CHAPTER

2

Frequency-Domain Models

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2.1 INTRODUCTION AND FUNDAMENTAL PRINCIPLES

The hydrodynamic interaction between wave energy converters (WECs) and ocean waves is a complex high-order nonlinear process that, under particular conditions, can be simplified. This is true for waves and small-amplitude device oscillatory motions. In this case the hydrodynamic problem is well characterized by a linear approach, which in general is acceptable throughout the device's operational regime (Falnes, 2002). In addition, whenever a linear representation of reactive forces, such as moorings (using a linear spring) and power take-off (using either a linear damper or a linear spring-damper system) is used, the original nonlinear WEC dynamics are completed described by linear equations. This means that the superposition principle applies[1] (Denis, 1973), and linear combinations of simple solutions can be used to form more complex solutions.

[1] In linear systems theory, the superposition property states that the resultant response caused by two or more stimuli (harmonic in frequency domain) is simply the algebraic sum of the responses caused by each stimulus individually. The principle holds only for linear equations. It does not apply if the equations are not linear.

Under these circumstances, the first step in modelling the WEC dynamics is traditionally carried out in the frequency domain, where the excitation is of a simple harmonic form. Accordingly, all the physical quantities vary sinusoidally in time with the same frequency of the incident wave. Therefore, the inhomogeneous equations of motion become a system of algebraic linear equations that may be solved straightforwardly. The main challenge in a frequency-domain analysis is the determination of the radiation and excitation loads on the captor (the body or bodies that interact directly with the waves). This typically relies on the application of boundary element methods (BEMs) (also referred to as boundary-integral equation methods (BIEMs) or panel methods) to estimate the hydrodynamic coefficients of added mass and damping and the excitation force per unit incident wave amplitude.

The BEM is used widely in computational solutions of a number of physical problems such as acoustics, stress analysis and potential flow. In wave-structure hydrodynamic interactions the fundamental basis of this method is a form of Green's theorem, where the velocity potential at any point on the body wetted surface is represented by distributions of singularities (sources or dipoles) over the body discretised surface (Newman, 1977; Linton and McIver, 2001). This

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leads to an integral equation that must be solved for the unknown source strength or dipole moment. Global quantities, including the hydrodynamic added mass and wave radiation damping coefficients and exciting force components, can then be obtained from the velocity potentials (Hess and Smith, 1994) for specified modes, frequencies and wave headings.

Accurate numerical approximations of the free-surface Green function, which are valid for all ranges of frequency and water depth, were developed by Newman (1985, 1992). Based on this development, and with the use of an iterative method of solution of the linear system developed by Lee (1988) and Korsmeyer et al. (1988), it is possible to determine the hydrodynamics of complex offshore structures. The numerical methodologies of Newman and Lee led to the development of the frequency-domain, free-surface, radiation/diffraction code WAMIT (Lee and Newman, 2013), which has been widely used for offshore and naval problems, wave-structure interaction and wave energy conversion. Subsequently, other BEM codes, such as ANSYS-AQWA (http://www.ansys.com/Products/Other+Products/ANSYS+AQWA), Moses (http://www.ultramarine.com) and the open source code NEMOH (http://www.lheea.ec-nantes.fr/cgi-bin/hgweb.cgi/nemoh) dedicated to the computation of first-order wave loads on offshore structures, have been developed. These codes have been essential, and typically the first step, in the evaluation of WEC technologies, due to their satisfactory accuracy and relatively low computation effort.

Since the 1980s a wide range of WECs with different working principles have been modelled using linear potential theory and BEM codes. In 1980, Standing (1980) predicted the power absorption efficiency and the reaction forces of a submerged pitching 'Duck'. In 1992, Pizer numerically modelled a pitching device called the Salter's Duck (Pizer, 1993). Later, Yemm et al. (1998) and Pizer et al. (2000) modelled the Pelamis wave energy converter, a hinged attenuator concept. In 1998, Brito-Melo et al. (1998) presented an adaptation of the BEM code AQUADYN (Delhommeau et al., 1992) to study the dynamic behaviour of oscillating water columns (OWCs). Babarit et al. (2005), Josset et al. (2007), and Ruellan et al. (2010), modelled the SEAREV, a floating oscillating body completely enclosed with an internal moving mass. Moreover, Folley et al. (2007a,b), and Renzi and Dias (2012) modelled a concept similar to Oyster, a bottom-hinged flap device. Farley et al. (2011) modelled the flexible Anaconda device, a submerged flooded rubber tube aligned with the predominant wave direction. Furthermore, Babarit et al. (2012) developed a numerical benchmark of a wide range of WECs.

The aforementioned list of works of modelling studies is nonexhaustive, since nowadays the first step in modeling WECs is almost universally a frequency domain analysis and the application of BEM codes. A more comprehensive review on the use of BEM codes to model wave energy devices is given by Payne et al. (2008).

2.2 PHENOMENOLOGICAL DISCUSSION

Essentially, the numerical modelling of WECs is based on Newton's second law, which states that the inertial force is balanced by the whole forces acting on the WEC, schematically represented in Fig. 2.1. These forces are usually split into external (hydrodynamic/hydrostatic) loads and reaction forces. The external (hydrodynamic/hydrostatic) loads include:

- *Hydrostatic force* caused by variation of the hydrostatic pressure distribution due to the oscillatory motion of the captor,
- *Excitation loads* due to the action of the incident waves on a motionless captor,

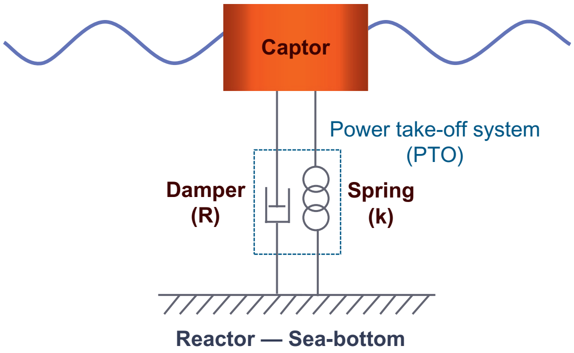

FIG. 2.1 Schematic representation of a generic wave energy converter (WEC).

- *Radiation force* corresponding to the force experienced by the captor due to the pressure field alteration as a result of the fluid displaced by its own oscillatory movement, in the absence of an incident wave field.

Furthermore, depending on the type of WEC, the reaction forces may be caused by the

- *Power take-off* (*PTO*) equipment, which converts mechanical energy (captor motions) into electricity or other useful energy vector,
- *Mooring/foundation system*, responsible for the WEC station-keeping,
- *End-stop* mechanism, used to decelerate the captor at the end of its stroke in order to dissipate the kinetic energy gently, and so avoid mechanical damage in the device.

2.3 POTENTIAL FLOW THEORY

This section provides an overview of the most important considerations and the fundamental equations of potential flow theory. This theory is considered in some depth here because of its fundamental relevance to a large number of other models. The section starts by developing the Laplace equation, which is fundamental to solving the potential flow. This is followed by defining the boundary conditions for the water surface, the body surface and the seabed. The solution of the Laplace equation is then defined for sinusoidal waves and finally the decomposition of the solution into incident, diffracted and radiated waves is described.

Potential flow theory is based on the assumption of ideal flow, ie, inviscid (frictionless) and irrotational. An inviscid flow is a flow in which there are no viscous shear stresses to deform fluid elements or cause fluid particle rotation; only normal stresses are observed. Furthermore, an irrotational flow is a flow where the fluid elements do not rotate relative to their own centre of gravity (although, they can describe circular trajectories). Therefore, in essence the potential theory states that if an inviscid flow is initially irrotational then it remains irrotational at all subsequent times.

