SEN1221 – PART I – CHOICE BEHAVIOUR MODELLING

FORMULA CHEAT SHEET

Linear-additive (or: linear-in-parameters) Random Utility Maximization-based Logit model

Systematic utility associated with alternative i:

$$V_i = \sum_m \beta_m \cdot x_{im}$$

Total utility associated with alternative i:

$$U_i = V_i + \varepsilon_i = \sum_m \beta_m \cdot x_{im} + \varepsilon_i$$

Condition for alternative i to be chosen:

$$\sum_m \beta_m \cdot x_{im} + \varepsilon_i > \sum_m \beta_m \cdot x_{jm} + \varepsilon_j, \forall j \neq i$$

Probability that alternative i is chosen, if $\varepsilon \sim EV\ Type\ I$ with variance $\pi^2/6$

(Note: in the denominator, J denotes choice set size. j runs from 1 to J, and includes i)

$$P(i) = P\left(V_i + \varepsilon_i > V_j + \varepsilon_j, \forall j \neq i\right) = \frac{\exp(V_i)}{\sum_{j=1..J} \exp(V_j)} = \frac{\exp(\sum_m \beta_m x_{im})}{\sum_{j=1..J} \exp(\sum_m \beta_m x_{jm})}$$

Marginal effect: percentage point change in alternative i's choice probability, caused by a one unit-change in its mth attribute

(measures absolute effect)

$$\partial P_i / \partial x_m = \beta_m P_i (1 - P_i)$$

Elasticity: percentage change in i's choice probability, caused by a percentage change in its mth attribute

(measures relative effect)

$$E_{im} = \beta_m x_m (1 - P_i)$$

Likelihood of a series of n choice-observations, given parameter set β

(Note: i denotes the alternatives in the choice set which is associated with observation n)

$$L(\beta) = \prod_n \prod_i P_n(i|\beta)^{y_n(i)}$$

where $y_n(i) = 1$ if i is chosen, 0 otherwise

Special case: likelihood of one observed choice for alternative i, given parameter set β:

$$P(i|\beta)$$

(Note: this is simply the predicted choice probability for the alternative, given parameter set β)

Null-Likelihood of a series of n choice-observations, given that choice set size = J

(Note: this is the Likelihood when all parameters are set to zero)

$$L(0) = \prod_n \prod_i P_n(i|0)^{y_n(i)} = \left(\frac{1}{J}\right)^n$$

Log-Likelihood of a series of n choice-observations, given parameter set β

$$LL(\beta) = \ln\left(\prod_n \prod_i P_n(i|\beta)^{y_n(i)}\right) = \sum_n \sum_i y_n(i) \cdot \ln\big(P_n(i|\beta)\big)$$

Null-Log-Likelihood of a series of n choice-observations, given that choice set size = J

(Note: this is the Log-Likelihood when all parameters are set to zero)

$$LL(0) = \sum_n \sum_i y_n(i) \cdot \ln\big(P_n(i|0)\big) = n \cdot \ln\left(\frac{1}{J}\right)$$

McFadden's rho-squared (LL_β is LL of the estimated model, LL_0 is LL of null-model)

$$\rho^2 = 1 - \frac{LL_\beta}{LL_0}$$

Likelihood Ratio Statistic: determines if model B's better fit (compared to model A) is due to chance

$$LRS = -2 \cdot (LL_A - LL_B)$$

t-ratio associated with comparison of an estimated parameter ($\hat{\beta}$) against some value (α)

$$t = (\hat{\beta} - \alpha)/SE(\hat{\beta})$$

Note: Special case: t-ratio associated with comparison of an estimated parameter ($\hat{\beta}$) against zero

$$t = \hat{\beta}/SE(\hat{\beta})$$

95%-confidence interval for an estimated parameter

Note: you also need to be able to derive the interval (see corresponding slide)

$$\hat{\beta} \pm 1.96 \cdot SE(\hat{\beta})$$

Economic appraisal with RUM-based Logit models

Willingness to pay (in terms of higher costs C) for improvement of an alternative's attribute x_m

$$WtP(x_m) = \frac{\frac{\partial V}{\partial x_m}}{\frac{\partial V}{\partial C}} = \frac{\beta_m}{\beta_C}$$

Special case of WtP: Value of Travel Time (TT) Savings, in terms of higher travel costs TC

$$VoTTS = \frac{\frac{\partial V}{\partial TT}}{\frac{\partial V}{\partial TC}} = \frac{\beta_{TT}}{\beta_{TC}}$$

Expected utility of a choice set (a.k.a. "the Logsum")

$$\ln\left(\sum_{j=1..J} \exp\left[V_j\right]\right)$$

Gain in welfare, in monetary terms, due to a policy implemented between $t = 0$ and $t = 1$

(a.k.a. the difference in Consumer Surplus CS)

(Note: γ is marginal utility of income. Not observed. Use $-\beta_{TC}$ instead.

$$\Delta CS = \frac{1}{\gamma}\left[\ln\left(\sum_{j=1..J} \exp\left[{V_j}^{t=1}\right]\right) - \ln\left(\sum_{j=1..J} \exp\left[{V_j}^{t=0}\right]\right)\right]$$