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E-commerce supply chain network design using on-demand warehousing system under uncertainty

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ABSTRACT
During the COVID-19 pandemic, e-commerce retailers have had trouble satisfying the growing demand because of limited warehouse capacity constraints. Fortunately, an on-demand warehousing system has emerged as a new alternative to mitigate warehouse capacity issues. In recent years, several studies have focused on the supply chain problem considering on-demand warehousing. However, there is no study that deals simultaneously with inherent uncertainties and the property of commitment, which is the main advantage of on-demand warehousing. To fill these research gaps, this paper presents an e-commerce supply chain network design problem considering an on-demand warehousing and decisions for commitment periods. We propose the two-stage stochastic programming model that captures the inherent uncertainties to formulate the presented problem. We solve the proposed model utilizing sample average approximation combined with the Benders decomposition algorithm. Of particular note, we develop a method to generate effective initial cuts for improving the convergence speed of the Benders decomposition algorithm. Computational results show that the developed method could find an effective feasible solution within a reasonable computational time for problems of practical size. Furthermore, we show the significant cost-saving effects, based on experiment results, that occur when an on-demand warehousing system is used for designing supply chain networks.

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On-demand warehousing system; supply chain network design; stochastic programming; sample average approximation; benders decomposition

1. Introduction

With the growth of both communications technology and contact-free delivery demand, e-commerce has grown significantly during the past few years (Goldberg 2022). Because of the rapid development of e-commerce marketplaces, the number of *e-commerce retailers* selling products online also has increased. Compared to the big capital-markets businesses, most e-commerce retailers run their businesses with low capital investment. Therefore, e-commerce retailers usually operate warehouses with small spaces for dealing with varying demands (Dunke et al. 2018; Paul et al. 2021).

In order to manage the problem of small warehouse capacity, e-commerce retailers have several older solutions at their disposal (Unnu and Pazour 2019). First, e-commerce sellers can build new warehouses or infrastructure for expanding capacity. However, a lot of capital investment is necessary to implement this solution. Another solution is to lease warehouses from traditional warehouse operators or third-party logistics providers (3PLs). The contract duration for leasing space from traditional providers is usually long and requires a long-term contract. Therefore, this way is not a suitable strategy for e-commerce sellers who need flexible solutions (Ceschia et al. 2022).

Because of such circumstances, the *on-demand warehousing* platform has emerged as a new alternative (Hahn 2020). In real cases, the platform *FLEXE* provides service for on-demand warehousing in the global market (Hass 2019). This platform connects warehouse providers who have excess space with e-commerce retailers who require empty space for the short term (Forger 2018). We present the three advantages of utilising an on-demand warehousing system (ODWS) as follows:

(1) *Saving setup cost:* Because an ODWS does not require additional infrastructure, it could be a low-risk strategy to match varying customer demands and prevent inventory overflow for seasonal products.
(2) *High flexibility:* By utilising an ODWS, e-commerce retailers could secure warehousing and fulfillment

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solutions instantly. Also, they could build a flexible distribution network according to the dynamic of retailers' businesses.

(3) *Delivery speed:* A same-day shipping service is almost impossible if the warehouses are far from the customer base. However, by employing the ODWS, e-commerce retailers can search the shared warehouse space that is located near the customer base.

In recent cases, because it cannot be sure how long the pandemic-driven consumer spending will last, many small-medium sized e-commerce companies prefer to utilise the ODWS (Leonard 2021). From the standpoint of the e-commerce retailer, the main advantage of the ODWS is that a short-term rent for warehouses is available (Tornese et al. 2020). Throughout this paper, we will use the terminology *commitment* to indicate the short-term rent contract for warehouses in the ODWS.

Because of the distinctive advantages of the ODWS, several recent studies have focused on solving the supply chain problem with the ODWS to derive a cost-saving strategy based on optimisation-based methods (Van der Heide et al. 2018; Shi, Yu, and Dong 2021; Tian and Zhang 2021; Ceschia et al. 2022; Unnu and Pazour 2022). Even though previous studies have dealt with the ODWS in various aspects, this study seeks to fill two research gaps in the ODWS research area. The first research gap is that previous studies did not address the main characteristic of the ODWS, the short-term rent contract (i.e.commitment), except (Unnu and Pazour 2022). Although Unnu and Pazour (2022) addressed the property of commitment, they did not deal with the decisions for the commitment period for using the ODWS because the available commitment period was a given parameter. The second research gap is scarcity studies that consider the inherent uncertainties systematically involved in making decisions that might occur in the supply chain with the ODWS. While several studies considered uncertainties of demand (Van der Heide et al. 2018; Shi, Yu, and Dong 2021), as far as we know, there was no research that dealt with the properties of commitment and inherent uncertainties simultaneously.

To fill these research gaps, this study aims to deal with the supply chain network design (SCND) problem considering the characteristics of the ODWS and the decisions for the commitment period. Furthermore, because demand and supply have inherent uncertainties, our research addresses the SCND problem with the ODWS under uncertain environments. To the best of our knowledge, this study is the first attempt to solve the problem considering the properties of commitment and uncertainties simultaneously in the ODWS research area. Of special note, we define the supply uncertainty form as *yield uncertainty*, which means the amount actually supplied is random and different from the amount ordered.

This study extends the conference paper (Lee, Park, and Moon 2021) by considering decisions for supplier selections and inherent uncertainties of demand and supply. Motivated by the above research gaps in existing ODWS literature, this study defines the following four research questions to address:

(1) How would it be best to consider the uncertainties for the SCND with the ODWS and devise the solution approach for reducing computational efforts?
(2) How does the ODWS affect the supply chain network and the total cost of the resulting supply chain?
(3) What impact does the total cost and utilisation of warehouses have when the commitment and stock-out costs vary?
(4) What impact does the lead time have in the supply chain with the ODWS?

The main contributions of this paper are threefold. First, we propose the two-stage stochastic programming model (TSSP) for an e-commerce SCND with the ODWS under uncertainties. To estimate the expected function in the proposed model, we employ the sample average approximation (SAA) method. Second, to alleviate the computational burden in SAA, we utilise the multi-cut version of the Benders decomposition (BD) algorithm. Furthermore, we develop the acceleration method for improving the convergence of bounds by focusing on the initial iteration in the BD algorithm. Third, we show the potential cost-saving effects of using the ODWS in the supply chain through computational experiments.

The remainder of this paper is organised as follows. In Section 2, we review related studies to our study and present novelties of our study in detail. We describe the definition and assumptions for the problems and propose the mathematical model in Section 3. In Section 4, we explain the solution methodology, including the SAA and BD algorithms. In Section 5, we present a computation study involving the comparison of algorithms and the analysis of adopting the ODWS. Finally, a brief summary of our research, along with ideas for future studies, are presented in Section 6.

2. Literature review

Our study is directly related to three streams of literature in operations management. First, we review the literature on the dynamic facility location model (DFLM), which is the general supply chain model of our study. Second, we investigate relevant literature on scenario-based stochastic programmes for the SCND within a methodological context for our research problem. Third, we review literature that considers the properties of ODWS in supply chain problems. In addition, we present distinctive features of our study compared to relevant studies on the ODWS.

2.1. Dynamic facility location and SCND under uncertainty

The facility location (FL) model is roughly categorised using six classifications, and detailed taxonomy is presented in Klose and Drexl (2005). Our SCND model is developed based on the multi-stage, capacitated, multiple-sourcing, multi-item, and dynamic FL model. Among several categories, the dynamic property is the most essential for accommodating the features of ODWS. The DFLM considers the multi-period problem, and the input parameters (e.g. cost, capacity, and demand) differ depending on the time period. Due to this property, facilities can be opened or closed in every period throughout a given planning horizon (Klose and Drexl 2005; Manzini and Gebennini 2008).

Instead of reviewing all the works related to the DFLM, we present three papers covering the capacity adjustment through the lens of opening or closing a facility, which is related to one of the properties of the ODWS. Melo, Nickel, and Da Gama (2006) proposed the DFLM that considered the gradual relocation of facilities over the planning horizon. In this model, the capacity could be transferred from existing facilities to new facilities. To accommodate fluctuations of demands, two extended mathematical models were suggested for dealing with scenarios of capacity expansion and reduction. In addition, because the above two scenarios considered capacity transfer size as continuous, the authors presented the modular case model that permits discrete amounts. However, they did not consider any commitment properties for opening or closing facilities.

Several related works considered different time resolutions for strategic and tactical periods over a planning horizon (Thanh, Bostel, and Péton 2008; Bashiri, Badri, and Talebi 2012; Badri, Bashiri, and Hejazi 2013; Fattahi, Mahootchi, and Moattar Husseini 2016). In this literature, the decision to open or close facilities could be allowed only in strategic periods. Badri, Bashiri, and Hejazi (2013) developed a mixed-integer linear programming (MILP) model for capacity expansion in four echelons of the multiple commodity supply chain. The budget constraint for the expansion of the supply chain was determined according to cumulative net profits and funds supplied by external sources. Two types of warehouses, private and public, were considered, and public warehouses could be used at any time if contracted to be utilised. However, they also did not accommodate commitment constraints for using public warehouses. Fattahi, Mahootchi, and Moattar Husseini (2016) proposed a multi-stage, multi-item, and DFLM, which considered price dependent demand. The authors also considered private and public warehouses, and decisions for product shipments were made in tactical periods. While public warehouses could be opened or closed at any time period, private warehouses could not be closed if opened once.

The FL problem is applied to various domains. Especially, SCND has been considered as an appropriate application area for the FL problem (Geoffrion and Graves 1974; Klose and Drexl 2005). In general, large investments are required to make strategic decisions for determining locations and the number of facilities in SCND. However, if these strategic decisions are made in a deterministic environment, a huge amount of costs can be incurred due to the fluctuations of demands and supplies. Therefore, in both practice and academia, the necessity of considering uncertainty in SCND has obtained substantial attention (Govindan, Fattahi, and Keyvanshokooh 2017). To cope with uncertainty in SCND, our study proposes a mathematical framework based on scenario-based stochastic programmes. Owing to the nature of scenario-based stochastic programmes, the problem size increases depending on the number of scenarios. The emphasis in our review of the literature is on how existing studies address the scenario-generation issue and solution approach for the proposed stochastic programming model.

Through reviewing several previous studies, we could observe that the SAA and scenario tree construction are broadly used for scenario generation. First, several studies adopting the SAA will be introduced. Santoso et al. (2005) dealt with the large-scale problem for the global SCND. They used the SAA method and single-cut BD algorithm. In the single-cut BD algorithm, only a single optimality cut is applied at each iteration. Schütz, Tomasgard, and Ahmed (2009) considered the SCND problem for the Norwegian meat industry. They used the SAA method and dual decomposition algorithm to solve the problem. Fazeli et al. (2021) proposed the two-stage stochastic mixed-integer nonlinear programming (MINLP) to design an electric vehicle charging station network. They compared the single-cut and multi-cut BD algorithms and showed that multi-cut BD outperformed single-cut BD. Different from the single-cut BD, several optimality cuts are generated at each iteration in the multi-cut BD. Nur et al. (2021) addressed a biofuel SCND incorporating biomass quality properties. They proposed a parallelised decomposition algorithm that combined the SAA and an enhanced progressive hedging algorithm to solve real-life problem instances in a reasonable time. Azaron, Venkatadri, and Farhang Doost (2021) developed a multi-objective TSSP for taking into account the decision about production, inventory, and shipping among the entities of the supply chain network. The $\epsilon-$constraint method and SAA were utilised to solve the proposed multi-objective TSSP.

To generate efficient scenarios, several studies utilised scenario tree construction. Khatami, Mahootchi, and Farahani (2015) addressed closed-loop supply chains and used the single-cut BD algorithm for the solution approach. They generated scenarios based on the demand distribution function using Chloesky's factorisation method. Fattahi and Govindan (2017) introduced the SCND problem for an integrated forward/reverse logistics setup over a planning horizon. The Latin Hypercube Sampling method generated a fan of scenarios for demand and potential return uncertainty. Zahiri et al. (2018) presented the multi-stage stochastic programming approach with a combined scenario tree for an integrated supply chain planning for blood products. The meta-heuristic algorithm was used to alleviate the high complexity of the model. Azizi, Hu, and Mokari (2020) addressed the SCND problem with multi-period reverse logistics with lot-sizing. Scenarios were generated with the moment matching technique, and the number of scenarios was reduced using forward selection. Ghorashi Khalilabadi, Zegordi, and Nikbakhsh (2020) developed the multi-stage stochastic integer programming model for prior planning for disruptions in the supply chain. A scenario tree was constructed, and a progressive hedging algorithm was used to alleviate the computational burden.

2.2. Supply chain problems in the ODWS and distinctive features of this study

The last few years have seen a huge growth in the problem of utilising different types of warehouses to mitigate capacity and demand shortage issues. In particular, the two warehouse system that utilises rented warehouse has become a central issue for reducing product shortage or expiration (Tiwari et al. 2017; Jonas 2019; Gupta, Tiwari, and Jaggi 2020). In addition, recent developments in third-party logistics and online platforms have led to many researchers proposing novel problems (Shi, Guo, and Yu 2018; Ponce, Contreras, and Laporte 2020; Ren et al. 2020). Although on-demand warehousing is a very popular trend in real business, it is underexplored, and only a few researchers dealt with the problems regarding the supply chain using the ODWS.

There are two significant characteristics of the ODWS compared to other warehouse systems: *capacity granularity* and *commitment granularity* (Pazour and Unnu 2018). *Capacity granularity* means the minimum capacity that can be acquired by a chosen distribution alternative (e.g. warehouses). In terms of the ODWS, the minimum capacity requirement is very small. *Commitment granularity* means the minimum commitment periods (in time units) a user of the system must maintain their decision. As mentioned in Section 1, the minimum commitment periods of the ODWS are usually very short (e.g.monthly or weekly commitments) compared to leasing warehouses. Throughout this study, we will use the term *duration constraint* to refer to the constraint that the firm must utilise the ODWS at least the minimum of specified commitment periods. On the other hand, the firm can commit for a period of use that is longer than the minimum commitment periods and shorter than the maximum commitment periods allowed by the ODWS. The cost structures for using the ODWS and other facilities are usually different, depending on the commitment periods. The term *period decision* will be used to indicate this decision for commitment periods.

We reviewed related studies that accommodated the properties of the ODWS. Thanh, Bostel, and Péton (2008) proposed a MILP model based on DFLM to design a production-distribution system in a deterministic demand setting. In their model, two types of warehouses, public and private warehouses, were considered. Even though the authors did not directly refer to the ODWS, the concept of public warehouses was similar to the ODWS. Public warehouses could be opened and closed multiple times, but their status only can be changed after at least two periods. This property was similar to the duration constraint in commitment granularity. Van der Heide et al. (2018) analysed the benefits of utilising dynamic shipments in shared warehouse and transportation networks motivated by the ODWS. They defined the model as a sequential decision making problem and accommodated the demand uncertainty. They applied a mathematical framework to compute optimal ordering and transportation decisions using the Markov decision process and value iteration method. Even though capacity and commitment granularity were not considered, several numerical experiments provided managerial insights into improving demand fulfillment and transport efficiency through dynamic shipments and a high degree of consolidation.

Tian and Zhang (2021) dealt with the problem of renting warehouses and allocating products among the warehouses in the e-commerce supply network with the ODWS. The authors suggested the MINLP model and converted the proposed MINLP to MILP form. However, when demand uncertainty is considered, it is impossible to convert the MINLP to MILP, as stated in Tian and Zhang (2021). Moreover, the commitment granularity of the ODWS was not accommodated because the problem was defined as the single-period setting. Shi, Yu, and Dong (2021) suggested a periodic review warehouse model that considers the ODWS and third-party retailers. They showed the optimality of base stock policy and monotonicity of optimal space allocation decisions in the suggested model. To address a multiple items situation that incurs the curse of dimensionality, the heuristic based on approximate dynamic programming was developed. However, the commitment granularity was also not considered. Ceschia et al. (2022) proposed the supply matching problem from the perspective of platform providers in the ODWS. In contrast to related studies in ODWS, the objective was to maximise the number of transactions between customers and warehouse space suppliers. In addition, they developed a list-based heuristic to reduce the time for solving the problem. However, because customer requests and supplier availability were given, it is necessary to consider the dynamic situation for enhancing the applicability in the ODWS.

