The impact of online sales on centralised and decentralised dual-channel supply chains

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Abstract: This paper studies a supply chain structure featuring two different types of distribution channels through which manufacturers sell products. The *centralised* and *decentralised* distribution channels considered in this study are affected by online sales outside the structured channels. In the centralised distribution channel, two retail stores located in geographically distinct markets are operated by a single owner. In the decentralised distribution channel, two retailers independently operate two retail stores. In the non-cooperative scenario, the manufacturer always prefers the decentralised distribution channel irrespective of whether an online channel is used. To achieve channel coordination, a revenue-sharing contract is applied, but it can be used to coordinate only the decentralised distribution system. Therefore, a modified revenue-sharing contract is proposed to coordinate the centralised distribution system. The analytical study reveals that without coordination among the channel members, the manufacturer always earns maximum profit in decentralised distribution systems. However, if the supply chain is coordinated, then the manufacturer receives more benefits from using the

centralised distribution systems under certain conditions. Propositions are presented to describe the characteristics of distribution structures, and to provide meaningful management guidelines for coordinating them. Extensive numerical investigations are also presented. [Received 20 January 2016; Revised 13 December 2016, 9 March, 4 August 2017; Accepted 25 September 2017]

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1   Introduction

The rapid changes in consumer behaviour over the last three decades, spurred by technology innovation, has forced retailers and manufacturers to rethink almost every aspect of their business operations. To maintain growth, several manufacturers have started selling products directly to the user through an online channel, and retailers have

introduced chain stores to expand into new territories. According to the Federal Trade Commission, a chain store is defined as "an organization owning a controlling interest in two or more establishments which sell substantially similar merchandise at retail prices". The presence of a retail chain is observed in almost every sector, including J Sainsbury (grocery), Religare wellness (health), lifestyle (consumer goods), Fabindia (garments), and body shop (skin care) among others. Chain stores look similar, and are horizontally integrated under centralised control. Because of the horizontal integration, the owner can take advantages of bulk purchasing. Similar to the strategic moves made by the manufacturer to improve profit by opening an online channel, a big retailer may also enhance wholesale-price negotiation power by operating chain stores and thereby increase profit. In this paper, we explore the profitability of supply chain members in the presence of both the chain store and dual channel.

To formulate an analytically tractable model for the introduction of the chain store, we consider two retail shops located in two different geographical territories. We conduct a comparison study on the profitability of the manufacturer in two different distribution structures. In the first distribution structure, two retail shops in different geographical territories are operated by two independent retailers. In the second distribution structure, retail shops in different geographical territories are operated by a single retailer as is seen in a retail chain. We call these two distribution structures *decentralised* and *centralised* distribution systems, respectively. Our objective is to verify the profitabilities and preferences of channel members under these two distribution structures. In addition, a coordination mechanism is required for achieving supply chain coordination. These management issues become more complicated and intensive due to price competition created when online channels are introduced. In this paper, we have addressed this complex real-world situation.

In the last two decades, the rapid development of e-commerce has encouraged many manufacturers to open up online channels. Thousands of companies, such as IBM, Cisco, and Nike, sell products online, at the same time they sell through traditional distribution channels (Cai et al., 2010). As a consequence, the properties of a dual channel have been important topics for exploration by both academicians and practitioners. Researchers have analysed various aspects of the dual channel: for example, disruptions management (Xiao and Qi, 2008; Huang et al., 2012), price strategy (Chen et al., 2013; Moon and Yao, 2013, Bai and Xu, 2016), channel coordination (Chen et al., 2012; Saha, 2015), power structure among channel members (Khouja et al., 2010; Lu and Liu, 2013), the effect of risk aversion (Xu et al., 2014; Liu et al., 2016). For more detail, one can refer to an extensive review of the literature provided by Shang and Yang (2015). Our study extends previous research by incorporating more than one retail channel in a study of supply chains with different distribution structures, and we also discuss channel coordination of this complicated retail situation.

A manufacturer establishing a dual-channel supply chain must address important issues of vertical and horizontal competition. Supply chain performance can be enhanced if participants in the cooperative game orchestrate their efforts and use contractual incentives so that each participant's objectives are aligned with the objectives of the entire supply chain (Cachon, 2003). Sharing sales revenues between channel members is one way to enhance supply chain performance (Cachon and Lariviere, 2005). Several authors have argued that a revenue-sharing (RS) contract is a relatively straightforward way to ensure aligned objectives, and numerous studies have focused on this contract mechanism (Zeng, 2013). For example, behaviour under the RS contract in specific

settings has been explained by Govindan and Popiuc (2014), who focused on the personal computer industry; Moon et al. (2015), who looked at multi-stage supply chains under budget constraints; and Palsule-Desai (2013), who examined the film industry. Cachon and Lariviere (2005) looked at contract strengths and limitations. Additional examples of contract studies include those by Saraswati and Hanaoka (2014), Feng et al. (2015), Luo and Chen (2016), He et al. (2016) and others. Unique compared to existing literature, our paper verifies the effectiveness of RS contracts under two relatively new dual-channel structures.

In this study, we merge and analyse two separate issues: a dual-channel supply chain with one or two retailers and coordination of members in the dual channel. We make an analytical assessment on the basis of preferred pricing strategies for a dual channel in decentralised and centralised distribution environments, which we refer to as scenario decentralised distribution (DD) and scenario centralised distribution (CD), respectively. We use the centralised decision model, referred to as scenario CC, because it is an idealised scenario in which the central planner makes all the decisions to maximise supply chain profit. Scenario CC serves as the benchmark to compare performances of two distribution structures under specific coordination mechanisms. To the best of our knowledge, pricing, procurement, and coordination in a dual-channel supply chain with the proposed distribution structures have not been studied previously.

The rest of the paper is organised as follows. The models are developed and results are compared for decentralised and centralised distribution scenarios in Section 2. Behaviour of contract mechanisms is analysed in Section 3. Finally, conclusions and future directions for study are discussed in Section 4. Tests for concavity and their results are presented in Appendices.

2   Mathematical model and analysis

The following notations are used to develop the models:

A_i overall size of the market potential in geographical territory $i, i = 1, 2$

p_i^j j= dd, cd, cc; price of the product in a retail channel under Scenario DD, Scenario CD, and Scenario CC, respectively

p_m^j price of the product in the online channel

w_i^j wholesale price of the manufacturer charged to the retailer per unit

h_i unit operational costs at in geographical territory $i, i = 1, 2$

b_i price sensitivity parameter of the retail channel demand in geographical territory $i, i = 1, 2$

b_3 price sensitivity parameter of the online channel demand

c_m unit cost of the manufacturer

c_i intensity of price competition between the retail and online channels in geographical territory $i, i = 1, 2$

π_m^j manufacturer's profit in Scenario DD and Scenario CD, respectively

π_{ri}^j retailer's profit in geographical territory $i, i = 1, 2$ in Scenario DD and Scenario CD, respectively

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In this paper, we describe a stylised two-stage dual channel supply chain under the deterministic environment consisting of a single manufacturer selling a homogeneous product through traditional retail channels in two different geographical territories with different economic-development levels. In addition, the manufacturer sells the product directly to the customers through an online channel. We have considered two different distribution systems. Pictorial representations of distribution systems are given in Figures 1(a)–1(b).

In the first DD scenario, two different retailers (R1 and R2) sell the product to the end consumers (C) with different retail prices (p_i^{dd}, i=1,2) at their respective retail shops located at geographically distinct markets and the manufacturer also sells the product directly to the end consumers by using his own online channel. The manufacturer adopts the differential wholesale prices (w_i^{dd}) for the retailers. The primary decision for the manufacturer is to set the wholesale prices (w_i^{dd}) and price of the online channel (p_m^{dd}). Wholesale price differentiation is practiced in several markets, examples include markets such as petroleum distribution, steel, heavy trucking, tobacco, dairy products, and pharmaceutical etc., and several author argues for wholesale price discrimination (Leng and Parlar, 2012; Brunner 2013). In the CD, a single retailer (R) sells the product to the end consumers through two retail shops (RS1 and RS2) in two different geographical territories with different retail prices (p_i^{cd}, i=1,2). Note that, if two markets are operated by single retailer (like chain shop owner), the manufacturer cannot adopt the differential wholesale prices and sets uniform wholesale price ($w_1^{cd} = w_2^{cd} = w^{cd}$). The primary decision for the manufacturer is to set the wholesale price (w^{cd}) and price of the online channel (p_m^{cd}).

Figure 1 (a) Decentralised distribution system with two independent retailer (b) Centralised distribution system with single retailer controlling two retail shops at different location

(a)   (b)
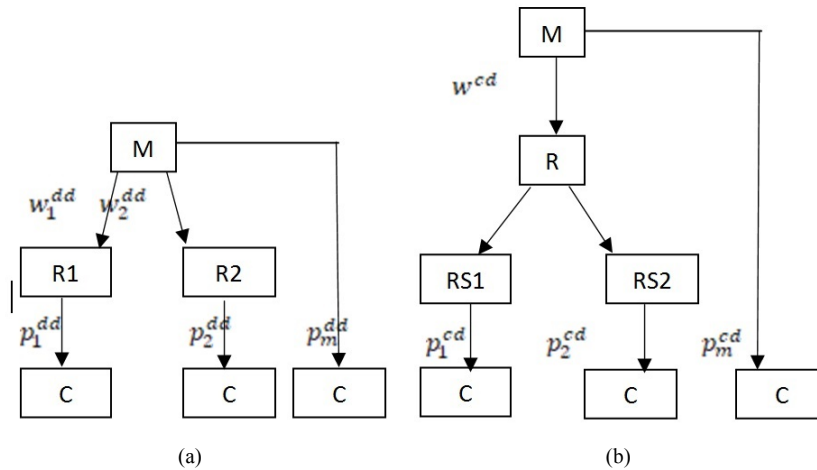
To obtain the general form of the demand functions for online (D_o) and two retail channels ($D_{ri}, i = 1, 2$), we followed the elegant framework established by Hua et al. (2010) and Lu and Liu (2015), who employed similar demand functions with linear in self- and cross-price effects: $D_o = a_3 - b_3 p_m + c_1 p_1 + c_2 p_2$ and $D_{ri} = a_i - b_i p_i -$

