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The sandpile model and empire dynamics

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ABSTRACT

According to the self-organized criticality theory, the sandpile model is built to investigate the evolutionary dynamics of empires in the history of China. The methods of agent-based modeling and simulations are applied to capture empires' mechanism of rising and falling cycles, and to obtain the observed life cycle pattern of empires in history. Under the self-organized criticality theory, natural systems and human empires systems have similar structures and mechanisms, which makes systems reaches the critical states automatically. Therefore, the rising and falling dynamics of empires can be reflected by the sandpile model as well. With the sandpile modeling and simulations, the optimal solution of parameters can be found, based on which the satisfactory fitness of results can be achieved. Under the optimal solution, we run the simulations for 1000 times to check the fitness and robustness. First, the number of empires can be matched. There were 22 empires in the history of China, and the same number of empires can be obtained via sandpile model simulations, and the amount of empires follows the normal distribution with the mean of 22 empires; Second, the distribution of empire durations follows the power-law distribution, for both simulated and historical empires; Third, for less than 22 simulated empires, we drop empires with tiny durations in history to compare the 19, 20, and 21 pairs of counterparts respectively, and the fitness can be guaranteed as well; finally, for more than 22 empires, we drop simulated empires with tiny durations to compare the 23, 24, and 25 pairs, the matching degree is satisfactory as well. It indicates that multiple simulations have more robust and stable outcomes than one, even the best, simulation.

1. Introduction

The empire is defined as sovereign political entities [1]. Although empires in history have different durations and growth-decline processes, the life cycle pattern of empires is robust and stable [2]. All the four stages, such as outbreak, growth, peak, and vanish, are included in the life cycle of empires in human history. We have the standard or traditional sociological researches and interdisciplinary quantitative researches. For standard sociological researches, they focus on the conceptions and growth-crisis transitions of the empires, by concretization of certain aspects or factors, other than the empire dynamics by using modeling approaches. For instances, Wimmer (2006) identify the wars (1816–2001) as the main contribution to regional conflicts, disintegration of empire and formation of the nation-state [3]. Norkus (2008) applies the concept of empire to define the medieval Lithuanian state (Grand Duchy of Lithuania, GDL). The GDL should exist as an empire, according to the theoretical framework of military power, territorial structure, the sphere of hegemony, and long-distance trade [4]. De Quiroga (2010) uses the freeman as the analysis unit to explore the rise and fall of the Italian Empire, and pointed out the importance of the social network constructed by the freeman in commerce, crafts, or banking [5]. Based on social fields and social spaces, Steinmetz (2016) presents a theory of the empire scale to distinguish the conceptions of social fields, subfields, and social spaces of empires [6]. These researches, at the macro-level, focus on the concretization of certain aspects or factors, other than modeling the whole process of rise-and-fall dynamics.

Quantitative research methods are often interdisciplinary, with mathematical modeling, statistical analysis, or big data mining of history. For mathematical modeling, Taacepera (1978) proposed a simple logistic pattern to model empires' growth dynamics [7]. The logistic growth model involves "a slow start, a speed of expansion, and finally a slow approach to a stable maximum size" [8,9]. By analyzing the population dynamical model, a mathematical theory has been built to explore the evolutions of human societies or empires [10]. It is found that spiral orbit and oscillations can explain the rise and fall evolutions of empires, and cultural resources for social evolution can be calculated and modeled. Chase-Dunn et al. (2006) explore the causal patterns of rises and falls since the Bronze Age, and the population and territorial sizes can explain

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the mechanism of empires [11]. Khmaladze (2007) believes that both Roman and China emperors follow the exponential distribution [12]. Chase-Dunn et al. (2010) examines the empires' dynamic patterns, using three quantitative components including the population, territorial sizes, and state-empire networks [13]. Furthermore, Arbesman (2011) has investigated the lifespan distribution of global empires and find the memoryless exponential distribution, which is the same as species and corporate firms [14]. Ausubel (2012) focuses on the quantitative dynamics of empire breakdown, which is the S-shape or Logistic Curves. Besides, the traffic speed determines the territory of empires [15]. Ferguson (2012) adopts the life circle perspective of civilizations or empires and deems that current global powers are in similar traces of global empires in history [16]. Yegorov (2019) formulates a dynamic optimization model that includes multiple dimensions like geography, economy, border dynamics, transport costs, objective and evolution of empires. It suggests that the multiplicity of equilibria can explain why empires and small states coexist in history [17]. However, they cannot vividly simulate the history process of empire dynamics.

Empires in China's history provide ideal target cases. Thus, here we construct and agent-based modeling and simulations to explore the internal mechanism, which shapes the life cycle dynamics of empires. Current researchers analyze empires based on geographical [18], economic [19], population [20], military [21], agrarian [22], and archeological factors [23,24].The long cycle theory shows that there is a regular cyclical pattern in the history of empires and interactions of politics in the world [25]. Therefore, there is also a certain distribution pattern in the history of Ancient Chinese Empires. In other words, China and global empires share the same macro-level patterns and micro-level mechanisms. The broader perspective of interdisciplinary researches is needed for empire researches. Some scholars build mathematical models that combine natural and societal phenomena. By these models, more mathematical approaches can be applied to examine the formation (rising) and breakdown (fall) of global empires. To investigate why empires in history repeatedly rose and fell, various methods have been applied. Based on the self-organized criticality theory, the sandpile model can be applied and the agent-based simulations should be conducted. Mathematical modeling and agent-based simulations are advanced tools, which can visualize, back-calculate, and predict the dynamic process of empires. Nevertheless, agent-based modeling and simulations have not been applied widely to the rise-and-fall mechanism of empires. In this work, we therefore propose the new perspective of agent-based modeling and simulations, to explore the constant rise and fall mechanism of empires in history.

2. The self-organized criticality theory

Bak et al. (1987) initially, proposed self-organized criticality (SOC), which exists in both natural and societal systems [26]. According to the self-organized criticality theory, the extended dissipative dynamic system spontaneously evolves into the critical state. The evolution to the critical state does not require special provisions, such as the initial state, and the process that critical state generates systemic chaos and disturbance is robust all the time. When the system deviates from the critical state, it will automatically return to the critical state again [27]. Self-organized criticality is believed to be the key concept that explains the general mechanism of various complex systems. Mathematically, the behavior of complex systems follows a simple pattern. The self-organized criticality discovers the behavior evolution model of dissipative systems, which has two essential characteristics: the 1/f noise in time effect and the scale-invariant self-similarity in spatial structure evolution [28].

The sandpile model is a typical tool of investigating the self-organized criticality. Held et al. (1990) conducted a serial of sandpile experiments. The sand particles with the size of 1–1.25 mm were selected to construct the sandpile on a disk with a radius of 2 cm. When the inclination angle is close to the critical state, the number of particles added generally equals the number of particles falling outside, and the sandpile stops growing. At this time, the response to the newly added sand will be unpredictable. The sand may be fixed on the sandpile, may cause small sliding, or even lead to a massive avalanche. However, the rule that the relationship between the scale of avalanche and the frequency follows the power-law distribution can be found [29]. In this avalanche process, the number of sand particles collapsed is called the avalanche size (S), and the duration is called relaxation time (T). The probability of avalanche occurrence D, avalanche size S, and relaxation time T satisfy the power-law distribution.

Under the interaction of multiple elements, the system automatically reaches the critical state, without related parameter adjustments. This means that even a small event will trigger the chain reaction process, which a large number of components in the system, leading to the occurrence of large-scale events. Although there are more small events than large events among systems, the chain reaction of all scales is an essential part of dynamic systems, hence the power-law can be used as the evidence of self-organized criticality [30]. The self-organized criticality theory has been used to explain many complex phenomena in seismology [31], geology [32], river branches [33], astrophysics [34], computer networks [35], transportation [36], biology [37], forest fire [38], and economics [39]. It is found that many cellular automata models exhibit similar self-organized critical behaviors, such as earthquakes, life games, forest fires, stock market shocks, and so forth. For instance, Bak et al. (1989) found that the distribution of energy released during earthquakes also follows the power-law distribution [31]. The crust is in motion and undergoes slow deformation, which leads to the stress accumulation in the rock. If the rock somewhere cannot bear it, the energy will be released. In other words, if the environment can afford this energy increase, it will be an isolated event; But if it is also in a critical state, chain reaction occurs, and isolated events will be connected as the macro-level turbulences.

The self-organized criticality has been extended to both social and historical topics. Roberts & Turcotte (1998) studied the wars in history and believed that world order is similar to the Forest Fire Model [36]. There are small conflicts in the world, which may or not develop into major wars. The world order is a self-organized system, which has nothing to do with the control efforts of countries [40]; Picoli et al. (2014) found that violent events in human conflicts may be related to a threshold mechanism, and SOC can be used to describe them [41]. Shimada & Koyama (2015) found crisis signs in the social dynamics systems, indicating that the system is ready for large-scale transformations. So detecting self-organized criticality can be used to diagnose the transformation potential of societies [22]. Zhukov et al. (2016) analyzed Russia's population and grain prices over the past two centuries and verified the 1/f noise rule as well [42]. As we investigate the rise and fall of human social systems, such as empires, the self-organized criticality theory and models can be therefore applied, which can be supported by these existing researches.