Unnu and Pazour (2022) proposed the MILP based on the DFLM that determines location-allocation decisions of three distribution warehouses types–self-distribution, 3PL/lease, and the ODWS. In the proposed MILP, the duration constraint in commitment granularity was considered, and the stochastic parameter was replaced with the expected value of demand. By using the obtained solution of this model, the authors evaluated distribution network design with and without the ODWS by adding the randomness of demand in the simulation. Although they tried to accommodate the demand uncertainty, it is difficult to confirm that the stochastic nature is properly considered. If a shortage of demand can occur, the quality of the solution from the MILP model replacing the stochastic parameter with the expected value could be poorer than the solution obtained by the stochastic approach (e.g. SAA+BD). We will show this stochastic solution gap in Section 5.3.

We show several distinctive features of our study in Table 1. As far as we know, this is the first study to consider the period decision for commitment in the ODWS. In addition, we consider a realistic situation in which the longer the commitment period, the greater the discount is that's applied. The novelties of our study can be summed up from three perspectives, as follows:

- *Modeling:* We develop a mathematical model based on the multi-stage, capacitated, multiple-sourcing, multi-item, and dynamic FL model. In the presented model, we accommodate the period decision in commitment granularity for the first time. In addition, we consider the aggregated customer demand to reflect the case of the e-commerce market supply chain in South Korea.
- *Uncertainty:* We propose the TSSP model that makes the decision considering the uncertainty of demand. Also, because supplier selections are included as decisions in our model, supply uncertainty (i.e. yield uncertainty) is also considered. We utilise the SAA method to estimate the expected function accurately with the reasonable size of scenarios.
- *Computational time:* Through our use of a commercial solver, the scenario-based model can be solved with a large number of scenarios. However, because the problem size increases depending on the number of scenarios, the solver cannot solve the practical large-scale problem in reasonable times. To alleviate the computational burden, we propose a methodology combined with SAA and a multi-cut version of BD.

3. Problem description and mathematical model

This section presents a problem and mathematical formulation for the supply chain considering the ODWS. The detailed problem description for the SCND utilising an ODWS is presented in Section 3.1. Section 3.2 presents the TSSP to represent the problem under uncertainty. In Section 3.3, we represent a compact formulation and explain the well-defined property briefly.

3.1. The supply chain with the ODWS

We describe the supply chain network for e-commerce retailers using the ODWS. We use the case of the e-commerce market in South Korea for the supply chain network description. From here forward, we will use the term *retailer* to indicate the e-commerce retailer and the term *provider* to indicate the warehouse operator who has excess capacity. We deal with the multi-items and multi-period problem, and the decision-maker corresponds to a retailer. An overview of the supply chain with an ODWS is shown in Figure 1.

We define the two types of decisions determined based on the before and after the realisation of uncertainties. Before the realisation of uncertainties, the decisions for choice of suppliers and warehouses are made because they are in the *strategic* levels of decision (Snyder and Shen 2019). First, we will illustrate the decisions for the selection of suppliers, y_j. Among many suppliers, $j \in \mathcal{J}$, the retailer tries to cooperate with suppliers who provide a better quality of items or who provide a number of items with low variability. Also, the locations of the suppliers

Table 1. Comparison of recent studies related to dynamic facility location and on-demand warehousing.

Author (year)	On-demand warehousing	Multi-item	Multi-period	Capacity granularity	Commitment granularity: duration constraint	Commitment granularity: period decision	Uncertainty (factors)	Solution methodology
Melo, Nickel, and Da Gama (2006)		✓	✓	✓				Solver (Cplex)
Thanh, Bostel, and Péton (2008)		✓	✓	✓	✓			Solver (Xpress)
Badri, Bashiri, and Hejazi (2013)		✓	✓	✓				LR[a]
Fattahi, Mahootchi, and Moattar Husseini (2016)		✓	✓	✓				Solver (Cplex)
Van der Heide et al. (2018)	✓		✓				✓(demand)	MDP[b], VI[c]
Shi, Yu, and Dong (2021)	✓		✓				✓(demand)	ADP[d]
Tian and Zhang (2021)	✓	✓		✓				Solver (Cplex)
Ceschia et al. (2022)	✓		✓					Heuristics
Unnu and Pazour (2022)	✓		✓	✓	✓			Solver (Cplex), SIM[e]
This paper	✓	✓	✓	✓	✓	✓	✓(demand, supply)	TSSP, SAA + BD

[a]Lagrangian relaxation; [b]Markov decision process; [c]Value iteration; [d]Approximate dynamic programming; [e]Simulation

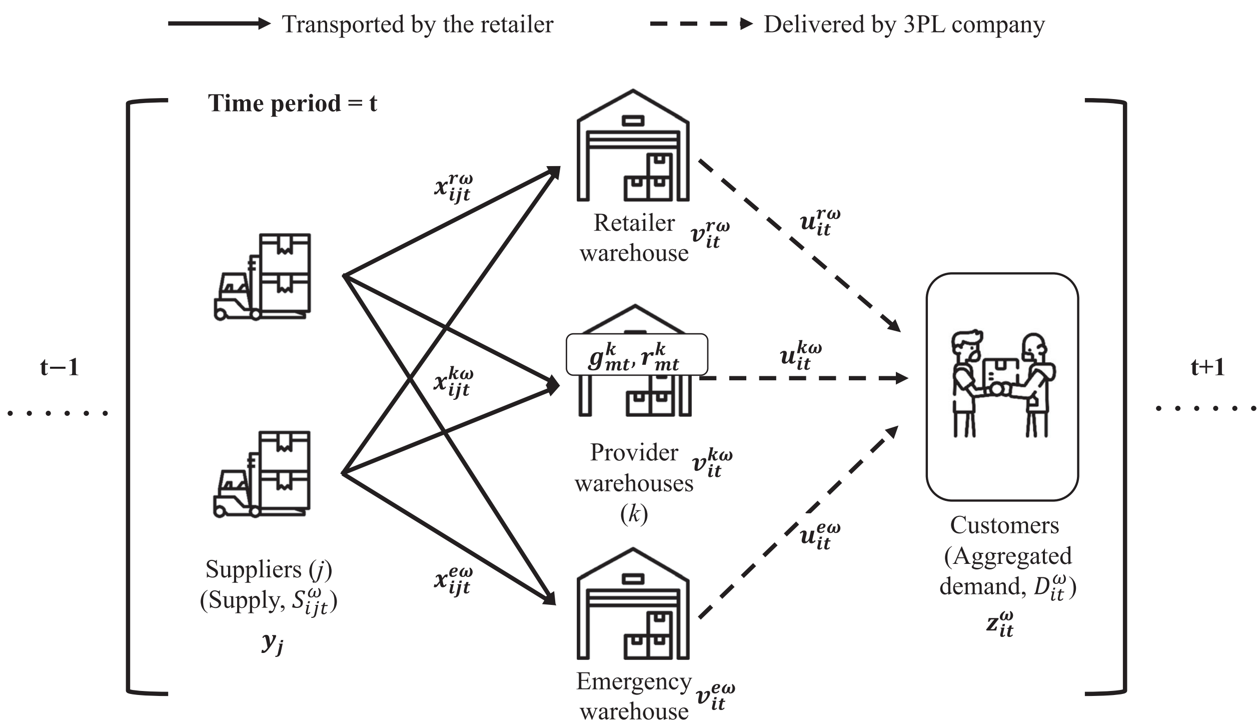

Figure 1. Overview of the supply chain with an ODWS.

are significant in order to minimise the transportation costs from suppliers to warehouses, c_j^r, c_j^k, and c_j^e. The different value of investment cost, F_j, is charged to engage cooperation according to suppliers.

We assume that the retailer can utilise three types of warehouses: (1) the retailer's own warehouse (*retailer warehouse*), (2) the warehouse of providers connected by the ODWS platform (*provider warehouse*), and (3) the warehouse that charges higher unit holding and transportation costs than other types of warehouses (*emergency warehouse*). We assume that there is one retailer warehouse, one emergency warehouse, and several provider warehouses, $k \in \mathcal{K}$. Note that the problem can easily be extended to multiple retailer and emergency warehouses by increasing the set size for warehouses. We propose the mathematical model and solution methodology considering multiple retailer and emergency warehouses, but every computational experiment is conducted in the setting of one retailer and one emergency warehouse. The transportation capacity from suppliers to warehouses, as well as the storage capacity, is assigned for every warehouse, C^r, C^k, and C^e. Every warehouse has the same role with distribution centers as follows:

(1) Shipments from the suppliers will be assembled, and vehicle loads will be de-aggregated.
(2) If the capacity of the warehouses is not full, every item can be held in warehouses for the short or long term.

(3) Items will be assorted according to customers' demands and will be processed or packaged for bringing to customers.

In the case of the provider warehouse, the above roles can only be applied when the retailer has committed to using the provider warehouse for a designated period. We introduce the detailed procedure for the commitment decisions, g^k_{mt} and r^k_{mt}, for the provider warehouse using the simple example that is depicted in Figure 2(a). For a brief explanation, we consider two types of commitments (2-period and 3-period) over a six-period planning time horizon with a provider warehouse, k. In period one, the retailer made the 3-period commitment; thus, the provider warehouse, k, can be used from period one to period three. However, because the commitment for using the warehouse in period four has not been made, the retailer cannot utilise the provider warehouse, k, at this period. On the other hand, the warehouse is available for use from period five to period six because the retailer made the 2-period commitment in period five.

We take into account the realistic situation in which the retailer takes a greater discount when a longer commitment period is made. Therefore, the cost function for committing warehouses for the m-period is defined as $m\alpha\gamma^m$, where γ is the discount factor, and α is the commitment cost to utilise a provider warehouse for a period. In Figure 2(b), we describe the effects of commitment periods on the cost. When retailers plan to utilise the provider warehouse from periods one to three, there are three ways to make the commitment in these periods. First, the retailer can use the warehouse from period one to three by making the 3-period commitment in period one (Case 1). Furthermore, the retailer can use 2-period and 1-period commitments (Cases 2 and 3) or make the 1-period commitment for each period to utilise the warehouse for three periods (Case 4). Because of the cost function for committing warehouses $m\alpha\gamma^m$, Case 1 is the cheapest way to utilise the warehouse (i.e. commitment cost: \$257.2). However, committing for a long period to use warehouses could incur unnecessary costs due to the long-term use of warehouses, although there is small customer demand.

After the realisation of uncertainties, operational decisions are made. We describe the decision procedure following the flow of items from suppliers to customers according to the process from left to right in Figure 1. In the beginning, the transportation decisions from suppliers to the arriving warehouses, $x^{r\omega}_{ijt}$, $x^{k\omega}_{ijt}$, and $x^{e\omega}_{ijt}$, are made for the ordered items. The lead time between suppliers and warehouses exists, L_s. After items have arrived at the designated warehouses, items are processed for sending to customers. Inventory holding decisions, $v^{r\omega}_{it}$, $v^{k\omega}_{it}$, and $v^{e\omega}_{it}$, and delivery decisions, $u^{r\omega}_{it}$, $u^{k\omega}_{it}$, and $u^{e\omega}_{it}$, will be made at warehouses. Depending on the type of warehouses, different inventory holding costs, h^r_i, h^k_i, and h^e_i, will be incurred. In particular, because most retailers commonly use the services of a logistics company for last-mile deliveries in the case of the South Korean e-commerce market, we consider the aggregated customer demand for the proposed model and assume that items will be delivered from warehouses to customers by the 3PL company. Furthermore, the delivery cost per parcel of items, b_i, is identical without taking into account the weights of items and locations of destinations. There exists lead time between warehouses and aggregated customer demands, L_d. Finally, in order to address the stock-out issue, we assume that unsatisfied demand will become lost sales, z^{ω}_{it}. This assumption is reasonable because customers are more likely to switch to another website to search for substitute items rather than wait for insufficient items to be stocked. Additionally, the corresponding penalty cost, β_i, for lost sales will be incurred.

We consider two additional assumptions. First, we exclude perishable items in the proposed problem. In order to deal with perishable products, it is necessary to install the cold storage system that is available to maintain the specific temperature and humidity conditions that do not alter the products' original characteristics. However, it is difficult to use this system in the ODWS because various users store heterogeneous products in the same space. Second, lateral transshipment between warehouses is not considered. The lateral transshipment could increase the complexity of the problem because the number of decision variables related to lateral transshipment could increase exponentially depending on the number of warehouses. Furthermore, because of the property that ensures that the provider warehouse can be opened or closed at each period, the connections for lateral transshipment can be negated.

3.2. The TSSP model

This section presents the SCND model, which is developed based on the multi-stage, capacitated, multiple-sourcing, multi-item, and dynamic FL model. In order to model the problem under uncertainties, we extend the deterministic model as the TSSP. As mentioned in Section 3.1, the operational decisions have been considered after the prior decisions. By considering these characteristics of decision-making, we employ the TSSP to represent the situation of the SCND with an ODWS. We assume that demands, D, and supplies, S, are random parameters with full knowledge of probability

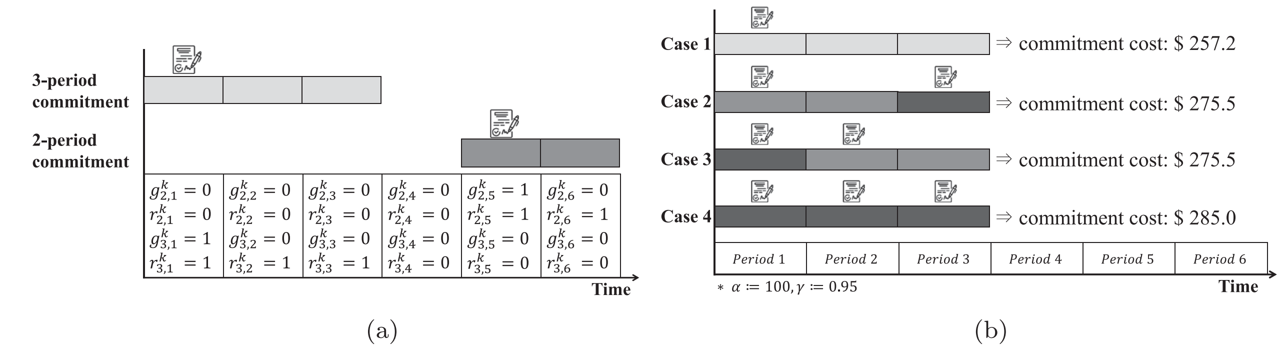

Figure 2. Simple example of commitment decisions for provider warehouse k. (a) Commitment decisions using two types of commitments and (b) Commitment cost using four cases.

distributions, defined as *stochastic parameters*. Therefore, we use $\zeta = (D, S)$, which stands for the stochastic parameters vector with finite and discrete support, which can be represented as a finite number of realizations (scenarios). Let Ω be a set of scenarios, and each scenario is denoted as ω. Then, $\zeta^{\omega}, \forall\omega \in \Omega$, is a particular realisation of stochastic parameters. The sample space of stochastic parameters is represented as set $\{\zeta^1, \ldots, \zeta^{|\Omega|}\}$ with the following probabilities, $p_1, \ldots, p_{|\Omega|}$.

In the proposed TSSP, decisions for supplier selection and commitments for the provider warehouses are made in the first-stage problem. The first-stage decisions are the *here-and-now* decisions that are determined before the realisation of stochastic parameters. Subsequently, in the second-stage, operational decisions such as transportation, inventory holding, and lost sales are made after realizations of stochastic parameters. The following notations are utilised in the proposed mathematical formulation.