$c_i p_m$. We assume x and y $(0 < \{x, y\} < 1)$ represent the compatibility of the product in the retail channels; that is, $a_1 = A_1 x$, $a_2 = A_2 y$ and $a_3 = (1 - x)A_1 + (1 - y)A_2$. The price sensitivity in the two retail channels and the online channel are respectively considered as b_1, b_2, and b_3. c_1 and c_2 measure the intensity of price competition between the retail and online channels in two locations, where $min\{b_1, b_2\} > max\{c_1, c_2\}$ and $b_3 > c_1 + c_2$. In the development of the model, we ignore the cross-price effect between two retail channels because they are presumably located in geographically separated markets (Inderst and Valletti, 2009; Arya et al., 2015). However, in the presence of the online channel, cross-price effects are considered. In the literature on the dual-channel supply chain, the conversion and holding costs of a retailer are sometimes assumed to be zero because of analytical tractability (Hua et al., 2010; Dan et al., 2012; Chen et al., 2012; Matsui, 2017). In the present study, we consider the unit operational costs of the retailers as h_1 and h_2, respectively, because two different locations are considered. The unit marginal cost of the manufacturer is c_m. Additional subscripts are used to differentiate the outcomes of the models under coordination. In the next subsection, the expressions of all the decision variables are derived for two different distribution systems in a non-cooperative environment.

2.1 The decentralised distribution system model

First, we explain the scenario in which two retail shops are operated by two independent retailers. We assume that the manufacturer acts as a Stackelberg leader and offers a wholesale price to each retailer. Based on the declared wholesale price, each retailer makes a decision on the quantity to order from the manufacturer and simultaneously sets the market price for the retail channel. In this scenario, the manufacturer determines the wholesale prices (w_i^{dd}, $i = 1, 2$) for the retail channels and the selling price for products sold in the online channel (p_m^{dd}), and then each retailer follows and makes decisions on the selling prices in their respective retail channel (p_i^{dd}, $i = 1, 2$). Profit functions of each retailer (π_{ri}^{dd}) and the manufacturer (π_m^{dd}) in this scenario are as follows:

$$\pi_{ri}^{dd} = (p_i^{dd} - w_i^{dd} - h_i)D_{ri}, \quad i = 1, 2 \tag{1}$$

$$\pi_m^{dd} = \sum_{i=1}^{2}(w_i^{dd} - c_m)D_{ri} + (p_m^{dd} - c_m)D_o \tag{2}$$

The first term of the manufacturer profit function represents the profit earns from two independent retailers and the last term represents the profit earns from selling products through the online channel. If $\Delta_1 = b_1 b_2 b_3 - b_2 c_1^2 - b_1 c_2^2 > 0$, then profit functions of each channel member are concave on the decision variables and there exists an optimal solution for each channel member. The optimal solution can be calculated as follows:

$$w_1^{dd} = \frac{1}{2\Delta_1}[a_1 b_2 b_3 + a_3 b_2 c_1 + a_2 c_1 c_2 - a_1 c_2^2 + \Delta_1(c_m - h_1)] \tag{3}$$

$$w_2^{dd} = \frac{1}{2\Delta_1}[a_2 b_1 b_3 - a_2 c_1^2 + a_3 b_1 c_2 + a_1 c_1 c_2 + \Delta_1(c_m - h_2)] \tag{4}$$

$$p_1^{dd} = \frac{1}{4b_1\Delta_1}[2b_1c_1(a_3b_2 + a_2c_2) + a_1(3b_1b_2b_3 - b_2c_1^2 - 3b_1c_2^2)$$
$$+\Delta_1((b_1 + c_1)c_m + b_1h_1)] \tag{5}$$

$$p_2^{dd} = \frac{1}{4b_2\Delta_1}[2b_2c_2(a_3b_1 + a_1c_1) + a_2(3b_1b_2b_3 - 3b_2c_1^2 - b_1c_2^2)$$
$$+\Delta_1((b_2 + c_2)c_m + b_2h_2)] \tag{6}$$

$$p_m^{dd} = \frac{1}{2\Delta_1}[a_3b_1b_2 + a_1b_2c_1 + a_2b_1c_2 + \Delta_1c_m] \tag{7}$$

Using equations (3)∼(7), the sales volume of the entire supply chain (Q^{dd}), the profit of the manufacturer, and each retailer are obtained as follows

$$Q^{dd} = \frac{1}{4b_1b_2}[2a_3b_1b_2 + a_1b_2(b_1 + c_1) + a_2b_1(b_2 + c_2) - (b_1b_2(b_1 + b_2 + b_3)$$
$$-2b_1b_2(c_1 + c_2) + \Delta_1)c_m - b_1b_2((b_1 - c_1)h_1 + (b_2 - c_2)h_2)] \tag{8}$$

$$\pi_m^{dd} = \frac{A_{11}}{8b_1b_2\Delta_1} \tag{9}$$

$$\pi_{ri}^{dd} = \frac{[a_i - (b_i - c_i)c_m - b_ih_i]^2}{16b_i}, i = 1, 2 \tag{10}$$

where $A_{11} = 2a_3^2b_1^2b_2^2 + \Delta_1(a_1^2b_2 + a_2^2b_1) - 2(a_1b_1c_2 - a_2b_1c_2)^2 + 4a_3b_1b_2(a_1b_2c_1 + a_2b_1c_2 - \Delta_1c_m) - 2(a_2b_1(b_2 + c_2) + a_1b_2(b_1 + c_1))c_m\Delta_1 + c_m((b_1b_2(b_1 + b_2 + b_3) + 2b_1b_2(c_1 + c_2) - \Delta_1)c_m)\Delta_1 - b_1b_2(2(a_1 - b_1c_m + c_1c_m)h_1 - 2(a_2 - b_2c_m + c_2c_m)h_2 - b_1h_1^2 - b_2h_2^2)\Delta_1$. For feasibility, the solution must satisfy the two conditions. First, the selling price in the online-selling channel must be greater than the wholesale price in the retail channels; otherwise, retailers will not participate in the system. Second, both the retailers must receive non-negative profits. To incorporate the optimal decision variables that satisfy these conditions, the following Proposition is made:

Proposition 1: There exists optimal game solutions when the degree of customer loyalty to the retail channels lies within the following ranges:

$$\Gamma_{11}^L = \frac{(b_1 - c_1)c_m + b_1h_1}{A_1} \le x \le$$
$$min\left\{\frac{(A_1 + A_2)b_1(b_2 - c_2) - h_2\Delta_1}{A_1(b_1 - c_1)(b_2 - c_2)}, \frac{(A_1 + A_2)b_2(b_1 - c_1) - h_1\Delta_1}{A_1(b_2(b_1 + b_3 - 2c_1) - c_2^2)}, 1\right\}$$
$$= \Gamma_{11}^U \tag{11}$$

$$\Gamma_{12}^L = \frac{(b_2 - c_2)c_m + b_2h_2}{A_2} \le y \le$$
$$min\left\{\frac{(A_1 + A_2)b_2(b_1 - c_1) - h_1\Delta_1}{A_2(b_2 - c_2)(b_1 - c_1)}, \frac{(A_1 + A_2)b_1(b_2 - c_2) - h_2\Delta_1}{A_2(b_1(b_2 + b_3 - 2c_2) - c_1^2)}, 1\right\} \tag{12}$$
$$= \Gamma_{12}^U$$

Proof of the Proposition 1 and detail derivation of decision variables are given in the Appendix A.

2.2 The centralised distribution system model

In this scenario, first the manufacturer determines the wholesale price (w^{cd}) and selling price of the online channel (p_m^{cd}). Then the retailer, as follower, makes decision on the sale prices of two retail shops (p_i^{cd}), $i = 1, 2$. In this scenario, profit functions of the retailer ($\pi_r^{cd} = \pi_{r1}^{cd} + \pi_{r2}^{cd}$) and the manufacturer ($\pi_m^{cd}$) are as follows:

$$\pi_r^{cd} = \sum_{i=1}^{2} (p_i^{cd} - w^{cd} - h_i) D_{ri} \tag{13}$$

$$\pi_m^{cd} = (w^{cd} - c_m) \sum_{i=1}^{2} D_{ri} + (p_m^{cd} - c_m) D_o \tag{14}$$