Although all real fluids are viscous, in particular conditions the effects of viscosity are sufficiently small that frictional effects may be negligible and so the fundamental hypothesis of potential flow theory may be considered valid. This is the case in high Reynolds number flows, where viscous forces are relatively insignificant and so the flow is essentially inviscid. An example of this type of flow is the hydrodynamic interaction between ocean waves and WECs for small-amplitude wave and body motions. However, it is important to recognize that whenever the WEC is interacting with extreme waves, viscous effects may become large and the body motions may become highly nonlinear and so potential theory become invalid.

2.3.1 Laplace Equation

Under the assumption of incompressible flow, which is an acceptable approximation for liquids, the continuity equation becomes

$$\nabla u = 0, \tag{2.1}$$

where $u(x, y, z)$ denotes the fluid velocity vector. In addition, assuming that the flow is irrotational, the curl of u is zero, ie,

$$\nabla \times u = 0. \tag{2.2}$$

Under these conditions and taking into account that the curl of a gradient vanishes, the water velocity can be expressed in terms of a velocity potential, $\phi(x,y,z,t)$, according to

$$u = \nabla\phi. \tag{2.3}$$

Then, introducing Eq. (2.3) into (2.1) it follows that in the fluid domain the velocity potential must satisfy the second-order partial differential equation

$$\nabla^2\phi = 0. \tag{2.4}$$

Eq. (2.4) is known as Laplace's equation, named in honour of Pierre-Simon Laplace, who first studied its properties.

2.3.2 Boundary Conditions

In the context of wave energy, the flow has to satisfy Laplace's equation in the fluid domain together with a set of appropriate additional constraints, called boundary conditions, defined at the free surface, seafloor and on the body (captor) surface.

On the water's free surface there are dynamic and kinematic boundary conditions. The dynamic boundary condition results from the free surface inability to withstand pressure differences. Applying the Bernoulli equation on the free surface, given that it reflects the fluid mechanical energy conservation and so it is valid in every point of the fluid domain, it follows that

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + \frac{p_0}{\rho} + g\eta = C \quad \text{on}\, z = -\eta(x,y,t), \tag{2.5}$$

where η denotes the elevation of the water free surface from its equilibrium position, being positive when the surface is above the xy plane. Commonly, p_0 is the atmospheric air pressure and the constant C set to be equal to this divided by the water density, ρ, in still water conditions so that Eq. (2.5) may be rewritten as

$$\frac{\partial\phi}{\partial t} + \frac{1}{2}(\nabla\phi)^2 + g\eta = 0 \quad \text{on } z = -\eta(x,y,t). \tag{2.6}$$

The kinematic boundary condition is basically an impermeable condition. It requires that the component of the fluid velocity normal to the surface must equal the surface velocity. This condition takes the form

$$\frac{\partial\eta}{\partial t} + \frac{\partial\phi}{\partial x}\frac{\partial\eta}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\eta}{\partial y} + \frac{\partial\phi}{\partial z} = 0 \quad \text{on } z = -\eta(x,y,t). \tag{2.7}$$

Eqs (2.6), (2.7) are nonlinear, as there are second-order terms resulting from the multiplication of unknowns, since they are applied over an unidentified position of the free surface. To solve this problem, linear theory is applied based on the assumption that the wavelength is much larger than the wave amplitude. As a result, the quadratic terms are an order of magnitude smaller than the remaining terms, and so can be neglected. Linear theory also considers that the boundary conditions on the water's free surface are applied at its equilibrium (undisturbed) position instead of on its instantaneous position. Subsequently, the linearization of Eq. (2.6) to (2.7) yields, respectively,

$$\frac{\partial\phi}{\partial t} + g\eta = 0 \quad \text{on } z = 0 \tag{2.8}$$

and

$$\frac{\partial\eta}{\partial t} + \frac{\partial\phi}{\partial z} = 0 \quad \text{on } z = 0. \tag{2.9}$$

The introduction of Eq. (2.9) into the resultant time derivative of Eq. (2.8) gives a general form of the free surface boundary condition, described by

$$\frac{\partial^2\phi}{\partial t} + g\frac{\partial\phi}{\partial z} = 0 \quad \text{on } z = 0. \tag{2.10}$$

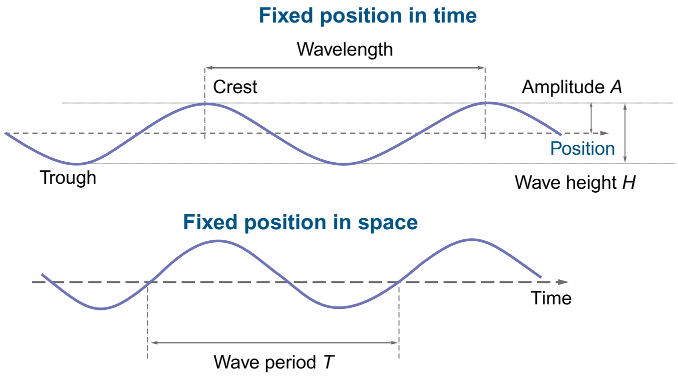

FIG. 2.2 Main characteristics of monochromatic waves.

The boundary condition over the body-fluid interface, the impermeable boundary condition, requires that the component of the fluid velocity normal to the body surface, u_n, must be equal to the body velocity on the direction normal to its surface. In a linearized form, this boundary condition (defined on the body equilibrium position, ie, the mean wetted body surface) is given by

$$\frac{\partial \phi}{\partial n} = u_n. \tag{2.11}$$

Moreover, assuming that the seafloor is flat at a depth $z=-h$, an additional impermeable boundary condition over the seafloor is required to impose a null vertical component of the fluid velocity. This condition is written as

$$\frac{\partial \phi}{\partial z} = 0 \quad \text{on } z=-h. \tag{2.12}$$

Finally, the last boundary condition that must also be satisfied is the radiation condition. As is physically expected, this boundary condition imposes that far from the oscillatory body the wave field should appear undisturbed or similar to the incident wave field. That is to say, the potential that satisfies the radiation condition must decay as the distance from the body increases. By applying the energy conservation principle, the magnitude of the potential should decrease with the inverse of the square root of the distance. Accordingly, the radiation boundary condition is stated as

$$\phi \propto (kr)^{-1/2} \mathrm{e}^{-\mathrm{i}kr} \quad \text{as } r \to \infty, \tag{2.13}$$

where r is the radial distance from the body and k the wave number, related to the wave frequency by the dispersion relation, given by

$$\frac{\omega^2}{g} = k \tanh kh. \tag{2.14}$$

2.3.3 Sinusoidal Waves

In infinite water depth, or water of arbitrary but uniform depth, the effortless solution of the previously discussed boundary value problem takes a sinusoidal form (see Fig. 2.2).[2]

This approach allows decomposing the problem into spatial and temporal dependencies, and so each quantity can be defined entirely by a frequency-dependent function or complex amplitude and a sinusoidal time dependence with unit amplitude, $\mathrm{e}^{\mathrm{i}\omega t}$. As a result, the velocity potential can be written as

$$\phi(x,y,z,t) = \mathrm{Re}\{\hat{\phi}(x,y,z)\mathrm{e}^{\mathrm{i}\omega t}\}, \tag{2.15}$$

[2] It is practical to consider single monochromatic waves as they correspond to the basic problem and also because afterwards it is straightforward representing, in accordance with the linear theory, irregular sea-states as a superposition of sinusoidal waves with different periods and amplitudes.

where the hat sign, $\hat{}$, denotes complex amplitudes. According to the previous decomposition, the equation acquires the form

$$\nabla^2\hat{\phi} = 0 \text{ in the fluid domain.} \tag{2.16}$$

Similarly, the boundary conditions become

$$\frac{\partial\hat{\phi}}{\partial n} = \hat{u}_n, \text{ over the body mean wetted surface,} \tag{2.17}$$

$$\frac{\partial\hat{\phi}}{\partial z} = 0 \text{ on the seafloor, } z = -h, \tag{2.18}$$

$$-\omega^2\hat{\phi} + g\frac{\partial\hat{\phi}}{\partial z} = 0 \text{ on the free surface, } z = 0. \tag{2.19}$$

