г) $\lim\limits_{x\to 0}(1+\operatorname{arctg}2x)^{\frac{1}{3x}} = 1^{\infty}$

$= (1+2x)^{\frac{1}{2x}\cdot\frac{2x}{3x}} = e^{\lim\frac{2x}{3x}} = e^{\frac{2}{3}}$

$= \left(1+\overset{\to 1}{\frac{\operatorname{arctg}2x}{2x}}\,2x\right)^{\frac{1}{3x}} = (1+2x)^{\frac{1}{2x}\cdot\frac{2x}{3x}} = e^{\lim\limits_{x\to 0}\frac{2x}{3x}} = e^{\frac{2}{3}} = \sqrt[3]{e^2}$

д) $\lim\limits_{x\to 0}(1-3\sin x)^{\frac{1}{6x}} = 1^{\infty}\ \left(1+(-3\sin x)\right)^{\frac{1}{-3\sin x}\cdot\frac{-3\sin x}{6x}} =$

$= e^{\lim -\frac{3x}{6x}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$

е) $\lim\limits_{x\to\infty}\left(\frac{x^2+3x+1}{x+x^2}\right)^{x^2+1} = 1^{\infty} = \lim\left(\frac{x^2+x+2x+1}{x+x^2}\right)^{x^2+1} =$

$= \left(\left(1+\frac{2x+1}{x+x^2}\right)^{\frac{x+x^2}{2x+1}}\right)^{\frac{(x^2+1)(2x+1)}{x+x^2}} = e^{\lim(\text{вышл бдуд } 0)} =$

$= e^{\infty} = \infty$

ж) $\lim\limits_{x\to 0}(1+\underset{1}{\cos x})^{\frac{x+2}{x-1}} = 2^{-2} = \frac{1}{4}$

з) $\lim\limits_{x\to 0}(1+5x)^{\frac{1}{3x}} = 1^{\infty} = e^{\lim\frac{5x}{3x}} = e^{\frac{5}{3}} = \sqrt[5]{e^5}$

г) $\lim\limits_{x\to 0} \frac{1-\cos 6x}{\sin x \cdot tg^3 x} = \left(\frac{0}{0}\right) = \frac{2\sin^2 3x}{\sin x \cdot \underbrace{\frac{\sin^3 x}{\cos^3 x}}_{=1}} = 2\lim\limits_{x\to 0} \frac{\sin^2 3x}{\sin^4 x}$

$\sin x \sim x \quad 2\lim\limits_{x\to 0} \frac{(3x)^2}{x^4} = \frac{9}{x^2} = \infty$

д) $\lim\limits_{x\to\frac{\pi}{4}} \frac{\sin\left(\frac{\pi}{4}-x\right)}{x^2-\frac{\pi^2}{16}} = \left(\frac{0}{0}\right) = \frac{\overbrace{\sin\left(\frac{\pi}{4}-x\right)}^{=1}}{-\left(-x+\frac{\pi}{4}\right)\underbrace{\left(x+\frac{\pi}{4}\right)}_{=\frac{\pi}{4}}} = -\frac{2}{\pi}$

$\cancel{\times}\ \frac{2\pi}{4} = \frac{\pi}{2}$

5) а) $\lim\limits_{x\to 0} \frac{1-\cos 5x}{x^2} = \left(\frac{0}{0}\right) = 2\frac{\sin^2\left(\frac{5}{2}x\right)}{x^2} = 2\frac{\overbrace{\frac{\sin\frac{5x}{2}\sin\frac{5}{2}x}{\frac{25x^2}{4}}}^{=1}\cdot\frac{25x^2}{4}}{x^2} =$

$= \frac{\cancel{2}\cdot 25\cancel{x^2}}{\cancel{4}\,\cancel{x^2}} = \frac{25}{2} = 12{,}5$

б) $\lim\limits_{x\to\infty}\left(\frac{3x-4}{3x+3}\right)^{2-5x} = 1^\infty = \left(\frac{3x+3-7}{3x+3}\right)^{2-5x} = \left(1+\left(-\frac{7}{3x+3}\right)\right)^{2-5x} =$

$\left(\left(1+\left(-\frac{7}{3x+3}\right)\right)^{-\frac{3x+3}{7}}\right)^{-\frac{7(2-5x)}{3x+3}} = e^{\lim\limits_{x\to\infty}\frac{-\overset{=0}{14}+35x}{\underset{=0}{3x+3}}} = e^{\frac{35}{3}}$

в) $\lim\limits_{x\to\pm\infty}\left(\frac{\overset{=1}{x^2}+\overset{=0}{3x}+\overset{=0}{1}}{\underset{=3}{3x^2}+\underset{=0}{x}}\right)^x = \left(\frac{\infty}{\infty}\right)^{\pm\infty} = \lim\left(\frac{1}{3}\right)^x = \frac{1}{3}^{\pm\infty}$ $\begin{cases} 0 \text{ если } x\to+\infty \\ +\infty \text{ если } x\to-\infty \end{cases}$