If $\Delta_2 = 2(b_1 + b_2)\Delta_1 + (b_1 c_2 - c_1 b_2)^2 > 0$, then profit functions of the retailer and the manufacturer are concave on the decision variables and there exists an optimal solution for each channel member. The optimal solution can be calculated as follows:

$$\begin{aligned}
w^{cd} = \frac{1}{2\Delta_2} &\left[2a_3 b_1 b_2 (c_1 + c_2) + a_2 (2b_1 b_2 b_3 - b_2 c_1^2 + b_1 c_1 c_2) \right. \\
&+ a_1 (2b_1 b_2 b_3 + b_2 c_1 c_2 - b_1 c_2^2) + \Delta_2 c_m - (2\Delta_1 - c_2(b_1 c_2 - b_2 c_1)) b_1 h_1 \\
&\left. - (2\Delta_1 - c_1(b_1 c_2 - b_2 c_1)) b_2 h_2 \right]
\end{aligned} \tag{15}$$

$$\begin{aligned}
p_m^{cd} = \frac{1}{2\Delta_2} &[2a_3 b_1 b_2 (b_1 + b_2) + a_1 b_2 (2b_1 c_1 + b_2 c_1 + b_1 c_2) \\
&+ a_2 b_1 (b_2 c_1 + b_1 c_2 + 2b_2 c_2) + \Delta_2 c_m - b_1 b_2 (b_2 c_1 - b_1 c_2)(h_1 - h_2)]
\end{aligned} \tag{16}$$

$$\begin{aligned}
p_1^{cd} = \frac{1}{4b_1 \Delta_2} &[a_1 (2b_1 b_2 (3b_1 + 2b_2) b_3 - b_2 (2b_1 + b_2) c_1^2 - 2b_1 b_2 c_1 c_2 \\
&- b_1 (3b_1 + 4b_2) c_2^2) + 2b_1 (a_2 b_1 b_2 b_3 + a_2 (b_1 + b_2) c_1 c_2 \\
&+ a_3 b_2 (2b_1 c_1 + b_2 c_1 + b_1 c_2)) + \Delta_2 (b_1 + c_1) c_m \\
&+ b_1 ((2b_1 b_2 (b_1 + 2b_2) b_3 - b_2 (2b_1 + b_2) c_1^2 - 4b_1 b_2 c_1 c_2 \\
&- b_1 (b_1 + 4b_2) c_2^2) h_1 + 2b_1 b_2 (c_2(c_1 + c_2) - b_2 b_3) h_2)]
\end{aligned} \tag{17}$$

$$\begin{aligned}
p_2^{cd} = \frac{1}{4b_2 \Delta_2} &[a_2 (b_2 (2b_1 (2b_1 + 3b_2) b_3 - (4b_1 + 3b_2) c_1^2) - 2b_1 b_2 c_1 c_2 \\
&- b_1 (b_1 + 2b_2) c_2^2) + 2b_2 (a_1 b_1 b_2 b_3 + a_1 (b_1 + b_2) c_1 c_2 \\
&+ a_3 b_1 (b_2 c_1 + b_1 c_2 + 2b_2 c_2)) + \Delta_2 (b_2 + c_2) c_m \\
&+ b_2 (2b_1 b_2 (b_1 b_3 - c_1(c_1 + c_2)) h_1 + (b_2 (4b_1 + b_2) c_1^2 \\
&- 2b_1 b_2 (2b_1 + b_2) b_3 + 4b_1 b_2 c_1 c_2 + b_1 (b_1 + 2b_2) c_2^2) h_2)]
\end{aligned} \tag{18}$$

Using equations (15)~(18), the sales volume of the entire channel (Q^{cd}); the profit of the manufacturer and the retailer are determined as follows:

$$
\begin{aligned}
Q^{cd} = \frac{1}{4b_1b_2}[&2a_3b_1b_2 + a_1b_2(b_1 + c_1) + a_2b_1(b_2 + c_2)\\
&-(b_1b_2(b_1 + b_2 + b_3) - 2b_1b_2(c_1 + c_2) + \Delta_1)c_m\\
&-b_1b_2((b_1 - c_1)h_1 + (b_2 - c_2)h_2)]
\end{aligned}
\tag{19}
$$

$$
\pi_m^{cd} = \frac{B_{11}}{8b_1b_2\Delta_2}
\tag{20}
$$

$$
\pi_r^{cd} = \sum_{i=1}^{2} \frac{1}{16b_i}[a_i - (b_i - c_i)c_m - b_ih_i + (-1)^iB_{22}]^2
\tag{21}
$$

where $B_{11} = 4a_3^2b_1^2b_2^2(b_1 + b_2) + 4a_1a_2b_1b_2(b_1b_2b_3 + (b_1 + b_2)c_1c_2) + a_1^2b_2(b_2^2c_1^2 + 2b_1b_2c_1(c_1 + c_2) + b_1^2(2b_2b_3 - c_2^2)) + a_2^2b_1(b_1^2c_2^2 - b_2^2c_1^2 + 2b_1b_2(b_2b_3 + c_2(c_1 + c_2))) - 2a_1b_2(b_1 + c_1)c_m\Delta_2 - c_m(2a_2b_1(b_2 + c_2) + (b_2(2b_1c_1 + c_1^2 - b_1(b_1 + b_2 + 2b_3)) + 2b_1b_2c_2 + b_1c_2^2)(h_1 + c_m))\Delta_2 - 4a_2b_1^3b_2^2b_3h_1 + 4a_2b_1^2b_2^2c_1^2h_1 + 4a_2b_1^2b_2^2c_1c_2h_1 + 4b_1^4b_2^2b_3c_mh_1 + 2b_1^4b_2^2b_3h_1^2 - 2b_1^3b_2^2c_1^2h_1^2 + b_1^2b_2^2c_1^2h_1^2 - 2b_1^3b_2^2c_1c_2h_1^2 + 4a_3b_1b_2(a_1b_2(2b_1c_1 + b_2c_1 + b_1c_2) + b_1b_2^2(b_1^2c_2^2 - b_2^2c_1^2 + 2b_1b_2(b_2b_3 - c_2(c_1 + c_2)))h_2^2a_2b_1(b_2c_1 + b_1c_2 + 2b_2c_2) - (h_2 + c_m)\Delta_2)$, and $B_{22} = 2b_1b_2[a_3(b_2c_1 - b_1c_2) + a_1(b_2b_3 - c_2^2 - c_1c_2) - a_2(b_1b_3 - c_1^2 - c_1c_2) - \Delta_1(h_1 - h_2)]/\Delta_2$

If $B_{22} = 0$, then the profit of the retailer will be converted into the sum of total profits of two independent retailers. To incorporate the optimal decision variables that satisfy feasibility conditions for existing online channel, the following Proposition is made:

Proposition 2: There exists optimal game solutions when the degree of customer loyalty to the retail channels lies within the following ranges:

$$
\Gamma_{21}^L = max\{\psi_{11}, \psi_{12}, 0\} \le x \le min\{\psi_{13}, 1\} = \Gamma_{21}^U
\tag{22}
$$

$$
\Gamma_{22}^L = max\{\psi_{21}, \psi_{22}, 0\} \le y \le min\{\psi_{23}, 1\} = \Gamma_{22}^U
\tag{23}
$$

where $\psi_{11} = \frac{1}{A_1[\Delta_2+N]}[M(b_1c_2 - b_2c_1) + (b_1 - c_1)\Delta_2c_m + b_1(3b_1^2(c_2^2 - 2b_2b_3)h_1 + b_2^2c_1^2(h_1 + 2h_2) + 2b_1b_2(3c_1^2h_1 + 3c_1c_2h_1 - b_2b_3(2h_1 + h_2) + c_2^2(2h_1 + h_2)))]$, $\psi_{12} = \frac{1}{A_1N}[M(b_1c_2 - b_2c_1) - (b_2 - c_2)\Delta_2c_m - b_2(3b_2^2c_1^2h_2 - b_1^2(2b_2b_3(h_1 + 2h_2) - c_2^2(2h_1 + h_2)) + 2b_1b_2(3b_2b_3h_2 - 3c_1c_2h_2 - 3c_2^2h_2 - c_1^2(h_1 + 2h_2)))]$, $\psi_{13} = \frac{M(b_1+b_2-c_1-c_2)+2\Delta_1(b_1h_1+b_2h_2)-(b_2c_1-b_1c_2)(h_1b_1(b_2-c_2)-h_2b_2(b_1-c_1))}{A_1[\Delta_1+b_2(2b_1^2+b_1(2b_2+b_3-4c_1-3c_2)-c_1(b_2-c_1-c_2))]}$, $\psi_{21} = \frac{1}{A_2(\Delta_2+O)}[M(b_2c_1 - b_1c_2) + (b_2 - c_2)\Delta_2c_m - b_2(3b_2^2c_1^2h_2 + b_1^2(c_2^2(2h_1 + h_2) - 2b_2b_3(h_1 + h_2)) + 2b_1b_2(3c_1c_2h_2 + 3(c_2^2 - b_2b_3)h_2 + c_1^2(h_1 + 2h_2)))]$, $\psi_{22} = $

$\frac{1}{A_2 O}[M(b_2 c_1 - b_1 c_2) - (b_1 - c_1)\Delta_2 c_m + b_1(b_1^2(6b_2 b_3 h_1 - 3c_2^2 h_1) - b_2^2 c_1^2(h_1 + 2h_2) - 2b_1 b_2(3c1^2 h_1 - 3c_1 c_2 h_1 + b_2 b_3(2h_1 + h_2) - c_2^2(2h_1 + h_2)))]$, $\psi_{23} = \frac{M(b_1 + b_2 - c_1 - c_2) + 2\Delta_1(b_1 h_1 + b_2 h_2) + (b_2 c_1 - b_1 c_2)(b_1(b_2 - c_2)h_1 - b_2(b_1 + c_1)h_2)}{A_2[\Delta_1 + b_1(b_2(2b_1 + 2b_2 + b_3 - 3c_1) - (b_1 + 4b_2 - c_1)c_2 + c_2^2)]}$, $M = 2(A_1 + A_2)b_1 b_2$, $N = 2b_1 b_2[b_2(b_3 - c_1) + b_1 c_2 - c_2(c_1 + c_2)]$, and $O = 2b_1 b_2[b_1(b_3 - c_2) + b_2 c_1 - c_1(c_1 + c_2)]$.

Proof of the Proposition 2 and detail derivations of the decision variables are given in Appendix B.

