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Hybrid NSGA-II for an imperfect production system considering product quality and returns under two warranty policies

Ata Allah Taleizadeh [a], Pooya Pourrezaie Khaligh [a], Ilkyeong Moon [b,*]

[a] School of Industrial Engineering, Collage of Engineering, University of Tehran, Tehran, Iran
[b] Department of Industrial Engineering, Seoul National University, Seoul 08826, South Korea

HIGHLIGHTS

- We developed a new bi-objective (profit and customer satisfaction) model for an imperfect production system.
- A non-dominated sorting genetic algorithm was developed.
- Uncertain customer demand is modeled as stochastic consumer behavior.
- Two optional pro-rata warranty and free replacement/repair warranty policies are considered.

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ABSTRACT

This study presents a new bi-objective mathematical model for an imperfect production system (IPS) that accounts for product returns and quality levels under two different warranty policies. We considered uncertain customer demand as stochastic behavior and product price as a function of return compensation, product quality level, and warranty-period length. We simultaneously looked at the pro rata warranty and free replacement/repair warranty policies and assumed that customers can choose the desired policy. A return policy for an online distributor was also included and two objective functions were used to address the problem. The first objective function maximized the total expected revenue, and the second objective function maximized customer satisfaction. The non-dominated sorting genetic *algorithm* (NSGA-II) and two others, the basic bee metaheuristic and teaching-learning-based optimization algorithms, were used to generate the initial solution for use in the NSGA-II algorithm. The results from the hybrid algorithms revealed that the proposed method improves the performance of the NSGA-II algorithm. Finally, several specific sensitivity analyses were conducted to determine the effects of the problem parameters.

1. Introduction

In this section, we have presented brief descriptions of the various dimensions of the proposed problem. Also, we have reviewed some related works in each category.

1.1. Imperfect production system

In this study, we considered an imperfect production system (IPS) in which two states were defined for a production system. In the normal state, the system is controlled but could shift to an out-of-control state. In the controlled state, the production of incompatible products is increased. Ben-Daya [1] developed a new mathematical model for integrating the combined determinations of an economic production quantity (EPQ) model and suggested a preventive maintenance level for the IPS. Chung and Hou [2] extended the basic EPQ model by considering an IPS for determining the optimal value for the run time. They showed that only one value for the length of a production process was optimal for minimizing the total cost of the proposed problem. A new mathematical model was developed by Sana [3] for finding the optimal product reliability and rate of production. To set up the problem, Sana considered the IPS. Pearn et al. [4] proposed a mathematical model for a two-stage production process that accounts for an IPS. Sarkar and Moon [5] analyzed the effect of inflation rate on a production inventory model. They considered stochastic demand and an IPS to explain the problem. Sarkar and Saren [6] presented an EPQ model that features process deterioration that leads to the production of imperfect items. To decrease the cost of product inspection, they proposed a new policy for selective, instead of comprehensive, inspections of products.

* Correspondence to: Department of Industrial Engineering and Center for Sustainable & Innovative Industrial Systems, Seoul National University, Seoul 08826, Korea.
E-mail address: ikmoon@snu.ac.kr (I. Moon).

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1.2. Warranty policies

The warranty period is a factor that exerts a great influence on customer decisions about buying a product. Several warranty policies, each of them with specific features, are used to garner customer favor. In this study, we considered two warranty policies: the free replacement/repair warranty (FRW) and the pro rate warranty (PRW). Yeh and Lo [7] proposed the *preventive maintenance warranty* policy for repairable items. They showed that only one policy proved optimal for the problem under study. The preventive maintenance policy was further developed by Jung and Park [8], who combined two warranty policies. They aimed to determine the optimal length of a maintenance period. Wu et al. [9] developed a decision model for use by manufacturers in a stable demand market. They sought to determine the optimal values of production rate, warranty-period length, and production price. A system maintenance policy proposed by Jung et al. [10] for which a renewable warranty policy and product lifecycle were determined by customer outlook.

Chien [11] investigated a mathematical formulation for combining a FRW policy with a PRW policy and analyzed the effect of this combination on the basic *age-replacement* policy. A new mathematical model was proposed by Bouguerra et al. [12] to investigate the effects of a new warranty that was created by extending a classical warranty policy between a buyer and a manufacturer; they investigated the model under different maintenance policies. Also, Su and Wang [13] looked at an extended warranty for the buyer and manufacturer by extending the model proposed by Bouguerra et al. [12], and they introduced a preventive maintenance policy with a two-stage model by considering a two-dimensional extended warranty. Chen et al. [14] reviewed the problems related to a manufacturer pricing strategy recommended for use in a two-echelon supply chain that featured warranty period dependent demands. Esmaeili et al. [15] presented warranty contracts for a three-level supply chain (customer, manufacturer, and agent) for which different warranty policies were defined and chosen by customers. Liu et al. [16] used a renewable FRW policy to investigate a warranty-cost model for a multi-component system. Lee et al. [17] considered a heterogeneous population from which an item was randomly selected. They proposed a mathematical model in which a warranty policy is based on information randomly obtained from this population.

1.3. Return policies

Yalabik et al. [18] developed an integrated approach for analyzing the logistic and marketing decisions made to design a suitable returns system for a retailer or manufacturer. The main contribution of their study was the analysis of the implications of using an optimal refund policy. Min et al. [19] developed a multi-echelon reverse-logistic network in which some return centers were used to collect the returned items from retailers and customers. Collected items were sorted and returned to the manufacturer or distributor. The aim of the Min et al. [19]'s study was determination of the optimal number and best locations for return centers. They proposed a genetic algorithm (GA) to solve the proposed problem. Also, to determine best number and location of return centers, Diabat et al. [20] presented a new mathematical model for product return in a multi-echelon reverse logistic network. They used a GA and an artificial immune system to solve the model, and they compared results of algorithms.

De Brito and Van Der Laan [21] analyzed the effect of imperfect information on the performance of inventory management with regard to product returns. Shaharudin et al. [22] used institutional theory to develop some research propositions for product return management. Specifically, they showed that product-return management platforms can help manufacturers to create a sustainable supply chain network. General assumptions of inventory modeling, shown by De Brito and Dekker [23], accounted for the return flows, and real data on these flows were analyzed. Zhou et al. [24] evaluated the impact of the return policy on the performance of a dynamic system in a three-echelon closed-loop supply chain.

1.4. Quality level

Ramezani et al. [25] proposed a stochastic multi-objective model for a reverse logistic network design considering uncertainty. They considered suppliers, plants, and distribution centers in the forward direction. Collection and disposal centers were considered in the backward direction. They obtained a closed-form solution for an optimal threshold and costs. They proposed a mathematical model that consisted of three objective functions. They considered the sigma quality level as an objective function and tried to increase this objective function by minimizing the total number of defective raw materials. Li et al. [26] developed several models to examine the impact of the return policy, quality of the product, and strategy of the pricing on the customers' purchases and decisions, which are related to the return policy. Their findings indicated that decisions related to the return policy are consistent with the quality of product and pricing strategies.

Jin et al. [27] proposed an assembly system of the modular product for using module returns by considering uncertainties in terms of timing, quantity, and quality in the remanufacturing environment. They analyzed the structure of an optimal policy and proposed a heuristic approach for comparing the performance of it with that of an optimal approach. Maiti and Giri [28] considered a *two-way product recovery* in a *two-echelon closed-loop supply chain* that consisted of one retailer and one manufacturer who trade a single product. They assumed that the market demand was a function of the selling price and quality of the product. They applied three decentralized game structures to analyze the proposed model, and compared the applied approach to find the best policy. Moreover some related researches can be found in Taleizadeh et al. [29,30]. A comparative analysis of published research, which shows the gap in the literature, is presented in Table 1.

To our knowledge, no study has been published that simultaneously takes into account all the features presented in Table 1. To fill this gap, we studied an IPS considering product quality levels and returns under two warranty policies. We solved the problem with three Metaheuristic algorithms. The main contributions of this study are the simultaneous considerations of

- the price of the product as a function of the warranty-period length, product quality level, and return compensation;
- two warranty policies offered to customers, FRW and PRW; and
- two hybrid metaheuristic algorithms to solve the problem.

The rest of this paper is structured as follows: The problem definition and formulation are provided in Section 2. The solution methods for the developed mathematical model are described in Section 3. The computational and practical results are given in Section 4 as are the parameter settings. The sensitivity analysis is explained in Section 5. Finally, Section 6 presents concluding remarks and directions for future study.

2. Mathematical model

2.1. Problem definition

We looked at the IPS and online direct selling for one company. Specifically, we considered the product selling price as a function of return compensation amount, quality level, and the warranty-period length. Customers choose the desired warranty

Table 1
A comparative analysis of research on imperfect systems, warranties, and details of products and returns.