2.3.4 Problem Decomposition

Under the linearity assumption it is possible to describe the resultant wave field around a WEC or any other floating body as a superposition of an incident, a diffracted and a radiated wave field. In this context, the incident field is defined as a plane propagating wave in the absence of the body. The diffracted wave field results from the interaction between the incident wave and a motionless body and, finally, the radiation wave field is produced by the body oscillatory motions in calm waters, ie, in the absence of an incident wave field. Therefore, the total velocity potential may be decomposed as

$$\phi = \phi_{\mathrm{D}} + \phi_{\mathrm{r}}, \tag{2.20}$$

where

$$\phi_{\mathrm{D}} = \phi_0 + \phi_{\mathrm{s}}. \tag{2.21}$$

In Eq. (2.21) ϕ_{s} and ϕ_0 represent the scattered and the incident-wave potential, respectively. The incident-wave potential does not satisfy the boundary condition on the body nor the radiation condition, since it represents the wave propagation in the absence of the body. The complex amplitude of the velocity potential associated with a propagating incident wave can be straightforwardly computed through the expression

$$\hat{\phi}_0 = \frac{\mathrm{i}gA}{\omega}e(kz)\exp\{-\mathrm{i}k(x\cos\beta + y\sin\beta)\}, \tag{2.22}$$

where β is the angle between the direction of propagation of the incident wave and the positive x-axis and $e(kz)$ is the decay function, which expresses the decay in the dynamic pressure with distance below the still water line. This function is given by

$$e(kz) = \frac{\cosh[k(z+h)]}{\cosh kh}, \tag{2.23}$$

in which h denotes the water depth. Note that when h tends to infinity (deep water), this function turns into the exponential function, e^{kz}.

The velocity potential of the scattered wave, φ_{s}, respects the boundary condition on the body, assuming that it is motionless, the homogeneous boundary conditions on the seafloor and on the water free surface. The scattered wave is generated by the interaction of the incident wave and the motionless body, when the incident wave does not satisfy the homogeneous boundary condition on the body over the fixed body wet surface, thus

$$-\frac{\partial\hat{\phi}_{\mathrm{s}}}{\partial n} = \frac{\partial\hat{\phi}_0}{\partial n}. \tag{2.24}$$

The latter component of the velocity potential, Eq. (2.20), is the radiation potential, φ_{r}, and corresponds to the radiated wave generated through the body motions in the absence of an incident wave field. This potential must satisfy the boundary condition on the body, assuming that it oscillates in any unconstrained degree-of-freedom (DoF). The net complex amplitude of the radiation potential is described through the superposition

$$\hat{\phi}_{\mathrm{r}} = \mathrm{i}\omega\sum_{j=1}^{N}\hat{\xi}_j\varphi_j, \tag{2.25}$$

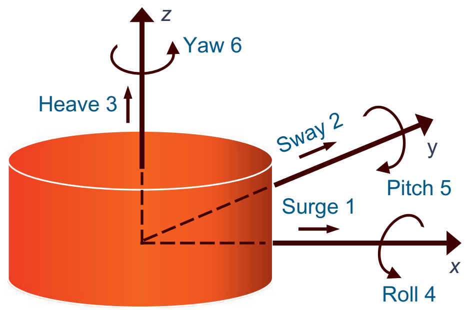

FIG. 2.3 Indices of the six modes of oscillation of a rigid body.

where N is the number of oscillatory modes, $\hat{\xi}_j$ is the complex amplitude of the harmonic body motion in mode j and φ_j is a complex coefficient of proportionality corresponding to the complex amplitude of the radiation potential due to the motion in mode j with unit amplitude. In the case of a rigid body motion, j identifies each one of the 6 rigid degrees of freedom, according to Fig. 2.3.

2.4 EQUATION OF MOTION: SINGLE DEGREE-OF-FREEDOM WEC

The generic equation of motion of a single DoF WEC, schematically represented in Fig. 2.4, is derived in this section in order to set the basis of a standard frequency-domain model. For this, it is necessary to simplify and to linearize the forces involved. Thus, in accordance with linear theory, we assume that the fluid motion and the motion amplitude of the device are sufficiently small for viscous effects to be neglected.

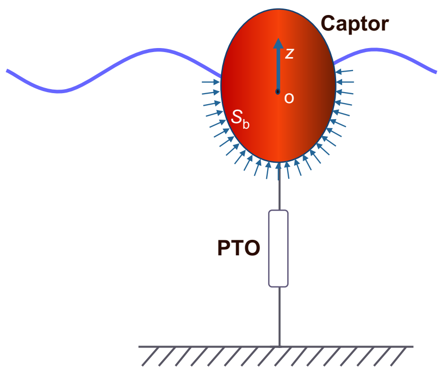

FIG. 2.4 Schematic representation of one single degree-of-freedom (heaving) WEC.

In the time domain, the general motion equation, according to the Newton's second law, is given by

$$m\ddot{\xi}(t) = F_{\mathrm{pe}}(t) + F_{\mathrm{re}}(t), \tag{2.26}$$

where m denotes the total inertia of the captor, ξ its displacement (and so $\ddot{\xi}$ the acceleration), F_{pe} the force due to the external pressure (hydrodynamic/hydrostatic) on the captor and F_{re} the reaction forces, which typically include the loads induced by the PTO equipment and, depending on the WEC working principle, the mooring/foundation system. In accordance with linear theory and considering the oscillatory motion of the waves and the device to be harmonic, it is possible and convenient to decompose each term of Eq. (2.26) in its spatial and temporal dependencies. Therefore, all forces acting on the device can be described by a complex amplitude and the sinusoidal time dependence, $e^{i\omega t}$. Hence, the device displacement vector becomes

$$\xi(t) = \mathrm{Re}\left\{\hat{\xi}(\omega)e^{i\omega t}\right\} \tag{2.27}$$

and, consequently, the velocity and the acceleration vectors result, respectively, from

$$\dot{\xi}(t) = \mathrm{Re}\left\{i\omega\hat{\xi}(\omega)e^{i\omega t}\right\} \tag{2.28}$$

and

$$\ddot{\xi}(t) = \mathrm{Re}\left\{-\omega^2\hat{\xi}(\omega)e^{i\omega t}\right\}, \tag{2.29}$$

where, as before, the hat symbol, $\hat{}$, denotes the complex amplitude. Thus, in the frequency domain, Eq. (2.26) may be written as

$$-\omega^2 m\hat{\xi}(\omega) = \hat{F}_{\mathrm{pe}}(\omega) + \hat{F}_{\mathrm{re}}(\omega), \tag{2.30}$$

in which we remove the exponential $e^{i\omega t}$ in both forces. The force due to the external pressure on the body, $\hat{F}_{\mathrm{pe}}$, may be decomposed into two components to differentiate the sources of hydrostatic and hydrodynamic pressure. Then,

$$\hat{F}_{\mathrm{pe}}(\omega)=\hat{F}_{\mathrm{hd}}(\omega)+\hat{F}_{\mathrm{hs}}(\omega), \tag{2.31}$$

where the first term represents the hydrodynamic force acting on the WEC captor and the second one the hydrostatic restoring force due to gravity and buoyancy that tend to bring the body back to its equilibrium position. In addition, the reaction forces, $\hat{F}_{\mathrm{re}}$, also usually comprises two components, ie,

$$\hat{F}_{\mathrm{re}}(\omega)=\hat{F}_{\mathrm{pto}}(\omega)+\hat{F}_{\mathrm{m}}(\omega), \tag{2.32}$$

where the first term refers to the reaction force from the PTO equipment and the second to the constraint caused by the mooring system design to hold the WEC in position.

2.4.1 Hydrodynamic Force

This section deals with the analysis of the hydrodynamic force acting on the WEC captor. This force is decomposed into the sum of two components: the wave excitation and the wave radiation forces. This decomposition helps clarify the benefits arising from solving the potential flow boundary value problem (obtaining the radiation and diffraction potentials), and the convenience of using BIEM codes for this purpose.

