SEN122A – Choice Behaviour Modelling
Formula Cheat Sheet

S. van Cranenburgh November 2025

RUM-MNL

Observed utility associated with alternative i:

$$V_i = f(\boldsymbol{\beta}, \boldsymbol{x_i})$$

Observed utility associated with alternative i under the assumption that utility is linear and additive:

$$\begin{aligned} V_i &= \sum_{m=1}^{M} \beta_m \, x_{im} \\ &= \beta_1 x_{i1} + \beta_2 x_{i2} + \ldots + \beta_M x_{iM} \end{aligned}$$

Total utility associated with alternative i:

$$U_i = V_i + \varepsilon_i$$

Condition for alternative i to be chosen:

$$U_i > U_j \quad \forall j \in C$$

Probability for alternative i to be chosen:

$$P_i = prob(V_i + \varepsilon_i > V_j + \varepsilon_j, \forall j \neq i)$$

MNL choice probability formula:

If $\varepsilon_i \sim$ i.i.d. Extreme Value Type I, with variance $\pi^2/6$:

$$P_i = \frac{\exp(V_i)}{\sum_{j=1}^{J} \exp(V_j)}$$

where J denotes the number of alternatives in the choice set.

The likelihood function

The likelihood of N observations given β:

$$L(\beta) = \prod_{n=1}^{N} \prod_{i \in C_n} P(i|\beta)^{y_{ni}}$$

where $y_{ni} = 1$ if i is chosen by individual n, and 0 otherwise.

Null-likelihood:

$$L_0 = \prod_{n=1}^{N} \left(\frac{1}{J_n} \right)$$

Log-likelihood:

$$\ln(L(\beta)) = \sum_{n=1}^{N} \sum_{i \in C_n} y_{ni} \cdot \ln P_n(i|\beta)$$

Null log-likelihood:

$$\ln(L_0) = \sum_{n=1}^{N} \ln \left(\frac{1}{J_n} \right)$$

McFadden's ρ^2:

$$\rho^2 = 1 - \frac{LL(\hat{\beta})}{LL_0}$$

Likelihood Ratio Statistic:

$$LR = -2 \cdot (LL_A - LL_B)$$

Determines if model B's better fit (compared to model A) is due to chance.

t-ratio for parameter $\hat{\beta}$ vs. value β_0:

$$t = \frac{\hat{\beta} - \beta_0}{\text{se}(\hat{\beta})}$$

For the special case that we test against zero:

$$t = \frac{\hat{\beta}}{\text{se}(\hat{\beta})}$$

95% confidence interval:

$$\hat{\beta} \pm 1.96 \times \text{se}(\hat{\beta})$$

Economic appraisal with RUM-MNL

Willingness to Pay (WTP) for a one-unit improvement in attribute x_k:

$$WTP_k = -\frac{\frac{\partial V}{\partial x_k}}{\frac{\partial V}{\partial x_c}}$$

where c is the cost attribute.

If utility is linear and additive:

$$WTP_k = -\frac{\beta_k}{\beta_c}$$

Value of Travel Time (VTT):

$$VTTS = -\frac{\beta_{tt}}{\beta_{tc}}$$

Expected maximum utility derived from a choice set by decision-maker ***n:***

$$\mathbb{E}[\max U_n] = \ln(\sum_{i \in C_n} e^{V_{in}}) + \gamma$$

where γ is the Euler-Mascheroni constant (0.5772)
This term is also known as the LogSum (LS).

Change in Consumer Surplus (CS) due to policy from the ***SQ*** **to** ***New*****:**

$$\Delta CS = \frac{-1}{\beta_c} \left[LS_{SQ} - LS_{New} \right]$$
$$\Delta CS = \frac{-1}{\beta_c} \left[\ln \left(\sum_{i \in C_n} \exp(V_i^{SQ}) \right) - \ln \left(\sum_{i \in C_n} \exp(V_i^{New}]) \right) \right]$$

Mixed Logit model

Choice probability with random error term

Choice probability of alternative i in the presence of a random error term υ with pdf $f(\upsilon)$:

$$P_i(\beta, \sigma) = \int_{\upsilon} \left[P_i \mid \upsilon \right] \, f(\upsilon|\sigma) \, d\upsilon$$

Simulated choice probability

Simulated choice probability of alternative i in the presence of υ, where $r = 1, \ldots, R$ denote draws from $f(\upsilon)$:

$$\breve{P}_i = \frac{1}{R} \sum_{r=1}^{R} \left[P_i \mid \upsilon^{(r)} \right]$$

Drawing from a Normal distribution

Draws $\theta \sim \mathcal{N}(\mu, \sigma)$ are generated by rescaling standard Normal draws $\eta \sim \mathcal{N}(0, 1)$:

$$\theta = \mu + \eta \, \sigma$$

Drawing from a LogNormal distribution

Draws $\tau \sim \text{LogNormal}(\mu, \sigma)$ are generated by exponentiating Normal draws $\theta = \mu + \eta \, \sigma$, where $\eta \sim \mathcal{N}(0, 1)$:

$$\tau = \exp(\theta) = \exp(\mu + \eta \, \sigma)$$

Drawing from a LogUniform distribution

Draws $\tau \sim \text{LogUniform}(a, b)$ are generated by exponentiating Uniform draws $U \sim \text{Uniform}(0, 1)$:

$$\tau = e^{\,a+U\,(b-a)}$$

Panel Mixed Logit choice probability

Panel Mixed Logit probability that a decision-maker n chooses alternative i in T consecutive choice situations, in the presence of taste heterogeneity:

$$P_{ni}(\boldsymbol{\beta}, \sigma) = \int_{\boldsymbol{\beta}_n} \left[\prod_{t=1}^{T} P_{ni}^{t} \mid \boldsymbol{\beta}_n \right] f(\boldsymbol{\beta}_n \mid \sigma)\, d\boldsymbol{\beta}_n$$