Reference	imperfect system	warranty		return	quality level	demand/return	
		one policy	multiple policy			deterministic	uncertain
Yalabik et al. [18]				✔		✔	
Chen and Lo [31]				✔		✔	
Min et al. [19]	✔	✔				✔	
Wu et al. [9]		✔				✔	
De Brito and Van Der Laan [21]				✔			✔
Jung et al. [10]		✔					
Pearn et al. [4]	✔					✔	
Sarkar and Moon [5]	✔						✔
Chen et al. [14]		✔					
Ramezani et al. [25]					✔		✔
Esmaeili et al. [15]			✔			✔	
Sarkar and Saren [6]	✔	✔				✔	
Lee et al. [17]		✔					✔
Taleizadeh et al. [32]		✔				✔	
This study	✔		✔	✔	✔		✔

policy among PRW and FRW policies. In addition to the warranty policy, we examined the return policy for an online distributor. We assumed that the returned quantity of the product is a function of quality level and return compensation. We sought to determine the optimal warranty-period length, duration of the production process, quality level, and return compensation amounts that maximize total profit. Also, we looked at customer satisfaction as a second objective function. We used standardization and customization concepts to calculate customer satisfaction. Fig. 1. presents the flow of information between a factory and customers.

The following assumptions characterize our study:

- The rate of production is deterministic, but the rate of demand is uncertain and shows stochastic behavior with normal distribution.
- The price of a constructed product is a function of return compensation amount, quality level, and warranty period.
- Shortages are allowed.
- The product lifetime follows an exponential distribution.
- Customers choose one of the following warranty policies:

  1. Under the FRW, no fees are charged to the purchasing customer for the replacement or repair of a product that fails within the warranty period. The related costs for a free-replacement warranty policy are incurred by the manufacturer, which must rectify the problems related to a defective product.
  2. Under the PRW, the product is replaced at a cost that depends on the portion of the lifetime that had elapsed at the time of failure. This type of warranty covers part of the initial cost incurred by the customer.

Table 2 lists the notations used in this paper.

2.2. Formulation

The price of each constructed product is a function of the return compensation amount, quality level of the product, and the warranty-period length for both types of proposed warranty policies and return compensation, we developed the formulation used in [33]. Eq. (1) shows this function.

$$P(w_1, w_2, q, r) = \left[\frac{(s_1+q)^{b_1} + (s_2+w_1)^{b_2} + (s_3+w_2)^{b_3}}{d}\right]^{\frac{1}{\alpha}} + \omega r \tag{1}$$

where $s_1, s_2, s_3 > 0, \alpha > 0$ and $0 < b_1, b_2, b_3 < 1$

R denotes the returned quantity, and a base return quantity is denoted by ϕ, which is independent of refund and quality factors. The sensitivity of the return quantity to the return policy is represented by ρ: When the return compensation, r, increases, the return quantity, R, increases. ϕ denotes the sensitivity of the return quantity to quality: By increasing the quality, q, the return quantity, R, decreases, we employed the formulation in [26]. Eq. (2) explains the returned quantity.

$$R = \upsilon + \rho r - \phi q \tag{2}$$

By multiplying the return quantity (R) by the return compensation, the return cost is obtained. Eq. (3) shows the return cost as follows:

$$RC_1 = rR \tag{3}$$

By multiplying the price $P(w)$ by pt(production quantity per cycle), the expected revenue per cycle (RPC) is obtained. To tackle the stochastic nature of demand, the expectation of demand, which is the mean of the normal distribution, was replaced by d. Eq. (4) shows the total expected revenue.

$$\begin{aligned} RPC &= E\left[P(w_1, w_2, q, r)\right] \\ &= \left(\left[\frac{(s_1+q)^{b_1} + (s_2+w_1)^{b_2} + (s_3+w_2)^{b_3}}{E\left[d\right]}\right]^{\frac{1}{\alpha}} + \omega r\right) pt \end{aligned} \tag{4}$$

Production cost is calculated by Eq. (5). S denotes the setup cost per cycle, and $c \times p \times t$ is the total production cost per cycle. This formulation can be referred to [34]

$$PC = S + cpt \tag{5}$$

Under the assumptions described, the rate of demand is stochastic. Hence, to obtain the value of the holding cost, an integral that shows the expected value of the inventory during the length of the production run is calculated. In this equation, $f_d(d)$ is the distribution function of the demand rate. By multiplying the integral value by the holding cost (h), the expected inventory holding cost is obtained and is shown as follows in Eq. (6):

$$HC = h\left(\int_{d=0}^{d=p} (p-d)t\, f_d(d) d_d\right) \tag{6}$$

Like the calculations for the expected inventory holding cost, the shortage cost is also determined. Eq. (7) shows the expected shortage cost as

$$SC = \pi\left(\int_{d=p}^{\infty} (d-p)t\, f_d(d) d_d\right) \tag{7}$$

The expected restoration cost is calculated as follows:

$$RC_2 = rs\left[P\left(Y < t\right)\right] = rs\left(1 - e^{-ut}\right) \tag{8}$$

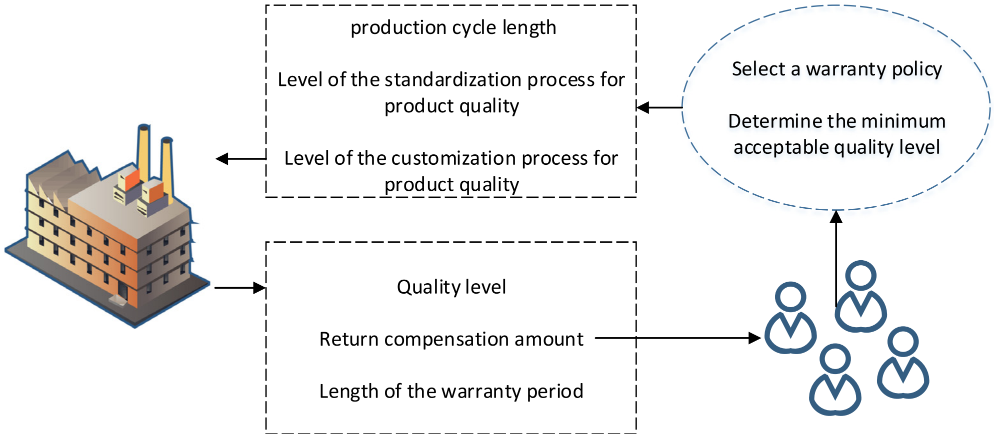

Fig. 1. Flow of information between a factory and customers.

Table 2
Notations.

Decision variables	
t	length of the production cycle
w_1	warranty-period length under free-replacement warranty policy
w_2	warranty-period length under pro rata warranty policy
q	quality level
r	return compensation
τ_s	level of standardization quality, $\tau_s > 0$
τ_c	level of customization quality, $\tau_c > 0$
Parameters:	
S	setup cost
d	rate of customer demand
p	rate of production
c	cost of product unit
h	holding cost
π	shortage cost
rs	restoration cost when systems is in an uncontrolled state
C_w	related cost to free-replacement warranty policy
λ	cost related to quality improvement
B	maximum budget
M	maximum capacity of return
υ	base return quantity
ω	sensitivity rate of price to return compensation
ρ	sensitivity rate of return quantity to return policy
ϕ	sensitivity rate of return quantity to quality
Y	random variable to show the time passed from a system in a controlled state until it shifts to an uncontrolled state.
$h(y)$	probability density function of Y, assumed to be an exponential distribution with parameter u
u	parameter of the exponential distribution $h(y)$
y_1	probability of incompatible products when the production process is in a controlled state
y_2	probability of incompatible products when the production process is in an uncontrolled state; $0 < y_1 < y_2 < 1$
α_1	expected post-sale incompatible product lifetime for a lifetime that follows an exponential distribution with parameter α_1
α_2	expected a post-sale compatible product lifetime for a lifetime that follows an exponential distribution with parameter α_2, $\alpha_2 > \alpha_1$
β	percentage of customers who choose the free-replacement warranty
ξ	relative importance to customers of customization process for product quality, $0 < \xi < 1$
δ_s	linear effect of the standardization process for product quality levels on costs, $\delta_s > 0$
η_s	acceleration rate of expected costs for standardization process of product quality, $\eta_s > 0$
η_c	acceleration rate of expected costs for standardization process of product quality, $\eta_c > 0$
θ	sensitivity rate of standardization and customization costs to level of standardization and customization quality

where $P(Y < t)$ denotes the probability that the time passed during a shift of the system from a controlled state to an uncontrolled state is less than the length of the production cycle. By multiplying this probability by the restoration cost, the expected restoration cost is obtained.