By combining Propositions 1 and 2, we can conclude that if the following inequalities holds then optimal game solutions exist for two distribution system under non-cooperative environment.

$$max\left\{\Gamma_{11}^L, \Gamma_{21}^L\right\} \le x \le min\left\{\Gamma_{11}^U, \Gamma_{21}^U\right\}$$
$$max\left\{\Gamma_{12}^L, \Gamma_{22}^L\right\} \le y \le min\left\{\Gamma_{12}^U, \Gamma_{22}^U\right\} \tag{24}$$

From equatio (24), one can conclude that the manufacturer cannot always introduce the online channel. Compatibility of the product plays an important role for the successful implementation of the online channel. Note that, if $A_1 = A_2$, $x = y$, $b_1 = b_2$, and $c_1 = c_2$ (that is, the market demand at two retail channels are identical), then $\pi_m^{dd} - \pi_m^{cd} = (h_1 - h_2)^2/(16b_2) > 0$ (that is, the manufacturer earns more profit in the decentralised distribution system than in the centralised distribution system due to operational cost differences). Moreover, in general $\frac{\partial(\pi_m^{dd} - \pi_m^{cd})}{\partial h_1} = \frac{b_1 b_2 \Delta_3}{\Delta_2} = -\frac{\partial(\pi_m^{dd} - \pi_m^{cd})}{\partial h_2}$, where $\Delta_3 = (b_1 c_2 - b_2 c_1)[a_2(b_1 b_3 - c_1(c_1 + c_2)) - a_1(b_2 b_3 - c_2(c_1 + c_2)) - a_3(b_2 c_1 - b_1 c_2) + \Delta_1(h_1 - h_2)]$. Therefore, the unit operation cost plays a significant role in the manufacturer's profitability under non-cooperative scenarios. The manufacturer can earn more profit in the decentralised distribution system as the operational cost difference increases. In the decentralised distribution system, the manufacturer can apply wholesale price differentiation, which becomes more effective as operational cost differences increase. Based on the profit structure of channel members under the two scenarios presented, one can easily integrate channel structures to a single-benchmark cooperative centralised decision model. In the next sub-section, we derive the expressions of decision variables for the centralised decision model.

2.3 Benchmark centralised decision model

In the benchmark model, all the channel members act as a vertically integrated firm and make decisions on the sale prices in the retail channel (p_i^{cc}, $i = 1, 2$) and the online channel (p_m^{cc}) simultaneously to maximise system performance. The benchmark model is controlled by a central planner, the wholesale price is not significant, and two distribution systems can be integrated into a unique distribution model. The profit function (π^{cc}) in the centralised decision model can be formulated as:

$$\pi^{cc} = \sum_{i=1}^{2}(p_i^{cc} - c_m - h_i)D_{ri} + (p_m^{cc} - c_m)D_o \tag{25}$$

Note that if $\Delta_1 > 0$, then the above profit function is also concave on the decision variables and there exists an optimal solution of the equation (25). From the first-order conditions, the optimal solution can be calculated as follows:

$$p_m^{cc} = \frac{1}{2\Delta_1}[a_3 b_1 b_2 + a_1 b_2 c_1 + a_2 b_1 c_2 + \Delta_1 c_m] \tag{26}$$

$$p_1^{cc} = \frac{1}{2\Delta_1}[a_1(b_2 b_3 - c_2^2) + a_3 b_2 c_1 + a_2 c_1 c_2 + \Delta_1(c_m + h_1)] \tag{27}$$

$$p_2^{cc} = \frac{1}{2\Delta_1}[a_2(b_1 b_3 - c_1^2) + a_3 b_1 c_2 + a_1 c_1 c_2 + \Delta_1(c_m + h_1)] \tag{28}$$

Using equations (26)~(28), the optimal value of the sales volume (Q^c) of the entire channel and the supply chain profit are obtained as follows:

$$Q^{cc} = \frac{1}{2}[a_1 + a_2 + a_3 - (b_1 + b_2 + b_3 - 2(c_1 + c_2))c_m$$
$$-(b_1 - c_1)h_1 - (b_2 - c_2)h_2] \tag{29}$$

$$\pi^{cc} = \frac{1}{4\Delta_1}[a_3^2 b_1 b_2 + a_2^2 b_1 b_3 + a_1^2 b_2 b_3 + 2a_1 a_3 b_2 c_1 + 2a_2 a_3 b_1 c_2$$
$$-(a_2 c_1 - a_1 c_2)^2 - 2(a_1 + a_2 + a_3)\Delta_1 c_m$$
$$+\Delta_1(b_1 + b_2 + b_3 - 2(c_1 + c_2))c_m^2$$
$$-(2(a_1 - b_1 c_m + c_1 c_m)h_1 + b_1 h_1^2$$
$$+2(a_2 - b_2 c_m + c_2 c_m)h_2 - b_2 h_2^2)\Delta_1] \tag{30}$$

The derivations of decision variables are similar to first scenario. Hence, we have omitted the proof. Note that the sales volume of the online channel $Q_o^{cc} = a_3 - b_3 p_m^{cc} + c_1 p_1^{cc} + c_2 p_2^{cc} = \frac{1}{2}[a_3 - b_3 c_m + c_1(c_m + h_1) + c_2(c_m + h_2)]$. Now $\frac{\partial Q_o^{cc}}{\partial h_1} = -(b_1 + c_1)/2$, $\frac{\partial Q_o^{cc}}{\partial h_2} = -(b_2 + c_2)/2$, $\frac{\partial Q_o^{cc}}{\partial h_1} = c_1/2$, and $\frac{\partial Q_o^{cc}}{\partial h_2} = c_2/2$. Therefore, the sales volume of the entire distribution system decreases as the operation costs of the two retail channel increase; however, the reverse is found for the online channel. The retailer needs to charge high retail prices to compensate for high operational costs. Therefore, demand of the retail channels decreases as the operational cost increases. In next sub-section, we explore the characteristics of the non-cooperative and cooperative models in detail.

2.4 Model analysis

In summary, results show that in a non-cooperative environment, the sales volume of the entire channel in the two distribution systems remains identical, but the optimal profits of channel members differ. By comparing profits of the manufacturer [equations (9) and (20)], obtained from two distribution systems, we find that

$$\Delta_{md} = \pi_m^{dd} - \pi_m^{cd} = \frac{b_1 b_2}{4\Delta_1 \Delta_2}[a_3(b_1 c_2 - b_2 c_1) + a_2(b_1 b_3 - c_1(c_1 + c_2))$$
$$-a_1(b_2 b_3 - c_2(c_1 + c_2)) + (h_1 - h_2)\Delta_1]^2 \tag{31}$$

From the equation (31), one can conclude that the manufacturer earns higher profit under decentralised distribution systems. Analytically this result is not surprising, it indicates that the downstream price discrimination is a preferred strategy for the manufacturer. Now, we work backwards to analyse the effects of distribution structure in the absence of the manufacturer online channel. In this situation, the demand of retail channels are $D_{ri} = A_i - b_i p_i, (i = 1, 2)$ and the corresponding profit functions of channel members under two distribution structures are as follows:

$$\pi_{ri}^{dds} = (p_i - w_i - h_i)D_{ri}, \quad i = 1, 2 \qquad \pi_m^{dds} = \sum_{i=1}^{2}(w_i - c_m)D_{ri}$$

$$\pi_r^{cds} = \sum_{i=1}^{2}(p_i - w - h_i)D_{ri} \quad \pi_m^{cds} = \sum_{i=1}^{2}(w - c_m)D_{ri}$$

The additional superscript is used to represent the single channel. The results under two different distribution structures are given in Table 1.

Table 1 Optimal solutions in the absence of the online channel

	Two retailer
wholesale price	$\frac{A_i + b_i c_m - b_i h_i}{2b_i}, i = 1, 2$
retail price	$\frac{3A_i + b_i(c_m + h_i)}{4b_1}, i = 1, 2$
retailer's profit	$\frac{(A_i - b_1(c_m + h_i))^2}{16b_i}, i = 1, 2$
manufacturer's profit	$\frac{b_2(A_1 - b_1(c_m + h_1))^2 + b_1(A_2 - b_2(c_m + h_2))^2}{8b_1 b_2}$
sales volume	$\frac{A_1 + A_2 - (b_1 + b_2)c_m - b_1 h_1 - b_2 h_2}{4}$
	Single retailer
wholesale price	$\frac{A_1 + A_2 + (b_1 + b_2)c_m - b_1 h_1 - b_2 h_2}{2(b_1 + b_2)}$
retail price	$\frac{2A_i(b_1 + b_2) + b_i(A_1 + A_2) + b_i(b_1 + b_2)(c_m + h_i) + (-1)^i b_1 b_2(h_2 - h_1)}{4b_i}, i = 1, 2$
retailer's profit	$\sum_{i=1}^{2}\frac{[(b_1 + b_2)(A_i - b_i(c_m + h_i)) + (-1)^i((b_1 A_2 - A_1 b_2) + b_1 b_2(h_1 - h_2))]^2}{16b_i(b_1 + b_2)^2}$
manufacturer's profit	$\frac{[A_1 + A_2 - (b_1 + b_2)c_m - b_1 h_1 - b_2 h_2]^2}{8(b_1 + b_2)}$
sales volume	$\frac{A_1 + A_2 - (b_1 + b_2)c_m - b_1 h_1 - b_2 h_2}{4}$

Like that found for the dual-channel systems, the sales volume of the entire channel remains identical, but the optimal profits of each supply chain member differ. The manufacturer earns higher profit under the decentralised distribution system; the amount of profit gain is $\Delta_{ms} = (A_2 b_1 - A_1 b_2 + b_1 b_2(h_1 - h_2))^2/[8b_1 b_2(b_1 + b_2)]$. Previously obtained results show that when operating an online channel, the manufacturer gains

maximum profit under the decentralised distribution system. Therefore, introduction of the manufacturer online channel is always profitable if profit of the manufacturer in the centralised distribution system is greater compared to decentralised single channel system, that is $\pi_m^{cd} > \pi_m^{dds}$. In particular, if $x = y$, $A_1 = A_2$, $b_1 = b_2$, and $c_1 = c_2$, then the profit difference is

$$
\begin{aligned}
\pi_m^{cd} - \pi_m^{cds} = \frac{1}{4b_2(b_2b_3 - 2c_2^2)} & [4A_2^2(4b_2^2(1-y)^2 - b2(1-y)(b_3 + b_3y - 8c_2y) \\
& + 2c_2^2(1+y^2)) + 4A_2(b_2b_3 - 2c_2^2) \\
& (b_2(h_1 + h_2 - 2c_m)(1-y) - 2c_2c_my) \\
& - (b_2b_3 - 2c_2^2)(4c_2^2c_m^2 + b_2^2(h_1 - h_2)^2 \\
& + 4b_2c_m(c_2(2c_m + h_1 + h_2) - b_3c_m))]
\end{aligned}
$$

