Microwave Engineering

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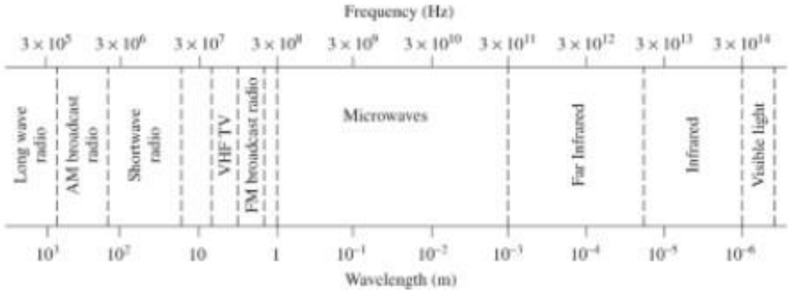
C h a p t e r   O n e

Electromagnetic Theory

We begin our study of microwave engineering with a brief overview of the history and major applications of microwave technology, followed by a review of some of the fundamental topics in electromagnetic theory that we will need throughout the book. Further discussion of these topics may be found in references [1–8].

1.1 INTRODUCTION TO MICROWAVE ENGINEERING

The field of radio frequency (RF) and microwave engineering generally covers the behavior of alternating current signals with frequencies in the range of 100 MHz (1 MHz = 10^6 Hz) to 1000 GHz (1 GHz = 10^9 Hz). RF frequencies range from very high frequency (VHF) (30–300 MHz) to ultra high frequency (UHF) (300–3000 MHz), while the term *microwave* is typically used for frequencies between 3 and 300 GHz, with a corresponding electrical wavelength between $\lambda = c/f = 10$ cm and $\lambda = 1$ mm, respectively. Signals with wavelengths on the order of millimeters are often referred to as *millimeter waves*. Figure 1.1 shows the location of the RF and microwave frequency bands in the electromagnetic spectrum. Because of the high frequencies (and short wavelengths), standard circuit theory often cannot be used directly to solve microwave network problems. In a sense, standard circuit theory is an approximation, or special case, of the broader theory of electromagnetics as described by Maxwell's equations. This is due to the fact that, in general, the lumped circuit element approximations of circuit theory may not be valid at high RF and microwave frequencies. Microwave components often act as *distributed elements*, where the phase of the voltage or current changes significantly over the physical extent of the device because the device dimensions are on the order of the electrical wavelength. At much lower frequencies the wavelength is large enough that there is insignificant phase variation across the dimensions of a component. The other extreme of frequency can be identified as optical engineering, in which the wavelength is much shorter than the dimensions of the component. In this case Maxwell's equations can be simplified to the geometrical optics regime, and optical systems can be designed with the theory of geometrical optics. Such



Typical Frequencies	
AM broadcast band	535–1605 kHz
Short wave radio band	3–30 MHz
FM broadcast band	88–108 MHz
VHF TV (2–4)	54–72 MHz
VHF TV (5–6)	76–88 MHz
UHF TV (7–13)	174–216 MHz
UHF TV (14–83)	470–890 MHz
US cellular telephone	824–849 MHz 869–894 MHz
European GSM cellular	880–915 MHz 925–960 MHz
GPS	1575.42 MHz 1227.60 MHz
Microwave ovens	2.45 GHz
US DBS	11.7–12.5 GHz
US ISM bands	902–928 MHz 2.400–2.484 GHz 5.725–5.850 GHz
US UWB radio	3.1–10.6 GHz

Approximate Band Designations	
Medium frequency	300 kHz–3 MHz
High frequency (HF)	3 MHz–30 MHz
Very high frequency (VHF)	30 MHz–300 MHz
Ultra high frequency (UHF)	300 MHz–3 GHz
L band	1–2 GHz
S band	2–4 GHz
C band	4–8 GHz
X band	8–12 GHz
Ku band	12–18 GHz
K band	18–26 GHz
Ka band	26–40 GHz
U band	40–60 GHz
V band	50–75 GHz
E band	60–90 GHz
W band	75–110 GHz
F band	90–140 GHz

FIGURE 1.1 The electromagnetic spectrum.

techniques are sometimes applicable to millimeter wave systems, where they are referred to as *quasi-optical*.

