time series data and past error terms, respectively. By estimating future demand with past data from the ARMA (p, q) process, the inventory manager can obtain the future demand as the affine function of the uncertain factors as follows (See and Sim, 2010):

$$d_t(\widetilde{\mathbf{z}}_t) \simeq \sum_{k=1}^{p} \alpha_j d_{t-k}(\widetilde{\mathbf{z}}) + \widetilde{z}_t + \sum_{k=1}^{\min(q,t-1)} \beta_j \widetilde{z}_{t-j} \tag{25}$$

The ARMA model projects the future values of a series based entirely on its inertia. In other words, there exists a limit to capture demand for other products that can affect each other. Thus, we adopted a model that can incorporate the correlation or dependency of different demands.

5.1.2. VARMA model

To capture the dependencies among the three types of demands, we adopted the *vector autoregressive moving − average* (VARMA) model. It is one of the stochastic processes characterizing the dependencies of the multivariate time series. By allowing the error terms to be autocorrelated on the *vector autoregressive* (VAR) model, the VARMA (p, q), which is a generalized model, can be developed. Unlike the univariate ARMA model, the VARMA model has more than one time-dependent variable. Each variable depends not only on its past values but also has some dependency on other variables. Denote by $\mathbf{D}_t \triangleq (\tilde{d}_t, \tilde{\phi}_t, \tilde{\xi}_t)$ as a vector of three types of demands at period t. Let $\mathbf{D}_t^0$ denote the vector of constant terms $(d_t^0, \phi_t^0, \xi_t^0)$. For simplicity, let VARMA (p, q) as VAR(1) which can be representable when the stochastic process is stable (Lütkepohl, 2005). Then, we can develop the demand model as follows:

$$\mathbf{D}_t = \mathbf{D}_t^0 + \Psi_{t-1}\mathbf{D}_{t-1} + \epsilon_t \tag{26}$$

where Ψ and ϵ_t indicate the coefficients matrices and unobservable uncertain factors, respectively. Uncertain factors follow a zero-mean normal distribution with time-invariant covariance $\mathbf{\Sigma}$. In detail, we developed the three types of demands as follows:

$$\begin{bmatrix} d_t \\ \phi_t \\ \xi_t \end{bmatrix} = \begin{bmatrix} d_t^0 \\ \phi_t^0 \\ \xi_t^0 \end{bmatrix} + \begin{bmatrix} \Phi_{1,1}^{t-1} & \Phi_{1,2}^{t-1} & \Phi_{1,3}^{t-1} \\ \Phi_{2,1}^{t-1} & \Phi_{2,2}^{t-1} & \Phi_{2,3}^{t-1} \\ \Phi_{3,1}^{t-1} & \Phi_{3,2}^{t-1} & \Phi_{3,3}^{t-1} \end{bmatrix} \begin{bmatrix} d_{t-1} \\ \phi_{t-1} \\ \xi_{t-1} \end{bmatrix} + \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{bmatrix} \tag{27}$$

The demand model represented in (27) can be induced to the factor-based demand model presented in (1). If the process is stable and invertible, the VARMA process has pure *moving − average* (MA) representation (Lütkepohl, 2005). As See and Sim (2010) and Lee and Moon (2020) utilized the demand process of Graves (1999) in their computational experiments, the MA process can likewise be regarded as a factor-based demand model. We provide an example in Section 5.1.3 in detail to make it easier for readers to understand. In general, the estimation of the VARMA (p, q) process provides the correlation or covariance matrix of residuals which follows a multivariate normal distribution with a zero mean and covariance matrix $\mathbf{\Sigma}$. Solving the problem based on this normal distribution leads to a multistage stochastic optimization problem. However, estimating the expectation of the objective function in the multistage setting cannot guarantee tractability. Also, in a normal distribution, which has infinite support for both negative and positive parts, larger values above a certain level are rarely realized. Therefore, we truncated the support as $[-3\sigma, 3\sigma]$ and alleviated information about a particular distribution (See Section 4 in Ang et al. (2012)). Consequently, we could obtain the inventory policy through (19) based on only information with a mean, support, and covariance of predefined uncertain factors.