2.4.1.1 Solving the Potential Flow Boundary Value Problem

Except for very simple geometries, it is not possible to analytically solve the boundary value problem described in Section 2.3. Therefore, numerical approaches are usually used to solve these wave diffraction–radiation problems. For analysing floating or bottom-fixed structures in the presence of ocean waves, robust and practical potential-flow methods are available and widely used. Those methods are based on numerical solution of a boundary-integral equation, formulated using the Green function, that satisfies the linear boundary conditions for the diffraction–radiation of harmonic waves.

The most widely used commercial code based on this approach is probably WAMIT (Lee and Newman, 2004, 2013), although open-source alternatives such as Nemoh (http://www.lheea.ec-nantes.fr/cgi-bin/hgweb.cgi/nemoh; Babarit and Delhommeau, 2015) are also available. The code is based on the linear potential theory (although recent versions include a second-order module that provides complete second-order nonlinear quantities) and uses the BIEM, also known as the panel method, to solve for the velocity potential on the mean wetted surface of the body. Distinct solutions are computed simultaneously for the diffraction problem, giving the effects of incident waves on the stationary body, and the radiation problem for each of the prescribed body motion modes. These solutions are then used to obtain relevant hydrodynamic parameters including the added mass and damping coefficients and the complex amplitude of the exciting forces/moments per unit wave amplitude.

The potential flow hydrodynamic force results from the integration of the dynamic pressure on the mean wetted body surface, S_{b}. The dynamic pressure, determined from the Bernoulli equation, may be written, disregarding second-order terms, as

$$p_{\mathrm{e}}=-\rho\left(\frac{\partial\phi}{\partial t}\right). \tag{2.33}$$

Therefore, the linear hydrodynamic force on a floating body is obtained from

$$F_{\mathrm{hd}}=\int_{S_{\mathrm{b}}} p_{\mathrm{e}} n dS_{\mathrm{b}}=\rho\int_{S_{\mathrm{b}}}\frac{\partial\phi}{\partial t} n dS_{\mathrm{b}}, \tag{2.34}$$

where n denotes the unit vector normal to the body surface. The decomposition of the velocity potential, discussed in Section 2.3.4, allows writing the complex amplitude of the hydrodynamic force as

$$\hat{F}_{\mathrm{hd}} = \hat{F}_{\mathrm{e}} + \hat{F}_{\mathrm{r}} = \mathrm{i}\omega\rho\int_{S_{\mathrm{b}}} (\hat{\phi}_0 + \hat{\phi}_{\mathrm{s}}) n dS_{\mathrm{b}} - \omega^2\rho\int_{S_{\mathrm{b}}} \sum_{j=1}^{N} \hat{\xi}_j \varphi_j n dS_{\mathrm{b}}. \tag{2.35}$$

The first term in Eq. (2.35) represents the excitation force, $\hat{F}_{\mathrm{e}}$, related to the effect of incident waves on the body and the second represents the radiation force, $\hat{F}_{\mathrm{r}}$, which arises from the change in momentum of the fluid caused by the motion of the body.

WAVE EXCITATION FORCE

The wave excitation force on the body, the first term in Eq. (2.35), is usually divided into two components: the Froude-Krylov force, $\hat{F}_{\mathrm{FK}}$, and the scatter or diffraction excitation term $\hat{F}_{\mathrm{s}}$. Namely,

$$\hat{F}_{\mathrm{e}} = \hat{F}_{\mathrm{FK}} + \hat{F}_{\mathrm{s}} = \mathrm{i}\omega\rho\int_{S_{\mathrm{b}}} \hat{\varphi}_0 n dS_{\mathrm{b}} + \mathrm{i}\omega\rho\int_{S_{\mathrm{b}}} \hat{\varphi}_{\mathrm{s}} n dS_{\mathrm{b}}. \tag{2.36}$$

The Froude-Krylov force is derived from the velocity potential of the undisturbed incident wave, considering the pressure distribution over the mean wetted surface of the motionless body. That is to say that the body is transparent to the incident wave and so the incident wave field is not disturbed by the presence of the body.

The scattering component of the exciting force results from the integration of the scattered wave potential over the mean wetted surface. This term may be seen as a correction to the Froude-Krylov term due to the effective presence of the body and the consequent disturbance of the incident wave field. It corresponds to the wave field that is scattered by the stationary body. Having identified the incident and scatter potentials, these solutions are used to obtain the complex amplitude of the exciting force, according to Eq. (2.36).

In the case of small (compared to the wavelength) surface-piercing bodies oscillating in heave, it is expected that the disturbance of the incident wave field is insignificant, suggesting that the Froude-Krylov force may represent a reasonable approximation to the vertical excitation force. It may be computationally convenient to use this approximation since it avoids the need to solve the boundary value problem for the scattered potential. It has been found that in the context of wave energy absorption the aforementioned Froude-Krylov approximation is typically acceptable for point absorbers.

RADIATION FORCE

The second term in Eq. (2.35) represents the wave radiation force. This force is caused by the displacement of water in the vicinity of the body when the body moves. The complex amplitude of the radiation force may be written as

$$\hat{F}_{\mathrm{r}} = -\mathrm{i}\omega Z\hat{\xi}, \tag{2.37}$$

where, using an electric analogy, Z denotes the radiation impedance, which, according to the second term in Eq. (2.35), is given by

$$Z = -\mathrm{i}\omega\rho\int_{S_{\mathrm{b}}} \varphi n dS_{\mathrm{b}}. \tag{2.38}$$

The concept of impedance in AC circuits includes the effects of resistance, related to power dissipation, and reactance, related to energy storage in components like inductors and capacitors. Impedance is a complex quantity, in which the real part represents the resistive effect and the imaginary part represents the reactive effect. In this context, one can also expand the analogous hydrodynamic impedance in real and imaginary parts,

$$Z = -\mathrm{i}\omega\rho\int_{S_{\mathrm{b}}} \varphi n dS_{\mathrm{b}} = R + \mathrm{i}X, \tag{2.39}$$

where the real part, R, is the so-called hydrodynamic damping coefficient, which refers to a dissipative effect related with the energy transmitted to the water by the body oscillations that propagate away from the body. Furthermore, the impedance imaginary part X represents the radiation reactance which refers to the difference between the average added kinetic energy, associated with the velocity of the water displaced, and the average added gravitational potential energy, associated with the deformation of the water surface when water is lifted from troughs to crests. Because the system response is harmonic, the energy stored in the water flows into the mechanical system itself and back out again into the surrounding water (reactive effect). The radiation reactance $X(\omega)$ is frequently written as ωA, where A represents the added mass coefficient, which corresponds to an inertial increase due to the water displaced in the body vicinity when the body moves. Therefore, the impedance, Eq. (2.39), may be rewritten as

$$Z = R + \mathrm{i}\omega A. \tag{2.40}$$

Accordingly, the hydrodynamic radiation force of a floating body is given by

$$\hat{F}_{\mathrm{r}} = -\mathrm{i}\omega R\hat{\xi} + \omega^2 A\hat{\xi}, \tag{2.41}$$

where the first term is a dissipative force, proportional to the captor velocity, and the second an inertial force, proportional to the captor acceleration.

