CHAPTER

4

Spectral-Domain Models

M. Folley

School of Planning, Architecture and Civil Engineering, Queen's University Belfast, Belfast, Northern Ireland

4.1 INTRODUCTION AND FUNDAMENTAL PRINCIPLES

The spectral domain differs fundamentally from the frequency domain and the time domain because it uses a probabilistic model of the system dynamics. Both frequency-domain and time-domain models of wave energy converters (WECs) use classical methods to solve the equation(s) of motion using a deterministic model. A deterministic model involves defining a specific wave that excites the WEC, whose dynamics are solved using classical equation(s) of motion to produce a specific system response. However, in contrast, a probabilistic model uses a statistical representation of the waves, which when passed through an appropriate transformation function produces a probabilistic estimate of the WEC response. If we accept that the energy spectrum is a reasonable statistical representation of the waves and that statistical responses such as average power capture are sufficient, then the challenge in developing spectral-domain models of WECs lies in defining an appropriate transformation function.

Before progressing it is worth reviewing the acceptability of using the energy spectrum as a reasonable statistical representation of the waves. For simplicity we will only consider one-dimensional spectra, where the energy density varies with frequency; however, the conclusions are equally valid for two-dimensional spectra, where the energy density also varies with direction. Fig. 4.1 illustrates a typical wave energy spectrum and shows how the energy in the waves varies with frequency. This energy spectrum can be considered to represent a large number of individual sinusoidal wave components, where the amplitude of each wave component depends on the spectral energy density at the wave frequency. From this an infinite number of different wave profiles can be generated as this is dependent on the phase relationship between the wave components. Thus, the energy spectrum does not provide a complete representation of the waves as the phase relationship is not included. However, where the phase relationships are unknown the energy spectrum does provide a reasonable statistical representation of the waves. Furthermore, if it is assumed that there is a random phase relationship between the wave components, then the waves can be modelled as a Gaussian process, which has well-known statistical characteristics.

The first probabilistic model of a floating structure was developed in 1953 to estimate

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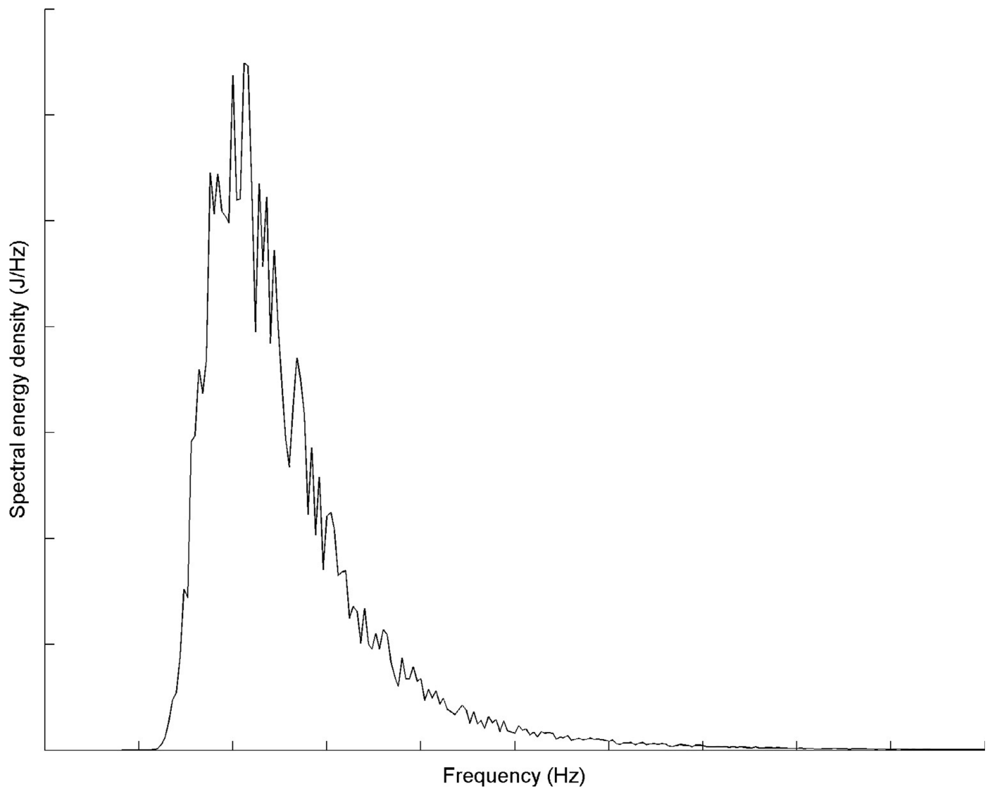

FIG. 4.1 Wave energy spectrum.

the motion of ships in 'confused seas' (St Denis and Pierson, 1953). This model used a linear model of the system dynamics so that the response to each wave component could be calculated independently as they were not influenced by the system response at other frequencies. Effectively, a frequency-domain model was used to calculate the system response at each frequency and the total response calculated by linear superposition of the individual frequency responses. The random-phase assumption was then used to infer that the incident waves are a Gaussian process and, because the system was linear, that the ship response was also a Gaussian process. Thus, St Dennis and Pierson were able to estimate a range of statistical properties of the ship response such as the probability of exceeding a particular maximum displacement.