5.1.3. Numerical example

Using the demand models (26) and (27), we provide an example based on the small data. Assume that the inventory manager has historical data, including three types of demands, d^-, ϕ^-, and ξ^-, from the past as follows:

$$\begin{aligned} d^- &= (22, 23, 21, 22, 25, 21, 20, 22, 24) \\ \phi^- &= (20, 17, 15, 17, 24, 22, 19, 21, 23) \\ \xi^- &= (17, 14, 16, 15, 14, 15, 16, 14, 13) \end{aligned}$$

We assume that the inventory manager establishes the inventory policy from $t = 1$ to $t = 3$. From the VARMA (1, 0) process, three types of demand models were estimated as follows:

$$\begin{aligned} \tilde{d}_t &= 22.25 + 0.08d_{t-1} - 0.18\phi_{t-1} + 0.11\xi_{t-1} + \epsilon_{1,t} \\ \tilde{\phi}_t &= 40.75 - 0.63d_{t-1} + 0.42\phi_{t-1} - 1.02\xi_{t-1} + \epsilon_{2,t} \\ \tilde{\xi}_t &= 15.38 + 0.11d_{t-1} - 0.06\phi_{t-1} - 0.14\xi_{t-1} + \epsilon_{3,t} \end{aligned} \tag{28}$$

The estimated covariance matrix of residuals, ϵ_t, was obtained as follows:

$$\begin{array}{c} \\ \epsilon_{1,t} \\ \epsilon_{2,t} \\ \epsilon_{3,t} \end{array} \begin{array}{c} \begin{array}{ccc} \epsilon_{1,t} & \epsilon_{2,t} & \epsilon_{3,t} \end{array} \\ \begin{pmatrix} 4.39 & 6.39 & -2.60 \\ 6.39 & 14.27 & -3.70 \\ -2.60 & -3.70 & 1.84 \end{pmatrix} \end{array}$$

If residuals $(\epsilon_{1,1}, \epsilon_{1,2}, \epsilon_{1,3}, \epsilon_{2,1}, \epsilon_{2,2}, \cdots)$ were redefined to $(\tilde{z}_1, \tilde{z}_2, \tilde{z}_3, \tilde{z}_4, \tilde{z}_5, \cdots)$, the demand model (28) could be induced to the factor-based demand model (29) as follows:

$$
\begin{aligned}
\tilde{d}_1 &= 21.46 + \tilde{z}_1 \\
\tilde{\phi}_1 &= 22.03 + \tilde{z}_2 \\
\tilde{\xi}_1 &= 14.82 + \tilde{z}_3 \\
\tilde{d}_2 &= 22.25 + 0.08(21.46 + \tilde{z}_1) - 0.18(22.03 + \tilde{z}_2) + 0.11(14.82 + \tilde{z}_3) + \tilde{z}_4 \\
&= 21.63 + 0.08\tilde{z}_1 - 0.18\tilde{z}_2 + 0.11\tilde{z}_3 + \tilde{z}_4 \\
\tilde{\phi}_2 &= 40.75 - 0.63(21.46 + \tilde{z}_1) + 0.42(22.03 + \tilde{z}_2) - 1.02(14.82 + \tilde{z}_3) + \tilde{z}_5 \\
&= 21.37 - 0.63\tilde{z}_1 + 0.42\tilde{z}_2 - 1.02\tilde{z}_3 + \tilde{z}_5 \\
\tilde{\xi}_2 &= 15.38 + 0.11(21.46 + \tilde{z}_1) - 0.06(22.03 + \tilde{z}_2) - 0.14(14.82 + \tilde{z}_3) + \tilde{z}_6 \\
&= 14.34 + 0.11\tilde{z}_1 - 0.06\tilde{z}_2 - 0.14\tilde{z}_3 + \tilde{z}_6 \\
\tilde{d}_3 &= 22.25 + 0.08(21.63 + 0.08\tilde{z}_1 - 0.18\tilde{z}_2 + 0.11\tilde{z}_3 + \tilde{z}_4) \\
&\quad - 0.18(21.37 - 0.63\tilde{z}_1 + 0.42\tilde{z}_2 - 1.02\tilde{z}_3 + \tilde{z}_5) \\
&\quad + 0.11(14.34 + 0.11\tilde{z}_1 - 0.06\tilde{z}_2 - 0.14\tilde{z}_3 + \tilde{z}_6) + \tilde{z}_7 \\
&= 21.71 + 0.13\tilde{z}_1 - 0.10\tilde{z}_2 + 0.18\tilde{z}_3 + 0.08\tilde{z}_4 - 0.18\tilde{z}_5 + 0.11\tilde{z}_6 + \tilde{z}_7 \\
\tilde{\phi}_3 &= 40.75 - 0.63(21.63 + 0.08\tilde{z}_1 - 0.18\tilde{z}_2 + 0.11\tilde{z}_3 + \tilde{z}_4) \\
&\quad + 0.42(21.37 - 0.63\tilde{z}_1 + 0.42\tilde{z}_2 - 1.02\tilde{z}_3 + \tilde{z}_5) \\
&\quad - 1.02(14.34 + 0.11\tilde{z}_1 - 0.06\tilde{z}_2 - 0.14\tilde{z}_3 + \tilde{z}_6) + \tilde{z}_8 \\
&= 21.47 - 0.43\tilde{z}_1 + 0.35\tilde{z}_2 - 0.35\tilde{z}_3 - 0.63\tilde{z}_4 + 0.42\tilde{z}_5 - 1.02\tilde{z}_6 + \tilde{z}_8 \\
\tilde{\xi}_3 &= 15.38 + 0.11(21.63 + 0.08\tilde{z}_1 - 0.18\tilde{z}_2 + 0.11\tilde{z}_3 + \tilde{z}_4) \\
&\quad - 0.06(14.34 + 0.11\tilde{z}_1 - 0.06\tilde{z}_2 - 0.14\tilde{z}_3 + \tilde{z}_6) \\
&\quad - 0.14(14.34 + 0.11\tilde{z}_1 - 0.06\tilde{z}_2 - 0.14\tilde{z}_3 + \tilde{z}_6) + \tilde{z}_9 \\
&= 14.47 + 0.03\tilde{z}_1 - 0.04\tilde{z}_2 + 0.09\tilde{z}_3 + 0.11\tilde{z}_4 - 0.06\tilde{z}_5 + 0.14\tilde{z}_6 + \tilde{z}_9
\end{aligned} \tag{29}
$$

