

TENTH EDITION IN SI UNITS

# VECTOR MECHANICS FOR ENGINEERS STATICS AND DYNAMICS

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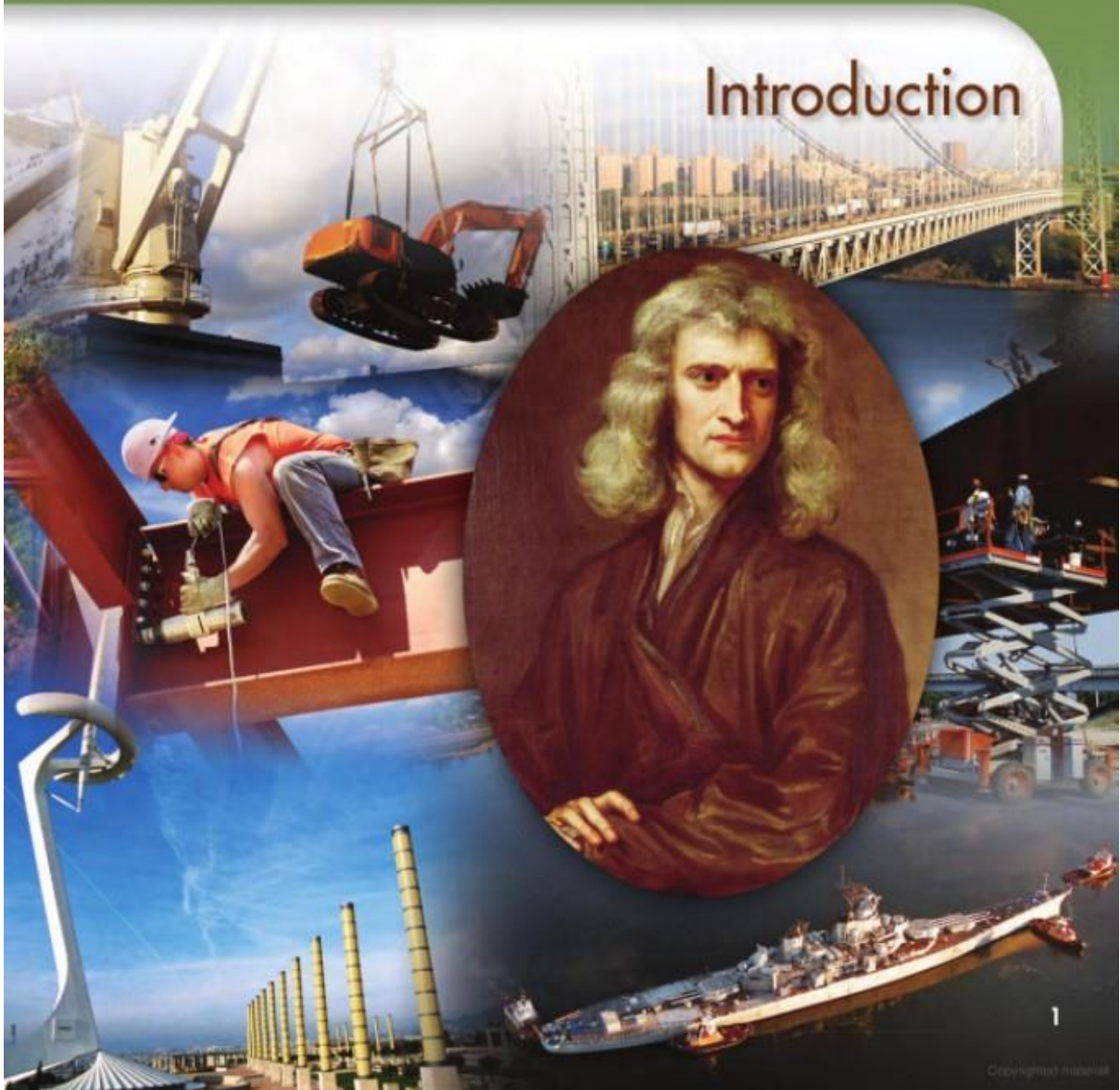
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# CHAPTER

## Introduction



## Chapter 1 Introduction

- 1.1 What Is Mechanics?
- 1.2 Fundamental Concepts and Principles
- 1.3 Systems of Units
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- 1.6 Numerical Accuracy

### 1.1 WHAT IS MECHANICS?

Mechanics can be defined as that science which describes and predicts the conditions of rest or motion of bodies under the action of forces. It is divided into three parts: mechanics of *rigid bodies*, mechanics of *deformable bodies*, and mechanics of *fluids*.

