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Pricing strategies in an M/G/m/m loss system: A case study of Incheon International Airport customer services

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ABSTRACT

This paper studied the optimal pricing strategy based on a highly realistic pricing scheme in order to maximize the revenue of a service modeled as an M/G/m/m loss system. The blocking probability is considered together to prevent compromising the quality of service. Customers' willingness-to-pay is regarded to be randomly distributed, indicating price-dependent arrival. Considering the severe complexity of the proposed model, the optimal pricing strategy is numerically investigated through computational experiments based on case studies of two actual services currently operated at Incheon International Airport (Seoul, South Korea). The two services represent a congested and quiet situation, allowing for the analysis of opposite cases. The results of the computational experiments clearly demonstrate distinct optimal strategies for the two contrasting situations. The performance of the two services with their current pricing strategies is evaluated, providing managerial insights for the service providers.

1. Introduction

Incheon International Airport (ICN), the biggest and best airport in South Korea, is also one of the best airports in the world. In particular, ICN, established in 2001, has consistently ranked within the top five of The World's Top 100 Airports (awarded by Skytrax) over the past two decades (https://www.worldairportawards.com/). Such a ranking implies the presence of various services prepared for travelers. The primary functions of airports are handling freight and passenger transportation-related operations such as gate allocation, air traffic control, immigration procedures, and security checks. Besides these operations, airports offer various other services, notably duty-free shops, stores, or restaurants within the airport. A variety of other services are available apart from these purchasable services, and we will focus on two specific services in this paper.

1.1. Parking lot (PL)

Public transportation has been developed in South Korea due to the limited land area and high population density. Consequently, various transportation options are available at ICN. However, according to the report jointly published by the Ministry of Land, Infrastructure, and Transport of South Korea and the Korea Civil Aviation Association (https://www.airportal.go.kr/knowledge/od/main/main.jsp), a significant proportion of South Korean residents drive themselves to ICN.

Fig. 1 presents the percentages of the final transportation options chosen for arrival at ICN and all the options utilized until reaching the airport, respectively. As one can transfer several times for the arrival, the sum of the ratios in the right graph exceeds

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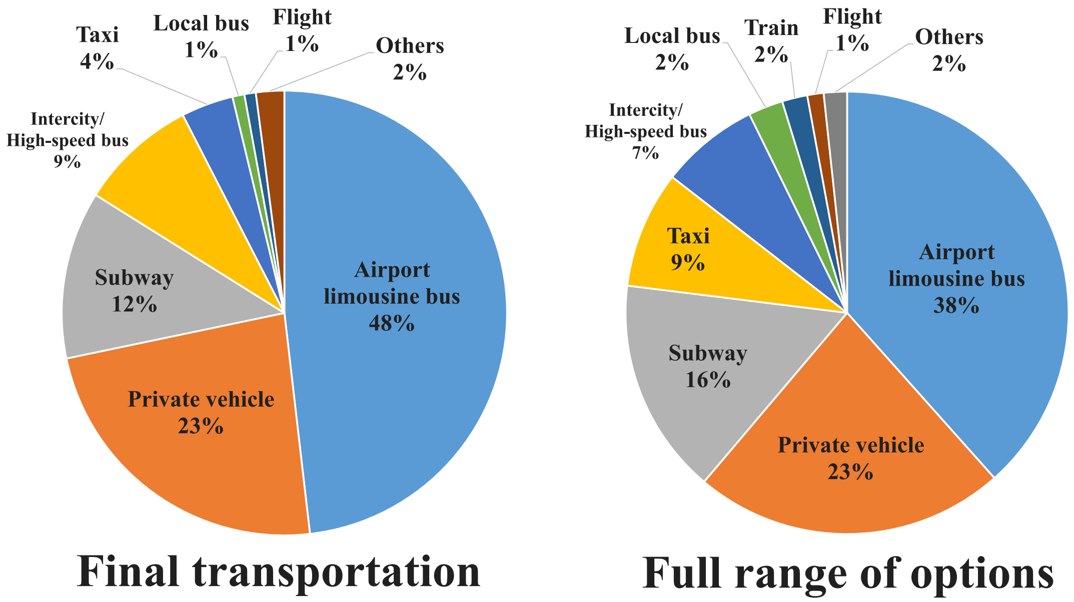


Fig. 1. Percentages of final transportation choices to ICN arrival and the full range of options employed en route to the airport.

Table 1
Percentages of the reasons for the choice of each transportation option.

Reason	Private vehicle	Airport limousine bus	Subway	Taxi	Intercity/High-speed bus	Local bus	Train	Flight
Convenient boarding location and transportation connections	42.52	49.93	42.70	51.57	49.34	50.88	41.67	37.70
Convenient boarding facilities	17.87	15.83	11.51	8.38	12.40	9.65	11.46	14.75
Short total travel time	16.20	11.15	14.71	15.97	9.23	7.89	26.04	31.15
Precise arrival time	7.75	13.93	12.34	6.81	13.72	0.88	13.54	8.20
Reasonable fare	5.99	7.26	17.08	4.45	14.25	28.07	3.13	–
Excessive luggage	4.93	0.29	–	8.38	–	–	–	1.64
Relatively safer	2.20	0.97	1.30	1.05	0.53	2.63	1.04	1.64
Accompanying passengers (e.g., children)	1.50	–	–	1.05	–	–	–	–
No alternative options	0.88	0.34	0.12	1.05	0.53	–	3.13	1.64
Included in the travel package	0.18	0.05	0.12	0.52	–	–	–	3.28
Inconvenience and expensive fee of the parking lot	–	0.24	0.12	0.79	–	–	–	–

Table 2
The numbers of transfers depending on the inclusion of private vehicles.

Number of transfers	Private vehicle included	Private vehicle not included
0	597	1,331
1	111	447
2	18	31

No data for more transfers than twice.

one. Table 1 shows the reasons for using private vehicles and other transportation surveyed in the report. On the other hand, the numbers of transfers for both those using private vehicles and those not are reported in Table 2. From both tables, it can easily be seen that a considerable number of individuals opt for alternative transportation, even tolerating transfers, possibly due to the absence of private vehicles or for various other reasons. However, the parking lot (PL) of ICN frequently experiences congestion, with an occupancy rate exceeding 105 percent. In response, Incheon International Airport Corporation often encourages public transportation usage (https://www.airport.kr/co/ko/index.do). An interesting observation from Table 1 is that some participants also responded that the reason for taking other transportation was the "inconvenience and expensive fee of the parking lot", despite other major selections being given as multiple choices. Moreover, "reasonable fare" was appreciably selected for the other transportation options. In other words, driving to the airport can be burdensome for some, and this signifies that controlling the demand through pricing of parking fees can be a solution to addressing parking congestion issues, assuming an unchangeable capacity. Furthermore, improved profitability can also be expected. Note that the long-term PL of ICN is currently charging a fee of KRW 9,000 per day.

Table 3
Prices for the CSSs at ICN.

Airline	Initial fare	Extra charge
Korean Air	Free for 5 days	KRW 2,500/day
Asiana Airlines	Free for 5 days	KRW 2,000/day
Jeju Air	KRW 9,000 up to 7 days	KRW 2,000/day
Eastar Jet	KRW 9,000 up to 7 days	KRW 2,000/day
T'way Air	KRW 9,000 up to 7 days	KRW 2,000/day

1.2. Coat storage service (CSS)

South Korea is known for its distinct four seasons, resulting in significant temperature variations between summer and winter. Considering monthly average high and low temperatures, the highest temperature throughout the year is 29.6 °C in August, while the lowest is −5.9 °C in January (https://www.weather-atlas.com/en/south-korea/seoul-climate). According to the previous report, Vietnam ranks third as the final destination country (8.3 percent) among South Korean residents, and Da Nang, a city in Vietnam, also ranks third as the arrival airport (3.8 percent). Located in Southeast Asia, Vietnam enjoys a warm climate throughout the year. When considering the monthly average high and low temperatures again, the highest temperature is 31.8 °C in June (30.8 °C in August), while the lowest is 19.3 °C in January (https://www.weather-atlas.com/en/vietnam/da-nang-climate). Consequently, the temperature difference in winter is significant when compared to summer. In other words, when traveling from South Korea to warm countries like Vietnam during the winter, travelers need thick winter clothing at the departure point. However, it becomes a space-consuming burden upon arrival. To relieve such inconvenience, services for storing winter clothing, which we shall refer to as the coat storage service (CSS), are provided in ICN. The CSS bears similarities to the PL. Customers of both services deposit their belongings for a specific duration, the travel period in this case, by occupying a designated capacity (e.g., a car to the PL or winter clothing to the CSS). Therefore, this study considers these two services together, based on their resemblance. Indeed, the two services are distinct in their representation of the situation. The PL exemplifies a substantially congested situation in which there is a lack of capacity regarding the potential demand. In contrast, the CSS illustrates a quiet situation with abundant capacity so that an arriving customer is not likely to expect service refusal. The current pricing of the CSS at ICN varies, depending on the service provider, and is documented in Table 3. Note that at ICN, airlines provide the CSS to customers, and that it is not limited to their own passengers.

1.3. Summary and contributions

• We investigate the optimal pricing strategy in order to maximize the revenue of a service modeled as an M/G/m/m loss system. Research on loss systems considering non-exponential service time is insufficient compared to other types of queuing models due to its complexity. Additional concerns about price-dependent arrival and a highly realistic pricing scheme complicate the problem even more. To address this issue, the optimal pricing strategy is numerically investigated through computational experiments. Therefore, this study plays a role in filling the gap in the body of literature related to loss systems.

• This study is motivated by two particular services currently operated at ICN, the PL and the CSS. Consequently, we focus on changing the service price rather than on deciding the capacity to maximize the revenue. The blocking probability (BP) is considered together to prevent compromising the quality of service. A two-step functioned pricing scheme is implemented to demonstrate the actual prices of the two services and to capture practicality, as such a pricing scheme is commonly adopted in practical industry settings. As this type of pricing is markedly distinct from that considered in previous literature, we present a novel formula for computing the expected revenue to reflect such a difference.

• We explicitly account for customers' willingness-to-pay (WTP). Assuming the customers' WTP to be randomly distributed, the arrivals depend on the price, illustrating a stochastic environment. Despite our assumption of a uniform or normal distribution, we additionally consider the stochasticity of the corresponding parameters. Eventually, we can additionally incorporate various skewed unimodal distributions in consideration due to the randomness of the parameters of a normal distribution.

• The results of computational experiments, based on case studies of the PL and the CSS, clearly demonstrate the optimal pricing strategy for two distinct situations. Regardless of whether the customers' WTP follows a uniform or normal distribution, the overarching goals remain the same. The optimal strategy for highly congested situations, represented by the PL, is to gentrify the service and to intensively target only a selected group of customers. However, the specifics of whether to benefit short-term or long-term customers depend on the distributions. On the contrary, the optimal strategy for less congested situations with ample capacity, exemplified by the CSS, is to charge moderate prices without specifically targeting short-term or long-term customers to secure sufficient demand. Indeed, in both situations, additional considerations of finely tuning the robustness of pricing are necessary.

• We evaluate the performance of the PL and the CSS using their current pricing strategies. As a result, both services are incurring losses and, thus, are supposed to have possibilities for improvements. In addition to potential loss, the PL probably suffers from unnecessarily excessive blockings. These results are likely due to prioritizing customer convenience at the service level over profit. The fact that both services are not the primary operation of each service provider (i.e., the airport and airlines) but rather are offered to enhance customer satisfaction explains the lack of efforts to maximize profitability. However, deviating from this exceptional situation, it is necessary to thoroughly inspect whether the optimal strategy has been appropriately applied.

The remainder of this study is organized as follows: Existing literature that previously studied related topics is reviewed in Section 2. In Section 3, we describe our problem, including the assumptions. Then, the requisite preliminaries for the investigation are discussed in Section 4. Computational experiments are reported in Section 5, based on case studies, and conclusions are offered in Section 6.

2. Literature review

2.1. Airport services

Aviation plays a key role in cross-border travel in modern society. Due to its growing importance, a wide range of research areas relevant to aviation and air transportation has been thoroughly considered. Such research considers not only the primary functions of airports but also various other services that airports provide to passengers and customers. Airports, performing as service providers, cannot be free from customer reviews. As a result, evaluating the service qualities of airports has also gained great interest. Specifically, Pabedinskaitė and Akstinaitė (2014) and Mainardes et al. (2021) focused on the primary functions of airports, which are directly related to air transport and are among the main concerns of passengers. However, the services considered in this paper stand quite apart from such a category.

Based on a comprehensive review of relevant literature, Fodness and Murray (2007) selected various services that passengers expect from an airport and evaluated the airport service quality. In that evaluation, both the aviation-related services (e.g., "I should be able to exit the airplane within ten minutes of landing" or "An airport's terminal should be designed so that waiting lines are minimized") and any other services (e.g., "National chain restaurants should be available at airports" or "I should be able to walk to the parking lot from the terminal at an airport") are simultaneously considered. Similarly, Bezerra and Gomes (2016) measured airport service quality through a multidimensional approach. Note that variables regarding the price or availability of a service are explicitly included, which are the main concerns of this paper as well. Prentice and Kadan (2019) not only ended up just evaluating the airport service quality but also showed that passengers utilize their past experiences when choosing another travel destination and whether to reuse a particular airport. Again, general services such as "Retail and dining options/Restaurants offered a wide range of products" were included in the variables. Certainly, pleasant and comfortable PL also plays a significant role when evaluating the service quality of an airport. Specifically, Chonsalasin et al. (2021) explicitly incorporate PL in their evaluation. It can be easily seen how much the customers value PLs, considering that measures such as "Sufficient parking spaces" or "Value for money of Parking facilities" are included. As Chonsalasin et al. (2021) have considered in part already, we also focus on the prices of the two aforementioned services in order to take into account the availability of the services that customers experience and the revenue received.

2.2. Pricing and WTP

Customers are affected by pricing, and thus, the revenue heavily relies on the pricing scheme. Revenue and profitability are the utmost concerns of a business, without any doubt, except for some special sectors such as public, nonprofit organizations, or humanitarian organizations. Consequently, pricing strategies have been extensively studied (Sammut-Bonnici and Channon, 2015; Li et al., 2022; Zhou et al., 2022). A pricing strategy includes a wide range of concepts. For example, it can be significant from the perspective of business and management, economics, operational research and management science, or psychology. In this paper, we will focus on the aspect of operational research and management science. In particular, we investigate the optimal pricing strategies to maximize revenue.

Research on dynamic pricing has also been extensively conducted with the development of computational capabilities. However, this is mainly employed in specific areas, such as airline tickets (Jo et al., 2024), accommodations (Mitra, 2020), or tailored services (Lin and Wang, 2022), and is challenging to use in general services. General services, like the PL or CSS shown above, predominantly involve predetermined prices rather than dynamically changing ones. Therefore, this paper considers fixed or static pricing rather than dynamic pricing. However, obtaining information on the degree to which customers tolerate price points is necessary in order to derive an optimal pricing strategy and to maximize profit, regardless of whether the pricing scheme is static or dynamic.