From the above expression, one can conclude that the introduction of online channel is not profitable for the manufacturer if

$$
\begin{aligned}
max & \left\{ \frac{\Theta_{11} - \sqrt{2\Theta_{22}}}{2A_2^2[b_2(4b_2 + b_3) - 8b_2c_2 + 2c_2^2]}, 0 \right\} \leq y \\
& \leq min \left\{ \frac{\Theta_{11} + \sqrt{2\Theta_{22}}}{2A_2^2[b_2(4b_2 + b_3) - 8b_2c_2 + 2c_2^2]}, 1 \right\}
\end{aligned} \tag{32}
$$

where $\Theta_{11} = 8A_2^2b_2(b_2 - c_2) - A_2(b_2b_3 - 2c_2^2)(b_2(2c_m - h_1 - h_2) - 2c_2c_m)$ and $\Theta_{22} = A_2^2(b_2b_3 - 2c_2^2)(2A_2^2(b_2(b_3 - 8c_2) + 2c_2^2) + 2A_2b_2(b_2(b_3(2c_m - h_1 - h_2) + 4c_2(h_1 + h_2)) - 2c_2^2(2c_m + h_1 + h_2)) + b_2^2(2b_2^2(h_1 - h_2)^2 - 2b_3^2c_m^2 + 4b_3c_2c_m(4c_m + h_1 + h_2) - 4c_2^2(7c_m^2 + h_1h_2 + 3c_m(h_1 + h_2)) + b_2(b_3(h_1^2 + h_2^2 - 6c_m^2 - 2c_m(h_1 + h_2)) + 4c_2(4c_m^2 - (h_1 - h_2)^2 + 2c - m(h_1 + h_2)))))$. Similarly one can verify that the profit difference of the manufacturer in decentralised distribution system ($\pi_m^{dd} - \pi_m^{dds}$) and centralised distribution system ($\pi_m^{cd} - \pi_m^{cds}$) are identical and its value is equal to

$$
\begin{aligned}
\pi_m^{dd} - \pi_m^{dds} = \frac{1}{4b_2(b_2b_3 - 2c_2^2)} & [A_2^2(4b_2^2(1-y)^2 - b_2(1-y)(b_3 + b_3y - 8c_2y) \\
& + 2c_2^2(1+y^2)) - A_2(b_2b_3 - 2c_2^2)(b_2(2c_m - h_1 - h_2)(1-y) \\
& + 2c_2c_my) + (b_2b_3 - 2c_2^2)c_m \\
& (b_2(b_3c_m - c_2(2c_m + h_1 + h_2) - c_2^2c_m))]
\end{aligned}
$$

From the above expression, one can conclude that the introduction of online channel is not profitable for the manufacturer under decentralised and centralised distribution system if

$$
\begin{aligned}
max & \left\{ \frac{\Theta_{33} - \sqrt{2\Theta_{44}}}{2A_2^2[b_2(4b_2 + b_3) - 8b_2c_2 + 2c_2^2]}, 0 \right\} \leq y \leq \\
& min \left\{ \frac{\Theta_{33} + \sqrt{2\Theta_{44}}}{2A_2^2[b_2(4b_2 + b_3) - 8b_2c_2 + 2c_2^2]}, 1 \right\}
\end{aligned} \tag{33}
$$

where $\quad \Theta_{33} = 8A_2^2 b_2(b_2 - c_2) - A_2(b_2 b_3 - 2c_2^2)(b_2(2c_m - h_1 - h_2) - 2c_2 c_m)$ and $\Theta_{44} = A_2^2(b_2 b_3 - 2c_2^2)(4A_2^2(b_2(b_3 - 8c_2) + 2c_2^2) + 4A_2 b_2(b_2(b_3(2c_m - h_1 - h_2) + 4c_2(h_1 + h_2)) - 2c_2^2(2c_m + h_1 + h_2)) + b_2^2(b_2(2c_m + h_1 + h_2)(16c_2 c_m - b_3(6c_m - h_1 - h_2)) - 2(2b_3^2 c_m^2 - 4b_3 c_2 c_m(4c_m + h_1 + h_2) + c_2^2(28c_m^2 + 12c_m(h_1 + h_2) + (h_1 + h_2)^2))))$. From the above analysis, the following Proposition is made:

Proposition 3: If $A_1 = A_2$, $x = y$, $b_1 = b_2$, and $c_1 = c_2$, i.e., the market demand of two retail channel remains identical then introduction of online channel is not profitable if the compatibility of the product satisfy equation (32).

Figure 2 Profits of the manufacturer (π_m^{dd} and π_m^{dds} ... ...)
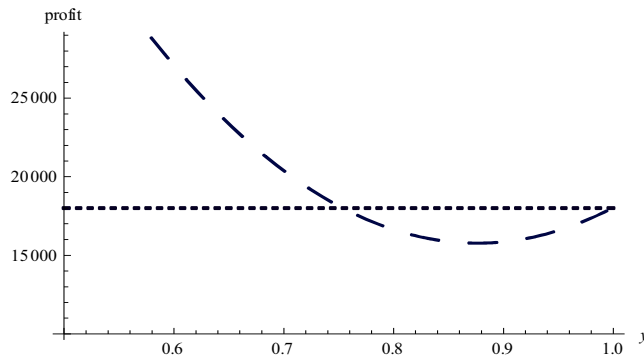
The graphical representation of profit of the manufacturer in centralised distribution system in presence of online channel and decentralised distribution system in absence of online channel are shown in Figure 2. The following parameters are used for illustration: $A_1 = A_2 = 200$, $b_1 = b_2 = 0.5$, $b_3 = 0.3$, $c_m = 20$, $h_1 = 0.5$, $h_2 = 0.6$, and $c_1 = c_2 = 0.1$.

Figure 3 Profit differences of the manufacturer [Δ_{md} (white) and Δ_{ms} (blue)] (see online version for colours)
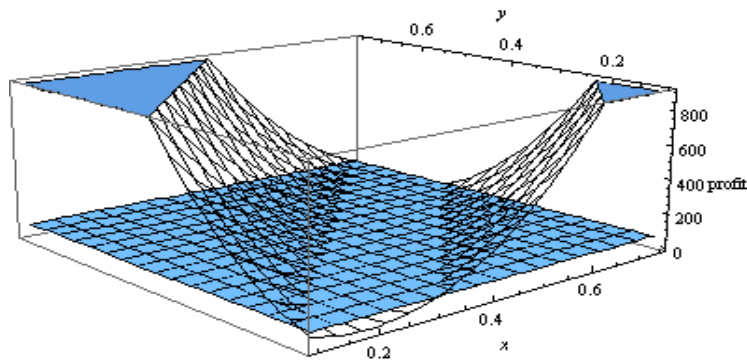
From Figure 2, it is found that if $y \in [0.75, 1]$, then operating dual channel is not profitable for the manufacturer. The lower limit of the interval indicates that if the customer loyalty is very high for retail channels, then only the online channel is not profitable for the manufacturer. It is also observed that profit gain of the manufacturer in presence of online channel (Δ_{md}) and in absence of online channel (Δ_{ms}) are different. In particular, if $x = y$, $b_1 = b_2$, $h_1 = h_2$, and $c_1 = c_2$, then $\Delta_{md} - \Delta_{ms} = -\frac{(A_1 - A_2)^2 (1 - y^2)}{16 b_2} < 0$, i.e., profit gain in absence of online channel is maximum. But if $A_1 = A_2$, $x = y$, $c_1 = c_2$ and $b_1 = b_2$, i.e., market potential at two locations are identical, then $\Delta_{md} - \Delta_{ms} = 0$ i.e., profit gain remains identical. But if $A_1 = A_2$, $x = y$, $b_1 = b_2$ and $h_1 = h_2$, then $\Delta_{md} - \Delta_{ms} = \frac{A_2^2 (c_1 - c_2)^2 [2 b_2 (1 - y) + (c_1 + c_2) y]^2}{4 b_2 (4 b_2 b_3 - 3 c_1^2 - 2 c_1 c_2 - 3 c_2^2)(b_2 b_3 - c_1^2 - c_2^2)} > 0$ as $b_3 > (c_1 + c_2)$, i.e., profit gain in presence of online channel is maximum. Therefore, online operation is not always profitable for the manufacturer. The graphical representation of profit gain of the manufacturer are shown in Figure 3. The following parameters are used for illustration: $A_1 = 200$, $A_2 = 180$ $b_1 = 0.5$, $b_2 = 0.4$, $b_3 = 0.3$, $c_m = 20$, $h_1 = 0.5$, $h_2 = 0.6$, and $c_1 = c_2 = 0.1$.

Figure 3 justifies the above analytical findings. Next, we have analysed the impact of an online channel. The sales volumes of the online channel under the decentralised (Q_o^{dd}) and centralised (Q_o^{cd}) distribution system are respectively $Q_o^{dd} = a_3 - b_3 p_m^{dd} + c_1 p_1^{dd} + c_2 p_2^{dd}$ and $Q_o^{cd} = a_3 - b_3 p_m^{cd} + c_1 p_1^{cd} + c_2 p_2^{cd}$. Substituting the corresponding optimal decision variables, one can obtain the following expression: $Q_o^{dd} = Q_o^{cd} = \frac{1}{4 b_1 b_2} [2 a_3 b_1 b_2 + a_1 b_2 c_1 + a_2 b_1 c_2 - (2 \Delta_1 + b_2 c_1^2 + b_1 c_2^2) c_m + b_1 b_2 (c_1 (c_m + h_1) + c_2 (c_m + h_2))]$. Similarly, the sales volume in the online channel under the benchmark model is $Q_o^{cc} = \frac{1}{2} [a_3 - b_3 c_m + c_1 (c_m + h_1) + c_2 (c_m + h_2)]$ By comparing optimal online sales volume, we have

$$Q_o^{dd} - Q_o^{cc} = \frac{1}{4 b_1 b_2} [a_1 b_2 c_1 + a_2 b_1 c_2 + (b_2 c_1^2 + b_1 c_2^2) c_m - b_1 b_2 (c_1 (c_m + h_1) + c_2 (c_m + h_2))] > 0 \tag{34}$$

That is, the sales volume of the online channel under the benchmark model is always less than it is in non-cooperative models, and by applying the centralised decision, then the manufacturer incurs loss in market share for operating an online channel. A comparison of equations (7) and (26) shows that $p_m^{dd} = p_m^{cc}$; that is, under the decentralised distribution system, the profit of the online channel is always greater than it is in the benchmark centralised-decision model. By comparing equations (16) and (26), we find

$$p_m^{cd} - p_m^{cc} = \frac{b_1 b_2 \Delta_3}{2 \Delta_1 \Delta_2} \tag{35}$$

i.e., $p_m^{cd} > p_m^{dc}$ if $\Delta_3 > 0$. From the above analysis, the following Proposition is proposed:

Proposition 4: If $\Delta_3 > 0$, then sales volume and profit for the online channel are always greater in two non-cooperative distribution model than they are in the benchmark centralised decision model.

