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Strategic inventory and pricing decision for substitutable products

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ABSTRACT

This study investigates the effect that strategic inventory (SI) has on a supply chain consisting of a common retailer and two manufacturers in a two-period decision setting. In each period, participating members set their respective prices to maximize their profits. Additionally, the retailer decides how many products need to be upheld as strategic inventory. We derive the equilibrium under the Bertrand competition and compare this with an equilibrium where two competing upstream manufacturers cooperate, and one of the manufacturers offers a long-term wholesale price commitment to the retailer. The aim is to investigate the following question: Is such a deal between two members beneficial for the overall supply chain? Results demonstrate that every member of the supply chain has the opportunity to receive higher profits if the holding cost is in a certain range for which the retailer can maintain strategic inventory for wholesale prices offered by two manufacturers. Cooperation between two upstream manufacturers can lead to a superior outcome in the absence of strategic inventory, which is not always true if the retailer upholds strategic inventory. The retailer may receive higher profits and consumers need to pay lower prices under commitment contract, and the strategic integration decision may worsen the performance. Moreover, the manufacturers sometimes are better off if they remain decentralized or offer a long-term wholesale price commitment. Cross-price elasticity and market share remain key parameters affecting the two manufacturer's strategic decisions.

1. Introduction

In a pragmatic business environment, retailers sell similar types of products, such as footwear from Nike and Adidas, toothpaste from Colgate and Sensodyne, sodas from Coca-Cola and Pepsi, apparel from Levi and Gap, and electronic gadgets from Apple and Samsung, to provide greater variety, increase profits, boost consumer reliance, and remain competitive. Similar types of products are sold through various retail outlets, including department stores, convenience stores, or customer electronics stores (Li & Chen, 2018). For example, Procter & Gamble and Unilever mainly dominate the global market of beauty products, cosmetics, toiletries, and the like, and a joint presence of products from these two manufactures are common in most convenience stores. The demand for substitute products has a negative correlation and creates rivalry among upstream manufacturers while they are trading with a common retailer. Therefore, it is challenging for a retailer that sells both products to set profitable price–quantity pairs while trading with substitute products. This study addresses price-strategic inventory decisions in a supply chain that consists of a common retailer and two competing manufacturers within three decision-making scenarios. It is noted in the existing literature that a retailer's use of strategic inventory (SI) in a multi-period supply chain interaction can reduce the double marginalization effect and potentially benefit all supply chain members. However, the effect of SI remains unexplored in a single-retailer-and-two-competing-manufacturers supply chain setting. In this paper, we consider the following questions:

- If the common retailer maintains SI, how should manufacturers react regarding their respective wholesale pricing decisions? In such a scenario, in other words, should the two upstream manufacturers cooperate or remain non-cooperative? Does the retailer's use of SI benefit the two competing manufacturers?
- Also if the market size of the two competing manufacturers is not uniform, how does cross-price elasticity of customers and the retailer's strategic decision affect participating supply chain members?
- Additionally, if one manufacturer offers a wholesale price commitment to the common retailer, then how does it affect the retailer's pricing and SI decisions, as well as the performance of other manufacturers?

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The literature on SI is comparatively new, and there are a limited number of articles that explore the characteristics of supply chain decisions in the presence of SI. Anand et al. (2008) first, identified the importance of a retailer's strategic decision by analyzing a supply chain model in a two-period setting with a manufacturer and a retailer, to pinpoint the advantages of the retailer's strategic decision to uphold inventory for wholesale price negation. The authors established that the retailer's adoption of SI can mitigate the effects of double marginalization. Arya et al. (2014) extended the work of Anand et al. (2008) to explore the effect of SI on centralized/decentralized procurement decisions in a supply chain consisting of a manufacturer and multiple retailers. Mantin and Jiang (2017) analyzed the effect of product deterioration and showed that the retailer can ensure strategic advantages if product deterioration is not too high. The manufacturer also receives higher profits in this scenario. Moon et al. (2018) extended the SI model in which the manufacturer or retailer makes investments to stimulate market demand. They showed that a manufacturer's higher investment effort can reduce the volume of SI; still, both supply chain members receive higher profits. However, the reverse scenario was reported when the retailer invested in stimulating market demand. In that circumstance, the retailer needed to increase the volume of SI to invest more in marketing efforts. Roy et al. (2018) investigated a supply chain model wherein the manufacturer had no foresight of the retailer's use of SI and showed that the manufacturer can receive higher profits even if the inventory is not fully observed. Dey and Saha (2018) established the effect of SI on green product manufacturing issues (development-intensive or marginal-cost-intensive green products) under the manufacturer and retailer Stackelberg game structure, and showed that the manufacturer's preferences were largely affected by the retailer's strategic decisions. Nielsen et al. (2019a) studied the influence of SI on a three-echelon supply chain, and found that the retailer needs to uphold more SI if the intermediaries dominate the market. We refer to the following articles where the influence of SI is also explored in different contexts (Hartwig et al., 2015; Mantin & Veldman, 2019; Nielsen et al., 2019b). Note that strategic use of inventory is common in practice, and anecdotal evidence is provided by Guan et al. (2019), Martínez-de-Albéniz and Simchi-Levi (2013). However, the influence of SI within a single-retailer-and-two-competing-manufacturers supply chain setting is still scanty.

Although much literature exists on single-retailer-and-two-competing-manufacturers supply chain settings, this study looks specifically at the topics of pricing under various game structures, product choices, and coordination issues. Trade with substitute products is more common, for example, within the confines of fast-moving consumer goods from two major competitors, such as Unilever and Procter & Gamble. This is also the case with electronic products from Canon and Epson, two product lines that commonly coexist in many retail outlets. When a retailer sells substitute products produced by two different manufacturers, the manufacturers have different degrees of market power, and manufacturers with more market power often announce price decisions first. Therefore, determining the influence of the decision-making sequence in a game-theoretic framework within a single-retailer-and-two-manufacturers supply chain setting remains an important research area (Edirisinghe et al., 2011; Fang et al., 2018; Lee & Staelin, 1997). The following Table 1 reflects some findings from existing literature on such a supply chain setting.

Table 1 demonstrates that researchers mostly focus on comparative analysis of optimal pricing decisions under various market power structures. However, there are also some exceptions. For instance, Cachon and Kök (2010) focused on coordinating contract choices, whereas Yang et al. (2020) considered when it is best for both manufacturers to invest in improving product quality. Giri et al. (2019) studied the influence of government subsidies when both manufacturers trade with green products. Perhaps, for purposes of analytical tractability, market sizes for two products are considered equal in most of the studies. However, one can identify the following gaps in existing research: (i) the effect of SI on a single-retailer-and-two-competing-manufacturers supply chain setting has not yet been explored; most of the studies explored pricing decisions in a single-period decision setting; (ii) except for Zhao et al. (2014), the influence of upstream cooperation between two manufacturers also has not been explored, nor is it compared against optimal decisions made within other scenarios; and (iii) the influence of one manufacturer's commitment on the wholesale price to the common retailer(MRC) has not been explored.

In this study, we explore optimal decisions in three decision-making contexts, which are further divided into two scenarios based on the manufacturer's decision to offer wholesale prices so that the retailer can uphold SI. Note that if the holding cost is too high, the retailer might not be able to maintain SI for the wholesale prices offered by the manufacturers. Therefore, we present two sets of optimal decisions for each decision making context, which leads to six scenarios. The first two decision-making scenarios are similar to the game structure studied by Bian et al. (2018), Gu et al. (2019), Zhao et al. (2014), and Ferrell et al. (2020), where two upstream manufacturers cooperate. Strategic cooperation between two competing firms is not rare in today's business world; stable collaborative relationships between Apple and Samsung (Garrett, 2019) justify this claim. Upstream manufacturers cooperate for many reasons, such as to increase the current joint market size, define their business territories (Piccolo & Reisinger, 2011; Sun & Yang, 2021)), develop products with new features to protect the present and future share of the market, improve logistic planning for reducing cost (Ferrell et al., 2020), conquer a larger share of the market that remains, and many other reasons. We refer to the studies by Cho (2014), Sudhir (2001), and Reisinger and Thomes (2017), Zhao et al. (2019) for a detailed discussion on the advantages and disadvantages in which colluding manufacturers might set wholesale prices that maximize industry profits. Moreover, in a two-manufacturers-two-retailers supply chain setting, several researchers recently studied the effect of integration between upstream manufacturers (Bian et al., 2018, 2020; Wei et al., 2019). Therefore, it is important to explore an optimal decision in the presence of SI when two manufacturers cooperate. In this scenario, the extent to which a retailer's SI affects the ultimate pricing decision is an important factor. In this circumstance, the retailer might uphold more or less SI. The third and fourth scenarios, which are perhaps studied for the first time in the proposed supply chain setting, are the long-term commitment contracts between both of the manufacturers with the common retailer. Although the impact of a manufacturer's commitment is, in practice, identified and supported by some scholarly articles (Tirole, 1988), an analytical result is still missing within the context of competition. In this context, one of the manufacturers commits to a uniform wholesale price for each period. Finally, in the fifth and sixth scenarios, we derive optimal decisions under the manufacturer-Bertrand competition setup, which has been studied by several researchers.

The contributions of this study are as follows: First, to the best of our knowledge, this article is the first to explore the impact of SI under competition in a single-retailer-and-two-manufacturers supply chain setting. Mathematical models are formulated, and corresponding two solutions are derived to study the effect of manufacturer's decision to offer wholesale prices under a certain threshold of holding cost so that a retailer might uphold SI under the manufacturer's leadership where demand functions are linear in retail prices, and market sizes are unequal. We found that the presence of SI has a significant impact on the profitability. Second, if the market sizes differ, and if the cross-price elasticity of customers is higher, then the optimal preferences of the three members are not concurrent. The profitability relationships are completely different according to the presence of SI. In the existing literature, researchers explored the optimal decision under the Bertrand competition setup, which may lead to a sub-optimal decision. As expected, if two competing upstream manufacturers cooperate, then not only do the two upstream manufacturers receive higher profits, the downstream retailer also can receive higher profits. However, the

Table 1
Supply chain models under a single-retailer-and-two-competing-manufacturer supply chain setting.

Study	Game type	Primary demand	Market sizes	SI	Key emphasis of the study
Choi (1991)	MB, RB,VN	price	equal	×	Derive optimal decision for both linear and non-linear demand
Choi (1996)	MB, RB, VN	price	equal	×	Analyze results for different supply chain structures
Cachon and Kök (2010)	MB	price	unequal	×	Analyze contract choice decision among wholesale-price, quantity-discount, and two-part tariff contracts
Pan et al. (2010)	MB, RB	price	equal	×	Analyze performance of revenue sharing contract by deriving results for one-manufacturer-two-retailer supply chain also
Zhao et al. (2012)	MB, RB, VN	price	unequal	×	Introduce fuzzy parameters in decision
Wang et al. (2013)	MS	price	equal	×	Non-linear demand function and markup pricing strategy
Wei et al. (2013)	MS, MB, RB, RS, VN	price	equal	×	Compare preferences among all games
Zhao et al. (2014)	MB, MC, MS,RS, RB, RC, VN	price	equal	×	Fuzzy parameters in demand function and results compare under various game
Shang et al. (2016)	MB	price	equal	×	Information asymmetry, demand uncertainty, and cost-sharing contract
Wang et al. (2017)	MS, MB	price	equal	×	Effect of dual channel supply chain
Luo et al. (2017)	MB, RB, VN	price	equal	×	Retailer's decision to sell single or both products
Li and Chen (2018)	MS, RS	price	equal	×	Quality-differentiated brands selection
Giri et al. (2019)	MB, RB, VN	price, greening level	unequal	×	Influence of government subsidy on green product development
Du et al. (2020)	MB	price, greening level	unequal	×	Two types of green product manufacturing
Yang et al. (2020)	MB	price, quality, and greening level	unequal	×	Manufacturers' green investment strategy
Present study	MB, MC, MRC	price	unequal	✓	Two period decision under influence of SI and wholesale price commitment contract

MB-manufacturer-Bertrand competition; RB-retailer-Bertrand competition; VN-Vertical Nash; MS-manufacturer-Stackelberg; RS-retailer-Stackelberg; MC-two manufacturer cooperate under the leadership of manufacturers; RC-two manufacturer cooperate under the leadership of retailer; MRC-wholesale price commitment between one of the manufacturers with the common retailer.

result changes significantly in the presence of SI. Finally, a comparative analysis conducted in this study demonstrates that the performance under a commitment contract can outperform the scenarios wherein two manufacturers cooperate. Therefore, this study provides a decision support scheme for competing manufacturers that helps them decide whether they should cooperate with competitors or with the retailer.

The rest of the paper is organized as follows. A problem description is given in Section 2 along with all assumptions. In Section 3, optimal solutions are presented and results are compared. Managerial insights are drawn in Section 4. Numerical illustrations also are presented in this section. Finally, in Section 5, the conclusion and future scope of the study are presented. Derivation of optimal decisions are presented in the Appendices.

2. Model description

We consider a supply chain with one common retailer, (R), and two competing manufacturers, $(M_j$, j=1, 2). The M_j produces jth product, and sells to customers through the common retailer. Two products are perfectly substitutable products that compete in price. The demand functions, (D_{ij}), for the jth product at ith, $i = 1, 2$ period are linear functions of market prices, (p_{ij}), a setup that is common in literature on operations management (Choi, 1991; Pan et al., 2010). The demand functions are defined as follows:

$$D_{i1} = a - p_{i1} + \theta p_{i2}, \quad i = 1,2 \tag{1}$$

$$D_{i2} = a\alpha - p_{i2} + \theta p_{i1}, (\alpha > 0) \quad i = 1,2 \tag{2}$$

Therefore, if $\alpha > 1$, then market sizes for the product produced by M_2 are higher compared to M_1, and vice versa. If $\alpha = 1$, the market sizes for the two products remain uniform Choi (1991, 1996), and Pan et al. (2010). The parameter $\theta (0 \le \theta < 1)$ reflects cross-price sensitivity (i.e., the degree of product substitution based on the market prices) (Choi, 1996; Zhao et al., 2014). If $\theta = 0$, then products are independent. For analytical tractability, it is assumed that the inventory holding cost is h per unit for both products.

In this study, we consider three decision-making contexts, and a corresponding graphical representation is presented in Fig. 1 below.

To explore the effect of SI on the performance of supply chain members, two separate scenarios are considered against each decision-making context. As presented in Fig. 1(a), in Scenario MMI, two upstream manufacturers cooperate and make wholesale pricing decisions to optimize the sum of profits for the two manufacturers (Bian et al., 2018, 2020; Gu et al., 2019). In this circumstance, we present the optimal decision in which both manufacturers offer the wholesale

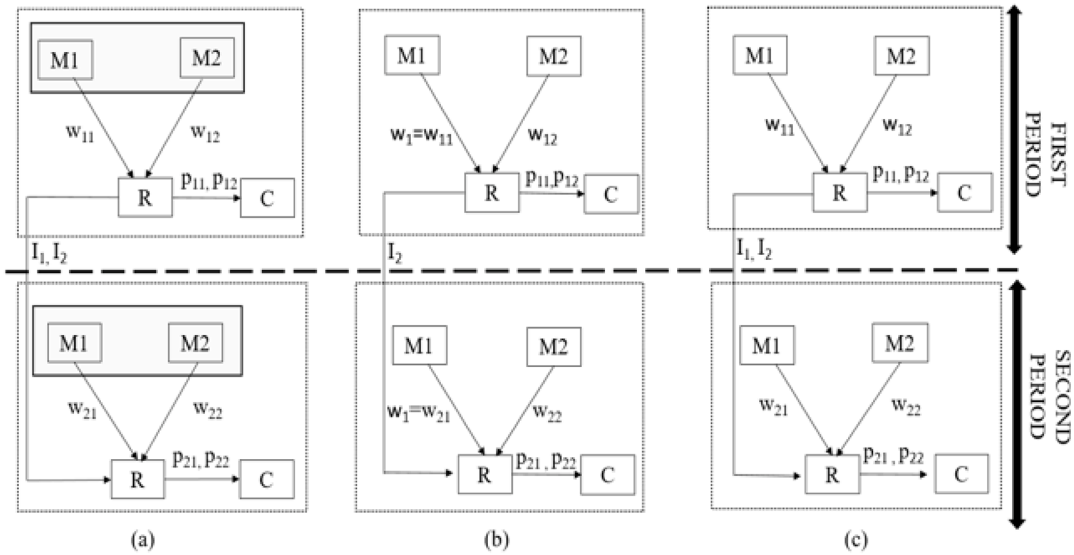

Fig. 1. Decision-making scenarios considered in this study, (a) Scenario MMI, (b) Scenario MRI, and (c) Scenario BI.

Table 2
Notations.

i	index for ith selling period, $i \in \{1, 2\}$
j	index for jth manufacturer, $j \in \{1, 2\}$
k	index for kth scenario, $k \in \{mmi, mmni, mri, mrni, bi, bni\}$
h	the unit holding cost
I_j^k	amount of SI
p_{ij}^k	market price
w_{ij}^k	wholesale price
π_{2r}^k	profits for the retailer in the second period
π_{2mj}^k	profits for the jth manufacturer in the second period
π_r^k	sum of profits in two consecutive periods for the retailer
π_{mj}^k	sum of profits in two consecutive periods for the manufacturer j
Q_j^k	total sales volume in two periods

prices so that the retailer can procure products in both periods and uphold SI. Such a setup, however, does restrict scenarios where it might be optimal for the manufacturers to offer wholesale prices for which the retailer might procure all the products in the first period to sell in both periods, or in scenarios where the retailer does not sell any products in the second period. We consider another scenario: we call it Scenario MMNI, and it is related to the first decision-making context, which is similar to the optimal solution setup under a single-period game, in keeping with most of the supply chain literature and similar to the solution of just-in-time (JIT) operations (Arya & Mittendorf, 2013). We use the notation 'NI' to represent such a scenario. In other word, it represents the scenario where both manufacturers do not offer dynamic contract while deciding their respective wholesale prices or holding cost is too high so that it is not feasible to offer dynamic contract. As mentioned earlier, apart from theoretical evidence (Wei et al. (2019), Zhao et al. (2014)), in practice, two rival firms sometimes cooperate for the betterment of both Hamel et al. (1989). As presented in Fig. 1(b), in Scenario MRI, one of the two upstream manufacturers offers a long-term commitment contract to the common retailer (Roy et al., 2018). In such a situation, we assume that the first manufacturer commits to charging a uniform wholesale price in two consecutive selling periods. Consequently, the retailer decides the amount of the second product to be carried forward as SI, and Scenario MRNI represent the decision where the retailer does not maintain SI for the second product. Finally, Scenario BI represents the Bertrand competition setup, while the common retailer acts as a follower (Shang et al., 2016; Zhao et al., 2012).

We assume that all the members of the supply chain have symmetric information about cost and demand parameters, and make decisions to maximize their respective profits (Choi (1996), Wei et al. (2019), Zhao et al. (2012)). Moreover, the unit marginal costs for both the manufacturers and the common retailer are assumed to be constant and normalized to zero (Ha et al., 2017). We summarize the additional notations used to represent optimal decisions under the six scenarios in Table 2.