The mechanics of rigid bodies is subdivided into *statics* and *dynamics*, the former dealing with bodies at rest, the latter with bodies in motion. In this part of the study of mechanics, bodies are assumed to be perfectly rigid. Actual structures and machines, however, are never absolutely rigid and deform under the loads to which they are subjected. But these deformations are usually small and do not appreciably affect the conditions of equilibrium or motion of the structure under consideration. They are important, though, as far as the resistance of the structure to failure is concerned and are studied in mechanics of materials, which is a part of the mechanics of deformable bodies. The third division of mechanics, the mechanics of fluids, is subdivided into the study of *incompressible fluids* and of *compressible fluids*. An important subdivision of the study of incompressible fluids is *hydraulics*, which deals with problems involving water.

Mechanics is a physical science, since it deals with the study of physical phenomena. However, some associate mechanics with mathematics, while many consider it as an engineering subject. Both these views are justified in part. Mechanics is the foundation of most engineering sciences and is an indispensable prerequisite to their study. However, it does not have the *empiricism* found in some engineering sciences, i.e., it does not rely on experience or observation alone; by its rigor and the emphasis it places on deductive reasoning it resembles mathematics. But, again, it is not an *abstract* or even a *pure* science; mechanics is an *applied* science. The purpose of mechanics is to explain and predict physical phenomena and thus to lay the foundations for engineering applications.

### 1.2 FUNDAMENTAL CONCEPTS AND PRINCIPLES

Although the study of mechanics goes back to the time of Aristotle (384–322 B.C.) and Archimedes (287–212 B.C.), one has to wait until Newton (1642–1727) to find a satisfactory formulation of its fundamental principles. These principles were later expressed in a modified form by d'Alembert, Lagrange, and Hamilton. Their validity remained unchallenged, however, until Einstein formulated his *theory of relativity* (1905). While its limitations have now been recognized, *newtonian mechanics* still remains the basis of today's engineering sciences.

The basic concepts used in mechanics are *space*, *time*, *mass*, and *force*. These concepts cannot be truly defined; they should be accepted on the basis of our intuition and experience and used as a mental frame of reference for our study of mechanics.

The concept of *space* is associated with the notion of the position of a point  $P$ . The position of  $P$  can be defined by three lengths measured from a certain reference point, or *origin*, in three given directions. These lengths are known as the *coordinates* of  $P$ .

To define an event, it is not sufficient to indicate its position in space. The *time* of the event should also be given.

The concept of *mass* is used to characterize and compare bodies on the basis of certain fundamental mechanical experiments. Two bodies of the same mass, for example, will be attracted by the earth in the same manner; they will also offer the same resistance to a change in translational motion.

A *force* represents the action of one body on another. It can be exerted by actual contact or at a distance, as in the case of gravitational forces and magnetic forces. A force is characterized by its *point of application*, its *magnitude*, and its *direction*; a force is represented by a *vector* (Sec. 2.3).

In newtonian mechanics, space, time, and mass are absolute concepts, independent of each other. (This is not true in *relativistic mechanics*, where the time of an event depends upon its position, and where the mass of a body varies with its velocity.) On the other hand, the concept of force is not independent of the other three. Indeed, one of the fundamental principles of newtonian mechanics listed below indicates that the resultant force acting on a body is related to the mass of the body and to the manner in which its velocity varies with time.

You will study the conditions of rest or motion of particles and rigid bodies in terms of the four basic concepts we have introduced. By *particle* we mean a very small amount of matter which may be assumed to occupy a single point in space. A *rigid body* is a combination of a large number of particles occupying fixed positions with respect to each other. The study of the mechanics of particles is obviously a prerequisite to that of rigid bodies. Besides, the results obtained for a particle can be used directly in a large number of problems dealing with the conditions of rest or motion of actual bodies.