Proposition 4 indicates that the manufacturer always loses market share and profit in the online channel under the benchmark centralised decision; however, the centralised decision yields maximum channel profit. Therefore, in the next section, we introduce the RS contract to remove channel inefficiency and subsequently provide a means to coordinate the supply chain.

3   Coordinating dual-channels by using an RS contract

In this section, the RS contract is discussed first for the decentralised distribution system. The RS contract can be described by four parameters, wholesale prices $w_i, i = 1, 2$, and revenue sharing fractions $\phi_i, (0 < \phi_i < 1, i = 1, 2)$. The RS contract entices each retailer to order the amount of product and set the prices that benefit the entire supply chain, and the manufacturer charges unit wholesale prices that are less than the marginal cost. In exchange, each retailer provides a fraction of revenue ϕ_i to the manufacturer. Under this contract mechanism, the profit functions of each retailer ($\pi_{ri}^{ddrs}, i = 1, 2$) and the manufacturer ($\pi_m^{ddrs}$) are as follows:

$$\pi_{ri}^{ddrs} = (\phi_i p_i - w_i - h_i) D_{ri} \tag{36}$$

$$\pi_m^{ddrs} = \sum_{i=1}^{2} (w_i + (1 - \phi_i) p_i - c_m) D_{ri} + (p_m - c_m) D_o \tag{37}$$

To verify whether the contract can coordinate the supply chain, it is necessary to determine response of each retailer by solving $\frac{d\pi_{ri}^{ddrs}}{dp_i} = 0, i = 1, 2$. After simplification, we have obtained

$$p_i = \frac{1}{2b_i\phi_i}[a_i\phi_i + b_i w_i + c_i p_m \phi_i + h_i b_i] \tag{38}$$

Based on the retailer's response, the manufacturer can maximise its own profit by maximising π_m^{ddrs} with respect to w_1, w_2, and p_m or the manufacturer can coordinate the retailer's decision on pricing (i.e., set $p_i = p_i^{cc}$, i=1,2). As our objective is to achieve channel coordination, we assume that the manufacturer will adopt the second option for achieving channel coordination. Therefore, the wholesale prices are obtained as follows:

$$\begin{aligned} w_i = \frac{1}{b_i \Delta_1}[&\phi_i(a_3 b_1 b_2 c_i + a_1 b_2 c_1 c_i + a_2 b_1 c_2 c_i) - (c_m + h_i) b_i \Delta_1 \\ &+ \phi_i b_i h_i \Delta_1 - c_i \Delta_1 p_m \phi_i] \end{aligned} \tag{39}$$

In this situation, the manufacturer has two alternatives:

1   Maximise profit by maximising $\pi_m^{ddr\ s}$ with respect to p_m^{cc}.

2 Set $p_m = p_m^{cc}$. If the manufacturer optimises π_m^{ddr} ^{s}:

Then by substituting values of w_i obtained into equation (39) to equation (37) and by solving $\frac{d\pi_m^{ddrs}}{dp_m} = 0$, the price of the product in the online channel is

$$p_m = \frac{1}{2\Delta_1\Delta_4}[a_3 b_1 b_2 \Delta_4 + a_1 b_2 c_1 (\Delta_4 - \Delta_1 \phi_1) + a_2 b_1 c_2 (\Delta_4 - \Delta_1 \phi_2)$$
$$+ b_1 b_2 \Delta_1 (b_3 c_m + c_1 (c_m + h_1)\psi_1 + c_2 (c_m + h_2)\psi_2)] \tag{40}$$

where $\Delta_4 = b_1 b_2 b_3 + b_2 c_1^2 \phi_1 + b_1 c_2^2 \phi_2$. By equating $p_m = p_m^{cc}$ for the channel coordination, one can obtain $\phi_1 b_2 c_1[a_1 - (b_1 - c_1)c_m - b_1 h_1] + \phi_2 b_1 c_2[a_2 - (b_2 - c_2)c_m - b_2 h_2] = 0$. Note that, both ϕ_1 and ϕ_2 are non-negative. The first retailer receives positive if $a_1 - b_1(c_m + h_1) + c_1 c_m > 0$ as $\text{Min}\{p_m, w_1\} > (c_m + h_1)$. Similarly, $a_2 - b_2(c_m + h_2) + c_2 c_m > 0$ for the second retailer. Therefore, sum of all positive quantities never be equal to zero and this relation is not feasible. If the manufacturer adopts second options, then one can easily verify that $\sum_{i=1}^{2} \pi_{ri}^{ddrs} + \pi_m^{ddrs} = \pi^{cc}$, i.e. the distribution channel becomes coordinated. Now the difference of profits of each retailer and the manufacturer obtained under the RS contract and non-cooperative decentralised distribution system are computed as follows:

$$\pi_{ri}^{ddrs} - \pi_{ri}^{dd} = \frac{1}{16 b_i}[a_i - (b_i + c_i)c_m - h_1 b_1]^2 (4\phi_i - 1), i = 1, 2 \tag{41}$$

$$\pi_m^{ddrs} - \pi_m^{dd} = \frac{1}{8 b_1 b_2}[b_1(1 - 2\phi_2)(a_2 - (b_2 - c_2)c_m - h_2 b_2)^2$$
$$+ b_2(1 - 2\phi_1)(a_1 - (b_1 - c_1)c_m - h_1 b_1)^2] \tag{42}$$

The win-win outcome can be achieved if $\pi_{ri}^{ddrs} \geq \pi_{ri}^{dd}(i = 1, 2)$, and $\pi_m^{ddrs} \geq \pi_m^{dd}$. On simplification, we have obtained following inequalities: $\phi_1 \geq 1/4$, $\phi_2 \geq 1/4$, and $b_1[a_2 - (b_2 - c_2)c_m - h_2 b_2]^2 + b_2[a_1 - (b_1 - c_1)c_m - h_1 b_1]^2 > 2b_1\phi_2[a_2 - (b_2 - c_2)c_m - h_2 b_2]^2 + 2b_2\phi_1[a_1 - (b_1 - c_1)c_m - h_1 b_1]^2$. These three inequalities represent a triangular feasible region, and the extreme points of the region are $A(1/4, 1/4)$; $B\left(1/4, 1/2 + \frac{b_2(a_1 - (b_1 - c_1)c_m - h_1 b_1)^2}{4b_1(a_2 - (b_2 - c_2)c_m - h_2 b_2)^2}\right)$, and $C\left(1/2 + \frac{b_1(a_2 - (b_2 - c_2)c_m - h_2 b_2)^2}{4b_2(a_1 - (b_1 - c_1)c_m - h_1 b_1)^2}, 1/4\right)$. Note that the manufacturer achieves it maximum profit under the RS contract at the point A and the maximum profit is

$$\pi_m^{ddrsmax} = \frac{1}{16 b_1 b_2 \Delta_1}[4 a_3^2 b_1^2 b_2^2 + 8 a_1 a_2 b_1 b_2 c_1 c_2$$
$$+ a_1^2 b_2(b_2(3b_1 b_3 + c_1^2) - 3b_1 c_2^2) + a_2^2 b_1(b_1(3b_2 b_3 + c_2^2) - 3b_2 c_1^2)$$
$$- 2a_2 b_1(3b_2 + c_2)c_m \Delta_1 - 2a_1 b_2(3b_1 + c_1)c_m \Delta_1$$
$$+ (3b_1 b_2(b_1 + b_2 + b_3 - 2(c_1 + c_1)) + \Delta_1)\Delta_1 c_m$$
$$+ 8 a_3 b_1 b_2(a_1 b_2 c_1 + a_2 b_1 c_2 - c_m \Delta_1)$$
$$- 3b_1 b_2(2h_1(a_1 - (b_1 - c_1)c_m) + 2h_2(a_2 - (b_2 - c_2)c_m)$$
$$+ b_1 h_1^2 + b_2 h_2^2)\Delta_1] \tag{43}$$

Similarly, the maximum profit of the retailers is obtained as follows:

$$
\begin{aligned}
\pi_{ri}^{ddrsmax} = \frac{1}{16b_1b_2}[&b_1(a_2 - (b_2 - c_2)c_m - h_2b_2)^2 \\
&+b_2(a_1 - (b_1 - c_1)c_m - h_1b_1)^2] \\
&+\frac{[(a_i - (b_i - c_i)c_m - h_ib_i)^2]}{16bi}
\end{aligned}
\tag{44}
$$