Accordingly, the demand model featured the affine function of the uncertain factors. Based on the derived demand model, the inventory manager established the inventory policy through the robust counterpart (19). We assumed that purchasing costs $(c_{n,t}, c_{o,t}, c_{q,t}, c_{m,t})$ were given as (6, 15, 2, 3) for periods $t = 1, 2,$ and 3. For the inventory holding and penalty costs, we assumed that $(h_{u,t}, h_{v,t}, h_{w,t}, h_{I,t})$ and (b_t, p_t) were given as (1.5, 1.2, 1, 1) and (10, 20), respectively, for periods $t = 1, 2,$ and 3. For the capacities of the manufacturer and remanufacturer, C_t and U_t, we assumed that each capacity was 60 and 40 for periods $t = 1, 2,$ and 3. For simplicity, we assumed that all lead times are zero. Based on the given costs and parameters, the inventory manager established the policy as follows:

$$
\begin{aligned}
x_t(\tilde{\mathbf{z}}) &= (56.67,\ 33.45 + 1.45\tilde{z}_1 + 1.24\tilde{z}_2 - 0.91\tilde{z}_3,\ 43.18 + 0.03\tilde{z}_1 + 0.38\tilde{z}_2 - 0.60\tilde{z}_3 + 0.12\tilde{z}_4 \\
&\qquad + 0.11\tilde{z}_5 - 0.65\tilde{z}_6) \\
y_t(\tilde{\mathbf{z}}) &= (0, 0, 0) \\
q_t(\tilde{\mathbf{z}}) &= (17.16,\ 11.99 + 0.11\tilde{z}_1 - 0.06\tilde{z}_2 + 0.86\tilde{z}_3,\ 14.47 + 0.07\tilde{z}_1 - 0.05\tilde{z}_2 - 0.03\tilde{z}_3 + 0.08\tilde{z}_4 \\
&\qquad - 0.05\tilde{z}_5 - 0.02\tilde{z}_6) \\
m_t(\tilde{\mathbf{z}}) &= (0, 0, 0)
\end{aligned}
$$

In this example, the order quantity for the remanufacturing process was not featured. The demand of the trade-in program was relatively high and sufficient end-of-use products were returned to the retailer. Since the refurbishing cost is less than the remanufacturing cost, the retailer did not send the end-of-life products to the remanufacturer. Also, refurbishment service did not encounter the backlog. As a result, the manufacturer did not produce the old-generation products. That is, the inventory policies for $y_t(\tilde{\mathbf{z}})$ and $m_t(\tilde{\mathbf{z}})$ were zero for all periods.