We use additional symbols to simplify mathematical expressions, and those are presented in the Supplementary file. In the next section, we discuss characteristics of optimal decisions in different scenarios.

3. Model solutions and discussions

We assume that the two competing manufacturers decide wholesale prices and that the downstream common retailer decides market prices and amount of SIs. In all decision making context, the two upstream manufacturers act as leaders and the common retailer acts as a follower.

3.1. Optimal decisions in scenarios MMI and MMNI

In this decision-making context, two upstream manufacturers cooperate with each other to maximize the sum of total upstream profits. In period $i(i = 1, 2)$, the retailer sources products from both manufacturers with wholesale prices w_{ij}^k, and sells those products at market prices p_{ij}^k, $(k = mmi, mmni)$. The manufacturers set wholesale prices so that the retailer may carry $I_j^k, (I_j^k \geq 0)$ amount of inventories from period 1 to period 2. According to Bian et al. (2020), the decision structure represents upstream 'collusion'. Therefore, all the members' profits in the second period are affected by the retailer's decision to carry inventory in period 1. Profit functions for the retailer and the sum of profits for the two manufacturers in the second period, $(\pi_{2m}^{mmi} = \pi_{2m1}^{mmi} + \pi_{2m2}^{mmi})$, and the sum of profits in the two consecutive periods for the retailer and the two manufacturers, ($\pi_m^{mmi} = \pi_{m1}^{mmi} + \pi_{m2}^{mmi}$), are presented as follows:

$$\pi_{2r}^{mmi}(p_{21}^{mmi}, p_{22}^{mmi}) = \sum_{j=1}^{2} p_{2j}^{mmi} D_{2j}^{mmi} - \sum_{j=1}^{2} w_{2j}^{mmi}(D_{2j}^{mmi} - I_j^{mmi}) \tag{3}$$

$$\pi_{2m}^{mmi}(w_{21}^{mmi}, w_{22}^{mmi}) = \pi_{2m1}^{mmi} + \pi_{2m2}^{mmi} = \sum_{j=1}^{2} w_{2j}^{mmi}(D_{2j}^{mmi} - I_j^{mmi}) \tag{4}$$

$$\pi_r^{mmi}(p_{11}^{mmi}, p_{12}^{mmi}, I_1^{mmi}, I_2^{mmi}) = \sum_{j=1}^{2}(p_{1j}^{mmi} - w_{1j}^{mmi})D_{1j}^{mmi} - \sum_{j=1}^{2}(w_{1j}^{mmi} + h)I_j^{mmi} + \pi_{2r}^{mmi} \tag{5}$$

$$\pi_m^{mmi}(w_{11}^{mmi}, w_{12}^{mmi}) = \pi_{m1}^{mmi} + \pi_{m2}^{mmi} = \sum_{j=1}^{2} w_{1j}^{mmi}(D_{1j}^{mmi} + I_j^{mmi}) + \pi_{2m}^{mmi} \tag{6}$$

Therefore, the sales volume for product j in two periods is $Q_j^{mmi} = \sum_{i=1}^{2} D_{ij}^{mmi}$. By maximizing the retailer's profits and joint profits for two manufacturers sequentially, we derive the optimal decision. The sequence of events in Scenario MMI is as follows (Anand et al., 2008; Dey & Saha, 2018):

In the first period, the following activities occur:

Stage 1. Two upstream manufacturers quote their respective wholesale prices, w_{11}^{mmi} and w_{12}^{mmi}, by maximizing their sum of upstream profits in two consecutive periods.

Stage 2. The common retailer chooses market prices, p_{11}^{mmi} and p_{12}^{mmi}, and decides the amount of SIs to be upheld $I_j^{mmi}, j = 1, 2$ by maximizing cumulative profits in two periods.

In the second period, the following activities occur:

Stage 1. Two upstream manufacturers quote wholesale prices, w_{21}^{mmi} and w_{22}^{mmi}, by maximizing the sum of upstream profits in the second period.

Stage 2. The common retailer chooses market prices, p_{21}^{mmi} and p_{22}^{mmi}, by maximizing the total second-period profits.

Proposition 3.1 summarizes the optimal decision in Scenario MMI and we refer to Appendix A for the detailed derivation.

Proposition 3.1. *Optimal decision in Scenario MMI is obtained as follows:*

$$w_{11}^{mmi} = \frac{9a(1+\alpha\theta) - 2h(1-\theta^2)}{17(1-\theta^2)};\ w_{12}^{mmi} = \frac{9a(\alpha+\theta) - 2h(1-\theta^2)}{17(1-\theta^2)};$$

$$w_{21}^{mmi} = \frac{6a(1+\alpha\theta) + 10h(1-\theta^2)}{17(1-\theta^2)};\ w_{22}^{mmi} = \frac{6a(\alpha+\theta) + 10h(1-\theta^2)}{17(1-\theta^2)};$$

$$p_{11}^{mmi} = \frac{13a(1+\alpha\theta) - h(1-\theta^2)}{17(1-\theta^2)};\ p_{12}^{mmi} = \frac{13a(\alpha+\theta) - h(1-\theta^2)}{17(1-\theta^2)};$$

$$p_{21}^{mmi} = \frac{23a(1+\alpha\theta) + 10h(1-\theta^2)}{34(1-\theta^2)};\ p_{22}^{mmi} = \frac{23a(\alpha+\theta) + 10h(1-\theta^2)}{34(1-\theta^2)};$$

$$I_1^{mmi} = \frac{5(a - 4h(1-\theta))}{34}; I_2^{mmi} = \frac{5(a\alpha - 4h(1-\theta))}{34};$$

$$\pi_r^{mmi} = \frac{155a^2(1+2\alpha\theta+\alpha^2) - 118ah(1+\alpha)(1-\theta^2) + 608h^2(1+\theta)(1-\theta)^2}{1156(1-\theta^2)};$$

$$\pi_{m1}^{mmi} = \frac{9a^2(1+\alpha\theta) - 2ah(1-\theta)(2-\theta+3\alpha\theta) + 8h^2(1-\theta)^2(1+\theta)}{34(1-\theta^2)};$$

$$\pi_{m2}^{mmi} = \frac{9a^2\alpha(\alpha+\theta) - 2ah(1-\theta)(2\alpha-\alpha\theta+3\theta) + 8h^2(1-\theta)^2(1+\theta)}{34(1-\theta^2)};$$

$$Q_1^{mmi} = \frac{19a - 8h(1-\theta)}{34};\ Q_2^{mmi} = \frac{19a\alpha - 8h(1-\theta)}{34}.$$

Note that in this scenario, both manufacturers can offer above wholesale prices, and the retailer carries SI and procures products in each period if $\min\{\frac{a}{4(1-\theta)}, \frac{a\alpha}{4(1-\theta)}\} \geq h(= h^{mmi}, say)$. If $\theta = 0$ and $\alpha = 1$, the limit becomes exactly the same, as noted in Anand et al. (2008). From Proposition 3.1, the differences between sales volume, amount of SIs, and profits for the two manufacturers are obtained as $Q_1^{mmi} - Q_2^{mmi} = \frac{19a(1-\alpha)}{34}$, $I_1^{mmi} - I_2^{mmi} = \frac{5a(1-\alpha)}{34}$, $\pi_{m1}^{mmi} - \pi_{m2}^{mmi} = \frac{a(1-\alpha)(9a(1+\alpha)-4h(1-\theta)(1-2\theta))}{34(1-\theta^2)}$, respectively, and the differences remain identical if $\alpha = 1$. The results are sensible, because if the market sizes are uniform, then the corresponding decision will be symmetric. The difference between the wholesale and market prices for both the products in between the first and second periods are obtained as $2(p_{11}^{mmi} - p_{21}^{mmi}) = (w_{11}^{mmi} - w_{21}^{mmi}) = \frac{3(a(1+\alpha\theta)-4h(1-\theta^2))}{17(1-\theta^2)}$ and $2(p_{12}^{mmi} - p_{22}^{mmi}) = (w_{12}^{mmi} - w_{22}^{mmi}) = \frac{3(a(\alpha+\theta)-4h(1-\theta^2))}{17(1-\theta^2)}$, respectively. Therefore, the common retailer has the second period wholesale price advantage by maintaining SI, if $min\left\{\frac{a(1+\alpha\theta)}{4-4\theta^2}, \frac{a(\alpha+\theta)}{4-4\theta^2}\right\} > h$. Note that the limits for the holding cost are not identical with non-negative strategic inventories. This implies that the retailer may hold inventories, but would not necessarily ensure a wholesale price advantage under the single-retailer-and-two-manufacturers supply chain setting. The amount of SI is directly proportional with the market size, $a\alpha$, for the second manufacturer (i.e., the retailer maintains a higher amount of product as SI for the second product, in anticipation of a better wholesale price deal).

We present the optimal decisions in a two period repeated game scenario or in a JIT environment (Arya & Mittendorf, 2013) in Table D.1 in Appendix D. Note that optimal decisions remain uniform in the first and second periods. By substituting $I_j^{mmi} = 0$ and $h = 0$ in Eqs. (3)–(6), one can obtain the profit functions for the supply chain members in Scenario MMNI. We propose the following Proposition 3.2 to compare profits functions for the supply chain members in two games.

Proposition 3.2. *In between Scenarios MMI and MMNI*

(i) The retailer receives higher profits in Scenario MMI if

$$h \notin \left(max\left\{0, \frac{59a(1+\alpha)(1-\theta) - a\sqrt{(1+\theta)\Gamma_1}}{608(1-\theta^2)}\right\}, min\left\{\frac{59a(1+\alpha)(1-\theta) + a\sqrt{(1+\theta)\Gamma_1}}{608(1-\theta^2)}, h^{mmi}\right\}\right)$$

(ii) The first manufacturer receives higher profits in Scenario MMI if

$$h \notin \left(max\left\{0, \frac{a[2+(3\alpha-1)\theta - \sqrt{(\alpha-1)\theta(8+(9\alpha-1)\theta)}]}{8(1-\theta^2)}\right\}, min\left\{\frac{a[2+(3\alpha-1)\theta + \sqrt{(\alpha-1)\theta(8+(9\alpha-1)\theta)}]}{8(1-\theta^2)}, h^{mmi}\right\}\right)$$

(iii) The second manufacturer receives higher profits in Scenario MMI if

$$h \notin \left(max\left\{0, \frac{a[2\alpha+(3-\alpha)\theta - \sqrt{(1-\alpha)\theta((9-\alpha)\theta+8\alpha)}]}{8(1-\theta^2)}\right\}, min\left\{\frac{a[2\alpha+(3-\alpha)\theta + \sqrt{(1-\alpha)\theta((9-\alpha)\theta+8\alpha)}]}{8(1-\theta^2)}, h^{mmi}\right\}\right)$$

We refer to Appendix E for the detailed proof. Therefore, Proposition 3.2 demonstrates the strategic advantage of the manufacturers to offer dynamic contract while holding cost for the retailer is low. Certainly, the profit for the retailer is decreased with respect to the holding cost, and it is profitable for the retailer to maintain SI if the holding cost is small (i.e., up to the lower limit). To explain this in detail, one can find the profit difference for the retailer and the two manufacturers for $\alpha = 1$ is $\pi_r^{mmi} - \pi_r^{mmni} = \frac{(21a-152h(1-\theta))(a-4h(1-\theta))}{1156(1-\theta)}$ and $\pi_{m1}^{mmi} - \pi_{m1}^{mmni} = \pi_{m2}^{mmi} - \pi_{m2}^{mmni} = \frac{(a-4h(1-\theta))^2}{68(1-\theta)}$, respectively. Therefore, both manufacturers always receive higher profits. However, the retailer receives higher profits if $\frac{21a}{152(1-\theta)} > h$ and if $h \in \left(\frac{21a}{152(1-\theta)}, \frac{a}{4(1-\theta)}\right)$, the retailer can maintain SI.

3.2. Optimal decisions in scenarios MRI and MRNI

In this section, we explore the effect of the wholesale price commitment of one of the manufacturers with the common retailer. If one of the manufacturers commits to set a uniform wholesale price for two consecutive periods, then there is no need for the retailer to maintain SI for wholesale price negotiation (Tirole, 1988). Because wholesale prices remain the same in two consecutive periods for the first product, we denote the wholesale price by $w_{21}^{mri} = w_{11}^{mri} = w_1^{mri}$. The profit functions for the retailer and two manufacturers in the second period, and the sum of profits in two consecutive periods for the retailer and two manufacturers in Scenario MRI are obtained as follows:

$$\pi_{2r}^{mri}(p_{21}^{mri}, p_{22}^{mri}) = \sum_{j=1}^{2} p_{2j}^{mri} D_{2j}^{mri} - w_1^{mri} D_{21}^{mri} - w_{22}^{mri}(D_{22}^{mri} - I_2^{mri}) \tag{7}$$

$$\pi_{2m1}^{mri} = w_1^{mri} D_{21}^{mri} \tag{8}$$

$$\pi_{2m2}^{mri}(w_{22}^{mri}) = w_{22}^{mri}(D_{22}^{mri} - I_2^{mri}) \tag{9}$$

$$\pi_r^{mri}(p_{11}^{mri}, p_{12}^{mri}, I_2^{mri}) = \sum_{j=1}^{2} p_{1j}^{mri} D_{1j}^{mri} - w_1^{mri} D_{11}^{mri} - w_{12}^{mri}(D_{12}^{mri} + I_2^{mri}) - hI_2^{mri} + \pi_{2r}^{mri} \tag{10}$$

$$\pi_{m1}^{mri}(w_1^{mri}) = w_1^{mri} D_{11}^{mri} + \pi_{2m1}^{mri} \tag{11}$$

$$\pi_{m2}^{mri}(w_{12}^{mri}) = w_{12}^{mri}(D_{12}^{mri} + I_2^{mri}) + \pi_{2m2}^{mri} \tag{12}$$

Note that the first manufacturer, (M_1), offers a wholesale price commitment to the common retailer. Therefore, the step wise decision sequence in Scenario MRI is mostly similar to that of Scenario MMI, with the main difference being that the first manufacturer postponed the second period wholesale pricing decision and wholesale prices remain identical in both periods. The optimal decision in Scenario MRI is presented in Proposition 3.3, and we refer to Appendix B for the detailed derivation of the optimal decision.

Proposition 3.3. *Optimal decision in Scenario MRI is obtained as follows:*

$$w_1^{mri} = \frac{a(34 + 15\alpha\theta) + 8h\theta}{68 - 15\theta^2}; \quad w_{12}^{mri} = \frac{18a(2\alpha + \theta) - h(8 - 6\theta^2)}{68 - 15\theta^2};$$

$$w_{22}^{mri} = \frac{12a(2\alpha + \theta) + h(40 - 6\theta^2)}{68 - 15\theta^2};$$

$$p_{11}^{mri} = p_{21}^{mri} = \frac{a(102 + \alpha(83 - 30\theta^2)\theta - 49\theta^2) + 8h\theta(1 - \theta^2)}{2(68 - 15\theta^2)(1 - \theta^2)};$$

$$p_{12}^{mri} = \frac{a(104\alpha - 51\alpha\theta^2 + 86\theta - 33\theta^3) - 2h(4 - 7\theta^2 + 3\theta^4)}{2(68 - 15\theta^2)(1 - \theta^2)};$$

$$p_{22}^{mri} = \frac{a(92\alpha - 39\alpha\theta^2 + 80\theta - 27\theta^3) + h(40 - 46\theta^2 + 6\theta^4)}{2(68 - 15\theta^2)(1 - \theta^2)};$$

$$I_2^{mri} = \frac{5a(2\alpha + \theta) - 10h(4 - \theta^2)}{68 - 15\theta^2};$$

$$\pi_r^{mri} = \frac{1}{2(68 - 15\theta^2)^2(1 - \theta^2)}[a^2(1156 + 4844\alpha\theta + 1240\alpha^2 + 3(523\alpha^2 + 534)\theta^2 + 774\alpha\theta^3 + 51\theta^4) + 2ah(1 - \theta^2) \times (508\theta + 2\alpha(236 + 87\theta^2) + 27\theta^3) - 32h^2(1 - \theta^2)(3\theta^4 - 35\theta^2 + 76)];$$

$$\pi_{m1}^{mri} = \frac{(a(34 + 15\alpha\theta) + 8h\theta)^2}{(68 - 15\theta^2)^2};$$

$$\pi_{m2}^{mri} = \frac{306a^2(2\alpha - \theta)^2 - 68ah(4 - 3\theta^2)(2\alpha - \theta) + 4h^2(21\theta^4 - 136\theta^2 + 272)}{(68 - 15\theta^2)^2};$$

$$Q_1^{mri} = \frac{a(34 + 15\alpha\theta) + 8h\theta}{68 - 15\theta^2}; \quad Q_2^{mri} = \frac{19a(2\alpha + \theta) + 8h(2 - \theta^2)}{68 - 15\theta^2}.$$

Note that the second manufacturers can offers above wholesale prices if the holding cost satisfies $\frac{a(2\alpha - \theta)}{2(4 - \theta^2)} \geq h(= h^{mri}, say)$, From Proposition 3.3, and in this circumstance the retailer buy products in both periods and upholds SI. The wholesale prices also less because $w_{12}^{mri} - w_{22}^{mri} = \frac{6a(2\alpha + \theta) - 2h(4 - \theta^2)}{68 - 15\theta^2} > 0$. Customers also receive products at a less price in the second period because $p_{12}^{mri} - p_{22}^{mri} = \frac{3a(2\alpha + \theta) - 6h(4 - \theta^2)}{68 - 15\theta^2} > 0$. Previously, in Scenario MMI, we found that the sales volume and profits for the two manufacturers remain uniform if market sizes are the same($\alpha = 1$), which is not true if one of the manufacturers cooperates with the retailer, because $Q_1^{mri} - Q_2^{mri} = \frac{a(34 - \alpha(38 + 15\theta) + 19\theta) + 8h(2 - \theta - \theta^2)}{68 - 15\theta^2}$ and $\pi_{m1}^{mri} - \pi_{m2}^{mri} = \frac{a^2(204\alpha\theta + 34(34 - 9\theta^2) - 9\alpha^2(136 - 25\theta^2)) + 4ah(\alpha(136 - 42\theta^2) - 51\theta(4 - \theta^2)) - 4h^2(4 - \theta^2)(68 - 21\theta^2)}{(68 - 15\theta^2)^2}$, respectively.

Note that the wholesale and market prices remain uniform for the product from the first manufacturer. Therefore, cooperation between the retailer and the first manufacturer creates an imbalance in the overall supply chain decision. If the market size remains the same, ($\alpha = 1$) and the consumer's cross-price elasticity (θ) is zero, then the commitment between the first manufacturer and the retailer is beneficial for the second manufacturer, because $\pi_{m1}^{mri} - \pi_{m2}^{mri} = -\frac{(a - 4h)^2}{68} < 0$. If the market price remains uniform, then the demand for the first product may decrease in the second period, because in the presence of SI for the second product, the second-period price should be less, compared to the price in the first period. This, perhaps, gives an indication of why the manufacturer does not commit to the retailer when cross-price elasticity is negligible.

Similar to Scenario MMNI, we present the optimal decision in Scenario MRNI in Table D.1 and we propose the following proposition to highlight strategic advantage of supply chain members.