In RF and microwave engineering, then, one must often work with Maxwell's equations and their solutions. It is in the nature of these equations that mathematical complexity arises since Maxwell's equations involve vector differential or integral operations on vector field quantities, and these fields are functions of spatial coordinates. One of the goals of this book is to try to reduce the complexity of a field theory solution to a result that can be expressed in terms of simpler circuit theory, perhaps extended to include distributed elements (such as transmission lines) and concepts (such as reflection coefficients and scattering parameters). A field theory solution generally provides a complete description of the electromagnetic field at every point in space, which is usually much more information than we need for most practical purposes. We are typically more interested in terminal quantities such as power, impedance, voltage, and current, which can often be expressed in terms of these extended circuit theory concepts. It is this complexity that adds to the challenge, as well as the rewards, of microwave engineering.

Applications of Microwave Engineering

Just as the high frequencies and short wavelengths of microwave energy make for difficulties in the analysis and design of microwave devices and systems, these same aspects provide unique opportunities for the application of microwave systems. The following considerations can be useful in practice:

- Antenna gain is proportional to the electrical size of the antenna. At higher frequencies, more antenna gain can be obtained for a given physical antenna size, and this has important consequences when implementing microwave systems.
- More bandwidth (directly related to data rate) can be realized at higher frequencies. A 1% bandwidth at 600 MHz is 6 MHz, which (with binary phase shift keying modulation) can provide a data rate of about 6 Mbps (megabits per second), while at 60 GHz a 1% bandwidth is 600 MHz, allowing a 600 Mbps data rate.
- Microwave signals travel by line of sight and are not bent by the ionosphere as are lower frequency signals. Satellite and terrestrial communication links with very high capacities are therefore possible, with frequency reuse at minimally distant locations.
- The effective reflection area (radar cross section) of a radar target is usually proportional to the target's electrical size. This fact, coupled with the frequency characteristics of antenna gain, generally makes microwave frequencies preferred for radar systems.
- Various molecular, atomic, and nuclear resonances occur at microwave frequencies, creating a variety of unique applications in the areas of basic science, remote sensing, medical diagnostics and treatment, and heating methods.

The majority of today's applications of RF and microwave technology are to wireless networking and communications systems, wireless security systems, radar systems, environmental remote sensing, and medical systems. As the frequency allocations listed in Figure 1.1 show, RF and microwave communications systems are pervasive, especially today when wireless connectivity promises to provide voice and data access to "anyone, anywhere, at any time."

Modern wireless telephony is based on the concept of *cellular frequency reuse*, a technique first proposed by Bell Labs in 1947 but not practically implemented until the 1970s. By this time advances in miniaturization, as well as increasing demand for wireless communications, drove the introduction of several early cellular telephone systems in Europe, the United States, and Japan. The *Nordic Mobile Telephone* (NMT) system was deployed in 1981 in the Nordic countries, the *Advanced Mobile Phone System* (AMPS) was introduced in the United States in 1983 by AT&T, and NTT in Japan introduced its first mobile phone service in 1988. All of these early systems used analog FM modulation, with their allocated frequency bands divided into several hundred narrow band voice channels. These early systems are usually referred to now as *first-generation* cellular systems, or 1G.

Second-generation (2G) cellular systems achieved improved performance by using various digital modulation schemes, with systems such as GSM, CDMA, DAMPS, PCS, and PHS being some of the major standards introduced in the 1990s in the United States, Europe, and Japan. These systems can handle digitized voice, as well as some limited data, with data rates typically in the 8 to 14 kbps range. In recent years there has been a wide variety of new and modified standards to transition to handheld services that include voice, texting, data networking, positioning, and Internet access. These standards are variously known as 2.5G, 3G, 3.5G, 3.75G, and 4G, with current plans to provide data rates up to at least 100 Mbps. The number of subscribers to wireless services seems to be keeping pace with the growing power and access provided by modern handheld wireless devices; as of 2010 there were more than five billion cell phone users worldwide.