3. Theoretical analogy

The sandpile and empire have the similarity of self-organized criticality. They both have hierarchical structures and reaction processes. Therefore, the SOC analogy between human conflict patterns and natural phenomena can be supported.

3.1. Hierarchical structures

The social hierarchy or social stratification always exist in human society since ancient civilizations such as China, Egypt, and India [43,44]. In antient China, the society was categorized as Social-bureaucrats, Gentry, working-class, lower classes when the time goes back to the Qing Dynasty [45]. Others divide them into four classes, such as bureaucrats, farmers, workers and the businessman. The hierarchy appeared in ancient Egypt, Greek, and India as well [46,47]. For antient Indian, the caste system (Brahmin, Kshatriya, Vaisya, and Shudra) provides crucial evidence of social hierarchy, which exist till now as well. From top to bottom, in ancient Egypt people, except the Pharaohs(king), were classified as Vizier, Nobles, Priests, Scribes, and Soldiers [48]. Showing the pyramid shape, the sandpile structure is similar to the social structure of empires. The bottom sand particles symbolize the ruled workers or peasants who produce energy (wealth) for the whole society. This is the “cornerstone” of the sandpile, because they support the whole sandpile. The sand particle in the middle means the businessmen and soldier class, while the top sand refers to the ruling class (scholar bureaucrat, king, and others). The sandpile with a critical state presents a standard pyramid shape. The process of adding sand drops is similar to the social development process of empires. In this process, social stratification and social contradictions will be gradually accumulated, until the outbreak of turbulence.

3.2. Similar processes

The accumulation and avalanche process of sandpile is similar to the rising and falling process of empires. Sinopoli (1994) generalized the growth and decline of empires into three stages, such as expansion, consolidation, and collapse [49]. The regionally fragmented or weak empire expands its territorial size through conquest and incorporation. The ultimate destiny of empires is to collapse, but nobody knows when it will happen. The collapse of empires has the nature of both unpredictability and determinism. The whole process of the sandpile model is divided into three stages as well: begin-input-collapse. In the beginning, we set the platform range, the number and distribution of initial sand particles. Then, we add sand one by one to the device. First the sand will stay near where they are located. Then they are stacked one by one, forming a slowly sloping sandpile. Finally, when the slope becomes too steep, the sand will slide down, leading to small avalanches. As more sand is added, the slope becomes steeper, forming more large or small avalanches. The avalanches would be unpredictable because they are a consequence of the complete history of the entire sandpile. No matter what the local dynamics are, the avalanches will mercilessly take places and persist under relative probabilities, and this process cannot be altered. With self-organization behaviors of agents, the sandpile model is an evolutionary system, which is driven by internal factors and individual behaviors. Although the factors are complex and diverse, such as economic activities, cultural system, technical level, geographical environment, military strength, etc. [50], the periodic evolutions of dynasties or empires is mainly driven by endogenous factors, such as degrees of corruption, military strength, harvest, and so on.

3.3. Self-organized criticality analogy

In addition to structural and process similarities, empires dynamics and sandpile models have the same nature, self-organized criticality. In the sandpile model, the self-organized process is reflected by the random accumulation and unpredictable avalanche, which is the metaphor of empires' growing, stabilizing, and unpredictable collapse. The sandpile model is an extended dissipative system with self-organized criticality. When the amount of sand added balances the amount of sand outside, the sandpile stops growing, and the system has reached a critical state. When more sand is added to a sandpile with a critical state, it causes avalanches of any size. Under the critical state, the sandpile maintains a constant height and slope. If the slope less than the critical value of the subcritical state, the avalanche will be less than that produced in the critical state. The subcritical reactor will grow to a critical state. If the slope is larger than the critical value, the avalanche will be much more massive. The collapse of the supercritical pile will occur before it reaches a critical state. Subcritical and supercritical piles are naturally attracted to the critical state. The development of empires can be deemed as the evolution of systems. When it reaches a critical point, there is a phase change for empires. Self-organized criticality of systems has been applied to analyze empires, civilizations, polities, or group behaviors. Iberall (1987) investigated the emergence of settled civilizations by using the matter condensation in physics [51]. Furthermore, Portugali (1997) uses the expressions of self-organizing criticality in the physical world to analyze the cycle and system for cities and large cultural entities [52]. The natural science (such as in physics) studies of civilization and have demonstrated the availability of self-organized criticality or other natural features in social organizations and human empires [53].

4. Methods and materials

Based on the self-organized criticality theory, the sandpile model is used to explore the periodic evolutions of empires. This sandpile model, as well as the agent-based simulations, are conducted via the Netlogo software, which is a tool to create multi-agent systems and simulations. During the 2132 years in Table 1, we have 22 empires or dynasties in China. Taking China as an example, we try to depict, back-calculate and predict the social and historical process of human societies. Real-time sandpile collapse percentage will be monitored. By setting different thresholds of crisis, empires or dynasties will be generated or identified, and the optimal solutions can be solved.

4.1. Multi-agent behavioral systems settings

Netlogo is a programmable modeling environment for simulations of natural and social phenomena. In Netlogo, there are dynamic and static agents, and both are agents. The static agent is called a patch, which is used to simulate immovable objects, such as buildings, mountains, rivers, land, and plants. The dynamic agents simulate mobile subjects, such as individuals, animals, particles, cars, etc. Netlogo is particularly suitable for modeling complex systems that evolve over time. It can well simulate behaviors of micro-level individuals and emergences of macro-level patterns. Based on the sandpile model, we set the static patch as the Empire territory and the dynamic subject as sand particles. The number of sand particles indicates social conflicts. The core goal is to match or fit the historical process of the rising and falling of empires in China from 221 BC to 1912 AD. Therefore, we set 2132 iterations for each simulation, and each iteration t refers to one year. The empire is set as a square, and the patch area refers to its territory. The total area is $S = 41 \times 41 = 1681$ patch^2. One sand particle will be added to the system constantly at each time t. The initial number of sand particles on each patch the same number of four, which can be adjusted by the interface slider "grains per patch". At each time t, one sand particle is added randomly onto one patch (the central patch). When the number of sand particles reaches the critical threshold of this central patch, the collapse happens. Four sand particles will be transferred from this central

Table 1
The 22 Empires in History of China.

Empires or Dynasties	Birth Year	Death Year	Duration (Y_i, years)
Qin	221 BC	206 BC	15
Western Han	206 BC	25 AD	231
Eastern Han	25 AD	220 AD	195
Three Kingdoms	220 AD	265 AD	45
Western Jin	265 AD	316 AD	51
Eastern Jin	316 AD	420 AD	104
Southern & Northern dynasties-Song	420 AD	479 AD	59
Southern & Northern dynasties-Qi	479 AD	502 AD	23
Southern & Northern dynasties-Liang	502 AD	557 AD	55
Southern & Northern dynasties-Chen	557 AD	589 AD	32
Sui	581 AD	618 AD	37
Tang	618 AD	907 AD	289
Five Dynasty & Ten Kingdom Late-Liang	907 AD	923 AD	16
Five Dynasty & Ten Kingdom Late-Tang	923 AD	936 AD	13
Five Dynasty & Ten Kingdom Late-Jin	936 AD	947 AD	11
Five Dynasty & Ten Kingdom Late-Han	947 AD	951 AD	4
Five Dynasty & Ten Kingdom Late-Zhou	951 AD	960 AD	9
Northern Song	960 AD	1127 AD	167
Southen Song	1127 AD	1279 AD	152
Yuan	1271 AD	1368 AD	97
Ming	1368 AD	1644 AD	276
Qing	1636 AD	1912 AD	276

Note: To make the total lifespan of all dynasties be 2133 years, the segmentation data deviate from the real historical data.

patch to four neighbor patches, and one neighbor obtains one particle. Then, neighborhood patches may become overloaded, which leads to new turns of collapse. The critical collapse threshold is controlled by the interface slider “threshold value”, which is set to be five. The boundaries are not connected, and once the sand particles leave the boundary, they disappear. At each iteration t, the whole process from the first avalanche to no further avalanche is called an avalanche. The number of collapsed patches C*t* is taken as the avalanche size. The collapsed percentage is calculated as P*t*=C*t*/S, where S is the total number of patches. At each time t, we calculate and record P*t*, comparing with the empire turnover threshold P_t.

