FIRST MULTICOLOUR EDITION

Theory of Machines

[A Textbook for the Students of B.E. / B.Tech., U.P.S.C. (Engg. Services); Section 'B' of A.M.I.E. (I)]

(S. I. UNITS)

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instants, at which the sun crosses a meridian on two consecutive days. This value varies slightly throughout the year. The average of all the solar days, during one year, is called the mean solar day.

1.13. Presentation of Units and their Values

The frequent changes in the present day life are facilitated by an international body known as International Standard Organisation (ISO) which makes recommendations regarding international standard procedures. The implementation of ISO recommendations, in a country, is assisted by its organisation appointed for the purpose. In India, Bureau of Indian Standards (BIS) previously known as Indian Standards Institution (ISI) has been created for this purpose. We have already discussed that the fundamental units in M.K.S. and S.I. units for length, mass and time is metre, kilogram and second respectively. But in actual practice, it is not necessary to express all lengths in metres, all masses in kilograms and all times in seconds. We shall, sometimes, use the convenient units, which are multiples or divisions of our basic units in tens. As a typical example, although the metre is the unit of length, yet a smaller length of one-thousandth of a metre proves to be more convenient unit, especially in the dimensioning of drawings. Such convenient units are formed by using a prefix in front of the basic units to indicate the multiplier. The full list of these prefixes is given in the following table.

With rapid development of Information Technology, computers are playing a major role in analysis, synthesis and design of machines.

Table 1.1. Prefixes used in basic units

Factor by which the unit is multiplied	*Standard form*	*Prefix*	*Abbreviation*
1 000 000 000 000	10^{12}	tera	T
1 000 000 000	10^{9}	giga	G
1 000 000	10^{6}	mega	M
1 000	10^{3}	kilo	k
100	10^{2}	hecto*	h
10	10^{1}	deca*	da
0.1	10^{-1}	deci*	d
0.01	10^{-2}	centi*	c
0.001	10^{-3}	milli	m
0. 000 001	10^{-6}	micro	μ
0. 000 000 001	10^{-9}	nano	n
0. 000 000 000 001	10^{-12}	pico	p

2.8. Equations of Linear Motion

The following equations of linear motion are important from the subject point of view:

1. $v = u + a.t$ 2. $s = u.t + \frac{1}{2}\, a.t^2$

3. $v^2 = u^2 + 2a.s$

4. $s = \frac{(u+v)}{2} \times t = v_{av} \times t$

where u = Initial velocity of the body,

v = Final velocity of the body,

a = Acceleration of the body,

s = Displacement of the body in time t seconds, and

v_{av} = Average velocity of the body during the motion.

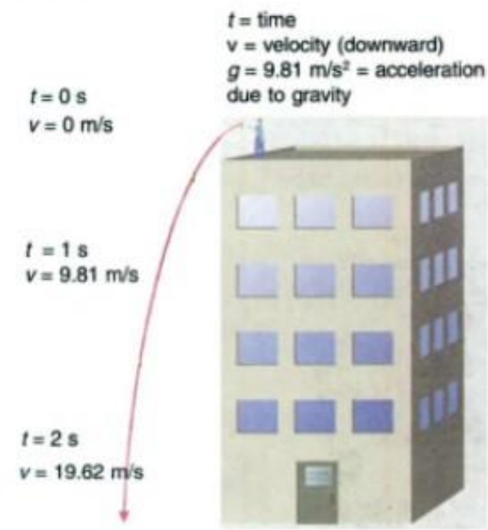
Notes: 1. The above equations apply for uniform acceleration. If, however, the acceleration is variable, then it must be expressed as a function of either t, s or v and then integrated.

2. In case of vertical motion, the body is subjected to gravity. Thus g (acceleration due to gravity) should be substituted for 'a' in the above equations.