According to the assumptions, the lifetimes of incompatible and compatible products are exponentially distributed with parameters α_1 and α_2. The expected post-sale warranty cost under the FRW policy, which can be found in [34], is

$$WC_1 = \beta\, C_w\, pt\left(\left[y_2 - (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_1}{\alpha_1} + \left[1 - y_2 + (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_1}{\alpha_2}\right) \tag{9}$$

We proposed the following formulation to calculate the expected post-sale warranty cost under the PRW policy, for which (1 - β) is the percent of customers who choose the PRW:

$$WC_2 = (1 - \beta)P(w_1, w_2, q, r)E\left(\frac{w_2 - X}{w_2}\right) \times pt\left(\left[y_2 - (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_2}{\alpha_1} + \left[1 - y_2 + (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_2}{\alpha_2}\right) \tag{10}$$

where $E\left(\frac{w_2 - X}{w_2}\right)$ is the elapsed portion of the product lifetime at the time of the failure.

Eq. (11) shows the related cost to quality of product which can be found in [26] as

$$QC = \lambda q^2 \tag{11}$$

It is assumed that customer satisfaction is a function of the standardization process of product quality and the customization process of product quality as shown in Eq. (12):

$$CS = (1 - \xi)\tau_s + \xi\tau_c \tag{12}$$

Standardization and customization costs are calculated by Eq. (13):

$$FC = -\delta_s\tau_s + \frac{\eta_s\tau_s^2}{2} + \frac{\eta_c\tau_c^2}{2} + \theta\tau_s\tau_c \tag{13}$$

Eq. (14) shows the total profit per cycle as

$$\begin{aligned} TPPC &= RPC - PC - HC - SC - RC_1 - WC_1 - WC_2 - RC_2 - QC - FC \\ &= \left(\left[\frac{(s_1 + q)^{b_1} + (s_2 + w_1)^{b_2} + (s_3 + w_2)^{b_3}}{E\,[d]}\right]^{\frac{1}{a}} + \omega r\right) \times pt - (S + cpt) \\ &\quad - h\left(\int_{d=0}^{d=p}(p - d)t\, f_d(d)d_d\right) - \pi\left(\int_{d=p}^{\infty}(d - p)t\, f_d(d)d_d\right) - rs\left(1 - e^{-ut}\right) \\ &\quad - \beta\, C_w\, pt\left(\left[y_2 - (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_1}{\alpha_1} + \left[1 - y_2 + (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_1}{\alpha_2}\right) \\ &\quad - (1 - \beta)P(w_1, w_2, q, r)E\left(\frac{w_2 - X}{w_2}\right) \times pt\left(\left[y_2 - (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_2}{\alpha_1} + \left[1 - y_2 + (y_2 - y_1)\frac{1 - e^{-ut}}{ut}\right]\frac{w_2}{\alpha_2}\right) - r(\upsilon + \rho r - \phi q) - \lambda q^2 \\ &\quad + \delta_s\tau_s - \frac{\eta_s\tau_s^2}{2} - \frac{\eta_c\tau_c^2}{2} - \theta\tau_s\tau_c \end{aligned} \tag{14}$$

$e^{-ut} \cong 1 - ut + \frac{u^2t^2}{2}$Taylor' s expansion => Using Taylor's expansion, Eq. (14) becomes

$$\begin{aligned} TPPC = &\left(\left[\frac{(s_1+q)^{b_1}+(s_2+w_1)^{b_2}+(s_3+w_2)^{b_3}}{E[d]}\right]^{\frac{1}{a}} + \omega r\right)pt - (S + cpt) \\ &-h\left(\int_{d=0}^{d=p}(p - d)t\, f_d(d)d_d\right) - \pi\left(\int_{d=p}^{\infty}(d - p)t\, f_d(d)d_d\right) \\ &-rs\left(ut - \frac{u^2t^2}{2}\right) \\ &-\beta\, C_w\, pt\left(\left[y_1 + \frac{(y_2-y_1)ut}{2}\right]\frac{w_1}{\alpha_1} + \left[1 - y_1 + \frac{(y_2-y_1)ut}{2}\right]\frac{w_1}{\alpha_2}\right) \\ &-(1 - \beta)P(w_1, w_2, q, r)E\left(\frac{w_2-X}{w_2}\right)pt\left(\left[y_1 + \frac{(y_2-y_1)ut}{2}\right]\frac{w_2}{\alpha_1}\right. \\ &\left.+\left[1 - y_2 + (y_2 - y_1)\frac{1-e^{-ut}}{ut}\right]\frac{w_2}{\alpha_2}\right) - r(\upsilon + \rho r - \phi q) - \lambda q^2 \\ &+\delta_s\tau_s - \frac{\eta_s\tau_s^2}{2} - \frac{\eta_c\tau_c^2}{2} - \theta\tau_s\tau_c \end{aligned}$$

Total profit per item is obtained by dividing *TPPC* over *pt* and shown in Eq. (15).

$$\begin{aligned} TPPC = &\left[\frac{(s_1+q)^{b_1}+(s_2+w_1)^{b_2}+(s_3+w_2)^{b_3}}{E[d]}\right]^{\frac{1}{a}} + \omega r - \frac{S}{pt} - c \\ &-h\left(\int_{d=0}^{d=p}\frac{(p-d)f_d(d)d_d}{p}\right) - \pi\left(\int_{d=p}^{\infty}\frac{(d-p)f_d(d)d_d}{p}\right) - \frac{rsu}{p} - \frac{rsu^2t}{2p} \\ &-\beta\, C_w\left(\left[y_1 + \frac{(y_2-y_1)ut}{2}\right]\frac{w_1}{\alpha_1} + \left[1 - y_1 + \frac{(y_2-y_1)ut}{2}\right]\frac{w_1}{\alpha_2}\right) \\ &-(1 - \beta)P(w_1, w_2, q, r)E\left(\frac{w_2-X}{w_2}\right)\left(\left[y_1 + \frac{(y_2-y_1)ut}{2}\right]\frac{w_2}{\alpha_1}\right. \\ &\left.+\left[1 - y_2 + (y_2 - y_1)\frac{1-e^{-ut}}{ut}\right]\frac{w_2}{\alpha_2}\right) - \frac{r(\upsilon+\rho r-\phi q)}{pt} - \frac{\lambda q^2}{pt} \\ &+\frac{\delta_s\tau_s}{pt} - \frac{\eta_s\tau_s^2}{2pt} - \frac{\eta_c\tau_c^2}{2pt} - \frac{\theta\tau_s\tau_c}{pt} \end{aligned} \tag{15}$$

The final mathematical model of the problem formulated is as follows:

$$Max\ TPPC = \frac{(RPC - PC - HC - SC - RC_1 - WC_1 - WC_2 - RC_2 - QC - FC)}{pt} \tag{16}$$

$$Max\ CS = (1 - \xi)\tau_s + \xi\tau_c \tag{17}$$

$s.t$

$$PC + HC + SC + RC_1 + WC_1 + WC_2 + RC_2 + QC + FC \le B \tag{18}$$

$$R \le M \tag{19}$$

The first objective function (16) maximizes the total expected profit and the second objective function (17) maximizes customer satisfaction. Eq. (18) explains the maximum available budget, and Eq. (19) shows the maximum capacity of return.

3. Solution method

In this section, we present the design of two metaheuristic methods used to solve the problem. One is the non-dominated sorting genetic *algorithm* II (NSGA-II) and the other is based on a hybrid method in which the output of one metaheuristic algorithm is provided for the NSGA-II as an initial population. The non-dominated sorting genetic *algorithm* has recently been applied to several areas including load balancing, supply chain design, and flexible manufacturing system [35–37]. The original bee algorithm (BA) and teaching-learning-based optimization (TLBO) algorithms were used as part of the NSGA II hybrid. This hybridization method makes NSGA-II a greedy algorithm.