The study of elementary mechanics rests on six fundamental principles based on experimental evidence.

**The Parallelogram Law for the Addition of Forces.** This states that two forces acting on a particle may be replaced by a single force, called their *resultant*, obtained by drawing the diagonal of the parallelogram which has sides equal to the given forces (Sec. 2.2).

**The Principle of Transmissibility.** This states that the conditions of equilibrium or of motion of a rigid body will remain unchanged if a force acting at a given point of the rigid body is replaced by a force of the same magnitude and same direction, but acting at a different point, provided that the two forces have the same line of action (Sec. 3.3).

**Newton's Three Fundamental Laws.** Formulated by Sir Isaac Newton in the latter part of the seventeenth century, these laws can be stated as follows:

**FIRST LAW.** If the resultant force acting on a particle is zero, the particle will remain at rest (if originally at rest) or will move with constant speed in a straight line (if originally in motion) (Sec. 2.10).



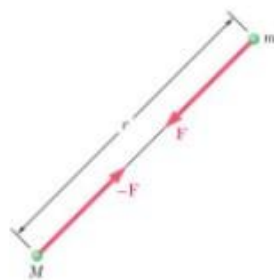


Fig. 1.1



**Photo 1.1** When in earth orbit, people and objects are said to be weightless even though the gravitational force acting is approximately 90% of that experienced on the surface of the earth. This apparent contradiction will be resolved in Chapter 12 when we apply Newton's second law to the motion of particles.

**SECOND LAW.** If the resultant force acting on a particle is not zero, the particle will have an acceleration proportional to the magnitude of the resultant and in the direction of this resultant force.

As you will see in Sec. 12.2, this law can be stated as

$$\mathbf{F} = m\mathbf{a} \quad (1.1)$$

where  $\mathbf{F}$ ,  $m$ , and  $\mathbf{a}$  represent, respectively, the resultant force acting on the particle, the mass of the particle, and the acceleration of the particle, expressed in a consistent system of units.

**THIRD LAW.** The forces of action and reaction between bodies in contact have the same magnitude, same line of action, and opposite sense (Sec. 6.1).

**Newton's Law of Gravitation.** This states that two particles of mass  $M$  and  $m$  are mutually attracted with equal and opposite forces  $\mathbf{F}$  and  $-\mathbf{F}$  (Fig. 1.1) of magnitude  $F$  given by the formula

$$F = G \frac{Mm}{r^2} \quad (1.2)$$

where  $r$  = distance between the two particles

$G$  = universal constant called the *constant of gravitation*

Newton's law of gravitation introduces the idea of an action exerted at a distance and extends the range of application of Newton's third law: the action  $\mathbf{F}$  and the reaction  $-\mathbf{F}$  in Fig. 1.1 are equal and opposite, and they have the same line of action.

A particular case of great importance is that of the attraction of the earth on a particle located on its surface. The force  $\mathbf{F}$  exerted by the earth on the particle is then defined as the *weight*  $\mathbf{W}$  of the particle. Taking  $M$  equal to the mass of the earth,  $m$  equal to the mass of the particle, and  $r$  equal to the radius  $R$  of the earth, and introducing the constant

$$g = \frac{GM}{R^2} \quad (1.3)$$

the magnitude  $W$  of the weight of a particle of mass  $m$  may be expressed as†

$$W = mg \quad (1.4)$$

The value of  $R$  in formula (1.3) depends upon the elevation of the point considered; it also depends upon its latitude, since the earth is not truly spherical. The value of  $g$  therefore varies with the position of the point considered. As long as the point actually remains on the surface of the earth, it is sufficiently accurate in most engineering computations to assume that  $g$  equals  $9.81 \text{ m/s}^2$ .

†A more accurate definition of the weight  $\mathbf{W}$  should take into account the rotation of the earth.