3. The value of g is taken as $+ 9.81$ m/s^2 for downward motion, and $- 9.81$ m/s^2 for upward motion of a body.

4. When a body falls freely from a height h, then its velocity v, with which it will hit the ground is given by

$$v = \sqrt{2\, g.h}$$



2.9. Graphical Representation of Displacement with Respect to Time

The displacement of a moving body in a given time may be found by means of a graph. Such a graph is drawn by plotting the displacement as ordinate and the corresponding time as abscissa. We shall discuss the following two cases :

1. *When the body moves with uniform velocity.* When the body moves with uniform velocity, equal distances are covered in equal intervals of time. By plotting the distances on Y-axis and time on X-axis, a displacement-time curve (*i.e.* s-t curve) is drawn which is a straight line, as shown in Fig. 2.1 (a). The motion of the body is governed by the equation $s = u.t$, such that

Velocity at instant $1 = s_1 / t_1$

Velocity at instant $2 = s_2 / t_2$

Since the velocity is uniform, therefore

$$\frac{s_1}{t_1} = \frac{s_2}{t_2} = \frac{s_3}{t_3} = \tan\theta$$

where $\tan\theta$ is called the slope of s-t curve. In other words, the slope of the s-t curve at any instant gives the velocity.

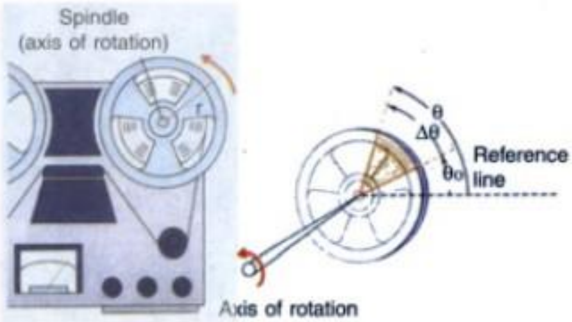
about a fixed shaft or a shaft rotating about its own axis, then the motion is said to be a ***plane rotational motion.***

Note: The motion of a body, confined to one plane, may not be either completely rectilinear nor completely rotational. Such a type of motion is called combined rectilinear and rotational motion. This motion is discussed in Chapter 6, Art. 6.1.

2.5. Linear Displacement

It may be defined as the distance moved by a body with respect to a certain fixed point. The displacement may be along a straight or a curved path. In a reciprocating steam engine, all the particles on the piston, piston rod and cross-head trace a straight path, whereas all particles on the crank and crank pin trace circular paths, whose centre lies on the axis of the crank shaft. It will be interesting to know, that all the particles on the connecting rod neither trace a straight path nor a circular one; but trace an oval path, whose radius of curvature changes from time to time.

The displacement of a body is a vector quantity, as it has both magnitude and direction. Linear displacement may, therefore, be represented graphically by a straight line.



2.6. Linear Velocity

It may be defined as the rate of change of linear displacement of a body with respect to the time. Since velocity is always expressed in a particular direction, therefore it is a vector quantity. Mathematically, linear velocity,

$$v = ds/dt$$

Notes: 1. If the displacement is along a circular path, then the direction of linear velocity at any instant is along the tangent at that point.

2. The speed is the rate of change of linear displacement of a body with respect to the time. Since the speed is irrespective of its direction, therefore, it is a scalar quantity.

2.7. Linear Acceleration

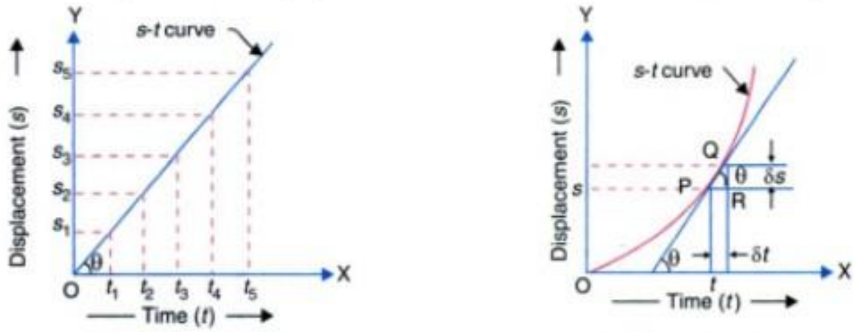
It may be defined as the rate of change of linear velocity of a body with respect to the time. It is also a vector quantity. Mathematically, linear acceleration,