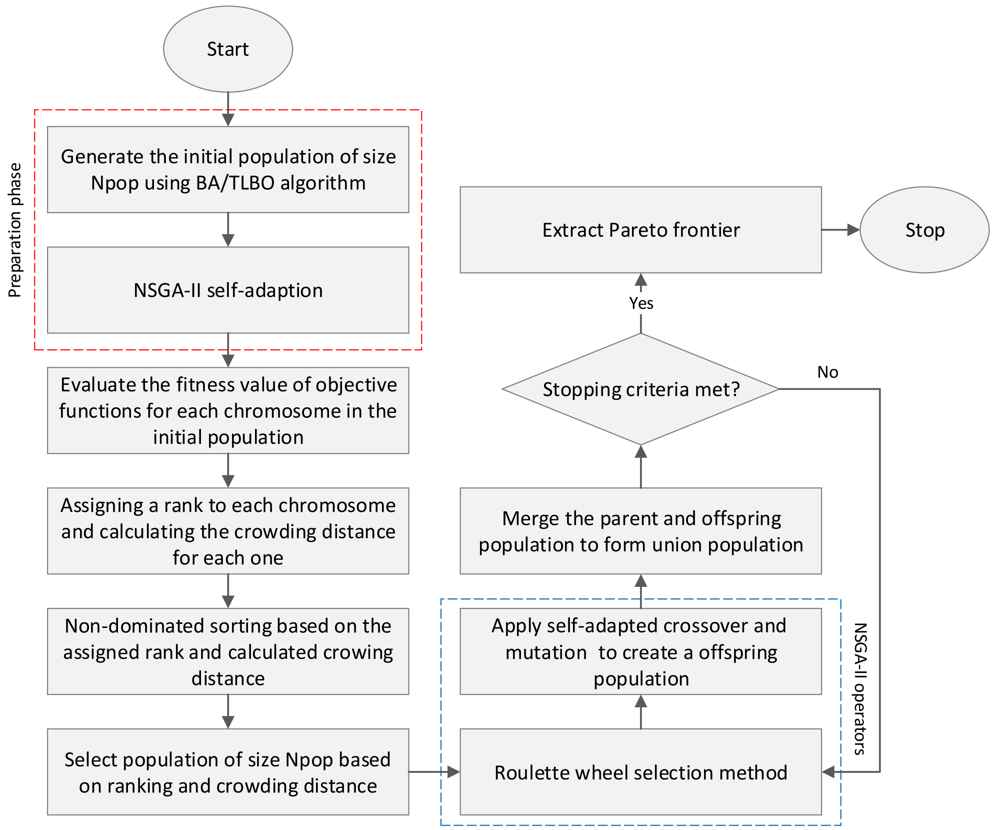

Fig. 2. Flowchart of the hybrid NSGA-II algorithm.

3.1. Multi-objective optimization

An optimization problem is one of the common and primary problems in areas such as engineering practice and scientific research [38]. Because of the number of objective functions, this type of problem can be separated in two types: the single objective optimization problem and multi-objective optimization problem (MOP), which involves a number of objective functions that are simultaneously maximized or minimized [38]. Like the single objective optimization problem, an MOP may also involve one or more constraints. In the MOP, objective functions are always contradictory to each other; therefore, a solution that is good for one objective function may be bad for another function. Finding the solution that satisfies all the objective functions is impossible. In this situation one set of feasible solutions is generated [38]. A common form of an MOP is as follows:

$$\min/\max\left(g_1(x), g_1(x), \ldots, g_{N_{obj}}(x)\right)$$

$$\begin{aligned} s.t:\quad & d_j(x) \geq 0 && j = 1, 2, \ldots, N_{const_1}; \\ & W_v(x) = 0 && v = 1, 2, \ldots, N_{const_2}; \\ & x_i^l \leq x_i \leq x_i^u && x_i \in x \quad i = 1, 2, \ldots, N; \end{aligned}$$

Where $g_k(k = 1, 2, \ldots, N_{obj})$ represents objective functions; $d_j(j = 1, 2, \ldots, N_{const_1})$ represents the inequality constraint functions; $W_v(v = 1, 2, \ldots, N_{const_2})$ represents the equality constraint functions; and x is a design vector consisting of n design variables, $x_j(j = 1, 2, \ldots, N)$. Also, x_i^l and x_i^u , $i = (1, 2, \ldots, N)$, are the lower bounds (LB) and upper bounds (UB) of the design variable [39].

3.2. Proposed NSGA-II

Deb et al. [40] introduced the NSGA-II, which has since become a well-known algorithm used in MOPs. The operators of the NSGA-II algorithm are the same as the genetic algorithm (GA) operators (i.e., mutation and crossover). Because the NSGA-II is an extension of a GA, the pseudocode for the NSGA-II is presented in Fig. 2 which is similar to that of a GA. As can be seen in this figure, in the preparation phase, the algorithm is started with an initial population of N_{pop} number of individuals (chromosomes) that had been generated by the other metaheuristic algorithm (BA/TLBO). Also, in this phase, the algorithm is run for a predefined number of iterations to obtain the self-adaption versions of mutation and crossover. After the preparation phase, the main loop of the NSGA-II is started. The fitness value of the objective function for each chromosome is evaluated, and a rank is assigned to each chromosome according to a non-dominated level. Rank one is assigned to the best solutions and so on. Also, the crowding distance is calculated for solutions of the same rank. This measure provides an approximation of the density of possible solutions along with a specific solution. The amount of crowding distance for a particular solution is the average distance from the two neighboring solutions for each objective function. Finally, the individuals are sorted based on the ranking and the calculated crowding distances in each Pareto front. The self-adaption version of mutation and crossover are applied to make the offspring populations and find a near-optimal Pareto solution. Also, the roulette wheel selection method is used to choose the individuals from a population. The algorithm will be stopped when the stopping criterion, which is the maximum number of iterations, will be met. In the last chosen population, the first rank solutions (individuals) are the Pareto optimal solutions of the proposed problem.

3.2.1. Solution representation

This study was based on 7 decision variables, so the chromosome consisted of 7 cells. Seven random numbers were generated in a range of [0,1] randomly, and each was assigned to one cell. These random numbers were transformed to numbers that represent the denoted value of each decision variable. Eq. (20) shows the

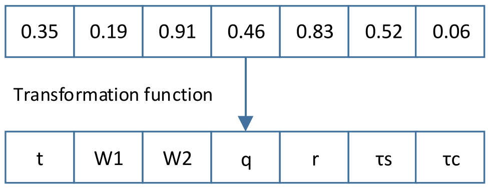

Fig. 3. Example of a chromosome.

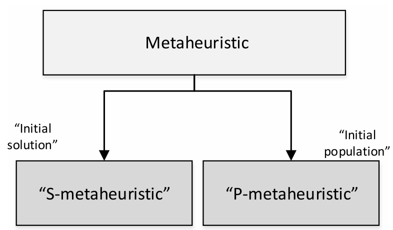

Fig. 4. High-level relay hybridization metaheuristic construction.

transformation function, and a simple example of a chromosome is shown in Fig. 3.

$$x = r \times (x_{\max} - x_{\min}) + x_{\min} \tag{20}$$

where r denotes the random number that should be transformed to a decision variable value, and x_{min} and x_{max} denote the bounds of the decision variable.

3.2.2. Initial solutions

In the population-based metaheuristic algorithms, the initial solution plays a great role in the efficiency of algorithm. Hence, high-level relay (HRH) hybridization Metaheuristic algorithms or greedy heuristic algorithms are applied to generate initial solutions of a population according to single Metaheuristic algorithms (P or S-Metaheuristic). Fig. 4 shows the HRH construction [41]. This method can generate a better initial population.

As described at the beginning of this section, under "Solution Method," we used basic BA and TLBO algorithms to construct an initial solution for the NSGA-II. For constructing the initial populations for the NSGA-II, a single objective version of the model (only the first objective function was considered) was solved by the BA and the TLBO. The algorithms were run for a predefined number of iterations, which were necessarily greater than the initial population of the NSGA-II. The n_{pop} (number of best iterations) of the BA and the TLBO was selected, and then, the selected solutions became the initial population of the NSGA-II. For the simple NSGA-II, the initial solutions were generated randomly.

3.2.2.1. Bee algorithm. The BA is a population-based optimization algorithm introduced by Pham et al. [42]. It was developed on the basis of food foraging behavior of honey bees in a colony. This algorithm performs as a neighborhood search combined with a global search. Fig. 5. presents a pseudocode of the BA.

3.2.2.2. Teaching-learning-based optimization algorithm. A new Metaheuristic algorithm was proposed by Rao et al. [43] that was inspired by the teaching-learning process, from which the name, *teaching-learning-based optimization (TLBO)* algorithm was derived. The TLBO has two main parts, known as the *teacher* and *learner* phases. The population of the learner is generated as the initial solutions. A pseudocode of this algorithm is presented in Fig. 6.