2.4.1.2 Haskind Relation

The excitation force requires knowing the diffraction or scattered potential, as discussed in section 'Wave Excitation Force'. However, the excitation force may be also derived from the radiation potential as formulated by Haskind (Haskind, 1957; Newman, 1962) for the case of a single body. Therefore, by using the so-called Haskind relation, it is possible to relate the hydrodynamic damping with the complex amplitude of the excitation force per unit incident wave amplitude. This relation can be written in the form

$$R = \frac{\omega k}{4k\rho g^2 D(kh)|A|^2}\int_{-\pi}^{\pi} \hat{F}_{\mathrm{e}}(\beta)\hat{F}_{\mathrm{e}}^{*}(\beta), \tag{2.42}$$

where the symbol * denotes a complex conjugate, β represents the angle that the direction of the propagating incident wave makes with the x-axis and $D(kh)$ is a depth function given by

$$D(kh) = \left[1 + \frac{2kh}{\sinh(2kh)}\right]\tanh(kh), \tag{2.43}$$

which defines the effect of the proximity of the sea bottom, thus $D(kh) \approx 1$ for $kh \gg 1$ ($\tanh(kh) \gg 1$). To some extent, the physical interpretation of Eq. (2.42) is fairly intuitive, since basically it decodes the expectable relation between the body's ability to radiate a wave into a certain direction and the excitation force that the body experiences from an incoming wave with the same direction.

For the particular case of an axisymmetric body (ie, independent of the incident direction), which is typically the case of point absorbers (however, note that it is not the axial symmetry that defines point absorption), Eq. (2.42) degenerates into a very simple expression,

$$R = \frac{2\omega k}{\rho g^2 D(kh)|A|^2}\left|\hat{F}_{\mathrm{e}}\right|^2. \tag{2.44}$$

2.4.1.3 Kramers-Kronig Relations

Besides the relation between the excitation and the radiation potential, stated by the Haskind relation, the hydrodynamic damping coefficient $R(\omega)$ may also be related to the frequency-dependent component of the added mass $A(\omega)$ through the Kramers-Kronig relations, expressed by

$$R(\omega) = \frac{2\omega^2}{\pi}\int_0^{\infty} \frac{A(\omega) - A_{\infty}}{\omega^2 - y^2}dy \tag{2.45}$$

and

$$A(\omega) - A_{\infty} = \frac{2}{\pi}\int_0^{\infty} \frac{-R(y)}{\omega^2 - y^2}dy. \tag{2.46}$$

The Kramers-Kronig relations are very useful for the derivation of the unknown hydrodynamic coefficient if only one of them is known.

However, the integrals in Eqs (2.45), (2.46) are improper integrals that require some care to be accurately calculated. To assign values to the improper integrals, which would otherwise be undefined, the Cauchy principal value method must be used. The method is defined by a limiting process where the singular point is approached simultaneously from left and right in a symmetric manner (Papoulis, 1962).

Essentially, the Kramers-Kronig relations reveal the relation between the body's ability to radiate waves and the frequency dependence of the inertia increase due to the water displaced by the body motion. In other words, the Kramers-Kronig relations reflect the inherent correlation between the reactive effect associated to the stored energy that flows into the mechanical system and back out again into the surrounding water and the dissipative effect related to the energy transmitted to the water by the body oscillations. This ability to radiate waves is an essential characteristic of WEC, linked to maximizing the capture of wave energy, as will be discussed later (Section 2.4.5).

2.4.2 Hydrostatic Force

The second term of Eq. (2.31) embodies the hydrostatic force resulting from the balance between buoyancy and gravity. The hydrostatic force follows from integration of the hydrostatic pressure distribution over the body wetted surface in undisturbed conditions. When the amplitudes of the body motions are small the linearization of the hydrostatic force provides a reasonably accurate approximation. In this case the hydrostatic force becomes proportional to the displacement and so its complex amplitude is simply given by

$$\hat{F}_{\mathrm{hs}} = -G\hat{\xi}, \tag{2.47}$$

where G is the hydrostatic spring stiffness, commonly called the hydrostatic coefficient. In the case of a heaving WEC the balance between gravity and buoyancy yields, after some effortless mathematical manipulation,

$$G = \rho g S. \tag{2.48}$$

Here S is the cross-sectional area at the undisturbed sea level. This area is assumed to be constant during the device excursions, according to the linear theory assumption of small-amplitude motions.

2.4.3 Reaction Forces

Typically the reaction forces acting on a WEC comprise the loads caused by the PTO equipment and the mooring system (or foundations) responsible for the WEC station-keeping. In addition, some concepts, such as point absorbers, also require the use of end-stop mechanisms to decelerate the captor at the end of its stroke in order to gently dissipate the kinetic energy and so prevent mechanical failures or structural damages.

In general, the PTO system will result in a complex nonlinear dynamic behaviour due to the complexity of the control strategy applied to maximize the wave energy capture. Nevertheless, in frequency domain the PTO reaction force must be linearized to keep the superposition principle valid. In its linear form, the PTO force is composed of two contributions: the first one, proportional to velocity, represents a purely resistive effect represented by a damper, and the second one, proportional to displacement of the body (with respect to the hydrostatic equilibrium condition), is represented by a spring. Accordingly, the complex amplitude of the force generated by the PTO is obtained from

$$\hat{F}_{\mathrm{pto}} = -\mathrm{i}\omega B_{\mathrm{pto}}\hat{\xi} - K_{\mathrm{pto}}\hat{\xi}. \tag{2.49}$$

This force can be adjusted in a way that seeks to continually tune the damping force and, in some concepts, also the spring force in order to suit the wave force that drives the converter.

The action generated by the mooring system on the floater, is, like the PTO, typically strongly nonlinear and dependent on the dynamics of the WEC and the mooring lines. Nevertheless, quasilinear mooring models are well-suited for small motions of the floater and for slack-moored designs with natural periods well below the wave peak period, which is typically the case of WECs. In this framework, the mooring line tensions in the frequency domain are often represented by a linear function of the captor displacement, where the constant of proportionality represents the stiffness characteristics of the mooring system. Therefore, the complex amplitude of the mooring force can be simply given by

$$\hat{F}_{\mathrm{m}} = -K_{\mathrm{m}}\hat{\xi}, \tag{2.50}$$

where K_{m} represents the mooring spring stiffness.

2.4.4 Complex Amplitude of the Body Motion

Introducing the resulting expression for the force caused by the external pressure, Eq. (2.35) (which includes the hydrostatic restoring force, Eq. 2.47, the hydrodynamic radiation, Eq. 2.41, and wave excitation loads, Eq. 2.36) together with the resulting expression for the reaction forces, Eq. (2.32) (comprising the force generated by the PTO, Eq. 2.49, and the action produced by the mooring system, Eq. 2.50), into the motion equation, Eq. (2.30), we obtain, after some elementary mathematical manipulation, the complex amplitude of the captor motion,

$$\hat{\xi} = \frac{\hat{F}_{\mathrm{e}}}{\left[-\omega^2(m+A)+G+K_{\mathrm{pto}}+K_{\mathrm{m}}\right]+\mathrm{i}\omega\left(R+B_{\mathrm{pto}}\right)}. \tag{2.51}$$

It is often convenient to identify two complex impedances that define the intrinsic impedance, Z_{i}, and the PTO impedance, Z_{pto}, as

$$Z_{\mathrm{i}} = R - \frac{\mathrm{i}}{\omega}\left[-\omega^2(m+A)+G+K_{\mathrm{m}}\right] \tag{2.52}$$

and

$$Z_{\mathrm{pto}} = B_{\mathrm{pto}} - \frac{\mathrm{i}}{\omega}K_{\mathrm{pto}}. \tag{2.53}$$

So, Eq. (2.51) becomes

$$\hat{\xi} = \frac{1}{\mathrm{i}\omega}\frac{\hat{F}_{\mathrm{e}}}{Z_{\mathrm{i}}+Z_{\mathrm{pto}}}. \tag{2.54}$$

It is worth noting that in this case the impedance is defined relative to velocity, but that it is equally possible to define it relative to displacement. Both definitions are used in the wave energy literature, although this definition is more common and is subsequently used in this chapter.

The effective resistance combines the hydrodynamic radiation damping (dissipative effect) and the energy dissipated in the mechanical damper, ie, the energy extracted from the mechanical system. Furthermore, the effective reactance merges the energy stored in the water that flows between the mechanical system and the surrounding water (inductive reactance), the potential energy stored in the hydrostatic spring (capacitive reactance) and the elastic energy stored in the mechanical equipment (PTO spring) and the mooring lines (capacitive reactance).

