Appendix A - MILP configuration formulation

Variables and indices

i, j: Node indices, where $i = 0$ is the transformer (root), and $i \in \{1, \dots, n\}$ are demand nodes

x_{ij}: Binary variable, equal to 1 if an edge from node i to node j is selected, 0 otherwise

d_i: Continuous variable, representing the cumulative LV path length from the transformer to node i

D_{ij}: Euclidean distance between node i and j

D^{max}: Maximum allowable cumulative LV path length from the transformer to any demand node

M: A large positive constant in the Big-M formulation

Objective function

The objective is to minimize the total length of the LV network, as shown in (1).

$$minimize \sum_{i=0}^{n} \sum_{j=0}^{n} D_{ij} \cdot x_{ij} \tag{1}$$

Constraints

Each node must have exactly one incoming edge, forming a tree structure, as shown in equation (2). The transformer, designated as node 0, receives electricity from MV line, so it has no incoming edge, as shown in equation (3).

$$\sum_{i \neq j} x_{ij} = 1 \quad for\ j \in \{1, \dots, n\} \tag{2}$$

$$\sum_{i \neq 0} x_{i0} = 0 \quad for\ i \in \{1, \dots, n\} \tag{3}$$

To enforce the path length constraint, Equation (4) defines the cumulative distance from the transformer to each node. If an edge from i to j is selected, the distance to j is at least the distance to i plus the length of that edge. A large constant M ensures this constraint is only active when the edge is selected.

$$d_j \geq d_i + D_{ij} - M \cdot (1 - x_{ij}) \quad for\ i \neq j \tag{4}$$

The transformer's path length is set to zero as the starting node of the tree.

$$d_0 = 0 \tag{5}$$

Each node path length must remain within the allowable distance, D^{max}, as enforced by Equation (6).

$$d_j \leq D^{max} \quad for\ j \in \{1, \ldots, n\} \tag{6}$$