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{ds}{dt}\right) = \frac{d^2s}{dt^2} \qquad \ldots\left(\because\ v = \frac{ds}{dt}\right)$$

Notes: 1. The linear acceleration may also be expressed as follows:

$$a = \frac{dv}{dt} = \frac{ds}{dt} \times \frac{dv}{ds} = v \times \frac{dv}{ds}$$

2. *When the body moves with variable velocity.* When the body moves with variable velocity, unequal distances are covered in equal intervals of time or equal distances are covered in unequal intervals of time. Thus the displacement-time graph, for such a case, will be a curve, as shown in Fig. 2.1 (*b*).



(*a*) Uniform velocity. (*b*) Variable velocity.

Fig. 2.1. Graphical representation of displacement with respect to time.

Consider a point P on the s-t curve and let this point travels to Q by a small distance δs in a small interval of time δt. Let the chord joining the points P and Q makes an angle θ with the horizontal. The average velocity of the moving point during the interval PQ is given by

$$\tan \theta = \delta s / \delta t \qquad \text{... (From triangle } PQR\text{)}$$

In the limit, when δt approaches to zero, the point Q will tend to approach P and the chord PQ becomes tangent to the curve at point P. Thus the velocity at P,

$$v_p = \tan \theta = ds/dt$$

where $\tan \theta$ is the slope of the tangent at P. Thus the slope of the tangent at any instant on the s-t curve gives the velocity at that instant.

2.10. Graphical Representation of Velocity with Respect to Time

We shall consider the following two cases :

1. *When the body moves with uniform velocity.* When the body moves with zero acceleration, then the body is said to move with a uniform velocity and the velocity-time curve (v-t curve) is represented by a straight line as shown by AB in Fig. 2.2 (*a*).

We know that distance covered by a body in time t second

= Area under the v-t curve AB

= Area of rectangle $OABC$

Thus, the distance covered by a body at any interval of time is given by the area under the v-t curve.

2. *When the body moves with variable velocity.* When the body moves with constant acceleration, the body is said to move with variable velocity. In such a case, there is equal variation of velocity in equal intervals of time and the velocity-time curve will be a straight line AB inclined at an angle θ, as shown in Fig. 2.2 (*b*). The equations of motion *i.e.* $v = u + a.t$, and

Integrating both sides,

$$\int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a.dt \quad \text{or} \quad v_2 - v_1 = \int_{t_1}^{t_2} a.dt$$

where v_1 and v_2 are the velocities of the moving body at time intervals t_1 and t_2 respectively.

The right hand side of the above expression represents the area (PQQ_1P_1) under the *a-t* curve between the time intervals t_1 and t_2. Thus the area under the *a-t* curve between any two ordinates represents the change in velocity of the moving body. If the initial and final velocities of the body are u and v, then the above expression may be written as

$$v - u = \int_0^t a.d\,t = \text{Area under } a\text{-}t \text{ curve } AB = \text{Area } OABC$$

Example 2.1. *A car starts from rest and accelerates uniformly to a speed of 72 km. p.h. over a distance of 500 m. Calculate the acceleration and the time taken to attain the speed.*

If a further acceleration raises the speed to 90 km. p.h. in 10 seconds, find this acceleration and the further distance moved. The brakes are now applied to bring the car to rest under uniform retardation in 5 seconds. Find the distance travelled during braking.

Solution. Given : $u = 0$; $v = 72$ km. p.h. = 20 m/s ; $s = 500$ m

First of all, let us consider the motion of the car from rest.

Acceleration of the car

Let $\quad a$ = Acceleration of the car.