Indices and sets

- $\mathcal{T}$ set of periods, $t \in \mathcal{T} = \{1, 2, \ldots, T\}$
- $\mathcal{I}$ set of items, $i \in \mathcal{I} = \{1, 2, \ldots, I\}$
- $\mathcal{J}$ set of suppliers, $j \in \mathcal{J} = \{1, 2, \ldots, J\}$
- $\mathcal{K}$ set of provider warehouses, $k \in \mathcal{K} = \{1, 2, \ldots, K\}$
- $\mathcal{R}$ set of retailer warehouses, $r \in \mathcal{R} = \{1, 2, \ldots, R\}$
- $\mathcal{E}$ set of emergency warehouses, $e \in \mathcal{E} = \{1, 2, \ldots, E\}$
- $\mathcal{M}$ set of available commitment periods, $m \in \mathcal{M} = \{1, 2, \ldots, M\}$
- Ω set of scenarios, $\omega \in \Omega$

Parameters

- D^{ω}_{it} aggregated demand of item i at period t under scenario ω
- S^{ω}_{ijt} supply of item i from supplier j at period t under scenario ω
- C^r capacity of the retailer warehouse r
- C^k capacity of the provider warehouse k
- C^e capacity of the emergency warehouse e
- L_s lead time between suppliers and warehouses
- L_d lead time between warehouses and customers
- F_j investment cost to select supplier j
- h^r_i inventory holding cost of the retailer warehouse r for a unit of item i per period
- h^k_i inventory holding cost of provider warehouse k for a unit of item i per period
- h^e_i inventory holding cost of the emergency warehouse e for a unit of item i per period
- α commitment cost to utilise provider warehouse for a period
- β_i lost sales cost for a unit of item i
- b_i cost of delivery for a unit of item i from warehouses to customers
- c^r_j transportation cost for a unit of item from supplier j to the retailer warehouse r
- c^k_j transportation cost for a unit of item from supplier j to provider warehouse k
- c^e_j transportation cost for a unit of item from supplier j to the emergency warehouse e
- γ discount factor of commitment cost
- p_{ω} probability that scenario ω occurred

Decision variables

- g^k_{mt} 1 if an m period commitment is made at period t for provider warehouse k, 0 otherwise
- r^k_{mt} 1 if provider warehouse k can be utilised because of the m period commitment at period t, 0 otherwise
- y_j 1 if supplier j is selected, 0 otherwise
- $v^{r\omega}_{it}$ number of item i held in inventory at the retailer warehouse r from period t to $t+1$ under scenario ω
- $v^{k\omega}_{it}$ number of item i held in inventory at provider warehouse k from period t to $t+1$ under scenario ω

$v_{it}^{e\omega}$ number of item i held in inventory at the emergency warehouse e from period t to $t+1$ under scenario ω

$x_{ijt}^{r\omega}$ number of item i transported from supplier j to the retailer warehouse r at period t under scenario ω

$x_{ijt}^{k\omega}$ number of item i transported from supplier j to provider warehouse k at period t under scenario ω

$x_{ijt}^{e\omega}$ number of item i transported from supplier j to the emergency warehouse e at period t under scenario ω

$u_{it}^{r\omega}$ number of item i delivered to satisfy aggregated demand from the retailer warehouse r at period t under scenario ω

$u_{it}^{k\omega}$ number of item i delivered to satisfy aggregated demand from provider warehouse k at period t under scenario ω

$u_{it}^{e\omega}$ number of item i delivered to satisfy aggregated demand from the emergency warehouse e at period t under scenario ω

z_{it}^{ω} lost sales of item i at period t under scenario ω

By considering the above problem descriptions and notations, the extensive form of the TSSP is formulated as follows:

First-stage problem

$$\min \quad \sum_{j\in\mathcal{J}} F_j y_j + \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} m\alpha\gamma^m g_{mt}^k + \mathbb{E}_\zeta\left[Q(y,r,\zeta^\omega)\right] \tag{1}$$

$$\text{s.t.} \quad \sum_{\tau=t}^{\min\{t+m-1,|\mathcal{T}|\}} g_{m\tau}^k \le 1, \quad \forall k\in\mathcal{K}, m\in\mathcal{M}, t\in\mathcal{T}, \tag{2}$$

$$\sum_{\tau=\max\{t-m+1,1\}}^{t} g_{m\tau}^k = r_{mt}^k, \quad \forall k\in\mathcal{K}, m\in\mathcal{M}, t\in\mathcal{T}, \tag{3}$$

$$\sum_{m\in\mathcal{M}} r_{mt}^k \le 1, \quad \forall k\in\mathcal{K}, t\in\mathcal{T}, \tag{4}$$

$$\sum_{m\in\mathcal{M}} g_{mt}^k \le 1, \quad \forall k\in\mathcal{K}, t\in\mathcal{T}, \tag{5}$$

$$r_{mt}^k, g_{mt}^k \in \{0,1\}, \quad \forall k\in\mathcal{K}, m\in\mathcal{M}, t\in\mathcal{T}, \tag{6}$$

$$y_j \in \{0,1\}, \quad \forall j\in\mathcal{J}. \tag{7}$$

where $Q(y,r,\zeta^\omega)$ is the value function for the optimal objective value of the second-stage problem with a given scenario ω. By applying the scenario-based approach, the expected second-stage cost can be denoted with $\sum_{\omega\in\Omega} p_\omega Q(y,r,\zeta^\omega)$. The objective function of the first-stage problem (1) minimises the total cost incurred in the supply chain. Constraint (2) ensures that other commitments for provider warehouses cannot be made until the ongoing commitment expires. Constraints (3) and (4) ensure that every provider warehouse can be utilised only in the case when commitments are made for the designated period. Constraint (5) guarantees that just one type of commitment can be made among available commitment periods for each provider warehouse at each period. Constraints (6) and (7) enforce that first-stage decision variables are binary variables. Given the values of y_j and r_{mt}^k and a scenario ω, the second-stage problem that determines the recourse function $Q(y,r,\zeta^\omega)$ is as follows:

Second-stage problem

$$Q(y,r,\zeta^\omega) = \min \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\Bigg(\sum_{r\in\mathcal{R}} h_i^r v_{it}^{r\omega} + \sum_{e\in\mathcal{E}} h_i^e v_{it}^{e\omega} + \sum_{k\in\mathcal{K}} h_i^k v_{it}^{k\omega} + b_i\left(\sum_{r\in\mathcal{R}} u_{it}^{r\omega} + \sum_{e\in\mathcal{E}} u_{it}^{e\omega} + \sum_{k\in\mathcal{K}} u_{it}^{k\omega}\right) + \beta_i z_{it}^\omega + \sum_{j\in\mathcal{J}}\left(\sum_{r\in\mathcal{R}} c_j^r x_{ijt}^{r\omega} + \sum_{e\in\mathcal{E}} c_j^e x_{ijt}^{e\omega} + \sum_{k\in\mathcal{K}} c_j^k x_{ijt}^{k\omega}\right)\Bigg) \tag{8}$$

$$\text{s.t.} \quad \sum_{r\in\mathcal{R}} x_{ijt}^{r\omega} + \sum_{k\in\mathcal{K}} x_{ijt}^{k\omega} + \sum_{e\in\mathcal{E}} x_{ijt}^{e\omega} \le S_{ijt}^\omega y_j, \quad \forall i\in\mathcal{I}, j\in\mathcal{J}, t\in\mathcal{T}, \tag{9}$$

$$u_{it}^{r\omega} + v_{it}^{r\omega} = v_{it-1}^{r\omega} + \sum_{j\in\mathcal{J}} x_{ijt-L_s}^{r\omega}, \quad \forall i\in\mathcal{I}, r\in\mathcal{R}, t\in\mathcal{T}, \tag{10}$$

$$u_{it}^{k\omega} + v_{it}^{k\omega} = v_{it-1}^{k\omega} + \sum_{j\in\mathcal{J}} x_{ijt-L_s}^{k\omega}, \quad \forall i\in\mathcal{I}, k\in\mathcal{K}, t\in\mathcal{T}, \tag{11}$$

$$u_{it}^{e\omega} + v_{it}^{e\omega} = v_{it-1}^{e\omega} + \sum_{j\in\mathcal{J}} x_{ijt-L_s}^{e\omega}, \quad \forall i\in\mathcal{I}, e\in\mathcal{E}, t\in\mathcal{T}, \tag{12}$$

$$\sum_{r\in\mathcal{R}} u_{it-L_d}^{r\omega} + \sum_{k\in\mathcal{K}} u_{it-L_d}^{k\omega} + \sum_{e\in\mathcal{E}} u_{it-L_d}^{e\omega} + z_{it}^\omega \ge D_{it}^\omega, \quad \forall i\in\mathcal{I}, t\in\mathcal{T}, \tag{13}$$

$$\sum_{i\in\mathcal{I}} v_{it}^{r\omega} \le C^r, \quad \forall r\in\mathcal{R}, t\in\mathcal{T}, \tag{14}$$

$$\sum_{i\in\mathcal{I}} v_{it}^{e\omega} \le C^e, \quad \forall e \in \mathcal{E}, t \in \mathcal{T}, \tag{15}$$

$$\sum_{i\in\mathcal{I}} v_{it}^{k\omega} \le C^k \sum_{m\in\mathcal{M}} r_{mt}^k, \quad \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{16}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} x_{ijt}^{r\omega} \le C^r, \quad \forall r \in \mathcal{R}, t \in \mathcal{T}, \tag{17}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} x_{ijt}^{e\omega} \le C^e, \quad \forall e \in \mathcal{E}, t \in \mathcal{T}, \tag{18}$$

$$\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}} x_{ijt}^{k\omega} \le C^k \sum_{m\in\mathcal{M}} r_{mt}^k, \quad \forall k \in \mathcal{K}, t \in \mathcal{T}, \tag{19}$$

$$x_{ijt}^{r\omega}, x_{ijt}^{k\omega}, x_{ijt}^{e\omega} \ge 0, \quad \forall i \in \mathcal{I}, j \in \mathcal{J}, r \in \mathcal{R}, k \in \mathcal{K}, e \in \mathcal{E}, t \in \mathcal{T}, \tag{20}$$

$$u_{it}^{r\omega}, u_{it}^{k\omega}, u_{it}^{e\omega}, v_{it}^{r\omega}, v_{it}^{k\omega}, v_{it}^{e\omega} \ge 0, \quad \forall i \in \mathcal{I}, r \in \mathcal{R}, k \in \mathcal{K}, e \in \mathcal{E}, t \in \mathcal{T}, \tag{21}$$

$$z_{it}^{\omega} \ge 0, \quad \forall i \in \mathcal{I}, t \in \mathcal{T}. \tag{22}$$

In the second-stage problem, every constraint is defined within the entire time horizon, $t \in \mathcal{T}$. For a realisation of ω, the objective function of the second-stage problem (8) minimises the costs for the inventory holding, delivery, stockout, and transportation within the entire time horizon. Constraint (9) requires that the total number of items transported from the supplier, j, to every warehouse should be less than the given supplies. Constraints (10), (11), and (12) are the balance equations representing the flow of items from retailer, provider, and emergency warehouses to customers, respectively. The inventories stored in warehouses during the previous period, $t-1$, are transferred to the current period, t. Moreover, these constraints ensure the lead time between suppliers and warehouses, L_s. Constraint (13) ensures that the demand is satisfied by delivered items from each warehouse and that the lead time between warehouses and customers, L_d, exists. Furthermore, this constraint enforces that unsatisfied demand is lost. Constraints (14), (15), and (16) express the storage capacity for the retailer, emergency, and provider warehouses, respectively. Constraints (17), (18), and (19) represent the transportation capacity between suppliers and the retailer, emergency, and provider warehouses, respectively. Finally, Constraints (20), (21), and (22) ensure that decision variables for the second-stage problems are non-negative real variables.

Because of the decision variables for lost sales, z_{it}^{ω}, the second-stage problem remains feasible under any first-stage feasible solution, y_j, g_{mt}^k, and r_{mt}^k, $\forall j$, m, and t. In this case, we say that the stochastic programming (1)–(22) has the property called *relatively complete recourse* (Birge and Louveaux 2011). This is a key property for implementing the SAA and BD algorithms, and we will explain this in detail in Section 4.

We now define six cost components as follows:

$$\text{Delivery cost} := \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega} p_\omega b_i \times \left(\sum_{r\in\mathcal{R}} u_{it}^{r\omega} + \sum_{e\in\mathcal{E}} u_{it}^{ew} + \sum_{k\in\mathcal{K}} u_{it}^{k\omega}\right) \tag{23}$$

$$\text{Commitment cost} := \sum_{k\in\mathcal{K}}\sum_{m\in\mathcal{M}}\sum_{t\in\mathcal{T}} m\alpha\gamma^m g_{mt}^k \tag{24}$$

$$\text{Stockout cost} := \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega} p_\omega \beta_i z_{it}^{\omega} \tag{25}$$

$$\text{Supplier investment cost} := \sum_{j\in\mathcal{J}} F_j y_j \tag{26}$$

$$\text{Transportation cost} := \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega} p_\omega \times \left(\sum_{r\in\mathcal{R}} c_j^r x_{ijt}^{r\omega} + \sum_{e\in\mathcal{E}} c_j^e x_{ijt}^{e\omega} + \sum_{k\in\mathcal{K}} c_j^k x_{ijt}^{k\omega}\right) \tag{27}$$

$$\text{Inventory holding cost} := \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}}\sum_{\omega\in\Omega} p_\omega \times \left(\sum_{r\in\mathcal{R}} h_i^r v_{it}^{r\omega} + \sum_{e\in\mathcal{E}} h_i^e v_{it}^{e\omega} + \sum_{k\in\mathcal{K}} h_i^k v_{it}^{k\omega}\right) \tag{28}$$

3.3. Compact formulation

For ease of the expositions, we represent the extensive form, (1)–(22), by the compact form using the concatenated vectors of decision variables, which are defined as follows:

Concatenated vectors of decision variables

- $\mathbf{g}$ Concatenated vector of the g_{mt}^k, $\forall k \in \mathcal{K}, m \in \mathcal{M}$, and $t \in \mathcal{T}$
- $\mathbf{r}$ Concatenated vector of the r_{mt}^k, $\forall k \in \mathcal{K}, m \in \mathcal{M}$, and $t \in \mathcal{T}$
- $\mathbf{u}_\omega$ Concatenated vector of the $(u_{it}^{r\omega}, u_{it}^{k\omega}, u_{it}^{e\omega})$, $\forall i \in \mathcal{I}, r \in \mathcal{R}, k \in \mathcal{K}, e \in \mathcal{E}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{v}_\omega$ Concatenated vector of the $(v_{it}^{r\omega}, v_{it}^{k\omega}, v_{it}^{e\omega})$, $\forall i \in \mathcal{I}, r \in \mathcal{R}, k \in \mathcal{K}, e \in \mathcal{E}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{v}_\omega^r$ Concatenated vector of the $v_{it}^{r\omega}$, $\forall i \in \mathcal{I}, r \in \mathcal{R}$, and $t \in \mathcal{T}$ under scenario ω

- $\mathbf{v}^k_\omega$ Concatenated vector of the $v^{k\omega}_{it}$, $\forall i \in \mathcal{I}, k \in \mathcal{K}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{v}^e_\omega$ Concatenated vector of the $v^{e\omega}_{it}$, $\forall i \in \mathcal{I}, e \in \mathcal{E}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{x}_\omega$ Concatenated vector of the $(x^{r\omega}_{ijt}, x^{k\omega}_{ijt}, x^{e\omega}_{ijt})$, $\forall i \in \mathcal{I}, j \in \mathcal{J}, r \in \mathcal{R}, k \in \mathcal{K}, e \in \mathcal{E}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{x}^r_\omega$ Concatenated vector of the $x^{r\omega}_{ijt}$, $\forall i \in \mathcal{I}, j \in \mathcal{J}, r \in \mathcal{R}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{x}^k_\omega$ Concatenated vector of the $x^{k\omega}_{ijt}$, $\forall i \in \mathcal{I}, j \in \mathcal{J}, k \in \mathcal{K}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{x}^e_\omega$ Concatenated vector of the $x^{e\omega}_{ijt}$, $\forall i \in \mathcal{I}, j \in \mathcal{J}, e \in \mathcal{E}$, and $t \in \mathcal{T}$ under scenario ω
- $\mathbf{z}_\omega$ Concatenated vector of the z^{ω}_{it}, $\forall i \in \mathcal{I}$, and $t \in \mathcal{T}$ under scenario ω

With the above vectors of decision variables, the extensive form can be simplified as follows:

Compact formulation

$$\min \quad \mathbf{f}^\top\mathbf{y} + \mathbf{e}^\top\mathbf{g} + \mathbb{E}_\zeta\left[Q(\mathbf{y}, \mathbf{r}, \zeta^\omega)\right] \tag{29}$$
$$\text{s.t.} \quad \mathbf{Ag} \le \mathbf{1}, \tag{30}$$
$$\mathbf{Bg} = \mathbf{r}, \tag{31}$$
$$\mathbf{Wr} \le \mathbf{1}, \tag{32}$$
$$\mathbf{y} \in \{0,1\}^{|\mathcal{J}|}, \tag{33}$$
$$\mathbf{g}, \mathbf{r} \in \{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{34}$$
$$\text{where} \quad Q(\mathbf{y}, \mathbf{r}, \zeta^\omega) = \min\ \mathbf{h}^\top\mathbf{v}_\omega + \mathbf{b}^\top\mathbf{u}_\omega + \boldsymbol{\beta}^\top\mathbf{z}_\omega + \mathbf{c}^\top\mathbf{x}_\omega \tag{35}$$
$$\text{s.t.} \quad \mathbf{Px}_\omega \le \mathbf{S}_\omega\mathbf{y}, \tag{36}$$
$$\mathbf{Uu}_\omega + \mathbf{Vv}_\omega - \mathbf{Tx}_\omega = \mathbf{0}, \tag{37}$$
$$\mathbf{Ku}_\omega + \mathbf{Jz}_\omega \ge \mathbf{D}_\omega, \tag{38}$$
$$\mathbf{Mv}^r_\omega \le \mathbf{C}^r, \tag{39}$$
$$\mathbf{Gv}^e_\omega \le \mathbf{C}^e, \tag{40}$$
$$\mathbf{Hv}^k_\omega \le \mathbf{C}^k\mathbf{r}, \tag{41}$$
$$\mathbf{Ex}^r_\omega \le \mathbf{C}^r, \tag{42}$$
$$\mathbf{Rx}^e_\omega \le \mathbf{C}^e, \tag{43}$$
$$\mathbf{Lx}^k_\omega \le \mathbf{C}^k\mathbf{r}, \tag{44}$$
$$\mathbf{v}_\omega, \mathbf{u}_\omega, \mathbf{z}_\omega, \mathbf{x}_\omega \ge \mathbf{0}. \tag{45}$$

In the case in which the objective function of the stochastic programming model is well-defined, the model possesses the optimal solution (Birge and Louveaux 2011). As mentioned earlier, because of the relatively complete recourse property, the feasibility of the proposed model (29)–(34) will always be guaranteed for all $\mathbf{y} \in \mathcal{Y}, \mathbf{g} \in \mathcal{G}, \mathbf{r} \in \mathcal{V}$ and $\omega \in \Omega$, where Y, G, and R are feasible sets of the corresponding decision variables. The objective function is to minimise the sum of the cost for the first-stage problem, $\mathbf{f}^\top\mathbf{y} + \mathbf{e}^\top\mathbf{g}$, and the expected cost for the second-stage problem, $\mathbb{E}_\zeta[Q(\mathbf{y}, \mathbf{r}, \zeta^\omega)]$. Therefore, the lost sales cost term, $\boldsymbol{\beta}^\top\mathbf{z}_\omega$, guarantees that $Q(\mathbf{y}, \mathbf{r}, \zeta^\omega) \le \infty$ for all $\mathbf{y}, \mathbf{r}$, and ω. Moreover, because we assume that all cost parameters are non-negative, it is obvious that $Q(\mathbf{y}, \mathbf{r}, \zeta^\omega) \ge -\infty$ for all $\mathbf{y}, \mathbf{r}$, and ω. Thus, $Q(\mathbf{y}, \mathbf{r}, \zeta^\omega)$ is finite for all $\mathbf{y}, \mathbf{r}$ and every realisation of ω, and it can be assumed that the expected value, $\mathbb{E}_\zeta[Q(\mathbf{y}, \mathbf{r}, \zeta^\omega)]$, is well defined. Finally, the objective function of the first-stage variables is well-defined in the proposed model (29)–(45), and the optimal solutions exist because the set $\mathcal{Y}, \mathcal{G}$, and $\mathcal{V}$ is nonempty and finite.

4. Solution methodology

In this section, we develop the solution methodology, specifically the SAA and BD algorithms, for solving the proposed TSSP. There are several advantages of using the SAA and BD algorithms compared to other methods (Verweij et al. 2003; Rahmaniani et al. 2017). First, the SAA approach is quite general, so that can be combined with various algorithms that are specialised in solving the deterministic optimisation problem. Also, the SAA approach has valuable convergence properties. The BD algorithm can efficiently solve complicated problems due to several variables, which makes the problem easier to handle when temporarily fixed. The BD algorithms converge to the optimal of the MILP rather than to a relaxation of the problem. Section 4.1 presents the concept and procedure of the SAA. Section 4.2 examines the BD algorithm, and Section 4.3 illustrates the acceleration method for the BD algorithm.

4.1. Sample average approximation

The fundamental difficulty of solving the *true problem* (29)–(45) is computing the expected value function, $\mathbb{E}_\zeta[Q(\mathbf{y}, \mathbf{r}, \zeta^\omega)]$. Let $\zeta^1, \dots, \zeta^N$ be an independently and identically distributed (i.i.d) random sample of N realizations (scenarios) of the stochastic parameter vector ζ. By solving the following *SAA problem* with a larger N, the objective function of the SAA problem converges to the true objective function with a probability of one (Kleywegt, Shapiro, and Homem-de Mello 2002).

$$\min_{\mathbf{y}\in\mathcal{Y}, \mathbf{g}\in\mathcal{G}, \mathbf{r}\in\mathcal{V}} \left\{ \mathbf{f}^\top\mathbf{y} + \mathbf{e}^\top\mathbf{g} + \frac{1}{N}\sum_{n=1}^{N} Q(\mathbf{y}, \mathbf{r}, \zeta^n) \right\} \tag{46}$$

Let $\hat{\psi}_N$ denote the optimal value of the SAA problem (46), and $\hat{\psi}_N$ is random because the value will be different depending on the corresponding random sample.

However, the computational complexity for solving the SAA problem (46) often increases exponentially with the size of N. In order to overcome these challenges, we utilise the SAA algorithm, which estimates the objective value of the true problem and requires less computational effort than solving the SAA problem with a large-sized N.

In the SAA algorithm, we employ the number M of replications, generating and solving the SAA problem with the same size N. It is more efficient to utilise several SAA problems with a smaller-sized N than it is to solve one SAA problem with a large-sized N. Based on the number M of SAA replications, the solution quality of each replication is measured with an optimality gap. In this paper, the *SAA gap* stands for an optimality gap used for stopping criteria in the SAA algorithm. When the SAA gap can not satisfy the predefined threshold ϵ_{SAA}, we increase the sample size N for every SAA replication to obtain solutions with better quality. The procedure for the SAA algorithm is described as follows:

SAA algorithm

(1) Generate i.i.d. samples with size N scenarios for each replication of m (i.e. $(\zeta_m^1, \ldots, \zeta_m^n)$, $\forall m \in \{1, \ldots, M\}$), and solve the corresponding SAA problem. Let $\hat{\psi}_N^m$ and $\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m$, and $\hat{\mathbf{r}}_N^m$ be the optimal objective value and the optimal solution of the mth SAA replication, respectively.

(2) Compute the following equation to obtain the statistical lower bound for ψ^*, where ψ^* is the optimal objective value for the true problem.

$$\bar{\psi}_{MN} := \frac{1}{M} \sum_{m=1}^{M} \hat{\psi}_N^m \tag{47}$$

It is well known that the expected value of the $\hat{\psi}_N$ is less than or equal to the ψ^* (Norkin, Pflug, and Ruszczyński 1998; Emelogu et al. 2016). Because $\bar{\psi}_{MN}$ is the unbiased estimator for the $\mathbb{E}[\hat{\psi}_N]$, it is clear that $\bar{\psi}_{MN}$ provides the statistical lower bound for ψ^*, $\mathbb{E}[\bar{\psi}_{MN}] \le \psi^*$. Let $\sigma^2_{\bar{\psi}_{MN}}$ be an estimate of the variance of $\bar{\psi}_{MN}$. It can be obtained by computing the following equation, which is derived from the *Central Limit Theorem*.

$$\sigma^2_{\bar{\psi}_{MN}} := \frac{1}{M(M-1)} \sum_{m=1}^{M} \left(\hat{\psi}_N^m - \bar{\psi}_{MN}\right)^2 \tag{48}$$

(3) Select a feasible first-stage solution, $\hat{\mathbf{y}} \in \mathcal{Y}, \hat{\mathbf{g}} \in \mathcal{G}$, and $\hat{\mathbf{r}} \in \mathcal{V}$. This feasible first-stage solution was determined from the obtained solution by solving the SAA problem for each replication, $\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m$, and $\hat{\mathbf{r}}_N^m$. With a newly-generated sample of N' scenarios, $(\zeta^1, \ldots, \zeta^{N'})$, the optimal value of the true problem is estimated from the following equation.

$$\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) := \mathbf{f}^{\mathsf{T}}\hat{\mathbf{y}} + e^{\mathsf{T}}\hat{\mathbf{g}} + \frac{1}{N'} \sum_{n=1}^{N'} Q(\hat{\mathbf{y}}, \hat{\mathbf{r}}, \zeta^n) \tag{49}$$

Note that the size of N' is much larger than the sample size of N used to obtain the estimate for the lower bound ($N \ll N'$). Among obtained solutions $\hat{\mathbf{y}}_N^m, \hat{\mathbf{g}}_N^m$, and $\hat{\mathbf{r}}_N^m, \forall m$, a solution that has the smallest value, $\bar{f}_{N'}$, is commonly chosen for $\hat{\mathbf{y}}, \hat{\mathbf{g}}$, and $\hat{\mathbf{r}}$ to estimate the upper bound. Let $f(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$ be the optimal objective value of the true problem with the solution $\hat{\mathbf{y}}, \hat{\mathbf{g}}$, and $\hat{\mathbf{r}}$. The inequality $\psi^* \le f(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$ holds because $\hat{\mathbf{y}}, \hat{\mathbf{g}}$, and $\hat{\mathbf{r}}$ are the feasible solutions of the true problem. Then, because $\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$ is the unbiased estimator of $f(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$, $\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$ provides an upper bound for ψ^*. Similar to the way of deriving $\sigma^2_{\bar{\psi}_{MN}}$, the estimate of the variance of $\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$ can be obtained by the following equation.

$$\sigma^2_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) := \frac{1}{N'(N'-1)} \times \sum_{n=1}^{N'} \left(\mathbf{f}^{\mathsf{T}}\hat{\mathbf{y}} + \mathbf{e}^{\mathsf{T}}\hat{\mathbf{g}} + Q(\hat{\mathbf{y}}, \hat{\mathbf{r}}, \zeta^n) - \bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})\right)^2 \tag{50}$$

(4) Obtain the SAA gap of the feasible solution $\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}$ and its variance by calculating the following equations:

$$Gap_{MNN'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) := \bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) - \bar{\psi}_{MN} \tag{51}$$

The relative SAA gap is computed by the following equation:

$$Gap^{rel}_{MNN'} := \frac{(\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) - \bar{\psi}_{MN})}{\bar{\psi}_{MN}} \times 100(\%) \tag{52}$$

The estimate of the variance of $Gap_{MNN'}$ can be calculated as follows:

$$\sigma^2_{Gap_{MNN'}} := \sigma^2_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}}) + \sigma^2_{\bar{\psi}_{MN}} \tag{53}$$

4.2. Benders decomposition algorithm

By applying the SAA algorithm, we can obtain a stochastic solution. However, for the large problem, a lot of computational effort is required to solve the SAA problem (46) even with the moderate size of N scenarios.

Therefore, we alleviate the computational burden by utilising a special property of the TSSP. It is well known that the TSSP has the block structure of the extensive form. When taking the dual of the extensive form, a *dual block-angular structure* appears, and the BD algorithm is a suitable approach to exploit this structure (Benders 1962; Birge and Louveaux 2011). As mentioned in Section 4.1, because the SAA problem (46) is itself the TSSP, we use the BD algorithm to solve the SAA problem.

Without loss of generality, we explain the BD algorithm with the model (29)–(45), which will be referred to as the *original problem*. As mentioned in Section 1, we present the multi-cut version of the BD algorithm, which generates several optimality cuts in one iteration. The $1/N$ and $\{\zeta^1,\ldots,\zeta^N\}$ replace p_ω and Ω, respectively, when applying the BD algorithm for the SAA problem. In order to devise the BD algorithm, the proposed stochastic mathematical model is decomposed into one master problem (MP) and several subproblems (SUB(ω), $\forall\omega\in\Omega$). MP and the corresponding SUB(ω), $\forall\omega\in\Omega$, in the (itr+1)th iteration are presented as follows:

MP

$$\min \quad \mathbf{f}^\intercal\mathbf{y}+\mathbf{e}^\intercal\mathbf{g}+\sum_{\omega\in\Omega}p_\omega\theta_\omega \tag{54}$$

$$\text{s.t.} \quad \mathbf{Ag}\le\mathbf{1}, \tag{55}$$

$$\mathbf{Bg}=\mathbf{r}, \tag{56}$$

$$\mathbf{Wr}\le\mathbf{1}, \tag{57}$$

$$\theta_\omega\ge(\mathbf{a}_\omega^{itr})^\intercal\mathbf{y}+(\mathbf{c}_\omega^{itr})^\intercal\mathbf{r}+d_\omega^{itr},\quad \forall itr\in\Im,\omega\in\Omega, \tag{58}$$

$$\mathbf{y}\in\{0,1\}^{|\mathcal{J}|}, \tag{59}$$

$$\mathbf{g},\mathbf{r}\in\{0,1\}^{|\mathcal{K}||\mathcal{M}||\mathcal{T}|}. \tag{60}$$

where $\Im:=\{1,\ldots,itr\}$ and θ_ω, $\forall\omega\in\Omega$, are free variables. Constraint (58) is called as *optimality cuts* at iteration itr, and coefficients $(\mathbf{a}_\omega^{itr})^\intercal$, $(\mathbf{c}_\omega^{itr})^\intercal$, and d_ω^{itr} will be explained in the latter part of this section. After solving the MP with current optimality cuts, obtained optimal solutions are denoted as $\bar{\mathbf{y}},\bar{\mathbf{g}},\bar{\mathbf{r}}$, and $\bar{\theta}_\omega$, $\forall\omega\in\Omega$. Because MP is the relaxed problem to the model (29)–(45), the optimal objective value of MP provides the lower bound, Z_{lb}, for the original problem.