If the application of probabilistic models was restricted to linear systems, their potential would be limited. However, probabilistic models can also be applied to nonlinear systems, although the methods of solution become significantly more complex. An early application of probabilistic models to nonlinear hydrodynamic problems was for the dynamics of a floating offshore structure due to the Morison equation based modelling of the wave-induced forces, including the influence of currents (Gudmestad and Connor, 1983). This nonlinear hydrodynamic problem was solved by generating an equivalent linear system based on the assumption of a Gaussian sea, which could then be solved in the frequency domain. The total response of the system was then used to define an equivalent linear system and the solution iterated to minimize the error in response induced by the equivalent linearization. It was found by Gudmestad and Connor that this procedure produced a good estimate of the statistical response of the system with relatively little computational effort. Subsequently, probabilistic methods have been used extensively for the analysis of nonlinear hydrodynamic problems for a wide range of scenarios.

It is also worth noting that probabilistic models are also fundamental to the representation of nonlinear processes in spectral wave models. A spectral wave model incorporates a range of physical processes, such as refraction, wind-growth, white-capping, etc. to model the evolution of the wave energy spectrum in both time and space. A random phase relationship between wave components is used to infer that the process is Gaussian, which is then used with probabilistic models of the physical processes to calculate the expected change to the wave spectrum. For example, the energy lost due to bottom friction is modelled to be proportional to the expected value of the square of the water particle velocity at the seabed, which itself is based on the assumption that the bottom friction energy loss is due to turbulence and thus proportional to the velocity squared. Significantly, a spectral wave model has been found to model the wave transformation process accurately and nowadays is the standard model used for large-scale modelling of wave propagation processes.

Given the success of probabilistic methods in solving nonlinear hydrodynamic problems, it is surprising that these methods have only recently been applied to WECs (Folley and Whittaker, 2010), where it has been termed 'spectral-domain modelling'. Folley and Whittaker used the same techniques as Gudmestad and Connor, statistical linearization (also sometimes called stochastic linearization or equivalent linearization), to model the effects of Morison's equation-type drag forces and large-angle rotation decoupling of the incident waves on a flap-type WEC. It was shown in these cases that statistical linearization produces a model that predicts a statistically very similar response to time-domain models even for relatively high levels of nonlinearity. Subsequently, a probabilistic model of a WEC with a Coulomb-friction type force has been developed and shown to provide a reasonable estimate of the statistical response of the system in the majority of cases (Folley and Whittaker, 2013).

The principal advantage of using a probabilistic, or spectral-domain, model of a WEC is that for complex nonlinear systems it is computationally more efficient at providing estimates of statistical parameters, such as the expected average power capture, than an equivalent deterministic model. However, it is important to recognize that a spectral-domain model cannot be used to calculate the temporal response of a system and consequently cannot provide values such as the maximum instantaneous displacement or power capture, although the maximum expected instantaneous displacement can be calculated. However, in many cases this is not significant because we are primarily interested in the expected average power capture and not the average power capture for the particular instance of a sea-state, which would be provided by a time-domain simulation. Indeed, it may take multiple time-domain simulations to produce a good estimate of the expected average power capture and its standard deviation, whilst these statistics are available directly from a spectral-domain model.

The high computational efficiency at producing an estimate of the expected response and mean power capture means that spectral-domain modelling is most suitable for tasks where a large number of simulations are required. Consequently, spectral-domain models are ideally suited to the calculation of the mean annual energy production as well as parametric design investigations. Conversely, because only a probabilistic estimate of the WEC response is available, they are less suitable for estimating the impact of transients and extreme events.

4.2 FORMULATION OF THE SPECTRAL-DOMAIN MODEL

The general structure of a spectral-domain model contains a probabilistic representation of the waves as input and a probabilistic representation of the model response as output. This is shown in Fig. 4.2, where the WEC system can invariably be represented as a set of coupled nonlinear ordinary differential equations (ODEs). A range of different techniques has been developed for solving the set of ODEs, including equivalent nonlinear equations, perturbation/functional series and Markov chains. Discussion of these techniques is outside the scope of this book because they have not yet been used for modelling WECs. However, for those interested, further information can be found in books on nonlinear random vibrations (see for example Cho, 2011), which is the more general field within which spectral-domain models exist. The technique that is detailed here, and that has been used for formulating and solving the set of ODEs of WECs, is commonly called statistical linearization, but is also known as stochastic or equivalent linearization.

Although not essential, it is useful in understanding spectral-domain models to outline the derivation. It is possible to derive the spectral-domain model from a number of different perspectives, and although these derivations may follow different paths the conclusions are necessarily the same. The derivation that is presented here is based on the derivation used to determine the effect of bottom friction (Hasselmann and Collins, 1968) and white-capping (Hasselmann, 1974) on wave growth in spectral wave models. This derivation assumes that both the input and output are Gaussian processes, which means that they can be represented statistically as the linear superposition of a set of frequency components with a random phase. The assumption that the response is also Gaussian is known as Gaussian closure.

In this derivation the calculation of an expected power is considered the output statistic of interest. This power could be the power capture of the WEC or a particular element such as the power loss due to vortex shedding. Power is the product of force and velocity and so the expected power P, denoted using the brackets $\langle \ldots \rangle$, is given by

$$P = \langle FU \rangle \tag{4.1}$$

As the velocity can be represented as a the linear sum of sinusoidal velocity components u_j, then the nonlinear force will be a functional of all these velocity components in addition to other parameters β such as the wave force. Substituting these representations into Eq. (4.1) gives

$$P = \sum_j \langle F(u_1, u_2, \ldots, u_j, \ldots, u_n, \beta_1, \ldots, \beta_n) u_j \rangle \tag{4.2}$$

It is now convenient to decompose the force functional Z into two constituents, one being the (infinitesimal) velocity component u_j and the other being the remainder of the force functional Z'

