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불확실한 수명주기의 제품에서의 경제적 주문량 모형

An Economic Order Quantity Model under Random Life Cycle

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Abstract

This paper considers an Ecnomic Order Quantity Model under random life cycle. It is assumed that the life cycle of the product is unknown; a random variable. Three cost parameters are considered; ordering cost, inventory carrying cost and salvage cost. Expected total cost is the optimization criterion. We show that the optimal cycle length is unique and finite, and present a simple line search method to find an optimal cycle length.

1. INTRODUCTION

Inventory is the stock of any commodity a company keeps to be used in the company's output. Inventorying of a commodity should be justified by benefits accruing from one or more functions served by inventories. Inventories are used to serve a variety of functions, chief of which are those related to economies of scale in production and procurement, to fluctuating requirements over time, to production smoothing, and to improving customer service. The problem of determining the most desirable order quantity under rather stable conditions is commonly known as the classical Economic Order Quantity(EOQ) problem.

There have been extensive discussions in the literautre for possible extensions of the basic EOQ Model to improve the practicality of the model. Cheng [2] studied an EOQ model with demand-dependent unit production cost and imperfect production processes. Lev *et al.* [8] considered an EOQ model in which one or more of the cost or demand parameters will change at some time in the future. Porteus [9, 10] developed an extension of the EOQ model in which the setup cost is veiwed as a decision variable, rathr than as a parameter, and the cost of selecting different values of the setup is included in the formulation explicitly. Trippi and Lewin [11] adopted the Discounted Cash-Flows(DCF) approach for the analysis of the basic EOQ model. Kim *et al.* [7] extended Trippi and Lewin's [11] work by applying the DCF approach to various inventory systems. Chung [3] studied the DCF approach for the

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analysis of the basic EOQ model in the presence of the trade credit. Gurnani [5] applied the DCF approach to the finite planning horizon EOQ model in which the planning horizon is a given constant. Gurnani [6]claimed that an infinite planning horizon does not exist in real life, and a finite horizon inventory model (accounting for the time value of money) is theoretically superior and of greater practical utility. Chung and Kim [4] proved that Gurnani's [5] model is essentially identical to an infinite planning horizon model since the planning horizon is assumed to be a given constant. They also suggested that the assumption of the infinite planning horizon is not realistic, and called for a new model which relaxes the assumption of the infinite planning horizon.

We consider a finite planning horizon EOQ model where the planning horizon is dependent on the product life cycle which is a random variable. The performance measure is the expected total cost. We prove that the objective function has a unique extreme point which results in an optimal cycle length. The optimal cycle length can be easily obtained by a line search. We also show how the optimal cycle length behaves as the parameters change. A Numerical example is included to illustrate the model and solution procedure.

2. MODELLING

The following notations are used;

Q = the order quantity
T = the cycle length
P = the product life cycle(random variable)
D = the demand rate per year
S = the ordering cost per order
h = the inventory carrying cost per unit per year
c = the salvage cost per unit

The assumptions are same as the basic EOQ model except the followings:

1. The product life cycle of the product P is a random variable which follows an exponential distribution with parameter λ.
2. At the end of the product life cycle, the remnant inventories, if any, can be sold instantaneuosly at a discount price, i.e. there is a salvage cost associated with each remnant inventory.

In order to compute the expected total cost, we proceed as follows; Suppose that the product life cycle P fully accommodates first k cycles, and ends during $(k+1)$th cycle(See Figure 1), then cost per cycle until the end of k