5.2. Experiment 1: tractability of the RIMRSTIP

We conducted Experiment 1 to identify the tractability of the RIMRSTIP. Based on the randomly generated data, we solved the problem by varying the planning horizon of the demand. In other words, the data size was gradually increased to examine whether the RIMRSTIP is solvable until a modest size, or not. The results of Experiment 1 are described in Table 2. As presented in Table 2, the RIMRSTIP could solve the problem with the planning horizon from $t = 20$ to $t = 55$. Experiment 1 addresses that the robust

Table 2
Results of Experiment 1.

	Data_Planning horizon of the data							
	A.1_20	A.2_25	A.3_30	A.4_35	A.5_40	A.6_45	A.7_50	A.8_55
Objective value	57,755	75,995	99,077	122,368	153,452	165,635	201,220	234,639
Computation time (s)	7.75	20.59	45.05	67.19	145.10	184.39	287.13	493.80

Table 3
Results of Experiment 2.

		Period of uncertain factors affects the demand function				
		10	15	20	25	30
B.1	EV|PI	141,329	141,331	141,330	141,330	141,336
	LDR	143,021	143,048	143,078	143,095	143,101
	Gap (%)	1.20	1.21	1.24	1.25	1.25
B.2	EV|PI	141,354	141,355	141,354	141,352	141,351
	LDR	144,533	144,613	144,709	144,743	144,774
	Gap (%)	2.25	2.30	2.37	2.40	2.42
B.3	EV|PI	141,376	141,371	141,372	141,369	141,392
	LDR	147,104	147,367	147,858	148,221	148,349
	Gap (%)	4.05	4.24	4.59	4.85	4.92
B.4	EV|PI	141,435	141,435	141,437	141,439	141,439
	LDR	149,737	150,820	152,968	153,266	153,435
	Gap (%)	5.87	6.64	8.15	8.36	8.48

counterpart which is approximated from the multistage stochastic optimization model retains the tractability. In other words, the stochastic model, which is intractable, is well reformulated as the deterministic second-order cone program. Analysis of the objective value lost in the approximation process was verified in Experiment 2.

5.3. *Experiment 2: approximation error from the EV|PI*

We performed Experiment 2 to verify whether the robust model provided reasonable upper bounds against the multistage stochastic optimization model, or not. Experiment 2 was conducted by increasing the length of the support of the uncertain factor from data set B.1 to B.4. We also varied the number of uncertain factors affecting the demand at time t on the same data set. To determine how the approximated objective value is affected by the uncertain factor, we introduced the concept of *expected value given perfect information* (EV|PI). By solving the deterministic model (3) with the generated uncertain factors recursively, the close bound from the stochastic optimization model (6) can be estimated. That is, we used EV|PI as an alternative to the multistage stochastic optimization model (Ang et al., 2012; Shin et al., 2019; Lee and Moon, 2020). We generated the uncertain factors 10,000 times for each experiment and regarded the objective value of the stochastic optimization model (6) by calculating the average with the results of 10,000 times of the deterministic model (3). The results of Experiment 2 are presented in Table 3. From Table 3, it is evident that when the period of uncertain factors that affect the demand function became longer, the gap between the objective value from the EV|PI and LDR formulation became larger. It could also be observed that the gap increased when the length of the support of the uncertain factor increased. The results of Experiment 2 address that the proper adjustment of the uncertain factor can reduce the loss in the approximation process.

5.4. *Experiment 3: protection against realized random factors*

Since the demand models were developed as affine functions of the uncertain factors, a great variety of results can be produced in accordance with the realized uncertain factors. When the inventory manager establishes the inventory policy through the RIMRSTIP, the policy should protect the variability when the uncertain data is realized. To verify the robustness, we established an inventory policy based on LDR and analyzed how the objective value varies when the uncertain data is generated. As a control group for the RIMRSTIP, we made a policy that replaces order quantities based on the mean of the demand model. Recall the example of the demand model (29). If the inventory manager regards the demand as an expected value, the demand for the new-generation product can be estimated as (21.46, 21.63, 21.71). We call the inventory policy planned in this manner as a *deterministic inventory policy* (DIP).