Satellite systems also depend on RF and microwave technology, and satellites have been developed to provide cellular (voice), video, and data connections worldwide. Two large satellite constellations, Iridium and Globalstar, were deployed in the late 1990s to provide worldwide telephony service. Unfortunately, these systems suffered from both technical communications systems and have thus provided an impetus for the continuing development of low-cost miniaturized microwave components. We refer the interested reader to references [1] and [2] for further historical perspectives on the fields of wireless communications and microwave engineering.

1.2 MAXWELL'S EQUATIONS

Electric and magnetic phenomena at the macroscopic level are described by Maxwell's equations, as published by Maxwell in 1873. This work summarized the state of electromagnetic science at that time and hypothesized from theoretical considerations the existence of the electrical displacement current, which led to the experimental discovery by Hertz of electromagnetic wave propagation. Maxwell's work was based on a large body of empirical and theoretical knowledge developed by Gauss, Ampere, Faraday, and others. A first course in electromagnetics usually follows this historical (or deductive) approach, and it is assumed that the reader has had such a course as a prerequisite to the present material. Several references are available [3–7] that provide a good treatment of electromagnetic theory at the undergraduate or graduate level.

This chapter will outline the fundamental concepts of electromagnetic theory that we will require later in the book. Maxwell's equations will be presented, and boundary conditions and the effect of dielectric and magnetic materials will be discussed. Wave phenomena are of essential importance in microwave engineering, and thus much of the chapter is spent on topics related to plane waves. Plane waves are the simplest form of electromagnetic waves and so serve to illustrate a number of basic properties associated with wave propagation. Although it is assumed that the reader has studied plane waves before, the present material should help to reinforce the basic principles in the reader's mind and perhaps to introduce some concepts that the reader has not seen previously. This material will also serve as a useful reference for later chapters.

With an awareness of the historical perspective, it is usually advantageous from a pedagogical point of view to present electromagnetic theory from the "inductive," or axiomatic, approach by beginning with Maxwell's equations. The general form of time-varying Maxwell equations, then, can be written in "point," or differential, form as

$$\nabla \times \bar{\mathcal{E}} = \frac{-\partial \bar{\mathcal{B}}}{\partial t} - \bar{\mathcal{M}}, \tag{1.1a}$$

$$\nabla \times \bar{\mathcal{H}} = \frac{\partial \bar{\mathcal{D}}}{\partial t} + \bar{\mathcal{J}}, \tag{1.1b}$$

$$\nabla \cdot \bar{\mathcal{D}} = \rho, \tag{1.1c}$$

$$\nabla \cdot \bar{\mathcal{B}} = 0. \tag{1.1d}$$

The MKS system of units is used throughout this book. The script quantities represent time-varying vector fields and are real functions of spatial coordinates x, y, z, and the time variable t. These quantities are defined as follows:

$\bar{\mathcal{E}}$ is the electric field, in volts per meter (V/m).[1]

$\bar{\mathcal{H}}$ is the magnetic field, in amperes per meter (A/m).

[1] As recommended by the *IEEE Standard Definitions of Terms for Radio Wave Propagation, IEEE Standard 211-1997*, the terms "electric field" and "magnetic field" are used in place of the older terminology of "electric field intensity" and "magnetic field intensity."

$\bar{\mathcal{D}}$ is the electric flux density, in coulombs per meter squared (Coul/m^2).

$\bar{\mathcal{B}}$ is the magnetic flux density, in webers per meter squared (Wb/m^2).

$\bar{\mathcal{M}}$ is the (fictitious) magnetic current density, in volts per meter (V/m^2).

$\bar{\mathcal{J}}$ is the electric current density, in amperes per meter squared (A/m^2).