4.2. Empire evolutions and dynasty recognitions

The sandpile model system initializes the continuous dynamic evolution process of human empires. In the beginning, the sand particle is added from a new stable state, representing the beginning of a dynasty. Therefore, the avalanche threshold for all patches is five particles. For the initial settings of sand particles, each patch has four sand particles on it. This means that all patches or regions are under the same degree of pressure, which makes the newly added sand particle quickly generate the sandpiles and avalanches. The dynamic process of sandpile (empire) dynamics is as follows: at each iteration or tick t, one sand drop is added onto one patch whose location is totally random. When the number of sand particles on the patch reaches the critical threshold of five, it will produce both direct and indirect collapses. The number of sand particles on the patch will be reduced to lower than the collapse threshold (=five) again, and each of four neighborhood patches obtains one sand particle from the central patch, respectively, which probably causes further cascading sandpile avalanches. The patch that is affected by the collapse will change its color into silver-white visually. In the Netlogo, each agent or patch has one unique ID, which is denoted as *i*. This is a dynamic system process, and Eq. (1) is applied to reflect the attribution changes of the patch. For an arbitrary patch *i*, the number of sand particles at the time *t* is denoted as z_{it}. At the next time $t+1$, the number of sand particles on it $z_{i,t+1}$ have three situations or possibilities. If the patch is the central patch whose number of sand particles at the time *t* has satisfies the threshold five, the patch will lose or transfer four sand particles to four neighbor patches at the time $t+1$; when the patch *i* is one of the neighbors of the central patch whose threshold has been satisfied or is the central patch whose number of sand particles at the time *t* has not satisfied the threshold five, the number of sand particles on it will be increased by one; otherwise, the number does not change, because these patches are outside of the direct and indirect avalanches.

$$z_{i,\ t+1} = \begin{cases} z_{it} & if\,outside \\ z_{it}+1 & if\,neighbor \\ z_{it}-4 & if\,center \end{cases} \tag{1}$$

At each time t, we calculate the collapsed percentage of $P_t = C_t/S$, which means how many patches are correlated to the avalanches. In the history of both China and the world, this percentage P_t captures the degree of social instability and system chaos, which significantly undermines empires. For strong and healthy regimes, weaker levels of chaos will be controlled or handled. However, more substantial chaos and instabilities will certainly overthrow existing regimes or empires, because it is beyond the controlling capability. There should be a threshold mechanism here as well. At each time, we calculate P_t, which is compared with the crisis threshold of empires P_T. If the $P_t \geq P_T$, the current dynasty or empire will end, and the new dynasty and empire begin. If the $P_t < P_T$, the chaos and risks are still under control, and the empire will survive accordingly. According to this rule, we are able to cut the whole 2132 iterations into several dynasties or empires. In China's long history of 2132 years, we generally have recorded about 22 dynasties or empires from Qin dynasty starting in 221 BC to Qing dynasty ending in 1912 AD, which is deemed as historical reality. Therefore, it is expected that 22 empires can be identified from simulations of 2132 years (ticks).

4.3. Finding optimal parameters and solutions

Our priority is to find optimal parameter settings $Paramters^*(\cdot)$, which can generate the outcomes best matching the historical reality. Mathematically, the 22 empires or dynasties in history is deemed as $(Y_1, Y_2,\ Y_3, \ldots Y_i,\ \ldots Y_{21}, Y_{22})$, which is an ordered serial of empires durations, and these empires composed the whole history of 2132 years. For each empire, Y_i is the duration or span, which is the crucial feature of empires. In general, the historical reality can also be deemed as the function $f_{real}(\cdot)$. In the simulations, the real-time scale or percentage of collapses P_t reflects the dynamic magnitude of Empire turbulence or risks. By setting different empire crisis thresholds P_T, the number of identified or recognized empires is various. The identified empires are separated and segmented outcomes, with a serial of empire lifespans as well, such as $(\widehat{Y_1}, \widehat{Y_2}, \widehat{Y_3}, \ldots \widehat{Y_k}, \ldots \widehat{Y_{21}}, \widehat{Y_{22}})$. In general, the simulated empire outcomes, such as durations, can be denoted as the function $f_{sim}(\cdot)$. In order to calculate the fitness, we will sort both $(Y_1, Y_2, Y_3, \ldots Y_i, \ldots Y_{21}, Y_{22})$ and $(\widehat{Y_1}, \widehat{Y_2}, \widehat{Y_3}, \ldots \widehat{Y_k}, \ldots \widehat{Y_{21}}, \widehat{Y_{22}})$, and then match them with pairs of simulated and historical empires. For different parameter settings, the simulation outcomes and fitness (empires and durations) are different. In China, most empires and dynasties are clearly divided and recorded by historians, but there are also chaotic periods, which is combined and deemed as the whole empire. For instances, the Five Dynasties and Ten Kingdoms period (907–960) have 15 overlapped kingdoms, and the Three Kingdoms period has 3 coexisting regimes. These inaccuracies of empires are inevitable and acceptable, and cannot substantially affect the whole simulations of empires in history.

$$Paramters^*(\cdot) = Argmin[f_{sim}(\cdot) - f_{real}(\cdot)] = Argmin \sum_1^{22} \frac{\left(Y_i - \widehat{Y_i}\right)^2}{n-1} \tag{2}$$

$$s.t. \sum Y_i = 2132$$

Under all the possible parameter settings and simulations, we calculate the outcome fitness, $\Delta = f_{sim}(\cdot) - f_{real}(\cdot)$, and finally obtain the optimal solution that minimizes the errors of differences between historical reality and simulations. There are three criteria of matching or fitness: (a) The basic standard is the amount matching. The number of simulated empires should match the records of historians, which means $max(i) = max(k) = 22$. In other words, the expectations or mean values of simulations should match 22, which implies that $E(k) = max(i) = 22$. If the multiple or repeated simulations under the optimal solution are run, the number of observations k, i.e. how many empires can be simulated, is expected to follow the normal distributions with the mean of 22 empires, which is recorded in history; (b) The middle standard is the distribution matching. Given the matching numbers of 22 empires, the distribution of 22 simulated empire durations should match the history as well. This includes both the discrete distribution form and kernel density distribution, which is the continuous form; and (c) the higher standard is the one-by-one duration matching of empires between simulations and history. This enables us to find the simulated empires and their counterparts in history and provides significance to related researches in social, natural, and system sciences. The optimal solution $Paramters^*(\cdot)$ can be solved in Eq. (2). Once the optimal parameters are found, we run 1000 simulations repeatedly to further validate the fitness and robustness. Finally, we visualize related outcomes of simulations in the following section.

5. Results

Before this optimal solution can be found or solved, we need to run multiple simulations under different parameter settings, which is time-consuming but necessary. Applying the parameter spectrum of 10,000 simulations, we finally obtain the optimal parameter solutions $Paramters^*(\cdot)$ with the empire crisis threshold 0.563. The starting sand number is the same of 4, the collapse threshold is 5, and the best delineation value is 0.563. Therefore, it has been proved that the sandpile model under the self-organized criticality theory can be used to analyze to unveil the rising and falling dynamics and mechanisms of human empires in history. Under the optimal parameter solutions, we provide the outcomes of 1000 repeated simulations.

5.1. Distributive fitness of models and simulations

The Fig. 1 below captures the simulation results of the sandpile model with the optimal parameters. We run the simulation repeatedly for 1000 times under the optimal parameter settings $Paramters^*(\cdot)$, and therefore in total have 1000 simulations or observations. As the process of both history and simulations are highly random, dynamic and complex, it is normal and inevitable that we witness some variations other than 22 empires, ranging from 12 to 33 simulated empires. However, the mean of 1000 observations is 22, which reinforces the fitness and robustness of sandpile models and the solution. The standard deviation of 1000 observations is 3.59801622. The percentages of $N = 15$ (3.10%), $N = 16$ (4.00%), $N = 17$ (4.20%), $N = 18$ (7.70%), $N = 19$ (7.00%), $N = 20$ (10.00%), $N = 21$ (10.20%), $N = 22$ (11.10%), $N = 23$ (10.70%), $N = 24$ (8.90%), $N = 25$ (7.30%), $N = 26$(5.00%), $N = 27$(4.30%), $N = 28$ (2.30%), and $N = 29$(1.70%) can be calculated and visualized in Fig. 1. The whole probability density function is well centered and symmetric to the mean value of 22 empires. The subfigure of Q-Q normal plot indicates that the distribution is close to the normal distribution. Hence, Fig. 1 indicates that the sandpile model is able to simulate and back-calculate the rise and fall dynamics of empires in history.