Based on the two types of inventory policies, LDR and DIP, we conducted simulation studies for the same data by generating uncertain factors recursively, 10,000 times. Also, we generated two types of data, C.1 and C.2, which have different support sets (the supports of C.1 are about three times larger than those of C.2). The planning horizon was set to 30, and the experiments were carried out with varying lengths of uncertain factors affecting the demand model with $t = 10$, 15, 20, 25, and 30. The results of Experiment 3 are illustrated in Fig. 4. As shown in Fig. 4, the objective values from the LDR formulation did not show significant variations against the realized factors. In the case of the DIP, values that deviate significantly from the average were observed for certain data. In other words, the worst-case scenario was not well protected. In practice, the inventory manager establishes the inventory policy by estimating future demand from past data. However, future data cannot be accurately predicted, which means that it is difficult to predict how uncertain data will be realized. It is also important to minimize the value of expectation, but in the event of a truly unexpected worst-case scenario, it might require a stable operation from a number of scenarios. In summary, the results of Experiment 3 examined the need for applying robust optimization to inventory management in response to correlated uncertain demands.

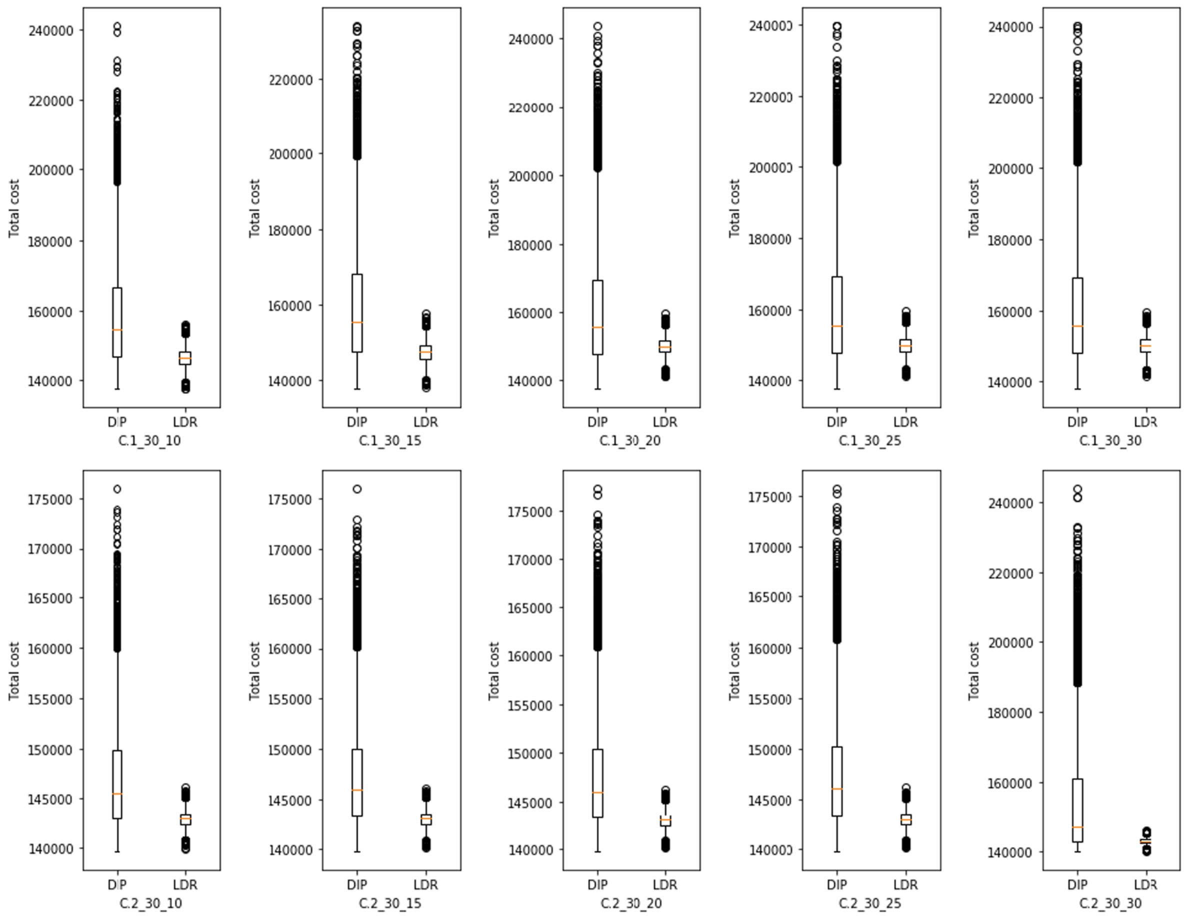

Fig. 4. Results of Experiment 3.