We know that $\quad v^2 = u^2 + 2\,a.s$

$\therefore \quad (20)^2 = 0 + 2a \times 500 = 1000\,a \quad$ or $\quad a = (20)^2/1000 = 0.4$ m/s^2 **Ans.**

Time taken by the car to attain the speed

Let $\quad t$ = Time taken by the car to attain the speed.

We know that $\quad v = u + a.t$

$\therefore \quad 20 = 0 + 0.4 \times t \quad$ or $\quad t = 20/0.4 = 50$ s **Ans.**

Now consider the motion of the car from 72 km.p.h. to 90 km.p.h. in 10 seconds.

Given : * $u = 72$ km.p.h. = 20 m/s ; $v = 96$ km.p.h. = 25 m/s ; $t = 10$ s

Acceleration of the car

Let $\quad a$ = Acceleration of the car.

We know that $\quad v = u + a.t$

$25 = 20 + a \times 10 \quad$ or $\quad a = (25 - 20)/10 = 0.5$ m/s^2 **Ans.**

Distance moved by the car

We know that distance moved by the car,

$$s = ut + \frac{1}{2}at^2 = 20 \times 10 + \frac{1}{2} \times 0.5(10)^2 = 225 \text{ m} \quad \textbf{Ans.}$$

Substituting the value of C_2 in equation (*iii*),

$$s = \frac{t^5}{20} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + 4$$

Substituting the value of $t = 2$ s, in this equation,

$$s = \frac{2^5}{20} - \frac{2^4}{4} + \frac{5 \times 2^2}{2} + 2 \times 2 + 4 = 15.6 \text{ m} \quad \textbf{Ans.}$$

Example 2.3. *The velocity of a train travelling at 100 km/h decreases by 10 per cent in the first 40 s after application of the brakes. Calculate the velocity at the end of a further 80 s assuming that, during the whole period of 120 s, the retardation is proportional to the velocity.*

Solution. Given : Velocity in the beginning (*i.e.* when $t = 0$), $v_0 = 100$ km/h

Since the velocity decreases by 10 per cent in the first 40 seconds after the application of brakes, therefore velocity at the end of 40 s,

$$v_{40} = 100 \times 0.9 = 90 \text{ km/h}$$

Let v_{120} = Velocity at the end of 120 s (or further 80s).

Since the retardation is proportional to the velocity, therefore,

$$a = -\frac{dv}{dt} = k.v \quad \text{or} \quad \frac{dv}{v} = -k.dt$$

where k is a constant of proportionality, whose value may be determined from the given conditions. Integrating the above expression,

$$\log_e v = -k.t + C \qquad \ldots (i)$$

where C is the constant of integration. We know that when $t = 0$, $v = 100$ km/h. Substituting these values in equation (*i*),

$$\log_e 100 = C \quad \text{or} \quad C = 2.3 \log 100 = 2.3 \times 2 = 4.6$$

We also know that when $t = 40$ s, $v = 90$ km/h. Substituting these values in equation (*i*),

$$\log_e 90 = -k \times 40 + 4.6 \qquad \ldots (\because C = 4.6)$$

$$2.3 \log 90 = -40k + 4.6$$

or

$$k = \frac{4.6 - 2.3 \log 90}{40} = \frac{4.6 - 2.3 \times 1.9542}{40} = 0.0026$$

Substituting the values of k and C in equation (*i*),

$$\log_e v = -0.0026 \times t + 4.6$$

or

$$2.3 \log v = -0.0026 \times t + 4.6 \qquad \ldots (ii)$$

Now substituting the value of t equal to 120 s, in the above equation,

$$2.3 \log v_{120} = -0.0026 \times 120 + 4.6 = 4.288$$

Planing Machine.

Example 2.5. *The cutting stroke of a planing machine is 500 mm and it is completed in 1 second. The planing table accelerates uniformly during the first 125 mm of the stroke, the speed remains constant during the next 250 mm of the stroke and retards uniformly during the last 125 mm of the stroke. Find the maximum cutting speed.*

Solution. Given : s = 500 mm ; t = 1 s ; s_1 = 125 mm ; s_2 = 250 mm ; s_3 = 125 mm

Fig. 2.4 shows the acceleration-time and velocity-time graph for the planing table of a planing machine.