5.2. The distribution matching of empire durations

As the sandpile model well unveils the empire dynamics, we explore the robust outcomes of 1000 simulations to check the matching degrees. Then, we focus on the exact matching cases where 22 dynasties are generated by simulations. According to Fig. 1, we have 110 cases with $N = 22$ dynasties, out of 1000 simulations. We check the fitness of both discrete and continuous distribution forms of empire durations: (a) For the discrete form of duration distributions, we compare the histograms of simulations with historical reality. Reviewing the history of 2132 years, most empires in China have durations or lifespans less than 40 years, and they account for nearly half of 22 empires. So we set 40 years as the segmentation unit to classify 22 empires into different intervals. Fig. 2 below indicates the high fitness of the sandpile simulations. The subfigure 2A1 shows the histogram of 22 dynasties or empires in China's history. It provides the exact numbers of empires within different ranges. For instance, we have 9 empires with durations less than 40 years, 4 empires less than 80 years, 2 empires less than 120 years, 1 empire less than 160 years, 2 empires less than 240 years, and 3 empires that last for over 240 years. There are also coexisting dynasties in history, and small durations errors caused between the real history and simulations will be handled in the following sections. The subfigure 2A2 refers to the best matching simulation with 22 empires, which has the same serial of values as subfigure A1 within all the intervals of empire duration. It indicates in Fig. 2A3 that the distributions of 22 empires ($N = 110$) are well matched because there are 9.154 (≈9), 3.667(≈4), 2.18(≈2), 1, 1.769(≈2), and 3 empires within respect intervals; and (b) the Kernel Density Estimation (KDE) is applied to estimate the continuous probability function of empire distributions. As a non-parametric pathway, the Kernel Density Estimation helps to understand the probability density more clearly by weighting the distances of all empire durations. We transform the discrete values of real empire lifespans into continuous values, and the probability density curves based on KDE function can be

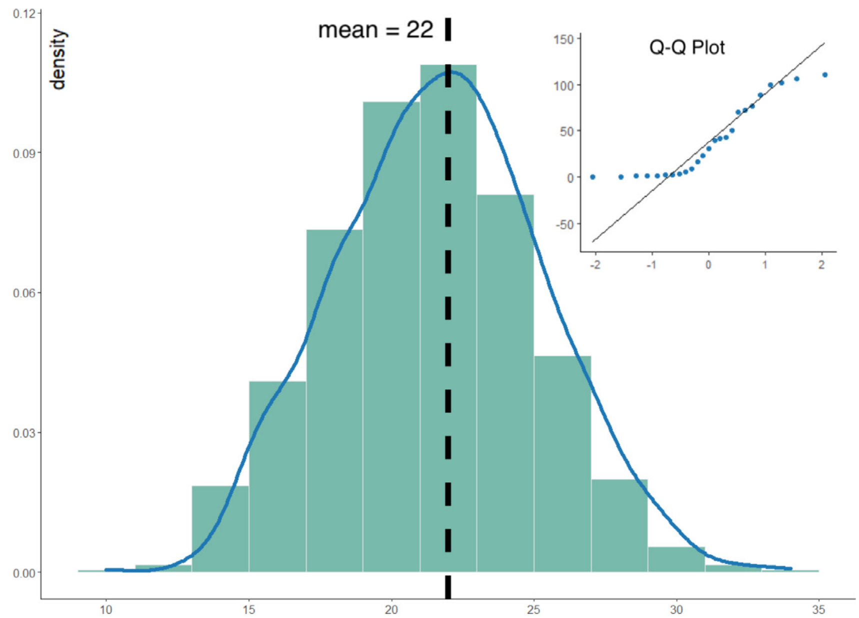

Fig. 1. The distribution of simulations under the optimal parameters ($N = 1000$). The main figure shows the probability density distribution of the empire number for the 1000 sandpile model simulations under the optimal solution of parameters. For each simulation, the generated number of empires may be different, and we have 1000 observations. The mean value of empires is 22, and the standard deviation is 3.598. The subfigure Q-Q normal plot check the normality. The whole process of 2132 years is divided into different empires, and it ranges from 12 to 33 dynasties. The x-axis coordinates show the number of simulated empires, and y-axis coordinates is the probability density. It presents a normal distribution.

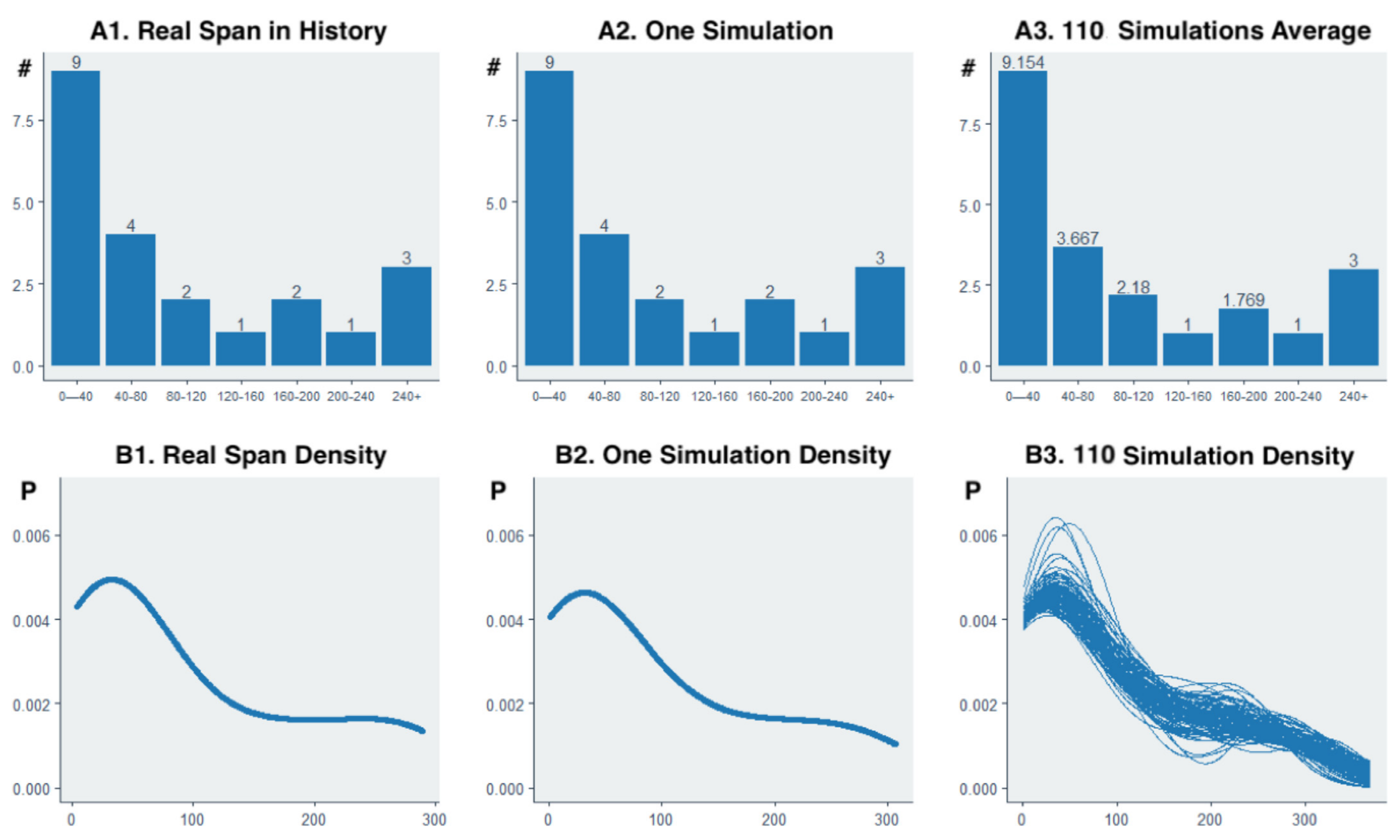

Fig. 2. The distribution matching of sandpile simulations. Subfigures 2A1, 2A2, and 2A3 show the distributions of real empires, one simulation empires, and 100 simulation empires. The y-axis is the number of empires, and x-axis refers to the span intervals of empires; Subfigures 2B1, 2B2, and 2B3 visualize the kernel density functions of real empires, one simulation empires, and multiple simulations. The y-axis refers to the probability, and the x-axis refers to the duration values of empires.

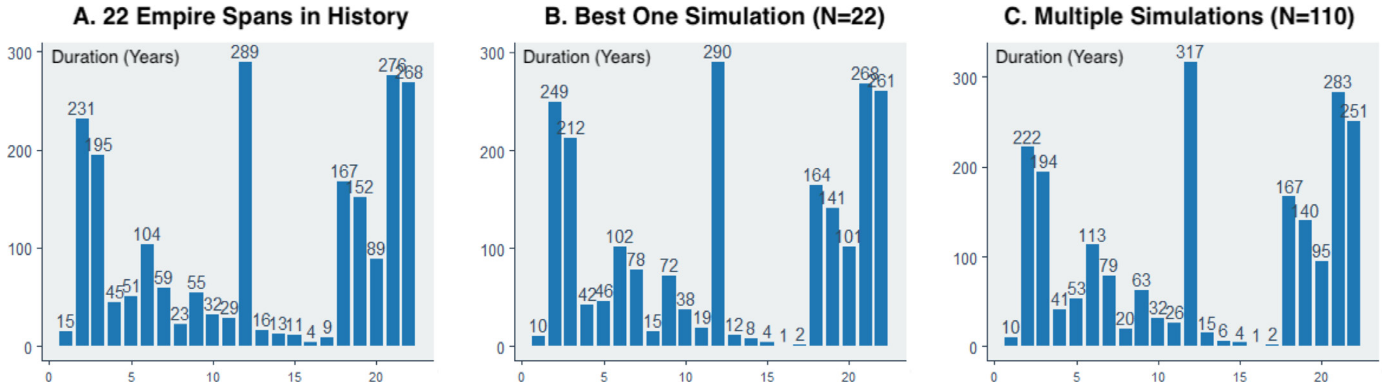

Fig. 3. The matching of 22 empire durations in simulations. We compare empires durations of empires in history and simulations. For each subfigure, the x-axis refers to the order of empires in history, and the y-axis refers to the duration (years). Fig. 3A visualizes the durations following the historical order of China; Fig. 3B captures the durations of one of the best simulations; and Fig. 3B provides outcomes of 100 simulations that have 22 empires.

obtained. Fig. 2B1 shows the kernel density function of real empires in history, and Fig. 2B2 refers to the best matching simulation. It seems that their kernel density function curves are quite similar, in terms of both shapes and heights. For Fig. 2B3, we draw the kernel density curves of 22 simulated empires ($N = 110$). To enhance the robustness and visual effects of the probability density curve, 100 smooth curves were drawn on a single graph and they overlap with each other. Line transparency is set to be 0.05. The recombined multiple curves naturally visualize the overall kernel density curve. It indicates obviously in Fig. 2B3 that the overall density curve of 110 simulations striking matches the curves of Fig. 2B1 and Fig. 2B2. Therefore, the smoothing density curve of the real empire spans in history is consistent with the sandpile model and simulations.