Proposition 3.4. *In between Scenarios MRI and MRNI*

(i) The common retailer always receives higher profits in Scenario MRI if

$$h \notin \left(max\left\{0, \frac{a(8 - 3\theta^2)\Gamma_2 - a(68 - 15\theta^2)\sqrt{\Gamma_3}}{32(76 - 35\theta^2 + 3\theta^4)(8 - 3\theta^2)}\right\}, min\left\{\frac{a(8 - 3\theta^2)\Gamma_2 + a(68 - 15\theta^2)\sqrt{\Gamma_3}}{32(76 - 35\theta^2 + 3\theta^4)(8 - 3\theta^2)}, h^{mri}\right\}\right)$$

(ii) The first manufacturer M_1 receives higher profits in Scenario MRI if

$$h \notin \left(max\left\{0, \frac{a(8 - 3\theta^2)(34 + 15\alpha\theta) - a(68 - 15\theta^2)(2 + \alpha\theta)\sqrt{4 - \theta^2}}{8\theta(8 - 3\theta^2)}\right\}, min\left\{\frac{a(8 - 3\theta^2)(34 + 15\alpha\theta) + a(68 - 15\theta^2)(2 + \alpha\theta)\sqrt{4 - \theta^2}}{8\theta(8 - 3\theta^2)}, h^{mri}\right\}\right)$$

(iii) The manufacturer M_2 receives higher profits in Scenario MRI if

$$h \notin \left(max\left\{0, \frac{34a(2\alpha + \theta)(8 - 3\theta^2)(4 - 3\theta^2) - a\theta(68 - 15\theta^2)\sqrt{\theta\Gamma_4}}{4(8 - 3\theta^2)(272 - 136\theta^2 + 21\theta^4)}\right\}, min\left\{\frac{34a(2\alpha + \theta)(8 - 3\theta^2)(4 - 3\theta^2) + a\theta(68 - 15\theta^2)\sqrt{\theta\Gamma_4}}{4(8 - 3\theta^2)(272 - 136\theta^2 + 21\theta^4)}, h^{mri}\right\}\right)$$

Therefore, a commitment between one of the competing manufacturers and the retailer may bring benefits for all the supply chain members. Because $\theta < 1$, one can obtain the feasible range for the holding cost for which the first manufacturer receives higher profits. However, for the second manufacturer, the feasible limits exist if $\Gamma_4 > 0$, i.e., $\alpha \in \left(\frac{1088 - 8\theta^2 - 84\theta^4 - 2(8 - 3\theta^2)\sqrt{17(272 - 136\theta^2 + 21\theta^4)}}{1344\theta - 472\theta^3 + 21\theta^5}, \frac{1088 - 8\theta^2 - 84\theta^4 + 2(8 - 3\theta^2)\sqrt{17(272 - 136\theta^2 + 21\theta^4)}}{1344\theta - 472\theta^3 + 21\theta^5}\right)$. Under such a situation the second manufacturer always receives higher profits. One can verify that if $\alpha = 1$ and $\theta = 0$, then the profits for the first manufacturer remain the same in Scenario MRI and Scenario MRNI. However, the second manufacturer will always receive higher profits in the presence of SI because $\pi_{m2}^{mri} - \pi_{m2}^{mrni} = \frac{(a - 4h)^2}{68}$. Therefore, the

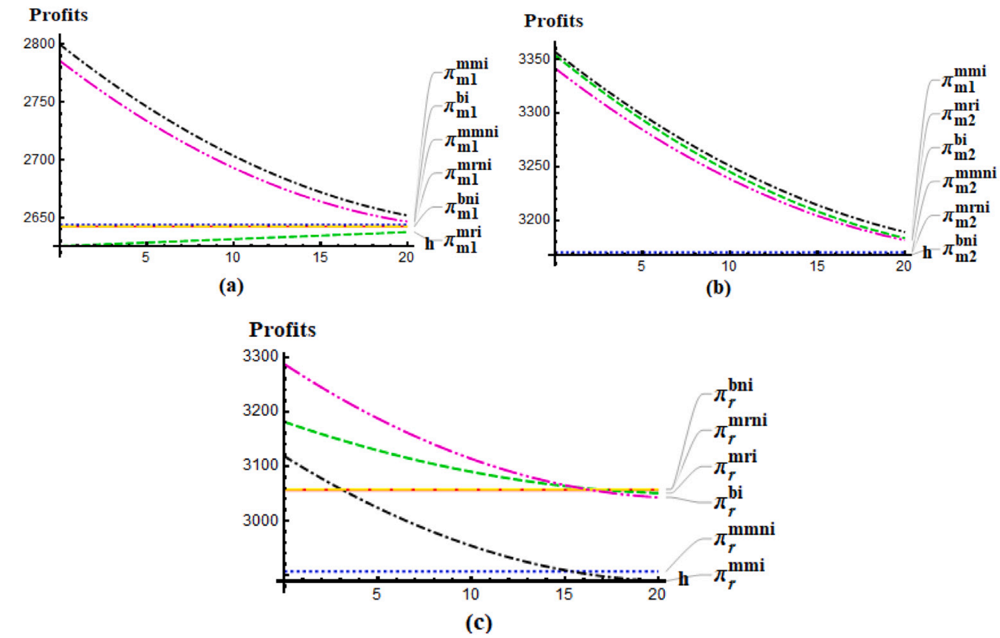

Fig. 2. Profits for (a) first manufacturer, (b) second manufacturer, and (c) common retailer in six scenarios.

first manufacturer needs to conduct extensive market analysis before making a wholesale price commitment.

3.3. Optimal decisions in scenario BI and BNI

Finally, in this subsection, we derive decision for the supply chain members under the Bertrand competition setup, which has extensively been studied in the existing literature. The profit functions for supply chain members in Scenario BI are obtained as follows:

$$\pi_{2r}^{bi}(p_{21}^{bi}, p_{22}^{bi}) = \sum_{j=1}^{2} p_{2j}^{bi} D_{2j}^{bi} - \sum_{j=1}^{2} w_{2j}^{bi}(D_{2j}^{bi} - I_j^{bi}) \tag{13}$$

$$\pi_{2m1}^{bi}(w_{21}^{bi}) = w_{21}^{bi}(D_{21}^{bi} - I_1^{bi}) \tag{14}$$

$$\pi_{2m2}^{bi}(w_{22}^{bi}) = w_{22}^{bi}(D_{22}^{bi} - I_2^{bi}) \tag{15}$$

$$\pi_r^{bi}(p_{11}^{bi}, p_{12}^{bi}, I_1^{bi}, I_2^{bi}) = \sum_{j=1}^{2}(p_{1j}^{bi} - w_{1j}^{bi})D_{1j}^{bi} - \sum_{j=1}^{2}(w_{1j}^{bi} + h)I_j^{bi} + \pi_{2r}^{bi} \tag{16}$$

$$\pi_{m1}^{bi}(w_{11}^{bi}) = w_{11}^{bi}(D_{11}^{bi} + I_1^{bi}) + \pi_{2m1}^{bi} \tag{17}$$

$$\pi_{m2}^{bi}(w_{12}^{bi}) = w_{12}^{bi}(D_{12}^{bi} + I_2^{bi}) + \pi_{2m2}^{bi} \tag{18}$$

The optimal decision in Scenario BI is presented in 3.5. By maximizing the retailer's and each manufacturer's profits sequentially, we obtain the optimal decision. Please see Appendix C for the detailed derivation of optimal decisions. The complete derivation for all possible solutions under this game structure is presented in a supplementary document.

Proposition 3.5. *Optimal decision in Scenario BI is obtained as given in Box I.*

Therefore, both manufacturers offer wholesale prices as presented and if $\min\{\frac{a(3060-561\theta^2+3\theta^4+2\theta^6-2\alpha\theta(9-\theta^2)(84-13\theta^2))}{(2-\theta)^2x_1(x_3+3(8-\theta^2))}, \frac{a(\alpha(3060-561\theta^2+3\theta^4+2\theta^6))-2(9-\theta^2)\theta(84-13\theta^2)}{(2-\theta)^2x_1(x_3+3(8-\theta^2))}\}$ $\geq h(= h^{bi}, say)$, then the retailer can procure products in both period and upholds SI. Note that the limits for the holding cost do not coincide and the amount of SI follows the opposite trend to α because $\frac{\partial I_1^{bi}}{\partial\alpha} = \frac{-a\theta(9-\theta^2)(84-13\theta^2)}{\Delta_{bi}}$ and $\frac{\partial I_2^{bi}}{\partial\alpha} = \frac{a(3060-561\theta^2+3\theta^4+2\theta^6)}{2\Delta_{bi}}$, which is completely different compared to Scenario MMI, because $\frac{\partial I_1^{mmi}}{\partial\alpha} = 0$ and $\frac{\partial I_2^{bi}}{\partial\alpha} = \frac{5a}{34}$. We can observe that the difference between sales volumes, SIs and profits for the two manufacturers are $Q_1^{bi} - Q_2^{bi} = \frac{a(1-\alpha)(57+18\theta-(4+\theta)\theta^2)}{x_1}$; $I_1^{bi} - I_2^{bi} = \frac{a(1-\alpha)(30+9\theta-\theta^2)}{2x_1}$; $\pi_{m1}^{bi} - \pi_{m2}^{bi} = \frac{2a(1-\alpha)[a(1+\alpha)(9-\theta^2)(153-28\theta^2+\theta^4)-h(2-\theta)(306-459\theta+162\theta^2-99\theta^3+(8-\theta)(3+\theta)\theta^4)]}{2\Delta_{bi}}$, respectively. Therefore, similar to Scenario MMI, if the market sizes remain the same ($\alpha = 1$), those remain uniform. The difference between wholesale prices and market prices for both the products are obtained as $2(p_{12}^{bi} - p_{22}^{bi}) = w_{12}^{bi} - w_{22}^{bi}$ $= \frac{2a(9-\theta^2)(102+9\theta^2-2\theta^4+\alpha\theta(81-8\theta^2))-h(2-\theta)(36-18\theta-3\theta^2+2\theta^3)x_1}{2\Delta_{bi}}$ and $2(p_{11}^{bi} - p_{21}^{bi}) = w_{11}^{bi} - w_{21}^{bi} = \frac{2a(9-\theta^2)(\alpha(102+9\theta^2-2\theta^4)+\theta(81-8\theta^2))-h(2-\theta)(36-18\theta-3\theta^2+2\theta^3)x_1}{2\Delta_{bi}}$, respectively. Therefore, wholesale and market price would be less in the second period compared to the first period if $h \leq min \left\{\frac{2a(9-\theta^2)(102+9\theta^2-2\theta^4+\alpha\theta(81-8\theta^2))}{(2-\theta)(36-18\theta-3\theta^2+2\theta^3)x_1}, \frac{2a(9-\theta^2)(\alpha(102+9\theta^2-2\theta^4)+\theta(81-8\theta^2))}{(2-\theta)(36-18\theta-3\theta^2+2\theta^3)x_1}\right\}$. Similar to the previous two decision scenarios, we present optimal decision in Scenario BNI in Table D.1 and propose the following proposition.

Proposition 3.6. *In between Scenarios BI and BNI*

(i) The common retailer always receives higher profits in Scenario BI if

$$h \notin \left(max\left\{0, \frac{a((1+\alpha)(2-\theta)x_1\Gamma_8 - x_2\sqrt{2\alpha\Gamma_9 - (1+\alpha^2)\Gamma_{10}})}{2(4-\theta^2)(2+\theta)x_1\Gamma_{11}}\right\}, \right.$$
$$\left. min\left\{\frac{a((1+\alpha)(2-\theta)x_1\Gamma_8 + x_2\sqrt{2\alpha\Gamma_9 - (1+\alpha^2)\Gamma_{10}})}{2(4-\theta^2)(2+\theta)x_1\Gamma_{11}}, h^{bi}, \right\}\right)$$

(ii) The manufacturer M_1 receives higher profits in Scenario BI if

$$h \notin \left(max\left\{0, \frac{2a(4-\theta^2)(\Gamma_{12} - \alpha\theta\Gamma_{13}) - ax_2\sqrt{2\theta(\alpha^2\theta\Gamma_{14} - 4\alpha\Gamma_{15} + 2\Gamma_{16})}}{2(4-\theta^2)(2+\theta)x_1\Gamma_{17}}\right\}, \right.$$
$$\left. min\left\{\frac{2a(4-\theta^2)(\Gamma_{12} - \alpha\theta\Gamma_{13}) + ax_2\sqrt{2\theta(\alpha^2\theta\Gamma_{14} - 4\alpha\Gamma_{15} + 2\Gamma_{16})}}{2(4-\theta^2)(2+\theta)x_1\Gamma_{17}}, h^{bi}\right\}\right)$$

(iii) The manufacturer M_2 receives higher profits in Scenario BI if

$$h \notin \left(max\left\{0, \frac{2a(4-\theta^2)(\alpha\Gamma_{12} - \theta\Gamma_{13}) - ax_2\sqrt{2\theta(\theta\Gamma_{14} - 4\alpha\Gamma_{15} + 2\alpha^2\Gamma_{16})}}{2(4-\theta^2)(2+\theta)x_1\Gamma_{17}}\right\}, \right.$$
$$\left. min\left\{\frac{2a(4-\theta^2)(\alpha\Gamma_{12} - \theta\Gamma_{13}) + ax_2\sqrt{2\theta(\theta\Gamma_{14} - 4\alpha\Gamma_{15} + 2\alpha^2\Gamma_{16})}}{2(4-\theta^2)(2+\theta)x_1\Gamma_{17}}, h^{bi}\right\}\right)$$

$$w_{11}^{bi}=\frac{2a(9-\theta^2)(2x_4+\alpha\theta x_5)-h(2-\theta)x_1x_3}{\Delta_{bi}};\ w_{12}^{bi}=\frac{2a(9-\theta^2)(2\alpha x_4-\theta x_5)-h(2-\theta)x_1x_3}{\Delta_{bi}};$$
$$p_{11}^{bi}=\frac{a(15912-11817\theta^2+2697\theta^4-248\theta^6+8\theta^8)-\alpha\theta(12942-8139\theta^2+1947\theta^4-206\theta^6+8\theta^8)-h(1-\theta^2)(2-\theta)x_1x_3}{2(1-\theta^2)\Delta_{bi}};$$
$$p_{12}^{bi}=\frac{a(\alpha(15912-11817\theta^2+2697\theta^4-248\theta^6+8\theta^8)-\theta(12942\theta-8139\theta^3+1947\theta^5-206\theta^7+8\theta^9))-h(1-\theta^2)(2-\theta)x_1x_3}{2(1-\theta^2)\Delta_{bi}};$$
$$I_1^{bi}=\frac{a(3060-561\theta^2+3\theta^4+2\theta^6+2\alpha(9-\theta^2)\theta(84-13\theta^2))-h(2-\theta)^2x_1(x_3+3(8-\theta^2))}{2\Delta_{bi}};$$
$$I_2^{bi}=\frac{a(\alpha(3060-561\theta^2+3\theta^4+2\theta^6)-2\theta(9-\theta^2)(84-13\theta^2))-h(2-\theta)^2x_1(x_3+3(8-\theta^2))}{2\Delta_{bi}};$$
$$w_{21}^{bi}=\frac{2a(9-\theta^2)((2x_5-3(26-\theta^2))-\alpha\theta(60-25\theta^2+2\theta^4))+h(2-\theta)x_1(x_3+3(8-\theta^2))}{\Delta_{bi}};$$
$$w_{22}^{bi}=\frac{2a(9-\theta^2)(\alpha(2x_5-3(26-\theta^2))-\theta(60-25\theta^2+2\theta^4))+h(2-\theta)x_1(x_3+3(8-\theta^2))}{\Delta_{bi}};$$
$$p_{21}^{bi}=\frac{a(3(4692-3313\theta^2+903\theta^4-102\theta^6+4\theta^8)-\alpha\theta(11484-6375\theta^2+1625\theta^4-190\theta^6+8\theta^8))+h(2-\theta)(1-\theta^2)x_1(x_3+3(8-\theta^2))}{2(1-\theta^2)\Delta_{bi}};$$
$$p_{22}^{bi}=\frac{a(3\alpha(4692-3313\theta^2+903\theta^4-102\theta^6+4\theta^8)-\theta(11484-6375\theta^2+1625\theta^4-190\theta^6+8\theta^8))+h(2+\theta)(1-\theta^2)x_1(x_3+3(8-\theta^2))}{2(1-\theta^2)\Delta_{bi}};$$
$$\pi_r^{bi}=\frac{\Gamma_5}{2(1-\theta^2)\Delta_{bi}^2};\ \pi_{m1}^{bi}=\frac{\Gamma_6}{2\Delta_{bi}^2};\ \pi_{m2}^{bi}=\frac{\Gamma_7}{2\Delta_{bi}^2};$$
$$Q_1^{bi}=\frac{2a(3(1938-451\theta^2+25\theta^4)-\alpha\theta(9-\theta^2)(309-59\theta^2+2\theta^4))-3h(1-\theta)(2-\theta)(8-\theta^2)x_1}{2\Delta_{bi}};$$
$$Q_2^{bi}=\frac{2a(3\alpha(1938-451\theta^2+25\theta^4)-\theta(9-\theta^2)(309-59\theta^2+2\theta^4))-3h(1-\theta)(2-\theta)(8-\theta^2)x_1}{2\Delta_{bi}}.$$

Box I.

Therefore, in all three decision-making contexts, if the manufacturers offer dynamic contract or commitment contract and holding cost for the retailer is within a certain limits, then every supply chain member has the opportunity to receive higher profits. The graphical representation of the profits of supply chain members in six scenarios is presented below in Fig. 2. The parameter values are considered as $a = 100$; $\theta = 0.05$; $\alpha = 1.1$.

From Fig. 2, we can observe that profits for each supply chain member decrease with respect to holding cost. As mentioned earlier, both manufactures can offer such wholesale prices if the holding cost is with in a threshold and the retailer can maintain SI. Although, the limits are not equal, the point of intersection represent the upper limit until which the manufactures offer such wholesale prices as mentioned earlier. It is also observed that the second manufacturer receives higher profits compared to the first manufacturer, due to higher market demand ($\alpha > 1$). Next, we explore the nature of market prices for two products.