ρ is the electric charge density, in coulombs per meter cubed (Coul/m^3).

The sources of the electromagnetic field are the currents $\bar{\mathcal{M}}$ and $\bar{\mathcal{J}}$ and the electric charge density ρ. The magnetic current $\bar{\mathcal{M}}$ is a fictitious source in the sense that it is only a mathematical convenience: the real source of a magnetic current is always a loop of electric current or some similar type of magnetic dipole, as opposed to the flow of an actual magnetic charge (magnetic monopole charges are not known to exist). The magnetic current is included here for completeness, as we will have occasion to use it in Chapter 4 when dealing with apertures. Since electric current is really the flow of charge, it can be said that the electric charge density ρ is the ultimate source of the electromagnetic field.

In free-space, the following simple relations hold between the electric and magnetic field intensities and flux densities:

$$\bar{\mathcal{B}} = \mu_0 \bar{\mathcal{H}}, \tag{1.2a}$$

$$\bar{\mathcal{D}} = \epsilon_0 \bar{\mathcal{E}}, \tag{1.2b}$$

where $\mu_0 = 4\pi \times 10^{-7}$ henry/m is the permeability of free-space, and $\epsilon_0 = 8.854 \times 10^{-12}$ farad/m is the permittivity of free-space. We will see in the next section how media other than free-space affect these constitutive relations.

Equations (1.1a)–(1.1d) are linear but are not independent of each other. For instance, consider the divergence of (1.1a). Since the divergence of the curl of any vector is zero [vector identity (B.12), from Appendix B], we have

$$\nabla \cdot \nabla \times \bar{\mathcal{E}} = 0 = -\frac{\partial}{\partial t}(\nabla \cdot \bar{\mathcal{B}}) - \nabla \cdot \bar{\mathcal{M}}.$$

Since there is no free magnetic charge, $\nabla \cdot \bar{\mathcal{M}} = 0$, which leads to $\nabla \cdot \bar{\mathcal{B}} = 0$, or (1.1d). The *continuity equation* can be similarly derived by taking the divergence of (1.1b), giving

$$\nabla \cdot \bar{\mathcal{J}} + \frac{\partial \rho}{\partial t} = 0, \tag{1.3}$$

where (1.1c) was used. This equation states that charge is conserved, or that current is continuous, since $\nabla \cdot \bar{\mathcal{J}}$ represents the outflow of current at a point, and $\partial\rho/\partial t$ represents the charge buildup with time at the same point. It is this result that led Maxwell to the conclusion that the displacement current density $\partial\bar{\mathcal{D}}/\partial t$ was necessary in (1.1b), which can be seen by taking the divergence of this equation.

The above differential equations can be converted to integral form through the use of various vector integral theorems. Thus, applying the divergence theorem (B.15) to (1.1c) and (1.1d) yields

$$\oint_S \bar{\mathcal{D}} \cdot d\bar{s} = \int_V \rho \, dv = Q, \tag{1.4}$$

$$\oint_S \bar{\mathcal{B}} \cdot d\bar{s} = 0, \tag{1.5}$$

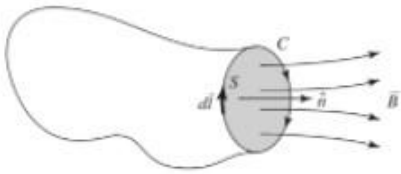


FIGURE 1.3 The closed contour C and surface S associated with Faraday's law.