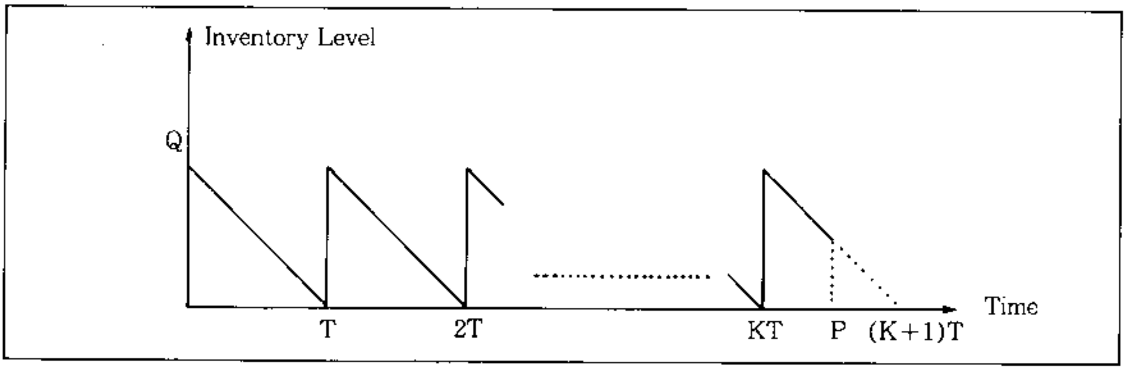


Figure 1. The Relationship between Product Life Cycle and Inventory Level

th cycle is the sum of an ordering cost and holding cost per cycle, that is,

$$S+\frac{hDT^2}{2}$$

Consequently, total cost up to the end of kth cycle is

$$kS+\frac{khDT^2}{2} \tag{1}$$

The total cost during the last cycle, i.e. $(k+1)$ th cycle can be obtained by summing up an ordering cost, inventory carrying costs, and salvage costs for all remnant inventories:

$$S+hD\left[T(P-kT)-\frac{(P-kT)^2}{2}\right] + cD[T-(P-kT)] \tag{2}$$

Now we can compute expected total cost from Equation (1) and (2). Since the product life cycle P follows an expontial distribution with parameter λ, the expected total inventory carrying costs, ordering costs and salvage costs, say $C(T)$, is

$$C(T)=\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}[(1)+(2)]\lambda e^{-\lambda P}\,dP \tag{3}$$

$C(T)$ can be represented as follows:

$$\begin{aligned}C(T)=&(S+cDT)\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}\lambda e^{-\lambda P}dP\\&+\left(S+\frac{hDT^2}{2}\right)\sum_{k=0}^{\infty}k\int_{kT}^{(k+1)T}\lambda e^{-\lambda P}dP\\&+(hDT-cD)\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}(P-kT)\lambda e^{-\lambda P}dP\\&-\left(\frac{hD}{2}\right)\sum_{k=0}^{\infty}\int_{kT}^{(k+1)T}(P-kT)^2\lambda e^{-\lambda P}dP\end{aligned} \tag{4}$$

We can use the following equations to simplify Equation (4)

$$\sum_{k=0}^{\infty}\left[\lambda e^{-\lambda kT}-e^{-\lambda(k+1)T}\right]=1$$

$$\sum_{k=0}^{\infty}k\left[\lambda e^{-\lambda kT}-e^{-\lambda(k+1)T}\right]=\sum_{k=0}^{\infty}e^{-\lambda(k+1)T}=\frac{1}{e^{\lambda T}-1}$$

$$\int_{kT}^{(k+1)T}(P-kT)\lambda e^{-\lambda P}dP=\frac{1}{\lambda}e^{-\lambda(k+1)T}(e^{\lambda T}-\lambda T-1)$$

$$\int_{kT}^{(k+1)T}(P-kT)^2\lambda e^{-\lambda P}dP=\frac{1}{\lambda^2}e^{-\lambda(k+1)T}(2e^{\lambda T}-2\lambda T-2-\lambda^2T^2)$$

Consequently, $C(T)$ can be simplified as follows;

$$\begin{aligned}C(T)=&S+cDT+\left(S+\frac{hDT^2}{2}\right)\frac{1}{e^{\lambda T}-1}\\&+\frac{(hDT-cD)(e^{\lambda T}-\lambda T-1)}{\lambda(e^{\lambda T}-1)}\\&-\frac{hD(2e^{\lambda T}-2\lambda T-2-\lambda^2T^2)}{2\lambda^2(e^{\lambda T}-1)}\\=&S+cDT\\&+\frac{[S\lambda^2+D(h\lambda Te^{\lambda T}-\lambda ce^{\lambda T}+\lambda c+\lambda^2cT-he^{\lambda T}+h)]}{\lambda^2(e^{\lambda T}-1)}\end{aligned} \tag{5}$$

Proposition 1.