Proposition 3.7. *Optimal market prices satisfy the following relationships:*

(i) In Scenarios MMI and MMNI, $\frac{\partial p_{11}^{mmi}}{\partial\alpha}=-\frac{\partial p_{12}^{mmi}}{\partial\alpha}=\frac{-13a\theta}{17(1-\theta^2)}<0$; $\frac{\partial p_{21}^{mmi}}{\partial\alpha}=-\frac{\partial p_{22}^{mmi}}{\partial\alpha}=\frac{-23a\theta}{34(1-\theta^2)}<0$; $\frac{\partial p_{11}^{mmni}}{\partial\alpha}=\frac{-3a\theta}{4(1-\theta^2)}$; $\frac{\partial p_{12}^{mmni}}{\partial\alpha}=\frac{3a}{4(1-\theta^2)}>0$

(ii) In Scenarios MRI and MRNI, $\frac{\partial p_{11}^{mri}}{\partial\alpha}=\frac{\partial p_{21}^{mri}}{\partial\alpha}=\frac{-a\theta(83-30\theta^2)}{2(68-83\theta^2+15\theta^4)}<0$; $\frac{\partial p_{12}^{mri}}{\partial\alpha}=\frac{a(104-51\theta^2)}{2(68-83\theta^2+15\theta^4)}>0$; $\frac{\partial p_{22}^{mri}}{\partial\alpha}=\frac{a(92-39\theta^2)}{2(68-83\theta^2+15\theta^4)}>0$; $\frac{\partial p_{11}^{mrni}}{\partial\alpha}=\frac{-5a\theta(2-\theta^2)}{2(8-3\theta^2)(1-\theta^2)}<0$; $\frac{\partial p_{12}^{mrni}}{\partial\alpha}=\frac{a(24-15\theta^2+\theta^4)}{4(8-3\theta^2)(1-\theta^2)}>0$

(iii) In Scenarios BI and BNI, $\frac{\partial p_{11}^{bi}}{\partial\alpha}=\frac{-a\theta(12942-8139\theta^2+1947\theta^4-206\theta^6+8\theta^8)}{2(1-\theta^2)\Delta_{bi}}<0$; $\frac{\partial p_{12}^{bi}}{\partial\alpha}=\frac{a(15912-11817\theta^2+2697\theta^4-248\theta^6+8\theta^8)}{2(1-\theta^2)\Delta_{bi}}>0$; $\frac{\partial p_{21}^{bi}}{\partial\alpha}=\frac{-a\theta(11484-6375\theta^2+1625\theta^4-190\theta^6+8\theta^8)}{2(1-\theta^2)\Delta_{bi}}<0$; $\frac{\partial p_{22}^{bi}}{\partial\alpha}=\frac{a(17748-15153\theta^2+4449\theta^4-512\theta^6+20\theta^8)}{2(1-\theta^2)\Delta_{bi}}>0$; $\frac{\partial p_{11}^{bni}}{\partial\alpha}=\frac{-a\theta(5-2\theta^2)}{2(4-\theta^2)(1-\theta^2)}<0$; $\frac{\partial p_{12}^{bni}}{\partial\alpha}=\frac{3a(2-\theta^2)}{2(4-\theta^2)(1-\theta^2)}>0$

Therefore, the pricing behaviors of the two products are opposite in nature. It is expected that the price of the second product will be increased to α, as the demand for the product increases. The results also demonstrate that fact. Therefore, for large α and small θ, profit differences for the two manufacturers are expected to be high. In this circumstance, the retailer's decision to uphold SI will be crucial, as in the presence of SI both manufacturers need to reduce their wholesale prices in the second period. A graphical representation of market prices for two products in the presence of SI is shown in Fig. 3.

As we found analytically, the market price for the product in the first period is always higher compared to the second period. The graphical representation also supports this fact. The main insight is that the price for the product in Scenario MMI is lower compared to the price in the other two scenarios. Therefore, customers can benefit if the two competing manufacturers cooperate. To compensate for increasing holding costs, the retailer needs to increase the market price. Fig. 3 also demonstrates this fact. In the next section, we will discuss the issue more elaborately and draw managerial insights about the study.

4. Model analysis and implications

In the previous section, we discussed the characteristics of the optimal decisions and we found that each member has the opportunity to gain higher profits. Now, we explore the relationship among the optimal profits for each member in single period game setting.

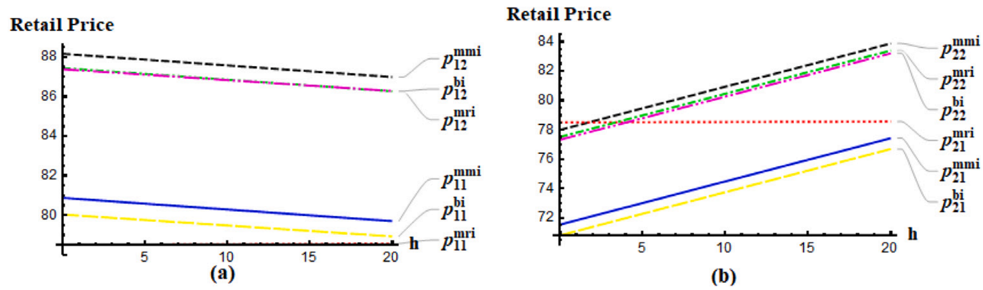

Fig. 3. Market prices for product from (a) first manufacturer and (b) second manufacturer in Scenarios MMI, MRI, and BI.

Proposition 4.1. *When market sizes are uniform ($\alpha = 1$) and the retailer does not uphold SI,*

(i) profits for the first manufacturer satisfy $\pi_{m1}^{mmni} \geq \pi_{m1}^{mrni} \geq \pi_{m1}^{bni}$
(ii) profits for the second manufacturer satisfy $\pi_{m2}^{mmni} \geq \pi_{m2}^{mrni} \geq \pi_{m2}^{bni}$
(iii) profits for the retailer satisfy $\pi_{r}^{bni} \geq \pi_{r}^{mrni} \geq \pi_{r}^{mmni}$.

We refer to Appendix F for a detailed derivation of the proof. From Proposition 4.1, we can conclude that upstream cooperation between two manufacturers always can benefit two manufacturers, not the retailer. Similarly, wholesale price commitment of the first manufacturer can benefit manufacturers, but not the retailer. Most of the existing studies discuss this decision under the Bertrand game, which leads to a sub-optimal outcome for the manufacturers. Therefore, the result suggests a notable consequence. Under competition, a manufacturer can make a deal based on the wholesale price with the retailer, to receive higher profits. Note that the difference in per-unit revenues for two products is ($R_{ij}^{k} = p_{ij}^{k} - w_{ij}^{k}$), each of which is obtained as $R_{11}^{bni} - R_{11}^{mmni} = R_{21}^{bni} - R_{21}^{mmni} = \frac{a\theta}{4(2-3\theta+\theta^2)} > 0$; $R_{11}^{mmni} - R_{11}^{mrni} = \frac{a(8-\theta^2-2\theta^3)}{4(1-\theta)(8-3\theta^2)} > 0$; $R_{12}^{mrni} - R_{12}^{mmni} = \frac{a\theta(4+3\theta-2\theta^2)}{4(1-\theta)(8-3\theta^2)} > 0$, respectively. Therefore, per-unit revenue for the retailer in different scenarios justifies the outcome of Proposition 4.1. Overall, we conclude that if two manufacturers sell their products through a common retailer, then it is always beneficial for them to cooperate. Next, we analyze the consequence when products are independent (i.e., $\theta = 0$). We refer to Appendix H for details. We find that the retailer can maintain SI to ensure higher profits and cooperation between upstream manufacturers also proves to be beneficial for all members, if $h < \frac{21a}{152}$. We propose the next proposition to explore the effect of SI.

Proposition 4.2. *If two manufacturers set their respective wholesale prices such that the retailer can keep SI and procures products in both periods, and market sizes are uniform ($\alpha = 1$), then*

(1) profits for the first manufacturer satisfy the following relations:

(i) $\pi_{m1}^{mmi} > \pi_{m1}^{mri}$

(ii) $\pi_{m1}^{mmi} \geq \pi_{m1}^{bi}$ *if* $h \in \left(max\left\{0, -\frac{2a(1-\theta)\phi_1 + ax_2\sqrt{17(1-\theta)\phi_2}}{(1-\theta)\phi_3}\right\}, min\left\{\frac{2a(1-\theta)\phi_1 + ax_2\sqrt{17(1+\theta)\phi_2}}{(1-\theta)\phi_3}, h^{bi}, h^{mmi}\right\}\right)$

(iii) $\pi_{m1}^{mri} \geq \pi_{m1}^{bi}$ *if* $h \in \left(max\left\{0, \frac{2a\phi_4 - a\theta(68-15\theta^2)x_2\sqrt{2\phi_5}}{\phi_6}\right\}, min\left\{\frac{2a\phi_4 + a\theta(68-15\theta^2)x_2\sqrt{2\phi_5}}{\phi_6}, h^{bi}, h^{mri}\right\}\right)$

(2) profits for the second manufacturer satisfy the following relations:

(i) $\pi_{m2}^{mmi} \geq \pi_{m2}^{mri}$ *if*
$h \in \left(max\left\{0, -\frac{a\theta\phi_7\sqrt{(1-\theta)} + a\theta(68-15\theta^2)\sqrt{17(68+1360\theta+3\theta^2(391+75\theta))}}{4\theta\sqrt{(1-\theta)}\phi_8}\right\}, min\left\{\frac{a\theta\phi_7\sqrt{(1-\theta)} + a\theta(68-15\theta^2)\sqrt{17(68-\theta(1360-3\theta(391-75\theta)))}}{4\theta\sqrt{(1-\theta)}\phi_8}, h^{mmi}, h^{mri}\right\}\right)$

(ii) $\pi_{m2}^{mmi} \geq \pi_{m2}^{bi}$ *if*
$h \in \left(max\left\{0, \frac{2a(1-\theta)\phi_1 - ax_2\sqrt{17(1-\theta)\phi_2}}{(1-\theta)\phi_3}\right\}, min\left\{\frac{2a(1-\theta)\phi_1 + ax_2\sqrt{17(1-\theta)\phi_2}}{(1-\theta)\phi_3}, h^{mmi}, h^{bi}\right\}\right)$

Table 3
The difference of per-unit revenue and SIs in different scenarios in the presence of SI.

Revenue difference	limits of holding cost
$R_{11}^{mri} - R_{11}^{mmi} = \frac{a(34+\theta(323+120\theta))-2h(68-\theta^2(83-15\theta))}{34(68-15\theta^2)(1-\theta)} > 0$	if $h > \frac{a(34+\theta(323+120\theta))}{2(68-\theta^2(83-15\theta))}$
$R_{11}^{bi} - R_{11}^{mmi} = R_{12}^{bi} - R_{12}^{mmi} = \frac{\theta(a(495-\theta(123+16(4-\theta)\theta))-h(1-\theta)(246-\theta(135+\theta(18-13\theta))))}{34(1+\theta)x_2} > 0$	if $h < \frac{a(495-\theta(123+16(4-\theta)\theta))}{(1-\theta)(246-\theta(135+\theta(18-13\theta)))}$
$R_{12}^{mri} - R_{12}^{mmi} = \frac{9\theta(a(34+19\theta)-8h\theta(1-\theta))}{34(1-\theta)(68-15\theta^2)} > 0$	–
$R_{21}^{mmi} - R_{21}^{mri} = \frac{a(170-323\theta-165^2)-2h(1-\theta)(340-68\theta-75\theta^2)}{34(1-\theta)(68-15\theta^2)}$	if $h < \frac{a(170-323\theta-165^2)}{2(1-\theta)(340-68\theta-75\theta^2)}$
$R_{22}^{mri} - R_{22}^{mmi} = \frac{3\theta(a(34+19\theta)-8h\theta(1-\theta))}{17(1-\theta)(68-15\theta^2)} > 0$	–
SI difference	**limits of holding cost**
$I_{2}^{bi} - I_{2}^{mmi} = \frac{2\theta(126-31\theta-20\theta^2+5\theta^3)(a-4h(1-\theta))+17h\theta^2(22+\theta(3-\theta(7+\theta)))}{34x_2} > 0$	–
$I_{2}^{mri} - I_{2}^{mmi} = \frac{5\theta(a(34+15\theta)-4h(68-(2+15\theta)\theta))}{34(68-15\theta^2)} > 0$	if $\frac{a(34+15\theta)}{272-8\theta-60\theta^2} > h$

(iii) $\pi_{m2}^{mri} \geq \pi_{m2}^{bi}$ *if*
$h \in \left(max\left\{0, \frac{2a\phi_9 - 2a(68-15\theta^2)x_1\sqrt{\phi_{10}}}{\phi_{11}}\right\}, min\left\{\frac{2a\phi_9 + 2a(68-15\theta^2)x_1\sqrt{\phi_{10}}}{\phi_{11}}, h^{mri}, h^{bi}\right\}\right)$

(3) profits for the common retailer satisfy the following relations:

(i) $\pi_{r}^{mri} \geq \pi_{r}^{mmi}$ *if* $h \in \left(max\left\{0, \frac{a(1-\theta)\phi_{12} - 17a(68-15\theta^2)\sqrt{(1-\theta)\phi_{13}}}{16(1-\theta)\phi_{14}}\right\}, min\left\{\frac{a(1-\theta)\phi_{12} + 17a(68-15\theta^2)\sqrt{(1-\theta)\phi_{13}}}{16(1-\theta)\phi_{14}}, h^{mmi}, h^{mri}\right\}\right)$

(ii) $\pi_{r}^{bi} \geq \pi_{r}^{mmi}$, $\forall \quad \theta \in (0, 1)$

(iii) $\pi_{r}^{mri} \geq \pi_{r}^{bi}$ *if* $h \in \left(max\left\{0, \frac{a\phi_{15} - a(68-15\theta^2)x_2\sqrt{\phi_{16}}}{\phi_{17}}\right\}, min\left\{\frac{a\phi_{15} + a(68-15\theta^2)x_2\sqrt{\phi_{16}}}{\phi_{17}}, h^{mri}, h^{bi}\right\}\right)$

We refer to Appendix G for the proof. Unlike the straightforward results obtained in the absence of SI, it is difficult to obtain such clear-cut preference in the presence of SI. Proposition 4.2 reflects that the opportunity exists for all members to gain higher profits in all the decision making context in the presence of SI. It also can be observed that the first manufacturer always receives higher profits by cooperating with the competitor, not with the retailer if market sizes are uniform. Note that the $68 + 1360\theta + 1173\theta^2 + 225\theta^3$ is positive. Therefore, the second manufacturer can also receive higher profits by cooperating with the competitor, not with the retailer. Therefore, in this scenario, if $h \leq min\{h^{mmi}, h^{mri}\}$, then both the manufacturers receive higher profits in Scenario MMI, compared to Scenario MRI. We compute the differences of per-unit revenues and SIs for the retailer in the presence of SI, and the differences are presented in the table below:

Table 3 also demonstrates that the holding cost for the retailer has a significant impact on the per-unit revenue difference. Graphical representation for the profit function in all six scenarios is presented in Figs. 4–5 and the amount of the SIs for two products in Fig. 6. Note

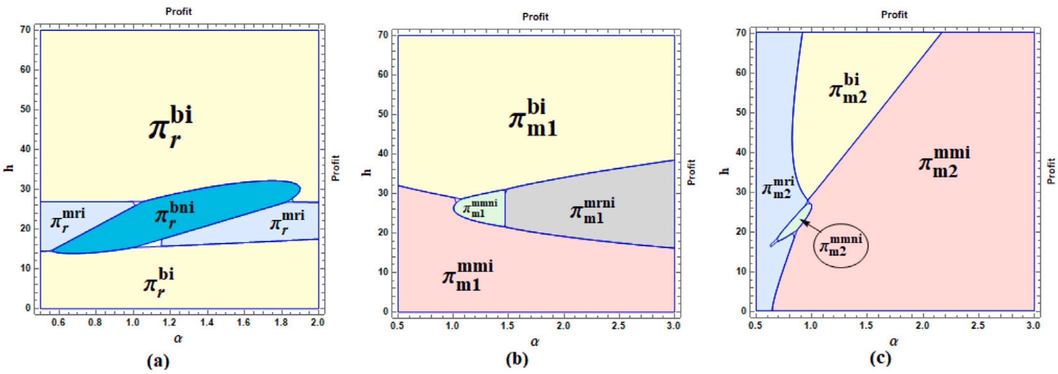

Fig. 4. Profits for (a) the common retailer, (b) first manufacturer and (c) second manufacturer for $a = 100,\ h \in (0, 70),\ \alpha \in (0.5, 2),\ \theta = 0.05$.

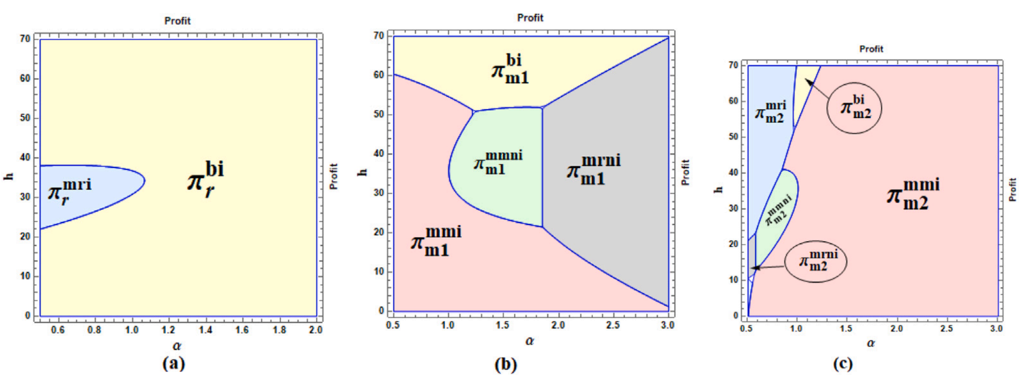

Fig. 5. Profits for (a) the common retailer, (b) first manufacturer and (c) second manufacturer for $a = 100,\ h \in (0, 70),\ \alpha \in (0.5, 2),\ \theta = 0.3$.

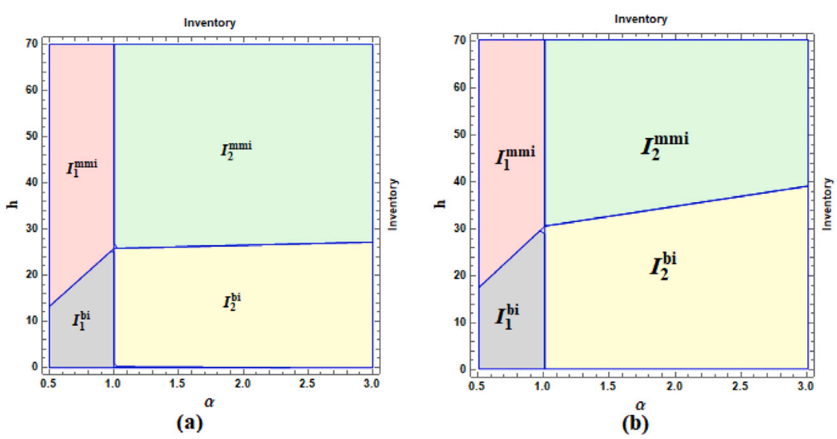

Fig. 6. Amount of strategic inventories for (a) $\theta = 0.05$ and (b) $\theta = 0.3$, where $a = 100,\ h \in (0, 70),\ \alpha \in (0.5, 2)$.

that the highlighted region in these figures represents the region where the profit is maximum compared to the other scenarios.