2.4.5 Power Absorption

This section presents the most common parameters used to evaluate the performance of WECs in terms of power capture. In this context the section discusses the characterization of the PTO equipment to maximize wave energy capture and the need of tuning the response of the device in order to make the device resonant with the incoming waves. This analysis is particularized for heaving WECs; however, the same approach is extensible to any other oscillatory mode.

2.4.5.1 Mean Power Absorption

The mean power absorbed of a WEC corresponds to the mean power consumed by the mechanical damper of the PTO equipment during a wave period. The average contribution of the mechanical spring is null over a wave period, since it refers to the energy that flows back and forth between the captor (kinetic energy) and the PTO spring (elastic potential energy). As a result, assuming sinusoidal waves, the mean absorbed power over a wave period is

$$P_a = \frac{1}{T}\int_0^T B_{\text{pto}} u^2 dt = \frac{1}{2} B_{\text{pto}} \omega^2 \left|\hat{\xi}\right|^2. \tag{2.55}$$

Inserting the complex amplitude of the captor displacement, Eq. (2.51), into Eq. (2.55) gives

$$\begin{aligned} P_a &= \frac{1}{2}\frac{B_{\text{pto}}\omega^2 \left|\hat{F}_{\text{e}}\right|^2}{\left[-\omega^2(m+A)+G+K_{\text{pto}}+K_{\text{m}}\right]^2 - \omega^2\left(R+B_{\text{pto}}\right)^2} \\ &= \frac{1}{2}\frac{B_{\text{pto}}\left|\hat{F}_{\text{e}}\right|^2}{\left|Z_{\text{i}}+Z_{\text{pto}}\right|^2}. \end{aligned} \tag{2.56}$$

2.4.5.2 *Optimal PTO Control*

The optimal PTO control is the control that maximises the power capture defined by Eq. (2.56). The maximization of Eq. (2.56) is essentially a double variable optimization problem, where it is found that the optimal couple of PTO parameters, K_{pto} and B_{pto}, are given by

$$K_{\text{pto}} = \omega^2(m+A) - G - K_{\text{m}} \tag{2.57}$$

and

$$B_{\text{pto}} = R. \tag{2.58}$$

The control strategy that provides these PTO parameters is typically called reactive or complex-conjugate control. Technically, reactive control refers only to the fact that the PTO reactance, K_{pto}/ω (the imaginary part of Z_{pto}), must cancel the inherent reactance (the imaginary part of Z_{i}), which in essence is stated by the condition Eq. (2.57). However, this does not highlight the fact that the PTO resistance and the hydrodynamic resistance must also be equal. Thus, complex-conjugate control is a more accurate description since it refers to the fact that the optimum PTO impedance equals the complex conjugate of the intrinsic impedance.

The optimal absorption conditions, stated by Eqs (2.57), (2.58), stress two major aspects of wave energy conversion, in particular regarding the design and control of WECs. The first condition states that power absorption is maximized at resonance, ie, when the effective total reactance of the system is zero. When this condition is achieved the velocity is in phase with the excitation force (see Eq. 2.51). The second condition states that the PTO damping must be equal to the hydrodynamic radiation damping. Fundamentally, this condition highlights the principle, apparently paradoxical, that 'to absorb a wave means to generate a wave'. This means that the ability to generate waves is a fundamental aspect in the design of WECs.

Introducing the optimal PTO coefficients, Eqs (2.57), (2.58), into Eq. (2.56) yields, after some manipulation, the expression for the maximum theoretical achievable mean power of a single mode WEC,

$$P_{a_{\max}} = \frac{\left|\hat{F}_{\text{e}}\right|^2}{8R}. \tag{2.59}$$

Similarly, inserting the optimal PTO coefficients into Eq. (2.51), the optimal displacement of the captor is given by

$$\hat{\xi}_{\text{opt}} = \frac{\hat{F}_{\text{e}}^{\,2}}{2\omega R}. \tag{2.60}$$

It can also be shown, using the Haskind relation (see Eq. 2.44), that for an axisymmetric heaving body Eq. (2.59) degenerates to

$$P_{a_{\max}} = \frac{\rho g^2 D(kh)}{4\omega k}\left|A_w\right|^2 = \frac{1}{k} J_w, \tag{2.61}$$

where $D(kh)$ is the depth function (see Eq. 2.43) and J_w is the mean incident power or, more precisely, the power transported per unit of time and unit width of wave frontage given, for a progressive plane harmonic wave, by

$$J_w = \left(\frac{\rho g^2 D(kh)}{4\omega}\right)|A_w|^2. \qquad (2.62)$$

The most common parameter to define the WEC's performance in converting energy from waves is the so-called capture width L_w, which is the ratio of the mean power absorption and the mean incident power, J_w. Hence, dividing Eq. (2.55) by (2.62), the capture width turns out to be given by

$$L_w = \frac{P_a}{J_w} = \frac{2\omega^2 B_{\text{pto}}}{\rho g}\left|\frac{\hat{\xi}}{A_w}\right|^2, \qquad (2.63)$$

in which the ratio $\hat{\xi}/A_w$ is commonly called the response amplitude operator (RAO). Combining Eqs (2.63), (2.59), it follows that in the case of an axisymmetric heaving body the maximum theoretical limit of capture width is simply obtained from

$$L_{w_{\max}} = \frac{P_{a_{\max}}}{J_w} = \frac{1}{k} = \frac{\lambda}{2\pi}, \qquad (2.64)$$

where λ is the wavelength. Eq. (2.64) shows that the optimal capture depends on the incident wavelength and not on the geometric characteristics of the device. Theoretically, any device, independent of its size, can reach the maximum capture level. However, in practice the required displacement for small bodies to achieve the maximum capture level would be unrealistically too high.

2.4.5.3 *Suboptimal PTO Control*

Often, the optimal absorption conditions, defined by Eqs (2.57), (2.58), are unattainable because the PTO system is unable to completely cancel the inherent reactance of the WEC through an effective spring mechanism. In many practical cases the PTO reactance is actually zero or negligible and so the optimal condition, Eq. (2.57), can be satisfied only at the resonant frequency. In this case the PTO load resistance to absorb the maximum power is no longer obtained from Eq. (2.58), but can be shown to be equal to the absolute value of the inherent impedance, that is

$$B_{\text{pto}} = |Z_{\text{i}}|. \qquad (2.65)$$

Therefore, according to Eq. (2.56), the power capture is

$$P_a = \frac{1}{2}\frac{|Z_{\text{i}}|\left|\hat{F}_{\text{e}}\right|^2}{\left|Z_{\text{i}} + |Z_{\text{i}}|\right|^2}. \qquad (2.66)$$

2.4.5.4 *Constrained Motion*

The motion of a WEC is considered to be constrained when the maximum allowable displacement amplitude is smaller than the optimum displacement amplitude. The allowable displacement amplitude is typically dependent on the dimensions of the device or mooring configuration beyond which the body cannot move or becomes decoupled from the incident wave due to submergence or similar. However, irrespective of what constrains a body's motion, it can be shown that the optimum method of achieving an acceptable displacement amplitude is to increase the PTO damping until the body has the maximum allowable displacement (Evans, 1981). Specifically, it is shown that for complex-conjugate control, if the displacement amplitude is limited to r of the optimum amplitude, then the proportion of power capture is given by

$$\frac{P_a}{P_{a_{\max}}} = 2r - r^2. \qquad (2.67)$$

Thus, it can be seen that the power capture reduces at a much slower rate than the proportion of optimum amplitude. For example, if the optimum amplitude is twice the allowable amplitude ($r = 0.5$) then the power capture is 0.75 of the maximum. Although the relationship for suboptimal PTO control is more complex, the type of relationship is similar in that the power capture reduces at a much slower rate than the degree of motion constraint.