WTP is defined as the highest price an individual is willing to accept to pay for some good or service (Breidert, 2007). Considering the definition of WTP, it plays a significant role in pricing, as previously mentioned. As a result, estimating the WTP of customers has been of great interest for some practical purposes. Specifically, topics considering WTP with regard to customer choices have been extensively studied in the fields of economics or business and management. According to Breidert (2007), diverse methods are implemented for estimating WTP. For example, Monty and Skidmore (2003) utilized the hedonic price model to evaluate customers' WTP for hotel bed and breakfast amenities. The authors proposed the evaluated mean and standard deviation for each variable rather than estimating the distribution for each of them. On the other hand, some studies explicitly derive the estimated distribution of the WTP. Jedidi and Zhang (2002) proposed a conjoint-based approach for the estimation. A right-skewed unimodal distribution resulted from the data on automobile batteries. Based on the contingent valuation method, Mmopelwa et al. (2007) applied the linear regression model to fit the WTP into a linear function. WTP decisions for both an entry fee and a camping fee in a game reserve in Botswana are considered, respectively. The overall expenditure of the trip, which serves as a proxy of income, significantly impacts the WTP decisions for both fees. In addition, the age of the respondent also considerably affected the WTP for the camping fee. Czajkowski et al. (2024) also utilized the contingent valuation method as Mmopelwa et al. (2007) did, but focused on determining the best fitting distribution among several candidates. Czajkowski et al. (2024) investigated two different data-sets and concluded that Weibull and gamma distributions fit well for the first one, while Birnbaum–Saunders and gamma distributions best describe the second one.

The studies mentioned above implement one single method each, whereas some other papers applied multiple approaches and compared their performances. Miller et al. (2011) considered four different approaches compared to a benchmark based on the real purchase data of customers. Though the authors did not specify the estimated distribution, the results implicitly indicate that the WTP nearly forms a normal distribution. Similarly, Tabasi et al. (2024) implemented three types of methods to estimate the WTP decisions for six different data sets. It was shown that customers' WTP does not follow a uniform distribution but rather demonstrates multimodal distributions.

Unfortunately, estimating the WTP requires ample, well-organized data. However, such data are high-value-added and, thus, are usually treated confidentially. Consequently, many studies assume a specific distribution for the WTP. For example, Weisbuch et al. (2008), Liu and Cooper (2015), Lobel (2020), and Jo et al. (2024) all assumed specific distributions for the numerical analyses, though their problems are not limited to those assumptions. Specifically, Weisbuch et al. (2008) adopted uniform and logistic distributions for the simulation, while Liu and Cooper (2015) considered uniform and beta distributions. Both Lobel (2020) and Jo et al. (2024) utilized uniform distribution only. It is evident that uniform distribution is a popular representative of designing the customers' WTP. More cases are reported in the following subsection.

In this paper, we also made an attempt to roughly estimate the customers' WTP through a simple approach considering insufficient data. The performances of the PL at ICN are evaluated based on this estimated WTP. Moreover, a case of maximizing the expected revenue is further analyzed when only the type of distribution of the customers' WTP is given without the specific values of the corresponding parameters.

2.3. Revenue or profit maximization in queueing models

Among various options from which to describe and model the problem, queueing theory is implemented in this paper for the examination. Queueing models has been widely adopted to analyze diverse subjects. For example, Keerthana et al. (2020) analyzed a stochastic inventory system by introducing a queueing-inventory system while Pourvaziri et al. (2024) combined queueing theory with mathematical modeling and a deep learning algorithm to decide the location and capacity of electric vehicle charging stations. As the queueing model allows us to design a problem that considers stochastic arrivals of entities, it enables us to analyze diverse performance measures. In this subsection, we focus on the studies that consider monetary gain among such diversity. Some other measures are further introduced in the following subsection.

The topic of applying queueing theory for the purpose of maximizing revenue or profit has been extensively studied. Lazov (2017) presented a profit maximization problem based on the M/M/m/m model (i.e., M/M/c/c or M/M/k/k). The main decision of this study was about determining the optimal capacity. On the other hand, some papers particularly considered pricing strategies for the profitability of a service, similar to our research. Jacob and Roet-Green (2021) presented two types of services, and depending on their combinations, the problem is modeled as M/M/m/m, M/G/m/m, or GI+M/M/m/m loss systems. The authors numerically examined the revenue-maximizing price-service menu through an iterative calculation process.

As we will further describe in more detail in Section 3, we also assume an M/G/m/m loss system (Ziya et al., 2006). Loss systems are common in various applications, such as telecommunications networks, computer systems, and call centers, where there is a maximum limit to the number of customers or transactions that can be handled simultaneously (Paschalidis and Tsitsiklis, 2000). Specifically, loss systems are commonly adopted for reusable resources. A reusable resource is a resource that can be utilized repeatedly by customers whenever it is idle, such as in accommodations or car rental services (Lei and Jasin, 2020). The PL and CSS considered in this study also belong to reusable resources. Accurate modeling and analysis of loss systems are essential for capacity planning and optimizing resource allocation to minimize loss and maintain acceptable service quality. It is evident that capacity is an essential consideration in a loss system. However, we will only focus on the prices under the assumption of an unchangeable capacity. Accordingly, the goal of this paper is to investigate the optimal pricing strategy in order to maximize the revenue of a service modeled as an M/G/m/m loss system. Besbes et al. (2022) and Elmachtoub and Shi (2023) showed that static pricing policies perform reasonably well compared to the optimal dynamic pricing policies for loss systems considering reusable resources. In addition, considering that Kim and Randhawa (2018) pointed out the possibility of customers feeling uncomfortable with dynamic prices, we adopt a static pricing scheme to capture practicality, as previously mentioned. In detail, a two-step functioned pricing scheme is implemented to demonstrate the actual prices of the PL and the CSS of ICN. Moreover, the customers' WTP is considered together, along with the impact on the demand. In other words, price-dependent arrival is considered.

Papers focusing on pricing strategies with a queueing model while considering price-dependent arrivals, which are the most relevant studies to ours, are summarized in Table 4. Note that columns indicating the decision for pricing and adoption of price-dependent arrival are omitted, as these two attributes belong to all the studies reported in Table 4. It is apparent that none of the papers considering delay-dependent arrival utilized a loss system. It is rather obvious due to the fact that a customer does not need to wait for any time once he/she succeeds in entering the system, which follows the characteristics of a ·/·/m/m queue. Table 4 also shows the lack of investigation on a loss system considering non-exponential service time. Although the queueing model is a powerful tool for designing a complicated practical situation, an exact analysis is difficult and challenging and often unavailable even under the assumption of exponential service time, which provides us additional convenience for examination (Maglaras and Zeevi, 2003; Ata and Shneorson, 2006; Chen et al., 2023). Consequently, relaxing the assumption of exponential service time aggravates such difficulties. Moreover, the pricing scheme of this paper essentially differs from those of the previous studies. Such a pricing scheme, combined with the complications raised above, significantly limits theoretical analyses and general considerations. To address this issue, we numerically investigated our problem through computational experiments as Jacob and Roet-Green (2021) did.

Table 4
Comparison of previous studies related to this research.

Authors	Objective[a]	Additional decision	Pricing type[b]	Queue type	Delay-dependent arrival	Assumption for the distribution of WTP[c]
Low (1974)	P	–	D	M/M/m/N	–	General (uniform)
Maglaras and Zeevi (2003)	P, SW	Capacity	S	M/M/∞, M/M/m	✓	General (uniform, exponential, iso-elastic)
Ata and Shneorson (2006)	SW	Service rate	D	M/M/1	–	General (uniform)
Besbes and Maglaras (2009)	REV	–	D	M/M/1	✓	General (exponential)
Kumar and Randhawa (2010)	P	Capacity	S	M/M/1, M/M/m	✓	General (uniform)
Haviv and Randhawa (2014)	REV	–	S	M/M/1	✓	General (uniform, exponential, triangular)
Kim and Randhawa (2018)	REV, SW	Capacity	S, D, DT	M/M/1, GI/GI/m	✓	General (exponential)
Lee and Ward (2019)	P	Capacity	S	M/GI/1+GI[d]	–	General (uniform)
Lin et al. (2021)	P, SW	Strategy	S	M/M/1, M/M/m	✓	Uniform
Hashemi Karouee (2021)	REV	–	S	M/M/1/1, M/M/m/m	–	Uniform
Zhao and Lee (2021)	QoS	–	D	M/M/m/N	–	Uniform
Chen et al. (2023)	P	Capacity	D	GI/GI/1	–	General (logistic)
Jia et al. (2024)	REV	–	D	M/M/m, M/M/m/m	–	Arrival rate corresponding to discrete price
This research	REV	–	ST	M/G/m/m	–	Uniform, normal

[a] P: profit, QoS: quality of service, REV: revenue, SW: social welfare.
[b] S: static single-price, D: dynamic single-price, DT: dynamic two-price, ST: static two-step function.
[c] Distributions in the parentheses indicate the ones utilized for the numerical analyses.
[d] "+GI" indicates the general distribution for patience times.

2.4. PLs

PLs can be easily found and accessed anywhere in the world. In particular, PLs in congested areas, such as metropolises or popular tourist spots, are especially essential (Falsafi et al., 2022; Choi and Lee, 2023; Zhang et al., 2024). Like many other services, PLs can be modeled by a queueing system to evaluate performance (Mai and Cuong, 2021). As a result, many papers studied and analyzed parking problems by applying queueing theory. Specifically, one of the main research streams is about evaluating or predicting the occupancy of a PL. In considerably congested urban areas, an empty parking space would probably be the most valuable attribute to drivers. Accordingly, calculating the probability of facing an empty parking space at the time of arrival considering the current location has been a popular research topic to meet such demand (Caliskan et al., 2007; Klappenecker et al., 2010; Xiao et al., 2018). Moreover, some studies recommend the optimal PL among a list of parking areas based on the probabilities mentioned previously (Du and Gong, 2016; Li et al., 2017). Furthermore, offering guidance for the optimal route considering multiple PLs simultaneously has gained great interest (Ma et al., 2017; Pel and Chaniotakis, 2017; Atif et al., 2020). The optimal route is provided, taking into account the possibility of entering a full PL. All the above studies are tailored to the needs of drivers who crave empty PLs, which also serve to relieve congestion.

An interesting attempt has been made to alleviate the same problem. Millard-Ball et al. (2014) and Du and Gong (2016) both considered the cruising (i.e., wandering) of drivers by adopting an M/M/m model. Wandering customers, who are waiting for their order for a given service, are usually not even a concern in the case of a general service. However, cruising or wandering drivers are definitely influential. As the drivers need to keep wandering for an empty parking space or an empty PL, such movements cause unintended and unnecessary congestion and pollution. Therefore, Millard-Ball et al. (2014) and Du and Gong (2016) analyzed and reduced such cruising. These two papers clearly show that drivers highly value parking availability

Apart from this, some studies analyze PLs with conventional measures that evaluate general queuing systems. For example, Dalla Chiara and Cheah (2017) measured the total time in the queue or the average queue length. Similarly, Millard-Ball et al. (2014) and Abhishek et al. (2021) employed the BP and utilization to evaluate a service. Both measures are utilized to present the service quality and system behavior. The utilization was defined as the expected proportion of busy servers, which also refers to the average occupancy. Note that both measures are also considered in this paper, and the details will be introduced later. Although dynamic pricing for electric vehicle charging stations has been widely studied recently (refer to Section 2.3), research on the pricing of traditional PLs based on queuing models is lacking. Unfortunately, as CSS hardly shows any distinct characteristics, no research has been conducted on such a service except for directly considering clothing.

3. Problem description

The notations used throughout this paper are given as follow:

μ : service rate of an M/G/m/m system

λ : arrival rate of an M/G/m/m system

ES : expected service time in general ($1/\mu$ in case of an M/M/m/m system)

ρ : offered load in general, $\rho = \lambda ES$

m : number of servers

$p_m(\rho)$: BP of the system with m servers and offered load given as ρ

$EB_m(\rho)$: expected number of busy servers out of m servers and offered load given as ρ

t : customers' service time

Λ : potential arrival rate of customers (i.e., market size)

Λ_t : potential arrival rate of customers with service time t, $\sum_t \Lambda_t = \Lambda$

V : customers' WTP for the service per unit time

f : probability density function of V

F : cumulative distribution function of V

r_0 : entry fee of the service (i.e., admission fee), $r_0 \geq 0$

T : starting time of the additional fee being charged, $T \geq 0$

r : additional fee per unit time being charged after T, $r \geq 0$

$c(T, r_0, r, t)$: total price charged to customers with service time t

$p(T, r_0, r, t)$: average price charged to customers with service time t, $p(t) = c(t)/t$

$\lambda(T, r_0, r)$: effective arrival rate of customers when the price per unit time is given as $p(T, r_0, r, t)$

$\lambda_t(T, r_0, r)$: effective arrival rate of customers with service time t when the price per unit time is given as $p(T, r_0, r, t)$, $\sum_t \lambda_t(T, r_0, r) = \lambda(T, r_0, r)$

$q_t(T, r_0, r)$: probability of service time being t upon a customer's arrival when the price per unit time is given as $p(T, r_0, r, t)$, $q_t(p) = \lambda_t(T, r_0, r)/\lambda(T, r_0, r)$

$ES(T, r_0, r)$: expected service time when the price per unit time is given as $p(T, r_0, r, t)$

$\rho(T, r_0, r)$: offered load when the price per unit time is given as $p(T, r_0, r, t)$

$\underline{V}$: lower bound of a uniform distribution of V

$\bar{V}$: upper bound of a uniform distribution of V

$\tilde{V}$: mean of a normal distribution of V

$\ddot{V}$: standard deviation of a normal distribution of V

We consider the problem of maximizing the revenue of a service provider. Though we focus on a PL and a CSS in this paper, any other service that fits into the problem situation can be fairly considered. Both the PL and the CSS can be modeled as an M/G/m/m loss system. Specifically, the act of parking and the act of storing clothes at the airport are both considered to be receiving the service. For example, a traveler comes into the PL. Then, such a situation can be regarded as a customer successfully entering the system. The traveler parks his or her own vehicle and goes on the trip, which represents a server being occupied. Eventually, the traveler will leave the PL with his or her vehicle after returning. This situation indicates that the customer is departing the system, and the corresponding server is being released. In this case, the period of parking, which is equivalent to the travel period, becomes the service time.

As numerous travelers visit the airport every day, it is reasonable to assume that the customer arrival follows a Poisson distribution. The data extracted from the aforementioned report will be used for the service time (t) distribution, which can be classified as a general distribution. The Poisson arrival process corresponding to the customer arrival can be decomposed into independent processes of customers with fixed service time (i.e., the travel period) considering the actual service time distribution. ICN serves two types of PLs: short-term and long-term. Commonly, the short-term PL is used by people sending off or picking up others. On the other hand, travelers who drive on their own usually visit the long-term PL. In this paper, we only consider the long-term PL. Then, for both services, customers exceeding the capacity of the service (e.g., the number of parking spaces) cannot be served nor wait in a queue. In other words, the size of the queue, including the servers, is the same as the number of servers. Therefore, we apply the M/G/m/m loss system to model the revenue maximization problem.