From Figs. 5 and 6, one can observe the following notable results in SI literature that consider the factor of competition. First, if two manufacturers set their respective wholesale prices so that the retailer can maintain SI and procure products in both periods when the holding cost lies within a certain threshold, then all the members of the supply chain may receive higher profits compared to the setup with a single-period game decision. These results differ from the single-retailer-and-single-manufacturer supply chain setting (Anand et al., 2008; Arya & Mittendorf, 2013), where the upstream manufacturer always receives higher profits in the presence of SI. Figs. 4 and 5 show that upstream cooperation between two manufacturers or wholesale price commitment is not always favorable; both manufacturers can receive higher profits in Scenario BI. Note that existing literature mostly highlights the performance for the supply chain members under the Bertrand competition setup, but in practice, two manufacturers cooperate to safeguard each other. Second, market sizes are important under scenarios that involve competition. Although researchers studied their models by assuming that the market sizes are uniform (Li & Chen, 2018; Pan et al., 2010), which confines the reality, the present study demonstrates that variations in market sizes have a significant impact on profitability. For example, if $\alpha > 1$ and consumers' cross-price sensitivity is low($\theta = 0.05$), then the second manufacturer receives higher profits in Scenario MRI, but results differ if cross-price sensitivity is high ($\theta = 0.3$). If θ is less, then price differences between the two products have less impact on the demand of others, and the manufacturer with a higher demand can set a higher price. Therefore, under

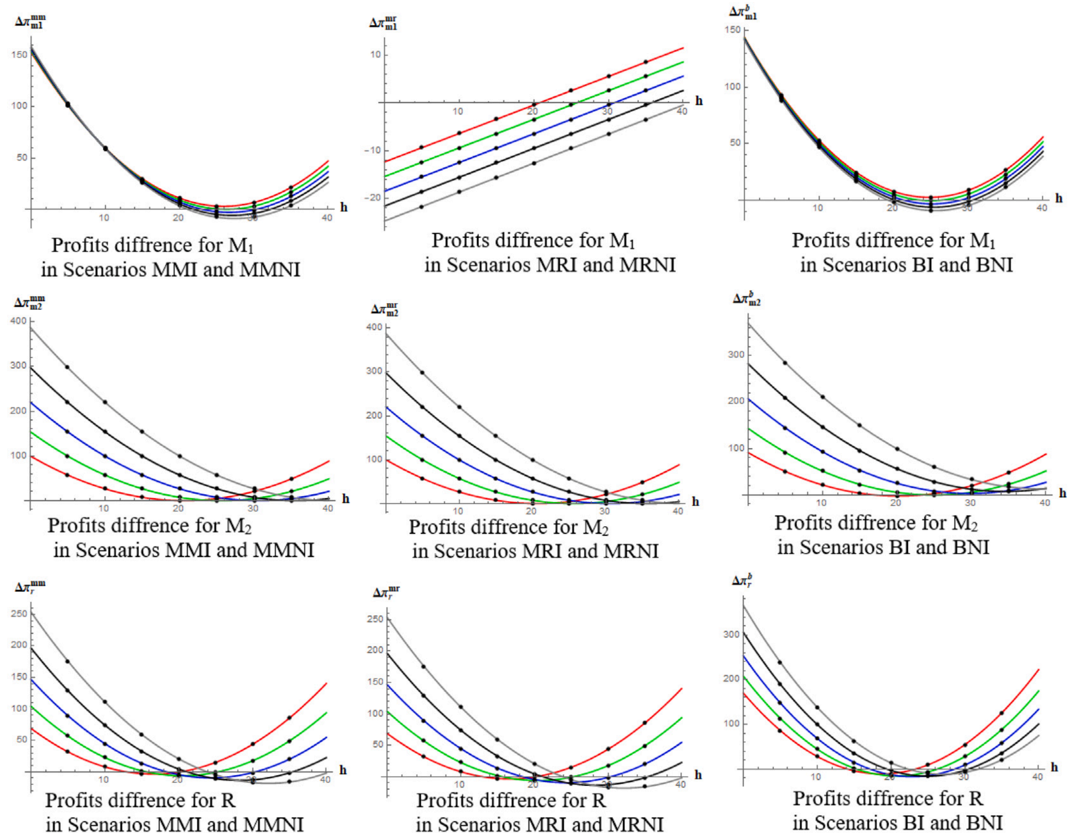

Fig. 7. Sensitivity analysis of profit differences for supply chain members for $a = 100$, $\theta = 0.05$, and $\alpha = 0.8$ (red), $\alpha = 1$ (green), $\alpha = 1.2$ (blue), $\alpha = 1.4$ (black), $\alpha = 1.6$ (gray).

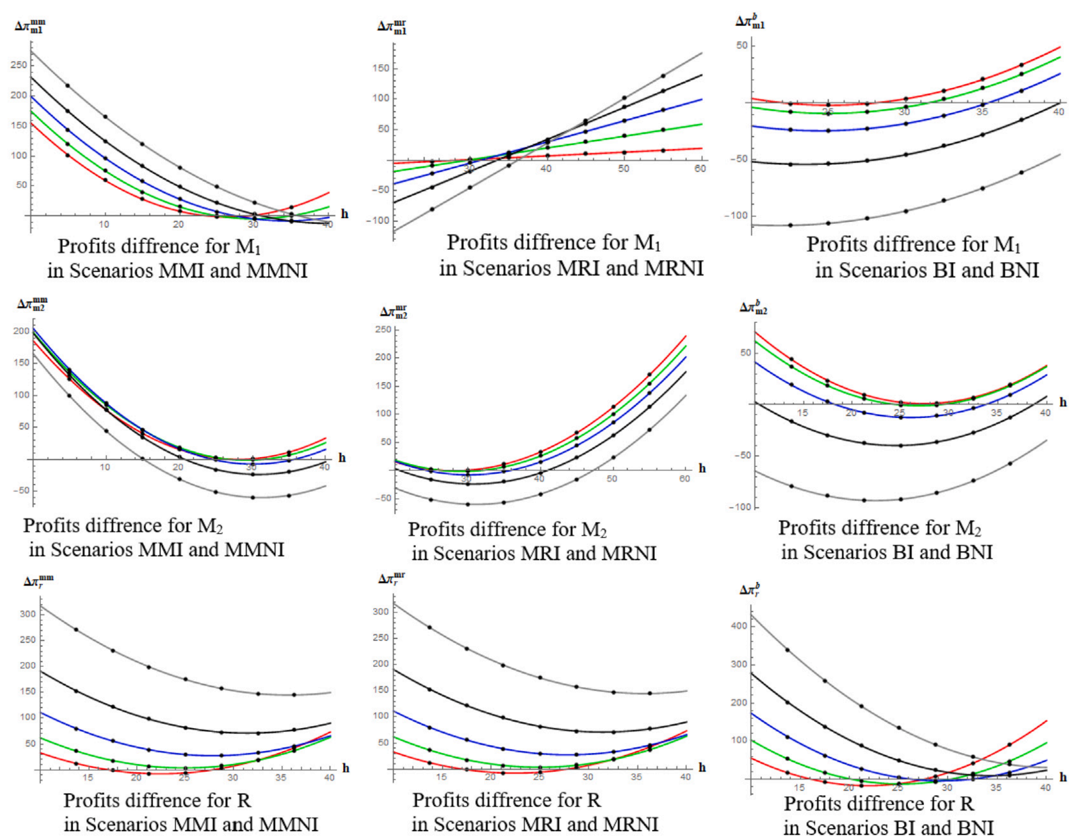

Fig. 8. Sensitivity analysis of profit differences for supply chain members for $a = 100$, $\alpha = 1.1$, and $\theta = 0.05$ (red), $\theta = 0.15$ (green), $\theta = 0.25$ (blue), $\theta = 0.35$ (black), $\theta = 0.45$ (gray).

competition, that manufacturer can lose some potential customers, as they will find alternative products at lower prices. In that circumstance, if manufacturers cooperate, the aggregated market size can reduce the effect of imbalance in the market sizes, and cooperation will benefit both. Last, but not least, Fig. 6 demonstrates a clear pattern of the amount of SI if $\alpha < 1$, the first manufacturer, has a higher market size. Therefore, I_1^k is expected to dominate and reverse for I_2^k when $\alpha > 1$. It is true to some extent. Although the second manufacturer has less demand, it can still hold higher SIs to ensure a wholesale price advantage. One thing is clear, as the region of higher profits and higher SIs do not coincide, from the perspective of the retailer we can conclude that higher volume SIs do not mean that the retailer can receive higher profits.

We conduct sensitivity analyses with respect to two major parameters to point to the profit gain for each member: ($\Delta_p^k = \pi_g^{k_1} - \pi_g^{k_2}, g = m_1, m_2, r; k_1 = mmi, mri, bi$; and $k_2 = mmni, mrni, bni$). The graphical representation of this is presented in Figs. 7 and 8 below:

From the expression of the optimal threshold of holding costs, one can find that the retailer has more flexibility to uphold inventories as demand increases. The limits of holding costs in Figs. 7 and 8 also justify that fact. The total market demand for the two products increases with α. Note that if α increased, then demand for the second product increased directly and the retailer would set a higher market price for that product. Consequently, some customers would prefer less expensive products and in this way raise demand for the alternative product. Therefore, profits for every member also will increase. The above figure also supports this fact. From the vertical axis, which represents the profit difference between the two scenarios, the upstream manufacturers' decisions have a significant impact on the profitability. The profit difference is at a higher level in the Bertrand competition for all three members. However, the impact of cross-price elasticity differs according to different decision-making contexts. As cross-price elasticity increases, the manufacturer with higher demand, along with the retailer, cannot freely increase the wholesale and market price for the product being considered, because these members of the supply chain can lose some potential customers. Consequently, demand decreases and the retailer's flexibility to uphold SIs also decreases. The results in Scenario MMI and BI also demonstrate this fact. Overall, cross-price elasticity and holding cost are important for both manufacturers to make decision-making.

Theoretical and empirical analyses that explore vertical relationships by emphasizing the effect of dependence on long-term orientation, transparency, and commitment have become increasingly popular in economics and supply chain management literature (Ganesan, 1994; Garrett, 2019; Geyskens et al., 1999; Lui & Ngo, 2012). In practice, a manufacturer's commitment to a retailer influences their long-term relationship in several ways. Commitment can reduce the perception of risk linked to opportunistic behaviors, can help meet targets, can improve supply chain competencies, and can increase price certainty, among other factors. On the other hand, we can observe many business scenarios where competition increasingly transformed from confrontation to cooperation in achieving economies of scale and range (Cygler & Sroka, 2017; Luo et al., 2007). Continuing in this direction, we made an effort to analyze such a possibility of cooperation and compared the corresponding equilibrium. Our analysis reveals that if the holding cost is less than a certain threshold, the manufacturer's strategic decision to offer dynamic wholesale prices is beneficial for all the members, compared to the outcomes in single-period decision models. In a two-period decision setting, if cross-price elasticity is high, then two competing manufacturers can receive higher profits through cooperation at the horizontal level because they can mitigate the price discrimination effect due to demand variation, to some extent. Moreover, the manufacturer also can be better off through wholesale price commitment.

5. Conclusions

This study highlights the effect of strategic inventories in a single-retailer-and-two-competing-manufacturers supply chain setting by also taking into account how cooperation between two manufacturers or wholesale price commitment between one of the manufacturers and the common retailer, affects performance in a two-period decision-making environment. Note that the presence of SIs provides the opportunity for the retailer to source products either from its stock, purchased in previous periods, or from the manufacturer in the present period. Therefore, manufacturers ultimately compete against their products. Therefore, it always remains challenging to study strategic measures from the perspective of upstream manufacturers.

The findings of the present study enable us to make managerial recommendations for competing manufacturers operating with a common retailer. First, we found that the commitment between the common retailer and manufacturer does not necessarily always improve their profits. Moreover, the manufacturer who offers a commitment may sometimes get hurt financially. This is because the manufacturer is unable to set a wholesale price dynamically. Thereby, if cross-price elasticity is low and the market size for the competitor is high, that manufacturer can lose the opportunity to improve profits. Still, in some market parameter settings, especially if cross-price sensitivity is high, a manufacturer can ensure higher profits by offering a pricing commitment to a retailer. Second, cooperation between the two manufacturers can be a potential strategic measure from the perspective of improving the performance of the overall supply chain, but does not always result in profits in the presence of SI. Customers also can receive products with lower prices under this scenario. If cross-price elasticity is negligible (i.e., products are independent), then manufacturers can use this strategic measure to ensure higher profits. The manufacturers still prefer Scenario BI to receive higher profits. Finally, single-retailer-and-two-competing-manufacturers supply chain models have, until the present study, been considered only in single-period game settings. However, at first we restricted our analysis to setups in which the manufacturers offered wholesale prices so that the retailer could procure products in both periods and maintain SI, and we found that in all three decision-making contexts, a two-period decision in the presence of SI can still outperform a single-period game decision if the holding cost for the product is low. Therefore, multi-period planning for supply chain members under upstream competition can be an excellent option to improve performance if holding cost is not high.

This study can be extended in several directions. Instead of deriving the complete solution under a dynamic wholesale price contract in a two-period supply chain (Anand et al., 2008; Mantin & Veldman, 2019; Roy et al., 2018), we presented the decision in which the manufacturers offer wholesale prices so that the retailer can procure products in two periods and carry SI. Therefore, one of the immediate extensions of this study is to analyze the behavior of equilibrium for each game structure for arbitrary values of holding cost or other parameters and compare the outcomes outside the threshold of the holding cost as presented in this study. Note that the thresholds of the holding costs are unequal, and vary with game structures. However, doing so could increase analytical computational complexity significantly. Although there are several factors at play in any supply chain, such as quality, reputation, and store location, all affecting the demand of the substitute products, one can extend the proposed model in this study by integrating some of the factors considered into the demand function. As we show in Table 1, researchers have analyzed the equilibrium under various game structures in a supply chain consisting of two manufacturers and a common retailer. Therefore, one can extend our study to explore the influence of SIs in other game structures. Finally, we assume that the upstream manufacturers cooperate to maximize their sum of profits(Bian et al., 2020; Ferrell et al., 2020; Gu et al., 2019; Sudhir, 2001). Therefore, one can study the effect of profit-sharing through a bargaining contract mechanism between two manufacturers.

CRediT authorship contribution statement

Subrata Saha: Conceptualization, Methodology, Software, Data curation, Writing – original draft, Visualization, Investigation, Software. **Izabela Ewa Nielsen:** Visualization, Investigation, Validation. **Ilkyeong Moon:** Conceptualization, Supervision, Validation, Writing – review & editing.

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Appendix A. Derivation of optimal decision in scenario MMI

The optimal response for the retailer's second period profit maximization problem defined in Eq. (3) is obtained by solving $\frac{\partial \pi_{2r}^{mmi}}{\partial p_{21}^{mmi}} = 0$ and $\frac{\partial \pi_{2r}^{mmi}}{\partial p_{22}^{mmi}} = 0$, simultaneously. After simplification, the retailer's responses on the second period market prices are obtained as

$$p_{21}^{mmi}(w_{21}^{mmi}) = \frac{a(1+\alpha\theta) + w_{21}^{mmi}(1-\theta^2)}{2(1-\theta^2)} \text{ and}$$
$$p_{22}^{mmi}(w_{22}^{mmi}) = \frac{a(\alpha+\theta) + w_{22}^{mmi}(1-\theta^2)}{2(1-\theta^2)}$$

Therefore, the market prices of products in the second period are not directly affected by the wholesale prices of the other products. The value of the determinant of the Hessian matrix (H_{2r}^{mmi})for the retailer's profit function is

$$H_{2r}^{mmi} = \begin{vmatrix} \frac{\partial^2 \pi_{2r}^{mmi}}{\partial p_{21}^{mmi\,2}} & \frac{\partial^2 \pi_{2r}^{mmi}}{\partial p_{21}^{mmi} \partial p_{22}^{mmi}} \\ \frac{\partial^2 \pi_{2r}^{mmi}}{\partial p_{21}^{mmi} \partial p_{22}^{mmi}} & \frac{\partial^2 \pi_{2r}^{mmi}}{\partial p_{21}^{mmi\,2}} \end{vmatrix} = \begin{vmatrix} -2 & 2\theta \\ 2\theta & -2 \end{vmatrix} = 4(1-\theta^2) > 0$$

Because all the diagonal elements are negative and the value of the determinant is positive, the profit function for the retailer in second period is always concave. Therefore, the order quantities in the second period are $q_{21}^{mmi} = \frac{a - 2I_1^{mmi} - w_{21}^{mmi} + w_{22}^{mmi}\theta}{2}$ and $q_{22}^{mmi} = \frac{a\alpha + w_{21}^{mmi}\theta - 2I_2^{mmi} - w_{22}^{mmi}}{2}$. Consequently, we obtain the following four cases. Case 1: The retailer may procure both products in the second period if (i) $a - w_{21}^{mmi} + w_{22}^{mmi}\theta \geq 2I_1^{mmi}$, and $a\alpha + w_{21}^{mmi}\theta - w_{22}^{mmi} \geq 2I_2^{mmi}$. Case 2: The retailer may procure only the first product in the second period if (i) $a - w_{21}^{mmi} + w_{22}^{mmi}\theta \geq 2I_1^{mmi}$, and $a\alpha + w_{21}^{mmi}\theta - w_{22}^{mmi} \leq 2I_2^{mmi}$. Case 3: The retailer may procure only the second product in the second period if (i) $a - w_{21}^{mmi} + w_{22}^{mmi}\theta \leq 2I_1^{mmi}$, and $a\alpha + w_{21}^{mmi}\theta - w_{22}^{mmi} \geq 2I_2^{mmi}$. Case 4: The retailer cannot buy anything in the second period due to the higher wholesale prices offered by two manufacturers in the second period if (i) $a - w_{21}^{mmi} + w_{22}^{mmi}\theta \leq 2I_1^{mmi}$, and $a\alpha + w_{21}^{mmi}\theta - w_{22}^{mmi} \leq 2I_2^{mmi}$. Therefore, the retailer will buy and sell products in both periods and might maintain strategic inventory (SI) only in Case 1. In all other cases, the retailer can either buy large amounts of products in the first period and sell those in the first and second periods, or the retailer can sell no products in the second period, and the game might terminate after the first period. To demonstrate the effect of two period procurement in the presence of SI, we restrict our analysis to the first case only.

Substituting the retailer's response in Eq. (4), the sum of profits for two manufacturers in the second period is obtained as

$$\pi_{2m}^{mmi} = \frac{a(w_{21}^{mmi} + w_{22}^{mmi}\alpha) - w_{22}^{mmi}(2I_2^{mmi} + w_{22}^{mmi}) - w_{21}^{mmi}(2I_1^{mmi} + w_{21}^{mmi} - 2w_{22}^{mmi}\theta)}{2}$$

Therefore, the optimal responses for the two manufacturers are obtained by solving $\frac{\partial \pi_{2m}^{mmi}}{\partial w_{21}^{mmi}} = 0$ and $\frac{\partial \pi_{2m}^{mmi}}{\partial w_{22}^{mmi}} = 0$, simultaneously. On simplification,

$$w_{21}^{mmi} = \frac{a(1+\alpha\theta) - 2(I_1^{mmi} + I_2^{mmi}\theta)}{2(1-\theta^2)} \quad \text{and}$$
$$w_{22}^{mmi} = \frac{a(\alpha+\theta) - 2(I_2^{mmi} + I_1^{mmi}\theta)}{2(1-\theta^2)}$$

One can observe that both manufacturers need to reduce wholesale prices in the presence of SI. The sum of profits for the two manufacturers is concave because $\frac{\partial^2 \pi_{2m}^{mmi}}{\partial w_{21}^{mmi\,2}} = -1 < 0$, and $\frac{\partial^2 \pi_{2m}^{mmi}}{\partial w_{21}^{mmi\,2}} \times \frac{\partial^2 \pi_{2m}^{mmi}}{\partial w_{22}^{mmi\,2}} - \left(\frac{\partial^2 \pi_{2m}^{mmi}}{\partial w_{21}^{mmi} \partial w_{22}^{mmi}}\right)^2 = 1 - \theta^2 > 0$, respectively.