where Q in (1.4) represents the total charge contained in the closed volume V (enclosed by a closed surface S). Applying Stokes' theorem (B.16) to (1.1a) gives

$$\oint_C \bar{\mathcal{E}} \cdot d\bar{l} = -\frac{\partial}{\partial t} \int_S \bar{\mathcal{B}} \cdot d\bar{s} - \int_S \bar{\mathcal{M}} \cdot d\bar{s}, \tag{1.6}$$

which, without the $\bar{\mathcal{M}}$ term, is the usual form of *Faraday's law* and forms the basis for *Kirchhoff's voltage law*. In (1.6), C represents a closed contour around the surface S, as shown in Figure 1.3. *Ampere's law* can be derived by applying Stokes' theorem to (1.1b):

$$\oint_C \bar{\mathcal{H}} \cdot d\bar{l} = \frac{\partial}{\partial t} \int_S \bar{\mathcal{D}} \cdot d\bar{s} + \int_S \bar{\mathcal{J}} \cdot d\bar{s} = \frac{\partial}{\partial t} \int_S \bar{\mathcal{D}} \cdot d\bar{s} + \mathcal{I}, \tag{1.7}$$

where $\mathcal{I} = \int_S \bar{\mathcal{J}} \cdot d\bar{s}$ is the total electric current flow through the surface S. Equations (1.4)–(1.7) constitute the integral forms of Maxwell's equations.

The above equations are valid for arbitrary time dependence, but most of our work will be involved with fields having a sinusoidal, or harmonic, time dependence, with steady-state conditions assumed. In this case phasor notation is very convenient, and so all field quantities will be assumed to be complex vectors with an implied $e^{j\omega t}$ time dependence and written with roman (rather than script) letters. Thus, a sinusoidal electric field polarized in the $\hat{x}$ direction of the form

$$\bar{\mathcal{E}}(x, y, z, t) = \hat{x} A(x, y, z) \cos(\omega t + \phi), \tag{1.8}$$

where A is the (real) amplitude, ω is the radian frequency, and ϕ is the phase reference of the wave at $t = 0$, has the phasor for

$$\bar{E}(x, y, z) = \hat{x} A(x, y, z) e^{j\phi}. \tag{1.9}$$

We will assume cosine-based phasors in this book, so the conversion from phasor quantities to real time-varying quantities is accomplished by multiplying the phasor by $e^{j\omega t}$ and taking the real part:

$$\bar{\mathcal{E}}(x, y, z, t) = \mathrm{Re}\{\bar{E}(x, y, z) e^{j\omega t}\}, \tag{1.10}$$

as substituting (1.9) into (1.10) to obtain (1.8) demonstrates. When working in phasor notation, it is customary to suppress the factor $e^{j\omega t}$ that is common to all terms.

When dealing with power and energy we will often be interested in the time average of a quadratic quantity. This can be found very easily for time harmonic fields. For example, the average of the square of the magnitude of an electric field, given as

$$\bar{\mathcal{E}} = \hat{x} E_1 \cos(\omega t + \phi_1) + \hat{y} E_2 \cos(\omega t + \phi_2) + \hat{z} E_2 \cos(\omega t + \phi_3), \tag{1.11}$$

has the phasor form

$$\bar{E} = \hat{x} E_1 e^{j\phi_1} + \hat{y} E_2 e^{j\phi_2} + \hat{z} E_3 e^{j\phi_3}, \tag{1.12}$$

can be calculated as

$$
\begin{aligned}
|\bar{\mathcal{E}}|^2_{\text{avg}} &= \frac{1}{T}\int_0^T \bar{\mathcal{E}} \cdot \bar{\mathcal{E}}\, dt \\
&= \frac{1}{T}\int_0^T \left[E_1^2 \cos^2(\omega t + \phi_1) + E_2^2 \cos^2(\omega t + \phi_2) + E_3^2 \cos^2(\omega t + \phi_3)\right] dt \\
&= \frac{1}{2}\left(E_1^2 + E_2^2 + E_3^2\right) = \frac{1}{2}|\bar{E}|^2 = \frac{1}{2}\bar{E} \cdot \bar{E}^*. \qquad (1.13)
\end{aligned}
$$

Then the root-mean-square (rms) value is $|\bar{E}|_{\text{rms}} = |\bar{E}|/\sqrt{2}$.