5.5. *Experiment 4: differences between modeling demands from VARMA and ARMA*

We conducted Experiment 4 to analyze how differences in inventory policies depend on the existence and nonexistence of inter-correlations among uncertain demands. We compared the demand model from the VARMA model, which captures the inter-relationship among three demands, with the ARMA model, which is affected only by its historical data. We refer to the inventory policies obtained through the VARMA and ARMA process as the *CORR* and *IDPT* policies, respectively (We used CORR and IDPT to emphasize the correlation across the demands and independent among the demands). We performed simulation experiments with these two types of inventory policies which were obtained through the LDR formulation. In a similar manner to Experiment 3, simulation studies were conducted based on the 10,000 sets of randomly generated values for the uncertain factors. The results of Experiment 4 are shown in Fig. 5. As can be observed in Fig. 5, the inventory policy from the CORR provides stable and robust solutions. The difference between the minimum and maximum of the objective value is significantly smaller than that of IDPT. However, the average value of the IDPT was smaller than that of CORR. It was also observed that the lowest cost occurred in IDPT.

5.6. *Experiments 5 and 6: comparisons of backlogged refurbishment service with or without trade-in program*

We performed Experiment 5 to identify the effect of the trade-in program on the acquisition of the returned products that are required for the refurbishment service. As an experimental group, we introduced *TIP* which is a supply chain system to which the trade-in program is considered. As a control group, we introduced *NO_TIP* where the trade-in program is not considered (See Fig. 6 in Appendix A). Four types of data sets, E.1 to E.4, were generated randomly based on periods 20, 25, 30, and 35, respectively. We first derived the order policy based on the LDR and conducted the simulation experiments by inputting the realized uncertain factors 10,000 times. Unlike other experiments in this study, which calculated the total cost over the entire planning horizon, Experiment 5 computed the sum of backlogged inventories over the entire period. The mean, minimum, and maximum values of the backlogged inventories for the 10,000 times simulation experiments are listed in Table 4 as Avg, Min, and Max, respectively. From Table 4, it can be identified that the introduction of the trade-in program reduced the backlog of the refurbishment service. The backlog of the refurbishment service not only incurs a high penalty cost but also triggers the production of old-generation products, which may

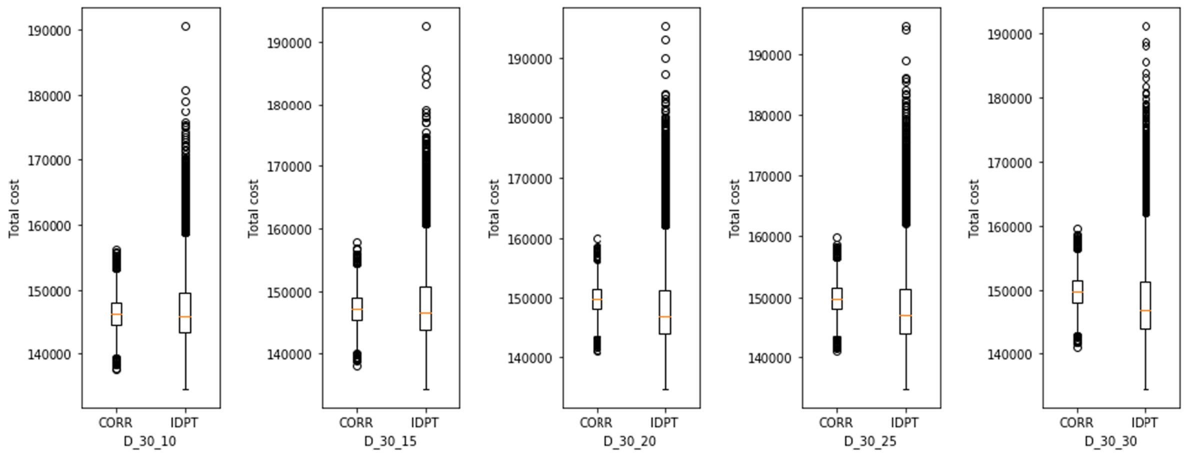

Fig. 5. Results of Experiment 4.

Table 4
Results of Experiment 5.

	E.1			E.2			E.3			E.4		
	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
TIP	79	67	92	90	78	103	72	61	83	57	43	71
NO_TIP	312	280	343	356	321	396	343	314	374	276	241	313

cause inefficiency in producing new-generation products. Experiment 5 addresses that the introduction of a trade-in program increases the complexity of the decision-making process for retailers; however, efficient management can lead to a stable operation that better meets customer needs in terms of refurbishment services.