в) $\lim\limits_{x\to0} \dfrac{\sqrt[3]{8+3x+x^2}-2}{x+x^2} = \left(\dfrac{0}{0}\right) = \dfrac{(8+3x+x^2-8)}{(x+x^2)\left(\underbrace{\sqrt[3]{(8+3x+x^2)^2}}_{4}+2\underbrace{\sqrt[3]{8+3x+x^2}}_{4}+4\right)}$

$\dfrac{1}{12}\lim\limits_{x\to0}\dfrac{\cancel{x}(3+x)^{\to0}}{\cancel{x}(1+x)_{\to0}} = \dfrac{3}{12} = \dfrac{1}{4}$

г) $\lim\limits_{x\to0}\dfrac{2x}{\sqrt{4+x}-\sqrt{4-x}} = \left(\dfrac{0}{0}\right) = \dfrac{2x(\sqrt{4+x}+\sqrt{4-x})}{\cancel{4}+x-\cancel{4}+x} = 4$

4) а) $\lim\limits_{x\to0}\dfrac{\sin 5x}{3x} = \dfrac{0}{0} = \overbrace{\dfrac{\sin 5x}{5x}}^{=1}\cdot\dfrac{5x}{3x} = \dfrac{5}{3}$

б) $\lim\limits_{x\to0}\dfrac{\sin(x+\delta)+\sin(x-\delta)}{2x} = \dfrac{0}{0} = \lim\limits_{x\to0}\dfrac{2\cos\left(\dfrac{x+\delta-x+\delta}{2}\right)\sin\left(\dfrac{x+\delta+x-\delta}{2}\right)}{2x}$

$= 2\cos\delta\,\dfrac{\sin x}{x} = \cos\delta$

в) $\lim\limits_{x\to0}\left(\dfrac{1}{\sin^2 x}-\dfrac{1}{4\sin\frac{x}{2}}\right) = (\infty-\infty) = \left(\dfrac{1}{\left(\frac{1}{2}\sin\frac{x}{2}\cos\frac{x}{2}\right)^2}-\dfrac{1}{4\sin\frac{x}{2}}\right) =$

$= \lim\limits_{x\to0}\dfrac{1}{\frac{1}{4}\sin^2\frac{x}{2}\underbrace{\cos^2\frac{x}{2}}_{1}} - \dfrac{1}{4\sin\frac{x}{2}} = \lim \dfrac{1}{\frac{1}{4}}\left(\dfrac{1}{\sin^2\frac{x}{2}}-\dfrac{1}{4\sin\frac{x}{2}}\right) =$

$= \dfrac{16-\sin\frac{x}{2}^{\to0}}{\underbrace{4\sin^2\frac{x}{2}}_{\to0}} = \dfrac{16}{0} = \infty$

Пределы 3:

2) а) $\lim\limits_{n \to \infty} \dfrac{n^2 - \sqrt{n^3+1}}{\sqrt{n^6+2} - n} = \dfrac{\infty}{\infty} = \dfrac{\frac{1}{n} - \sqrt{\frac{1}{n} + \frac{1}{n^6}}\ \sqrt{\frac{1}{n^3} + \frac{1}{n^6}}}{\sqrt{1 + \frac{2}{n^6} - \frac{1}{n^5}}} = \dfrac{0}{1} = 0$

б) $\lim\limits_{n \to \infty} \dfrac{\overset{=0}{(n+1)^2} + \overset{=0}{(n-1)^2} - \overset{=-1}{(n+2)^3}}{\underset{=1}{4 - n^3}} = \left(\dfrac{\infty}{\infty}\right)$ (ст. степ будет n^3 дем n^3) = будет везде 0 где нет n^3

$= \dfrac{-1}{-1} = 1$

е) $\lim\limits_{x \to \frac{1}{3}} \dfrac{3x^2 - 2x + 1}{\underset{=0}{x + \frac{1}{3}}} = \dfrac{\cdot}{0} = \infty$

3) а) $\lim\limits_{n \to \infty} \left(n\sqrt{n} - \sqrt{n(n+1)(n+2)}\right) = \dfrac{\left(n^2 n - n(n+1)(n+2)\right)}{n\sqrt{n} + \sqrt{n(n+1)(n+2)}} =$

$= \dfrac{n^3 - n(n^2 + 2n + n + 2)}{\cdot} = \dfrac{\cancel{n^3} - \cancel{n^3} - \overset{=-3}{3n^2} - \overset{=0}{2n}}{\underset{=0}{n\sqrt{n}} + \underset{=0}{\sqrt{n(n+1)(n+2)}}} = \dfrac{-3}{0} = -\infty$

$=$

б) $\lim\limits_{x \to \infty} \left(\sqrt{x^2 + 5x} - x\right) = (\infty - \infty) = \lim\limits_{x \to \infty} \left(\dfrac{\cancel{x^2} + 5x - \cancel{x^2}}{x^2 + 5x + x}\right) = \dfrac{5\cancel{x}}{\cancel{x}(x+6)} = \dfrac{5}{x+6}$

$= \cancel{5 \lim\limits_{x \to \infty} \dfrac{1}{x+6}}$ $\quad 5 \lim \dfrac{\overset{=1}{x}}{\underset{=1}{\sqrt{x^2+5x}} + \underset{=1}{x}} = 5 \cdot \dfrac{1}{2} = 2{,}5$