18) $\lim_{x\to 1}\left(\frac{3x-1}{x+1}\right)^{\frac{1}{\sqrt[3]{x}-1}} = \left(1+\left(\frac{3x-1}{x+1}\right)-1\right)^{\frac{1}{\sqrt[3]{x}-1}} = \lim_{x\to 1}\left(1+\frac{3x-1-x-1}{x+1}\right) =$

$= \left(\left(1+\frac{2x-2}{x+1}\right)^{\frac{x+1}{2x-2}}\right)^{\frac{2(x-1)}{(x+1)(\sqrt[3]{x}-1)}} = e^{\lim_{x\to 1}\frac{x-1}{\sqrt[3]{x}-1}} =$

$= e^{\lim \frac{(x-1)(\sqrt[3]{x^2}+\sqrt[3]{x}+1)}{(x-1)}=3} = e^3$

19) $\lim_{x\to e}\left(\frac{\ln x-1}{x-e}\right)^{\sin\frac{\pi x}{2e}+1} = \lim_{x\to 1}\frac{\ln x-1}{x-e} = \left(\frac{0}{0}\right) =$

$= t = x-e;\ x = t+e,\ t\to 0 \quad \lim_{t\to 0}\frac{\ln(t+e)-1}{t+\cancel{e}-\cancel{e}} =$

$= \frac{\ln(t+e)-\ln e}{t} = \frac{\ln\left(1+\frac{t}{e}\right)}{\frac{t}{e}\cdot e} = \frac{1}{e}$

20) $\lim_{x\to 0}\left(\sqrt{4\cos 3x} + x\,\operatorname{arctg}\left(\frac{1}{x}\right)\right) = \sqrt{4} = 2$

$\cos 3x \to 1$

$x\,\operatorname{arctg}\frac{1}{x} \to 0$

14) $\lim\limits_{x\to 0} \frac{7^{2x}-5^{3x}}{2x-\operatorname{arctg}3x} = \left(\frac{0}{0}\right) = \frac{\left(\frac{7^{2x}-1}{2x}\right)\cdot 2x - \left(\frac{5^{3x}-1}{3x}\right)\cdot 3x}{2x-\operatorname{arctg}3x} =$

$= \frac{\ln 7\cdot 2x - \ln 5\cdot 3x}{2x-3x} = \frac{\cancel{x}(2\ln 7 - 3\ln 5)}{\cancel{x}(2-3)} =$

$= -2\ln 7 + 3\ln 5 = \ln 125 - \ln 49 = \ln\frac{125}{49}$

15) $\lim\limits_{x\to 0} \frac{e^{x}+e^{-x}-2}{\sin^2 x} = \left(\frac{0}{0}\right) = \frac{\left(e^{\frac{x}{2}}-e^{-\frac{x}{2}}\right)^2}{\sin^2 x} = \frac{\left(\left(\frac{e^{\frac{x}{2}}-1}{\frac{x}{2}}\right)\cdot\frac{x}{2} - \left(\frac{e^{-\frac{x}{2}}-1}{-\frac{x}{2}}\right)\cdot\left(-\frac{x}{2}\right)\right)^2}{\sin^2 x} =$

$= \frac{\left(\frac{x}{2}+\frac{x}{2}\right)^2}{\sin^2 x} = \lim\limits_{x\to 0}\frac{x^2}{\sin^2 x} = \lim \frac{x\cdot x}{\sin x\,\sin x} = 1\cdot 1 = 1$

16) $\lim\limits_{x\to 0} \left(1-\ln(1+x^3)\right)^{\frac{3}{x^2\arcsin x}} = 1^{\infty} = \left(\left(1+\left(-\ln(1+x^3)\right)\right)^{\frac{1}{-\ln(1+x^3)}}\right)^{\frac{-3\cdot\ln(1+x^3)}{x^2\arcsin x}} =$

$= e^{\lim\limits_{x\to 0}\frac{-3\ln(1+x^3)}{x^2\arcsin x}} = e^{-3\lim\frac{x\,\ln(1+x^3)}{x^3\,\arcsin x}} = \frac{x}{x} = e^{-3}$

17) $\lim\limits_{x\to 0}\left(\frac{\sin 2x}{x}\right)^{1+x^{2}=0=1} = \frac{\sin 2x}{x} = 2\left(\frac{\sin 2x}{2x}\right)^{=1} = 2$

18) $\lim\limits_{x\to 4}$ ~~$(1+$ $3x+1$~~

10) $\lim\limits_{x\to 4} \frac{\sqrt{1+2x}-3}{\sqrt{x}-2} = \left(\frac{0}{0}\right) = \frac{(1+2x-9)(\sqrt{x}+2)}{(x-4)(\sqrt{1+2x}+3)} =$