2.4.5.5 *Absorption Bandwidth*

The absorption bandwidth is a relevant parameter to assess the frequency dependence of a WEC's performance. To define this quantity it is convenient to rewrite the expression for the power absorption, given by Eq. (2.56), which, after some basic mathematical manipulation, takes the form

$$P_a = \frac{B_{\mathrm{pto}}\left|\hat{F}_{\mathrm{e}}\right|^2}{2\left(R+B_{\mathrm{pto}}\right)^2}\left[1+\left(\frac{\omega_0}{2\delta}\right)^2\left(\frac{\omega_0}{\omega}+\frac{\omega}{\omega_0}\right)^2\right]^{-1}, \tag{2.68}$$

where ω_0 is the natural (resonant) angular frequency and the parameter δ the so-called damping coefficient of the oscillator, given by

$$\delta = \frac{B_{\mathrm{pto}}+R}{2(M+A)}. \tag{2.69}$$

The relative absorbed power response, given by the ratio between the mean power absorption, Eq. (2.68), and the maximum theoretical achievable mean power absorption, Eq. (2.59), which occurs at resonance (ie, $\omega=\omega_0$), may be written in the form

$$P_a^* = \frac{P_a(\omega)}{P_a(\omega_0)}\frac{\left|\hat{F}_{\mathrm{e}}(\omega_0)\right|^2}{\left|\hat{F}_{\mathrm{e}}(\omega)\right|^2} = \left[1+\left(\frac{\omega_0}{2\delta}\right)^2\left(\frac{\omega_0}{\omega}-\frac{\omega}{\omega_0}\right)^2\right]^{-1}, \tag{2.70}$$

in which 1 is naturally the maximum value (achieved at resonance). Defining the absorption bandwidth as the frequency range, $\omega_{\mathrm{l}}<\omega<\omega_{\mathrm{r}}$, where the relative absorbed power response, P_a^*, exceeds 1/2, we have from Eq. (2.70) that

$$\Delta\omega = \omega_{\mathrm{r}} - \omega_{\mathrm{l}} = 2\delta = \frac{B_{\mathrm{pto}}+R}{M+A}. \tag{2.71}$$

A WEC with a narrow bandwidth, $\Delta\omega$, can absorb wave energy efficiently from only a small part of the wave spectrum, unlike a wide bandwidth device, as illustrated in Fig. 2.5. In general, the reactive term $M+A$ is dominant in small devices leading to a narrow bandwidth, which makes wave energy point absorbers strongly dependent on the efficiency of the control strategy to maximize energy extraction. Overall, the bandwidth tends to increase with the device size (due to an increase of the resistive term).

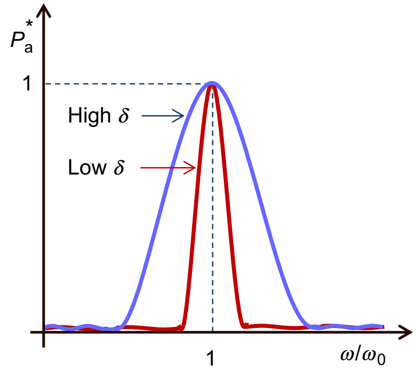

FIG. 2.5 Absorbed power for different values of the damping coefficient of the oscillator, δ.

2.5 EQUATION OF MOTION: MULTIPLE DEGREE-OF-FREEDOM WEC

The generic equation of motion of a multiple degree-of-freedom (DoF) WEC is in essence an extension of that for a single DoF WEC (see Eq. 2.51). It may be written in a matrix form as

$$\hat{\boldsymbol{\xi}} = A_w\hat{\mathbf{f}}_{\mathrm{e}}\left[-\omega^2(\mathbf{M}+\mathbf{A})+\mathbf{G}+\mathbf{K}_{\mathrm{pto}}+\mathbf{K}_{\mathrm{m}}+\mathrm{i}\omega\left(\mathbf{R}+\mathbf{B}_{\mathrm{pto}}\right)\right]^{-1}, \tag{2.72}$$

where the bold font denotes matrix or vector. In Eq. (2.72) $\hat{\mathbf{f}}_{\mathrm{e}}$ embodies the vector of complex amplitudes of the excitation wave loads on the mean body wetted surface, for unit amplitude waves (normally computed with standard 3D BEM radiation/diffraction numerical codes). Moreover, in Eq. (2.72) the matrices within the square brackets comprise:

- The *mass matrix*, **M**. This matrix is symmetric and for a 6 DoF freely floating rigid body is described by

$$\mathbf{M}=\begin{bmatrix} m & 0 & 0 & 0 & mz_g & -my_g \\ 0 & m & 0 & -mz_g & 0 & mx_g \\ 0 & 0 & m & my_g & -mx_g & 0 \\ 0 & -mz_g & my_g & I_{11} & I_{12} & I_{13} \\ mz_g & 0 & -mx_g & I_{21} & I_{22} & I_{23} \\ -my_g & mx_g & 0 & I_{31} & I_{32} & I_{33} \end{bmatrix}, \tag{2.73}$$

where m is the mass of the body (equivalent to the mass of the displaced water in the free flotation condition, ie, $m=\rho V$, x_g, y_g, z_g the coordinates of the centre of gravity and I_{ij} the moments of inertia defined, in terms of the corresponding radius of gyration, r_{ij}, by the relation

$$I_{ij}=\rho V r_{ij}|r_{ij}|. \tag{2.74}$$

- The *matrix of hydrostatic restoring coefficients*, G. This matrix is also symmetric and for the 6 rigid DoF the nonzero entries are given by

$$\begin{cases} G_{33}=\rho g\int_{S_b} n_3 dS_b \\ G_{34}=\rho g\int_{S_b} y n_3 dS_b \\ G_{35}=-\rho g\int_{S_b} x n_3 dS_b \\ G_{44}=\rho g\int_{S_b} y^2 n_3 dS_b+\rho g V z_b-mgz_g, \\ G_{45}=-\rho g\int_{S_b} xy n_3 dS_b \\ G_{55}=\rho g\int_{S_b} x^2 n_3 dS_b+\rho g V z_b-mgz_g \\ G_{56}=-\rho g V y_b+mgy_g \end{cases} \tag{2.75}$$

where x_b, y_b, z_b are the coordinates of the buoyancy centre and S_b the wetted surface of the body.

- The *damping*, **R**, and *added mass*, **A**, *matrices*. As discussed in section 'Radiation Force', the damping coefficient is related to the waves generated by the body oscillatory motions and the energy transported away from the body. On the other hand, the added mass accounts for the fact that when accelerating a water particle from rest the surrounding fluid is also accelerated and so it corresponds to an additional inertial effect due to the water displaced in the body vicinity when the body moves. The added mass coefficient has two components: one is a frequency-dependent term and the other is the so-called infinite added mass, a positive constant component that represents the added inertia at infinite frequency (where there are no radiated waves from the body), ie, $\mathbf{A}_\infty=\lim_{\omega\to\infty}\mathbf{A}$. Further discussion of the infinite added mass coefficient is provided in Chapter 3, where it is used in the Cummins equation when time-domain modelling of WECs is considered in detail.

  In the case of multiple DoF systems, radiation forces felt by the body in a particular direction (DoF) may also occur due to the alteration of the wave field in the body vicinity as it moves in another direction. For instance, heaving forces acting upon a body will occur due to acceleration and velocity of the body in the surge and other directions. This mode-coupling is reflected in the off-diagonal terms of the added mass and wave-damping matrices, which generate cross-coupled force components and whose coefficients can be both positive and negative. However, the hydrodynamic reciprocity relationships mean that these matrices are symmetrical and, in addition, due to the far-field radiation boundary condition the added damping matrix must also be positive definite (Fossen, 2002).
- The *PTO mechanical spring*, $\mathbf{K}_{pto}$, and *damping*, $\mathbf{B}_{pto}$, *matrices*. These matrices characterize the PTO system and typically only contain nonzero entries along their diagonal. This means that usually the PTO forces applied in one direction are due solely to movement in that direction.
- The *mooring spring stiffness matrix*, $\mathbf{K}_m$. This matrix contains the stiffness coefficients that

typify the elastic properties of the mooring lines, which depend on the physical configuration of the mooring system.