The principles we have just listed will be introduced in the course of our study of mechanics as they are needed. The study of the statics of particles carried out in Chap. 2 will be based on the parallelogram law of addition and on Newton's first law alone. The principle of transmissibility will be introduced in Chap. 3 as we begin the study of the statics of rigid bodies, and Newton's third law in Chap. 6 as we analyze the forces exerted on each other by the various members forming a structure. In the study of dynamics, Newton's second law and Newton's law of gravitation will be introduced. It will then be shown that Newton's first law is a particular case of Newton's second law (Sec. 12.2) and that the principle of transmissibility could be derived from the other principles and thus eliminated (Sec. 16.5). In the meantime, however, Newton's first and third laws, the parallelogram law of addition, and the principle of transmissibility will provide us with the necessary and sufficient foundation for the entire study of the statics of particles, rigid bodies, and systems of rigid bodies.

As noted earlier, the six fundamental principles listed above are based on experimental evidence. Except for Newton's first law and the principle of transmissibility, they are independent principles which cannot be derived mathematically from each other or from any other elementary physical principle. On these principles rests most of the intricate structure of newtonian mechanics. For more than two centuries a tremendous number of problems dealing with the conditions of rest and motion of rigid bodies, deformable bodies, and fluids have been solved by applying these fundamental principles. Many of the solutions obtained could be checked experimentally, thus providing a further verification of the principles from which they were derived. It is only in the twentieth century that Newton's mechanics was found at fault, in the study of the motion of atoms and in the study of the motion of certain planets, where it must be supplemented by the theory of relativity. But on the human or engineering scale, where velocities are small compared with the speed of light, Newton's mechanics has yet to be disproved.

### 1.3 SYSTEMS OF UNITS

With the four fundamental concepts introduced in the preceding section are associated the so-called *kinetic units*, i.e., the units of *length*, *time*, *mass*, and *force*. These units cannot be chosen independently if Eq. (1.1) is to be satisfied. Three of the units may be defined arbitrarily; they are then referred to as *basic units*. The fourth unit, however, must be chosen in accordance with Eq. (1.1) and is referred to as a *derived unit*. Kinetic units selected in this way are said to form a *consistent system of units*.

**International System of Units (SI Units)<sup>†</sup>.** In this system, which will be in universal use after the United States has completed its conversion to SI units, the base units are the units of length, mass, and time, and they are called, respectively, the *meter* (m), the *kilogram* (kg), and the *second* (s). All three are arbitrarily defined. The second,

<sup>†</sup>SI stands for *Système International d'Unités* (French).

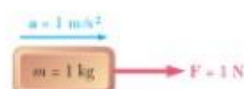


Fig. 1.2

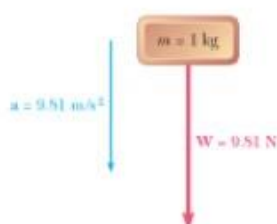


Fig. 1.3

which was originally chosen to represent  $1/86\,400$  of the mean solar day, is now defined as the duration of  $9\,192\,631\,770$  cycles of the radiation corresponding to the transition between two levels of the fundamental state of the cesium-133 atom. The meter, originally defined as one ten-millionth of the distance from the equator to either pole, is now defined as  $1\,650\,763.73$  wavelengths of the orange-red light corresponding to a certain transition in an atom of krypton-86. The kilogram, which is approximately equal to the mass of  $0.001\text{ m}^3$  of water, is defined as the mass of a platinum-iridium standard kept at the International Bureau of Weights and Measures at Sèvres, near Paris, France. The unit of force is a derived unit. It is called the *newton* (N) and is defined as the force which gives an acceleration of  $1\text{ m/s}^2$  to a mass of  $1\text{ kg}$  (Fig. 1.2). From Eq. (1.1) we write

$$1\text{ N} = (1\text{ kg})(1\text{ m/s}^2) = 1\text{ kg} \cdot \text{m/s}^2 \quad (1.5)$$

The SI units are said to form an *absolute* system of units. This means that the three base units chosen are independent of the location where measurements are made. The meter, the kilogram, and the second may be used anywhere on the earth; they may even be used on another planet. They will always have the same significance.