(a) $C(T)$ has a unique local minimum on $[0,\infty]$.

(b) The optimal cycle length, T^*, satisfies

$$e^{\lambda T^*}-\lambda T^*-1=\frac{S\lambda^2}{D(h+\lambda C)} \tag{6}$$

Proof. (a) The first derivative of $C(T)$, say $C'(T)$, is as follows:

$$\begin{aligned}C'(T)=&cD\\&+\frac{[-S\lambda^3e^{\lambda t}+\lambda D(-\lambda hTe^{\lambda T}+\lambda ce^{\lambda T}-\lambda c-\lambda^2cTe^{\lambda T}+he^{2\lambda T}-he^{\lambda T})]}{\lambda^2(e^{\lambda T}-1)^2}\\=&\frac{e^{\lambda T}[\lambda cDe^{\lambda T}-\lambda cD-S\lambda^2+D(-\lambda hT-\lambda^2cT+he^{\lambda T}-h)]}{\lambda(e^{\lambda T}-1)^2}\end{aligned}$$

Let

$$f(T)=\lambda cDe^{\lambda T}-\lambda cD-S\lambda^2+D(-\lambda hT-\lambda^2cT+he^{\lambda T}-h),$$

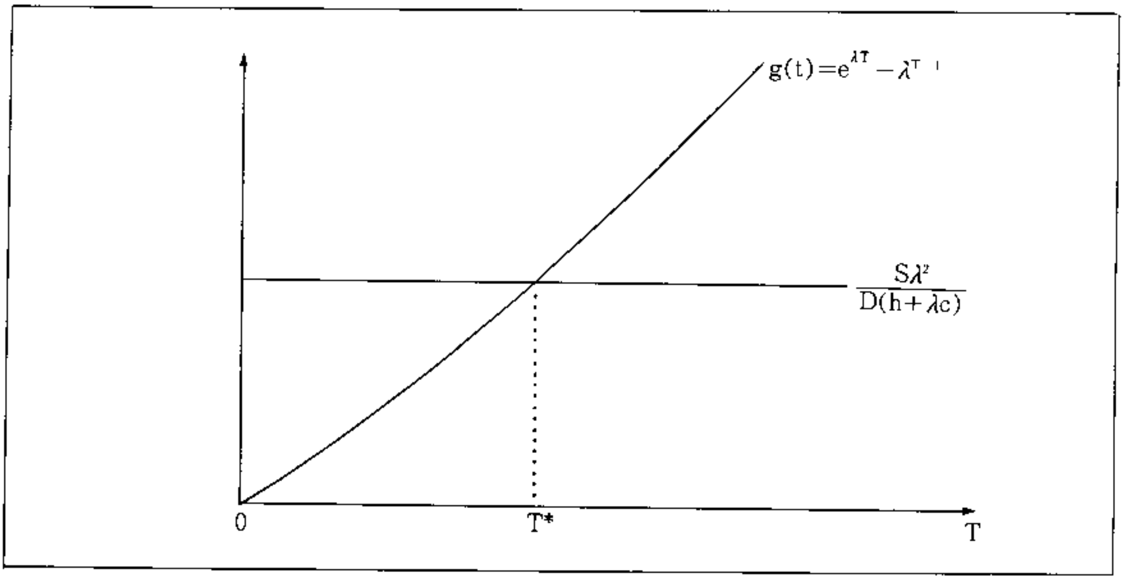
then $C'(T)$ has the same sign as $f(T)$. $f(T)$ is a strictly increasing function since

$$f'(T)=\lambda^2cDe^{\lambda T}-\lambda hD-\lambda^2cD+\lambda he^{\lambda T}=\lambda D(h+\lambda c)(e^{\lambda T}-1)>0.$$

Since $f(0)=-S\lambda^2<0$, and $f(\infty)>0$, there exists a unique local minimum on $(0, \infty]$.
(b) From $C(0)=C(\infty)=\infty$ and the result of (a), the optimum cycle length T^* satisfies $C'(T)=0$. The equation $C'(T^*)=0$ can be rewritten as Equation (6).

Corollary 1. The optimal cycle length increases (decreases) as ordering cost increases (decreases). It decreases (increases) as holding cost and/or demand rate increases (decreases).