Based on the obtained solution from MP, we solve the SUB(ω) for each $\omega\in\Omega$. *SUB*(ω) is presented as follows:

SUB(ω)

$$\min \quad \mathbf{h}^\intercal\mathbf{v}_\omega+\mathbf{b}^\intercal\mathbf{u}_\omega+\boldsymbol{\beta}^\intercal\mathbf{z}_\omega+\mathbf{c}^\intercal\mathbf{x}_\omega \tag{61}$$

$$\text{s.t.} \quad \mathbf{Px}_\omega\le\mathbf{S}_\omega\bar{\mathbf{y}},\quad(\boldsymbol{\pi}_\omega), \tag{62}$$

$$\mathbf{Uu}_\omega+\mathbf{Vv}_\omega-\mathbf{Tx}_\omega=\mathbf{0},\quad(\boldsymbol{\mu}_\omega), \tag{63}$$

$$\mathbf{Ku}_\omega+\mathbf{Jz}_\omega\ge\mathbf{D}_\omega,\quad(\boldsymbol{\nu}_\omega), \tag{64}$$

$$\mathbf{Mv}_\omega^r\le\mathbf{C}^r,\quad(\boldsymbol{\lambda}_\omega), \tag{65}$$

$$\mathbf{Gv}_\omega^e\le\mathbf{C}^e,\quad(\boldsymbol{\tau}_\omega), \tag{66}$$

$$\mathbf{Hv}_\omega^k\le\mathbf{C}^k\bar{\mathbf{r}},\quad(\boldsymbol{\rho}_\omega), \tag{67}$$

$$\mathbf{Ex}_\omega^r\le\mathbf{C}^r,\quad(\boldsymbol{\delta}_\omega), \tag{68}$$

$$\mathbf{Rx}_\omega^e\le\mathbf{C}^e,\quad(\boldsymbol{\iota}_\omega), \tag{69}$$

$$\mathbf{Lx}_\omega^k\le\mathbf{C}^k\bar{\mathbf{r}},\quad(\boldsymbol{\kappa}_\omega), \tag{70}$$

$$\mathbf{v}_\omega,\mathbf{u}_\omega,\mathbf{z}_\omega,\mathbf{x}_\omega\ge\mathbf{0}. \tag{71}$$

where the Greek bold-faced terms in parenthesis denote the corresponding vectors of the optimal dual solution with appropriate dimensions. Let $Q(\bar{\mathbf{y}},\bar{\mathbf{r}},\zeta^\omega)$ denote the optimal objective value of SUB(ω) with first-stage variables $\bar{\mathbf{y}}$, and $\bar{\mathbf{r}}$ under the scenario ω. The optimal objective value and solutions can be derived easily because every SUB(ω) is a simple linear programming model. Furthermore, the optimal primal solution SUB(ω) for each $\omega\in\Omega$ is feasible for the original problem. Hence, the following equation provides the upper bound, Z_{ub}, for the original problem:

$$Z_{ub}:=\mathbf{f}^\intercal\bar{\mathbf{y}}+\mathbf{e}^\intercal\bar{\mathbf{g}}+\sum_{\omega\in\Omega}p_\omega Q(\bar{\mathbf{y}},\bar{\mathbf{r}},\zeta^\omega) \tag{72}$$

If for every scenario $\omega\in\Omega$, $Q(\bar{\mathbf{y}},\bar{\mathbf{r}},\omega)$ is less than or equal to $\bar{\theta}_\omega$ from MP, then the current solution is optimal to the original problem (i.e.$Z_{ub}=Z_{lb}$). Otherwise, if the SUB(ω) corresponding to some ω has $Q(\bar{\mathbf{y}},\bar{\mathbf{r}},\omega)$ greater than $\bar{\theta}_\omega$, the corresponding optimality cuts are added to the MP. An optimality cut for scenario ω is generated as follows:

$$\theta_\omega\ge(\mathbf{a}_\omega^{itr+1})^\intercal\mathbf{y}+(\mathbf{c}_\omega^{itr+1})^\intercal\mathbf{r}+d_\omega^{itr+1} \tag{73}$$

Coefficients of the optimality cut are calculated as below:

$$(\mathbf{a}_\omega^{itr+1})^\intercal:={\boldsymbol{\pi}_\omega}^\intercal\mathbf{S}_\omega \tag{74}$$

$$(\mathbf{c}_\omega^{itr+1})^\intercal:=(\boldsymbol{\rho}_\omega+\boldsymbol{\kappa}_\omega)^\intercal\mathbf{C}^k \tag{75}$$

$$d_\omega^{itr+1}:={\boldsymbol{\nu}_\omega}^\intercal\mathbf{D}_\omega+(\boldsymbol{\lambda}_\omega+\boldsymbol{\delta}_\omega)^\intercal\mathbf{C}^r+(\boldsymbol{\tau}_\omega+\boldsymbol{\iota}_\omega)^\intercal\mathbf{C}^e \tag{76}$$

This procedure is implemented iteratively until the condition $(Z_{ub}-Z_{lb})/Z_{lb}<\epsilon_{BD}$ is satisfied, where ϵ_{BD} is the pre-determined control parameter. It is worth mentioning that because the proposed stochastic model has a relatively complete recourse, we do not consider the feasibility cut, which is necessary for the case in which some SUB(ω) are infeasible according to the optimal solution of MP.

4.3. Acceleration method

At the beginning of the typical BD algorithm (TBD), MP is initially solved with an empty set of optimality cuts. Then, based on the optimal dual solution of SUB(ω), optimality cuts at the first iteration are added to the MP. We refer to these optimality cuts as *initial optimality cuts*, which are generated in the first iteration.

However, it is obvious that the MP with an empty set of optimality cuts could provide a poor feasible solution (e.g. $\bar{\mathbf{y}}, \bar{\mathbf{g}}$, and $\bar{\mathbf{r}}$ are zero and $\bar{\theta}_\omega$ are negative in value). In this case, initial optimality cuts cannot contribute to creating a better lower bound because poor solutions tend to generate ineffective cuts (Kiya and Davoudpour 2012). Consequently, the TBD algorithm could incur a lot of iterations until the termination condition and naturally increase the total computation time. Therefore, we devise a simple method for accelerating the convergence of bounds in the BD algorithm and for reducing the number of required iterations by generating effective initial optimality cuts at the first step.

Prior to presenting the acceleration method, let us first introduce the *expected value problem* (EVP). This simple model is obtained by replacing all stochastic parameters with their expected values, and the optimal solution to the EVP is called the *expected value solution* (EVS) (Birge and Louveaux 2011). The EVS can sometimes be a high-quality solution to the true problem. By utilising this property, we utilise the EVS to generate better initial cuts than the typical one at the initialisation step of the BD algorithm. The detailed procedure for the acceleration method is as follows:

Acceleration method

(1) Obtain the expected value solution $\bar{\mathbf{y}}, \bar{\mathbf{g}}, \bar{\mathbf{r}}$, by solving the EVP with a commercial solver until the computation time falls within 30 seconds or until the gap between the best solution and the best bound falls within 5%.
(2) Solve SUB(ω) for each ω based on the obtained expected value solution in Step 1. Then, obtain the optimal objective value $Q(\bar{\mathbf{y}}, \bar{\mathbf{g}}, \omega)$ and optimal dual solution $\boldsymbol{\pi}_\omega, \boldsymbol{\mu}_\omega, \boldsymbol{\nu}_\omega, \boldsymbol{\lambda}_\omega, \boldsymbol{\tau}_\omega, \boldsymbol{\rho}_\omega, \boldsymbol{\delta}_\omega, \boldsymbol{\iota}_\omega$, and $\boldsymbol{\kappa}_\omega$ for all ω.
(3) Generate initial optimality cuts with the obtained objective value and optimal dual solution in Step 2. After generating the initial optimality cuts, the subsequent procedure is the same as the BD algorithm.

In Step 1, we set the stopping criteria as 30 seconds and the gap within 5% because it costs a computational burden to solve the EVP to get the optimal solution costs in a large-sized problem. Moreover, by implementing a lot of computational experiments, we observed that there was no obvious performance difference between the optimal solution and the sub-optimal solution of the EVP for improving the final computation time of the BD algorithm. Finally, the BD algorithm with the acceleration method (ABD) is presented in Algorithm 1.

Algorithm 1 Benders decomposition algorithm (Acceleration method)

```
Initialization:
Z_ub ← ∞, Z_lb ← −∞, itr ← 1
solve EVP and get ȳ, ḡ, r̄
for ω ∈ Ω do
    solve SUB(ω) based on ȳ, ḡ, and r̄
    get (a_ω^itr)ᵀ, (c_ω^itr)ᵀ, d_ω with optimal dual solutions
    add initial optimality cuts to MP
end
while Z_ub − Z_lb ≥ ε_BD × Z_lb do
    solve MP and get ȳ, ḡ, r̄, θ̄_ω, ∀ω ∈ Ω
    Z_lb ← max{Z_lb, fᵀȳ + eᵀḡ + Σ_{ω∈Ω} p_ω θ̄_ω}
    for ω ∈ Ω do
        solve SUB(ω) and get dual solution
        get (a_ω^itr)ᵀ, (c_ω^itr)ᵀ, d_ω with optimal dual solutions
        store the optimal objective value Q(ȳ, r̄, ζ^ω)
        if θ̄_ω < Q(ȳ, r̄, ζ^ω) then
            add an optimality cut to MP
        end
    end
    if Z_ub > fᵀȳ + eᵀḡ + Σ_{ω∈Ω} p_ω Q(ȳ, r̄, ζ^ω) then
        Z_ub ← fᵀȳ + eᵀḡ + Σ_{ω∈Ω} p_ω Q(ȳ, r̄, ζ^ω)
        y* ← ȳ, g* ← ḡ, r* ← r̄
        u*_ω ← ū_ω, v*_ω ← v̄_ω, x*_ω ← x̄_ω, ∀ω ∈ Ω
    end
    itr ← itr + 1
end
Return: Z_ub, Z_lb, y*, g*, r*, u*_ω, x*_ω, v*_ω, ∀ω ∈ Ω;
```

5. Computational experiments

In this section, we conducted three types of computational experiments to answer the research questions in Section 1. Research question (1) is answered by the results of experiments in Sections 5.2 and 5.3. Four types of computational experiments were implemented in Section 5.4. The first experiment result answers the Research question (2), and the second and third experiments answer the Research question (3). Research question (4) is answered by the results of the fourth experiment. We suggest several managerial insights in Section 5.5 based on the computational results. All the experiments were conducted on a PC with an AMD Ryzen 2700X 8-Core CP, 3.60 GHz processor, and 16GB of RAM with a Windows 10 64-bit system. Test instances were generated using Python 3.8, and every solution approach was developed with FICO Xpress 8.5 and Xpress-Optimizer version 33.01.02.

5.1. Description of the test instances

To validate the performance of the proposed algorithms, we need benchmark instances. However, as far as we know, there are no existing benchmark instances corresponding to our problem. Therefore, we rely on real-world information for determining the values of the parameters. At first, inventory holding costs, h_i^r and h_i^k, were generated on the basis of the article by Hass (2019). The cost of delivery, b_i, was determined based on the cost of the parcel delivery service in South Korea. To cover various cases, other deterministic parameters were randomly generated with the range of uniform distributions detailed in Table A1.

In order to estimate the distributions of stochastic parameters, we used the e-commerce public dataset (Kaggle 2018), which consists of demand data for 614 time periods, from September 4, 2016 to September 3, 2018. Then, we fitted the normal distribution to this dataset to estimate the distributions of demands, D_{it}^{ω}. We set the negative-value of realised demands or supplies to zero and adopted the same distribution of D_{it}^{ω} for S_{ijt}^{ω}. Consequently, every random sample of N scenarios is realised based on the estimated distribution of stochastic parameters shown in Table A2.

The locations of suppliers and warehouses are uniformly distributed over the pre-specified width and height of the XY plane. Moreover, the unit transportation costs, c_j^r, c_j^k, and c_j^e, are assumed to be proportional to the Euclidean distance in the XY plane. Because of the assumption that it is expensive to use the emergency warehouse, the values of c_j^e and h_i^e are significantly larger than the cost of the retailer or provider warehouses.

Based on the model given, the size of a problem is determined by $|\mathcal{I}|, |\mathcal{J}|, |\mathcal{T}|, |\mathcal{K}|$, and $|\mathcal{M}|$. We produced test instances ranging from small to large sizes. In particular, we classified the mathematical model using the test instances 13–15 for input as the large-sized problem. Every test instance is generated randomly according to the uniform distribution in Table A1. The detailed characteristics of test instances are indicated in Table 2. The columns labeled 'XY' represents the width and height of the XY plane. The number of variables (Vars) and constraints (Cons) are calculated for a scenario size $N = 40$.

We conducted every computational experiment considering one retailer warehouse, one emergency warehouse, and multiple provider warehouses according to assumptions in Section 3.1. Through implementing a lot of experiments, we observed that the emergency warehouse was rarely used because of the high operational cost compared to the stockout cost. Hence, in Sections 5.2 and 5.3, we accommodated the problem in which the emergency warehouse has unlimited capacity for storage and transportation. However, in Section 5.4, we considered the limited capacity of the emergency warehouse in order to analyse the effects of lead time and the lost sales cost parameter. The two types of lead time, L_s and L_d, were set to zero in Sections 5.2 and 5.3, but we evaluated the impacts of these two types of lead time by varying values in Section 5.4.

5.2. Performance analysis of the proposed algorithms

As mentioned in Section 4.2, the SAA problem with the moderate size N could suffer from the computational burden. In this section, we conducted computational experiments to compare the three solution approaches: TBD, ABD, and Solver (solving the given problem with an Xpress-Optimizer). Test instances with different sizes of N were employed to evaluate the performance of the proposed algorithms. For each size of N and solution approach, ten experiments were conducted with different samples of N scenarios. An average of ten experiments has been reported in Table A3 with comparison results among Solver, TBD, and ABD. The columns labeled 'CPUs' and 'Itr' represent the computation times in seconds and the number of iterations required to make the optimality gap of BD algorithms (TBD and ABD) less than the pre-determined threshold, ϵ_{BD}. The 'Gap' is defined as follows:

$$\text{Gap} := \left(\frac{\text{Best solution (OBJ by each approach)}}{\text{Best bound}\left(\max\left\{Z_{lb}\text{by ABD}, Z_{lb}\text{ by TBD}\right\}\right)} - 1 \right) \times 100(\%) \tag{77}$$

If the maximum time limit (i.e. 3600 seconds) was reached, algorithms were terminated, and they output the Gap, CPUs, and Itr obtained so far. We set the ϵ_{BD} for 10^{-4} for both TBD and ABD.

The computational results of all test instances in Table A3 were averaged in terms of 'Gap', 'CPUs', 'Itr', and the number of times each algorithm reached the time limit are depicted in Figure 3. Figure 3 indicates that the ABD outperformed the TBD and Solver in terms of every evaluation measure. Furthermore, Figure 3 shows that more computation time was required to solve the problem as the size of N increased. However, the computation time of the BD algorithms increased more slowly when compared to the Solver. On the other hand, a small number of iterations was required as the size of N increased for both BD algorithms. The results appeared because as the N increased, it required more time to implement one iteration compared to the smaller size of N.

Table 2. Test instances specifications ($N = 40$).

No.	XY	Total Vars	Binary Vars	Cont Vars	Cons	$\|\mathcal{I}\|$	$\|\mathcal{J}\|$	$\|\mathcal{T}\|$	$\|\mathcal{K}\|$	$\|\mathcal{M}\|$
1	100 × 100	29,103	153	28,950	14,000	2	3	10	5	3
2		32,899	131	32,768	15,104	2	3	8	8	2
3		73,844	324	73,520	26,000	3	4	10	8	4
4		88,204	304	87,900	30,000	3	4	10	10	3
5		88,803	483	88,320	34,800	3	3	12	10	4
6	300 × 300	146,859	471	146,388	51,168	4	3	12	13	3
7		149,404	544	148,860	48,480	3	4	12	15	3
8		289,355	680	288,675	76,200	4	5	15	15	3
9		434,705	1355	433,350	109,080	5	5	18	15	5
10		467,405	1205	466,200	112,200	5	5	15	20	4
11	500 × 500	532,806	906	531,900	114,600	5	6	15	20	3
12		561,605	1805	559,800	135,360	5	5	18	20	5
13		1,046,606	2506	1,044,100	210,800	6	6	20	25	5
14		1,049,606	4006	1,045,600	213,800	6	6	20	25	8
15		1,808,006	4506	1,803,500	345,500	7	6	25	30	6

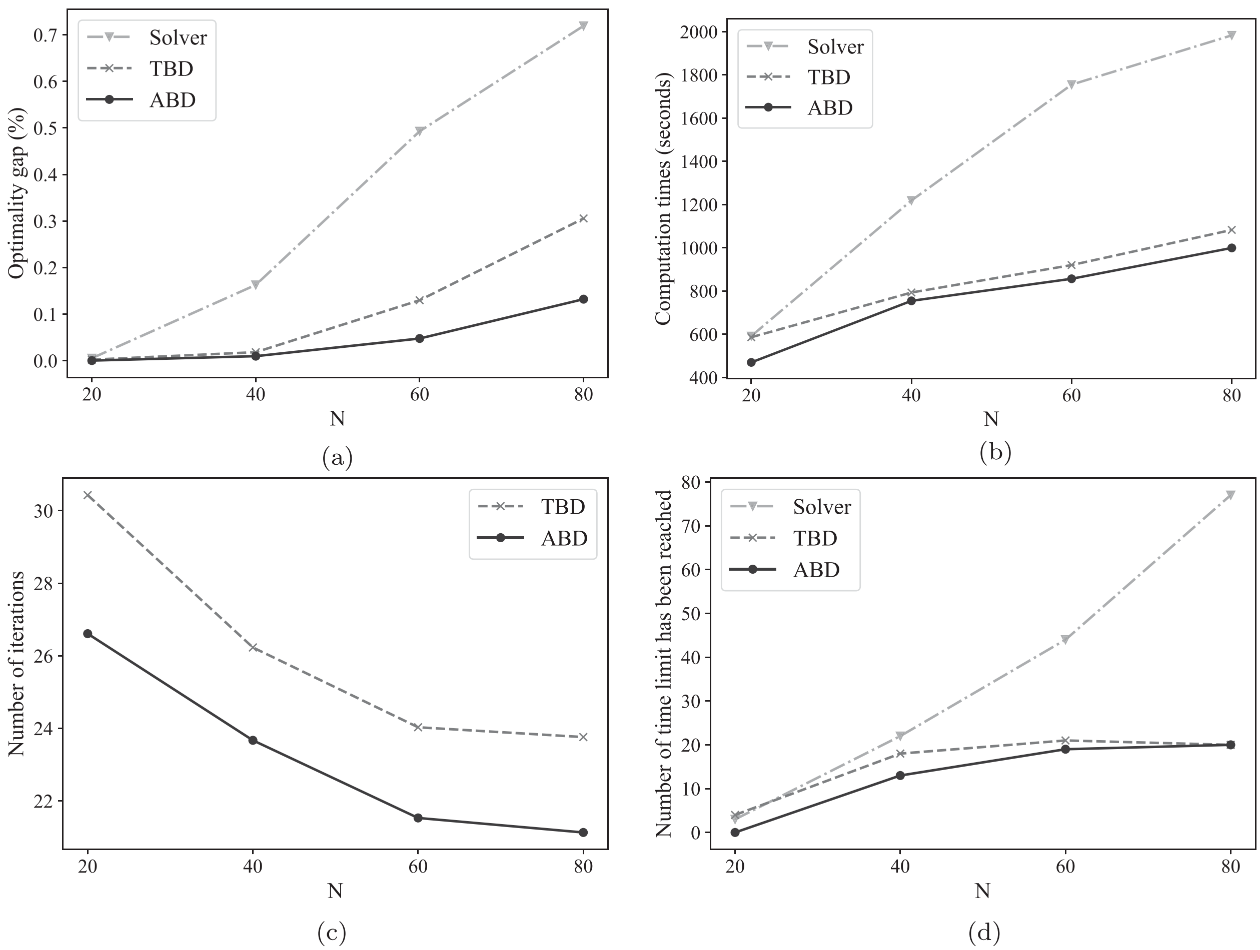

Figure 3. Comparisons between algorithms in terms of four performance measures. (a) Optimality gap (%). (b) Computation times (seconds). (c) Number of iterations% and (d) Number of time limit has been reached.