The *weight* of a body, or the *force of gravity* exerted on that body, should, like any other force, be expressed in newtons. From Eq. (1.4) it follows that the weight of a body of mass  $1\text{ kg}$  (Fig. 1.3) is

$$\begin{aligned} W &= mg \\ &= (1\text{ kg})(9.81\text{ m/s}^2) \\ &= 9.81\text{ N} \end{aligned}$$

Multiples and submultiples of the fundamental SI units may be obtained through the use of the prefixes defined in Table 1.1. The multiples and submultiples of the units of length, mass, and force most frequently used in engineering are, respectively, the *kilometer* (km) and the *millimeter* (mm); the *megagram*† (Mg) and the *gram* (g); and the *kilonewton* (kN). According to Table 1.1, we have

$$\begin{aligned} 1\text{ km} &= 1000\text{ m} & 1\text{ mm} &= 0.001\text{ m} \\ 1\text{ Mg} &= 1000\text{ kg} & 1\text{ g} &= 0.001\text{ kg} \\ 1\text{ kN} &= 1000\text{ N} \end{aligned}$$

The conversion of these units into meters, kilograms, and newtons, respectively, can be effected by simply moving the decimal point three places to the right or to the left. For example, to convert  $3.82\text{ km}$  into meters, one moves the decimal point three places to the right:

$$3.82\text{ km} = 3820\text{ m}$$

Similarly,  $47.2\text{ mm}$  is converted into meters by moving the decimal point three places to the left:

$$47.2\text{ mm} = 0.0472\text{ m}$$

†Also known as a *metric ton*.

**TABLE 1.1 SI Prefixes**

Multiplication Factor	Prefix†	Symbol
1 000 000 000 000 = $10^{12}$	tera	T
1 000 000 000 = $10^9$	giga	G
1 000 000 = $10^6$	mega	M
1 000 = $10^3$	kilo	k
100 = $10^2$	hecto‡	h
10 = $10^1$	deka‡	da
0.1 = $10^{-1}$	deci‡	d
0.01 = $10^{-2}$	centi‡	c
0.001 = $10^{-3}$	milli	m
0.000 001 = $10^{-6}$	micro	$\mu$
0.000 000 001 = $10^{-9}$	nano	n
0.000 000 000 001 = $10^{-12}$	pico	p
0.000 000 000 000 001 = $10^{-15}$	femto	f
0.000 000 000 000 000 001 = $10^{-18}$	atto	a

†The first syllable of every prefix is accented so that the prefix will retain its identity. Thus, the preferred pronunciation of kilometer places the accent on the first syllable, not the second.

‡The use of these prefixes should be avoided, except for the measurement of areas and volumes and for the nontechnical use of centimeter, as for body and clothing measurements.

Using scientific notation, one may also write

$$\begin{aligned} 3.82 \text{ km} &= 3.82 \times 10^3 \text{ m} \\ 47.2 \text{ mm} &= 47.2 \times 10^{-3} \text{ m} \end{aligned}$$

The multiples of the unit of time are the *minute* (min) and the *hour* (h). Since 1 min = 60 s and 1 h = 60 min = 3600 s, these multiples cannot be converted as readily as the others.

By using the appropriate multiple or submultiple of a given unit, one can avoid writing very large or very small numbers. For example, one usually writes 427.2 km rather than 427 200 m, and 2.16 mm rather than 0.002 16 m.†

**Units of Area and Volume.** The unit of area is the *square meter* ( $\text{m}^2$ ), which represents the area of a square of side 1 m; the unit of volume is the *cubic meter* ( $\text{m}^3$ ), equal to the volume of a cube of side 1 m. In order to avoid exceedingly small or large numerical values in the computation of areas and volumes, one uses systems of subunits obtained by respectively squaring and cubing not only the millimeter but also two intermediate submultiples of the meter, namely, the *decimeter* (dm) and the *centimeter* (cm). Since, by definition,

$$\begin{aligned} 1 \text{ dm} &= 0.1 \text{ m} = 10^{-1} \text{ m} \\ 1 \text{ cm} &= 0.01 \text{ m} = 10^{-2} \text{ m} \\ 1 \text{ mm} &= 0.001 \text{ m} = 10^{-3} \text{ m} \end{aligned}$$