It also suggests in Fig. 2 that the three distributions all have longer right tails. Targowski (2009) believes that there are some rules for civilizations or empires in human history [54]. The distribution of empire durations should fit some kind of exponential distribution. Fig. 2 conforms the conclusions of Arbesman (2011) that empire lifetimes follows the memoryless exponential distribution [14]. The trend of distribution has been further examined in the probability density graph. The overall trend for empire duration sequences can be drawn as the smooth curves in Fig. 2B1, B2, and B3, showing the right long-tail distributions of empire duration. The smooth probability density of real durations in Fig. 2A2 and Fig. 2B1 approximatively conforms to an exponential distribution, and so do the smooth curves of once simulation in Fig. 2A2 and Fig. 2B2 and the multiple ($N = 110$) simulations in Fig. 2A3 and Fig. 2B3.

5.3. Exact matching of 22 empire durations

Then we check the one-by-one matching between simulated empires and empires in history. We first extract the 110 cases with exactly 22 empires from the 1000 simulations. In history, the durations of the 22 empires are fixed. Based on the distribution matching of empire durations, we further validate the sandpile model's robustness in terms of the detailed durations of empires. First, we sort durations of both historical and simulate empires in order to exactly match their durations. After that, the 22 pairs of empires can be generated, and each pair has two durations that are close to each other. Finally, we restore the order of empires in history, based on which the simulate empires are also restored. To satisfy the total historical duration that is 2133 years, we may need to optimized the duration of some empires and eliminated some overlapped years. For example, the Yuan Dynasty was founded in 1271, but the Song Dynasty perished in 1279, so we calculate the span of the Yuan Dynasty from 1279. However, this sort of micro operations have little effect on the whole history.

Fig. 3A shows the historical sequence or process of 22 dynasties or empires in China from the Qin Dynasty to Qing Dynasty. The sequence of empire durations in Table 1 are as follows: Qin Dynasty(15 years), Western Han Dynasty(231 years), Eastern Han Dynasty(195 years), Three Kingdoms period (45 years), Western Jin Dynasty(51 years), Eastern Jin Dynasty(104 years), Liu Song Dynasty(59 years), Southern Qi Dynasty(23 years), Southern Liang Dynasty(55 years), Southern Chen Dynasty(32 years), Sui Dynasty(29 years), Tang Dynasty(289 years), Late Liang Dynasty(16 years), Late Tang Dynasty(13 years), Late Jin Dynasty(11 years), Late Han Dynasty(4 years), Late Zhou Dynasty(9 years), Northern Song Dynasty(167 years), Southern Song Dynasty(152 years), Yuan Dynasty(89 years), Ming Dynasty (276 years), and Qing Dynasty(268 years). Among these 22 empires, the Tang Dynasty had the longest duration, which lasted for 289 years. Late Han Dynasty in the period of Five Dynasties and Ten Kingdoms lasted for merely 4 years, the shortest one in history.

Fig. 3B shows the historical trace of the empire drawn according to the optimal simulation data and results, which corresponds to the state and historical process between the Qin Dynasty and the Qing Dynasty. The simulation results match well with China's history in 2133 years. After one-to-one matching, we find that the simulation results can basically restore the historical process, specifically reflected in (1) Historical trend matching. Between the Han and Tang Dynasties, there are many dynasties with a short span. In the second half of history, there are several dynasties with a span of more than 100 years. As shown in the figure. (2) One-to-one empire span match. After the simulation results are matched with the real dynasty one-to-one, the difference between them is not very large, and the error is generally within 10 years. Fig. 3C shows the average results of 100 simulations, which can still well restore the historical evolution process. Fig. 3C shows the averaged results of 100 simulations, which still matches well and reflects the historical process.

5.4. Duration matchings of fewer than 22 empires

The Agent-based modeling and simulations of sandpile model backwards calculate the historical process, which shows the preferable applicability between the empire durations and the natural phenomena. The previous is about the exact duration matching of empires, which means we have 22 empires in both simulations and history. The robustness has been supported as we obtain the exact duration match. To further validate the model's robustness, we loosen the conditions and explore the fitness under other quantities of empires, such as 19, 20, and 21 empires or 23, 24, 25 empires. Under different perspectives or standards of historians, the divisions and classifications of empires are different, especially for some empires with tiny durations less than 10 years. Therefore, we are able to find 19, 20, 21, 23, 24, and 25 pairs of empires from simulations in Fig. 4 and historical records. For each pair of simulated empire and empire in history, we calculate the duration difference as the errors. As the criterion for selecting the best simulation, the aggregated error is calculated by Eq. (2) to measure the general fitness of simulations.

(a) In the 1000 simulation under the optimal parameters, there are 70 cases that generate 19 empires. In history, it is also plausible to review the whole process as is consisted of 19 empires. We drop three empires or dynasties with societal chaos and tiny durations, such as the Late-Han with 4 years, Late-Zhou with 9 years, and Late-Jin with 11 years. Hence, the ensemble of 19 empires in history can be obtained. The remaining 19 empires are listed in Table 2. We compare the 19 simulated empires and 19 empires in history to validate the fitness. Fig. 4A1 visualizes the 19 empires from the earliest Qin empire to the latest Qing empire, and all the empires are following the order in China's history. Fig. 4A2 shows the best one simulation out of the 70 simulations. Generally speaking, the simulation fits the reality quite well. Especially for empires with longer durations, the simulation matches the outcomes very well. For instances, the second and third empires have longer durations 231 and 195 years, which is well-matched in simulations with 237 and 209 years; the largest lifespan in history is the 12th empire that lasts for 289 years, which coincides with the simulations that the longest duration is 305 years; the last three empires have durations of 89 (Yuan), 276 (Ming), and 268 years (Qing), which are also reflected in simulations by 87, 281, and 262 years. In detail, Fig. 4A3 calculates and visualizes the fitness errors for 19 pairs of empires from simulation and history. The estimated smoothing error curve is around zero because the horizontal line (0) is within the area of the smoothing curve. Besides one simulation, we check the outcomes of multiple simulations. As we have $N = 70$ simulations that generate 19 empires, we take the averaged values of empire durations as the robust outcomes and explore the fitness of 70 simulations. Before this, we need to rank the durations of 19 empires, which is the same for situations of 19, 20, and 21 empires. It indicates in Fig. 4A4 that the outcome of multiple simulations fits the reality in history as well. For instances, the second and third empire has 221 and 197 years, the longest duration is 334 years, the last three empires is 99, 290, and 257 years, which is not for away from the reality in history (231, 195, 289, 89, 276, 268). As well, we check the distribution of errors in Fig. 4A5, and it seems the errors fluctuates around the zero values, as the estimated smoothing curve is very close to zero.

(b) In 1000 simulation under the optimal parameters, we have 100 cases that generate 20 empires. If we review the whole history of China as the process containing 20 empires, the Later-Han with 4 years and Later-Zhou with 9 years should be dropped and the Later-Jin with 11 years will be kept. Then, we obtain the 20 empires, which are listed in Table 2. Similarly, we compare the 20 simulation empires and 20 empires in China's history to validate the fitness of the optimal parameters. Fig. 4B1 visualizes the 20 empires in history from the beginning empire (Qin) to the last empire Qing. Fig. 4B2 shows the best simulation result of 20 empires. Generally speaking, the simulation fits the reality quite well. Especially for empires with longer durations, the simulation matches the outcomes very well. For instances, the second (Western Han) the third empire (Eastern Han) in simulations last for 235 and 216 years respectively, and in history, they last for 231 and 195 years; the empires have the longest duration of 308 years, and in history, the highest value is 289 years for Tang Empire; the last three empires in the simulation have 104, 273, and 263 years of duration, and in history, the values are 89, 276, and 268 years, quite close. Fig. 4B3 checks the distribution of error, and it indicates that the mean of errors should be zero. The estimated smoothing curves are covering the horizontal zero line. Besides the best simulation, we validate the overall fitness of 100 simulations with 20 empires. It shows in Fig. 4B4 that they fit the history very well. For instances, the second and third empires in simulations last for 226 and 194 years, and in history, they have durations of 231 and 195 years; the last three empires in the simulation have 96, 289, and 255 years, and in history, they have 89, 276, and 268 years. Therefore, the distances between each pair are quite close. We visualize the fitness errors or 100 simulations in Fig. 4B5, and it suggests that the estimated curve of errors is very close to the horizontal zero line.