3.2.3. NSGA-II operators

Generally, search progress in the NSGA-II algorithm is done by two well-known operations called *crossover* and *mutation*. In this study, these two operators were used to generate feasible chromosomes and find better solutions. Generally, the crossover operator uses two chromosomes of an initial population to make a new chromosome with new features. Several types of crossover (e.g. single point, double point, three point, and uniform) and mutation operators (e.g. 2-opt, 3-optm and insertion) can be used to make new offspring in the NSGA-II algorithm. Using all the operators at each iteration of the NSGA-II significantly increases the CPU time. Therefore, in this study, we used self-adaptive versions of the crossover and mutation operators to obtain and exploit the best solutions. A pseudocode of the self-adaptive NSGA-II, which is a part of the preparation phase, is presented in Fig. 7. In the proposed method, all described operators are applied without increasing the CPU time. First, the NSGA-II parameters are set, and then, the initial population is generated using the BA and the TLBO algorithms (for the simple NSGA-II, the initial solutions are generated randomly), and the mean of the Pareto optimal solutions is calculated. Each objective function is normalized according to the maximum and minimum values found. Second, the summation of the objective functions is calculated, and the obtained value is normalized. The fitness function for each solution is calculated by using Eq. (21). Third, for each crossover and mutation operator, a score is calculated and compared to other operators on the basis of whether or not a specific operator be able to find a better solution at each iteration of algorithm. This process is repeated for a predefined number of iterations, and at the end of the preparation phase, a selection probability (SP) is calculated by using Eqs. (22) and (23) for each crossover and mutation operator. The sum of all of the selection probabilities is equal to 1. Figs. 8 and 9 present examples of the single-point crossover and all types of mutation used in this study, respectively.

$$F = \frac{Z^1_{normalized} - Z^2_{normalized}}{2} \tag{21}$$

where

$$Z^1_{normalized} = \frac{Z^1_{mean} - Z^1_{\min}}{Z^1_{\max} - Z^1_{\min}}$$

$$Z^2_{normalized} = \frac{Z^2_{mean} - Z^2_{\min}}{Z^2_{\max} - Z^2_{\min}}$$

$$SP_i^C = \frac{SC(i)}{\sum_i SC(i)}, \quad i \in All\ crossovers - C \tag{22}$$

$$SP_i^M = \frac{SM(i)}{\sum_i SM(i)}, \quad i \in All\ Mutations - M \tag{23}$$

4. Computational results

This section presents a comparison of results from the proposed algorithms used with different parameter values. For a fair comparison, the best parameter values of each algorithm were tuned. For this purpose, we used the Taguchi method for parameter tuning, and then, all algorithms were run with predetermined problem sizes so the results can be compared with each other.

4.1. Parameter setting

Because the NSGA-II algorithm is common to every proposed algorithm, we only the Taguchi method to find the best parameter values only of the NSGA-II. Different levels were defined for each parameter, and a four-level Taguchi method was used to find the best parameter values for the NSGA-II. Finally, after the best value of each parameter was determined, each algorithm was run with the best values for problems of different sizes. Table 2 shows the defined levels for each parameter. The average of the Pareto solutions for each objective function and the CPU time were

Set the BA parameters:

1. Give the maximum number of iterations (It_{max})
2. Give the number of scout bees ($n_{ScoutBee}$)
3. Give the neighborhood radius damp rate (r_{damp})
4. Calculate the number of selected sites, $n_s = round\ (p_s * n_{ScoutBee})$
5. Calculate the number of selected elite sites, $n_e = round\ (p_e * n_s)$
6. Calculate the number of recruited bees for selected sites, $n_{sb} = round(p_{sb} * n_{ScoutBee})$
7. Calculate the number of recruited bees for elite sites, $n_{eb} = 2 * n_{sb}$

Generate the initial population of $n_{ScoutBee}$ scout bees.

Evaluate cost function of all generated scout bees.

Sort initial population based on the cost function.

For $i = 1$ to It_{max} {

For $j = n_{sb} + 1$ to n_s {

1. Select non-elite sites as best patches for vicinity search.
2. Employ some bees at elite patches and non-elite patches.
3. Evaluate the cost function of each patch.
4. Sort obtained results based on the cost function.
5. Assign the remaining bees to the non-best locations.
6. Sort overall obtained results.
7. If terminating condition is met, stop running.

}

}

Fig. 5. Pseudocode for the bee algorithm.

Generate number of students in the initial population.

Modify the other "values of variables" based on the best found solution.

Set t=0

For t to Max_{It} {

1. **If** a new obtained solution is better than the best found solution, **Then**
   replace the new solution
   Else
   keep the best found solution.
   EndIf
2. Randomly select two solutions, T_1 and T_2.
3. **If** T_1 is better than T_2, **Then**
   calculate the new value of T_1
   Else
   calculate the new value of T_2
   EndIf
4. **If** the obtained solution in step 3 is better than the best found solution, **Then**
   Replace it
   Else
   keep the best found solution.
   EndIf

}

Report the best found solution.

Fig. 6. Pseudocode for the TLBO algorithm.

considered as the evaluation criteria for the Taguchi method. Minitab software was used for the implementation of the Taguchi design for 5 th test instance. The Taguchi average signal-to-noise chart is presented in Fig. 10. According to this chart, the best parameter values for the NSGA-II were found and are reported in Table 3. Also, the parameter values of the bee and TLBO algorithms were set according to the available guidelines in the literature and our computational experiments in which these algorithms were run. The adjusted parameter values for these algorithms are shown in Table 4.

Set the NSGA-II parameters (N_{pop}, Max_{It}).
Create an initial population using the BA and TLBO algorithms.
SC(i)=0 ($i \in$ All crossovers-C)
SM(i)=0 ($i \in$ All mutations-M)
Evaluate fitness function (F) for each solution (Equation (21)).
Set $t = 0$ (iteration).
For t to Max_{It} {

Select parents using the roulette wheel selection method.
Apply all crossover operators.

- Single-point (C1)
- Double-point (C2)
- Uniform (C3)

Evaluate fitness function (F) of the obtained solutions (Equation (21)).
If $F_{C(j)}$ dominates $F_{C(i)}$, $i \neq j$ **then**
SC(j)=SC(j)+1
EndIf

Select a sample using the roulette wheel selection method.
Apply all mutation operators.

- 2-opt (M1)
- 3-opr (M2)
- insertion (M3)

Evaluate fitness function (F) of the obtained solutions (Equation (21)).
If $F_{M(j)}$ dominates $F_{M(i)}$, $i \neq j$ **then**
SM(j)=SM(j)+1
EndIf

}

Calculate selection probabilities of operators (Equations (22) and (23)).

Fig. 7. Pseudocode for the self-adaptive NSGA-II.

Table 3
Defined levels for each parameter.

Level	Parameters			
	n_{pop}	p_c	p_m	It_{max}
1	50	0.2	0.3	50
2	75	0.4	0.5	100
3	100	0.6	0.7	150
4	125	0.8	0.9	200

4.2. Comparison metrics

Many criteria have been introduced to evaluate the performance of multi-objective evolutionary algorithms. In this study, we employed the following five comparison metrics for assessing the performance of proposed algorithms:

- CPU time
- Ratio of non-dominated individuals (RNI)
- Diversity metric (DM)
- Spacing metric (SM)
- Generational distance (GD)

CPU time refers to the run time of the main loop of the proposed algorithm for each run. RNI is used as a quantity metric to measure the ratio of the non-dominated individuals in an optimal Pareto front to the population size. This metric is calculated by using Eq. (24) as follows:

$$RNI = \frac{|\bar{F}|}{N_{pop}} \tag{24}$$

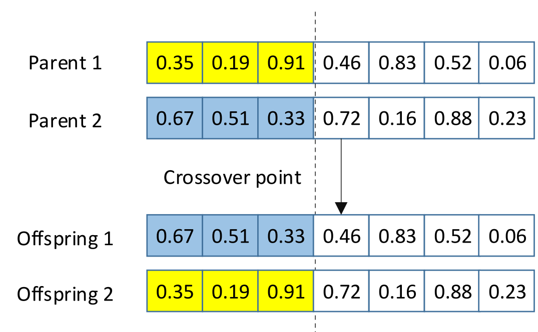

Fig. 8. Example of a single-point crossover.

where $|\bar{F}|$ and N_{pop} denote the number of non-dominated individuals in the optimal Pareto front and the size of the population, respectively. A higher value of these metrics is desirable. The diversity metric denotes the spread of non-dominated solutions and is calculated by using Eq. (25). A higher value is better than a lower value for this metric, because it represents the diversification of the optimal Pareto solutions and a larger space of solutions that can be covered by the algorithm.

$$DM = \sqrt{\sum_{i=1}^{N} \max \left\| x_i^k - y_i^k \right\|} \tag{25}$$

where N denotes the number of the Pareto optimal solutions, and $\max \left\| x_i^k - y_i^k \right\|$ shows the maximum Euclidean distance between

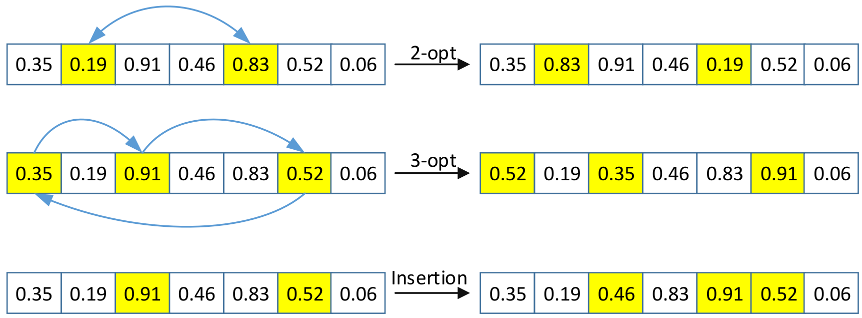

Fig. 9. Examples of the three types of mutations used.