Experiment 6 was then conducted to figure out how the trade-in program affects the input of the products used for the refurbishment service. Recall that the production of the old-generation products, refurbishing process, and remanufacturing process are input in the balance equation relevant to the refurbishment service (Fig. 3). In the same manner as Experiment 5, 10,000 times simulation experiments were conducted to identify the variations with respect to the production amounts of the refurbishment service. The total production amount of the refurbishing, remanufacturing, and old-generation products, which are defined as RF, RM, and OG, respectively, are provided in Table 5. As indicated in Table 5, the production of the old-generation product was not featured in the TIP system. Meanwhile, the remanufacturing process was not featured in NO_TIP. The introduction of the trade-in program reduced the productions of the remanufacturing processes and old-generation products, and increased the productions of the refurbishing processes. Since the costs and lead times for each process are assumed as $RF < RM < OG$, the introduction of the trade-in program has enabled efficient and stable operation in terms of inventory management.

6. Concluding remarks

In this study, we considered the multiperiod inventory model which incorporates the refurbishment service and trade-in program simultaneously. Due to the reverse flows of the products, the inventory model features a closed-loop supply chain system. To capture the correlations among the three types of demands, including new-generation, refurbishment service, and trade-in program demands, we adopted the factor-based demand model, which only requires the first and second moments of the uncertain factors. By

Table 5
Results of Experiment 6.

		E.1			E.2			E.3			E.4		
		Avg	Min	Max	Avg	Min	Max	Avg	Min	Max	Avg	Min	Max
TIP	OG	–	–	–	–	–	–	–	–	–	–	–	–
	RM	535	483	582	420	379	466	518	465	564	685	631	743
	RF	562	529	592	896	832	950	1134	1081	1195	1291	1236	1358
NO_TIP	OG	232	223	242	290	276	304	306	294	317	335	342	369
	RM	849	776	911	1027	958	1106	1345	1258	1444	1615	1537	1715
	RF	–	–	–	–	–	–	–	–	–	–	–	–

approximating the upper bound of the multistage stochastic optimization model, we could derive the robust counterpart. The computational results showed that the robust counterpart retains tractability until a modest size. It also provided the reasonable upper bound from the multistage stochastic optimization model.

6.1. Managerial insights

This study offered managerial insights, which could be instructive to inventory managers. By analyzing the results of the computational experiments, we derived the following managerial insights as follows:

(i) The inventory policy derived from the LDR formulation provides robust and stable solutions. Meanwhile, the accuracy lost through approximation was reasonable and tractability was greatly improved. That is, the robust model provided a much better policy than the deterministic model and was much more efficient than the multistage stochastic optimization model.
(ii) Depending on the propensity of the inventory manager, different inventory policies might be preferred. If the inventory manager pursues to minimize the expected cost for various scenarios, it would be better to establish an inventory policy through the IDPT, which incorporates only its own past data.
(iii) If the inventory manager seeks to operate in a more stable manner, it would be better to establish an inventory policy through the CORR, which incorporates the inter-correlation among demands as well as from its own past data.
(iv) The introduction of a trade-in program not only plays a role in a sales promotion to increase customer demand but also plays a role in the efficient acquisition of the returned products for a refurbishment service. By reducing the number of backlogged inventories for the refurbishment service, the retailer can improve a customer's service level. Furthermore, reducing the number of backlogged inventories mitigates the inefficient production of discontinued old-generation products and enhances the refurbishment process of products that can be produced efficiently.

6.2. Implications to emerging markets

The mathematical model presented in this study can be used as a decision support system for the operation of a trade-in program and refurbishment service. A retailer of the smartphone, such as *Apple* or *Samsung*, is a typical example of a retailer that uses such an operation (This claim can be confirmed by visiting the website www.apple.com/shop/trade-in or www.samsung.com/us/trade-in). With the commercialization of 5G, sales of the smartphone are expected to increase in developed markets. Meanwhile, smartphone sales are expected to continue to grow in emerging markets until 2024. In other words, it is predicted that the emerging market has a strong potential for smartphone market share growth (Arunachalam et al., 2019; Shankar and Narang, 2019). This study addresses that research on the operation of customer service in retail shops selling smartphones, such as the trade-in program or refurbishment service, should receive more attention.

Through an integrated system, it is possible for retailers to maintain a high service level by sufficiently preparing extra refurbished products. In addition, the remaining refurbished products can be sold at a low price in the second-hand sector of an emerging market. As can be identified from Remark 1 and Remark 2, the returned products would be surplus in this integrated system and could be sold in emerging markets, as well as used as the stock for the refurbishment service. As GSMA (2017) reported, smartphone penetration in emerging markets is steadily increasing, but low-income groups have difficulty in affording smartphones. If a retailer utilizes the surplus of returned products according to the system presented in this study, smartphones would be available to everyone in this market space. Consequently, no longer would the ownership of smartphones only be the privilege of those in developed countries. The selling of refurbished products, rather than new products, within the second-hand market, would also accompany sustainability, cleaner production, and positive environmental effects (Zhou et al., 2020).