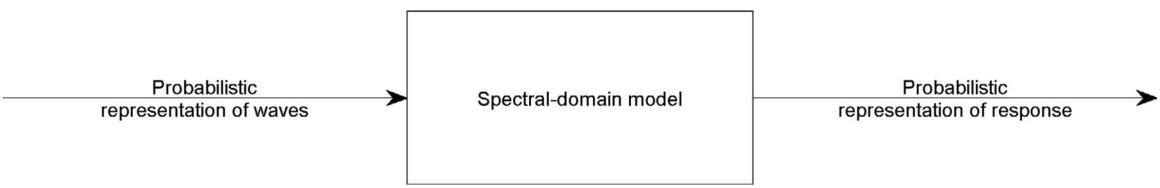

FIG. 4.2 General structure of a spectral-domain model.

$$F(u_1, u_2, \ldots, u_j, \ldots, u_n, \beta_1, \ldots, \beta_n) = F(Z) = F(Z' + u_j) \tag{4.3}$$

If the nonlinear force is differentiable then the first two terms of a Taylor series can be used to separate the two constituents so that

$$F(Z' + u_j) = F(Z') + \frac{\partial F}{\partial u_j}(Z')u_j \tag{4.4}$$

Putting this back into Eq. (4.2) gives

$$P = \sum_j \left\langle \left(F(Z') + \frac{\partial F}{\partial u_j}(Z')u_j \right) u_j \right\rangle \tag{4.5}$$

This equation can be simplified by applying two properties of expected values. The first property is that the expected value of a sum of terms is equal to the sum of the expected values of the terms $\langle A + B \rangle = \langle A \rangle + \langle B \rangle$, and the second property is that the expected value of the product of uncorrelated terms is equal to the product of the expected values of the terms $\langle AB \rangle = \langle A \rangle \langle B \rangle$. Using these properties Eq. (4.5) can be shown to be given by

$$P = \sum_j \langle F(Z') \rangle \langle u_j \rangle + \left\langle \frac{\partial F}{\partial u_j}(Z') \right\rangle \left\langle u_j^2 \right\rangle \tag{4.6}$$

This can be further simplified because the first term on the right-hand side of Eq. (4.6) disappears as the expected value of the sinusoidal velocity component u_j is zero. In addition, because the remainder of the functional Z' only differs from the complete functional Z by an infinitesimal amount, then this can be replaced by the full functional so that the power is given by

$$P = \sum_j \left\langle \frac{\partial F}{\partial u_j}(Z) \right\rangle \left\langle u_j^2 \right\rangle \tag{4.7}$$

Finally, the expected value of a sinusoidal velocity component squared is equal to half of the velocity component's magnitude squared $\left\langle u_j^2 \right\rangle = 0.5|U_j|^2$ and in Eq. (4.7) it is possible to replace the functional derivative with a quasi-linear coefficient $\left\langle \frac{\partial \mathrm{F}}{\partial u_j}(\mathrm{Z}) \right\rangle = D_j$ so that the power has a similar form to that for a frequency-domain model

$$P = \sum_j \frac{1}{2} D_j |U_j|^2 \tag{4.8}$$

Thus, the nonlinear force can be represented as a quasilinear coefficient within a frequency-domain model of the WEC. This representation can theoretically be applied to any nonlinear force that is differentiable with respect to the velocity component and remains accurate for even locally highly nonlinear forces provided that the response remains Gaussian, thereby ensuring that the velocity components at different frequencies remain uncorrelated. Consequently, a spectral-domain model can be formulated based on a frequency-domain model of a WEC where the nonlinear forces are replaced by quasilinear coefficients. However, the quasilinear coefficients will in general depend on the total response of the system, including the response at other frequencies, which means that the spectral-domain model needs to be solved iteratively.

As an example of the formulation of a spectral-domain model consider a single degree-of-freedom WEC as shown in Fig. 4.3. In this model it is assumed that the principal hydrodynamic force, F, and coefficients (k - hydrostatic stiffness, B - added damping, M_a - added mass) are linear and that they can be derived from linear potential flow theory. In addition, it is assumed that an external force, equivalent to a linear damper Λ, is applied to the WEC, together with a nonlinear force, F_{NL}, that opposes the WEC's motion and is proportional to its velocity squared. This nonlinear force is equivalent to the drag term of the Morison equation and thus represents a type of nonlinear force that is likely to be very important in the dynamics of many WECs.

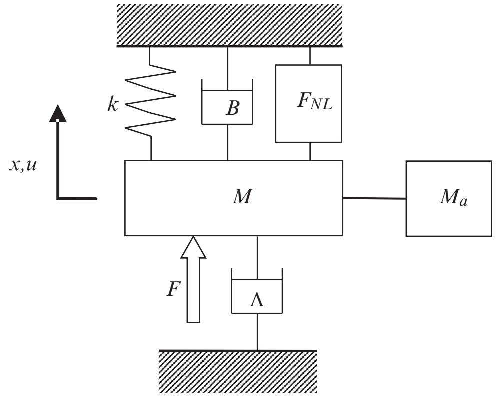

FIG. 4.3 Single degree-of-freedom WEC.

Decomposing the WEC's velocity into a set of velocity components allows the nonlinear force to be given by

$$F_D = C_D \mathbf{U}|\mathbf{U}| = C_D(u_1 + \cdots + u_j + \cdots + u_n)\,|u_1 + \cdots + u_j + \cdots + u_n| \tag{4.9}$$

The quasilinear coefficient can be obtained from the partial differentiation of this force by each velocity component so that

$$\begin{aligned} D_j &= \left\langle \frac{\partial}{\partial u_j} C_D(u_1 + \cdots + u_j + \cdots + u_n)\,|u_1 + \cdots + u_j + \cdots + u_n| \right\rangle \\ D_j &== 2C_D\langle |u_1 + \cdots + u_j + \cdots + u_n| \rangle \end{aligned} \tag{4.10}$$

This shows that the quasilinear drag coefficient is independent of frequency and proportional to the expected value of the WEC's absolute velocity. It is important to remember that this quasilinear drag coefficient is dependent on each velocity component making only a small contribution to the total response and consequently is inappropriate for use with monochromatic waves. Significantly, this quasilinear drag coefficient differs from one that can be derived from Lorentz's linearization for a monochromatic wave (see for example Terra et al., 2005; Folley et al., 2007) which is given by

$$D = \frac{8}{3\pi} C_D |U| \tag{4.11}$$

Lorentz's linearization is based on ensuring that the average power dissipation is maintained in the linearized model. The reason that this differs from that just derived here is that the velocity distribution for a monochromatic wave is not Gaussian and so contravenes one of the fundamental requirements for the derivation.