(c) Within 1000 simulations under the optimal parameters, we have 102 simulations that have generated 21 empires. Reviewing the whole history process that is composed of 21 empires, we merely drop one empire, the Later-Han with 4 years, and keep the Later-Zhou with 9 years and the Later-Jin with 11 years. Listed in Table 2, we obtain the 21 empires. Then, we compare the 21 simulation empires and 21 empires in China's history to check the robustness and fitness of the optimal parameters. Fig. 4C1 visualizes the 21 empires in history from the beginning empire (Qin) to the last empire Qing. Fig. 4C2 shows the 19 empires for the best simulation. Generally speaking, the simulation fits the reality quite well. Especially for empires with longer durations, the simulation matches the outcomes very well. For example, the second and third empires have durations of 245 and 203 years, the errors are 14 and 8 years; the last three empires have 88, 269, and 253 years, which generates errors of 1, 7, and 15 years. It indicates in Fig. 4C3 that most errors fluctuate around the zero values. In Fig. 4C4 we take the average values of 102 simulations that have 21 empires; the results are closer to the historical reality. For instance, the second and third empires in 102 simulations have 230 and 198 years, and they have 231 and 195 years in history. Their distances are merely 1 and 3 years; the last five empires have 167 (Northern Song), 152 (Southern Song), 89 (Yuan), 276 (Ming), and 268 (Qing) years in the history, and in 100 simulations, they have 170, 142, 95, 285, and 257 years. The paired distances are merely 3, 10, 6, 9, and 9 years. It indicates in Fig. 4C5 that the estimated trend of errors is close to the horizontal zero line.

5.5. Duration matchings of more than 22 empires

During the 1000 simulations, as the distribution of empires' quantity is close to the normal distribution (mean=22), we also have situations that generate more than 22 empires. We have 107 cases with 23 empires, 89 cases with 24 empires, and 73 cases with 25 empires. It indicates that 23, 24, and 25 empires have no significant difference with the typical 22 empires because the least durations out of 23–25 empires are less than 5 years, such as 1, 2, or 4 years. Therefore, we can drop smaller durations accordingly and render 23–25 to be 22 empires in Table 3, without losing essential information of China's history.

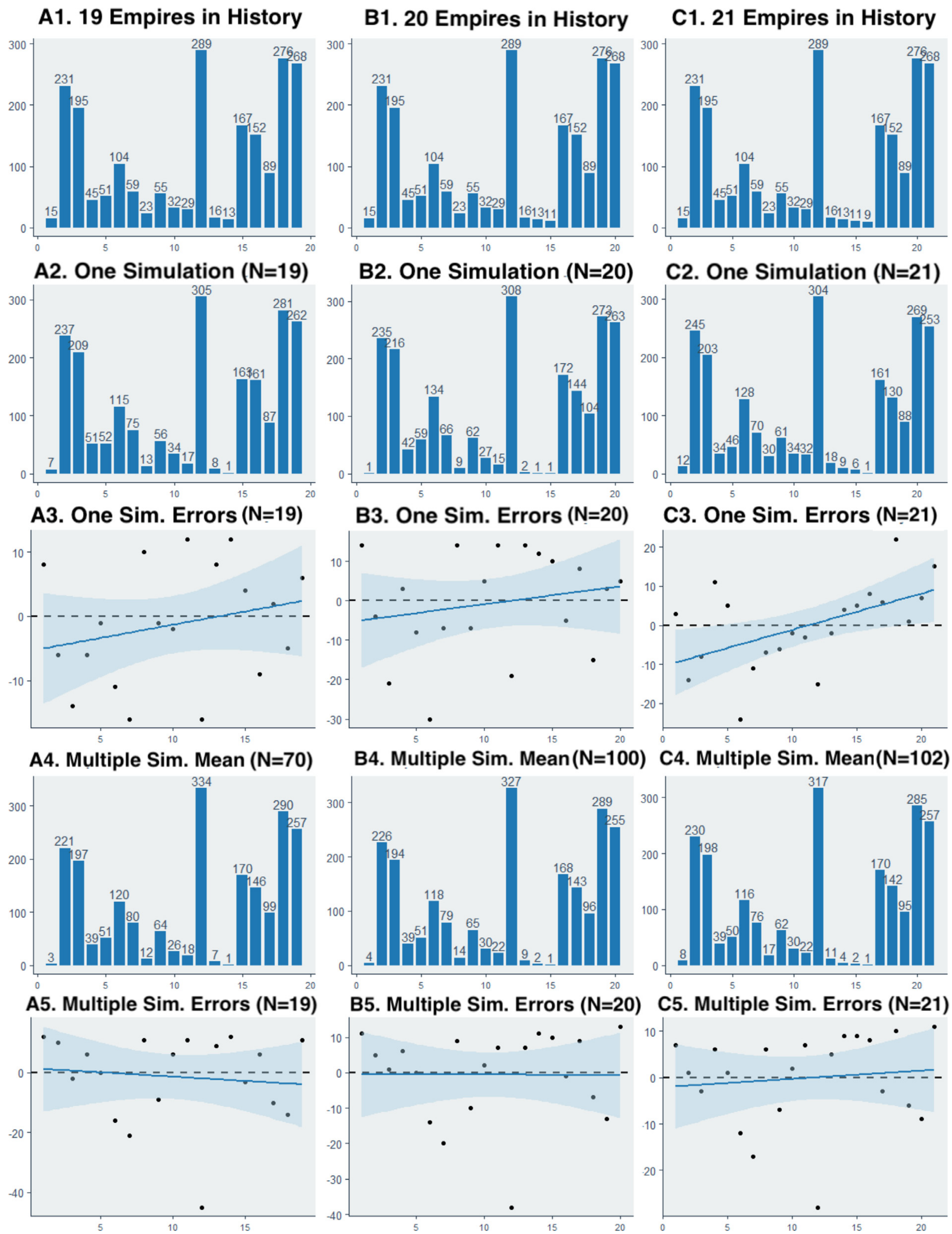

Fig. 4. The matching of fewer than 22 empires. Fig. 4A1 to 4A5 checks the fitness of 19 empires in simulations; Fig. 4B1 to 4B5 visualize the fitness of 20 empires in simulations; and Fig. 4C1 to 4C5 validate the fitness of 21 empires in simulations. For certain numbers (19, 20, and 21) of empires, we check the matching degrees for both the best one and multiple simulations, and we plot the residuals or errors in subfigures A3, B3 & C3, and A5, B5 & C5. For each subfigure, the x-axis refers to the historical order of dynasties, and the y-axis refers to the durations or errors.

Table 2
Simulation Fitness of 19–21 Empires in China.

19 Dynasties Scenario	Durations (History)	Best One Simulation	Best One Difference	Simulation Mean ($N = 70$)	Mean Difference
Qin	15	7	8	3	12
Western Han	231	237	−6	221	10
Eastern Han	195	209	−14	197	−2
Three Kingdoms	45	51	−6	39	6
Western Jin	51	52	−1	51	0
Eastern Jin	104	115	−11	120	−16
Song	59	75	−16	80	−21
Qi	23	13	10	12	11
Liang	55	56	−1	64	−9
Chen	32	34	−2	26	6
Sui	29	17	12	18	11
Tang	289	305	−16	334	−45
Late-Liang	16	8	8	7	9
Late-Tang	13	1	12	1	12
Northern Song	167	163	4	170	−3
Southern Song	152	161	−9	146	6
Yuan	89	87	2	99	−10
Ming	276	281	−5	290	−14
Qing	268	262	6	257	11
20 Dynasties Scenario	**Durations (History)**	**Best One Simulation**	**Best One Difference**	**Simulation Mean ($N = 100$)**	**Mean Difference**
Qin	15	1	4	14	11
Western Han	231	235	226	−4	5
Eastern Han	195	216	194	−21	1
Three Kingdoms	45	42	39	3	6
Western Jin	51	59	51	−8	0
Eastern Jin	104	134	118	−30	−14
Song	59	66	79	−7	−20
Qi	23	9	14	14	9
Liang	55	62	65	−7	−10
Chen	32	27	30	5	2
Sui	29	15	22	14	7
Tang	289	308	327	−19	−38
Late-Liang	16	2	9	14	7
Late-Tang	13	1	2	12	11
Later Jin	11	1	1	10	10
Northern Song	167	172	168	−5	−1
Southen Song	152	144	143	8	9
Yuan	89	104	96	−15	−7
Ming	276	273	289	3	−13
21 Dynasties Scenario	**Durations (History)**	**Best One Simulation**	**Best One Difference**	**Simulation Mean ($N = 102$)**	**Mean Difference**
Qin	15	12	8	3	7
Western Han	231	245	230	−14	1
Eastern Han	195	203	198	−8	−3
Three Kingdoms	45	34	39	11	6
Western Jin	51	46	50	5	1
Eastern Jin	104	128	116	−24	−12
Song	59	70	76	−11	−17
Qi	23	30	17	−7	6
Liang	55	61	62	−6	−7
Chen	32	34	30	−2	2
Sui	29	32	22	−3	7
Tang	289	304	317	−15	−28
Late-Liang	16	18	11	−2	5
Late-Tang	13	9	4	4	9
Late-Jin	11	6	2	5	9
Late-Zhou	9	1	1	8	8
Northern Song	167	161	170	6	−3
Southen Song	152	130	142	22	10
Yuan	89	88	95	1	−6
Ming	276	269	285	7	−9
Qing	268	253	257	15	11