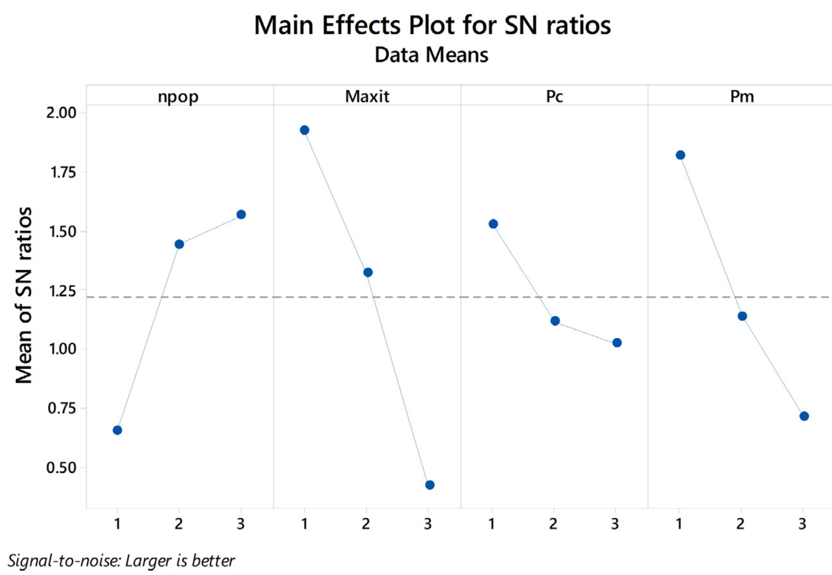

Fig. 10. Results of the Taguchi method.

non-dominated solutions. The spacing metric gives us an information about the distribution of non-dominated solutions using the obtained Pareto front. This metric is calculated by Eqs. (26) and (27) as follows:

$$SM = \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}\left(d_i - \bar{d}\right)^2} \tag{26}$$

$$d_i = \min_{j\neq i}\sum_{k=1}^{K}\left|f_j^k - f_i^k\right| \qquad \forall i, j \in \{1, \ldots, N\} \tag{27}$$

where d_i is the minimum value of a non-dominated solution i among all the obtained solutions; $\bar{d}$ is the average of all non-dominated solutions; and f_i^k is the value of the kth objective function of the non-dominated solution i. The lower value of this metric indicates a uniform dispersion of the optimal Pareto solutions. GD is used to measure the closeness of the solutions to the true Pareto front. This metric is calculated by using Eq. (28). The lower value of this metric denotes a relatively good performance.

$$GD = \frac{\sqrt{\sum_{i=1}^{N} d_i^2}}{N} \tag{28}$$

4.3. Computational and practical results

To compare the proposed algorithms, we generated 10 test instances for which the specific parameter values of each were defined. For this purpose, we chose the parameters that in the real world can be controlled and that an organization can increase or decrease. To address the lack of benchmark instances in the literature, we defined three levels for the value of each parameter on the basis of our experiments, which are shown in Table 5. With respect to the defined levels, 10 test instances were generated randomly using Microsoft Excel software. The chosen test instances are shown in Table 6. To evaluate the quality of the proposed algorithms, each test instance was solved by all the algorithms. Each test was run 5 times, and for each run, the average of all metrics was calculated. The final result was found by averaging all the obtained results from these 5 runs.

Table 7 presents the results from solving the proposed problem with the simple NSGA-II and hybrid algorithms. Also, the results of the proposed algorithms, in terms of the applied metrics, were compared with each other, and these comparisons are reported in Figs. 11–15. The results of the run times showed that the NSGA-II algorithm had the shortest run time. This outcome is attributable to the hybridization process because two separate metaheuristic algorithms were run such that the run time was the sum of both run times. With regard to the obtained results, the performances of the solution approaches for exploring more Pareto solutions, which were calculated by RNI, were close to each other but insignificant; the hybrid algorithms, BA-NSGA_II and TLBO-NSGA-II, showed better performance than the simple NSGA_II in more test instances. In terms of the DM and SM, the obtained results showed that the hybrid algorithms found the Pareto optimal solutions with more uniformity and diversity, respectively, than the simple NSGA-II did in the solution space. Like for the DM and SM, the obtained results

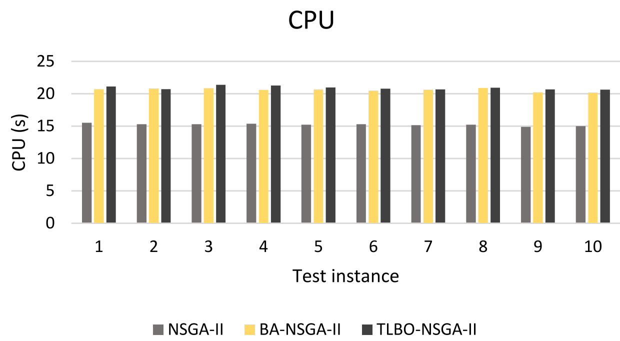

Fig. 11. Comparison of the proposed algorithms in terms of CPU time.

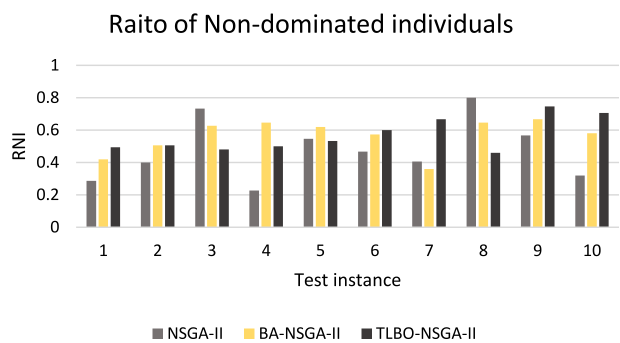

Fig. 12. Comparison of the proposed algorithms in terms of RNI.

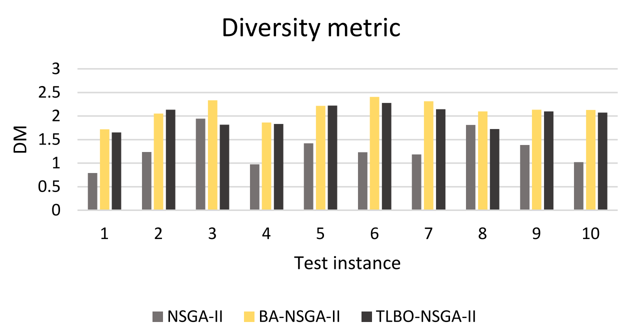

Fig. 13. Comparison of the proposed algorithms in terms of DM.

showed that the hybrid algorithms had better performances than the simple NSGA-II did in terms of the GD.

The results showed that the hybrid algorithms gave the best performances for improving the quality of Pareto optimal solutions. Therefore, we concluded that HRH hybridization is a good way to improve the performance of metaheuristic algorithms.

The findings showed no difference between the two hybrid algorithms in comparison metrics, but in some metrics, the BA-NSGA-II performed better than the TLBO-NSGA-II did. To compare the hybrid algorithms in more detail, a two-sample t test was designed for each metric. The analytical results of these tests are presented in Table 8. According to Table 8, the p values related to the mean of the RNI and DM were greater than 0.05, and the alternative hypothesis was not rejected. Therefore, we concluded that the optimal approach in terms of the RNI and DM was the TLBO-NSGA-II and the BA-NSGA-II, respectively. The results showed that the hypothesis of the mean inequality, in terms of the SM and GD, was rejected. Therefore, we concluded that both approaches were optimal in terms of the SM and GD. In terms of the CPU time, the alternative hypothesis was accepted; therefore, the BA-NSGA-II had a better performance than the TLBO-NSGA-II did.

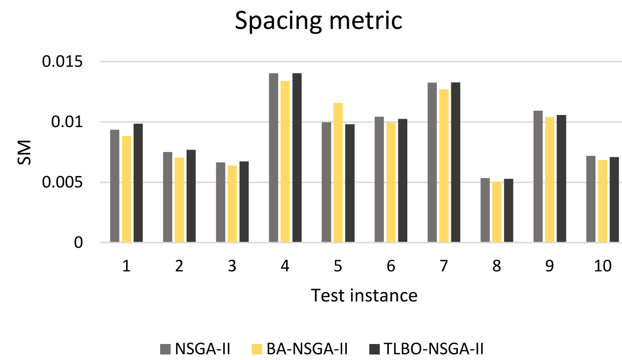

Fig. 14. Comparison of the proposed algorithms in terms of SM.

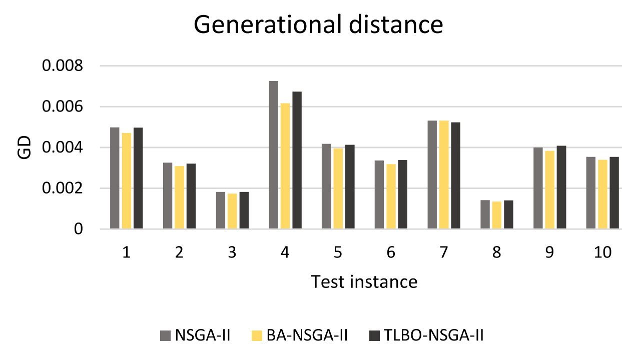

Fig. 15. Comparison of the proposed algorithms in terms of GD.