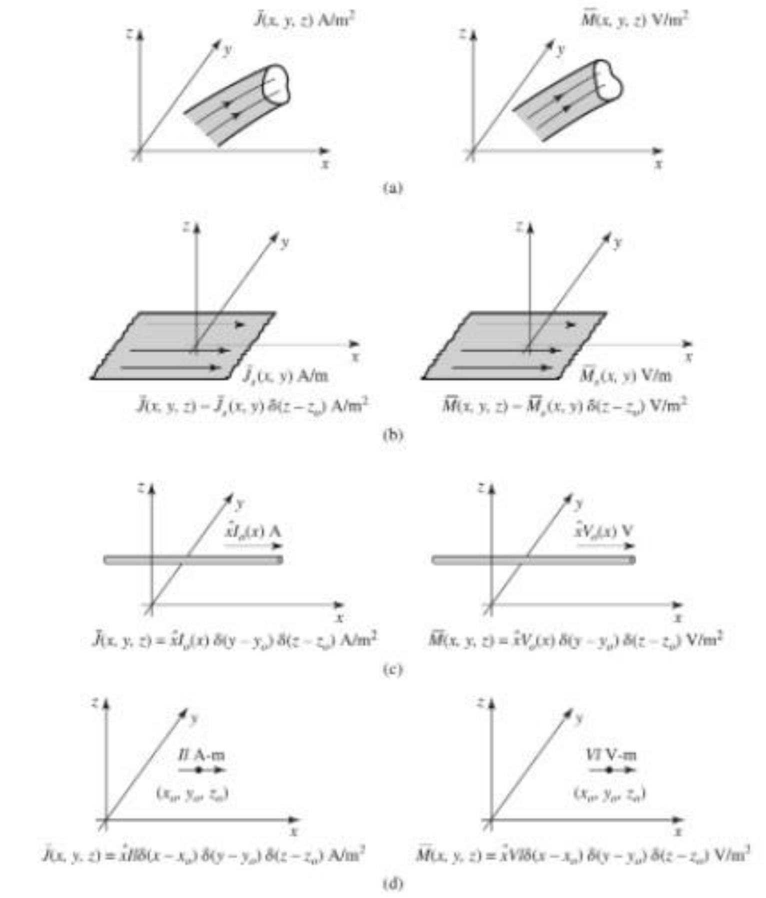


FIGURE 1.4 Arbitrary volume, surface, and line currents. (a) Arbitrary electric and magnetic volume current densities. (b) Arbitrary electric and magnetic surface current densities in the $z = z_0$ plane. (c) Arbitrary electric and magnetic line currents. (d) Infinitesimal electric and magnetic dipoles parallel to the x-axis.

Assuming an $e^{j\omega t}$ time dependence, we can replace the time derivatives in (1.1a)–(1.1d) with $j\omega$. Maxwell's equations in phasor form then become

$$\nabla \times \bar{E} = -j\omega \bar{B} - \bar{M}, \tag{1.14a}$$

$$\nabla \times \bar{H} = j\omega \bar{D} + \bar{J}, \tag{1.14b}$$

$$\nabla \cdot \bar{D} = \rho, \tag{1.14c}$$

$$\nabla \cdot \bar{B} = 0. \tag{1.14d}$$

The Fourier transform can be used to convert a solution to Maxwell's equations for an arbitrary frequency ω into a solution for arbitrary time dependence.

The electric and magnetic current sources, $\bar{J}$ and $\bar{M}$, in (1.14) are volume current densities with units A/m^2 and V/m^2. In many cases, however, the actual currents will be in the form of a current sheet, a line current, or an infinitesimal dipole current. These special types of current distributions can always be written as volume current densities through the use of delta functions. Figure 1.4 shows examples of this procedure for electric and magnetic currents.

1.3 FIELDS IN MEDIA AND BOUNDARY CONDITIONS

In the preceding section it was assumed that the electric and magnetic fields were in free-space, with no material bodies present. In practice, material bodies are often present; this complicates the analysis but also allows the useful application of material properties to microwave components. When electromagnetic fields exist in material media, the field vectors are related to each other by the constitutive relations.