Let

v = Maximum cutting speed in mm/s.

Average velocity of the table during acceleration and retardation,

$$v_{av} = (0 + v)/2 = v/2$$

Time of uniform acceleration $t_1 = \dfrac{s_1}{v_{av}} = \dfrac{125}{v/2} = \dfrac{250}{v}$ s

Time of constant speed, $t_2 = \dfrac{s_2}{v} = \dfrac{250}{v}$ s

and time of uniform retardation, $t_3 = \dfrac{s_3}{v_{av}} = \dfrac{125}{v/2} = \dfrac{250}{v}$ s

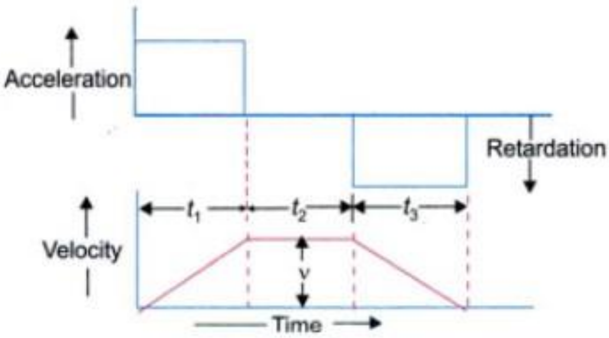


Fig. 2.4

Since the time taken to complete the stroke is 1 s, therefore

$$t_1 + t_2 + t_3 = t$$

$$\frac{250}{v} + \frac{250}{v} + \frac{250}{v} = 1 \text{ or } v = 750 \text{ mm/s } \textbf{Ans.}$$

Fig. 2.5. If this line moves from OB to OC, through an angle $\delta\theta$ during a short interval of time δt, then $\delta\theta$ is known as the ***angular displacement*** of the line OB.

Since the angular displacement has both magnitude and direction, therefore it is also a ***vector quantity***.

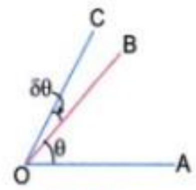


Fig. 2.5. Angular displacement.

2.13. Representation of Angular Displacement by a Vector

In order to completely represent an angular displacement, by a vector, it must fix the following three conditions :

1. ***Direction of the axis of rotation***. It is fixed by drawing a line perpendicular to the plane of rotation, in which the angular displacement takes place. In other words, it is fixed along the axis of rotation.

2. ***Magnitude of angular displacement***. It is fixed by the length of the vector drawn along the axis of rotation, to some suitable scale.

3. ***Sense of the angular displacement***. It is fixed by a right hand screw rule. This rule states that if a screw rotates in a fixed nut in a clockwise direction, *i.e.* if the angular displacement is clockwise and an observer is looking along the axis of rotation, then the arrow head will point away from the observer. Similarly, if the angular displacement is anti-clockwise, then the arrow head will point towards the observer.

2.14. Angular Velocity

It may be defined as the rate of change of angular displacement with respect to time. It is usually expressed by a Greek letter ω (omega). Mathematically, angular velocity,

$$\omega = d\theta / dt$$

Since it has magnitude and direction, therefore, it is a vector quantity. It may be represented by a vector following the same rule as described in the previous article.

Note : If the direction of the angular displacement is constant, then the rate of change of magnitude of the angular displacement with respect to time is termed as ***angular speed***.

2.15. Angular Acceleration

It may be defined as the rate of change of angular velocity with respect to time. It is usually expressed by a Greek letter α (alpha). Mathematically, angular acceleration,

$$\alpha = \frac{d\omega}{dt} = \frac{d}{dt}\left(\frac{d\theta}{dt}\right) = \frac{d^2\theta}{dt^2} \qquad \ldots \left(\because \omega = \frac{d\theta}{dt}\right)$$

It is also a vector quantity, but its direction may not be same as that of angular displacement and angular velocity.