To find the effect of the two warranty policies on the first objective function, other decision variables, assumed as parameters of problem, were each assigned a specific value. The range was determined for each warranty variable and the objective function was calculated with all defined values. The result revealed that by increasing the value of the warranty-period lengths of free-replacement and pro rata warranty policy the objective function was increased and decreased respectively. Fig. 16 presents the effect of the warranty variables on the first objective function. We concluded that increasing the warranty-period length of free-replacement is in the interest of the organization, but each company needs to determine the optimal value of the warranty-period lengths to increase the expected revenue. Also, we conducted the same analysis for the effect of the standardization and customization quality levels on the first objective function and found that by increasing the value of these decision variables, the first objective Function was decreased. Fig. 17 shows the effect of the quality level variables.

5. Sensitivity analysis

In this section, we thoroughly explain the findings from a relative sensitivity analysis that was used to find the effects of the various parameters on the mathematical model and reveal an optimal solution for the proposed problem. Several sensitivity analyses can be designed on the basis of different parameter combinations, but some reasonable parameter combinations were predefined and a sensitivity analysis was conducted on those parameters. All sensitivity analyses were undertaken for NSGA-II algorithm outputs.

To understand the effect of the sensitivity of the rate of return quantity to the return policy (ρ) and the sensitivity of the rate of return quantity to the quality level ϕ, a sensitivity analysis was conducted. For each parameter, a specific range was defined

Table 4
Adjusted Parameters for NSGA-II, BA and TLBO.

Algorithm	Parameter								
	n_{pop}	p_c	p_m	It_{max}	r_{damp}	p_c	p_c	p_{sb}	TF
NSGA-II	75	0.4	0.9	200	–	–	–	–	–
BA	60	–	–	100	0.99	0.5	0.4	0.5	–
TLBO	50	–	–	100	–	–	–	–	1 or 2

TF: The teaching factor of TLBO.

Table 5
Defined levels for each parameter.

Parameter	Random distribution	Random distribution		
		Level 1	Level 2	Level 3
S	Setup cost	10	15	20
p	Production rate	27	30	33
c	Cost of product unit	0.8	1	1.2
h	Holding cost	0.2	0.4	0.6
π	Shortage cost	0.3	0.5	0.7
rs	Restoration cost	0.4	0.7	1
C_w	Related cost to free-replacement warranty policy	0.1	0.3	0.5
B	Maximum budget	770	800	850
M	Maximum capacity of return	3	7	9
λ	Quality cost	3	5	8

Table 6
Chosen test instances.

NO.	Parameters									
	S	p	c	h	π	rs	C_w	B	M	λ
1	15	30	1.2	0.6	0.3	0.7	0.5	850	3	5
2	15	27	1	0.2	0.5	0.4	0.1	770	8	5
3	15	30	1	0.2	0.3	0.4	0.3	770	3	8
4	10	30	1.2	0.2	0.5	0.7	0.1	800	5	3
5	20	30	0.8	0.2	0.7	0.4	0.5	850	5	8
6	15	30	0.8	0.4	0.3	0.4	0.3	850	5	5
7	10	33	1	0.3	0.5	1	0.5	850	3	3
8	10	27	0.8	0.4	0.5	0.7	0.3	850	5	3
9	15	27	1	0.6	0.3	0.4	0.1	770	8	5
10	11	30	1	0.4	0.3	0.7	0.5	800	5	8

Table 7
Results of the ten test instances using the NSGA II and the hybridized NSGA II.

NO.	NSGA-II					BA-NSGA-II					TLBO-NSGA-II				
	CPU (s)	RNI	DM	SM	GD	CPU (s)	RNI	DM	SM	GD	CPU (s)	RNI	DM	SM	GD
1	15.541	0.286	0.787	0.009	0.004	20.7	0.42	1.715	0.008	0.004	21.105	0.493	1.654	0.009	0.004
2	15.291	0.4	1.234	0.007	0.003	20.817	0.506	2.053	0.007	0.003	20.704	0.506	2.132	0.007	0.003
3	15.305	0.733	1.944	0.006	0.001	20.875	0.626	2.331	0.006	0.001	21.389	0.48	1.813	0.006	0.001
4	15.358	0.226	0.976	0.014	0.007	20.601	0.646	1.864	0.013	0.006	21.264	0.5	1.831	0.014	0.006
5	15.208	0.546	1.419	0.009	0.004	20.681	0.62	2.214	0.011	0.003	20.984	0.533	2.222	0.009	0.004
6	15.31	0.466	1.233	0.01	0.003	20.477	0.573	2.405	0.009	0.003	20.8	0.6	2.277	0.01	0.003
7	15.135	0.406	1.185	0.013	0.005	20.645	0.36	2.311	0.012	0.005	20.676	0.666	2.146	0.013	0.005
8	15.222	0.8	1.811	0.005	0.001	20.88	0.646	2.099	0.005	0.001	20.946	0.46	1.723	0.005	0.001
9	14.896	0.566	1.383	0.010	0.003	20.227	0.666	2.131	0.01	0.003	20.672	0.746	2.097	0.01	0.004
10	14.984	0.32	1.018	0.007	0.003	20.201	0.58	2.126	0.006	0.003	20.619	0.706	2.071	0.007	0.003

Table 8
The analytical results of two sample t-tests.

Metric	Optimal approach	Average results		Two sample *t-test*	
		BA-NSGA-II	TLBO-NSGA-II	Alternative hypothesis	p-value
CPU	BA-NSGA-II	20.6109	20.9163	$\mu_1 < \mu_2$	0.008
RNI	TLBO-NSGA-II	0.5646	0.5693	$\mu_1 < \mu_2$	0.46
DM	BA-NSGA-II	2.1254	1.9972	$\mu_1 > \mu_2$	0.101
SM	Both methods	0.0092	0.0094	$\mu_1 \neq \mu_2$	0.815
GD	Both methods	0.0036	0.0038	$\mu_1 \neq \mu_2$	0.778

1: BA-NSGA-II, 2: TLBO-NSGA-II.

and the first objective function was calculated by the BA-NSGA-II algorithm. Each combination of parameters was run 5 times, and the average of the obtained numbers was calculated. According to Fig. 18, we concluded that by increasing the values of ρ and ϕ, the value of the first objective function was increased. However, the first objective function was decreased for some parameters; therefore, the corresponding parameter value can be considered the best parameter tuning for the presented problem.

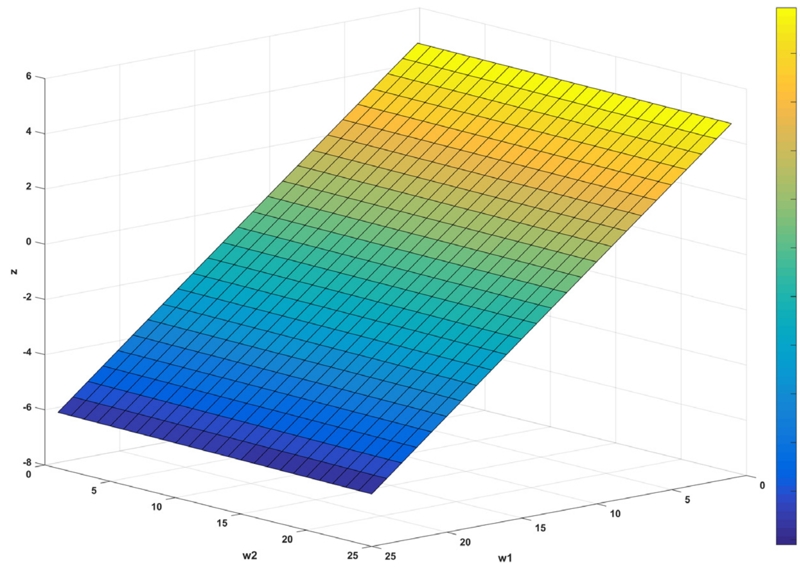

Fig. 16. Effects of the two warranty policies on the first objective function.