Returning to the formulation of the spectral-domain model, by exploiting the absence of correlation between the individual velocity components it is possible to calculate the expected absolute velocity and show that the quasilinear drag coefficient is given by

$$D_j = 2C_D \sqrt{\frac{1}{\pi} \sum_j |U_j|^2} = 2C_D \sqrt{\frac{1}{\pi} \sum_j \omega_j^2 |X_j|^2} \tag{4.12}$$

The spectral-domain model is then formulated using the same construction as a frequency-domain model except that the nonlinear force is replaced by the quasilinear coefficient D_j, which is given by Eq. (4.12). Thus, the spectral-domain model of the response of the WEC at each wave frequency can be obtained from

$$F_j = \left[\left(k - (M + M_{aj})\omega_j^2\right) + i\omega_j \left(B_j + \Lambda + 2C_D \sqrt{\frac{1}{\pi} \sum_j \omega_j^2 |X_j|^2} \right) \right] X_j \tag{4.13}$$

Total response can then be calculated by superposition of all the wave components, remembering that all the components are uncorrelated. Although this representation of a spectral-domain model appears very similar to a frequency-domain model, it differs in a significant way. That is, the response at each frequency is dependent not only on hydrodynamic coefficients at that frequency, but also the response of the WEC at all frequencies due to the nonlinear force.

4.3 SOLVING A SPECTRAL-DOMAIN MODEL

In general there is no analytical solution to a spectral-domain model, so typically an iterative solver has to be used. Iterative solvers use an initial guess and then generate successive approximations until the result converges and its termination criteria meet a predefined condition. A large number of iterative solvers exist; however, generally it has been found that for the spectral-domain models of WECs a very simple iterative solver is adequate.

The standard iterative method for solving spectral-domain models involves the following steps. An initial guess is generated by solving the WEC dynamics assuming that the nonlinear forces are negligible. By removing the nonlinear forces from the model, the system becomes linear and can be easily solved using the same techniques as for a frequency-domain model. Once an initial estimate of the response of the WEC has been produced, this can be used to estimate the quasilinear coefficient associated with the nonlinear forces. By approximating the nonlinear forces using quasilinear coefficients, the system is again linear and can again be easily solved. The revised response of the WEC can then be used again to refine the estimate of the quasilinear coefficients and the procedure repeated until an appropriate predefined condition is reached. An appropriate predefined condition to terminate iteration is that the WEC response has converged. Typically the solution is considered to have converged when the residual error, which equals the difference between the response used to calculate the quasilinear coefficients and the response estimated using the same quasilinear coefficients, is less than a specified amount such as 0.1% of the response.

This relatively simple iteration procedure is normally adequate for the spectral-domain modelling of WECs because either the nonlinear forces are not dominant and/or the response is well conditioned. For a range of different WEC models it has been found that, using the preceding method, the solution typically converges after 3–10 iterations. Although the described iteration method has to date always been found to be adequate, it is possible that models of WECs exist for which this iteration procedure does not converge. In these cases convergence may be achieved by either improving the initial guess and/or using an iteration relaxation method. Because the circumstances in which the described iteration procedure does not converge are likely to be variable, it is inappropriate to be overprescriptive. However, it may be that an improved initial guess can be identified by considering the underlying dynamics or from the response of the WEC where the nonlinear force is less dominant. With respect to suitable relaxation methods, a common method that is simple to implement is to define the refined estimate of the quasilinear coefficients D_j as the weighted sum of the previous quasilinear coefficients D_j^- and the newly calculated quasilinear coefficients D_j^+, where the weighting factor r defines the relaxation rate so that

$$D_j = rD_j^- + (1-r)D_j^+ \tag{4.14}$$

In general as the weighting factor increases towards unity the convergence rate becomes slower (requiring more iterations) but also more stable. Conversely, when the weighting factor is zero there is no relaxation and the solver becomes equivalent to the originally described iteration procedure.

The final output of the spectral-domain model is the spectral response of the WEC, which can be used to calculate a statistical estimate of performance. It has already been shown in Section 4.2 that due to the assumption of uncorrelated velocity components the expected average power capture is simply the sum of the average power capture at each frequency (see Eq. 4.8). In addition, the total variance of the WEC response is simply the sum of the variance at each frequency so that the expected standard deviation of the WEC's response σ_X is given by

$$\sigma_X = \sqrt{\frac{1}{2}\sum_j |X_j|^2} \tag{4.15}$$

In the vast majority of cases, the expected average power capture and standard deviation of response are likely to be the only statistical outputs required. However, because the spectra response of the WEC is available it is also possible to calculate a range of spectral parameters using the spectral moments, where the spectral moment is defined as

$$m_n = \frac{1}{2}\sum_j f_j^n |X_j|^2 \tag{4.16}$$

For example, to generate some data on power fluctuations the mean period T_m can be easily be calculated as it is simply the zeroth moment of the spectrum divided by the first moment of the spectrum

$$T_m = m_0 / m_1 \tag{4.17}$$

Application of the commonly reasonable approximation that the spectrum is narrow banded and because the WEC response is Gaussian, the WEC excursions can be shown to have a Rayleigh distribution. From this it is possible to calculate the expected groupiness of the WEC response (Liu et al., 1993), which may also provide some useful data on power fluctuations.