For a dielectric material, an applied electric field $\bar{E}$ causes the polarization of the atoms or molecules of the material to create electric dipole moments that augment the total displacement flux, $\bar{D}$. This additional polarization vector is called $\bar{P}_e$, the *electric polarization*, where

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}_e. \tag{1.15}$$

In a linear medium the electric polarization is linearly related to the applied electric field as

$$\bar{P}_e = \epsilon_0 \chi_e \bar{E}, \tag{1.16}$$

where χ_e, which may be complex, is called the *electric susceptibility*. Then,

$$\bar{D} = \epsilon_0 \bar{E} + \bar{P}_e = \epsilon_0(1 + \chi_e)\bar{E} = \epsilon \bar{E}, \tag{1.17}$$

where

$$\epsilon = \epsilon' - j\epsilon'' = \epsilon_0(1 + \chi_e) \tag{1.18}$$

is the complex permittivity of the medium. The imaginary part of ϵ accounts for loss in the medium (heat) due to damping of the vibrating dipole moments. (Free-space, having a real ϵ, is lossless.) Due to energy conservation, as we will see in Section 1.6, the imaginary part of ϵ must be negative (ϵ'' positive). The loss of a dielectric material may also be considered as an equivalent conductor loss. In a material with conductivity σ, a conduction current density will exist:

$$\bar{J} = \sigma \bar{E}, \tag{1.19}$$

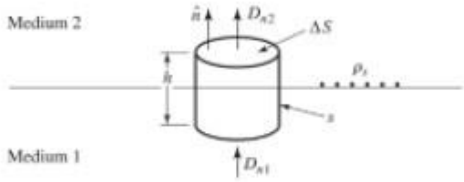


FIGURE 1.6 Closed surface S for equation (1.29).

fields at this interface. The time-harmonic version of (1.4), where S is the closed "pillbox"-shaped surface shown in Figure 1.6, can be written as

$$\oint_S \bar{D} \cdot d\bar{s} = \int_V \rho \, dv. \tag{1.29}$$

In the limit as $h \to 0$, the contribution of D_{tan} through the sidewalls goes to zero, so (1.29) reduces to

$$\Delta S D_{2n} - \Delta S D_{1n} = \Delta S \rho_s,$$

or

$$D_{2n} - D_{1n} = \rho_s, \tag{1.30}$$

where ρ_s is the surface charge density on the interface. In vector form, we can write

$$\hat{n} \cdot (\bar{D}_2 - \bar{D}_1) = \rho_s. \tag{1.31}$$

A similar argument for $\bar{B}$ leads to the result that

$$\hat{n} \cdot \bar{B}_2 = \hat{n} \cdot \bar{B}_1, \tag{1.32}$$

because there is no free magnetic charge.

For the tangential components of the electric field we use the phasor form of (1.6),

$$\oint_C \bar{E} \cdot d\bar{l} = -j\omega \int_S \bar{B} \cdot d\bar{s} - \int_S \bar{M} \cdot d\bar{s}, \tag{1.33}$$

in connection with the closed contour C shown in Figure 1.7. In the limit as $h \to 0$, the surface integral of $\bar{B}$ vanishes (because $S = h\Delta\ell$ vanishes). The contribution from the surface integral of $\bar{M}$, however, may be nonzero if a magnetic surface current density $\bar{M}_s$ exists on the surface. The Dirac delta function can then be used to write

$$\bar{M} = \bar{M}_s \delta(h), \tag{1.34}$$

where h is a coordinate measured normal from the interface. Equation (1.33) then gives

$$\Delta\ell E_{t1} - \Delta\ell E_{t2} = -\Delta\ell M_s,$$

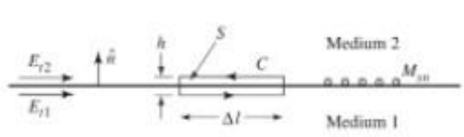


FIGURE 1.7 Closed contour C for equation (1.33).