Focusing only on the revenue may lower customer satisfaction. Hence, the quality of service is also considered in the form of the BP. The BP, known as the Erlang B formula, is the probability that an arriving customer is lost to the service because the capacity is full. Considering the definition of the BP, limiting its value to a certain level is important. In an M/M/m/m loss system, the BP $p_m(\lambda/\mu)$ is calculated as Eq. (1).

$$p_m(\lambda/\mu) = \frac{\frac{(\lambda/\mu)^m}{m!}}{\sum_{k=0}^{m} \frac{(\lambda/\mu)^k}{k!}} \tag{1}$$

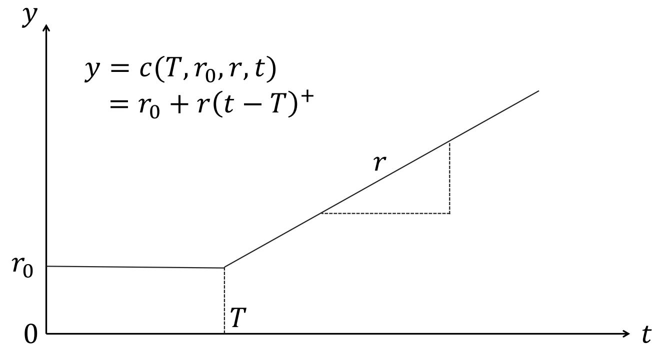


Fig. 2. The two-step function-shaped price for the service with fixed T, r_0, and r.

The inclusion of μ indicates that the service rate is mandatory for calculating $p_m(\lambda/\mu)$. However, the BP of an M/G/m/m loss system can also be calculated similarly, even if the service time distribution follows a general distribution. Though, the expected service time (ES) should be preinformed. Eq. (2) shows how the BP of an M/G/m/m loss system is calculated, where $\rho = \lambda ES$.

$$p_m(\rho) = \frac{\frac{\rho^m}{m!}}{\sum_{k=0}^{m} \frac{\rho^k}{k!}} \tag{2}$$

Four assumptions are made, as follows:

- The price for the service is charged as a two-step function, considering the service time t. A constant amount (r_0) is charged until a certain period (T), followed by a linearly increasing part (with a rate of r), with respect to the service time t. Therefore, the total price $c(T, r_0, r, t) = r_0 + r(t - T)^+$ is charged to customers with service time t. As the price is determined by the values of T, r_0, and r, the pricing strategy indicates making decisions about these values. See Fig. 2.
- The unit time of the problem is a day, and all the time-related attributes are treated on a daily basis. For example, the service time t of customers is drawn from a discrete probability distribution, resulting in t having a positive integer value. Similarly, T will only have an integer value and $p(T, r_0, r, t)$ is calculated as a daily average price.
- Customers' WTP for the service per unit time (V, which can be stated as the daily WTP due to the above assumption) is independent and identically distributed across the customers, having a probability density function (PDF) of f and a cumulative distribution function (CDF) of F. Then a customer's total WTP is presented as a cumulative amount along with the service time t, which equals to Vt. Therefore, if the service provider sets the price $c(T, r_0, r, t)$, only the customers whose WTP is no lower than that price visit the service. In other words, a customer with service time t will not visit the service if $Vt < c(T, r_0, r, t)$ (or $V < p(T, r_0, r, t)$).
- The capacity (m) is fixed and unchangeable, considering that it is usually determined before launching the service. As we are considering actual services currently operating, we will focus on the pricing strategy, which is more flexible.

Note that the actual prices of both services introduced in Section 1 correspond to our pricing scheme. In particular, the price of the ICN PL is a special case in which $T = r_0 = 0$.

4. Preliminaries

4.1. Constant WTP

We first assume that every customer is willing to pay the same unit cost and that the value is known. Specifically, V is constant and every customer wants to pay the same amount v every day (i.e., $\Pr\{V = v\} = 1$). Some preliminary analyses are made based on such assumptions, and we then extend to a general WTP. The situation can be classified in two cases, considering the price and customers' WTP. The first one represents a case in which the price and customers' WTP partially overlap ($r_0 = vT$ and $r = v$), and the other one includes all the other conditions except the previous one.

The first case demonstrates the situation in which the price for the customers having service periods longer than or equal to T equals their WTP (i.e., $c(T, vT, v, t) = vt$ for $t \geq T$). Thus, only the customers corresponding to the right side of T can offer the price and will visit the service. As v is predetermined, the only decision in this case is determining the value of T considering that the entry fee (r_0) is dependent on the value of T. As the customers with shorter service times compared to T are simply lost, the effective arrival rates for those service times become zero (i.e., $\lambda_t(T, vT, v) = 0$ for $t < T$). Note that every visiting customer pays the same amount every day, which is equal to their WTP v, regardless of the service time. Therefore, the expected revenue per day can be simply calculated as v multiplied by the expected number of busy servers. The expected number of busy servers $EB_m(\rho)$ is expressed as a function of the offered load ρ as Eq. (3) (Medhi, 2002).

$$EB_m(\rho) = \rho\left(1 - p_m(\rho)\right) \tag{3}$$

Lemma 1. $p_m(\rho) \geq max\left\{0, 1 - \frac{m}{\rho}\right\}$

Proof. See Theorem 2 of Sobel (1980). □

Corollary 1. *$p_m(\rho)$ is an increasing function with respect to ρ*

Proof.

$$\frac{\partial}{\partial\rho}p_m(\rho) = \frac{\frac{\rho^{m-1}}{(m-1)!} - \frac{\rho^m}{m!}}{\sum_{k=0}^m \frac{\rho^k}{k!}} + \left(\frac{\frac{\rho^m}{m!}}{\sum_{k=0}^m \frac{\rho^k}{k!}}\right)^2 = \frac{m}{\rho}p_m(\rho) - p_m(\rho) + (p_m(\rho))^2 = p_m(\rho)\left(\frac{m}{\rho} - 1 + p_m(\rho)\right)$$

Since $p_m(\rho) \geq \max\left\{0, 1 - \frac{m}{\rho}\right\}$ from Lemma 1, $\frac{\partial}{\partial\rho}p_m(\rho) \geq 0$. □

Considering the definitions of $EB_m(\rho)$ and ρ, it is natural that $EB_m(\rho)$ is an increasing function with respect to ρ, and the upper bound of $EB_m(\rho)$ is set to the capacity m. This behavior is related to the concept of system saturation, which is intuitive and aligns with common sense. More customers entering the system and requiring service leads to more servers becoming occupied with customers. This provides a conceptual understanding of why the expected number of busy servers increases with the offered load. As a result, such a relationship is generally accepted. Recall that the second term in the right-hand side of Eq. (3) is a decreasing function of ρ, as shown by Corollary 1. However, the first term in the right-hand side of Eq. (3) is an increasing function of ρ, as it is an identity function. Thus, some may wonder if $EB_m(\rho)$ is truly a monotonically increasing function with ρ. To address this issue, we provide two different proofs.

Lemma 2. *$\lambda(1 - p_m(\lambda/\mu))$ (the throughput of the M/G/m/m system) is jointly concave in (λ, μ)*

Proof. See Corollary 1 of Harel (1990). □

Corollary 2. *$EB_m(\rho)$ is concave in ρ*

Proof. Since $\lambda(1 - p_m(\lambda/\mu))$ is jointly concave in (λ, μ) from Lemma 2, $\lambda(1 - p_m(\lambda))$ is concave in λ by fixing $\mu = 1$.

Therefore, $\rho(1 - p_m(\rho))$ is also concave in ρ by replacing λ with ρ. □

Proposition 1. *$EB_m(\rho)$ is an increasing function with respect to ρ*

Proof. $\lim_{\rho\to\infty} p_m(\rho) = 1, \lim_{\rho\to\infty} EB_m(\rho) = m$

$\lim_{\rho\to\infty} \partial EB_m(\rho)/\partial\rho = \lim_{\rho\to\infty}\{1 - p_m(\rho) + \rho(1 - p_m(\rho))p_m(\rho) - mp_m(\rho)\} = 0 + m - m = 0$

Since $EB_m(\rho)$ is concave in ρ from Corollary 2 and $\lim_{\rho\to\infty} \partial EB_m(\rho)/\partial\rho = 0$, $EB_m(\rho)$ is an increasing function with respect to ρ. □

The second proof directly demonstrates the non-negativity of the derivative of $EB_m(\rho)$ and is provided in the online appendix. As υ is a constant value and $EB_m(\rho)$ is an increasing function with respect to ρ, the expected revenue per day is also an increasing function with respect to ρ. Therefore, the bigger the value of ρ, the higher the revenue. However, as Corollary 1 indicates, $p_m(\rho)$ is also an increasing function with respect to ρ. Hence, carelessly increasing the value of ρ may improve the revenue but also worsen customer satisfaction. To prevent such an issue, maximizing the value of ρ until $p_m(\rho)$ reaches a certain threshold can be a rational decision. Recall that the only decision in this case was T. The effective arrival rates change in response to the decision of T, and thus, $\rho(T, \upsilon T, \upsilon)$ also changes. As T increases, more customers are repelled, and thus, the demand decreases. However, such a situation also leads to the increment of the expected service time. Considering that $\rho(T, r_0, r)$ is calculated by multiplying $\lambda(T, r_0, r)$ and $ES(T, r_0, r)$, the two components shift to opposite directions. Note that $\rho(T, r_0, r)$ can be expressed alternatively as Eq. (4).

$$\rho(T, r_0, r) = \lambda(T, r_0, r)ES(T, r_0, r) = \lambda(T, r_0, r)\sum_t tq_t(T, r_0, r) = \lambda(T, r_0, r)\sum_t t\frac{\lambda_t(T, r_0, r)}{\lambda(T, r_0, r)} = \sum_t t\lambda_t(T, r_0, r) \tag{4}$$

Eq. (4) indicates that more and more $\lambda_t(T, \upsilon T, \upsilon)$s from the front will have the value of zero as T increases, and thus, $\rho(T, \upsilon T, \upsilon)$ decreases. Therefore, the optimal strategy is to increase T until $p_m(\rho(T, r_0, r))$ drops below a certain threshold set by the decision maker.

In the other case, the problem becomes more complicated than the former one. Not only did the previous case require only one decision, but calculating both the revenue and BP was simple. However, the effective arrival rates can now have zero values for both the head and the tail. Moreover, the customers no longer pay the same amount every day on average. Therefore, the expected revenue per day should be modified. The expected revenue per day of a single server is first formulated as Eq. (5) to calculate the expected revenue per day.

$$\text{Expected revenue per day of a single server} = \sum_{\tau} p(T, r_0, r, \tau)\Pr\{X = 1\}\Pr\{t = \tau | X = 1\} \tag{5}$$

$\Pr\{X = 1\}$ indicates the probability of a server being busy, while $\Pr\{t = \tau | X = 1\}$ presents the probability of the service time of the customer currently occupying that server to be τ. Both probabilities are calculated following Eqs. (6) and (7).

$$\Pr\{X = 1\} = \frac{\rho(T, r_0, r)(1 - p_m(\rho(T, r_0, r)))}{m} \tag{6}$$

$$\Pr\{t = \tau | X = 1\} = \frac{\tau q_\tau(T, r_0, r)}{\sum_t t q_t(T, r_0, r)} = \frac{\tau q_\tau(T, r_0, r)}{ES(T, r_0, r)} = \frac{\tau \lambda_\tau(T, r_0, r)}{\rho(T, r_0, r)} \tag{7}$$

As all servers are independent and the arrival distribution (Poisson distribution) has the memoryless property, the expected revenue per day for all servers just needs to be multiplied by the number of servers. Therefore, the expected revenue per day can be written as Eq. (8).

$$\begin{aligned} m \times \sum_{\tau} p(T, r_0, r, \tau)\Pr\{X = 1\}\Pr\{t = \tau | X = 1\} &= m \sum_{\tau} p(T, r_0, r, \tau) \frac{\rho(T, r_0, r)(1 - p_m(\rho(T, r_0, r)))}{m} \frac{\tau \lambda_\tau(T, r_0, r)}{\rho(T, r_0, r)} \\ &= (1 - p_m(\rho(T, r_0, r))) \sum_{\tau} c(T, r_0, r, \tau)\lambda_\tau(T, r_0, r) \end{aligned} \tag{8}$$

The parameters corresponding to the first case were implemented to validate Eq. (8). By applying $r = v$ and $r_0 = vT$, the expected revenue per day for the first case can be calculated as Eq. (9), equal to that previously achieved. For convenience, we will replace $1 - p_m(\rho(T, vT, v))$ with $\bar{p}_m$.

$$\begin{aligned} (1 - p_m(\rho(T, vT, v))) \sum_{\tau} c(T, vT, v, \tau)\lambda_\tau(T, vT, v) &= \bar{p}_m \sum_{\tau} (vT + v(\tau - T)^+)\lambda_\tau(T, vT, v) = v\bar{p}_m \sum_{\tau} \max\{\tau, T\} \times \lambda_\tau(T, vT, v) \\ &= v\bar{p}_m \left(\sum_{\tau < T} T\lambda_\tau(T, vT, v) + \sum_{\tau \geq T} \tau\lambda_\tau(T, vT, v) \right) = v\bar{p}_m \sum_{\tau \geq T} \tau\lambda_\tau(T, vT, v) = v\bar{p}_m \sum_{\tau} \tau\lambda_\tau(T, vT, v) = v\bar{p}_m \sum_{\tau} \tau q_\tau(T, vT, v)\lambda(T, vT, v) \\ &= v\bar{p}_m ES(T, vT, v)\lambda(T, vT, v) = v(1 - p_m(\rho(T, vT, v)))\rho(T, vT, v) = vEB_m(\rho(T, vT, v)) \end{aligned} \tag{9}$$

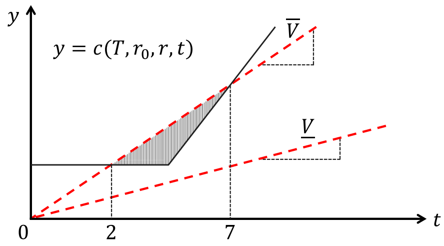
Note that Eq. (9) is driven based on the fact that $\lambda_\tau(T, vT, v) = 0$ for $\tau < T$. Eq. (8) explicitly contains the respective effective arrival rates. Therefore, affecting the arrival rates not only changes the value of $\rho(T, r_0, r)$, which was the only interest in the first case, but also directly affects the revenue. Thus, investigating the condition for maximizing the revenue cannot be analyzed simply nor explicitly, as it was in the previous case. This difficulty is discussed again in Sections 4.2 and 5.

4.2. Randomly distributed WTP

Assuming the customers' WTP to be identical is rather impractical. Therefore, we now relax such an assumption. Customers' WTP forms a probability distribution with PDF f and CDF F. Then, the total WTP of a customer equals the unit price multiplied by the period, as previously regarded. As the customers' WTP got more complicated than previously, the effective arrival rates are also obtained in a more complicated manner. Particularly, not only can the head and tail of the arrival rates be trimmed off, but the values themselves can also change besides being zero. Fig. 3 shows an example assuming a uniform distribution. The red dotted lines indicate the range of customers' total WTP.

$$\lambda_t(T, r_0, r) = \Lambda_t \Pr\left\{Vt \geq c(T, r_0, r, t)\right\} = \Lambda_t \left(1 - F(p(T, r_0, r, t))\right) \tag{10}$$

The effective arrival rates are calculated following Eq. (10), and the expected revenue per day can be obtained equally as Eq. (8) by utilizing those effective arrival rates. Therefore, the expected revenue can be easily computed if Λ_ts and F are given and T, r_0, r are fixed. However, an exact analysis of the revenue maximizing strategy is unavailable, as theoretical analyses and general considerations are limited, given the problem situation. Consequently, our problem is numerically investigated through computational experiments in Section 5.



t	Λ_t
1	Λ_1
2	Λ_2
3	Λ_3
4	Λ_4
5	Λ_5
6	Λ_6
7	Λ_7
⋮	
$\bar{t}$	$\Lambda_{\bar{t}}$

➡

t	$\lambda_t(T, r_0, r)$
1	0
2	0
3	$\lambda_3(T, r_0, r)$
4	$\lambda_4(T, r_0, r)$
5	$\lambda_5(T, r_0, r)$
6	$\lambda_6(T, r_0, r)$
7	0
⋮	
$\bar{t}$	0

Fig. 3. An example of determining the effective arrival rates. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

5. Computational experiments

5.1. Setting of the experiments

For clarity, the expected revenue per day is hereafter referred to as revenue. The PL and CSS are implemented for the computational experiments. Particularly, the actual data of ICN has been utilized. The arrival rates for both services are reported in the Appendix A (Tables A.13 and A.14). The original value of ρ for each service is 701,481 and 106,957, respectively. The capacity of the PL is set to be the actual value (12,560). On the other hand, the capacity of the CSS is assumed to be sufficient (100,000) regarding the demand, compared to the PL which represents a congested situation.