Hence, one can conclude that the RS contract can coordinate the channel and provide win-win outcomes. By comparing equations (10) and (44), one can find that the maximum profit gain of each retailer under the RS contract is uniform, and equal to $\frac{b_2[a_1 - (b_1 - c_1)c_m - h_1b_1]^2 + b_1[a_2 - (b_2 - c_2)c_m - h_2b_2]^2}{16b_1b_2}$. It implies that the manufacturer penalises the more profitable retailer by applying an RS contract. Similarly, in the centralised distribution system, the RS contract can be described by two parameters, wholesale price w and revenue sharing fraction ψ ($0 < \psi < 1$). Under this scenario, the profit functions of the retailer (π_r^{cdrs}) and the manufacturer (π_m^{cdrs}) are obtained as follows:

$$
\pi_r^{cdrs} = \sum_{i=1}^{2}(\psi p_i - w - h_i)D_{ri}
\tag{45}
$$

$$
\pi_m^{cdrs} = (1 - \psi)\sum_{i=1}^{2}p_iD_{ri} + (w - c_m)\sum_{i=1}^{2}D_{ri} + (p_m - c_m)D_o
\tag{46}
$$

Similar to previous scenario, by solving $\frac{\partial \pi_r^{cdrs}}{\partial p_i} = 0, i = 1, 2$; the retailer's responses in the RS contract can be obtained as follows:

$$
p_i = \frac{1}{2b_i\psi}[a_i\psi + b_iw_i + c_ip_m\psi - h_ib_i], i = 1, 2
\tag{47}
$$

After getting the retailer's response, the manufacturer can maximise its own profit with respect to w and p_m or coordinate the retailer's decision on pricing (i.e. sets $p_i = p_i^{cc}$). If the manufacturer coordinates the retailer's decision, then the following wholesale prices are obtained

$$
\begin{aligned}
w = \frac{1}{b_i\Delta_1}[&\psi(a_3b_1b_2c_i + a_1b_2c_1c_i + b_1a_2c_2c_i) \\
&+b_i(h_i + c_m)\psi\Delta_1 - c_i\Delta_1p_m - b_ih_i\Delta_1], i = 1, 2
\end{aligned}
\tag{48}
$$

The value of wholesale price must be independent of i, which is possible if $\psi = 0$ or $p_m = \frac{1}{\Delta_1}[a_3b_1b_2 + a_1b_2c_1 + a_2b_1c_2] + \frac{b_1b_2(h_1-h_2)}{(b_2c_1 - b_1c_2)}$. But, $\psi = 0$ is not feasible and the optimal channel profit cannot be achieved if the manufacturer sets the above price for the online channel. Hence, we can conclude that the RS contract cannot coordinate the channel. From above analysis, the following Proposition is made:
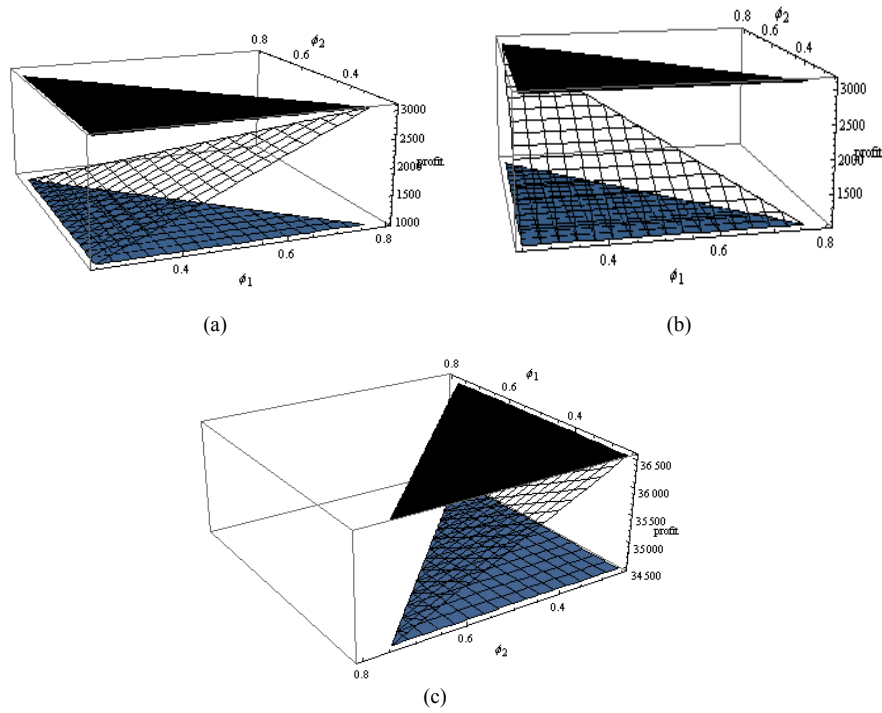
Proposition 5

- The RS contract can be used to coordinate the decentralised distribution system, but it fails to coordinate the centralised distribution system.

- The manufacturer penalises the more profitable retailer by applying an RS contract.

The graphical representation of profit functions of the manufacturer and two retailers under the RS contract (x=y=0.5) are shown in Figures 4(a)–4(c).

From Figures 4(a)–4(c), one can easily observe that as ϕ_i increases, the profit of each retailer also increases but the profit of the manufacturer decreases. The profit structures in equations (36) and (37) also justify the analytical findings.

Figure 4 (a) Profits of the first retailer under the RS contract [Π_{r1}^{dd} (grey), Π_{r1}^{ddrs} (white), $\Pi_{r1}^{ddrsmax}$ (black)] (b) Profits of the second retailer under the RS contract [Π_{r2}^{dd} (grey), Π_{r2}^{ddrs} (white), $\Pi_{r2}^{ddrsmax}$ (black)] (c) Profits of the manufacturer under the RS contract [Π_{m}^{dd} (grey), Π_{m}^{ddrs} (white), $\Pi_{m}^{ddrsmax}$ (black)] (see online version for colours)

(a)

(b)

(c)

3.1 Modified RS contract

Previous observations suggest that the RS contract cannot be used to coordinate the centralised distribution system. Therefore, to achieve channel coordination, we have modified the RS contract mechanism. Previous findings show that the manufacturer

sometimes earns more profit from the online channel ($p_m^{cd} > p_m^{dd} = p_m^{cc}$). Hence, to entice the retailer to order larger quantities of product and set prices that benefit the entire supply chain, the modified RS contract is used. When the manufacturer charges a unit wholesale price that is lower than the marginal cost and shares a fraction of revenue θ, ($0 < \theta < 1$), it earns from the online channel as does the retailer. In exchange, the retailer provides a fraction of revenue λ ($0 < \lambda < 1$) to the manufacturer. Under this scenario, the profit functions of the retailer (π_r^{cdrsm}) and the manufacturer (π_m^{cdrsm}) are as follows:

$$\pi_r^{cdrsm} = \sum_{i=1}^{2} \lambda(p_i - h_i)D_{ri} - w\sum_{i=1}^{2} D_{ri} + \theta(p_m - c_m)D_o \tag{49}$$

$$\pi_m^{cdrsm} = (1-\lambda)\sum_{i=1}^{2}(p_i - h_i)D_{ri} + (w - c_m)\sum_{i=1}^{2} D_{ri}$$
$$+ (1-\theta)(p_m - c_m)D_o \tag{50}$$

By solving $\frac{\partial \pi_r^{cdrsm}}{\partial p_i} = 0, i = 1, 2$; the retailer's responses for the prices in retail channels under the modified RS contract are obtained as follows:

$$p_i = \frac{1}{2b_i\lambda}[(a_i + b_i h_i)\lambda + b_i w_i + c_i p_m \lambda + c_i\theta(p_m - c_m)], i = 1, 2 \tag{51}$$

If the manufacturer coordinates the retailer's decision on pricing (i.e. sets $p_i = p_i^{cc}$), then the wholesale price of the product becomes

$$w = \frac{1}{b_i\Delta_1}[(a_3 b_1 b_2 c_i + a_1 b_2 c_1 c_i + a_2 b_1 c_2 c_i + \Delta_1(b_i c_m - c_i p_m))\lambda$$
$$+ c_i(c_m - p_m)\Delta_1\theta], i = 1, 2 \tag{52}$$

But the value of wholesale price must be unique and independent of i, which is possible if

$$p_m = \frac{1}{\Delta_1(\lambda + \theta)}[(a_3 b_1 b_2 + a_1 b_2 c_1 + a_2 b_1 c_2)\lambda + \Delta_1 c_m\theta] \tag{53}$$

For the channel coordination, by equating $p_m = p_m^{cc}$, one can obtain $\lambda = \theta$ and $w = \lambda c_m$. Under this circumstance, one can easily verify that $\pi_r^{cdrsm} + \pi_m^{cdrsm} = \pi^{cc}$, i.e. the system becomes coordinated. The difference of profits of the retailer and the manufacturer obtained under modified RS contract and corresponding non-cooperative scenario are computed as follows:

$$\pi_r^{cdrsm} - \pi_r^{cd} = \lambda\pi^{cc} - \pi_r^{cd} \tag{54}$$

$$\pi_m^{cdrsm} - \pi_m^{cd} = (1-\lambda)\pi^{cc} - \pi_m^{cd} \tag{55}$$

The win-win outcome can be achieved if

$$\frac{\pi^{cc} - \pi_m^{cd}}{\pi^{cc}} \geq \lambda \geq \frac{\pi_r^{cd}}{\pi^{cc}} \tag{56}$$

That is, if the RS parameter falls within the range given in equation (56), the contract between the manufacturer and the retailer can coordinate the non-cooperative centralised-distribution system, which results in a win-win situation. According to the contract structure, the manufacturer charges a wholesale price that is smaller than the marginal cost. The contract allows the manufacturer and the retailer to earn as much profit as in the non-cooperative centralised distribution structure. Under the modified RS contract, the manufacturer shares a percentage of the revenue with the retailer, which is a typical practice; for example, revenues from the IBM and HP e-commerce websites are reportedly shared with their respective retailers (Tsay and Agrawal, 2004). This profit-sharing structure suggests that a larger λ benefits the retailer, whereas a smaller λ benefits the manufacturer. The value of λ depends on the respective bargaining power of the parties in the negotiation. Thus, the modified RS contract can be used to coordinate the centralised distribution system. Under this contract mechanism the maximum profit of the manufacturer is $\pi_m^{cdrsmmax} = \pi^{cc} - \pi_r^{cd}$. From this analysis, the following proposition is made:

Proposition 6: The modified RS contract can be used to coordinate the centralised distribution system.