†It should be noted that when more than four digits are used on either side of the decimal point to express a quantity in SI units—as in 427 200 m or 0.002 16 m—spaces, never commas, should be used to separate the digits into groups of three. This is to avoid confusion with the comma used in place of a decimal point, which is the convention in many countries.



the submultiples of the unit of area are

$$1 \text{ dm}^2 = (1 \text{ dm})^2 = (10^{-1} \text{ m})^2 = 10^{-2} \text{ m}^2$$

$$1 \text{ cm}^2 = (1 \text{ cm})^2 = (10^{-2} \text{ m})^2 = 10^{-4} \text{ m}^2$$

$$1 \text{ mm}^2 = (1 \text{ mm})^2 = (10^{-3} \text{ m})^2 = 10^{-6} \text{ m}^2$$

and the submultiples of the unit of volume are

$$1 \text{ dm}^3 = (1 \text{ dm})^3 = (10^{-1} \text{ m})^3 = 10^{-3} \text{ m}^3$$

$$1 \text{ cm}^3 = (1 \text{ cm})^3 = (10^{-2} \text{ m})^3 = 10^{-6} \text{ m}^3$$

$$1 \text{ mm}^3 = (1 \text{ mm})^3 = (10^{-3} \text{ m})^3 = 10^{-9} \text{ m}^3$$

It should be noted that when the volume of a liquid is being measured, the cubic decimeter ( $\text{dm}^3$ ) is usually referred to as a *liter* (L).

Other derived SI units used to measure the moment of a force, the work of a force, etc., are shown in Table 1.2. While these units will be introduced in later chapters as they are needed, we should note an important rule at this time: When a derived unit is obtained by dividing a base unit by another base unit, a prefix may be used in the numerator of the derived unit but not in its denominator. For example, the constant  $k$  of a spring which stretches 20 mm under a load of 100 N will be expressed as

$$k = \frac{100 \text{ N}}{20 \text{ mm}} = \frac{100 \text{ N}}{0.020 \text{ m}} = 5000 \text{ N/m} \quad \text{or} \quad k = 5 \text{ kN/m}$$

but never as  $k = 5 \text{ N/mm}$ .

**TABLE 1.2 Principal SI Units Used in Mechanics**

Quantity	Unit	Symbol	Formula
Acceleration	Meter per second squared	...	$\text{m/s}^2$
Angle	Radian	rad	†
Angular acceleration	Radian per second squared	...	$\text{rad/s}^2$
Angular velocity	Radian per second	...	$\text{rad/s}$
Area	Square meter	...	$\text{m}^2$
Density	Kilogram per cubic meter	...	$\text{kg/m}^3$
Energy	Joule	J	$\text{N} \cdot \text{m}$
Force	Newton	N	$\text{kg} \cdot \text{m/s}^2$
Frequency	Hertz	Hz	$\text{s}^{-1}$
Impulse	Newton-second	...	$\text{kg} \cdot \text{m/s}$
Length	Meter	m	‡
Mass	Kilogram	kg	‡
Moment of a force	Newton-meter	...	$\text{N} \cdot \text{m}$
Power	Watt	W	$\text{J/s}$
Pressure	Pascal	Pa	$\text{N/m}^2$
Stress	Pascal	Pa	$\text{N/m}^2$
Time	Second	s	‡
Velocity	Meter per second	...	$\text{m/s}$
Volume			
Solids	Cubic meter	...	$\text{m}^3$
Liquids	Liter	L	$10^{-3} \text{ m}^3$
Work	Joule	J	$\text{N} \cdot \text{m}$

†Supplementary unit (1 revolution =  $2\pi$  rad =  $360^\circ$ ).

‡Base unit.

Many engineering problems can be solved by considering the equilibrium of a "particle." In the case of this excavator, which is being loaded onto a ship, a relation between the tensions in the various cables involved can be obtained by considering the equilibrium of the hook to which the cables are attached.