Exploiting the properties of a Rayleigh distribution also allows probability of an extreme response of the WEC to be calculated; however, this would probably push the reasonableness of the model too far. The calculation of the extreme response based on a Rayleigh distribution is likely to be unreasonable because during extreme events the nonlinear forces are likely to be most significant and cause the greatest deviation from a quasilinear response of the WEC. However, although it may be that the extreme responses may not be well represented using a spectral-domain model, the following examples demonstrate its accuracy for more typical modelling requirements.

4.4 EXAMPLES OF SPECTRAL-DOMAIN MODELLING

As a technique that is relatively new to the modelling of WECs, there are only a few examples of the application of spectral-domain models that can be used to illustrate its suitability. Specifically, the following applications of spectral-domain models are known to have been published in the literature:

A. a flap-type oscillating wave surge converter with vortex shedding (Folley and Whittaker, 2010)

B. a flap-type oscillating wave surge converter with wave torque decoupling (Folley and Whittaker, 2010)

C. an oscillating water column (OWC) with vortex shedding and orifice damping (Folley and Whittaker, 2014)

D. an array of heaving buoys with Coulomb friction damping (Folley and Whittaker, 2013)

In examples A and B the spectral-domain model is compared to a time-domain model to assess the spectral-domain model's validity. In these instances the time-domain model is considered to provide the true value of average power capture of the WEC against which the accuracy of the spectral-domain model is measured. Of course, in actuality the time-domain model does not provide the true value of average power capture not only because of the errors inherent in time-domain models (see Chapter 3), but also because a time-domain model will only converge on the average power capture as the simulation time tends to infinity. However, it is reasonable to expect that a well-designed time-domain model will provide an estimate of the average power capture that is accurate to within a few percent. In these examples the spectral-domain models use linear hydrodynamics coefficients derived from linear potential flow theory and the nonlinearity is limited to an additional vortex shedding force and a rotation-dependent attenuation of the linear wave force respectively.

Comparison of the estimated average power capture for the spectral-domain and time-domain models of a flap-type oscillating wave surge converter with vortex shedding, which is represented as a force that opposes motion and is proportional to the flap velocity squared, is shown in Fig. 4.4. It is clear that there is no significant difference between the average power estimates of the spectral-domain and time-domain models for a very wide range of quadratic damping coefficients (noting that the damping coefficient axis has a logarithmic scale). An equal degree of similarity was found for the spectral-domain model of a flap-type oscillating wave surge converter with wave torque decoupling, where the instantaneous wave torque is assumed to reduce in proportion to the cosine of the angle of rotation of the flap.

In example C the spectral-domain model of a bottom-mounted OWC is compared to data

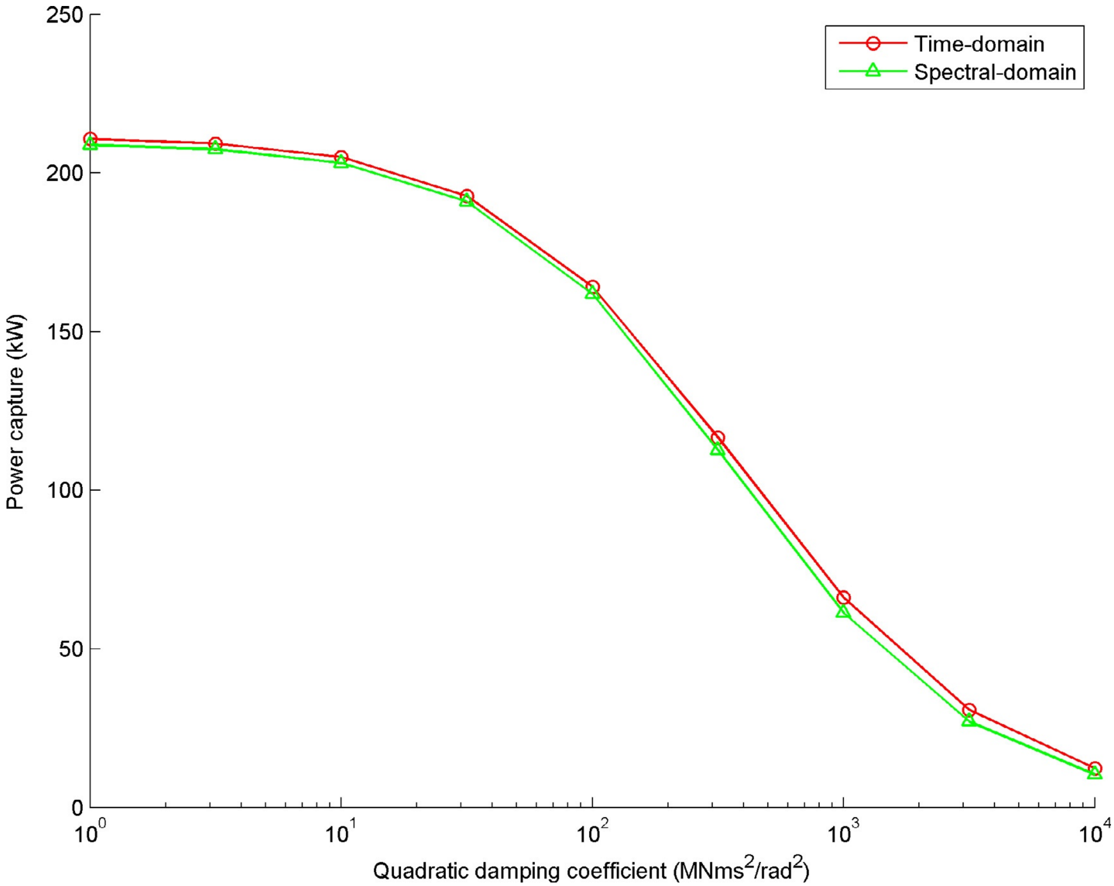

FIG. 4.4 Comparison of power capture and loss for spectral-domain and time-domain models *(reproduced from Folley and Whittaker, 2010).*