$= \frac{(2x-8)\cdot 4}{(x-4)\cdot 6} = \frac{8}{6}\lim\limits_{x\to 4}\frac{x-4}{x-4} = \frac{4}{3}$

11) $\lim\limits_{x\to 0} \frac{\ln(1+\sin x)}{\sin 4(x-\pi)} = \left(\frac{0}{0}\right) = \frac{\ln(1+\sin x)}{\sin x} \cdot \frac{\sin x}{\sin 4(x-\pi)} =$

$= \frac{\sin x}{\sin 4(x-\pi)} = \frac{\sin x}{2\cdot 2\sin(x-\pi)\underbrace{\cos(x-\pi)}_{=-1}} = -\frac{1}{4}\lim \frac{\sin x}{-\sin(\pi - x)} =$

$= \frac{1}{4}\lim\limits_{x\to 0}\frac{\sin x}{\sin x} = \frac{1}{4}$

12) $\lim\limits_{x\to 1}\frac{x^2-1}{\ln x} = \lim\limits_{x\to 1}\frac{(x-1)\overbrace{(x+1)}^{=2}}{\ln x} = 2\lim\limits_{x\to 1}\frac{x-1}{\ln x} = 2\lim\limits_{t\to 0}\underbrace{\frac{t+1-1}{\ln(t+1)}}_{=1} = 2$

$t = x-1,\ x = t+1,\ t\to 0$

13) $\cancel{\lim\limits_{x\to\frac{\pi}{2}}\frac{1-\sin^2 x}{\ln\sin x}}$ $\lim\limits_{x\to\frac{\pi}{2}}\frac{2^{\cos^2 x}-1}{\ln\sin x} = \left(\frac{0}{0}\right) = \frac{(2^{\cos^2 x}-1)\cos^2 x}{\cos^2 x\ \ln\sin x} =$

$= \frac{2\ln 2\cos^2 x}{\ln\sin^2 x} = \ln 2\lim\limits_{x\to\frac{\pi}{2}}\left(\frac{1-\sin^2 x}{\ln\sin x}\right) = \frac{(1-\sin x)(1+\sin x)}{\ln\sin x} =$

$= \frac{-(1-\sin x)\overbrace{(1+\sin x)}^{=2}}{\ln(1+(\sin x-1))} = -2\ln 2\lim\limits_{x\to\frac{\pi}{2}}\frac{\sin x-1}{\ln(1+(\sin x-1))} = 1.$

$= -\ln 4$

8) Д-ть непр. в т.

$f(x) = 5x^2 - 1$, $x_0 = 6$ ... $\varepsilon > 0$

$|5x^2 - 1 - 179| < \varepsilon$

$|5x^2 - 180| < \varepsilon$

$5|x^2 - 36| < \varepsilon$

$|(x-6)(x+6)| < \frac{\varepsilon}{5}$

$\delta\, |x+6| < \frac{\varepsilon}{5}$

$0 < \delta < \frac{\varepsilon}{5|x+6|}$ – такая δ естеств нашли, знач f(x) непр в т. $x_0 = 6$

9) $\lim\limits_{x \to -1} \frac{(x^3 - 2x - 1)(x+1)}{x^4 + 4x^2 - 5} =$

$$\begin{array}{r|l} x^3 + 0x^2 - 2x - 1 & x+1 \\ \cline{2-2} \underline{x^3 + x^2} \quad\quad\quad\quad & x^2 - x - 1 \\ -x^2 - 2x \quad\quad & \\ \underline{-x^2 - x} \quad\quad & \\ -x - 1 & \\ \underline{-x - 1} & \\ 0 & \end{array}$$

$= \frac{(x+1)^2 (x^2 - x - 1)}{\cancel{(x+1)}(x^3 - x^2 + 5x - 5)} =$

$= \frac{1 \cdot 0}{-12} = 0$

$$\begin{array}{r|l} x^4 + 0x^3 + 4x^2 + 0x - 5 & x+1 \\ \cline{2-2} \underline{x^4 + x^3} \quad\quad\quad\quad\quad\quad & x^3 - x^2 + 5x - 5 \\ -x^3 + 4x^2 \quad\quad\quad\quad & \\ \underline{-x^3 - x^2} \quad\quad\quad\quad & \\ 5x^2 + 0x \quad\quad & \\ \underline{5x^2 + 5x} \quad\quad & \\ -5x - 5 & \\ \underline{-5x - 5} & \\ 0 & \end{array}$$

в) $\lim\limits_{n\to\infty}\left(\frac{n+1}{n-1}\right)^n = \lim\left(\cancel{n+1}\ \frac{n-1+2}{n-1}\right)^n = \left(\left(1+\frac{2}{n-1}\right)^{\frac{n-1}{2}}\right)^{\frac{n2}{n-1}} =$