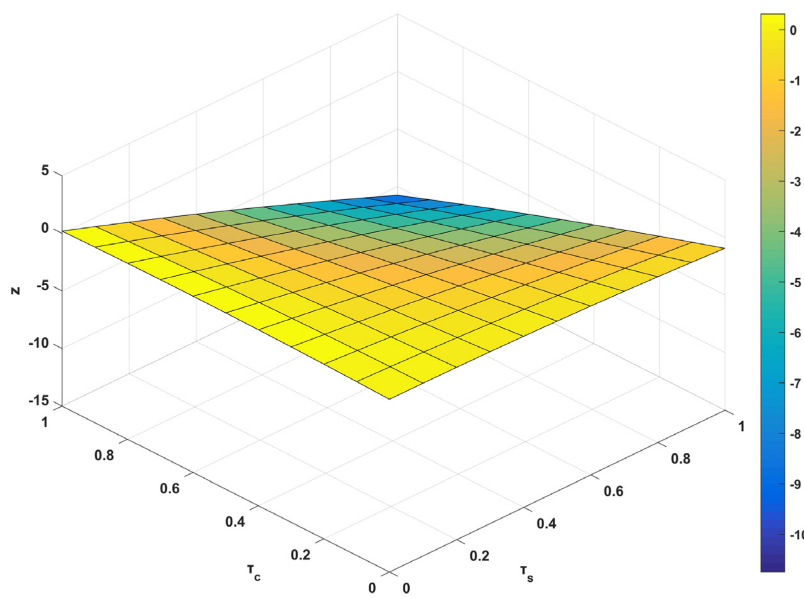

Fig. 17. Effect of standardization and customization quality levels on the first objective function.

The sensitivity analyses of holding and shortage costs were implemented and the results are presented in Fig. 19. As can be seen, the total expected revenue decreased by increasing h and π. It can also be seen that at low levels of h and π the gradient of the diagram was approximately 0, but when the h and π are increased this gradient was decreased.

A sensitivity analysis was conducted to find the relative importance to customers (ζ) of the customization process for product quality on the second objective function. Fig. 20 presents the result of this sensitivity analysis. As can be seen, two trends are featured in Fig. 20. First, the objective function was decreased until ζ was 0.5. After this point, increasing the value of ζ was related to an objective function increase. Therefore, we concluded that the standardization process is more important than the customization process for an organization. Another sensitivity analysis was designed to understand the effect of the percentage of customers who choose the FRW(β). Fig. 21 shows the analysis results from which we concluded that expected revenue decreased when a greater percentage of customers chose the FRW.

6. Conclusions

This study proposed a new mathematical model for an IPS with regard to product quality and returns under two warranty policies. The PRW and FRW policies were considered for the proposed problem. We assumed that the demand of customers was uncertain and had a stochastic behavior. The product prices were considered as functions of return compensation, product quality level, and warranty-period length. We assumed that customers can choose between the PRW and FRW policies. We considered a limitation on the available budget and capacity of return. Two objective functions were used to formulate the problem. The first objective function was utilized to maximize the total expected revenue per cycle, and the second objective function was utilized to maximize customer satisfaction. A NSGA-II algorithm and two hybrid algorithms – BA-NSGA-II and TLBO-NSGA-II – were used to solve the problem for various tests, which had been generated randomly. In the hybrid algorithms, the initial population of the NSGA-II was generated by the BA or the TLBO algorithm. Also, the self-adaptive

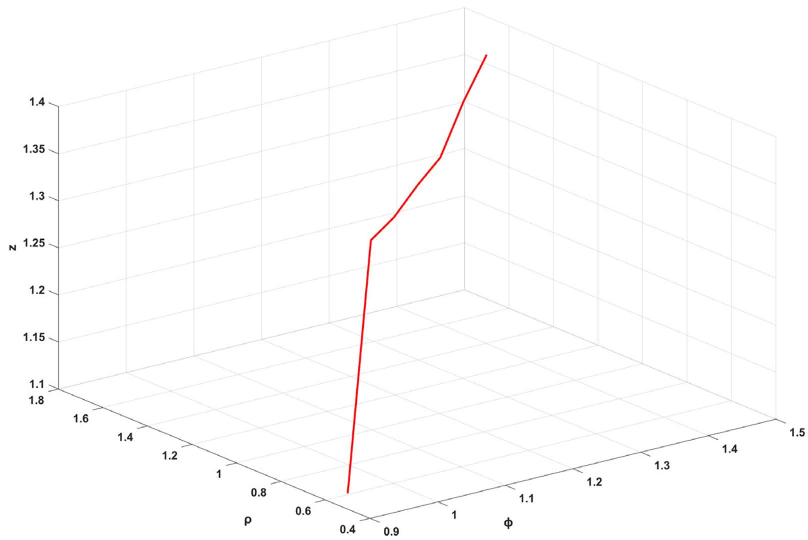

Fig. 18. Optimal value of the first objective function based on different values of ρ and ϕ.

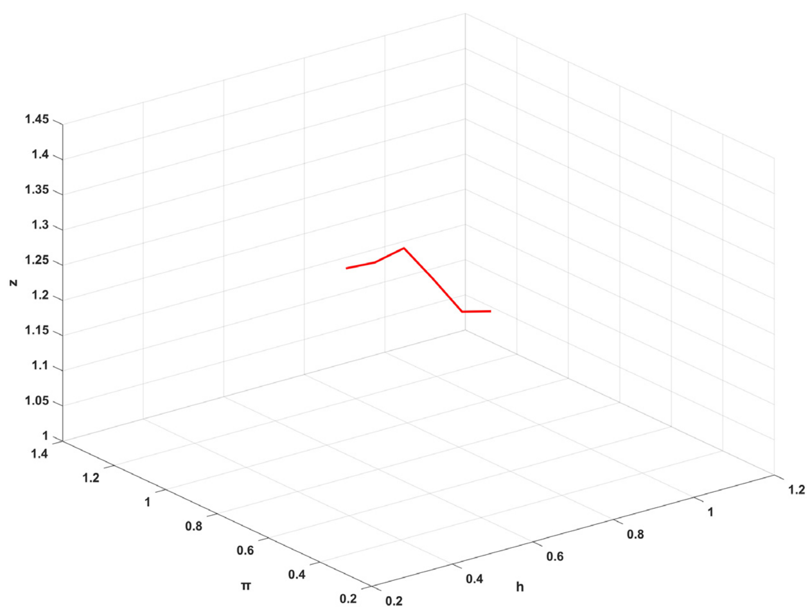

Fig. 19. Optimal value of the first objective function based on different values of h and π.

version of the crossover and mutation operators were applied to obtain and exploit the best solutions. Five metrics were used to compare the performances of the proposed solution approaches. The results indicated that the hybrid algorithms performed the best in improving the quality of Pareto optimal solutions. The findings revealed no difference between the two hybrid algorithms in the comparison metrics, but in some metrics, the BA-NSGA-II gave a better performance than the TLBO-NSGA-II did.

Several sensitivity analyses were designed on the basis of different parameter combinations to find the effects of the various parameters on the mathematical model and reveal an optimal solution for the proposed problem. In future studies, researchers might develop the proposed model by considering different warranty policies with more uncertain parameters for use in the problem. In terms of the solution approach, heuristics and other metaheuristics algorithms could be applied to solve the proposed problem. The NSGA-II is well known for solving the multi-objective optimization problem. However, it cannot guarantee better performances when more than three objectives are presented [44]. The NSGA-III is

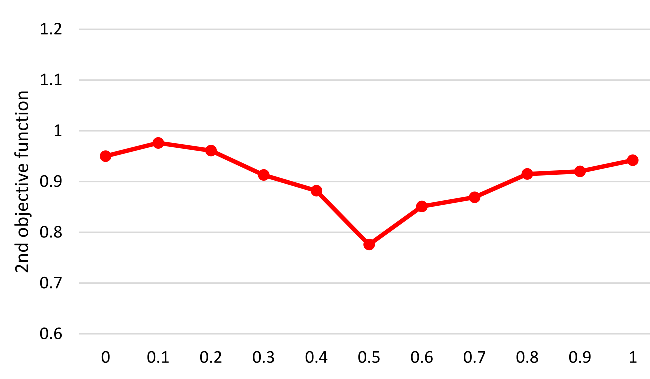

Fig. 20. Optimal value of the second objective function based on different values of ζ.

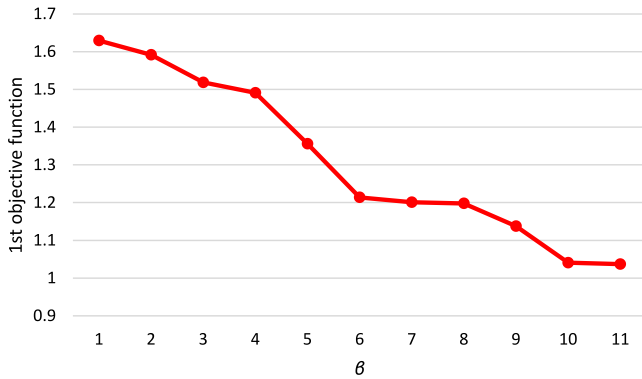

Fig. 21. Optimal value of the first objective function based on different values of β.

an extension of the NSGA-II and a powerful approach to handle more than three objectives, called Many-objective optimization problems. This approach has widely been used in the various types of industry including software and energy where multi-objectives are highly encouraged to improve their quality assurances [45,46]. The comparison between the NSGA-II and the NSGA-III for the problem studied in this paper might be an interesting research topic.

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