The BP basically has a value between zero and one, as it is a probability. However, it consumes significant computation, as the formula contains both exponential functions and factorials. To overcome such an issue, we implemented a well-known recursive algorithm to calculate the BPs efficiently (Shortle et al., 2018). For a given value of ρ and m, the BP $p_m(\rho)$ can be derived by recursively applying Eq. (11) with an initialization of $p_0(\rho) = 1$.

$$p_n(\rho) = \frac{\rho p_{n-1}(\rho)}{n + \rho p_{n-1}(\rho)} \tag{11}$$

As previously mentioned, the optimal pricing strategy is numerically examined. Consequently, the parameters T, r_0, and r are explored through a grid search. The entry fee r_0 is changed between 0 and 40, only taking an integer value while the daily additional fee r takes the value from 0.0 to 4.0, increasing by 0.1. Consequently, r can also be mapped to the integer values from 0 to 40. Unlike these two parameters, the range of T differs for both services. Basically, T takes only integer values. For the PL, the range consists of the numbers between 0 and 50 with additional periods of 60, 90, 120, 150, 180, 270, and 365. On the other hand, the CSS is composed of the values within 0 and 40, with 60 and 90 attached. The two ranges differ because of the difference between the characteristics of the corresponding services. The CSS targets customers traveling in winter, whereas the potential PL users are irrelevant to the season. The actual CSSs also serve only during December, January, and February. Therefore, the maximum service time of the CSS is set to be 90 days, while the PL reaches up to a year.

Various distributions were introduced to describe the customers' WTP, as explored in Section 2. Among a wide range of options, we implement uniform distribution, which is the most common and normal distribution. Despite the type of distribution being assumed, the corresponding parameters are uncertain. For example, the lower and upper bounds of the uniform distribution are not determined, although we assume such distribution. Instead, numerous pairs of parameters are considered, with each pair possessing its own probability of realization. The normal distribution is treated the same way, as it also requires two parameters, the mean and the standard deviation.

The realization probability of a parameter pair is calculated by considering each parameter as a random variable. Specifically, the parameters are drawn from two different distributions, which are a uniform distribution and a truncated normal distribution. In the case of assuming uniformly distributed WTP, the two parameters are drawn together from $U[0.0, 4.0]$ or truncated $N(2, (2/3.3)^2)$, resulting in two different scenarios. For instance, the first scenario represents the situation in which $V \sim U[\underline{V}, \bar{V}]$, while $\underline{V} \sim U[0.0, 4.0]$ and $\bar{V} \sim U[0.0, 4.0]$. On the other hand, assuming normally distributed WTP, the mean follows $U[0.0, 4.0]$ or truncated $N(2, (2/3.3)^2)$, while the standard deviation is drawn independently from $U[0.1, 4.0]$ or truncated $N(2, (2/3.3)^2)$, resulting in four scenarios. In other words, the two parameters of a uniform distribution follow the same distribution, whereas the mean and the standard deviation of a normal distribution may not. Note that the parameters are treated as discrete random variables despite the origin distribution being continuous. Specifically, all four parameters can have the same values with r, which increases by 0.1 from 0.0 to 4.0, except for the standard deviation of the normal distribution, which starts from 0.1, to prevent having a value of zero. Thus, the average revenue and BP can be calculated if a distribution and the corresponding parameters are given.

Table 5
Results of the best estimations.

Distribution	Parameters	$1-F(r)$	MSE	MAE	Revenue	BP (%)
Uniform :$(r,\underline{V},\bar{V})$	(2.5,0.0,3.4) (2.6,0.1,3.5) ⋮ (3.1,0.6,4.0)	0.2647	8,838.50	47.78	31,399.82 32,655.81 ⋮ 38,935.78	93.24
Normal :$(r,\tilde{V},\ddot{V})$	(2.2,0.0,3.5) (2.3,0.1,3.5) ⋮ (4.0,1.8,3.5)	0.2648	8,837.96	47.77	27,631.84 28,887.83 ⋮ 50,239.71	93.24

Considering that r is between 0.0 and 4.0, $U[0.0,4.0]$ and $N(2,(2/3.3)^2)$ were assumed so that both distributions have the mean of 2.0, and the extracted parameters generally lie within the interval [0.0,4.0]. However, as for a random variable $X \sim N(2,(2/3.3)^2)$, $\Pr\{0.0 \leq X \leq 4.0\} = \Pr\{-3.3 \leq Z \leq 3.3\} = 0.9990$ where $Z \sim N(0,1)$. Therefore, there exists a low probability of being out of the range [0.0,4.0], and, thus, we consider a truncated normal distribution. As previously mentioned, the parameters are regarded discretely. Extracting from a uniform distribution presents no problem for such discreteness, whereas a normal distribution necessitates additional assumptions. The two bounds of a uniform distribution and the mean of a normal distribution can have one of the values among 41, from 0.0 to 4.0. Consequently, we assume a truncated $N(2,(2/3.3)^2)$, which lies within the interval (−0.05,4.05) for the three parameters so that the probability of a parameter X having the of value of x, which is one of the 41 values, is calculated as $\Pr\{x-0.05 \leq X \leq x+0.05\}$. Indeed, when assuming $V \sim U[\underline{V},\bar{V}]$, the smaller and bigger (or maybe the same) values are regarded as the lower and upper bounds, respectively. In the case of the standard deviation of a normal distribution, it takes a value from 40 candidates, from 0.1 to 4.0. Accordingly, a truncated $N(2,(2/3.3)^2)$ lying within the interval (0.0,4.0) is assumed so that the probability of $\ddot{V}$ having the of value of x, which is one of the 40 values, is calculated as $\Pr\left\{x-0.1 \leq \ddot{V} \leq x\right\}$. The aforementioned six scenarios are examined based on the realization probability of each parameter pair, considering the extraction probabilities of respective parameters. Despite a normal distribution being implemented, we can additionally consider diverse unimodal distributions, including skewed ones, as $\tilde{V}$ has a value in the range of [0.0,4.0], and r shares the same range. For example, the skewness increases as $\tilde{V}$ deviates from 2.0.

5.2. Estimation of customers' WTP for the PL

The PL has data on both the effective arrival rates under the actual current prices and the potential arrival rates (Tables A.13 and A.15). At the outset, the customers' WTP for the PL is estimated based on these arrival rates. In detail, the best fitting distribution is evaluated. Unfortunately, the estimation of WTP for CSS is omitted due to the absence of data for the effective arrival rates. Various methods have been developed to estimate customers' WTP precisely (see Breidert (2007)). However, those methods that require sufficient data are impracticable, as only one pair of data is available. Consequently, customers' WTP is estimated based on measuring both the mean absolute error (MAE) and the mean squared error (MSE), which is a simple, basic, and classical approach.

As noted in Section 1.1, the price of ICN PL is quite simple: $r_0 = T = 0$ and $r > 0$. In fact, this type of price corresponds to the static single-price explored in the existing literature. However, such types of prices impact the demand naively. Similar to the situation explored in Section 4.1, every visiting customer is charged the same price every day (i.e., $p(0,0,r,t) = r$). Consequently, the effective arrival rates are calculated as $\lambda_t(0,0,r) = \Lambda_t(1-F(r))$. It is apparent that all effective arrival rates have the same ratio of $1-F(r)$ for each corresponding potential arrival rate. Thus, note that one can always find an optimal r that minimizes the MAE or MSE, regardless of the assumed distribution. Certainly, the optimal ratio $1-F(r)$ minimizing the MAE or MSE can be directly derived. As a result, the MAE and MSE are minimized, having the values of 47.33 and 8,836.75 (the corresponding MSE and MAE are 9,964.68 and 47.76, respectively) each when $1-F(r) = 0.2818$ and $1-F(r) = 0.2654$, respectively. Given this, it is plausible to select the ratio that minimizes the MSE (i.e., $1-F(r) = 0.2654$), considering the gaps between the corresponding values. Accordingly, the best fitting uniform and normal distributions are reported in Table 5.

It is evident that several combinations of the parameters result, considering that they share the same value of the cumulative probability. However, although the values of r correspond to the daily parking fee (KRW 9,000), the values of the other parameters are not specifiable, given that the selected candidates are not further discriminable. For example, if (3.0,0.5,3.9) is chosen, we can conclude that $\underline{V} = 1,500$ and $\bar{V} = 11,700$ from the fact that $r = 9,000$, in which such a choice has no evidence. Fig. 4 visualizes the effective arrival rates based on the real data and the two estimations. As both Table 5 and Fig. 4 clearly show, the two estimations are not discriminable. Moreover, we can roughly conclude that both estimations describe the real data fairly well.

The revenue and BP for each case are briefly documented in Table 5. These results indicate the estimated revenue that the PL is currently earning. Apparently, the highest revenue among the selected candidates represents the case with the highest value of r, as all the cases commonly assume $r_0 = T = 0$. However, assuming that each estimated distribution is true, the genuine highest revenues were all achieved by some other pricing strategies. Table 6 reports these revenues and their corresponding BPs, along with a comparison with their original prices.

It is evident that both the revenue and the BP can be improved by adopting a different strategy. Moreover, Table 6 clearly demonstrates several tendencies. Assuming a uniform distribution, the value of r increases accordingly as the distribution shifts rightward. Specifically, as $r = \bar{V} - 0.1$ and $r_0 > rT$ holds, only the customers with $V > r$ will visit. Such a situation causes an

Table 6
Comparison between the original price and the optimal price.

Distribution	Optimal pricing strategy	Optimal revenue	BP (%)	Absolute revenue gain	Relative revenue gain (%)	Reduction amount of BP (%p)
U(0.0,3.4)	(2,7,3.3)	41,592.16	18.20	10,192.34	32.46	75.03
U(0.1,3.5)	(4,14,3.4)	42,847.72	18.20	10,191.90	31.21	75.03
	(2,7,3.3)[a]	41,696.65	64.93	9,040.84	27.69	28.30
U(0.2,3.6)	(1,4,3.5)	44,093.86	12.62	10,182.06	30.03	80.61
	(2,7,3.3)[a]	41,727.02	77.75	7,815.22	23.05	15.49
U(0.3,3.7)	(1,4,3.6)	45,358.82	18.20	10,191.02	28.98	75.03
	(2,7,3.3)[a]	41,741.05	83.70	6,573.25	18.69	9.53
U(0.4,3.8)	(1,4,3.7)	46,612.58	24.21	10,188.79	27.97	69.02
	(2,7,3.3)[a]	41,749.14	87.15	5,325.35	14.62	6.09
U(0.5,3.9)	(2,8,3.8)	47,869.92	18.20	10,190.14	27.04	75.03
	(2,7,3.3)[a]	41,754.40	89.39	4,074.62	10.81	3.85
U(0.6,4.0)	(4,16,3.9)	49,125.47	18.20	10,189.70	26.17	75.03
	(2,7,3.3)[a]	41,758.10	90.96	2,822.32	7.25	2.27
N(0.0, 3.5^2)		57,425.68	72.83	29,793.83	107.82	20.41
N(0.1, 3.5^2)		57,475.31	74.17	28,587.48	98.96	19.07
N(0.2, 3.5^2)		57,525.74	75.42	27,381.91	90.84	17.81
N(0.3, 3.5^2)		57,576.96	76.60	26,177.14	83.37	16.64
N(0.4, 3.5^2)		57,629.00	77.71	24,973.19	76.47	15.53
N(0.5, 3.5^2)		57,681.88	78.75	23,770.07	70.09	14.49
N(0.6, 3.5^2)		57,735.61	79.73	22,567.81	64.17	13.50
N(0.7, 3.5^2)		57,790.21	80.66	21,366.42	58.66	12.58
N(0.8, 3.5^2)		57,845.69	81.53	20,165.91	53.52	11.71
N(0.9, 3.5^2)	(0,40,4.0)	57,902.08	82.34	18,966.31	48.71	10.89
N(1.0, 3.5^2)		57,959.39	83.12	17,767.63	44.21	10.12
N(1.1, 3.5^2)		58,017.65	83.85	16,569.89	39.98	9.39
N(1.2, 3.5^2)		58,076.86	84.53	15,373.11	36.00	8.71
N(1.3, 3.5^2)		58,137.05	85.18	14,177.30	32.25	8.06
N(1.4, 3.5^2)		58,198.24	85.79	12,982.50	28.71	7.44
N(1.5, 3.5^2)		58,260.44	86.37	11,788.71	25.37	6.87
N(1.6, 3.5^2)		58,323.67	86.92	10,595.95	22.20	6.32
N(1.7, 3.5^2)		58,387.96	87.44	9,404.25	19.20	5.80
N(1.8, 3.5^2)		58,453.33	87.93	8,213.62	16.35	5.31

[a] The optimal strategy of the first uniform distribution was additionally considered to propose a robust strategy.

enhancement in revenue as the value of r increases, but compared to the original price, the absolute difference remains almost constant at around 10,190. In other words, the relative difference decreases. On the other hand, for a normal distribution, the optimal strategy is the same regardless of the value of the mean. However, it appears that the values of r_0 and r are restricted to the upper limits of the grid, which are 40 and 4.0, respectively. Considering the uniform distributions together, it is expected that the prices would have become increasingly expensive if the upper limits were relaxed, as in the previous case. For a fixed T, the price gets more expensive, and the demand decreases as the value of r_0 increases. In contrast, for a fixed r_0, the price gets cheaper, and the demand increases as the value of T increases. Therefore, the strategy (0,40,4.0) represents the most uncrowded situation among all. Note that only customers with $V > 4.0$ can afford the prices. Consequently, the revenue increases tardily as the value of $\tilde{V}$ increases, whereas the BP grows relatively faster. This is because the proportion of customers with $V > 4.0$ increases as the distribution shifts rightward while $\dot{V}$ remains still but with the price nonetheless fixed. Accordingly, the absolute revenue gain gradually decreases, which is also for the relative revenue gain and the gap between the BPs. All of this lends support to the aforementioned speculation that the prices would have been increasingly expensive without restrictions. In fact, the main difference between a uniform distribution and a normal distribution is clearly demonstrated. A uniform distribution limits a customer's WTP to its upper bound, whereas a normal distribution does not.

Based on the above observations, we can conclude that charging extremely high prices (larger values of r) to reduce the demand and increase the profitability is preferable for the PL, as it suffers from a lack of capacity. In other words, gentrifying the service can be regarded as the optimal strategy. In reality, we can consider a policy that adopts a strategy of (2,7,3.3) or (0,40,4.0) when assuming uniform or normal distribution, respectively, as it is uncertain which one of the several candidates is true. Then, at least 7.25 percent of revenue compared to the current situation, can be additionally earned while the BP can be reduced by at least 2.27 percentage points. The relative revenue gain can be raised to at least 16.35 percent, and the BP can drop by at least 5.31 percentage points if the distribution can be specified. Therefore, the PL is missing the opportunity to gain extra revenue and is suffering from unnecessary blockings. To further complement these analyses and examine the CSS, the average revenue and BP for each pricing strategy considering the realization probabilities are investigated in the following subsection.