obtained from wave-tank testing of a model of the OWC. Although in this example the wave-tank model of the OWC is considered to provide the true value of response, the effect of wave-tank aberrations such as wave-tank wall reflections are not included in the spectral-domain model. Consequently, some difference between the results is to be expected even if the spectral-domain model were a perfect representation of the OWC. The spectral-domain model uses linear hydrodynamic coefficients obtained from a linear potential flow solver, with the addition of two nonlinear elements to represent vortex shedding from the lip of the OWC and the flow through the orifice, which is used to extract energy from the movement of the water column. Both of these nonlinear forces are represented as a force that opposes motion of the water column and is proportional to the water column velocity squared.

Comparison of the output of the spectral-domain model with the wave-tank model tests results are made for 15 different irregular sea-states with a range of different spectral shapes and scales, including bimodal sea-states. For these sea-states the average error in the spectral-domain capture factor is 4.4%, with a maximum error of 11.3%, as shown in Fig. 4.5, which provides reasonably strong validation of the spectral-domain model.

The spectral variance density of the spectral-domain model and the wave-tank water column response are also compared in Fig. 4.6, together with an estimate of the response that would be predicted by a frequency-domain model. This shows that the spectral-domain model not only produces a good estimate of the capture factor, but also that the spectral response is well represented.

These examples illustrate that at least in some circumstances a spectral-domain model is able to produce estimates of the statistical response of a WEC as accurately as a time-domain model. In addition, spectral-domain models have been validated using wave-tank testing, indicating that the combination of the linear hydrodynamic coefficients and nonlinear forces in a spectral-domain model can produce an accurate estimate of expected response and power capture. However, the examples presented only represent a small number of possible WECs and the success of the current spectral-domain models does not imply that all such models will be acceptable. The potential inadequacy of current spectral-domain models, which use statistical linearization with Gaussian closure, can be seen in the modelling of a heaving-buoy WEC with an external Coulomb damping force used to extract power. On the left-hand side of Fig. 4.7 the Coulomb damping force is low to moderate and it can be seen that the spectral response of the WEC is reasonably well represented by the spectral-domain model. However, on the right-hand side of Fig. 4.7 the Coulomb damping force is large and it can be seen that the spectral response is not correctly replicated by the spectral-domain model close to the peak of the system response, which leads to an increasing error in the estimation of the expected average power capture.

Consequently, it can be seen that the current generation of spectral domain models, which use statistical linearization and Gaussian closure, are capable of modelling many WECs accurately, but there are cases where they appear to be inadequate. However, it remains to be seen whether these inadequacies are a fundamental limitation of spectral-domain models, or a consequence of how the current generation of spectral-domain models are formulated.

4.5 FURTHER DEVELOPMENTS

Spectral-domain models of WECs are effectively specific solutions to nonlinear random vibration problems. Currently only a single method of solving the nonlinear random vibration problems, statistical linearization with Gaussian closure, has been used to formulate