$= e^{\lim\limits_{n\to\infty}\frac{2n}{n-1}} = e^{2\lim\frac{n}{n-1}} = e^2$

г) Д-ть $\lim\limits_{x\to -3}\frac{2x^2+5x-3}{x+3} = -8$

Треб. дов. що: $(\forall\varepsilon>0)(\exists\delta>0) \mid (\forall x : |x-x_0|<\delta)\ |f(x)-A|<\varepsilon$

$\varepsilon>0$

$0<\delta<\frac{\varepsilon}{2}$

$\left|\frac{2x^2+5x-3}{x+3}+7\right|<\varepsilon$

$\left|\frac{2x^2+5x-3+7x+21}{x+3}\right|<\varepsilon$

$\left|\frac{2x^2+12x+18}{x+3}\right|<\varepsilon$

$2\left|\frac{x^2+6x+9}{x+3}\right|<\varepsilon$

$2\left|\frac{(x+3)^{\cancel{2}}}{\cancel{x+3}}\right|<\varepsilon$

$|x+3|<\frac{\varepsilon}{2}$

x

2) $\lim\limits_{n\to\infty} \dfrac{(3-n)^2+(3+n)^2}{(3-n)^2-(3+n)^2} = \left(\dfrac{\infty}{\infty}\right) = \dfrac{9-\cancel{6n}+n^2+9+\cancel{6n}+n^2}{\cancel{9}-6n+\cancel{n^2}-\cancel{9}-6n-\cancel{n^2}} =$

$= \dfrac{18+2n^2}{-12n} = \lim\limits_{n\to\infty}\left(-\underset{=0}{\dfrac{3}{2n}} - \underset{=\infty}{\dfrac{n}{6}}\right) = -\infty$

3) $\lim\limits_{n\to\infty} \dfrac{n\sqrt[3]{5n^2}+\sqrt[4]{9n^8+1}}{(n+\sqrt{n})\sqrt{7-n+n^2}} = \left(\dfrac{\infty}{\infty}\right) = \dfrac{\sqrt[3]{\dfrac{5}{n}}+\sqrt[4]{9+\dfrac{1}{n^8}}}{\left(1+\underset{\to 0}{\sqrt{\dfrac{1}{n}}}\right)\left(\sqrt{\underset{\to 0}{\dfrac{7}{n^2}}-\underset{\to 0}{\dfrac{1}{n}}+1}\right)} =$

$= \dfrac{\sqrt[4]{9}}{1\cdot 1} = \sqrt[4]{9} = \sqrt{3}$

4) $\lim\limits_{n\to\infty} n\left(\sqrt{n^2+1}-\sqrt{n^2-1}\right) = \infty(\infty-\infty) =$

$= n\dfrac{(\cancel{n^2}+1-\cancel{n^2}+1)}{\sqrt{n^2+1}+\sqrt{n^2-1}} = \dfrac{2n}{\sqrt{n^2+1}+\sqrt{n^2-1}} = \dfrac{1}{\sqrt{1+\dfrac{1}{n^2}}+\sqrt{1-\dfrac{1}{n^2}}} =$

$= 2\cdot\dfrac{1}{2} = 1$

5) $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^2}+\dfrac{2}{n^2}+\ldots+\dfrac{n-1}{n^2}\right) = \lim \dfrac{1}{n^2}(1+2+3+\ldots+n-1)$

$S = \dfrac{1+n-1}{2}\cdot(n-1) = \dfrac{n^2-n}{2}$; $\lim\limits_{n\to\infty}\left(\dfrac{1}{n^2}\cdot\dfrac{n^2-n}{2}\right) =$

$= \dfrac{1-\dfrac{1}{n}}{2} = \dfrac{1}{2}$

14) $\lim\limits_{x\to 0} \dfrac{\sqrt[3]{tg\,x}\;\; arctg\,\frac{1}{x} + 3}{2 - tg(1+\sin x)} = \dfrac{3}{2 - tg\,1}$

13.02.24.

1) Д-ть что

$a_n = \dfrac{3n-2}{2n-1} \quad a = \dfrac{3}{2} \qquad \lim\limits_{n\to\infty} \dfrac{3n-2}{2n-1} = \dfrac{3}{2}$

$\left| \dfrac{3n-2}{2n-1} - \dfrac{3}{2} \right| < \varepsilon$

$\left| \dfrac{2(3n-2) - 6n + 3}{2(2n-1)} \right| < \varepsilon$

$\left| \dfrac{6n-4-6n+3}{4n-1} \right| < \varepsilon$

$\dfrac{1}{2\,|2n-1|} < \varepsilon$

$|\underbrace{2n-1}_{>0}| > \dfrac{1}{2\varepsilon}$

$2n-1 > \dfrac{1}{2\varepsilon}$

$2n > \dfrac{1}{2\varepsilon} + 1$

$n > \dfrac{1}{2}\left(\dfrac{1}{2\varepsilon} + 1\right)$