(a) Out of 1000 simulations under the optimal parameters, there are 107 simulations that have 23 empires. We drop tiny durations such as 1, 2, and 4 years, and review the history as a consistence of 22 empires as well, which can be shown in Fig. 3A1. This dynasty reduction process can be denoted as 23 → 22. It indicates the best simulation with the least total errors. Comparing the Fig.s 3A1 and 5A1, the fitness is not bad. For instance, the second and third empires have 226 and 213 years, and the errors are 5 and 18 years, within the 10% of empire spans. For the simulated Northern Song empire, the duration is 165 years, and it is 167 years in history. The distribution of 22 errors is within the domain of zero values, which is shown in Fig. 5A2. Taking the average values, we obtain the robust outcomes of 22 empires, reduced from the 107 simulations of 23 empires. Fig. 5A3 visualizes the reduced 22 empires, and the errors are not significant as well. For instances, the first empire

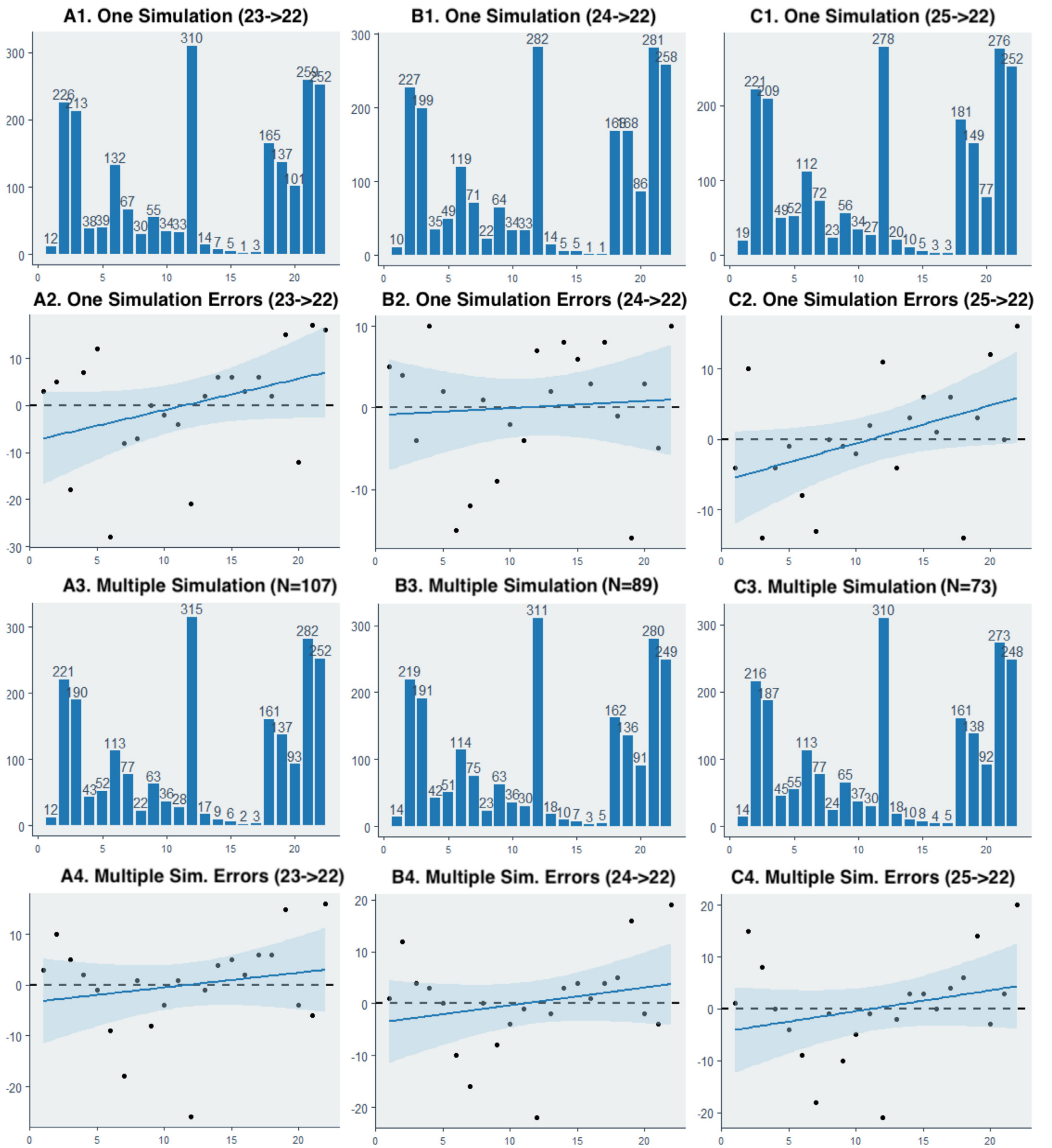

Fig. 5. The matching of more than 22 empires. Fig. 5A1 to 5A4 checks the fitness of 23 empires; Fig. 5B1 to 5B4 visualizes the fitness of 24 empires; and Fig. 5C1 to 4C4 validate the fitness of 25 empires. For 23, 24, and 25 empires, we check the matching degrees for both one and multiple simulations. We plot the residuals or errors in subfigures A2, B2 & C2, and A4, B4 & C4. For each subfigure, the x-axis refers to the historical order of dynasties, and the y-axis refers to the durations or errors.

have 12 years, and the error is 3 years; the second empire has 221 years with an error of 10 years; the third empire with 190 years has the error of 5 years; the third empire from the bottom (Yuan) generates the error of 4 years; the second empire (Ming) from the bottom has an error of 6 years. The results of multiple simulations ($N = 107$) are robust as well, which can be validated in Fig. 5A4.

(b) Under the optimal parameters, we have 89 simulations with 24 empires. Eliminating tiny durations or empires (1 or 2 years), the whole history of 2132 years can also be deemed as the

Table 3
Simulation Fitness of 22–25 Empires in China.

Dynasty Durations		22 Empires		23 Empires		23 Empires		25 Empires	
		Best Sim	Simulations	Best Sim	Simulations	Best Sim	Simulations	Best Sim	Simulations
Qin	15	10	10	12	12	10	14	19	14
Western Han	231	249	222	226	221	227	219	221	216
Eastern Han	195	212	194	213	190	199	191	209	187
3 Kingdoms	45	42	41	38	43	35	42	49	45
Western Jin	51	46	53	39	52	49	51	52	55
Eastern Jin	104	102	113	132	113	119	114	112	113
Song	59	78	79	67	77	71	75	72	77
Qi	23	15	20	30	22	22	23	23	24
Liang	55	72	63	55	63	64	63	56	65
Chen	32	38	32	34	36	34	36	34	37
Sui	29	19	26	33	28	33	30	27	30
Tang	289	290	317	310	315	282	311	278	310
Late-Liang	16	12	15	14	17	14	18	20	18
Late-Tang	13	8	6	7	9	5	10	10	10
Late-Jin	11	4	4	5	6	5	7	5	8
Late-Han	4	1	1	1	2	1	3	3	4
Late-Zhou	9	2	2	3	3	1	5	3	5
Northern Song	167	164	167	165	161	168	162	181	161
Southen Song	152	141	140	137	137	168	136	149	138
Yuan	89	101	95	101	93	86	91	77	92
Ming	276	268	283	259	282	281	280	276	273
Qing	268	261	251	252	252	258	249	252	248

entity consisting of 22 empires as well, which is denoted as $24 \to 22$. By comparing the 22 empires in history in Fig. 3A1, we are able to investigate the robustness of simulations. For the best simulation in Fig. 5B, the fitness is well supported. For instance, the first empire has an error of 5 years, the second has 5 years, and the third has 4 years. For the longest empire (Tang), the error is $7 = 289–282$ years. For the 18th empire (Northern Song), the error is merely one year. The errors of the 20th (Yuan) and 21st (Ming) empire are 3 and 5 years. By plotting the distribution of errors in Fig. 5B2, it is found that the estimated curve is around the horizontal zero line. Then we explore the robust (averaged) outcome of 89 simulations, and it seems in Fig. 5B4 that errors are close to zero as well. Fig. 5B3 provides the list of averaged empire durations according to the historical order. The first empire has an error of 1, and the third has 8 years. For the 18th empire (Northern Song), the error is 5 years. The errors of the 20th (Yuan) and 21st (Ming) empire are 2 and 4 years, respectively.