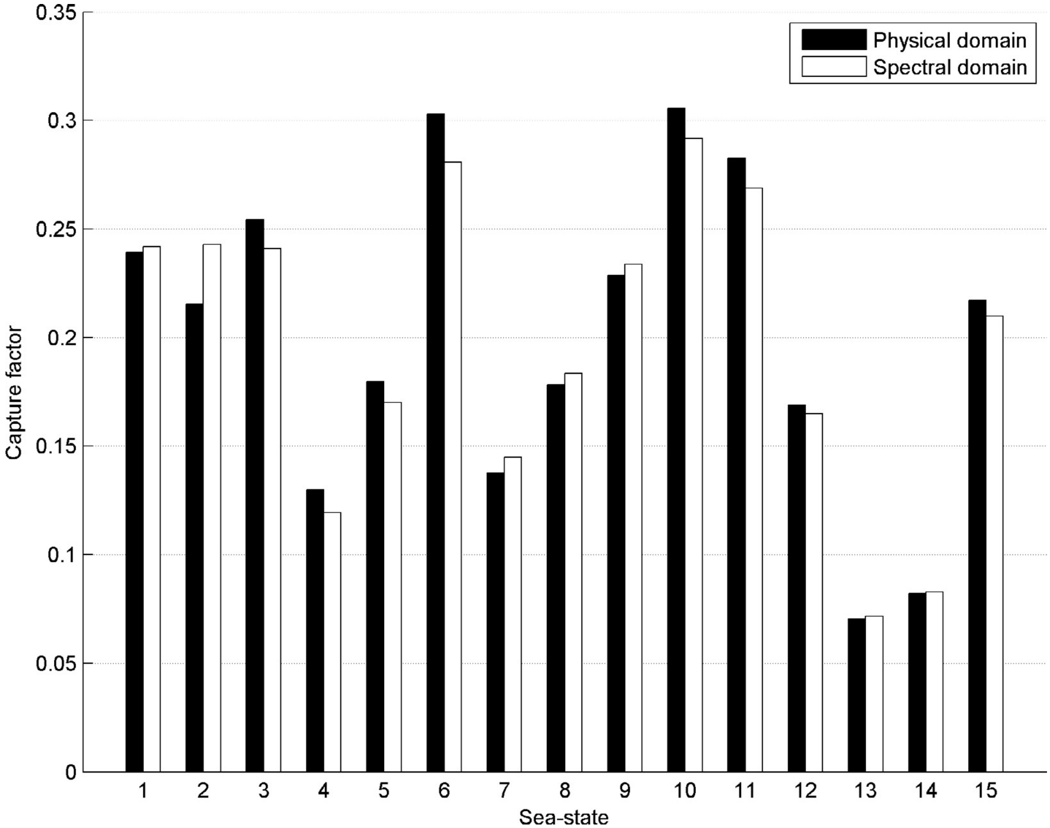

FIG. 4.5 Comparison of average power capture for spectral-domain and wave-tank models *(reproduced from Folley and Whittaker, 2014).*

the spectral-domain models. However, during the last 30 years significant progress has been made in developing solutions to nonlinear random vibration problems; all of these solutions may be applicable to the formulation of spectral-domain models of WECs. A simple lack of time and effort explains why no attempt has been made to apply these alternative solutions to WECs (spectral-domain models were only identified as possible solvers for WECs 5 years ago). Thus, there remains ample opportunity for the further development of alternative formulations of spectral-domain models, which may be expected to increase the utility of this type of model in the future.

Within the pantheon of solutions to nonlinear random vibration problems currently available, a particular promising area for the spectral-domain modelling of WECs is in the use of non-Gaussian closure. In these solutions the requirement that the response needs to be Gaussian is relaxed, thereby allowing a potentially more accurate approximation of the true response to be generated. It is argued that if it is possible to calculate moments of the response such as mean, mean squared and mean cubed, then these can be used to obtain estimates of the response's probability distribution. Then using either cumulant closure (Wu and Lin, 1984) or quasimoment closure (Bover, 1978),

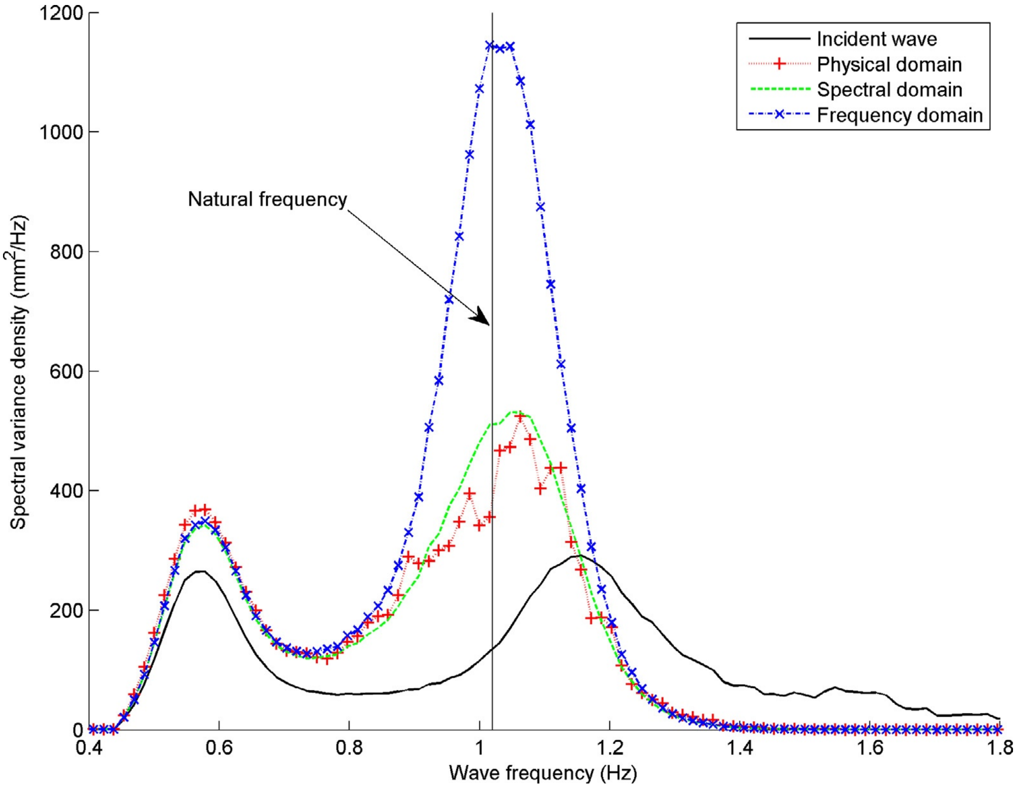

FIG. 4.6 Comparison of spectral variance density for spectral-domain and wave-tank models *(reproduced from Folley and Whittaker, 2014).*