In order to analyse the effects of the initial optimality cuts of ABD, we compared the convergence of bounds for the TBD and ABD. We used test instances 12–15 with $N = 40$ and set the ϵ_{BD} to 0.03 for visualising the apparent convergence. Figure 4 represents a comparison between TBD and ABD concerning the Z_{ub}, and the Z_{lb}. As the number of iterations increased, the upper bound decreased, and the lower bound increased for both algorithms until the value of $(Z_{ub} - Z_{lb})/Z_{lb}$ within ϵ_{BD}. At the first iteration, the gap between the upper and lower bound of ABD was clearly smaller than the gap of TBD, which meant that ABD created effective initial optimality cuts. Finally, ABD converged faster than TBD with a small number of iterations. Even though the results in

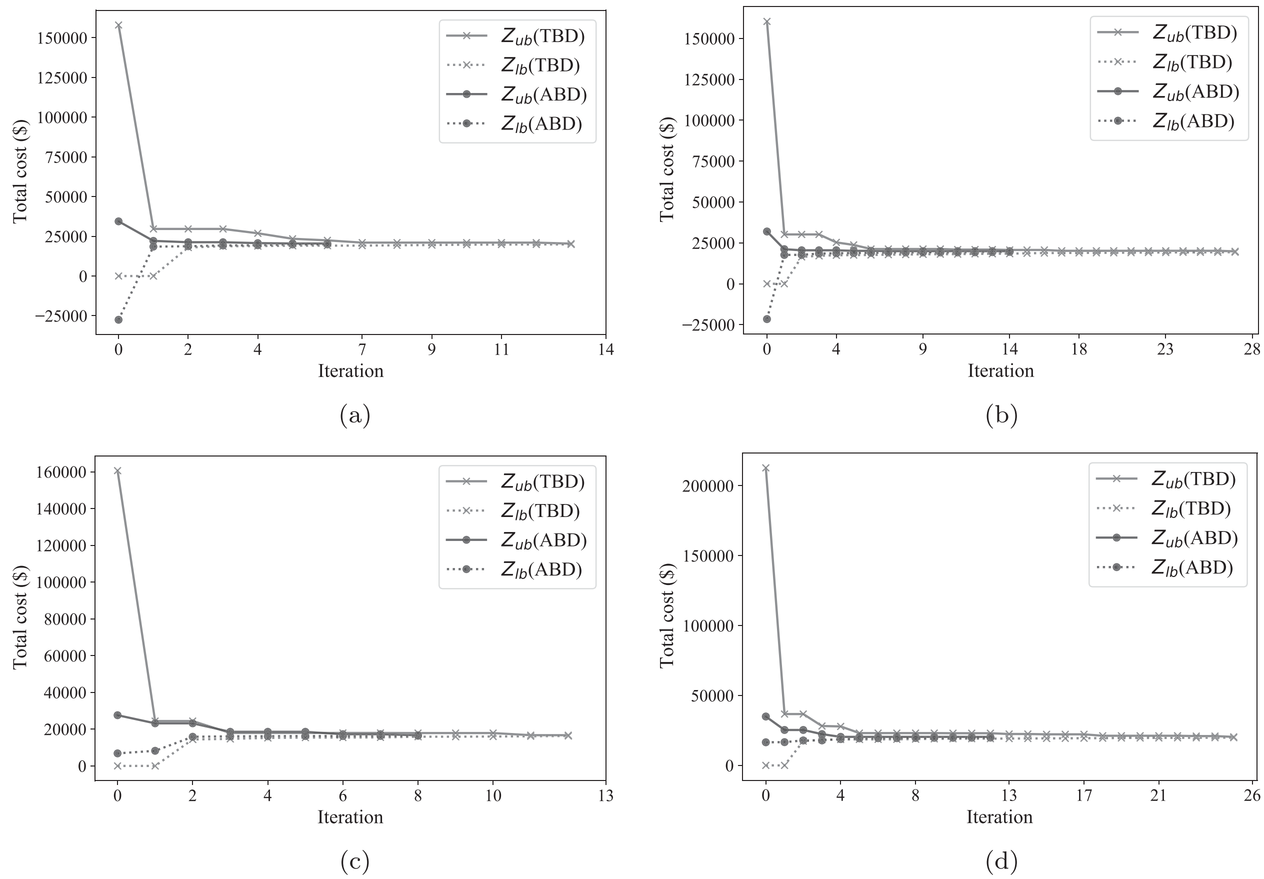

Figure 4. Comparison between TBD and ABD in terms of upper and lower bound. (a) Instance 12 ($N = 40$). (b) Instance 13 ($N = 40$). (c) Instance 14 ($N = 40$) and (d) Instance 15 ($N = 40$).

Figure 4 correspond to test instances 12–15 with $N = 40$, similar behaviour could be observed for other instances with different sizes of N.

5.3. Performance analysis of the stochastic solution

In this section, the quality of the stochastic solution is evaluated through several performance metrics. As mentioned in Section 4.1, the stochastic solution is derived from that which has the lowest upper bound, $\bar{f}_{N'}(\hat{\mathbf{y}}, \hat{\mathbf{g}}, \hat{\mathbf{r}})$, value among the number of M SAA replications. We utilise the *value of stochastic solution* (VSS),which is well-known performance metrics in stochastic programming research area (Birge and Louveaux 2011). VSS can be calculated as:

$$VSS = EEV - RP \tag{78}$$

where the EEV is the expected result of the EVP optimal solution and RP is the optimal objective value of the recourse problem.

For every test instance, we carried out the SAA algorithm in Section 4.1 with $N' = 3000, M = 20$, and $\epsilon_{SAA} = 1$. Therefore, the SAA algorithm terminates whenever the relative SAA gap, $Gap^{rel}_{MNN'}$, is within 1%. To compute the statistical lower bound, we progressively increased the number of scenarios in samples from 20 to 200 until the predetermined threshold ϵ_{SAA} was satisfied, $N \in \{20, 40, 60, 80, 100, 200\}$. Every SAA problem was computed by ABD with $\epsilon_{BD} = 10^{-3}$.

Table 3 presents the experiment results from the SAA algorithm. The upper bound values equal to RP, EEV, and WS were derived from the same sample with N' scenarios. By checking the results of the EEV, we could know that the EVS incurred a much higher total cost than the stochastic solution. The values of the VSS showed the performance of stochastic solutions compared with the EVS, which indicated the importance of capturing the stochastic nature of demands and supply for designing the supply chain. For every test instance, a size of N less than 100 was necessary to obtain the stochastic solution with the SAA gap less than 1%. In particular, we could get the high quality stochastic solution only with $N = 20$ for test instances 8–15.

In Table 4, we present the SAA gap estimates from the stochastic solution derived from the SAA algorithm and the EVS. As anticipated, the SAA gap estimates of

Table 3. Experiment results and statistics of the SAA algorithm.

No.	N	LB	σ_{LB}	UB	σ_{UB}	EEV	σ_{EEV}	WS	VSS
1	80	13,384.3	110.7	13,449.1	69.3	16,720.6	127.1	12,044.5	3271.5
2	40	10,923.4	89.3	11,023.1	53.4	20,079.1	143.5	10,099.1	9056.0
3	40	11,147.4	74.3	11,238.7	42.2	23,088.7	138.0	10,459.0	11,850.0
4	80	16,346.0	77.7	16,433.5	67.0	29,244.1	159.6	15,316.3	12,810.6
5	40	10,643.5	82.0	10,685.9	40.0	23,951.2	148.4	10,109.7	13,265.3
6	80	13,820.6	57.0	13,835.7	40.5	26,936.8	132.4	13,108.1	13,101.1
7	60	13,472.3	74.8	13,520.1	41.7	24,377.1	129.1	12,622.3	10,857.0
8	20	16,515.2	97.6	16,663.7	36.4	36,252.4	172.7	15,602.0	19,588.7
9	20	13,189.4	50.2	13,200.8	27.8	14,202.4	47.9	12,703.1	1001.6
10	20	14,883.5	106.8	15,017.9	32.4	16,158.7	60.8	14,396.0	1140.8
11	20	15,672.7	91.5	15,771.3	31.0	26,593.1	97.1	15,284.8	10,821.8
12	20	20,379.2	73.5	20,384.4	35.9	21,901.6	59.7	19,913.7	1517.2
13	20	19,762.1	70.7	19,765.8	28.5	19,888.2	33.9	19,401.7	122.4
14	20	16,568.0	96.5	16,641.7	30.6	16,740.0	34.0	16,329.5	98.3
15	20	20,224.4	67.8	20,228.8	23.8	20,585.9	27.7	20,110.6	357.1

Table 4. SAA gap estimates from stochastic and EVS.

	Stochastic solution			EVS		
	$Gap_{MNN'}$	$Gap^{rel}_{MNN'}$	$\sigma_{Gap_{MNN'}}$	$Gap_{MNN'}$	$Gap^{rel}_{MNN'}$	$\sigma_{Gap_{MNN'}}$
1	64.8	0.48	130.60	3336.3	24.93	168.51
2	99.7	0.91	104.08	9155.7	83.82	169.06
3	91.3	0.82	85.46	11,941.3	107.12	156.71
4	87.5	0.54	102.61	12,898.1	78.91	177.50
5	42.4	0.40	91.22	13,307.7	125.03	169.52
6	15.1	0.11	69.88	13,116.2	94.90	144.10
7	47.7	0.35	85.69	10,904.8	80.94	149.20
8	148.5	0.90	104.14	19,737.2	119.51	198.34
9	11.4	0.09	57.40	1013.0	7.68	69.41
10	134.4	0.90	111.56	1275.2	8.57	122.87
11	98.7	0.63	96.56	10,920.4	69.68	133.41
12	5.2	0.03	81.81	1522.4	7.47	94.69
13	3.7	0.02	6.29	126.1	0.64	78.45
14	73.7	0.44	101.24	172.0	1.04	102.33
15	4.4	0.02	71.83	361.5	1.79	73.22

the stochastic solution were less than 1%. On the other hand, the SAA gap of the EVS was much greater compared to the stochastic solution. In addition, the stochastic solution showed better performance than the EVS in terms of the standard deviation of the SAA gap, σ_{Gap}. For test instances 2–8 and 11, the SAA gap of the EVS was greater than 50%, which meant the provided EVS could not accommodate uncertainty for decision-making. In comparing the cost components derived from the EVS in Table 5, we found that the stockout costs absorbed a larger share of the total cost when the SAA gap estimate of the EVS was relatively high. Of special note, the stockout costs accounted for more than 50% of the total cost for test instances 2–8 and 11.

Table 5. Cost components derived from EVP solution.

	Delivery		Commitment		Stockout		Supplier investment		Transportation		Inventory holding	
No.	Cost (\$)	%	Cost (\$)	%	Cost (\$)	%	Cost (\$)	%	Cost (\$)	%	Cost (\$)	%
1	4869.9	29.13	1875.9	11.22	8076.3	48.30	1526.2	9.13	309.6	1.85	62.7	0.37
2	4079.1	20.32	1453.2	7.24	13,572.8	67.60	525.2	2.62	397.0	1.98	51.8	0.26
3	5219.6	22.61	1488.6	6.45	15,210.7	65.88	606.1	2.62	484.8	2.10	79.0	0.34
4	4654.6	15.92	3096.1	10.59	20,458.9	69.96	572.7	1.96	385.5	1.32	76.3	0.26
5	5337.5	22.28	788.3	3.29	16,838.7	70.30	517.2	2.16	351.0	1.47	118.4	0.49
6	5759.9	21.38	2491.6	9.25	16,568.2	61.51	532.5	1.98	1467.6	5.45	116.8	0.43
7	4168.7	17.10	2961.2	12.15	15,285.8	62.71	694.6	2.85	1152.5	4.73	114.3	0.47
8	6901.8	19.04	2126.1	5.86	24,009.7	66.23	889.1	2.45	2154.6	5.94	171.1	0.47
9	7726.1	54.40	945.8	6.66	2514.1	17.70	1197.9	8.43	1679.8	11.83	138.8	0.98
10	7592.7	46.99	2129.4	13.18	3603.6	22.30	1134.8	7.02	1587.0	9.82	111.2	0.69
11	7094.0	26.68	3042.4	11.44	14,002.4	52.65	990.8	3.73	1283.6	4.83	179.9	0.68
12	9560.4	43.65	2278.0	10.40	4337.7	19.81	1556.2	7.11	4044.9	18.47	124.5	0.57
13	11,459.6	57.62	2378.7	11.96	1976.1	9.94	1469.9	7.39	2504.7	12.59	99.2	0.50
14	9907.7	59.19	838.8	5.01	3101.3	18.53	1159.8	6.93	1612.7	9.63	119.8	0.72
15	12,178.9	59.16	2565.0	12.46	1761.2	8.56	1342.8	6.52	2597.5	12.62	140.4	0.68

Table 6. Impact of different number of available provider warehouses on utilisation and total cost.

K_{max}	$\mathcal{K}$	Utilisation	Warehouses	Total cost ($)
1	$\{1\}$	15	1	32,406.9
2	$\{1,2\}$	25	1,2	16,360.0
3	$\{1,2,3\}$	30	1,2,3	15,532.5
4	$\{1,2,\ldots,4\}$	30	1,2,3,4	15,259.9
5	$\{1,2,\ldots,5\}$	30	1,2,3,4	15,259.9
6	$\{1,2,\ldots,6\}$	30	1,2,3,4	15,259.9
7	$\{1,2,\ldots,7\}$	30	1,2,3,4,7	15,098.2
8	$\{1,2,\ldots,8\}$	30	1,2,3,4,7	15,098.2
9	$\{1,2,\ldots,9\}$	29	1,2,3,4,7,9	14,986.9
15	$\{1,2,\ldots,15\}$	29	1,2,3,4,7,9	14,986.9
20	$\{1,2,\ldots,20\}$	29	1,2,3,4,7,9	14,986.9

5.4. Effects of the ODWS on the supply chain

In this section, we conducted four types of experiments to explore the effects of the ODWS on the supply chain by solving the test instance 10 with $N = 40$. In the first experiment, we investigated the impact that available provider warehouses had on the resulting supply chain. We analysed the total cost and utilisation of provider warehouses by varying the number of available provider warehouses $K_{\max}$, which indicates the size of set $\mathcal{K}$. We use the term 'Utilisation' to refer to the utilisation of provider warehouses within the entire time horizon, and it is defined as:

$$\text{Utilisation} := \sum_{k\in\mathcal{K}} \sum_{m\in\mathcal{M}} \sum_{t\in\mathcal{T}} r^k_{mt} \tag{79}$$

Table 6 represents the utilisation of provider warehouses and the total cost varying $K_{\max}$. As the $K_{\max}$ increased until eight, the utilisation of provider warehouses also increased. However, the utilisation decreased from 30 to 29 when the $K_{\max}$ was bigger than eight. On the other hand, the total cost decreased when the $K_{\max}$ was increased. In the case where only one provider warehouse was available, it incurred the highest total cost because satisfying demands with only one provider warehouse capacity was challenging. Note that in the case in which $K_{\max}$ was bigger than nine, the utilisation and total cost did not change. It meant that utilising provider warehouses from 10 to 20 could not contribute to better solutions for reducing the total cost.