The DoF for WECs that consist of multiple bodies can be different from those for a rigid body, with the number of DoF depending on the actual WEC and the focus of the modelling. A common multibody WEC consists of a floating and a submerged body that are constrained to move relative to each other along the heave axis (eg, WaveBob (Previsic et al., 2004), PowerBuoy (http://www.oceanpowertechnologies.com/powerbuoy/), etc). In this case there are seven DoF: six for the whole body response plus one for the relative motion. However, to reduce the computational effort it is common to reduce this to two DoF, which is typically the heave motion of the two bodies. The reduction to two DoF can often be justified when movement in the other modes does not influence the response or power capture. However, whatever the final formulation of the multiple DoF model, the WEC dynamics can in general be solved using Eq. (2.72).

In addition, Eq. (2.72) can be used where the DoF have been defined by 'generalised modes' following the theory developed by Newman (1993, 1994). 'Generalised modes' were originally developed to allow the modelling of deformable structures, where the deformation can be defined by any continuous function. Thus, they could be used for continuous structural deflections where the total deflection is defined by the use of multiple modes that are added together to create the complex response of the beam. In wave energy, they are more commonly used to model the deformation of membranes used in WECs such as the Anaconda (Chaplin et al., 2007) or the Bombora WEC (http://www.bomborawavepower.com.au). However, 'generalised modes' can also be used for articulated WECs such as Pelamis (Yemm et al., 2012), where this type of representation can simplify the equations of motion because it becomes unnecessary to model all the 6 DoF for each body and then constrain the motions based on the physical configuration of the joints. By using 'generalised modes' for hinge-articulated bodies the number of DoF that need to be modelled can be reduced from $6N$ to $5+N$, where N is the number of bodies.

In a similar fashion to the equations of motion, the power absorption of the multiple DoF WEC can be defined using a matrix version of the equivalent single DoF equation. The single DoF equation for power absorption is given by Eq. (2.55) and the matrix equivalent is

$$P_a = \frac{1}{2}\omega^2 \hat{\boldsymbol{\xi}}^{\mathrm{T}} \mathbf{B}_{\mathrm{pto}} \hat{\boldsymbol{\xi}}^{*}, \tag{2.76}$$

where the superscripts T and * represent the transpose and the complex conjugate, respectively. In many cases it should be possible to formulate the PTO damping matrix so that it only contains nonzero terms along its diagonal. If this is the case, then an alternative equation for the power absorption is given by

$$P_a = \sum_{i=1}^{n} \frac{1}{2}\omega^2 B_{\mathrm{pto}_i} \left|\hat{\xi}_i\right|^2, \tag{2.77}$$

which can be seen to be even more closely related to the single DoF equation for power absorption, Eq. (2.55).

2.6 OWCS

OWC plants are one of the most popular categories of wave energy devices. Prototypes of OWC units have been operating in many parts of the world since the first OWC concept was developed in the 1940s by Yoshio Masuda (Falcao, 2010). Essentially, OWCs consist of a fixed or floating hollow structure, open to the sea below the water surface, that traps air in a chamber above the inner free surface. The wave action alternately compresses and decompresses the trapped air, forcing an air flow moving back and forth through a turbine that drives a

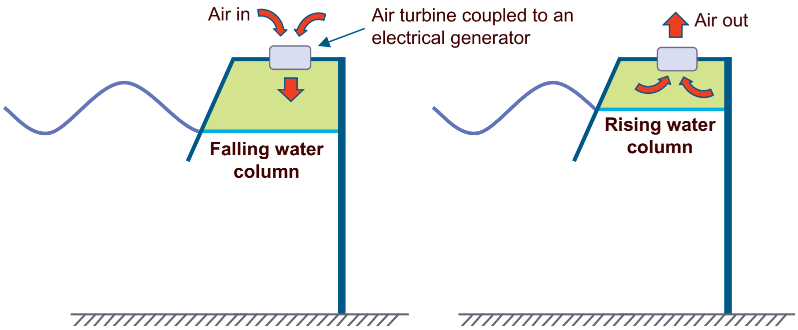

FIG. 2.6 Working principle of classical OWC plants.

generator and produces electricity. The working principle of OWC plants is illustrated in Fig. 2.6.

The most common models available to represent the hydrodynamics of an OWC device are based on a pressure distribution over the inner free surface (Evans, 1982; Sarmento and Falcao, 1985) or based on the assumption that the internal free surface behaves as a massless rigid piston (Evans, 1978). In the latter alternative, the piston mode may be complemented by a number of additional high-order modes (sloshing modes) to better satisfy the internal free-surface boundary condition (Lee and Nielsen, 1996). However, In the case of OWC point absorbers, a sole piston mode is normally an acceptable approach, since the physical dimensions of this type of WEC are generally much smaller than the typical incident wavelength. Although the piston mode is the only mode that contributes to energy absorption, the piston approach is, however, insufficient to describe the dynamic behaviour of the inner free surface of larger OWC plants, where the occurrence of sloshing is very likely.

An additional factor to consider in the numerical modelling of OWCs is the compressibility of the air in the plenum chamber above the water surface. The compressibility of the air acts like a spring so that the air velocity that is driving the turbine is out-of-phase with the velocity of the water surface. Although the thermodynamic equations for the air are fundamentally nonlinear, they can be linearized using the same assumptions that are used for the hydrodynamics; specifically that the amplitude of motion is small. In this case it can be shown that the effect of air compressibility can be represented as a linear spring (Sarmento and Falcao, 1985; Brendmo et al., 1996) with the spring rate given by

$$k_{\text{air}} = \frac{\gamma p S^2}{V}, \tag{2.78}$$

where the OWC is modelled as a piston with area, S, γ represents the ratio of specific heats and p the ambient pressure in the plenum chamber with volume, V.

2.7 LIMITATIONS

As discussed previously, frequency-domain models have limited applicability, essentially restricted to linear problems. Linearity holds only approximately in waves with small amplitudes relative to their wavelengths and oscillatory motions of small amplitudes. In this context the main limitation of the frequency approach is the accuracy around resonance, where typically the amplitude becomes too high so that it is well outside of the range of applicability of linear theory. In addition, nonlinear viscous effects, which may give rise to flow separation and vortex shedding around the floating body, are not accounted for, based on the assumption of inviscid flow. In addition, the frequency domain approach does not account for wave breaking and the wave steepness is assumed to be well below the point of collapse or break, given the requirement that the wave amplitude is small relative to the wavelength.

A further limitation of the frequency approach is related to the impossibility of drawing meaningful conclusions about the controllability of WECs, since control strategies able to maximize the power output are often highly nonlinear and a linear representation of the reaction force from the PTO is not sufficiently realistic. Indeed, it is not possible to implement any real control strategy other than constant PTO coefficients (which is not really a control strategy) in the frequency domain because the strategy must operate based on information received in real time, which is not available from a frequency-domain model.

A final limitation of frequency models is the fact that a monochromatic frequency-domain wave field is not an accurate representation of real waves, which are shaped by the superposition of a large number of regular sinusoidal waves of varying frequencies and amplitudes.

2.8 SUMMARY

- Frequency-domain models of WECs are based in a linearization of the hydrodynamics and response
- Linearization assumes that the waves and body response are harmonic and small and the flow is inviscid
- A frequency-domain model calculates the response of the WEC as a function of frequency
- The hydrodynamic coefficients required for frequency-domain models can be produced relatively easily using a linear potential flow solver and BEMs
- Frequency-domain models are relatively fast to calculate response and power capture
- Multiple degrees-of-freedom are easy to incorporate in a frequency-domain model
- Frequency-domain models may be relatively inaccurate for large waves, at frequencies close to resonance or where viscous forces are significant.

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