5.3. Analysis of average revenue

As previously mentioned in Section 5.1, a total of six scenarios are considered for both services. Table 7 reports the most profitable pricing strategy, on average, and the corresponding average BP for each scenario concerning the PL. The results of Table 7 align

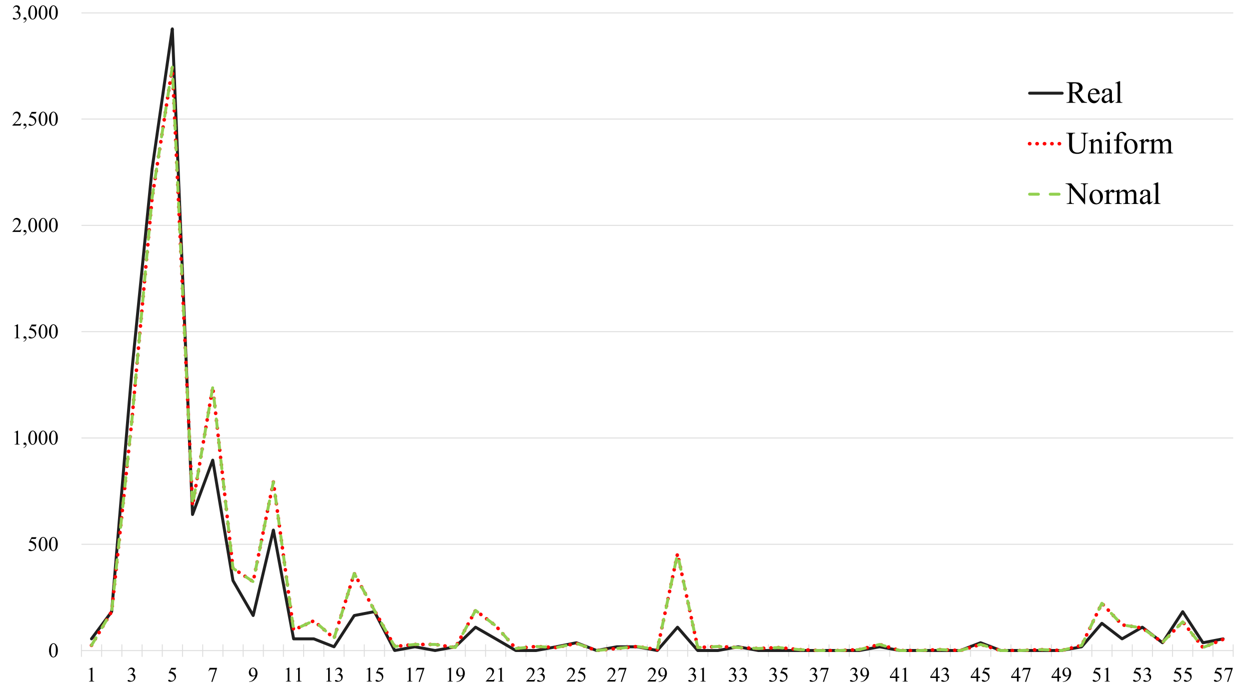


Fig. 4. Effective arrival rates based on the real data and the two estimations.

Table 7
Most profitable pricing strategy for each scenario; PL.

	Scenario			T	r_0	r	Average revenue	Average BP (%)
1	$V \sim U[\underline{V}, \bar{V}]$,	$\underline{V} \sim U[0.0, 4.0]$,	$\bar{V} \sim U[0.0, 4.0]$	3	4	3.1	22,888.54	65.59
2		$\underline{V} \sim N[2, (2/3.3)^2]$,	$\bar{V} \sim N[2, (2/3.3)^2]$	2	2	2.5	22,737.54	74.46
3	$V \sim N[\tilde{V}, \ddot{V}]$,	$\tilde{V} \sim U[0.0, 4.0]$,	$\ddot{V} \sim U[0.1, 4.0]$	0	19	4.0	42,636.01	58.70
4		$\tilde{V} \sim U[0.0, 4.0]$,	$\ddot{V} \sim N[2, (2/3.3)^2]$	0	14	4.0	49,405.10	66.50
5		$\tilde{V} \sim N[2, (2/3.3)^2]$,	$\ddot{V} \sim U[0.1, 4.0]$	0	18	4.0	42,520.80	58.07
6		$\tilde{V} \sim N[2, (2/3.3)^2]$,	$\ddot{V} \sim N[2, (2/3.3)^2]$	0	12	4.0	51,620.95	62.67

with the tendencies observed for the optimal strategy for each candidate discussed in Section 5.2. When customers' WTP follows a uniform distribution, an upper bound $\bar{V}$ is clearly defined, thus preventing the excessive increase of r. On the other hand, as a normal distribution does not limit customers' WTP, some degree of demand remains, even for expensive prices. Consequently, the value of r remains at 4.0 regardless of the distributions of $\tilde{V}$ and $\ddot{V}$.

Considering the top 100 pricing strategies (out of a total of $58 \times 41 \times 41$) with the highest average revenue of Scenario 1, Fig. 5 illustrates the frequencies of each parameter value appearing (the heatmap), the relationship between r_0 and rT (the scatter plot), and the box plot summarizing the corresponding average BPs. Although r ranges between 0.0 and 4.0, it can be mapped to the integer values between 0 and 40 and, thus, can be placed on the same grid with r_0. In other words, the ith number for r indicates $r = i/10$. Note that the dotted line of the scatter plot refers to $r_0 = rT$.

In the first scenario, T and r_0 have values from a wide range, whereas the values of r are concentrated around 3.2. Additionally, it is evident that r_0 is smaller than rT in all 100 cases. Such a relationship indicates an intention to embrace customers with $V < r$. Unlike in Section 5.2, the exact distribution (i.e., the values of the parameters) of customers' WTP is unidentified. Therefore, excessively increasing the value of r could result in failing to secure adequate demand. To prevent such a situation, r is set robustly, and moreover, $r_0 < rT$ to retain more customers. When $t \leq T$, $p(T, r_0, r, t) = r_0/t$, and, thus, the daily price decreases as service time t increases and is minimized at $t = T$ with r_0/T. On the other hand, when $t \geq T$ ($t = T$ can be included in both cases as $r(t-T)^+ = 0$), $p(T, r_0, r, t) = (r_0 + r(t-T)^+)/t = r + (r_0 - rT)/t$. Thus, as $r_0 < rT$ holds, $p(T, r_0, r, t)$ increases with t, starting from $p(T, r_0, r, T) = r_0/T$. Consequently, such pricing strategies can be regarded as advantageous for short-term customers and disadvantageous for long-term customers. The average BPs are arranged between 54 percent and 70 percent, which are relatively higher than those in Table 6. This result also indicates that the prices are set robustly.

Scenario 2 shows some resemblances to Scenario 1 (Fig. 6). Again, T and r_0 possess a wide range of values, but the values of r cluster around 2.5. In addition, r_0 is smaller than rT in all cases, with average BPs ranging between 67 percent and 85 percent.

Table 8
Expected values of $\Pr\{V \geq v\}$ for each scenario (%); PL.

Scenario	3.1	3.2	3.3	3.4	3.5
1	23.10	18.52	14.43	10.85	7.76
2	1.54	0.93	0.54	0.31	0.16
Scenario	4.0	5.0	6.0	7.0	8.0
3	18.75	10.58	6.03	3.37	1.83
4	19.69	10.33	4.96	2.22	0.94
5	15.91	9.15	5.18	2.86	1.52
6	16.76	8.24	3.75	1.60	0.65

Thus, it is apparent that the most significant differences are the values of r and the average BPs. Encompassing more customers due to Scenario 2 demonstrates relatively smaller values of r compared to Scenario 1, naturally resulting in higher BP. The rationale for charging lower prices in Scenario 2 lies in the distribution of the parameters. As the two bounds in the first scenario follow a uniform distribution, having a high or low upper bound (and similarly for the lower bound) is equally probable. However, as the parameters are now extracted from a normal distribution in the second scenario, it is more likely for both of them to have values near 2.0. Therefore, Scenario 2 has relatively fewer customers who can afford higher prices, necessitating a reduction in prices. Additionally, Scenario 2 is more likely to have a narrower gap between the two bounds, leading to higher BP.

To explicitly verify such results, the expected values of $\Pr\{V \geq v\}$ for each scenario are calculated and reported in Table 8. Considering the values of r in Table 7, the inputs from $v = 3.1$ to $v = 3.5$ were applied for Scenarios 1 and 2. Similarly, v having the values of 4.0, 5.0, 6.0, 7.0, and 8.0 was the setup considered for Scenarios 3 to 6. As shown in Table 8, Scenario 2 shows the expectation of meeting customers with a higher WTP (i.e., $v \geq 3.1$) less frequently, confirming that a lower price is required.

Scenarios 3 to 6 (Figs. 7–10) exhibit several common characteristics. In all four scenarios, without exception, r takes the value of 4.0 only and T is no bigger than 5. In addition, unlike the previous scenarios, r_0 is bigger than rT in all cases. This indicates that Scenarios 3 to 6 target only customers with $V > 4.0$. Recall that a uniform distribution limits the value of V, whereas a normal distribution does not. Consequently, assuming a uniform distribution involves a robust pricing strategy, which is not necessary for a normal distribution. In the same vein, some demand remains (though the proportion might decrease) regardless of how expensive the prices are, under the assumption of a normal distribution. However, targeting only customers with $V > 4.0$ requires securing adequate demand. As previously mentioned in Section 5.2, there are two ways to control the demand. Combining both of them, deviating from a given pricing strategy by simultaneously increasing or decreasing the values of T and r_0 preserves the pricing structure while allowing fine control of demand. As T is no bigger than 5, we can conclude that simultaneously increasing the values is avoided. Note that increasing (or decreasing) T by one and r_0 by four produces similar effects, assuming $r = 4.0$, though the effects are not exactly identical. For example, consider $c(1, 17, 4.0, t) = 17 + 4(t-1)^+$ and $c(2, 21, 4.0, t) = 21 + 4(t-2)^+$. Then, $c(1, 17, 4.0, t) = c(2, 21, 4.0, t)$ for $t \geq 2$, but the prices differ for $t = 1$, which are 17 and 21, respectively. Therefore, only the effective arrival rate of customers with $t = 1$ changes. In detail, lowering (raising) the values of T and r_0 increases (decreases) the demand, respectively. As a result, each scatter plot demonstrates the respective parallelogram partially. In other words, the points closest to $r_0 = rT$ satisfy the condition $r_0 = 13 + rT$ in Scenario 3. Moreover, Table A.13 clearly shows that the potential arrival rate with $t = 5$ is the highest. Thus, maintaining $T \leq 5$ ensures minimum disruption to that demand.

Assuming a uniform distribution (i.e., in Scenarios 1 and 2), $r_0 < rT$ holds, which represents benefiting short-term customers while disadvantaging long-term customers. In contrast, Scenarios 3 to 6 exhibit the relationship $r_0 > rT$, benefiting long-term customers. Despite $p(T, r_0, r, t) = r_0/t$ for $t \leq T$ decreasing as t gets bigger, the minimum daily price r_0/T (when $t = T$) is already bigger than r, given that $r_0 > rT$. On the other hand, when $t \geq T$, $p(T, r_0, r, t) = (r_0 + r(t-T)^+)/t = r + (r_0 - rT)/t$ decreases with increasing t, having the maximum value of r_0/T when $t = T$. In other words, the daily price for $t > T$ is always smaller compared to $t \leq T$, resulting in a monotonically decreasing $p(T, r_0, r, t)$. Moreover, considering that the values of r_0/t ($> rT/t$) are too excessive for relatively small values of t, such as $t = 1$ or $t = 2$, a monotonically decreasing $p(T, r_0, r, t)$ indicates a strategy disadvantageous for short-term customers but beneficial for long-term customers. Consequently, a focus on long-term customers is added to the gentrifying strategies.

It is clear that Scenarios 1 and 2 focus on short-term customers, while Scenarios 3 to 6 mainly target long-term customers. When assuming a uniform distribution, the prices had to be set robustly to effectively respond to the uncertain customers' WTP. The potential demand for the PL can be summarized as customers with short travel periods outnumbering those with long travel periods. Therefore, along with the robust prices, offering relative benefits to short-term customers to secure adequate demand can be regarded as the optimal strategy. On the contrary, assuming a normal distribution ensures some demand, even for high prices. Thus, in general, only customers with $V > 4.0$ are targeted. To further gentrify the service, only those capable of paying even higher prices are focused on among short-term customers, considering that the potential demand for short service time is sufficient. These differences result in both strategies being based on gentrification but with differing in specifics.

Compared to Scenario 3, Scenario 4 shows a significant reduction in the values of r_0 and a clearer shape of the parallelogram in the scatter plot. The difference between the two scenarios is that the standard deviation $\ddot{V}$ is now drawn from a normal distribution and, thus, is likely to have a value near 2.0. However, the mean $\tilde{V}$ still follows a uniform distribution, implying a high probability of having a relatively small value. Consequently, fewer customers are willing to afford expensive prices compared to Scenario 3. Note that the value of r is restricted to an upper limit of 4.0, and that there are many more customers with a high WTP than in Scenarios

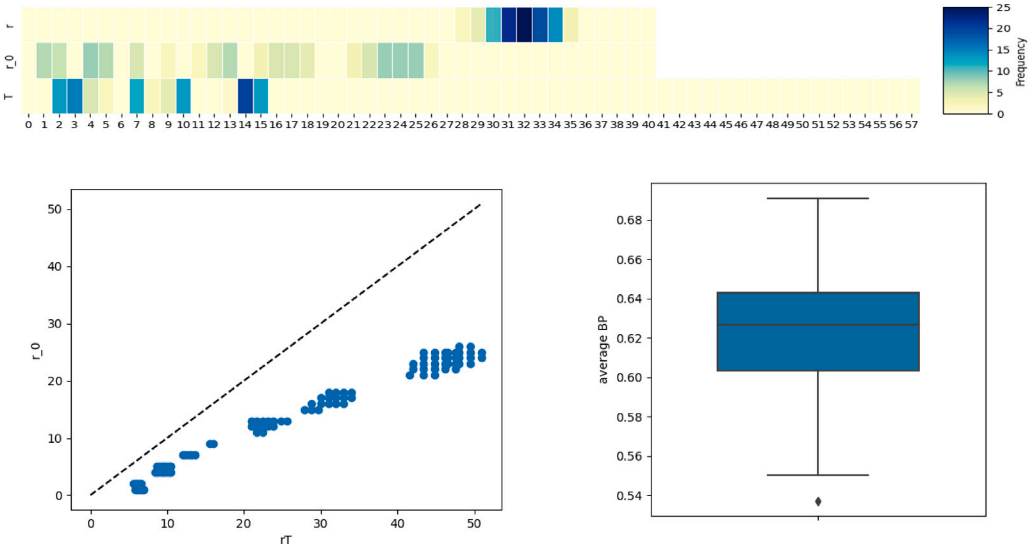


Fig. 5. Results of Scenario 1; PL.