Substituting a response for the second period in Eq (5), the profit function for the retailer in two consecutive periods is obtained as follows:

$$\pi_r^{mmi} = (p_{11}^{mmi} - w_{11}^{mmi})D_{11}^{mmi} + (p_{12}^{mmi} - w_{12}^{mmi})D_{12}^{mmi} - w_{11}^{mmi} I_1^{mmi} - w_{12}^{mmi} I_2^{mmi} - h(I_1^{mmi} + I_2^{mmi})$$
$$+ \frac{a^2(1+\alpha^2+2\alpha\theta) + 12a(I_1^{mmi}(1+\alpha\theta) + I_2^{mmi}(\alpha+\theta)) - 12(I_1^{mmi\,2} + I_2^{mmi\,2} + 2I_1^{mmi} I_2^{mmi}\theta)}{16(1-\theta^2)}$$

Therefore, optimal response for the retailer is obtained by solving $\frac{\partial \pi_r^{mmi}}{\partial p_{11}^{mmi}} = 0$, $\frac{\partial \pi_r^{mmi}}{\partial p_{12}^{mmi}} = 0$, $\frac{\partial \pi_r^{mmi}}{\partial I_1^{mmi}} = 0$, $\frac{\partial \pi_r^{mmi}}{\partial I_2^{mmi}} = 0$, simultaneously. On simplification,

$$p_{11}^{mmi} = \frac{a(1+\alpha\theta) + w_{11}^{mmi}(1-\theta^2)}{2(1-\theta^2)} \quad \text{and}$$
$$I_1^{mmi} = \frac{3a - 4(w_{11}^{mmi} + h - (w_{12}^{mmi} + h)\theta)}{6}$$

$$p_{12}^{mmi} = \frac{a(\alpha+\theta) + w_{12}^{mmi}(1-\theta^2)}{2(1-\theta^2)} \quad \text{and}$$
$$I_2^{mmi} = \frac{3a\alpha - 4(w_{12}^{mmi} + h - (w_{11}^{mmi} + h)\theta)}{6}$$

Note that the amounts of ST decrease with holding cost. Therefore, if the wholesale price offered by the manufacturers is high, it might be infeasible for the retailer to maintain SI. To verify concavity, we evaluated the Hessian matrix(H_r^{mmi}) for the retailer's profit function, which is obtained as

$$H_r^{mmi} = \begin{bmatrix} \frac{\partial^2 \pi_r^{mmi}}{\partial p_{11}^{mmi\,2}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{11}^{mmi} \partial p_{12}^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{11}^{mmi} \partial I_1^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{11}^{mmi} \partial I_2^{mmi}} \\ \frac{\partial^2 \pi_r^{mmi}}{\partial p_{11}^{mmi} \partial p_{12}^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{12}^{mmi\,2}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{12}^{mmi} \partial I_1^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{12}^{mmi} \partial I_2^{mmi}} \\ \frac{\partial^2 \pi_r^{mmi}}{\partial p_{11}^{mmi} \partial I_1^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{12}^{mmi} \partial I_1^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial I_1^{mmi\,2}} & \frac{\partial^2 \pi_r^{mmi}}{\partial I_1^{mmi} I_2^{mmi}} \\ \frac{\partial^2 \pi_r^{mmi}}{\partial p_{11}^{mmi} \partial I_2^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial p_{12}^{mmi} \partial I_2^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial I_1^{mmi} I_2^{mmi}} & \frac{\partial^2 \pi_r^{mmi}}{\partial I_2^{mmi\,2}} \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 2\theta & 0 & 0 \\ 2\theta & -2 & 0 & 0 \\ 0 & 0 & \frac{-3}{2(1-\theta^2)} & \frac{-3\theta}{2(1-\theta^2)} \\ 0 & 0 & \frac{-3\theta}{2(1-\theta^2)} & \frac{-3\theta}{2(1-\theta^2)} \end{bmatrix}$$

The cumulative profit function for the retailer is concave as the values of principal minors are $H_{r1}^{mmi} = -2 < 0$; $H_{r2}^{mmi} = 4(1-\theta^2) > 0$; $H_{r3}^{mmi} = -6 < 0$ and $H_{r4}^{mmi} = 9 > 0$, respectively.

Therefore, the amount of SI will be zero, if the wholesale prices set by the manufacturers in the first period are greater than $w_{11}^{minI1} = \frac{3(a+a\alpha\theta)}{4(1-\theta^2)} - h$, $w_{12}^{minI1} = \frac{3a(\alpha+\theta)}{4(1-\theta^2)} - h$, respectively. Similarly, the quantity in the first period is obtained as $Q_{11} = \frac{a - w_{11}^{mmi} + w_{12}^{mmi}\theta}{2}$ and $Q_{12} = \frac{a\alpha + w_{11}^{mmi}\theta - w_{12}^{mmi}}{2}$. Substituting the optimal response for the retailer in Eq. (6), the cumulative profit function for two manufacturers in two consecutive periods is obtained as follows:

$$\pi_m^{mmi} = \frac{18a(w_{11}^{mmi} + w_{12}^{mmi}\alpha) + 4h(1-\theta)(2h - w_{11}^{mmi} - w_{12}^{mmi}) - 17(w_{11}^{mmi\,2} - 2w_{11}^{mmi} w_{12}^{mmi}\theta + w_{12}^{mmi\,2})}{18}$$

Finally, the wholesale prices for two products for the first period are obtained by solving $\frac{\partial \pi_m^{mmi}}{\partial w_{11}^{mmi}} = 0$ and $\frac{\partial \pi_m^{mmi}}{\partial w_{21}^{mmi}} = 0$, simultaneously. The simplified value of the optimal solution is presented in Proposition 3.1. The cumulative profit function for the two manufacturers is concave because $\frac{\partial^2 \pi_m^{mmi}}{\partial w_{11}^{mmi\,2}} = \frac{\partial^2 \pi_m^{mmi}}{\partial w_{21}^{mmi\,2}} = -\frac{17}{9} < 0$, and $\frac{\partial^2 \pi_m^{mmi}}{\partial w_{11}^{mmi\,2}} \times \frac{\partial^2 \pi_m^{mmi}}{\partial w_{21}^{mmi\,2}} - \left(\frac{\partial^2 \pi_m^{mmi}}{\partial w_{11}^{mmi} \partial w_{21}^{mmi}}\right)^2 = \frac{289(1-\theta^2)}{81} > 0$. Therefore, optimal decision always exists in Scenario MMI if $\min\{\frac{a}{4(1-\theta)}, \frac{a\alpha}{4(1-\theta)}\} \geq h (= h^{mmi}, say)$.

From the expressions of the optimal decision in Proposition 3.1, we find that the order quantities for the common retailer for two consecutive periods are $D_{21}^{mmi} - I_1^{mmi} = \frac{3a+5h(1-\theta)}{17}$; $D_{22}^{mmi} - I_2^{mmi} = \frac{3a\alpha+5h(1-\theta)}{17}$; $D_{11}^{mmi} + I_1^{mmi} = \frac{13a-18h(1-\theta)}{34}$; $D_{12}^{mmi} + I_2^{mmi} = \frac{13a\alpha-18h(1-\theta)}{34}$, respectively. Therefore, order quantities in the first period are decreasing with respect to the holding cost, but the reverse trend follows for the second period. It is intuitive that the amount of SI decreases if the holding cost is high, and the results demonstrate this fact. Moreover, we can see from the simplified version of profit functions that those are convex with respect to the holding cost because $\frac{\partial^2 \pi_{m1}^{mmi}}{\partial h^2} = \frac{\partial^2 \pi_{m2}^{mmi}}{\partial h^2} = \frac{8(1-\theta)}{17} > 0$, and $\frac{\partial^2 \pi_r^{mmi}}{\partial h^2} = \frac{304(1-\theta)}{289} > 0$, respectively. It is expected that as the holding cost increases the profit should decrease, and after a certain limits, either the amount of SI or the order quantity become negative.

Appendix B. Derivation of optimal decision in scenario MRI

The optimal response for the retailer's second period profit maximization problem defined in Eq. (7) is obtained by solving $\frac{\partial \pi_{2r}^{mri}}{\partial p_{21}^{mri}} = 0$ and $\frac{\partial \pi_{2r}^{mri}}{\partial p_{22}^{mri}} = 0$, simultaneously. After simplification, the retailer's responses on market prices are obtained as $p_{21}^{mri} = \frac{a(1+\alpha\theta)+w_1^{mri}(1-\theta^2)}{2(1-\theta^2)}$ and $p_{22}^{mri} = \frac{a(\alpha+\theta)+w_{22}^{mri}(1-\theta^2)}{2(1-\theta^2)}$. The retailer's second period profit function is concave because the value of determinant of the Hessian matrix (H_{r2}^{mri}) is

$$H_{r2}^{mri} = \begin{vmatrix} \frac{\partial^2 \pi_{2r}^{mri}}{\partial p_{21}^{mri\,2}} & \frac{\partial^2 \pi_{2r}^{mri}}{\partial p_{21}^{mri} \partial p_{22}^{mri}} \\ \frac{\partial^2 \pi_{2r}^{mri}}{\partial p_{21}^{mri} \partial p_{22}^{mri}} & \frac{\partial^2 \pi_{2r}^{mri}}{\partial p_{21}^{mri\,2}} \end{vmatrix} = \begin{vmatrix} -2 & 2\theta \\ 2\theta & -2 \end{vmatrix} = 4(1-\theta^2) > 0$$

and the diagonal elements are negative.

Substituting these values in Eqs. (8) and (9), the second period profit functions for two manufacturers are obtained as follows:

$$\pi_{2m1}^{mri} = \frac{w_1^{mri}(a - w_1^{mri} + w_{22}^{mri}\theta)}{2} \quad \text{and}$$

$$\pi_{2m2}^{mri} = \frac{w_{22}^{mri}(a\alpha - 2I_2^{mri} - w_{22}^{mri} + w_1^{mri}\theta)}{2}$$

Therefore, wholesale price for the second manufacturer in the second period is obtain by solving $\frac{\partial \pi_{2m2}^{mri}}{\partial w_{22}^{mri}} = 0$. On simplification, $w_{22}^{mri} = \frac{a\alpha+w_1^{mri}\theta-2I_2^{mri}}{2}$. The profit function for the second manufacturer in the second period is also concave because $\frac{\partial^2 \pi_{2m2}^{mri}}{\partial w_{22}^{mri\,2}} = -1 < 0$. Similar to Scenario MMI, the wholesale price decreases as SI increase.

Substituting response in the second period in Eq. (10), the profit function for the retailer in two period is obtained as given in Box II.

Therefore, the optimal decision for the retailer is obtained by solving $\frac{\partial \pi_r^{mri}}{\partial p_{11}^{mri}} = 0$, $\frac{\partial \pi_r^{mri}}{\partial p_{12}^{mri}} = 0$, and $\frac{\partial \pi_r^{mri}}{\partial I_2^{mri}} = 0$, simultaneously. On simplification, the following solution is obtained:

$$p_{11}^{mri} = \frac{a(1+\alpha\theta) + w_1^{mri}(1-\theta^2)}{2(1-\theta^2)}; \quad p_{12}^{mri} = \frac{a(\alpha+\theta) + w_{12}^{mri}(1-\theta^2)}{2(1-\theta^2)};$$

$$I_2^{mri} = \frac{3a\alpha - 4h - 4w_{12}^{mri} + 3w_1^{mri}\theta}{6}$$

To verify concavity for the retailer's profit function, we compute the Hessian matrix(H_r^{mri}) for the retailer's cumulative profits as follows:

$$H_r^{mri} = \begin{bmatrix} \frac{\partial^2 \pi_r^{mri}}{\partial p_{11}^{mri\,2}} & \frac{\partial^2 \pi_r^{mri}}{\partial p_{11}^{mri} \partial p_{12}^{mri}} & \frac{\partial^2 \pi_r^{mri}}{\partial p_{11}^{mri} \partial I_2^{mri}} \\ \frac{\partial^2 \pi_r^{mri}}{\partial p_{11}^{mri} \partial p_{12}^{mri}} & \frac{\partial^2 \pi_r^{mri}}{\partial p_{12}^{mri\,2}} & \frac{\partial^2 \pi_r^{mri}}{\partial p_{12}^{mri} \partial I_2^{mri}} \\ \frac{\partial^2 \pi_r^{mri}}{\partial p_{11}^{mri} \partial I_2^{mri}} & \frac{\partial^2 \pi_r^{mri}}{\partial p_{12}^{mri} \partial I_2^{mri}} & \frac{\partial^2 \pi_r^{mri}}{\partial I_2^{mri\,2}} \end{bmatrix} = \begin{bmatrix} -2 & 2\theta & 0 \\ 2\theta & -2 & 0 \\ 0 & 0 & -\frac{3}{2} \end{bmatrix}$$

The cumulative profit function for the retailer is concave as the values of principal minors are $H_{r1}^{mri} = -2 < 0$; $H_{r2}^{mri} = 4(1-\theta^2) > 0$; $H_{r3}^{mri} = -6(1-\theta^2) < 0$, are alternative in sign.

Substituting the retailer's response, the profit functions for two manufacturers are obtained as:

$$\pi_{m1}^{mri} = \frac{w_1^{mri}(6a - 6w_1^{mri} + 2h\theta + 5w_{12}^{mri}\theta)}{6} \quad \text{and}$$

$$\pi_{m2}^{mri} = \frac{4h^2 - 4hw_{12}^{mri} - 17w_{12}^{mri\,2}}{18} + w_{12}^{mri}(a\alpha + w_1^{mri}\theta)$$

and

Therefore, by solving $\frac{\partial \pi_{m1}^{mri}}{\partial w_1^{mri}} = 0$ and $\frac{\partial \pi_{m2}^{mri}}{\partial w_{12}^{mri}} = 0$, one can obtain the optimal wholesale prices as shown in Proposition 3.3. The cumulative profit function for each manufacturers are concave as $\frac{\partial^2 \pi_{m1}^{mri}}{\partial w_1^{mri\,2}} = -2 < 0$, and $\frac{\partial^2 \pi_{m2}^{mri}}{\partial w_{12}^{mri\,2}} = -\frac{17}{9} < 0$, respectively.

One can find that the order quantity for the common retailer for two consecutive periods are $D_{21}^{mri} = \frac{a(34+3\theta(3\alpha-\theta))+\theta(32h-6h\theta^2)}{2(68-15\theta^2)}$, $D_{22}^{mri} - I_2^{mri} = \frac{6a(2\alpha+\theta)+h(20-3\theta^2)}{68-15\theta^2}$, $D_{11}^{mri} = \frac{a(34+3\theta(7\alpha+\theta))-2h\theta(8-3\theta^2)}{2(68-15\theta^2)}$, and $D_{12}^{mri} + I_2^{mri} = \frac{13a(2\alpha+\theta)-h(36-11\theta^2)}{68-15\theta^2}$, respectively, which demonstrates a similar pattern. It is intuitive that the amount of SI decreases if the holding cost is high, and the results demonstrate this fact. Moreover, from a simplified version of profit functions as presented in Proposition 3.3, the optimal profit functions are concave with respect to the holding cost, because $\frac{\partial^2 \pi_r^{mri}}{\partial h^2} = \frac{32(76-35\theta^2+3\theta^4)}{(68-15\theta^2)^2} > 0$; $\frac{\partial^2 \pi_{m1}^{mri}}{\partial h^2} = \frac{128\theta^2}{(68-15\theta^2)^2} > 0$; $\frac{\partial^2 \pi_{m2}^{mri}}{\partial h^2} = \frac{8(272-136\theta^2+21\theta^4)}{(68-15\theta^2)^2} > 0$, respectively. Therefore, the optimal profit functions are convex with respect to h.

Appendix C. Derivation of optimal decision in scenario BI

The optimal solution of the retailer's optimization problem defined in Eq. (13) is obtained by solving $\frac{\partial \pi_{r2}^{bi}}{\partial p_{21}^{bi}} = 0$ and $\frac{\partial \pi_{r2}^{bi}}{\partial p_{22}^{bi}} = 0$, simultaneously. After simplification, the retailer's responses on market prices are obtained as $p_{21}^{bi}(w_{21}^{bi}) = \frac{a(1+\alpha\theta)+w_{21}^{bi}(1-\theta^2)}{2(1-\theta^2)}$ and $p_{22}^{bi}(w_{22}^{bi}) = \frac{a(\alpha+\theta)+w_{22}^{bi}(1-\theta^2)}{2(1-\theta^2)}$, respectively. Therefore, the market prices of products in the second period is not directly affected by wholesale prices of the other products. The value of determinant of the Hessian matrix(H_{r2}^{bi}) for the retailer's profit function in Eq. (13) in the second period is

$$H_{r2}^{bi} = \begin{vmatrix} \frac{\partial^2 \pi_{r2}^{bi}}{\partial p_{21}^{bi\,2}} & \frac{\partial^2 \pi_{r2}^{bi}}{\partial p_{21}^{bi} \partial p_{22}^{bi}} \\ \frac{\partial^2 \pi_{r2}^{bi}}{\partial p_{21}^{bi} \partial p_{22}^{bi}} & \frac{\partial^2 \pi_{r2}^{bi}}{\partial p_{21}^{bi\,2}} \end{vmatrix} = \begin{vmatrix} -2 & 2\theta \\ 2\theta & -2 \end{vmatrix} = 4(1-\theta^2) > 0$$

Because diagonal elements are also negative, the profit function for the retailer in second period is always concave.

Substituting, retailer's response in Eqs. (14) and (15), the second period profit functions for two manufacturers are obtained as

$$\pi_{m21}^{bi}(w_{21}^{bi}) = \frac{w_{21}^{bi}(a - 2I_1^{bi} - w_{21}^{bi} + w_{22}^{bi}\theta)}{2} \quad \text{and}$$

$$\pi_{m22}^{bi}(w_{22}^{bi}) = \frac{w_{22}^{bi}(a\alpha - 2I_2^{bi} - w_{22}^{bi} + w_{21}^{bi}\theta)}{2}$$

Therefore, wholesale prices for each manufacturer are obtained by solving $\frac{d\pi_{m21}^{bi}}{dw_{21}^{bi}} = 0$ and $\frac{d\pi_{m22}^{bi}}{dw_{22}^{bi}} = 0$, simultaneously. On simplification,

$$\pi_r^{mri} = (p_{12}^{mri} - w_{12}^{mri})D_{12}^{mri} - (w_{12}^{mri} + h)I_2^{mri} - (p_{11}^{mri} - w_1^{mri})D_{11}^{mri} + \frac{(6a\alpha(2I_2^{mri} - w_1^{mri}\theta) - 4w_1^{mri}(2a - w_1^{mri}) - 3(2I_2^{mri} - w_1^{mri}\theta)^2)((1-\theta^2)) + a^2(4 + \alpha(\alpha + 8\theta + 3\alpha\theta^2))}{16(1-\theta^2)}$$

Box II.