From the above derivation, one can see that the manufacturer can coordinate distribution systems described herein by applying suitable coordination mechanisms, but the maximum profits of the manufacturer under each coordinated environment differ from each other. The graphical representation of the manufacturer's maximum profit under the two scenarios is shown in Figure 5.

Figure 5   The maximum profits of the manufacturer under coordination [Π_m^{ddmax} (grey) and Π_m^{cdmax} (black)] (see online version for colours)
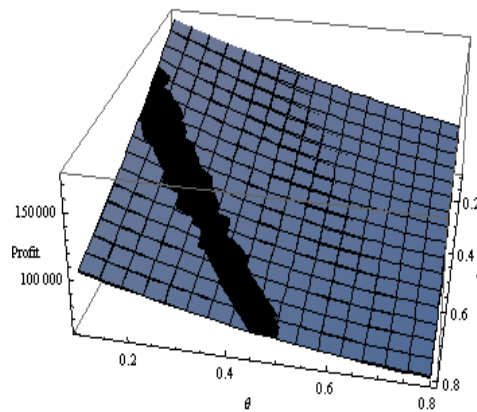
Figure 5 shows that under a coordinated system, the manufacturer's maximum profit is sometimes identical in two different control structures. In general, the manufacturer earns the maximum profit under the coordinated environment if two different retailers sell the product. In particular, if $A_1 = A_2$, $b_1 = b_2$, $c_1 = c_2$, and $x = y$ (i.e., if the market conditions are identical in two locations), then the maximum profits difference will be equal to zero. We can conclude that the maximum profit of the manufacturer under the coordinated environment is sometimes independent from the control structures of retail channels.

A chain store system is a network of branch shops situated at different parts of a country under centralised management. Because of competitive advantages for chain retailers, this trend of integration has grown rapidly. Examples include, but are not limited to, Coop in Switzerland, Crai in Italy, Big C in Thailand, and E-mart in South Korea. Chains provide the opportunity not only to share a common operational platform but also to negotiate price. In this study, we analyse the performance of the manufacturer under such an integration. Our study reveals that under the non-cooperative scenario such an integration is always harmful to the manufacturer. Although the manufacturer can improve its profit by opening an online channel, this action is insufficient. Moreover, the RS contract studied extensively in the literature is also insufficient to ensure cooperation and achieve maximum supply chain efficiency. Under cooperation, the manufacturer sometimes earns less profit than when operating independently.

4 Summary and concluding remarks

In this paper, we investigated the optimal product distribution and coordination strategy for manufacturers that use online sales. In contrast to existing studies, in which the effect of the online channel is studied as part of a single retail channel, we have analysed it in two different distribution systems. We call these two distribution structures, respectively, *decentralised* and *centralised* distribution systems. In the first scenario, two retail channels are operated by two independent retailers, and in the second scenario, two retail channels are operated by a single retailer. In the non-cooperative scenario, the manufacturer always prefers the decentralised distribution system to earn maximum profit whether an online channel is used or not. Moreover, operating an online channel is not always profitable for the manufacturer. Therefore, we identified the profitability of operating an online channel. Furthermore, we found that the RS contract can be used to coordinate the channel and distribute the fixed amount of profit arbitrarily for the decentralised distribution system; however, in this case, the manufacturer penalises the more profitable retailer through application of an RS contract. Our analysis reveals that the RS contract cannot be used to coordinate the centralised distribution system, and we proposed a modified RS contract mechanism through which the manufacturer shares a percentage of the revenue earned from the online channel with the retailer. The modified mechanism is not only used effectively to coordinate the channel but also to encourage retailers to cooperate their businesses.

Despite its importance and contribution, the present study has some limitations. Other researchers may want to generalise our analysis by considering more retail shops, but such an enhancement would require extraordinary computational effort. Moreover, we examined the supply chain structure in which the manufacturer is considered the leader. Therefore, another study is needed to compare decisions on the procurement

strategy when the retailer is the leader. For analytical tractability, we have excluded detailed cost parameters of the retailer, so a study that takes into account cost parameters to verify of our conclusions and to explain the effects of disruptions would extend the contributions of our initial findings. Finally, it will be worthwhile to use surveys to estimate the range of parameter values.

Acknowledgements

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Appendix A

Proof of the Proposition 1

The optimal solution for each retailer are obtained by solving $\frac{d\pi_{ri}^{dd}}{dp_i} = 0$, $i = 1, 2$. On simplification we have

$$p_i = \frac{1}{2b_i}[a_i + c_i p_m + b_i w_i + b_i(w_i + h_i)]$$

Since $\frac{d^2\pi_{ri}^{dd}}{dp_i^2} = -2b_i < 0$, i.e. the profit functions are concave with respect to p_i. Substituting p_i in Equation (2), we have obtained the profit function of the manufacturer as follows:

$$\pi_m^{dd} = \sum_{i=1}^{2} \frac{(w_i - c_m)}{2}(a_i + c_i p_m - b_i(w_i + h_i)) + (p_m - c_m)$$

$$\left[a_3 - b_2 p_m + \sum_{i=1}^{2} \frac{c_i}{2b_i}(a_i + c_i p_m + b_i(w_i + h_i))\right]$$

As the profit function of the manufacturer is a function of three variables, we have computed the Hessian matrix of the manufacturer's profit function as follows:

$$H_m = \begin{pmatrix} \frac{\partial^2\pi_m^{dd}}{\partial w_1^2} & \frac{\partial^2\pi_m^{dd}}{\partial w_1 \partial w_2} & \frac{\partial^2\pi_m^{dd}}{\partial w_1 \partial p_m} \\ \frac{\partial^2\pi_m^{dd}}{\partial w_1 \partial w_2} & \frac{\partial^2\pi_m^{dd}}{\partial w_2^2} & \frac{\partial^2\pi_m^{dd}}{\partial w_2 \partial p_m} \\ \frac{\partial^2\pi_m^{dd}}{\partial w_1 \partial p_m} & \frac{\partial^2\pi_m^{dd}}{\partial w_2 \partial p_m} & \frac{\partial^2\pi_m^{dd}}{\partial p_m^2} \end{pmatrix} = \begin{pmatrix} -b_1 & 0 & c_1 \\ 0 & -b_2 & c_2 \\ c_1 & c_2 & -2b_3 + c_1^2/b_1 + c_2^2/b_2 \end{pmatrix}$$

The values of principle minors of the above Hessian matrix, $m_1 = -b_1 < 0$, $m_2 = b_1 b_2 > 0$ and $m_3 = -2\Delta_1$ will be alternative in sign if $\Delta_1 > 0$. Therefore, one can conclude that the profit function of the manufacturer is concave if $\Delta_1 > 0$.

Each retailer can gain non-negative profits if $p_1^{dd} - w_1^{dd} - h_1 = (A_1 x - c_m(b_1 - c_1) - b_1 h_1)/(4b_1) \geq 0$ and $p_2^{dd} - w_2^{dd} - h_2 = (A_2 y - c_m(b_2 - c_2) - b_2 h_2)/(4b_2) \geq 0$. On simplification we can obtain the lower limits of inequalities (11) and (12). Similarly, each retailer will participate in dual channel if $p_m^{dd} - w_1^{dd} - h_1 = (A_1((2b_2 c_1 - b_2 b_3 + c_2^2)x + b_1(b_2 - b_2 x) - b_2 c_1) + A_2(b_1 - c_1)(b_2 - b_2 y + c_2 y) - h_1 \Delta_1)/2\Delta_1 \geq 0$ and $p_m^{dd} - w_2^{dd} - h_2 = ((b_2 - c_2)(A_2 b_1 + A_1(b_1 - b_1 x + c_1 x)) + A_2(c_1^2 - b_1(b_2 + b_3 - 2c_2))y - h_2 \Delta_1)/2\Delta_1 \geq 0$. After simplification we get the upper limits of inequalities (11) and (12).

Appendix B

Proof of the Proposition 2

The optimal values for the decision variables of the retailer are obtained by solving $\frac{\partial \pi_r^{cd}}{\partial p_i} = 0$, $i = 1, 2$. On simplification we have

$$p_i = \frac{1}{2b_i}[a_i + c_i p_m + b_i(w + h_i)]$$

As $\frac{\partial^2 \pi_r^{cd}}{\partial p_1^2} \frac{\partial^2 \pi_r^{cd}}{\partial p_2^2} - (\frac{\partial^2 \pi_r^{cd}}{\partial p_1 \partial p_2})^2 = 4b_1 b_2 > 0$, i.e. the profit function of the retailer is concave with respect to p_i. Substituting p_i in the Equation (14), we have obtained the profit function of the manufacturer as follows:

$$\pi_m^{cd} = \sum_{i=1}^{2} \frac{(w - c_m)}{2}(a_i + c_i p_m - b_i(w + h_i)) + (p_m - c_m)$$

$$\left[a_3 - b_3 p_m + \sum_{i=1}^{2} \frac{c_i}{2b_i}(a_i + c_i p_m + b_i(w + h_i)) \right]$$

As $\frac{d^2 \pi_m^{cd}}{dw^2} = -(b_1 + b_2) < 0$ and $\frac{\partial^2 \pi_m^{cd}}{\partial w^2} \frac{\partial^2 \pi_m^{cd}}{\partial p_m^2} - (\frac{\partial^2 \pi_m^{cd}}{\partial w \partial p_m})^2 = \Delta_2/b_1 b_2 > 0$ if $\Delta_2 > 0$ i.e. the profit function of the manufacturer is concave if $\Delta_2 > 0$.
Similar to appendix A, the retailer can gain non-negative profit by operating each retail channel if $p_1^{cd} \geq w$ and $p_2^{cd} \geq w^{cd}$, and the retailer will participate in dual channel if $p_m^{cd} \geq w^{cd}$. After simplification we get equations (22) and (23).