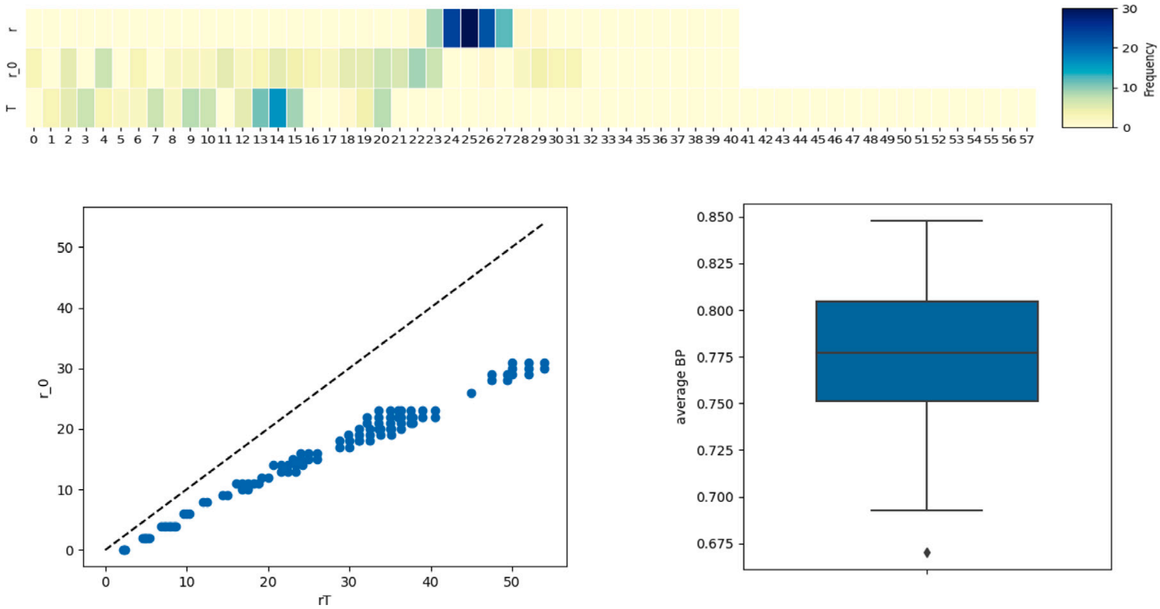


Fig. 6. Results of Scenario 2; PL.

1 or 2. Accordingly, the price is basically quite expensive and is only relatively cheaper compared to Scenario 3. Table 8 confirms this result, as the expected proportion of customers with a high WTP differs for each scenario. Specifically, the expected value of $\Pr\{V \geq v\}$ declines steeply in Scenario 4 and eventually drops below the values of Scenario 3. Therefore, relatively lower prices were adopted to respond robustly, resulting in an average BP range of approximately 62 percent to 70 percent, which is higher than in Scenario 3, where it is 55 percent to 61 percent.

Comparing the fifth scenario to the third rather than to the fourth is plausible because Scenarios 4 and 5 do not share any parameters. Scenario 5 demonstrates similar results to Scenario 3, except that the values of r_0 are slightly smaller. Extracting $\ddot{V}$ from a uniform distribution results in highly fluctuating situations regardless of the distribution of $\tilde{V}$. Given the high probability of $\tilde{V}$ taking the value around 2.0, the expected proportion of customers with a high WTP is fairly high even without excessively large values of $\ddot{V}$. Table 8 clearly supports this result, as the gap between Scenarios 3 and 5 is smaller than the gap between Scenarios 3 and 4 for $v \geq 6.0$, justifying high prices. As a result, the average BPs also lie on a similar range for Scenario 3.

For the last of the PL, Scenario 6 resembles Scenario 4. Similar to the significant (slight) reduction in the values of r_0 from Scenario 3 to Scenario 4 (Scenario 5), respectively, Scenario 6 shows a significant reduction compared to Scenario 5 and a slight

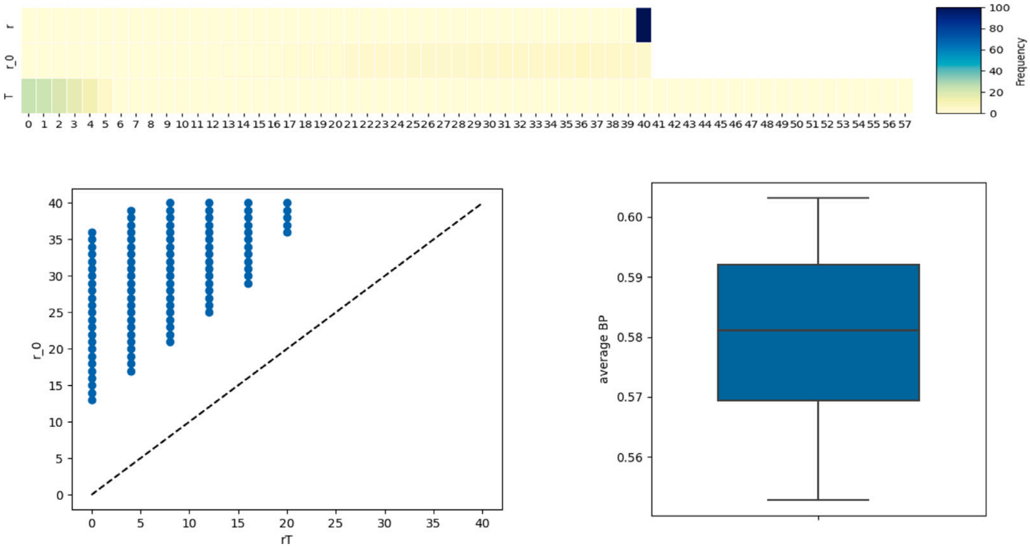


Fig. 7. Results of Scenario 3; PL.

Table 9
Most profitable pricing strategy for each scenario; CSS.

Scenario	T	r_0	r	Average revenue	Average BP (%)
1	2	4	1.7	105,532.73	2.02
2	1	2	1.5	129,160.37	3.72
3	0	0	2.5	110,156.10	0.23
4	0	0	2.5	111,085.05	0.01
5	0	0	2.0	106,592.40	0.17
6	0	0	2.3	108,187.59	0.00

reduction compared to Scenario 4. Consequently, the prices are influenced more by the distribution of $\ddot{V}$ rather than by that of the $\tilde{V}$. Particularly, $\ddot{V}$ following a normal distribution results in a tendency to set the values of r_0 robustly, which aligns with the result of Table 8. As the values of r_0 in Scenario 6 are smaller than in Scenario 4, the average BPs also construct a higher range than in Scenario 4.

Recall that each scatter plot partially illustrates a respective parallelogram. Moreover, a parallelogram can be represented as a bundle of points corresponding to a range of r_0. For example, the points of Scenario 6 are located between $r_0 = 7+rT$ and $r_0 = 24+rT$, in which the range of r_0 can be regarded as [7,24]. Similarly, Scenarios 3, 4, and 5 correspond to the ranges of [13,36], [8,26], and [12, 33], respectively. Apparently, all four scenarios share the range [13,24]. Therefore, a pricing strategy with $r = 4.0$, $T \leq 5$, and $13 + rT \leq r_0 \leq 24 + rT$ can be regarded as profitable and also robust at the same time, assuming that customers' WTP forms a normal distribution.

In conclusion, Table 7 and Figs. 5 to 10 indicate the proportion of customers with a high WTP for each scenario, thus visualizing the results of Table 8. Scenarios 1, 3, and 5 are more likely to encounter customers with a high WTP, while the other scenarios have relatively lower chances. As a result, the optimal strategy for the PL, representing a congested situation in which the capacity is insufficient regarding the demand, is gentrifying the service and focusing on the appropriate response to customers with a high WTP. In other words, the shrewd strategy would be to focus on those who can afford the expensive prices. Concentrating only on the most profitable customers still leaves sufficient demand regarding the capacity and, thus, triggers such a strategy.

Table 9 reports the most profitable pricing strategies for the CSS and their corresponding average BPs across the six scenarios. The results clearly contrast with those observed for the PL. First, the values of r for the CSS are notably lower than those of the PL. When the demand was excessive regarding the capacity, the congested situation represented by the PL, focusing only on the most profitable customers was the most attractive strategy. However, in the situation in which the capacity is sufficient, given an example of the CSS, charging a moderate price to secure enough customers is essential. Nevertheless, the average BPs remain quite low. Such consequence is caused by the gap in the actual loads. The CSS has a smaller offered load with a relatively ample capacity. Accordingly, the actual load of the CSS is substantially less, which results in low levels of BPs.

Direct comparisons between the average revenues of the six scenarios were avoided for the PL due to the obvious influence of the upper bound of a uniform distribution on revenues. Such an impact is particularly significant for the PL, in which the optimal strategy is based on gentrification and focuses on customers with a high WTP. However, Table 9 demonstrates that the difference between assuming a uniform or normal distribution is minimal for the CSS. This is because the CSS adopts a moderate pricing strategy, attracting a similar proportion of customers regardless of the distribution of their WTP. Notably, Scenario 2 exhibits relatively

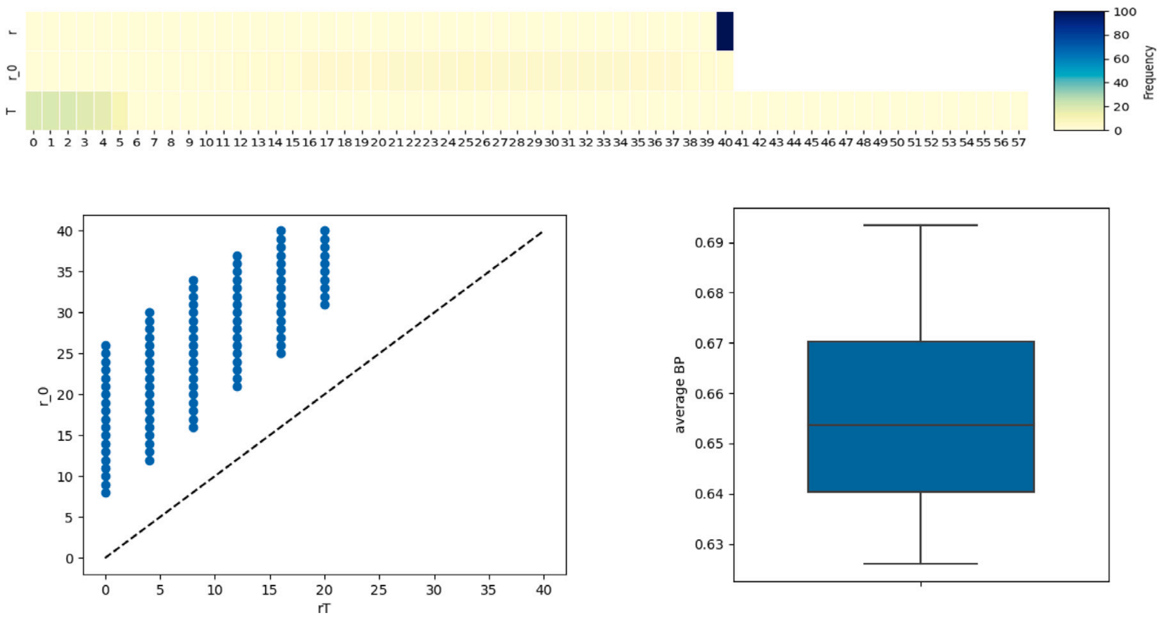


Fig. 8. Results of Scenario 4; PL.

higher average revenue compared to the other scenarios. Again, this is due to the fact that the CSS targets regular customers, unlike the PL. One possibility of having a high average revenue is concentrating on increasing the profitability of the favorable situations while abandoning the opposite ones. In Scenario 2, obtaining the two bounds of the uniform distribution having values close to 2.0 is likely. Thus, setting $r = 1.5$ substantially increases the probability of securing sufficient demand. In other words, the realization probability of a parameter pair resulting in poor revenue is relatively lower when compared to the other scenarios. Indeed, such a situation does not apply to all scenarios. For instance, in Scenario 1, both bounds follow a uniform distribution, and, thus, the realization probabilities of favorable and unfavorable situations are balanced. Consequently, r must be set lower than in Scenario 2 to robustly capture a sufficient number of customers. However, it is not a preferable decision from an average perspective, as it causes unnecessary losses in favorable situations. Rather, focusing on and prioritizing favorable cases is more effective in enhancing average revenue. The details will be discussed later.

Similarities with the PL are also noticeable. Scenarios 1 and 2, which assume a uniform distribution, set r lower than do Scenarios 3 to 6, with Scenario 1 having a higher r than Scenario 2. Moreover, T takes a small value for all scenarios, as in the case of the PL. Specifically, $T = 0$ holds for Scenarios 3 to 6 for both services. To further investigate the optimal pricing strategy of the CSS, the top 100 pricing strategies with the highest average revenue for each scenario are analyzed. Note that the total number of strategies is reduced to $43 \times 41 \times 41$ considering the range of T.

The distribution of parameters also significantly differs from that of the PL. In Scenario 1, the values of r cluster around 1.8 (Fig. 11). Additionally, the most noticeable difference from the PL is the relationship between r_0 and rT. Unlike in the PL, in which one of them evidently dominated the other one depending on the distribution of the customers' WTP, no such relationship is demonstrated for the CSS. Basically, the preferable strategies of the PL revolve around gentrification and adjusting the specifics based on the distribution of the customers' WTP. In contrast, the CSS, with sufficient capacity, does not require such strategies. Instead, the CSS focuses on maintaining moderate prices while securing ample demand without specifically targeting short-term or long-term customers. Such an observation is also confirmed by the range of T and r_0. Particularly, T possessing values no bigger than 6, while mostly less than 6, suggests a strong focus on securing the demand of $t = 5$, similar to what is true in Scenarios 3 to 6 of the PL. Moreover, unlike the PL, the values of r_0 do not exceed 9, representing the tendency to set lower prices. The average BPs were less than 2.5 percent, which are remarkably lower than those of the PL.

Similar to the PL, Scenario 2 resembles Scenario 1 (Fig. 12). Specifically, no clear hierarchy between r_0 and rT exists. In addition, the ranges of T and r_0 are respectively identical, though r takes lower values to aim for more robust pricing. As previously mentioned, in Scenario 2, slightly lowering the value of r allows for a much more robust response. Table 10 corresponds to Table 8 and reports the expected values of $\Pr\{V \geq v\}$ for each scenario, but with different values of v. Table 10 clearly shows that Scenario 2 has considerably lower chances to meet customers with $V \geq 2$ than does Scenario 1 and, thus, verifies the necessity of robust prices. Consequently, Scenario 2 exhibits average BPs approximately ranging from 1 percent to 5 percent, which is much wider than that exhibited in Scenario 1.