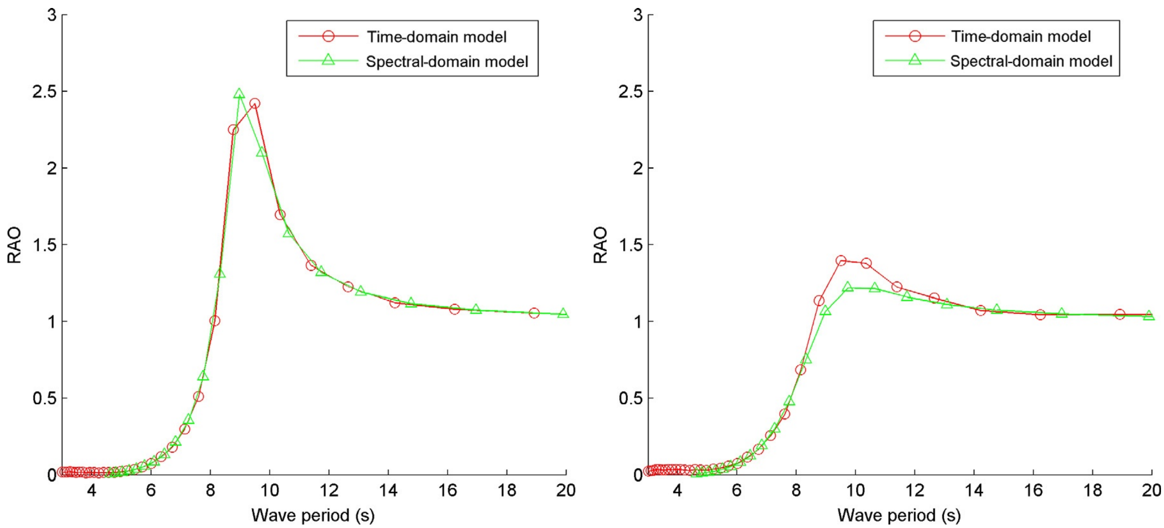

FIG. 4.7 Effect of strength of coulomb damping on spectral response of the Response Amplitude Operator (RAO) of a WEC.

the non-Gaussian response can be estimated. However, whilst these techniques may have been successful for improving the solution to other nonlinear random vibration problems, they need to be applied for WECs to determine their suitability and this remains an open area of research.

Related to the application of more sophisticated methods for formulating the nonlinear force in spectral-domain models of WECs is the requirement to develop suitable formulations for a wider range of nonlinear forces. Currently, good formulations have been identified for quadratic damping and cosine-type wave force decoupling, together with a potentially reasonable formulation for Coulomb friction; however, there is the need to expand the range of nonlinear forces for which formulations exist. Additional nonlinear forces with useful formulations are likely to include nonlinear hydrostatic stiffness, abrupt end-stop forces and complex mooring characteristics. Whether accurate WEC responses can be obtained using formulations based on statistical linearization with Gaussian closure, or whether other techniques from the field of nonlinear random vibrations are required is unknown. However, undoubtedly the ability to accurately represent a wider range of nonlinear forces will represent a significant development in the spectral-domain modelling of WECs.

4.6 LIMITATIONS

The fundamental limitation of spectral-domain models is that they can only provide statistical estimates of a WEC's response. This includes the expected average power capture and the spectral response amplitude operator, but does not include deterministic estimates such as the peak WEC displacement or power capture. Similarly, spectral-domain models are not suitable for modelling the response of a WEC during a transient, such as a power outage or wave impact event.

Although not a fundamental characteristic of spectral-domain models, it seems likely that they will always use hydrodynamic coefficients that have been derived from linear potential flow theory. Consequently, spectral-domain models are typically limited to cases where the hydrodynamics of the WEC remains approximately linear. In particular, this means that spectral-domain models are limited to sea-states where the wave steepness is moderate and so are not suited for modelling WECs in extreme waves.

In addition, the current formulation of spectral-domain models, which use statistical linearization with Gaussian closure to model nonlinear forces, also imposes a number of additional limitations on their usage. Whilst other formulations of spectral-domain models of WECs are likely to be possible they have not been produced. Furthermore, it cannot be known how easy it may be to generate additional formulations, or how effective they may be. Thus, it is prudent to base the current limitations of spectral-domain modelling on the currently available formulation. To consider these limitations it is convenient to consider each element of the current formulation separately: that is, first consider the limitations associated with statistical linearization and subsequently the limitations associated with Gaussian closure.

The use of statistical linearization to model nonlinear forces requires that these forces can be differentiated to enable a quasilinear coefficient to be calculated. Currently, quasilinear coefficients have only been determined for three types of nonlinear force: quadratic damping, cosine wave force decoupling and Coulomb damping. Thus, spectral-domain models are currently limited to those WECs whose nonlinear forces can be represented by one of these three types.

The use of Gaussian closure requires that the response of the WEC remains Gaussian. The greater the deviation from a Gaussian response, the less accurate the spectral-domain model becomes. Not surprisingly, the response typically becomes less Gaussian as the relative strength of the nonlinear force becomes greater. Unfortunately, although in many cases the response may remain effectively Gaussian, this cannot be assessed without resorting to another model to provide justification for the assumption of a Gaussian response. Because the use of spectral-domain models of WECs is relatively new, there is currently a lack of relevant experience to assess the reasonableness of the Gaussian response assumption for many WECs. Thus, there are two aspects of the use of Gaussian closure that result in limitations to spectral-domain models: a fundamental limitation and an epistemological limitation. The fundamental limitation is that the response of some WECs will not be sufficiently Gaussian so the spectral-domain model becomes invalid. The epistemological limitation is that it is difficult to assess whether a Gaussian response of the WEC is valid, which limits the application of spectral-domain models. Of course, it could be argued that all models require validation and so this is not a limitation exclusive to spectral-domain models; however, it is their lack of extensive application that makes this limitation particularly relevant for spectral-domain models at this time.

4.7 SUMMARY

- Spectral-domain models are probabilistic models that produce an estimate of the expected response of the WEC, for example, the expected average power capture
- Spectral-domain models are not suitable for the estimation of extreme responses
- The hydrodynamic forces in spectral-domain models are typically obtained using a linear potential flow solver
- Current spectral-domain models assume that the nonlinear force can be linearized and the WEC response is Gaussian
- The solution of a spectral-domain model typically involved iteration of a linearized frequency-domain model
- Nonlinear forces that can currently be modelled in spectral-domain models are quadratic damping, angular wave force decoupling and Coulomb friction
- It is possible that further developments in spectral-domain modelling may allow the WEC response to be non-Gaussian

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