$w_{21}^{bi} = \frac{a(2+\alpha\theta)-4I_1^{bi}-2I_2^{bi}\theta}{4-\theta^2}$ and $w_{22}^{bi} = \frac{a(2\alpha+\theta)-4I_2^{bi}-2I_1^{bi}\theta}{4-\theta^2}$. The profit functions for each manufacturer are also concave because $\frac{d^2\pi_{m21}^{bi}}{dw_{21}^{bi\,2}} = -1 < 0$, and $\frac{d^2\pi_{m22}^{bi}}{dw_{22}^{bi\,2}} = -1 < 0$, respectively.

Substituting these values in Eq. (16), the total profit function for the retailer in two consecutive period is obtained as follows:

$$\begin{aligned}\pi_r^{bi}(p_{11}^{bi}, p_{12}^{bi}, I_1^{bi}, I_2^{bi}) &= \frac{1}{4(1-\theta^2)(4-\theta^2)^2}[4a(1-\theta^2)((I_1^{bi} + I_2^{bi}\alpha) \\ &\times (12-\theta^2) + (I_2^{bi} + I_1^{bi}\alpha)(8\theta - \theta^3)) + 4(1-\theta^2)((I_1^{bi\,2} + I_2^{bi\,2}) \\ &\times (12-\theta^2) - 2I_1^{bi}I_2^{bi}\theta(8-\theta^2)) + a^2(2\alpha\theta(8+\theta^2) + (1+\alpha^2)(4+5\theta^2))] \\ &+ (p_{11}^{bi} - w_{11}^{bi})D_{11}^{bi} + (p_{12}^{bi} - w_{12}^{bi})D_{12}^{bi} - h(I_1^{bi} + I_2^{bi}) - w_{12}^{bi}I_2^{bi} - w_{11}^{bi}I_1^{bi}\end{aligned}$$

Therefore, optimal decision for the retailer is obtained by solving $\frac{\partial\pi_r^{bi}}{\partial p_{11}^{bi}} = 0$, $\frac{\partial\pi_r^{bi}}{\partial p_{12}^{bi}} = 0$, $\frac{\partial\pi_r^{bi}}{\partial I_1^{bi}} = 0$, and $\frac{\partial\pi_r^{bi}}{\partial I_2^{bi}} = 0$, simultaneously. On simplification,

$$p_{11}^{bi} = \frac{a(1+\alpha\theta) + w_{11}^{bi}(1-\theta^2)}{2(1-\theta^2)} \quad \text{and}$$

$$I_1^{bi} = \frac{a}{2} - \frac{(w_{11}^{bi} + h)(12-\theta^2) - (w_{12}^{bi} + h)\theta(8-\theta^2)}{2(9-\theta^2)}$$

$$p_{12}^{bi} = \frac{a(\alpha+\theta) + w_{12}^{bi}(1-\theta^2)}{2(1-\theta^2)} \quad \text{and}$$

$$I_2^{bi} = \frac{a\alpha}{2} - \frac{(w_{11}^{bi} + h)\theta(8-\theta^2) + (w_{12}^{bi} + h)(12-\theta^2)}{2(9-\theta^2)}$$

To verify concavity, we determine the Hessian matrix for the retailer's profit function(H_r^{bi}) in two consecutive period, which is obtained as

$$H_r^{bi} = \begin{bmatrix} \frac{\partial^2\pi_r^{bi}}{\partial p_{11}^{bi\,2}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{11}^{bi}\partial p_{12}^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{11}^{bi}\partial I_1^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{11}^{bi}\partial I_2^{bi}} \\ \frac{\partial^2\pi_r^{bi}}{\partial p_{11}^{bi}\partial p_{12}^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{12}^{bi\,2}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{12}^{bi}\partial I_1^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{12}^{bi}\partial I_2^{bi}} \\ \frac{\partial^2\pi_r^{bi}}{\partial p_{11}^{bi}\partial I_1^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{12}^{bi}\partial I_1^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial I_1^{bi\,2}} & \frac{\partial^2\pi_r^{bi}}{\partial I_1^{bi}I_2^{bi}} \\ \frac{\partial^2\pi_r^{bi}}{\partial p_{11}^{bi}\partial I_2^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial p_{12}^{bi}\partial I_2^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial I_1^{bi}I_2^{bi}} & \frac{\partial^2\pi_r^{bi}}{\partial I_2^{bi\,2}} \end{bmatrix} = \begin{bmatrix} -2 & 2\theta & 0 & 0 \\ 2\theta & -2 & 0 & 0 \\ 0 & 0 & -\frac{2(12-\theta^2)}{(4-\theta^2)^2} & \frac{-2\theta(8-\theta^2)}{(4-\theta^2)^2} \\ 0 & 0 & \frac{-2\theta(8-\theta^2)}{(4-\theta^2)^2} & -\frac{2(12-\theta^2)}{(4-\theta^2)^2} \end{bmatrix}$$

The cumulative profit function for the retailer is concave as the values of principal minors for the Hessian matrix (H_r^{bi}) are $H_{r1}^{bi} = -2 < 0$; $H_{r2}^{bi} = 4(1-\theta^2) > 0$; $H_{r3}^{bi} = -\frac{8(1-\theta^2)(12-\theta^2)}{(4-\theta^2)^2} < 0$ and $H_{r4}^{bi} = \frac{16(9-\theta^2)(1-\theta^2)(4-\theta^2)^2}{(4-\theta^2)^4} > 0$, respectively. Substituting these values in Eqs. (17) and (18), the profit functions for two manufacturers are obtained as follows:

$$\begin{aligned}\pi_{m1}^{bi}(w_{11}^{bi}) &= \frac{1}{2(9-\theta^2)^2}[w_{11}^{bi}(2a - w_{11}^{bi} + w_{12}^{bi}\theta)(9-\theta^2)^2 \\ &+ ((6-\theta^2)(h + w_{11}^{bi}) - (h + w_{12}^{bi})\theta)^2 \\ &- w_{11}^{bi}(9-\theta^2)((12-\theta^2)(h + w_{11}^{bi}) - \theta(h + w_{12}^{bi})(8-\theta^2))]\end{aligned}$$

$$\begin{aligned}\pi_{m2}^{bi}(w_{12}^{bi}) &= \frac{1}{2(9-\theta^2)^2}[w_{12}^{bi}(2a\alpha + w_{11}^{bi}\theta - w_{12}^{bi})(9-\theta^2)^2 \\ &+ ((6-\theta^2)(h + w_{12}^{bi}) + (h + w_{11}^{bi})\theta)^2 \\ &- w_{12}^{bi}(9-\theta^2)((12-\theta^2)(h + w_{12}^{bi}) - \theta(h + w_{11}^{bi})(8-\theta^2))]\end{aligned}$$

Finally, solving $\frac{d\pi_{m1}^{bi}}{dw_{11}^{bi}} = 0$ and $\frac{d\pi_{m2}^{bi}}{dw_{21}^{bi}} = 0$, one can obtain the optimal wholesale prices as given in Proposition 3.5. Profit function for each manufacturer is also concave as $\frac{d^2\pi_{m1}^{bi}}{dw_{11}^{bi\,2}} = \frac{d^2\pi_{m2}^{bi}}{dw_{21}^{bi\,2}} = -\frac{153-27\theta^2+\theta^4}{(9-\theta^2)^2} < 0$.

From the expressions of the optimal decision in Proposition 3.5, we find that the order quantity for the common retailer for two consecutive periods are $D_{21}^{bi} - I_1^{bi} = \frac{2a(9-\theta^2)(204-63\theta^2+4\theta^4+\alpha\theta(60-25\theta^2+2\theta^4))-h(2-\theta)(30-9\theta-3\theta^2+\theta^3)x_1}{\Delta_{bi}}$, $D_{22}^{mri} - I_2^{bi} = \frac{2a(9-\theta^2)(\alpha(204-63\theta^2+4\theta^4)+\theta(60-25\theta^2+2\theta^4))-h(30+9\theta-3\theta^2-\theta^3)x_1}{\Delta_{bi}}$, $D_{11}^{bi} + I_1^{bi} = \frac{2a(3978-582\theta^2-24\theta^4+4\theta^6-\alpha\theta(9-\theta^2)(249-34\theta^2))-h(2+\theta)(54+33\theta-6\theta^2-4\theta^3)x_1}{\Delta_{bi}}$ and $D_{12}^{bi} + I_2^{bi} = \frac{2a(2241\theta-555\theta^3+34\theta^5-\alpha(3978-582\theta^2-24\theta^4+4\theta^6))-h(2+\theta)(54+33\theta-6\theta^2-4\theta^3)x_1}{\Delta_{bi}}$, respectively, which demonstrates a similar pattern. It is intuitive that the amount of SI decreases if the holding cost is high, and the results demonstrate this fact. Moreover, from the simplified version of the profit functions as presented in Proposition 3.5, the optimal profit functions are concave with respect to the holding cost because $\frac{\partial^2\pi_r^{bi}}{\partial h^2} = \frac{\partial^2\pi_{m1}^{bi}}{\partial h^2} = \frac{(2+\theta)^2(1224+1224\theta+162\theta^2-246\theta^3-78\theta^4+12\theta^5+5\theta^6)}{(102+81\theta+9\theta^2-8\theta^3-2\theta^4)^2} > 0$; and $\frac{\partial^2\pi_r^{bi}}{\partial h^2} = \frac{(2+\theta)^2(2736+2664\theta+432\theta^2-372\theta^3-171\theta^4-9\theta^5+10\theta^6+2\theta^7)}{(102+81\theta+9\theta^2-8\theta^3-2\theta^4)^2} > 0$, respectively.

Appendix D. Optimal decision in the absence of strategic inventory

We present the optimal decisions in Scenarios MMNI, MRNI, and BNI in Table D.1.

From Table D.1, we observe the following:

- The differences between profits for the two manufacturers and sales volumes of two products in Scenario MMNI are obtained as $\pi_{m1}^{mmni} - \pi_{m2}^{mmni} = \frac{a^2(1-\alpha^2)}{4(1-\theta^2)}$ and $Q_1^{mmni} - Q_2^{mmni} = \frac{a(1-\alpha)}{2}$, respectively. As expected, if α increases, then the differences also increase, and

Table D.1
Optimal decision in Scenarios MMNI, MRNI, and BNI.

Decision	Scenario MMNI	Scenario MRNI	Scenario BNI
w_{i1}^k	$\frac{a(1+\alpha\theta)}{2(1-\theta^2)}$	$\frac{2a(2+\alpha\theta)}{8-3\theta^2}$	$\frac{a(2+\alpha\theta)}{4-\theta^2}$
w_{i2}^k	$\frac{a(\alpha+\theta)}{2(1-\theta^2)}$	$\frac{a(\alpha(8-\theta^2)+4\theta)}{2(8-3\theta^2)}$	$\frac{a(2\alpha+\theta)}{4-\theta^2}$
p_{i1}^k	$\frac{3a(1+\alpha\theta)}{4(1-\theta^2)}$	$\frac{a(12-7\theta^2+5\alpha\theta(2-\theta^2))}{2(8-3\theta^2)(1-\theta^2)}$	$\frac{a(6-3\theta^2+\alpha\theta(5-\theta^2))}{2(4-\theta^2)(1-\theta^2)}$
p_{i1}^k	$\frac{3a(\alpha+\theta)}{4(1-\theta^2)}$	$\frac{a(\alpha(24-15\theta^2+\theta^4)+10\theta(2-\theta^2))}{4(8-3\theta^2)(1-\theta^2)}$	$\frac{a(6\alpha-3\alpha\theta^2+5\theta-2\theta^3)}{2(4-\theta^2)(1-\theta^2)}$
π_r^k	$\frac{a^2(1+2\alpha\theta+\alpha^2)}{8(1-\theta^2)}$	$\frac{a^2\Lambda}{8(8-3\theta^2)^2(1-\theta^2)}$	$\frac{a^2(4+5\theta^2+2\alpha\theta(8+\theta^2)+\alpha^2(4+5\theta^2))}{2(4-\theta^2)^2(1-\theta^2)}$
π_{m1}^k	$\frac{a^2(1+\alpha\theta)}{4(1-\theta^2)}$	$\frac{a^2(4-\theta^2)(2+\alpha\theta)^2}{(8-3\theta^2)^2}$	$\frac{a^2(2+\alpha\theta)^2}{(4-\theta^2)^2}$
π_{m2}^k	$\frac{a^2\alpha(\alpha+\theta)}{4(1-\theta^2)}$	$\frac{a^2(8\alpha-\alpha\theta^2+4\theta)^2}{4(8-3\theta^2)^2}$	$\frac{a^2(2\alpha+\theta)^2}{(4-\theta^2)^2}$
Q_1^k	$\frac{a}{2}$	$\frac{a(4-\theta^2)(2+\alpha\theta)}{2(8-3\theta^2)}$	$\frac{a(2+\alpha\theta)}{(4-\theta^2)}$
Q_2^k	$\frac{a\alpha}{2}$	$\frac{a(8\alpha-\alpha\theta^2+4\theta)}{2(8-3\theta^2)}$	$\frac{a(2\alpha+\theta)}{(4-\theta^2)}$

where $\Lambda = \alpha^2(3\theta^6 - 31\theta^4 + 64(1+\theta^2)) + 8\alpha(32-7\theta^2)\theta - 12\theta^4 + 48\theta^2 + 64$.

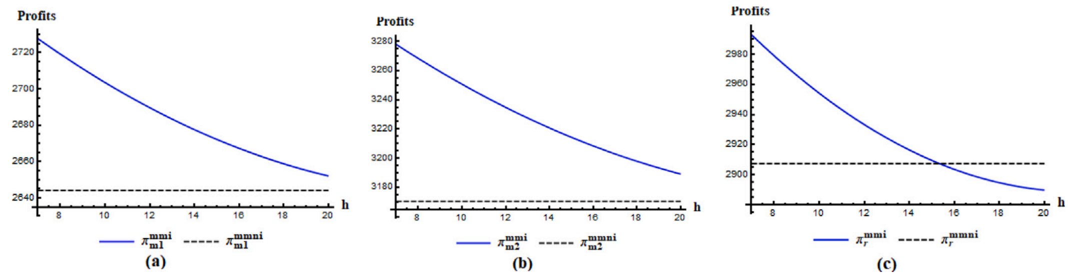

Fig. E.1. Profits for (a) first manufacturer, (b) second manufacturer, and (c) the retailer in Scenarios MMI and MMNI.

profits for the manufacturer and sales volume remain the same if $\alpha = 1$

- The differences between profits for the two manufacturers and sales volumes in Scenario MRNI are obtained as $\pi_{m1}^{mrni} - \pi_{m2}^{mrni} = \frac{a^2(8\alpha\theta^3+32(2-\theta^2)-\alpha^2(64-32\theta^2+5\theta^4))}{4(8-3\theta^2)^2}$ and $Q_1^{mrni} - Q_2^{mrni} = \frac{a(8-2(2-\theta)\theta+\alpha(8+\theta(4-\theta-\theta^2)))}{2(8-3\theta^2)}$, respectively. Therefore, if $\alpha = 1$, then the two manufacturers receive different amounts of profits, and the difference is $\pi_{m1}^{mrni} - \pi_{m2}^{mrni} = \frac{a^2(8-5\theta)\theta^3}{4(8-3\theta^2)^2} > 0$. Consequently, the commitment contract is profitable for the first manufacturer.
- The difference between sales volumes and profits for the two manufacturers in Scenario BNI are $Q_1^{bni} - Q_2^{bni} = \frac{a(1-\alpha)}{2-\theta}$ and $\pi_{m1}^{bni} - \pi_{m2}^{bni} = \frac{a^2(1-\alpha^2)}{(4-\theta^2)}$, respectively. Therefore, both profits for the manufacturer and sales volumes remain the same if $\alpha = 1$.

Appendix E. Proof of Proposition 3.2

The difference between the profits for the common retailer in Scenarios MMI and MMNI is
$\pi_r^{mmi} - \pi_r^{mmni} = \frac{21a^2(1-2\alpha\theta+\alpha^2)-236ah(1+\alpha)(1-\theta^2)+1216h^2(1-\theta)(1+\theta)^2}{2312(1-\theta^2)}$.

The numerator of above difference is a quadratic function of h, and is of the form $\beta_1 h^2 - \beta_2 h + \beta_3 = \beta_1\left(\left(h+\frac{\beta_2}{2\beta_1}\right)^2 + \frac{4\beta_3\beta_1-\beta_2^2}{4\beta_1^2}\right)$, where $\beta_1 = 1216(1-\theta)(1+\theta)^2$, $\beta_2 = 236a(1+\alpha)(1-\theta^2)$, and $\beta_3 = 21a^2(1-2\alpha\theta+\alpha^2)$, respectively.

Therefore, the expression will remain positive if $4\beta_1\beta_3-\beta_2^2 = 16a^2(1-\theta)(1+\theta)^2((2903+3481\theta)(1+\alpha^2)-\alpha(6962+5806\theta)) \geq 0$. Accordingly, we conclude that if $\Gamma_1 = (2903+3481\theta)(1+\alpha^2)-\alpha(6962+5806\theta) > 0$, the retailer always receive higher profits Scenario MMI compared to MMNI. Otherwise, i.e $4\beta_1\beta_3 \leq \beta_2^2$, one can obtain the interval as presented in Proposition 3.2.

Similarly, the differences between the profits for first manufacturers in Scenario MMI and MMNI are obtained as $\pi_{m1}^{mmi} - \pi_{m1}^{mmni} = \frac{a^2(1-\alpha\theta)+4ah(1+\theta)((3\alpha-1)\theta-2)+16h^2(1-\theta)(1+\theta)^2}{68(1-\theta^2)}$. Therefore, the first manufacturer will always receive higher profit in Scenario MMI if $16a^2(1-\alpha)\theta(1+\theta)^2((9\alpha-1)\theta-8) \geq 0$, i.e. $(1-\alpha)((9\alpha-1)\theta-8) \geq 0$.

The graphical representation of the profits of supply chain members in Scenario MMI and MMNI are presented below in Fig. E.1. The parameter values are considered as $a = 100$, $\theta = 0.05$, and $\alpha = 1.1$.

Note that the value of $(1-\alpha)\theta((9\alpha-1)\theta-8) = 0.042$, therefore the first manufacturers receives higher profits if the retailer able to carry SI as shown in Fig. E.1. However, $(1+\theta)\Gamma_1 = 1373.29$, therefore, the limits of retailer exist, and the retailer can receive higher profits if $h \in (0, 15.34)$. Note that derivation of Propositions 3.4 and 3.6 are similar in nature, and hence we omitted the detail.