For Scenarios 3 to 6, all the average BPs are below 0.3 percent, and, thus, the corresponding box plots are omitted. In addition, the scatter plots are provided in the Appendix A (Fig. A.14), as they do not provide any additional key information other than the information conveyed by the heatmaps. Fig. 13 clearly illustrates that the values of T are now all less than or equal to 5, unlike in

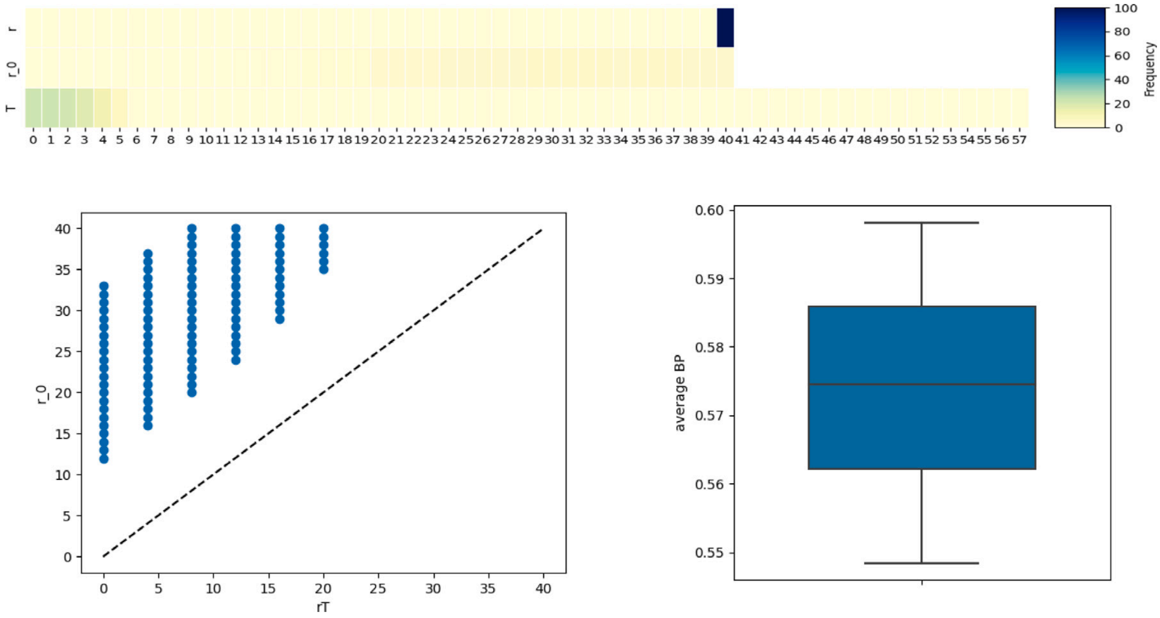


Fig. 9. Results of Scenario 5; PL.

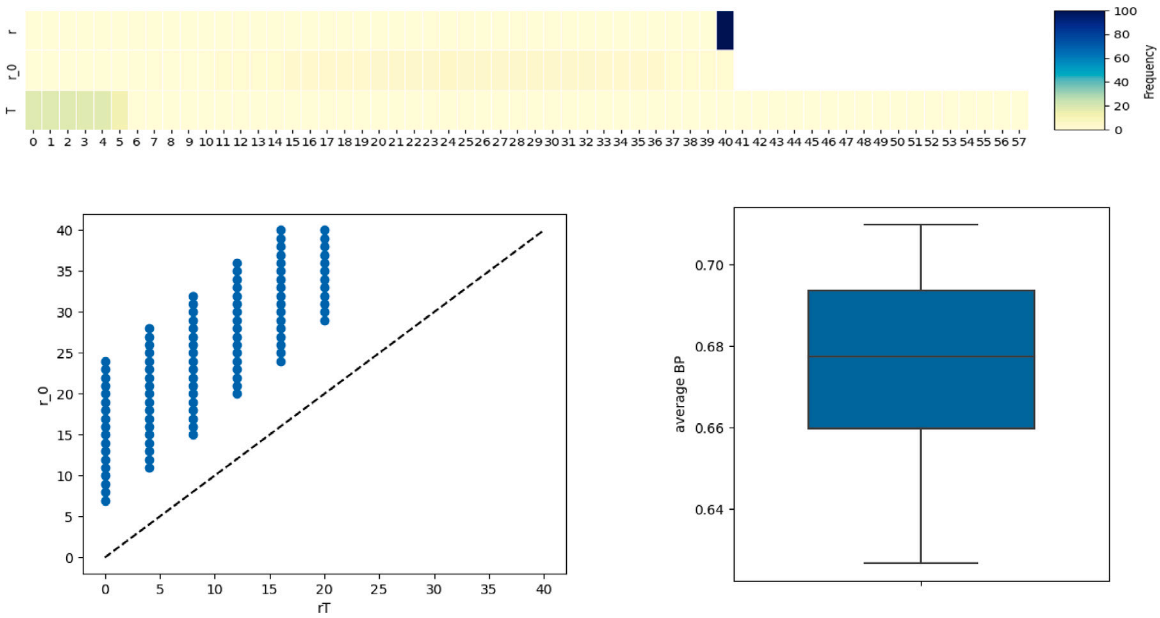


Fig. 10. Results of Scenario 6; PL.

Scenarios 1 and 2. Moreover, the values of r are bigger than in Scenarios 1 and 2, as previously mentioned. Unlike the PL, in which Scenarios 3 and 5 are similar, and in which Scenario 4 differs significantly, the CSS shows similarities between Scenarios 3 and 4, with Scenario 5 differing substantially. Specifically, Scenario 5 features the smallest values of r and relatively lower values of r_0. Considering that the value of r was all fixed to 4.0 in the PL, the values of r_0 indicated the differences between scenarios, which aligns with the results in Table 8. As the CSS can set the value of r more flexibly, the differences between scenarios are demonstrated more clearly through r rather than through r_0. Again, such differences reflect the results of Table 10. Unlike the PL, in which the prices became cheaper in the order of Scenarios 3, 5, 4, and 6, the prices of the CSS are hardly distinguishable for Scenarios 3 and 4, and then the prices become cheaper in the order of Scenarios 6 and 5.

For Scenarios 5 and 6, which assume normally distributed means, it is preferable to set the values of r robustly, similar to the setup in Scenario 2. Given the high probabilities of $\tilde{V}$ taking the values near 2.0, slightly lowering the value of r ensures considerable

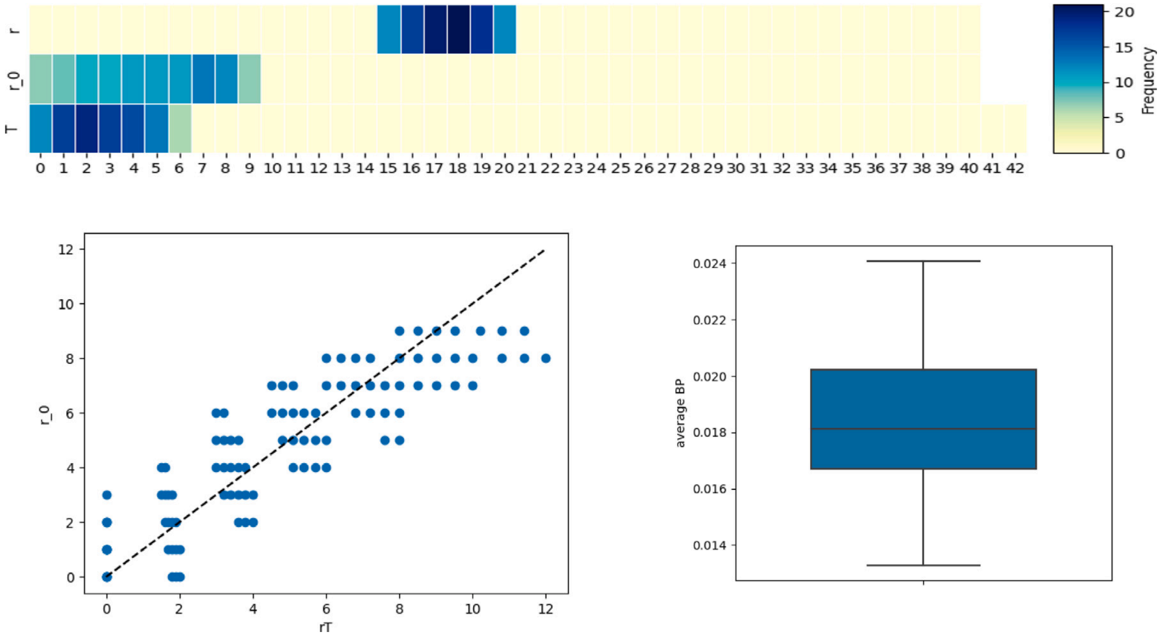


Fig. 11. Results of Scenario 1; CSS.

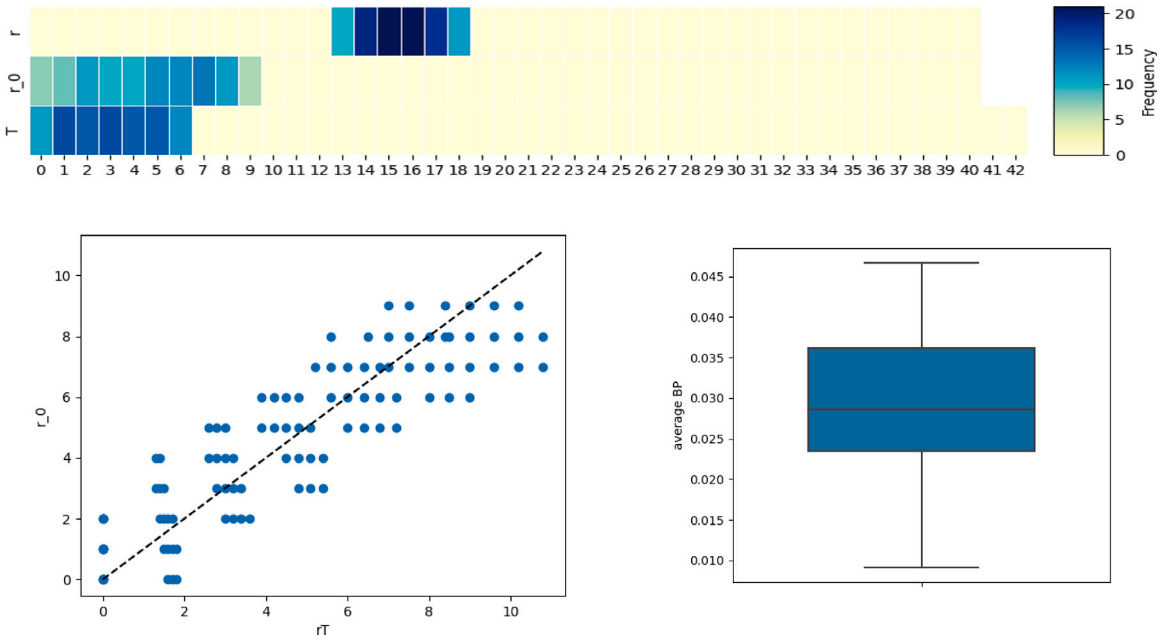


Fig. 12. Results of Scenario 2; CSS.

demand even under adverse situations, as shown by the distinctions between Scenarios 3, 4, and 5, 6. In the meantime, Scenario 5 faces an appreciable chance of having a big standard deviation, as $\ddot{V}$ follows a uniform distribution. Therefore, Scenario 5 requires more robust pricing than does Scenario 6 to secure sufficient demand even under unfavorable conditions. Considering that $\tilde{V}$ follows a uniform distribution in Scenarios 3 and 4, r should be set more robustly than in Scenarios 5 or 6 to ensure enough demand under adverse conditions. However, such a decision is not recommended for the same reason as in Scenario 1. Consequently, as in Scenario 1, prioritizing revenues in favorable situations by sacrificing some robustness in unfavorable ones can raise average revenue.

The aforementioned differences stem from the distinct target customers for the PL and the CSS. In the case of the PL, targeting customers with a high WTP results in focusing on the tail of the distribution of customers' WTP. In contrast, the CSS embraces normal

Table 10
Expected values of $\Pr\{V \geq v\}$ for each scenario (%); CSS.

Scenario	2.0	2.1	2.2	2.3	2.4	2.5
1	50.00	46.60	43.20	39.84	36.50	33.22
2	50.00	41.96	34.25	27.17	20.93	15.65
Scenario	2.5	2.6	2.7	2.8	2.9	3.0
3	41.42	39.73	38.05	36.38	34.74	33.11
4	41.56	39.90	38.26	36.64	35.05	33.48
5	37.93	35.75	33.68	31.72	29.88	28.15
6	40.05	38.13	36.25	34.42	32.63	30.90

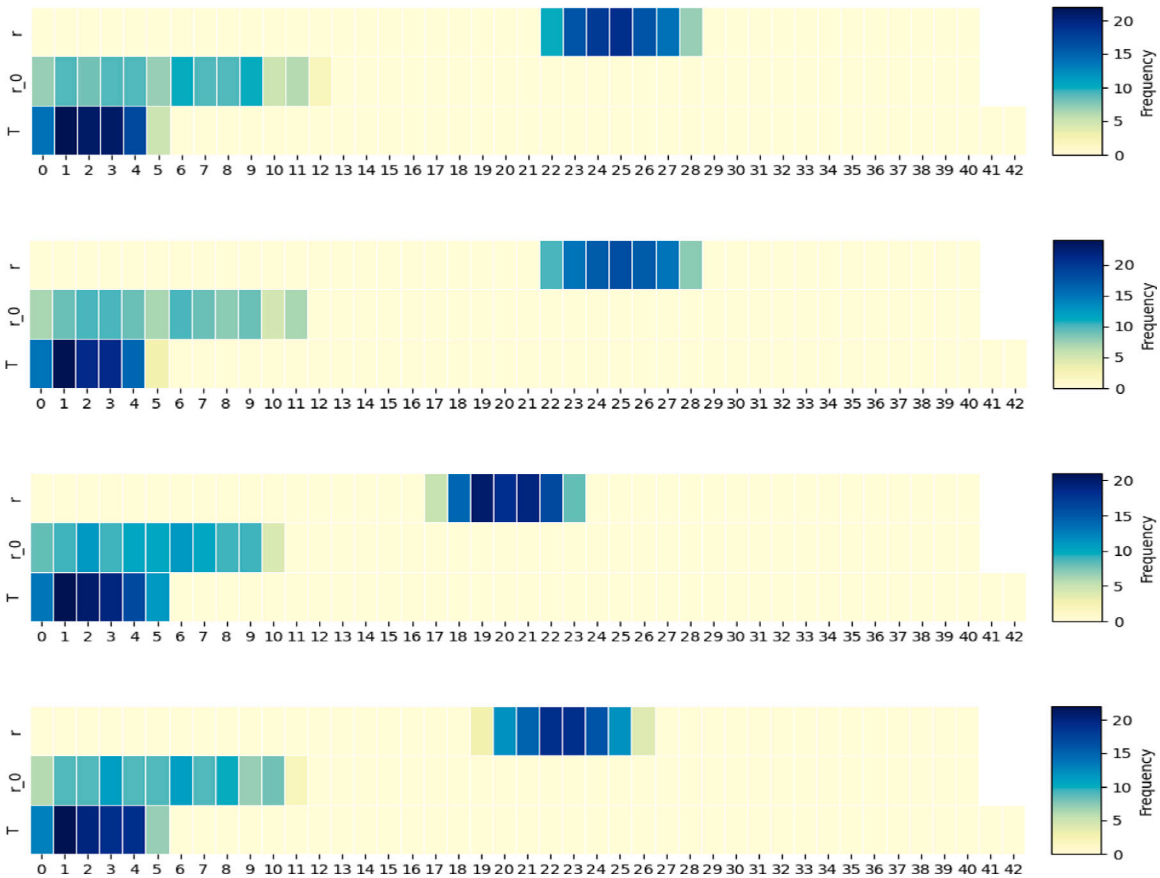


Fig. 13. Heatmaps of Scenarios 3–6; CSS.

Table 11
Most profitable case under the current pricing strategy for each scenario; PL.