In the second experiment, a sensitivity analysis on the commitment cost of parameter α was conducted to explore the effects on solutions. Figure 5 represents the changes of utilisation and total cost brought about by varying the value of α. As the α increased, utilisation decreased and total cost increased. Because of the expensive cost of commitment, utilising provider warehouses for the SCND was avoided. When the α was larger than 7500, the total cost did not vary, and the utilisation became zero, which meant provider warehouses were not used.

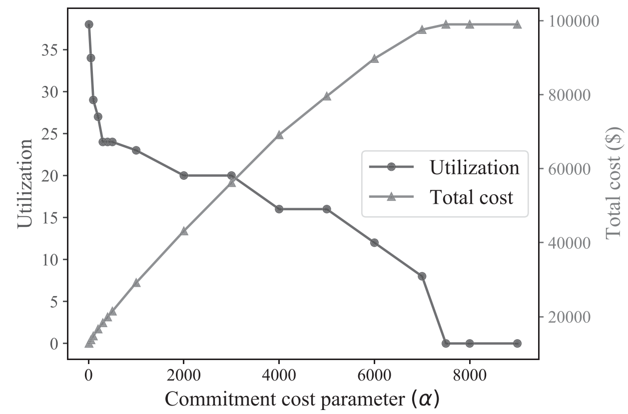

Figure 5. Changes of total cost and utilisation varying the commitment cost parameter α.

Figure 6 shows the share of the total cost according to different cost components. Increasing the α resulted in decreasing the percentage of transportation, delivery, and supplier investment costs. Because inventory holding cost parameters, h^r_i, h^r_i, and h^e_i, were much smaller than other cost parameters, the percentage of inventory holding cost was negligible. The percentage of commitment cost increased and then decreased at the point when the α was larger than 5000. On the contrary, the percentage of the stockout cost decreased and then increased at the same point for the commitment cost. Like the utilisation and total cost in Figure 5, the percentage of each cost component did not change when the α was larger than 7500. Furthermore, the stockout cost accounted for a disproportionately large share of the total cost. Based on this result, we could observe that allowing for the condition of stockout for most of the demands is a better cost-saving strategy compared to using provider warehouses when the α is significantly higher than the stockout cost parameter β_i.

In the third experiment, a sensitivity analysis on the lost sales cost parameter, β_i, was conducted to observe the relationship between utilisation of the emergency warehouse and stockout. In the third and fourth experiments, we assumed that the emergency warehouse is capacitated ($C^e = 70$). The average number of items delivered from the emergency warehouse to customers within the entire time horizon is used to refer to the utilisation of the emergency warehouse (UEW), and it is defined as:

$$\text{UEW} := \frac{1}{|\Omega|} \sum_{i\in\mathcal{I}} \sum_{e\in\mathcal{E}} \sum_{t\in\mathcal{T}} \sum_{\omega\in\Omega} u^{e\omega}_{it} \tag{80}$$

Figure 7 represents the changes in UEW, total cost, and stockout cost brought about by varying the value of β_i. Until the value of β_i was 150, UEW was zero, which meant the emergency warehouse was not used. Instead,

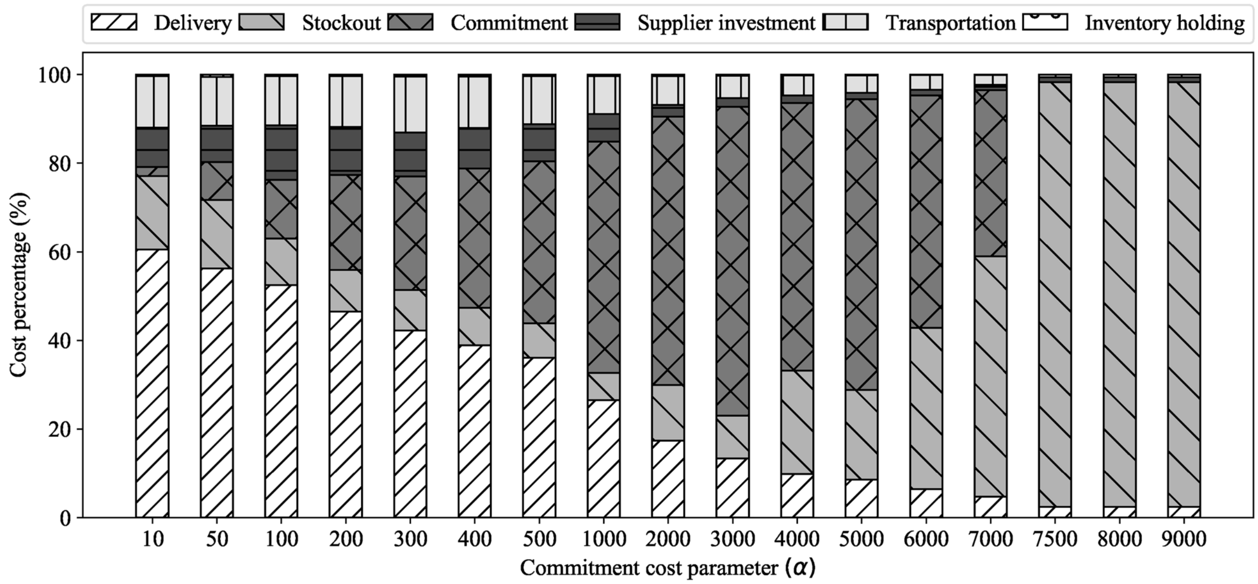

Figure 6. Share of total cost for each cost component varying the commitment cost parameter α.

every unsatisfied demand was addressed through the lost sales. At the point β_i was 160, UEW increased dramatically from zero to about 32, which means the emergency warehouse was used to satisfy demand. However, UEW slightly increased when β_i was bigger than 160.

The total cost and stockout cost increased rapidly until the value of β_i was 150. When the β_i was bigger than 160, the total cost increased slightly. On the other hand, the stockout cost decreased steeply at the point β_i was 160. After that, when the β_i was bigger than 160 and smaller than 600, the stockout cost increased slightly. The stockout cost became zero when the β_i was bigger than 650, which means that every demand was satisfied. In addition, when the β_i was bigger than 650, the UEW, total cost, and stockout cost did not vary.

Figure 8 depicts the share of the total cost according to different cost components for the lost sales cost parameter, β_i. The percentage of inventory holding cost was negligible in the same manner as is shown in Figure 6. Depending on the value of β_i, the percentage of stockout cost and the percentage of transportation cost tended to move into the opposite directions. In detail, as the β_i increased to 150, the percentage of stockout cost increased, and the percentage of delivery and transportation cost decreased. At the point when the β_i was 160, the percentage of stockout cost decreased rapidly, and the percentage of transportation cost increased significantly. This result means that the emergency warehouse was used to satisfy demand as much as possible to avoid stockouts because of the high cost of lost sales. When the β_i was 160 to 600, the percentage of stockout cost increased slightly, but the stockout cost did not account for any share of the total cost when the β_i was larger than 650. In addition, when the β_i was larger than 650, the percentage of supplier cost increased, which meant that every demand was satisfied by adopting additional suppliers.

In the fourth experiment, we evaluated the effects of lead times when utilising the ODWS in the supply chain by varying the values of lead times between suppliers and warehouses, L_s, and between warehouses and customers, L_d. In cases in which the lead time exists in the supply chain, a lot of stockout costs can be incurred at the beginning of the planning horizon if the retailer does not hold initial inventory. Therefore, in this experiment, we assumed that the initial inventory is equal to the expected value of demand. Based on the detailed results in Table A4, we presented in Figure 9 the impacts of each type of lead time on total cost, stockout cost, delivery cost, and commitment cost.

In Figure 9, we varied the value of one type of lead time, and the other one was fixed to zero to compare the impacts of each type of lead time. For both types of lead time, total cost and stockout cost increased as the value of lead time increased. On the other hand, because the total amount of stockout increased, the percentage of delivery cost decreased. Commitment cost also decreased as the lead time increased, which meant that the utilisation of the provider warehouse decreased as well. Finally, by observing that the total cost increased more rapidly when increasing the value of L_d than when increasing the value of L_s, we could know that the length of lead time between warehouses and customers severely affected the cost incurred in the supply chain.

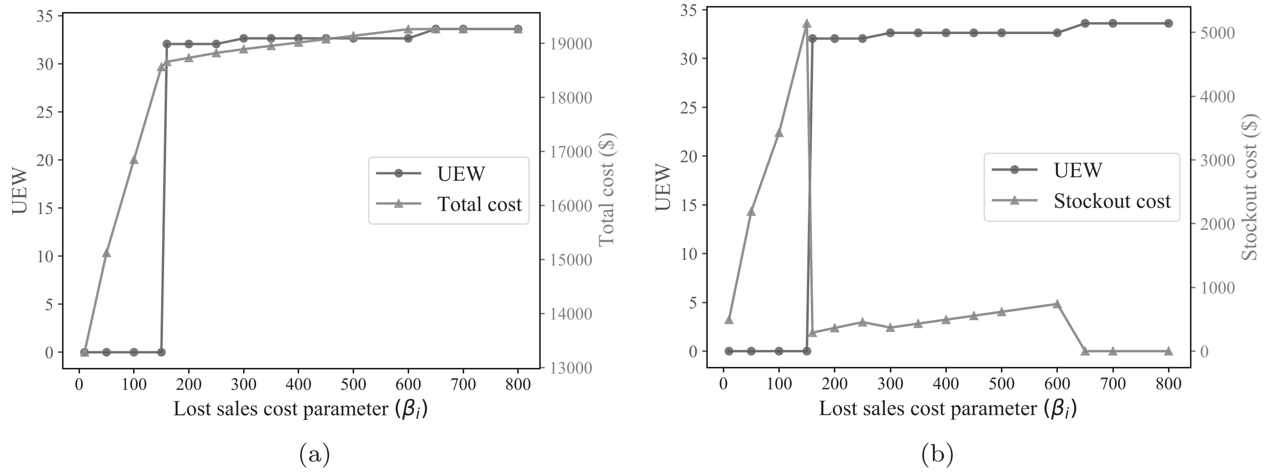

Figure 7. Changes of cost and UEW varying the lost sales cost parameter β_i. (a) Total cost and (b) Stockout cost.

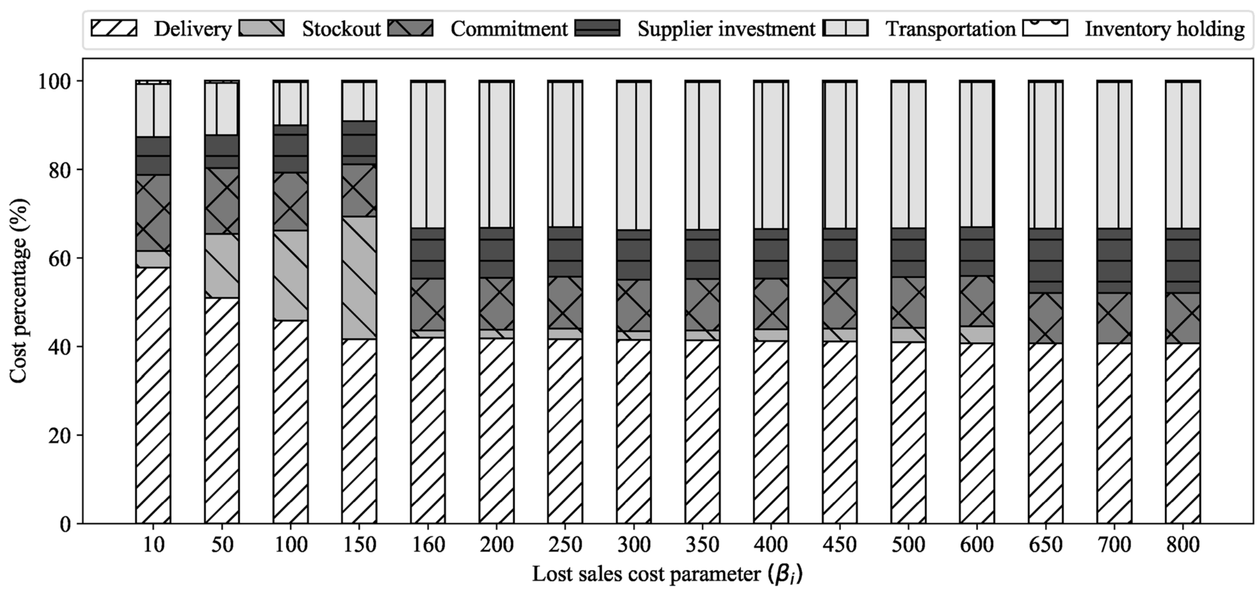

Figure 8. Share of total cost for each cost component varying the stockout cost parameter β_i.

5.5. Managerial insights

The proposed model and stochastic solution approach could contribute to e-commerce retailers who plan to build the supply chain network flexibly during the COVID-19 pandemic. After analysing the computational results, we can offer several managerial insights that could be instructive to e-commerce retailers who suffer from the limited space of warehouses. The proposed managerial insights are as follows:

(1) Utilizing the ODWS can save on the total cost of the supply chain because it has a similar effect as expanding capacity flexibly. Even though most of the demands can be satisfied with enough provider warehouses, we could observe that using a moderate number of provider warehouses is a good strategy for minimising total cost. Hence, considering the locations of suppliers and provider warehouses and the appropriate number of provider warehouses would be helpful to retailers when constructing an efficient supply chain with the ODWS.

(2) Our proposed model is very sensitive to uncertainty because frequent stockouts could occur when insufficient provider warehouses are committed to being used. Even though simple solution approaches could solve the problem (e.g. EVP), most obtained solutions are imprecise for acceptable decision-making. By analysing the value of the VSS, we could observe that it is important to deal with uncertainty accurately regarding the SCND problem with the ODWS. Therefore, we suggest that retailers who need to address frequent stockouts because

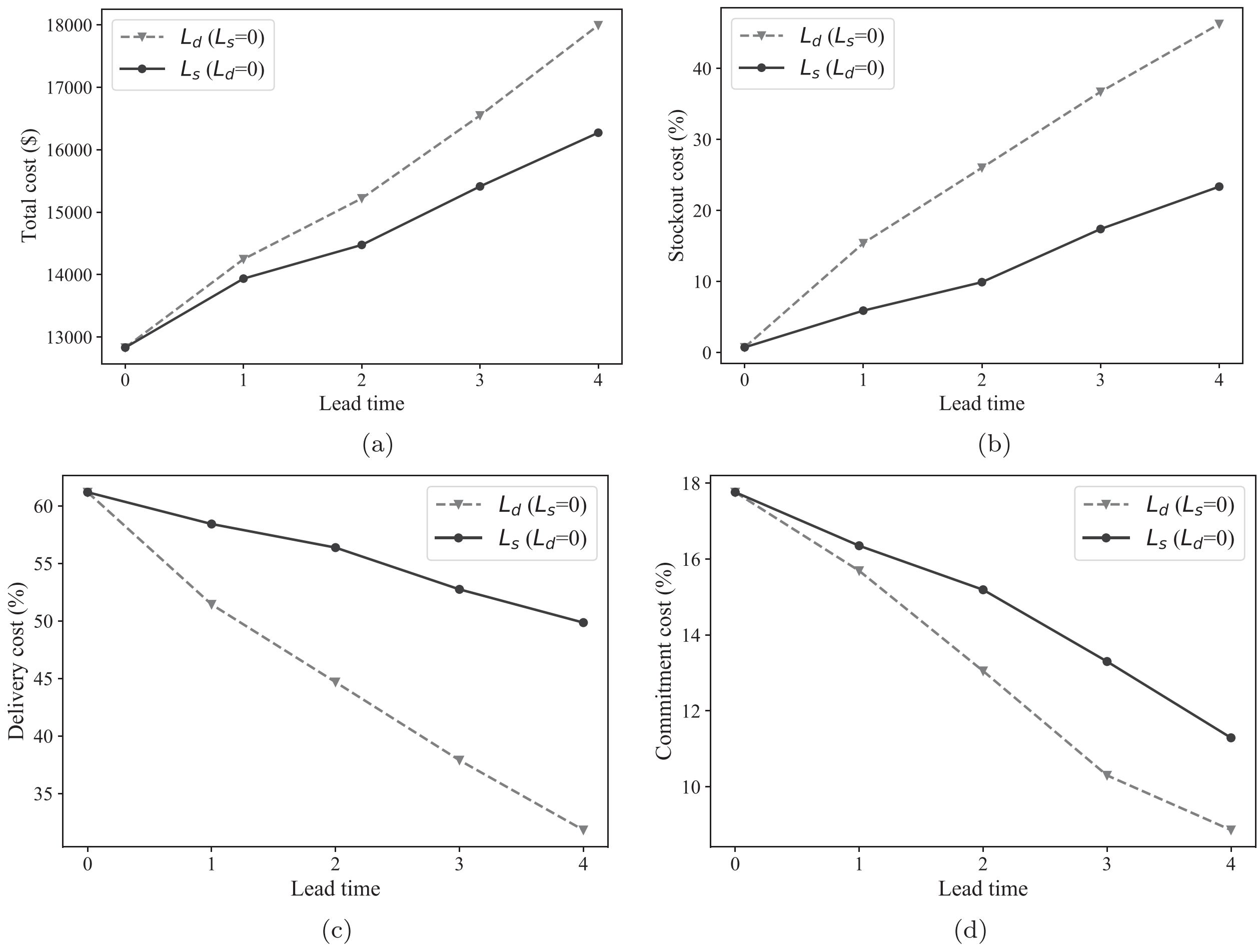

Figure 9. Comparisons between two types of lead time in terms of cost. (a) Total cost ($). (b) Share of total cost of stockout cost (%). (c) Share of total cost of delivery cost (%) and (d) Share of total cost of commitment cost (%).

of limited capacity should develop an efficient way for accommodating the uncertainty of demand and supply.