Appendix F. Proof of Proposition 4.1

The profit differences for the common retailer among Scenarios MMNI, MRNI, and BNI are obtained as follows: $\pi_r^{bni} - \pi_r^{mmni} = \frac{a^2\theta(4-\theta)}{4(1-\theta)(2-\theta)^2} > 0$, $\pi_r^{bni} - \pi_r^{mrni} = \frac{a^2\theta^2(64+32\theta-28\theta^2-12\theta^3+3\theta^4)}{8(2-\theta)^2(8-3\theta^2)^2} > 0$, and $\pi_r^{mrni} - \pi_r^{mmni} = \frac{a^2\theta(128+80\theta-40\theta^2-21\theta^3+3\theta^4)}{8(1-\theta)(8-3\theta^2)^2} > 0$. The above relations justify the claim for retailer's profit.

Similarly, the profit differences for first manufacturer among Scenarios MMNI, MRNI, and BNI are obtained as $\pi_{m1}^{mrni} - \pi_{m1}^{bni} = \frac{a^2\theta^4(3-\theta^2)}{(2-\theta)^2(8-3\theta^2)^2} > 0$, $\pi_{m1}^{mmni} - \pi_{m1}^{bni} = \frac{a^2\theta^2}{4(2-\theta)^2(1-\theta)} > 0$, and $\pi_{m1}^{mmni} - \pi_{m1}^{mrni} = \frac{a^2\theta^2(16-3\theta^2-4\theta(4-\theta^2))}{4(1-\theta)(8-3\theta^2)^2} > 0$, respectively. Note that there exist only one real root for the equation $16(1-\theta)-\theta^2(3-4\theta) = 0$, and its value is $\theta = 1.11181 \notin (0,1)$. Therefore, $\pi_{m1}^{mmni} \geq \pi_{m1}^{mrni}$. By combining three inequalities, we establish the relationship among profits for the first manufacturer.

Finally, the profit differences for second manufacturer among Scenarios MMNI, MRNI, and BNI are obtained as $\pi_{m2}^{mmni} - \pi_{m2}^{bni} = \frac{a^2\theta^2}{4(1-\theta)(2-\theta)^2} > 0$, $\pi_{m2}^{mmni} - \pi_{m2}^{mrni} = \frac{a^2\theta^2(16+8\theta+\theta^3)}{4(1-\theta)(8-3\theta^2)^2} > 0$, and $\pi_{m2}^{mrni} - \pi_{m2}^{bni} = \frac{a^2\theta^3(32-12\theta^2+\theta^3)}{4(2-\theta)^2(8-3\theta^2)^2} > 0$, respectively. By combining three inequalities, we establish the relationship among profits for the second manufacturer.

Appendix G. Proof of Proposition 4.2

(i) Difference between profits for the first manufacturer in Scenarios MMI and MRI is $\pi_{m1}^{mmi} - \pi_{m1}^{mri} = \frac{\beta_4 h^2-\beta_5 h+\beta_6}{34(1-\theta)(68-15\theta^2)^2}$, where $\beta_4 = 8(1-\theta)(4624-4624\theta-2312\theta^2+2040\theta^3+225\theta^4-225\theta^5)$; $\beta_5 = 4a(1-\theta)(4624+4624\theta+225\theta^4)$; and $\beta_6 = a^2(2312+4624\theta+8670\theta^2+7650\theta^3+2025\theta^4)$. Therefore, profits for the manufacturer always satisfy $\pi_{m1}^{mmi} \geq \pi_{m1}^{mri}$ because $4\beta_4\beta_6-\beta_5^2 = 2448a^2\theta^2(1-\theta)(68-15\theta^2)^2(68-75\theta^2+25\theta^3) > 0$, $\beta_4 > 0$, and $\theta \in (0,1)$.

However, difference between profits for the second manufacturer in Scenarios MMI and MRI is $\pi_{m2}^{mmi} - \pi_{m2}^{mri} = \frac{\theta(\beta_7+\beta_8 h-\beta_9 h^2)}{34(1-\theta)(68-15\theta^2)^2}$, where $\beta_7 = 9a^2\theta(1428+1156\theta+225\theta^2)$; $\beta_8 = 4a(1-\theta)(2312-1428\theta-1734\theta^2-225\theta^3)$; and $\beta_9 = 8(1-\theta)(4624-272\theta-2040\theta^2+132\theta^3+225\theta^4)$. Therefore, $4\beta_7\beta_9-\beta_8^2 = -272a^2\theta(1-\theta)(68-15\theta^2)^2(\theta(1360+3\theta(391+75\theta))+68) \geq 0$, which is not positive always for $\theta \in (0,1)$, and we obtain the range shown in Proposition 4.2.

Similarly, the difference between profits for the retailer in Scenarios MMI and BI is $\pi_r^{bi} - \pi_r^{mmi} = \frac{\theta(\beta_{13}h^2-\beta_{14}h+\beta_{15})}{578(1-\theta)x_2^2}$ where $\beta_{13} = (1-\theta)(1943712-3206304\theta+1660536\theta^2+434280\theta^3-320076\theta^4+85497\theta^5+69696\theta^6-5742\theta^7+638\theta^8)$, $\beta_{14} = 4a(1-\theta)(305388-275022\theta+165420\theta^2+49179\theta^3-13806\theta^4-1115\theta^5+944\theta^6-118\theta^7)$, and $\beta_{15} = a^2(1687284-1389969\theta+87474\theta^2+239427\theta^3-77164\theta^4+3184\theta^5-4960\theta^6+620\theta^7)$, respectively. Therefore, profits for the retailer always satisfy $\pi_r^{mmi} \geq \pi_r^{bi}$ because $4\beta_{13}\beta_{15}-\beta_{14}^2 = 1156a^2(1-\theta)x_2^2(966672-814968\theta+585512\theta^2+138960\theta^3-43981\theta^4-2141\theta^5+2646\theta^6-294\theta^7) > 0$, and $\beta_{13} > 0$. In a similar fashion, we derive other ranges.

Appendix H. Profits for the supply chain members for $\theta = 0$

Optimal decision in six scenarios for $\theta = 0$, i.e. when products are independent is presented in Table H.1.

Therefore, profit difference for the retailer are obtained as $\pi_r^{mmi} - \pi_r^{mri} = \pi_r^{bi} - \pi_r^{mri} = \frac{(21a-152h)(a-4h)}{2312}$, i.e. the retailer always receives a higher profit in Scenario MMI compared to MRI if $a > \frac{152h}{21}$. Similarly, the retailer receives higher profits under the wholesale price commitment contract compared to the scenarios where the retailer does

Table H.1
Optimal profits for the supply chain members in six scenarios for $\theta = 0$.

Decision	Scenario MMI and BI	Scenario MRI	Scenarios BNI, MMNI, and MRI
π_r^k	$\frac{155a^2(1+\alpha^2)-118ah(1+\alpha)+608h^2}{1156}$	$\frac{a^2(289+310\alpha^2)-236ah\alpha+608h^2}{2312}$	$\frac{a^2(1+\alpha^2)}{8}$
π_{m1}^k	$\frac{9a^2-4ah+8h^2}{34}$	$\frac{a^2}{4}$	$\frac{a^2}{4}$
π_{m2}^k	$\frac{9a^2\alpha^2-4ah\alpha+8h^2}{34}$	$\frac{9a^2\alpha^2-4ah\alpha+8h^2}{34}$	$\frac{a^2\alpha^2}{4}$

not maintain SI if $a > \frac{152h}{21\alpha}$, because $\pi_r^{mri} - \pi_r^{bni} = \frac{(21a\alpha-152h)(a\alpha-4h)}{2312}$. Similarly, upstream cooperation can bring benefits for the retailer if $h \notin \left(\frac{59a(1+\alpha)-a\sqrt{(6962-2903\alpha)(\alpha-2903)}}{608}, \frac{59a(1+\alpha)-a\sqrt{(6962-2903\alpha)\alpha-2903}}{608}\right)$ because $\pi_r^{mmi} - \pi_r^{bni} = \frac{21a^2(1+\alpha^2)-236ah(1+\alpha)+1216h^2}{2312}$. However, the first manufacturer always receives higher profits in Scenario MMI compared to the rest of the scenarios because $\pi_{m1}^{mmi} - \pi_{m1}^k = \frac{(a-4h)^2}{68}$, $k = mmni, mri, mrni, bi, bni$. Similarly, the second manufacturer always receives higher profits if in Scenarios MMI or MRI compared to rest of the scenarios because $\pi_{m2}^{mmi} - \pi_{m2}^k = \pi_{m2}^{mri} - \pi_{m2}^k = \frac{(a\alpha-4h)^2}{68}$, $k = mmni, mrni, bi, bni$.

Appendix I. A list of additional symbols

List of additional symbols used are presented below:

$\Gamma_1 = (2903 + 3481\theta)(1+\alpha^2) - \alpha(6962 + 5806\theta)$,
$\Gamma_2 = 472\alpha + 174\alpha\theta^2 - 508\theta - 27\theta^3$,
$\Gamma_3 = 4\alpha^2(64 - 304\theta^2 - 823\theta^4 + 378\theta^6 - 18\theta^8) + 4\alpha\theta(192 + 3248\theta^2 - 1173\theta^4) - 9152\theta^2 + 1776\theta^4 + 513\theta^6$,
$\Gamma_4 = 4\theta(272 - 69\theta^2) + \alpha^2\theta(1344 - 472\theta^2 + 21\theta^4) - 8\alpha(272 - 2\theta^2 - 21\theta^4)$,
$\Delta_{bi} = x_1 x_2$; $x_1 = 102 - 81\theta + 9\theta^2 + 8\theta^3 - 2\theta^4$,
$x_2 = 102 + 81\theta + 9\theta^2 - 8\theta^3 - 2\theta^4$,
$x_3 = 6 + 9\theta - \theta^3$, $x_4 = 153 - 27\theta^2 + \theta^4$,
$x_5 = 141 - 33\theta^2 + 2\theta^4$,
$\Gamma_5 = a^2((1+\alpha^2)(29027160 + 27222642\theta^2 - 16146459\theta^4 + 3119130\theta^6 - 315183\theta^8 + 22698\theta^{10} - 1324\theta^{12} + 40\theta^{14}) - 2\alpha\theta(59398272 - 16798374\theta^2 - 893070\theta^4 + 478602\theta^6 - 64839\theta^8 + 4594\theta^{10} - 252\theta^{12} + 8\theta^{14})) - ah(1+\alpha)(1-\theta^2)x_1^2(2124 + 1260\theta - 189\theta^2 - 150\theta^3 + 9\theta^4 - 2\theta^6) + h^2(1-\theta^2)(2+\theta)^2 x_1^2(2736 + 2664\theta + 432\theta^2 - 372\theta^3 - 171\theta^4 - 9\theta^5 + 10\theta^6 + 2\theta^7)$,
$\Gamma_6 = 4a^2(9-\theta^2)(1591812 - \theta(4\alpha(17-\theta^2)x_4 x_5\theta + \alpha^2\theta(9-\theta^2)x_4(253 - 60\theta^2 + 4\theta^4) + \theta(665856 - 108153\theta^2 + 8781\theta^4 - 384\theta^6 + 8\theta^8)) - 4ah(2+\theta)x_1(31212 + 2754(17-\theta)\theta + \alpha\theta(9-\theta^2)(18 + 19\theta + \theta^3(14409 + 1035\theta - 1293\theta^2 - 159\theta^3 + 12\theta^4 + 6\theta^5 + 2\theta^6))) + h^2(2+\theta)^2 x_1^2(5\theta^6 + 12\theta^5 - 78\theta^4 - 246\theta^3 + 162\theta^2 + 1224\theta + 1224))$,
$\Gamma_7 = 4a^2(9-\theta^2)(\theta^2(9-\theta^2)x_4(253 - 60\theta^2 + 4\theta^4) - 4\alpha\theta x_5 x_4(17-\theta^2) + \alpha^2(1591812 - 665856\theta^2 + 108153\theta^4 - 8781\theta^6 + 384\theta^8 - 8\theta^{10})) + h^2(2+\theta)^2 x_1^2(1224 + 1224\theta + 162\theta^2 - 246\theta^3 - 78\theta^4 + 12\theta^5 + 5\theta^6) - 4ah(2+\theta)x_1\theta(9-\theta^2)(18 + 19\theta - 2\theta^3)x_4 - \alpha(31212 + 46818\theta - 2754\theta^2 - 14409\theta^3 + 1035\theta^4 + 1293\theta^5 + 159\theta^6 - 12\theta^7 - 6\theta^8 - 2\theta^9)$,
$\Gamma_8 = 2124 + 1260\theta - 189\theta^2 - 150\theta^3 + 9\theta^4 - 2\theta^6$,
$\Gamma_9 = 18045504 + 34852032\theta + 12015216\theta^2 + 9482400\theta^3 - 20497860\theta^4 - 10852020\theta^5 + 7457337\theta^6 + 2450160\theta^7 - 1119834\theta^8 - 191400\theta^9 + 58089\theta^{10} - 3476\theta^{11} + 3160\theta^{12} + 1248\theta^{13} - 492\theta^{14} - 48\theta^{15} + 16\theta^{16}$,
$\Gamma_{10} = 15049152 + 33618240\theta + 35594640\theta^2 - 21578400\theta^3 - 13509180\theta^4 + 10401480\theta^5 - 1761210\theta^6 + 1753632\theta^7 + 701358\theta^8 - 495552\theta^9 - 123849\theta^{10} + 59052\theta^{11} + 8964\theta^{12} - 3424\theta^{13} - 244\theta^{14} + 80\theta^{15}$,
$\Gamma_{11} = 2736 + 2664\theta + 432\theta^2 - 372\theta^3 - 171\theta^4 - 9\theta^5 + 10\theta^6 + 2\theta^7$,
$\Gamma_{12} = 31212 + 46818\theta - 2754\theta^2 - 14409\theta^3 - 1035\theta^4 + 1293\theta^5 + 159\theta^6 - 12\theta^7 - 6\theta^8 - 2\theta^9$,
$\Gamma_{13} = 24786 + 26163\theta - 7128\theta^2 - 10278\theta^3 + 648\theta^4 + 1476\theta^5 - 18\theta^6 - 91\theta^7 + 2\theta^9$,
$\Gamma_{14} = 2820096 + 4406400\theta - 938304\theta^2 - 1873368\theta^3 + 1093680\theta^4 + 500166\theta^5 - 436338\theta^6 + 17910\theta^7 + 62371\theta^8 - 18446\theta^9 - 2281\theta^{10} + 2068\theta^{11} - 182\theta^{12} - 72\theta^{13} + 12\theta^{14}$,
$\Gamma_{15} = 1498176 + 9694080\theta - 7344\theta^2 - 2135592\theta^3 + 829116\theta^4 + 501606\theta^5 - 385239\theta^6 + 36957\theta^7 + 52989\theta^8 - 22290\theta^9 - 1041\theta^{10} + 2352\theta^{11} - 265\theta^{12} - 79\theta^{13} + 14\theta^{14}$,
$\Gamma_{16} = 2996352 + 2026944\theta - 4736880\theta^2 + 1110960\theta^3 + 971244\theta^4 - 673272\theta^5 + 117936\theta^6 + 89181\theta^7 - 52338\theta^8 + 24\theta^9 + 5226\theta^{10} - 662\theta^{11} - 170\theta^{12} + 31\theta^{13}$,
$\Gamma_{17} = 1224 + 1224\theta + 162\theta^2 - 246\theta^3 - 78\theta^4 + 12\theta^5 + 5\theta^6$,
$\phi_1 = 4284 + 1242\theta - 2070\theta^2 - 570\theta^3 + 332\theta^4 + 91\theta^5 - 15\theta^6 - 4\theta^7$,
$\phi_2 = 2448 - 4896\theta - 5142\theta^2 + 2740\theta^3 - 1210\theta^4 + 172\theta^5 - 75\theta^6 - 16\theta^7$,
$\phi_3 = 48960 + 843120\theta + 506880\theta^2 + 4902\theta^3 - 8058\theta^4 - 3350\theta^5 - 64\theta^6 + 203\theta^7 + 32\theta^8$,
$\phi_4 = 2829888(1+\theta) - 1872720\theta^2 - 3408432\theta^3 - 560508\theta^4 + 928264\theta^5 + 355639\theta^6 - 72532\theta^7 - 48649\theta^8 - 1088\theta^9 + 2055\theta^{10} + 225\theta^{11}$,
$\phi_5 = 332928 + 277440\theta - 177920\theta^2 + 50944\theta^3 + 72584\theta^4 - 12558\theta^5 - 12152\theta^6 + 780\theta^7 + 675\theta^8$,
$\phi_6 = 22639104 + 45278208\theta + 19975680\theta^2 - 17984640\theta^3 - 19023552\theta^4 - 1944288\theta^5 + 3997664\theta^6 + 1448072\theta^7 - 215214\theta^8 - 184126\theta^9 - 12962\theta^{10} + 7200\theta^{11} + 1125\theta^{12}$,
$\phi_7 = 2312 + 1428\theta - 1734\theta^2 + 225\theta^3$,
$\phi_8 = 4624 + 272\theta - 2040\theta^2 - 132\theta^3 + 225\theta^4$,
$\phi_9 = 2580192 + 3459024\theta + 391680\theta^2 - 1170144\theta^3 - 477734\theta^4 + 38495\theta^5 + 56422\theta^6 + 13407\theta^7 + 376\theta^8 - 873\theta^9 - 183\theta^{10}$,
$\phi_{10} = 41616 + 117504\theta - 19992\theta^2 - 24344\theta^3 + 21657\theta^4 + 2546\theta^5 - 4311\theta^6 - 164\theta^7 + 232\theta^8$,
$\phi_{11} = 9321984 + 14355072\theta + 2487168\theta^2 - 7029024\theta^3 - 3913536\theta^4 + 552472\theta^5 + 838584\theta^6 + 82794\theta^7 - 65590\theta^8 - 12802\theta^9 + 1824\theta^{10} + 453\theta^{11}$,
$\phi_{12} = 136408 + 146812\theta - 170646\theta^2 + 78030\theta^3 + 13275\theta^4$,
$\phi_{13} = 1156 - 353736\theta - 450831\theta^2 + 502207\theta^3 - 31413\theta^4 - 33975\theta^5$,
$\phi_{14} = 43928 + 87856\theta - 18530\theta^2 - 38760\theta^3 + 2541\theta^4 + 4275\theta^5$,
$\phi_{15} = 4910688 + 3312144\theta - 2586384\theta^2 - 1510464\theta^3 + 567810\theta^4 + 242907\theta^5 - 43389\theta^6 + 2981\theta^7 + 2965\theta^8 - 2780\theta^9 - 282\theta^{10} + 108\theta^{11}$,
$\phi_{16} = 41616 + 58752\theta - 395208\theta^2 - 549168\theta^3 + 151089\theta^4 + 253204\theta^5 - 5154\theta^6 - 45328\theta^7 - 3992\theta^8 + 3287\theta^9 + 280\theta^{10} - 102\theta^{11}$,
$\phi_{17} = 25302528 + 59692032\theta + 38820096\theta^2 - 11705856\theta^3 - 24075168\theta^4 - 6756576\theta^5 + 3487424\theta^6 + 2476136\theta^7 + 209852\theta^8 - 235244\theta^9 - 72503\theta^{10} + 1623\theta^{11} + 3666\theta^{12} + 450\theta^{13}$.

Appendix J. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.cie.2021.107570.

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