Proof. Let the left hand side of Equation (6) as $g(T)$. Note that $g(T)$ is a strictly increasing function of T since $g'(T)=\lambda(e^{\lambda T}-1)>0$ for all $T>0$. The optimal cycle length is an intersection of $g(T)$, which is strictly increasing, and $\frac{S\lambda^2}{D(h+\lambda C)}$ (See Figure 2.) The results follow directly from the behavior of the optimal cycle length as $\frac{S\lambda^2}{D(h+\lambda C)}$ changes.



Fogure 2. The Behavior of the Optimal Cycle Lenght

From Proposition 1 and Corollary 1, the optimal cycle length can be easily obtained by an one-dimensional line search method. Then, an optimal lot size is computed using $Q^*=DT^*$.
(*Remark*)If $\lambda T<<1$, that is

$$\frac{T}{1/\lambda}=\frac{\text{cycle length}}{\text{average product life cycle}}<<1,$$

then we can substitute

$$e^{\lambda T}\approx 1+\lambda T+\frac{\lambda^2 T^2}{2}$$

into Equation (6) to get an approximate optimal cycle length $\hat{T}$ and an approximate lot size $\hat{Q}$ as follows:

$$\hat{T}=\sqrt{\frac{2S}{D(h+\lambda c)}}$$

$$\hat{Q}=\sqrt{\frac{2DS}{h+\lambda c}}$$

3. A NUMERICAL EXAMPLE

Suppose $S=\$200$ per order, $D=1{,}000$ units per year, $h=\$10$ per unit per year, $c=\$20$ per unit, and $\lambda=0.5$. Then, an optimum cycle length 0.1398 years is obtained using the algorithm, and an optimum expected total cost is $\$3508$.

4. CONCLUSION

We have studied the basic EOQ model in which the finite planning horizon is a random variable. This approach improves the practicality of the assumption on the planning horizon. We proved that there exists a unique local minimum which becomes a global optimal solution. A simple line search is used to find out the optimal solution. One interesting area of extending this study is to apply the random planning horizon approach to the other types of inventory systems, which we are currently investigating.

REFERENCES

1. Carlson, M.L., and Rousseau, J.L., "EOQ under Date-terms Supplier Credit," *Journal of the Operational Research Society*, Vol.40, pp.451-460, 1989.
2. Cheng, T.C., "An Economic Order Quantity Model with Demand-dependent Unit Production Cost and Imperfect Production Process," *IIE Transactions*, Vol.23, pp.23-28, 1991.
3. Chung, K.H., "Inventory Control and Trade Credit Revisited," *Journal of the Operational Research Society*, Vol.40, pp.495-498, 1989.
4. Chung, K.H., and Kim, Y.H., "Economic Analysis of Inventory Systms: A Rejoinder," *Engineering Economist*, Vol.35, pp. 75-80, 1989.
5. Gurnani, C., "Economic Analysis of Inventory Systems," *International Journal of Production Research*, Vol.21, pp.261-277, 1983.
6. Gurnani, C., "Economic Analysis of Inventory Systems: A Reply," *International Journal of Production Research*, Vol.23, pp.771-772, 1985.
7. Kim, Y.H., Philippatos, G.C. and Chung, K. H., "Investments in Inventory Systems: A Net Present Value Framework," *Engineering Economists*, Vol.31, pp.119-136, 1986.
8. Lev, B., Weiss, H.J., and Soyster, A.L., "Optimal Ordering Policies When Anticipating Parameter Changes in EOQ Systems", *Naval Research Logistics Quaterly*, Vol.2, pp.267-279, 1981.
9. Porteus, E.L., "Investing in Reduced Sctmps in the EOQ Model," *Management Sciences*, Vol.31, pp.998-1010, 1985.
10. Porteus, E.L., "Investing in New Parameter Values in the Discounted EOQ Model," *Naval Research Logistics Quarterly*, Vol. 33, pp.39-48, 1986.
11. Trippi, R.R., and Lewin, D.E., "A Present Value Formulation of the Classical EOQ Problem," *Decision Secience*, Vol.5, pp.30 -35, 1974.