Scenario	T	r_0	r	Average revenue	Average BP (%)	Absolute revenue loss	Relative revenue loss (%)	Increment of average BP (%p)
1	0	0	2.3	19,164.64	63.08	3,723.90	16.27	−2.51
2	0	0	1.8	19,319.30	82.84	3,418.24	15.03	8.38
3	0	0	4.0	40,489.17	66.63	2,146.84	5.04	7.93
4	0	0	4.0	47,184.17	75.49	2,220.93	4.50	8.99
5	0	0	4.0	40,502.30	66.18	2,018.50	4.75	8.11
6	0	0	4.0	48,615.37	80.46	3,005.58	5.82	17.79

customers in order to secure sufficient demand and, thus, should focus on the main body of the distribution of customers' WTP. Due to such divergence, Tables 8 and 10 represent different tendencies, though the same scenarios are considered. Accordingly, the robustness of pricing for each scenario depends on the type of service.

Table 12
Most profitable case under the current pricing strategy for each scenario; CSS.

Scenario	Korean Air				Asiana Airlines				LCCs			
	Strategy	Average revenue	Absolute revenue loss	Relative revenue loss (%)	Strategy	Average revenue	Absolute revenue loss	Relative revenue loss (%)	Strategy	Average revenue	Absolute revenue loss	Relative revenue loss (%)
1	(5,0,2.5)	77,335.53	28,197.20	26.72	(5,0,2.0)	76,114.26	29,418.47	27.88	(7,9,2.0)	103,482.03	2,050.70	1.94
2	(5,0,2.5)	79,161.73	49,998.64	38.71	(5,0,2.0)	77,470.51	51,689.86	40.02	(7,9,2.0)	105,861.09	23,299.28	18.04
3	(5,0,3.6)	81,100.24	29,055.86	26.38	(5,0,2.9)	80,403.16	29,752.94	27.01	(7,13,2.9)	107,520.29	2,635.81	2.39
4	(5,0,3.6)	81,914.27	29,170.78	26.26	(5,0,2.9)	80,898.98	30,186.07	27.17	(7,13,2.9)	108,247.07	2,837.98	2.55
5	(5,0,2.8)	78,142.84	28,449.56	26.69	(5,0,2.2)	75,375.03	31,217.37	29.29	(7,10,2.2)	104,087.21	2,505.19	2.35
6	(5,0,2.8)	79,487.23	28,700.36	26.53	(5,0,2.2)	74,360.28	33,827.31	31.27	(7,10,2.2)	104,011.32	4,176.27	3.86

The potential improvements compared to the current pricing strategies of the PL and the CSS are additionally analyzed. The PL is currently adopting a pricing strategy of $T = r_0 = 0$. Among the strategies satisfying the condition $T = r_0 = 0$, the ones generating the highest average revenue for each scenario are documented in Table 11. Table 11 also compares those average revenues to those of Table 7, which are the authentic highest average revenues for each scenario, in a similar manner to what is shown in Table 6. Consequently, the potential losses induced by the current pricing strategies are calculated.

Assuming that the customers' WTP follows a uniform distribution, the PL is currently losing more than 15 percent of its average revenue. Though applying the optimal pricing strategy raises the average BP by 2.51 percentage points in Scenario 1, the result of 65.59 percent is quite tolerable. However, maintaining the current strategy in Scenario 2 results in the PL suffering 82.84 percent BP on average, which could have been reduced by 8.38 percentage points. For Scenarios 3 to 6, the highest average revenues all equally occurred at $(T, r_0, r) = (0, 0, 4.0)$. Considering the impracticality of adopting a strategy of $(T, r_0, r) = (0, 0, 4.0)$, as mentioned in Section 5.2, such a strategy still incurs losses greater than or equal to 4.5 percent. Moreover, the losses will become severe, considering that the value of r is likely to be smaller in reality, referring to Section 5.2. It is noticeable that the prices shown in Table 7 are more expensive than those shown in Table 11. In other words, the values of r for Scenarios 1 and 2 and the values of r_0 for the other scenarios are bigger in Table 7. These results indicate that the current pricing strategy of the PL cannot appropriately gentrify the service and, thus, incurs potential loss and probably unnecessarily more blockings.

The CSS slightly differs depending on the airline. Korean Air and Asiana Airlines, the only two full-service carriers (FSCs) in South Korea, adopt $r_0 = 0$ and $T = 5$, whereas Jeju Air, Eastar Jet, and T'way Air, the low-cost carriers (LCCs) in South Korea, apply $r_0 > 0$ and $T = 7$ (refer to Table 3). Let the setup assume that Asiana Airlines sets $r = \gamma$ (KRW 2,000/day), Korean Air implements $r = 1.25\gamma$ (KRW 2,500/day), and the three LCCs employ $r = \gamma$ (KRW 2,000/day) and $r_0 = 4.5\gamma$ (KRW 9,000). As T, r_0, and r are explored on a grid, the combinations of strategies that at least approximately satisfy the aforementioned conditions can be nominated as follows: ((5,0,1.1), (5,0,0.9), (7,4,0.9)), ((5,0,1.4), (5,0,1.1), (7,5,1.1)), ((5,0,2.5), (5,0,2.0), (7,9,2.0)), ((5,0,2.8), (5,0,2.2), (7,10,2.2)), ((5,0,3.6), (5,0,2.9), (7,13,2.9)), ((5,0,3.9), (5,0,3.1), (7,14,3.1)). Among the six candidates, the combination yielding the biggest sum of three average revenues is reported for each scenario in Table 12. In addition, similar to what is shown in Table 11, Table 12 demonstrates the potential losses incurred by adopting current pricing strategies. Note that all the average BPs are below 5 percent and, thus, omitted.

Similar to the PL, all airlines are facing potential losses. The two FSCs are losing at least 26 percent, and the three LCCs are losing roughly more than 2 percent. Note that these losses are the minimum amounts, given that the best combinations are considered. In particular, Scenario 2 shows the greatest discrepancy due to its notably higher maximum average revenue. The primary cause for the losses expected for the airlines appears to be the value of T. Despite the LCCs setting $T = 7$, which was not even explored before, their estimated potential losses are substantially less than those of the FSCs. The existence of the entry fee (i.e., $r_0 > 0$) offsets the impact caused by the decision for T. Figs. 11, 12, and A.14 clearly illustrate that it is only profitable when the value of r_0 is correspondingly big to the value of T. Conversely, the two FSCs apply $T = 5$ and $r_0 = 0$, which are far from the aforementioned condition. As a result, they are currently experiencing significant potential losses.

In the case of the two actual services at ICN, such estimated potential losses appears to be a result of sacrificing some profit as part of the offering, for the convenience of customers, rather than pursuing only on the profit and the BP. The observation that the two FSCs are enduring much bigger losses than the LCCs also supports the presumption that the actual services are sacrificing some proportion of their profits. LCCs offer relatively cheaper ticket prices, but most extra services are additionally charged. Consequently, it can be seen that the LCCs are generating revenues comparable to the optimum from the CSS. In contrast, as the FSCs provide more convenience services with higher ticket prices, bearing much bigger losses in an extra service is certainly plausible. The fact that both services are not the primary operation of each service provider (i.e., the airport and airlines) but rather are offered to enhance customer satisfaction explains the lack of efforts to maximize profitability.

6. Concluding remarks

In this paper, we studied the optimal pricing strategies for customer service that can be modeled by an M/G/m/m loss system. To maximize the revenue, we focused on changing the service price rather than on deciding the capacity, because the latter is usually determined before launching the service. In addition, we also addressed the concern that there might be a risk of compromising the quality of service while striving to maximize revenue. The BP is considered together to prevent such an issue. We have considered a highly realistic pricing scheme used in practical industry settings. Furthermore, we explicitly accounted for customers' WTP. In particular, we took into account the fact that the demand (i.e., the arrival rate) changes with respect to the price, considering the customers' WTP. Additionally, we conducted case studies on the PL and CSS, currently operated at ICN, based on actual data.

Table A.13
Potential arrival rates of customers for the PL.

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Λ_t	91	695	4,094	8,042	10,364	2,577	4,679	1,462	1,225	2,998	366	530	219	1,371	713	73	110	110	55
t	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
Λ_t	713	439	37	73	55	128	0	37	73	18	1,718	55	73	55	37	55	18	0	0
t	39	40	41	42	43	44	45	46	47	48	49	50	60	90	120	150	180	270	365
Λ_t	18	110	0	0	18	0	110	0	0	18	0	91	841	457	402	146	512	55	201

Establishing an appropriate strategy by comparing the capacity and demand is crucial. While it would have been ideal to accurately predict the demand and determine the optimal capacity from the beginning, improving the current situation for existing services is possible by adjusting the pricing strategy. Regardless of whether customers' WTP is uniformly or normally distributed, the overarching goals remain the same. The BP is a crucial consideration in congested situations represented by the PL. The BP is regarded as a performance measure representing the quality of service, but on the other hand, it also reflects whether an appropriate strategy is being adopted. Intensively targeting only a selected group of customers becomes the superior strategy, as reducing the BP is naturally preferred. Consequently, the optimal strategy is gentrifying the service and focusing on the appropriate response to customers with a high WTP. However, the specifics of whether to benefit short-term or long-term customers depend on the distributions. In contrast, the BP is not a relevant consideration for less congested situations, as exemplified by the CSS. Hence, securing sufficient demand is essential to achieving higher revenues, unlike with the PL. Naturally, the optimal strategy is charging moderate prices without specifically targeting short-term or long-term customers. Indeed, in both situations, additional considerations of finely tuning the robustness of pricing are necessary. Based on the case studies, both services are supposed to have possibilities for improvements. Considering the actual pricing strategies applied for both services, it appears that both services are incurring losses, which is likely due to prioritizing customer convenience at the service level. In addition to potential loss, it is likely that the PL is also suffering from unnecessarily excessive blockings. However, deviating from this exceptional situation, it is necessary to thoroughly inspect whether the optimal strategy has been appropriately applied.

A single service provider has, naturally, been considered, as our research dealt with customer service that can be modeled as an M/G/m/m loss system. However, there are multiple options to access an airport apart from driving. In other words, customers can alternatively choose among various transportation options, such as the subway or bus. Hence, implementing a customer choice model might be considered in future research. On the other hand, in the case of the CSS, ICN does not demonstrate a monopoly. Multiple companies coexist, enabling a game theoretic approach to be applied. Furthermore, optimization or reinforcement learning methodologies can be employed to model the problem, instead of the queueing theory. Altering the pricing scheme or the distribution of customers' WTP is another possibility. In this paper, we assumed the price to be charged as a practical two-step function. Although implementing dynamic pricing, as in the case of airline tickets or accommodations, may be challenging due to the different characteristics of the service, introducing segment-wise pricing inspired by those businesses can be further studied. Regarding the distribution of customers' WTP, not only a uniform or normal distribution but also other types, such as an exponential distribution or even the distributionally robust optimization method, can be considered. We hope that this research can serve as a foundation for these various extensions and enrich the literature on pricing strategies.

CRediT authorship contribution statement

Junseok Park: Writing – review & editing, Writing – original draft, Visualization, Software, Methodology, Investigation, Data curation, Conceptualization. **Ilkyeong Moon:** Writing – review & editing, Validation, Supervision, Conceptualization.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A

See Tables A.13–A.15 and Fig. A.14

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.tre.2024.103821.

Table A.14
Potential arrival rates of customers for the CSS.

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Λ_t	50	119	725	1,600	1,868	696	1,163	318	417	974	109	139	79	308
t	15	16	17	18	19	20	21	22	23	24	25	26	27	28
Λ_t	209	20	30	10	10	209	79	10	20	10	10	0	0	0
t	29	30	31	32	33	34	35	36	37	38	39	40	60	90
Λ_t	10	388	20	30	20	20	10	10	0	0	0	30	238	109

Table A.15
Effective arrival rates of customers for the PL under the actual current prices.

t	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
λ_t	55	183	1,316	2,266	2,925	640	896	329	165	567	55	55	18	165	183	0	18	0	18
t	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38
λ_t	110	55	0	0	18	37	0	18	18	0	110	0	0	18	0	0	0	0	0
t	39	40	41	42	43	44	45	46	47	48	49	50	60	90	120	150	180	270	365
λ_t	0	18	0	0	0	0	37	0	0	0	0	18	128	55	110	37	183	37	55

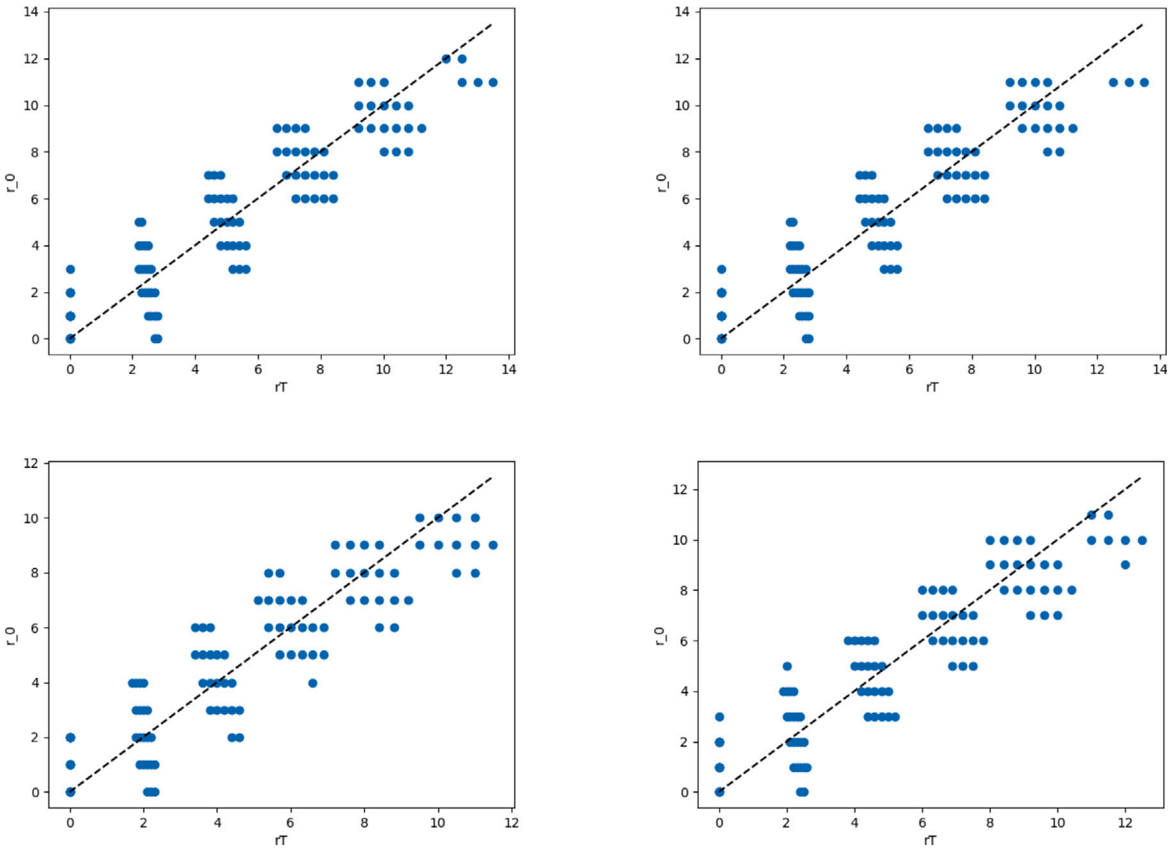

Fig. A.14. Scatter plots of Scenarios 3–6; CSS.

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