(3) In the actual case, the value of α is determined by the warehouse operators or the ODWS platform company. However, the value of β_i can be estimated by the e-commerce retailers. As shown in Figures 6–8, the estimated value of β_i has much influence on the quality of solutions. In terms of obtained solutions, if the β_i is estimated to be larger than the true value, it results in excess utilisation of provider and emergency warehouses. Otherwise, when β_i is estimated to be smaller, it could incur a lot of stockouts because of insufficient utilisation of provider warehouses and the expensive cost of utilising the emergency warehouse. Hence, we recommend that retailers conduct an accurate estimation for the value of β_i beforehand and then implement our proposed approach.

(4) Through several computational experiments, we observed that the lead time increased the total amounts of stockout, which incurred additional costs. The lead time between warehouses and customers was more significant than between suppliers and warehouses in terms of costs incurred in the supply chain. Therefore, when e-commerce retailers design the supply chain with the ODWS, we recommend choosing a 3PL company operating with short lead times even though the delivery cost is slightly higher. This strategy would be helpful to retailers in minimising the total cost incurred in the supply chain.

6. Conclusions

With e-commerce set to expand rapidly in the coming decades, the ODWS has emerged as a new alternative for satisfying growing demand. By utilising the ODWS in the supply chain, e-commerce retailers can flexibly respond to demand changes because this service makes short-term rent of warehouses available. However, a high degree of uncertainty regarding demand and supply exists in the e-commerce marketplace, which influences decision-making for the SCND. To the best of our knowledge, there is no existing research dealing with the problem of the SCND with the ODWS under uncertainty. Therefore, we propose the two-stochastic programming model, which reflects the supply chain network of the e-commerce marketplace in South Korea.

Because of the high computational complexity of the proposed model, a solution approach combining the SAA and BD algorithms was presented to solve the proposed model. Of special note, a method to accelerate the convergence of bounds in the BD algorithm, referred to as ABD, was developed. The ABD outperforms the typical version of the BD algorithm and Xpress-Optimizer with regard to the optimality gap and computation times. In addition, the quality of stochastic solutions derived from the SAA algorithm is better than the solutions from the EVP.

Through conducting computational experiments, we could observe that utilising the ODWS for the SCND saves on the total cost compared to using a small number of warehouses with limited capacity. Furthermore, through our sensitivity analysis, we could see the relationship between parameters of commitment cost and stockout cost for a decision about using the provider and emergency warehouses. We observed the impacts of two types of lead time on the cost incurred in the supply chain considering the ODWS. At last, we present several managerial insights that are helpful for e-commerce retailers who aim to design their supply chain networks with the ODWS.

For further research, we intend to extend our study by using multi-stage stochastic programming, which has an advantage for dealing with uncertainty under a multi-period setting. The nature of TSSP enables the stochastic parameters to become known in a single moment. However, regarding the problem with a planning horizon with multiple periods, the uncertainty can be dealt with more accurately when the stochastic parameters have been realised progressively in each period. Therefore, through utilising the above scheme, some decisions will be made before the realisation of uncertainty, and other decisions are made after the realisation in each period (Govindan, Fattahi, and Keyvanshokooh 2017).

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Data availability statement

The data that support the findings of this study are available from the author, Junhyeok Lee (ljh9533@snu.ac.kr), upon reasonable request.

Disclosure statement

No potential conflict of interest was reported by the authors.

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Appendices

Appendix 1. Parameter information

Table A1. Ranges of the deterministic parameters.

F_j	α	β_i	γ	C^k&C^r
$\mathcal{U}(500, 1000)$	$\mathcal{U}(100, 200)$	$\mathcal{U}(30, 70)$	$\mathcal{U}(0.80, 0.99)$	$\mathcal{U}(50, 100)$

Table A2. Probability distributions for stochastic parameters.

D_{it}^w	S_{ijt}^w
$\mathcal{N}\left(\frac{179.06}{\lvert\mathcal{I}\rvert}, \left(\frac{91.18}{\lvert\mathcal{I}\rvert}\right)^2\right)$	$\mathcal{N}\left(\frac{179.06}{\lvert\mathcal{I}\rvert}, \left(\frac{91.18}{\lvert\mathcal{I}\rvert}\right)^2\right)$

Appendix 2. Comparison of performance for solving SAA problems

Table A3. Comparison of performance between Xpress Solver, TBD, and ABD for solving SAA problems.

		N											
		20			40			60			80		
No.	Method	Gap	CPUs	Itr	Gap	CPUs	Itr	Gap	CPUs	Itr	Gap	CPUs	Itr
1	Solver	0.00	11.86	–	0.00	18.39	–	0.00	30.47	–	0.00	44.40	–
	TBD	0.00	4.39	13.7	0.00	3.87	12.1	0.00	5.35	12.1	0.00	6.90	11.9
	ABD	0.00	4.46	11	0.00	3.46	9.4	0.00	4.68	9.7	0.00	6.29	9.6
2	Solver	0.00	8.88	–	0.00	16.04	–	0.00	22.96	–	0.00	33.82	–
	TBD	0.00	4.30	13.4	0.00	3.72	13.3	0.00	4.59	12.5	0.00	7.45	13.5
	ABD	0.00	3.96	11.2	0.00	3.56	11.6	0.00	3.74	9.3	0.00	7.01	11.4
3	Solver	0.00	37.08	–	0.00	77.97	–	0.00	112.47	–	0.00	191.44	–
	TBD	0.00	28.05	20.0	0.00	39.45	18.1	0.00	34.47	15.7	0.00	64.91	17.4
	ABD	0.00	24.64	17.4	0.00	30.81	14.7	0.00	31.46	13.6	0.00	56.24	14.3
4	Solver	0.00	30.21	–	0.00	54.99	–	0.00	110.47	–	0.00	138.46	–
	TBD	0.00	16.57	19.3	0.00	21.74	16.4	0.00	32.91	17.5	0.00	44.50	17.7
	ABD	0.00	14.80	16.2	0.00	18.04	13.4	0.00	33.00	16.2	0.00	38.51	15.0
5	Solver	0.00	61.87	–	0.00	91.22	–	0.00	180.89	–	0.00	301.62	–
	TBD	0.00	30.64	18.3	0.00	32.89	15.9	0.00	62.89	17.8	0.00	71.48	16.5
	ABD	0.00	27.51	15.1	0.00	31.38	13.8	0.00	54.98	14.1	0.00	56.07	12.7
6	Solver	0.00	38.60	–	0.00	94.68	–	0.00	153.23	–	0.00	232.66	–
	TBD	0.00	18.68	16.0	0.00	31.89	15.5	0.00	47.82	15.9	0.00	63.51	16.1
	ABD	0.00	16.16	12.9	0.00	24.08	11.3	0.00	38.06	12.2	0.00	48.28	12.2
7	Solver	0.00	66.07	–	0.00	137.47	–	0.00	242.08	–	0.00	342.36	–
	TBD	0.00	79.86	27.3	0.00	62.83	22.4	0.00	102.06	23.4	0.00	162.87	24.7
	ABD	0.00	77.83	25.5	0.00	69.86	20.9	0.00	102.76	21.2	0.00	167.03	22.5
8	Solver	0.00	578.07	–	0.00	1496.01	–	0.20	3231.62	–	0.38	3599.15	–
	TBD	0.00	576.96	39.6	0.00	746.29	42.1	0.00	838.18	32.5	0.00	1531.10	36.5
	ABD	0.00	613.06	38.6	0.00	755.93	38.7	0.00	839.66	29.9	0.00	1372.98	33.9
9	Solver	0.00	837.11	–	0.00	1995.72	–	0.11	3340.58	–	1.45	3600*	–
	TBD	0.00	730.23	37.2	0.00	935.83	31.5	0.00	1260.52	30.8	0.00	1445.09	29.6
	ABD	0.00	870.76	34.5	0.00	862.95	27.9	0.00	1057.39	26.7	0.00	1187.31	25.4
10	Solver	0.00	438.08	–	0.02	1628.80	–	0.02	2453.07	–	0.25	3419.30	–
	TBD	0.00	323.73	33.9	0.00	598.13	31.7	0.00	1055.44	31.5	0.00	1318.84	32.6
	ABD	0.00	324.92	33.3	0.00	559.46	29.4	0.00	842.94	28.5	0.00	1193.71	29.3
11	Solver	0.00	488.98	–	0.00	981.57	–	0.00	2046.29	–	0.02	3441.95	–
	TBD	0.00	151.46	31.0	0.00	212.41	25.6	0.00	306.67	25.3	0.00	439.46	26.2
	ABD	0.00	147.37	27.7	0.00	198.26	22.3	0.00	303.81	22.6	0.00	455.59	24.4
12	Solver	0.00	732.77	–	0.00	1806.96	–	0.03	3600*	–	0.26	3600*	–
	TBD	0.00	733.32	39.6	0.00	651.79	30.6	0.00	1043.33	29.8	0.00	1290.18	29.1
	ABD	0.00	576.05	34.5	0.00	611.11	26.8	0.00	825.19	25.5	0.00	1129.70	24.9
13	Solver	0.05	1938.19	–	0.34	3600*	–	4.07	3600*	–	4.20	3600*	–
	TBD	0.00	2422.31	65.8	0.05	3578.91	53.6	0.43	3600*	43.8	1.72	3600*	37.2
	ABD	0.00	1747.20	52.7	0.04	3313.95	51.5	0.34	3600*	42.4	1.44	3600*	33.6
14	Solver	0.00	905.16	–	0.00	2682.87	–	0.32	3600*	–	1.34	3600*	–
	TBD	0.00	617.21	28.1	0.00	1375.98	25.6	0.00	1806.77	24.0	0.00	2594.65	24.6
	ABD	0.00	492.04	21.8	0.00	1234.17	21.2	0.00	1511.06	19.1	0.00	2071.29	19.3
15	Solver	0.01	2697.94	–	2.07	3600*	–	2.62	3600*	–	2.87	3600*	–
	TBD	0.01	3053.00	53.3	0.20	3600*	39.0	1.50	3600*	27.9	2.85	3600*	22.8
	ABD	0.00	2104.27	46.7	0.08	3600*	42.1	0.35	3600*	31.9	0.53	3600*	28.4

* Time limit was reached

Appendix 3. Computational results about the two types of lead time

Table A4. Impacts of lead time on cost incurred in supply chain with the ODWS.

			Delivery		Commitment		Stockout		Supplier investment		Transportation		Inventory holding	
L_s	L_d	Total cost ($)	Cost ($)	%	Cost ($)	%	Cost ($)	%	Cost ($)	%	Cost ($)	%	Cost ($)	%
0	0	12,830.6	7849.4	61.18	2278.4	17.76	93.0	0.72	1134.8	8.84	1368.0	10.66	107.1	0.83
0	1	14,246.8	7326.8	51.43	2235.7	15.69	2191.4	15.38	1134.8	7.97	1271.4	8.92	86.8	0.61
0	2	15,220.2	6799.9	44.68	1986.2	13.05	3958.3	26.01	1134.8	7.46	1266.2	8.32	74.9	0.49
0	3	16,548.1	6270.3	37.89	1704.8	10.30	6072.4	36.70	1134.8	6.86	1285.0	7.77	80.8	0.49
0	4	17,992.7	5725.5	31.82	1593.4	8.86	8321.8	46.25	1134.8	6.31	1157.7	6.43	59.5	0.33
1	0	13,935.3	8140.5	58.42	2278.4	16.35	820.1	5.89	1134.8	8.14	1482.9	10.64	78.7	0.56
1	1	15,287.0	7618.1	49.83	2166.9	14.17	2894.1	18.93	1134.8	7.42	1409.0	9.22	64.0	0.42
1	2	16,284.1	7075.4	43.45	1900.2	11.67	4720.8	28.99	1134.8	6.97	1400.1	8.60	52.8	0.32
1	3	17,627.9	6527.6	37.03	1656.4	9.40	6898.1	39.13	1134.8	6.44	1350.3	7.66	60.7	0.34
1	4	19,163.1	6000.0	31.31	1450.2	7.57	9233.1	48.18	1134.8	5.92	1302.4	6.80	42.7	0.22
2	0	14,475.1	8159.9	56.37	2198.1	15.19	1433.5	9.90	1134.8	7.84	1484.6	10.26	64.2	0.44
2	1	16,009.7	7611.5	47.54	1900.2	11.87	3819.7	23.86	1134.8	7.09	1482.2	9.26	61.3	0.38
2	2	16,989.9	7081.0	41.68	1693.9	9.97	5589.3	32.90	1134.8	6.68	1436.6	8.46	54.3	0.32
2	3	18,262.0	6542.5	35.83	1545.0	8.46	7690.3	42.11	1134.8	6.21	1304.0	7.14	45.3	0.25
2	4	19,733.2	6021.5	30.51	1353.1	6.86	9844.6	49.89	1134.8	5.75	1324.7	6.71	54.3	0.28
3	0	15,412.1	8127.7	52.74	2049.2	13.30	2677.9	17.38	1134.8	7.36	1353.7	8.78	68.9	0.45
3	1	16,782.2	7599.4	45.28	1774.2	10.57	4837.7	28.83	1134.8	6.76	1374.1	8.19	62.0	0.37
3	2	17,884.8	7055.7	39.45	1550.7	8.67	6782.4	37.92	1134.8	6.35	1306.5	7.31	54.7	0.31
3	3	18,924.5	6547.9	34.60	1359.1	7.18	8500.7	44.92	1134.8	6.00	1324.1	7.00	58.1	0.31
3	4	20,331.2	6030.3	29.66	1290.3	6.35	10,614.5	52.21	1134.8	5.58	1215.6	5.98	45.8	0.23
4	0	16,271.7	8112.0	49.85	1837.2	11.29	3799.9	23.35	1134.8	6.97	1320.2	8.11	67.6	0.42
4	1	17,830.6	7574.2	42.48	1725.8	9.68	6091.1	34.16	1134.8	6.36	1243.5	6.97	61.3	0.34
4	2	18,646.1	7056.9	37.85	1482.0	7.95	7721.5	41.41	1134.8	6.09	1195.9	6.41	55.1	0.30
4	3	19,696.8	6542.8	33.22	1349.6	6.85	9451.2	47.98	1134.8	5.76	1163.1	5.90	55.4	0.28
4	4	20,991.6	6022.4	28.69	1221.0	5.82	11,488.2	54.73	1134.8	5.41	1076.3	5.13	49.0	0.23