(c) We have 73 cases with 25 empires among the 1000 simulations. After excluding three tiny durations less than 5 years, we obtain the 22 dynasties in Fig. 5C1 that match the history records. By comparing Fig. 1A1 and Fig. 5C1, it suggests that the reduced 22 empires in simulations fit the historical reality quite well. For instance, the first, second and third empires have the errors of 4, 10, and 14 years. The 19th empire has an error of 3 years (152–149), and the 21st empire has 0 error (276–276). The distribution of 21 errors is visualized in Fig. 5C2, and it seems that errors are fluctuating around the mean of 0. Then, we calculate the outcomes of multiple simulations ($N = 73$) as well, which is shown in Fig. 5C3. We check the fitness by comparing it with Fig. 1A1. The first, second, third empires have the errors of 4, 15, and 8 years. The 18th empire has an error of 6 years, and the 20th empire is with the error of 6 years, and 21st empire is 3 years of error.

6. Conclusions and discussions

The life cycle of empires is robust and solid in history. For quantitative researches of empires, the application of agent-based modeling is the principal methodology contributions of this work. Previously, many models were developed to explain the life cycle dynamics of human empires. Taacepera (1978) formulates the growth and decline pattern of empires from history [7]. Khmaladze (2007) presented the exponential distribution on Roman Emperors [12], and Arbesman (2011) demonstrated the memoryless exponential distribution of empires [14]. Marchetti (2012) as well developed a logistic model for the formation and breakdown of empires, with the territorial expansion as the basic parameter [15]. Taking taxation, public expenditure, and corruption as main indicators, Chen (2014) introduced the stochastic growth model to analyze the dynastic cycles in Imperial China [55]. Existing researches have provided necessary parameters and diverse models, which facilitate the agent-based modeling and simulations of empires. We apply the self-organized criticality and sandpile model to explore the rise and fall mechanism of empires in history. The Agent-based Modeling has reinforced and confirmed the self-organizing criticality, showing the similarity between natural and social systems. The rising and falling of empires follow the self-organized criticality rule. When the degree of system chaos and social instability reaches the threshold or tipping point, the phase change or collapse of empires will certainly take place. The evolution system of empire is a typical extended dissipative dynamic system, which is far away from the state of equilibrium. The evolution system of empire has great complexity and periodicity. Evolutions of empires with the critical states can be investigated using the sandpile modeling. Our research shows that:

(a) The social system and the physical system are isomorphic. Both natural and societal systems have a similarity [56]. The similarity between the sandpile model and the empire system is obvious because of the internal linkages between the dynamic mechanism of the empire and the sandpile. First, the process of sand accumulation has a similarity to the social stress accumulation within the stratification structure of empires. This accumulation process may be ~~linearor~~ nonlinear; Secondly, the critical states will be reached automatically for sandpile and empire. As a complex system that contains many components and their interactions and factors, researchers have found the simple and ideal model to capture the basic characteristics of human empire systems. When the sandpile reaches the critical state, the sand particles are in the state of short-term equilibrium. The more sand particles added may lead to large-scale collapse or even a series of chain reactions, which is similar to the chaotic falling of empires. When the social problems accumulated to certain degrees, a small crisis may result in the collapse of the existing empires.

(b) The life cycles of empires are solid and robust across the whole history. Just as the individual has the life cycle of birth, youth, adult, and aging, the empires also have the life cycle of birth, growth, peak, and vanish. Besides, the firms, industries, and rumors and infections also share a similar life cycle pattern [25,57].For all the empires in both China and the world, the rising and falling of empires will definitely take place, which is not affected by any set of factors. In history, the destruction of the old empire always come up with the rising of new empires. Therefore, the historical trend or pattern that old empires will be replaced by new empires are also inevitable. No matter what measures the ruler will take and what initial conditions it has, the empires in history will automatically fall and vanish eventually. The solid life cycle pattern of empires in history is persistent and stable, which, therefore, can be explained by the self-organized criticality and investigated by the sandpile modeling and simulations.

(c) The rising and falling of empires follow the self-organized criticality rule. Each empire has a unique duration or lifespan. With the growth of empires, the social structure problem continues to be deepened, which eventually weakens the foundation and leads to the definite collapse [58]. When a society is under a completely chaotic state, it is impossible to handle or control. As the problem of societies remains unsolved, the destiny of death and rebirth of empires will be persistent. Therefore, it is inevitable that empires have life cycles and old empires will be replaced by new ones. For the self-organized criticality, the sandpile model captures the automatic process of empires to evolve into the critical state without adjusting parameters. The evolution of the system is the interactions of elements, which caused the self-organized criticality. The empire system is also influenced by social classification, fiscal revenue, foreign threats, population growth, cultivated land, climate, and their interactions [59,60].

(d) The sandpile modeling and simulations robustly back-calculate and predict the dynamics of empires. Several tools of calculating fitness have been applied here to find the optimal solution and validate the robustness of the sandpile model. The matching degree between simulated empires and empires in history is high enough to reach this conclusion. First, the number of empires in history can be matched. Under the optimal parameters, 1000 simulations are conducted and the distribution is close to the normal distributions ($N = 1000$), and the mean value is 22 empires, which coincides with the history; second, the distribution of durations can be perfectly matched. In history, the duration distribution of empires has a longer right tail. Many empires have durations shorter than 40 years, and others have longer lifespans. The distribution of simulated empires also has the longer right tails. Their continuous density functions also well match each other. The lifespan distribution of both historical and simulation empires follow the power exponential distribution. Besides of the macro-level distribution match, the detailed observations of empire durations can be matched as well. We can find matchings for 19, 20, 21, 22, 23, 24, and 25 pairs of simulated and historical empires.

(e) Multiple simulations have higher fitness than One simulation. In 1000 simulations of the optimal parameters, we have multiple simulations for situations of 19, 20, 21, 22, 23, 24, and 25 empires. Both the outcomes of one simulation and multiple simulations are provided, and the averaged multiple values are taken as stable and overall values. Generally speaking, the robustness of multiple simulations is better off than the one (best) simulation. In Fig. 2, curves of multiple simulations can visualize the possible boundary of the duration's distribution; In Fig. 4, the error distribution of multiple simulations is closer to the zero than one simulation if we compare Fig. 4A3 & 4A5, 4B3 & 4B5, and 4C3 & 4C5; this is also supported in Fig. 4, the multiple simulations have more zero errors than one simulation, if we compare Fig. 5A2 & 5A4 and 5C2 & 5C4. Therefore, it can be concluded that multiple simulations have more robust results than one simulation, as the estimated smoothing curve is closer to the horizontal zero line.

(f) The agent-based modeling and simulations support previous theories of empire dynamics, since the beginning of empires. Ancient Chinese scholars or elites used the Five Elements Theory (五德终始说) to justify or explain the replacement of old empires by new empires [61]. Interpreted as mysterious legends, the Five Elements Theory was not a scientific way of thinking and understanding the world. Fan (2010) suggested that the cycle of empires in ancient China was mainly caused by climate change [62]. Modelski (1988) found that empires' maritime hegemonies conform to the long cycle theory [57]. Each empire or hegemon dominants for certain phases and doomed to collapse. Each empire has four stages, such as global war, the emergence of world power, delegitimating of power, and deconcentrating of power. The long cycle theory can also explain the cyclical pattern in international relations [25]. Likewise, Barfield (1991) suggested that the relationship of the unified Chinese dynasties and the nomadic empires was a regular cycle [63]. The dynasty cycle of China and Nomadic Empires takes the forms of the Han & Hsiung-Nu (匈奴), Tang & Turkish (突厥, Song &Yuan (Mongol), and Ming & Qing dynasty. Although these theories can explain the rises and falls of empires, they lack the broader perspective to predict the whole history. Unified by the self-organized criticality, we consider natural systems and human empires as the same process and mechanism, such as resonance or synchronization, which provides the scientific way to think the society and history.

7. Limitations and future directions

Although the self-organized criticality and sandpile model can explain the growth and decline of empires, some high-definition process still remains beyond the capability of our modeling and simulations. We cannot simulate or draw the historical serial of 22 empires for one time. For now, the duration matching is achieved by first ranking simulated durations and then match the real historical durations accordingly. Besides, there exist different versions of empires in history, such as 22, 23, or 24 empires, which causes more complexity. The theory and method of self-organized criticality are very enlightening for the study of the historical process of empire evolution. For future researches, we plan to extend this research from China empires to global empires in human history. The next research will introduce the historical evolution of other empires; each country is a sandpile, the whole world forms a large sandpile system, the local sandpile evolution eventually leads to the collapse of the whole world. Besides, the life cycle pattern can be applied to other social systems or entities, such as organizations, firms, diseases, rumor spreading, and industries, where the self-organized criticality and life cycle theory can be applied.

Declaration of Competing Interest

The authors declare that there are no interest conflicts exist.

CRediT authorship contribution statement

Peng Lu: Conceptualization, Supervision, Formal analysis, Funding acquisition, Writing - original draft. **Hou Yang:** Data curation, Methodology, Investigation. **Mengdi Li:** Software, Validation. **Zhuo Zhang:** Visualization, Software, Writing - review & editing, Methodology.

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