6.3. Directions of future research

In practice, a retailer operates the trade-in program with an online store as well as with a physical "bricks and mortar" store. For future research, an inventory model considering the trade-in program can be extended to a multi-channel or an omnichannel system. Alternatively, the trade-in program can be run by returning points to the customer which have the same function with cash. In this case, a wide range of purchasing options would be available for the customers rather than the existing and more limiting option of purchasing discounted new-generation products. Considering the uncertain environment, we focused on the uncertainty in demands. Depending on the condition of returned products, the cost of refurbishing or remanufacturing process can vary. In this study, we divided the condition of the returned products into two types and considered them as deterministic manners. For further study, the condition of returned products could be regarded as uncertain values.

CRediT authorship contribution statement

Youngchul Shin: Conceptualization, Software, Methodology, Investigation, Data curation, Writing - original draft, Writing - review & editing, Visualization. **Sangyoon Lee:** Methodology, Investigation, Writing - review & editing. **Ilkyeong Moon:** Conceptualization, Validation, Writing - review & editing, Supervision.

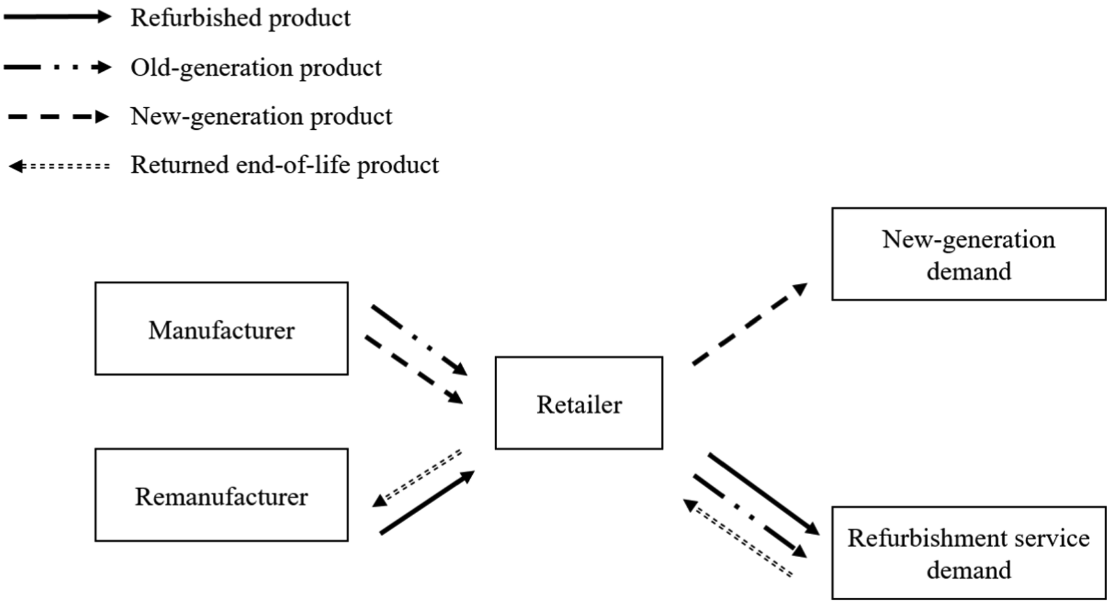

Fig. 6. Flow of products when the trade-in program is not considered.

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Appendix A

When a trade-in program is introduced, distinct differences exist from the model which only considers the refurbishment service. For the previous refurbishment service, customers return end-of-life products and they are sent to the remanufacturer. After being remanufactured, the refurbished products are sent to the retailer. For the customer who has submitted the end-of-life product, the refurbished product should be provided immediately. If a refurbished product is not available, discontinued old-generation products may be pre-produced to prevent backlog. However, this process produces discontinued products, which incurs a relatively higher cost including opportunity cost. Also, in the case of out of stock, dissatisfaction with the warranty service will have a significant detrimental effect on the brand's image, which can lead to a high penalty cost for the retailer. Fig. 6 is provided to illustrate the differences from the IMRSTIP in terms of product flow.

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