$$\sum_{n=1}^{\infty} \frac{1}{(2n+1)(2n+3)}$$

$$\frac{x}{2n+1} + \frac{y}{2n+3} = \frac{1}{(2n+1)(2n+3)} \quad \cdot (2n+1)(2n+3)$$

$$(2n+3)x + (2n+1)y = 1$$

$$\begin{aligned} &\text{по } n: \\ &\text{по } C: \end{aligned} \begin{cases} 2x + 2y = 0 \\ 3x + y = 1 \end{cases}$$

$$\begin{cases} x = -y \\ 3 \cdot (-y) + y = 1 \end{cases}$$

$$-2y = 1$$

$$y = -\frac{1}{2}$$

$$x = \frac{1}{2}$$

$$\frac{\frac{1}{2}}{2n+1} + \frac{-\frac{1}{2}}{2n+3} = \frac{1}{2}\left(\frac{1}{2n+1} - \frac{1}{2n+3}\right) \Rightarrow$$

$$\frac{1}{2}\sum_{n=1}^{\infty}\left(\frac{1}{2n+1} - \frac{1}{2n+3}\right) = \frac{1}{2}\cdot\left(\frac{1}{3} - \cancel{\frac{1}{5}} + \cancel{\frac{1}{5}} - \cancel{\frac{1}{7}} + \cancel{\frac{1}{7}} - \underset{\to 0}{\frac{1}{9}}\right)$$

$$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6}$$

$$\sum_{n=1}^{\infty} \frac{1}{(5n-4)(5n+1)}$$

$$\frac{X}{5n-4} + \frac{Y}{5n+1} = \frac{1}{(5n-4)(5n+1)}$$

$$(5n+1)x + (5n-4)y = 1$$

$$\begin{cases} 5x + 5y = 0 \\ x - 4y = 1 \end{cases} \qquad \begin{cases} x = -y \\ -y - 4y = 1 \end{cases}$$

$$y = -\frac{1}{5}$$

$$x = \frac{1}{5}$$

$$\frac{\frac{1}{5}}{5n-4} + \frac{-\frac{1}{5}}{5n+1} = \frac{1}{5}\left(\frac{1}{5n-4} - \frac{1}{5n+1}\right) \Rightarrow$$

$$\frac{1}{5}\sum_{n=1}^{\infty}\left(\frac{1}{5n-4} - \frac{1}{5n+1}\right) = \frac{1}{5}\cdot\left(\frac{1}{1} - \cancel{\frac{1}{6}} + \cancel{\frac{1}{6}} - \cancel{\frac{1}{11}} + \cancel{\frac{1}{11}} - \overset{\to\infty}{\frac{1}{16}} \cdots\right) = \frac{1}{5}$$

$$\sum_{n=1}^{\infty} \frac{6}{9n^2+12n-5}$$

$$9n^2+12n-5 = 9\cdot\left(n+\frac{5}{3}\right)\cdot\left(n-\frac{1}{3}\right) = (3n+5)(3n-1)$$

$$D = 144 + 4\cdot 9\cdot 5 = 324$$

$$n = \frac{-12 \pm 18}{18}$$

$$n_1 = \frac{-30}{18} = -\frac{5}{3}$$

$$n_2 = \frac{1}{3}$$

$$\sum_{n=1}^{\infty} \frac{6}{(3n+5)(3n-1)}$$

$$\frac{x}{3n+5} + \frac{y}{3n-1} = \frac{6}{(3n+5)(3n-1)}$$

$$(3n-1)x + (3n+5)y = 6$$

$$\begin{cases} 3x + 3y = 0 \\ -x + 5y = 6 \end{cases} \quad \begin{cases} x = -y \\ y + 5y = 6 \end{cases} \quad \begin{cases} x = -y \\ y = 1 \end{cases} \begin{cases} x = -1 \\ y = 1 \end{cases}$$

$$\frac{-1}{3n+5} + \frac{1}{3n-1} = \frac{1}{3n-1} - \frac{1}{3n+5}$$

$$\sum_{n=1}^{\infty}\left(\frac{1}{3n-1} - \frac{1}{3n+5}\right) = \frac{1}{2} - \cancel{\frac{1}{8}} + \frac{1}{5} - \cancel{\frac{1}{11}} + \cancel{\frac{1}{8}} - \cancel{\frac{1}{14}} + \cancel{\frac{1}{11}} -$$

$$- \frac{1}{17} + \cancel{\frac{1}{14}} \ldots = \frac{1}{2} + \frac{1}{5} = 0{,}7$$

$$\sum_{n=2}^{\infty} \frac{2}{n^2-1} = \sum_{n=2}^{\infty} \frac{2}{(n-1)(n+1)}$$

$$\frac{x}{n-1} + \frac{y}{n+1} = \frac{2}{(n-1)(n+1)}$$

$$(n+1)x + (n-1)y = 2$$

$$\begin{cases} x + y = 0 \\ x - y = 2 \end{cases} \quad \begin{cases} x = -y \\ y = -1 \end{cases} \quad \begin{cases} x = 1 \\ y = -1 \end{cases}$$

$$\frac{1}{n-1} + \frac{-1}{n+1} = \frac{1}{n-1} - \frac{1}{n+1}$$

$$\sum_{n=2}^{\infty} \left(\frac{1}{n-1} - \frac{1}{n+1} \right) = \frac{1}{1} - \cancel{\frac{1}{3}} + \frac{1}{2} - \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \frac{1}{5} + \cancel{\frac{1}{4}} \dots =$$

$$= 1 + \frac{1}{2} = 1,5$$

$$\sum_{n=3}^{\infty} \frac{3n-1}{n^3-n} = \sum_{n=3}^{\infty} \frac{3n-1}{n(n^2-1)} = \sum_{n=3}^{\infty} \frac{3n-1}{n(n-1)(n+1)}$$

$$\frac{x}{n} + \frac{y}{n-1} + \frac{z}{n+1} = \frac{3n-1}{n(n-1)(n+1)}$$

$$(n-1)(n+1)x + n(n+1)y + n(n-1)z = 3n-1$$

$$(n^2-1)x + (n^2+n)y + (n^2-n)z = 3n-1$$

$$\begin{cases} x+y+z=0 \\ y-z=3 \\ -x=-1 \end{cases} \quad \begin{cases} 1+y+z=0 \\ y=3+z \\ x=1 \end{cases} \quad \begin{cases} y=-z-1 \\ y=3+z \end{cases}$$

$$3+z=-z-1$$

$$2z=-4$$

$$\begin{cases} z=-2 \\ y=1 \\ x=1 \end{cases}$$

$$\frac{1}{n} + \frac{1}{n-1} + \frac{-2}{n+1}$$

$$\sum_{n=3}^{\infty}\left(\frac{1}{n} + \frac{1}{n-1} - \frac{2}{n+1}\right) = \frac{1}{3} + \cancel{\frac{1}{2}} - \cancel{\frac{2}{4}} + \cancel{\frac{1}{4}} + \cancel{\frac{1}{3}} - \cancel{\frac{2}{5}} +$$

$$+ \cancel{\frac{1}{5}} + \cancel{\frac{1}{4}} - \cancel{\frac{2}{6}} + \cancel{\frac{1}{6}} + \cancel{\frac{1}{5}} - \cancel{\frac{2}{7}} + \cancel{\frac{1}{7}} + \cancel{\frac{1}{6}} - \frac{2}{8} \ldots =$$

$$\frac{1}{3}+\frac{1}{2}+\frac{1}{3}=\frac{2}{3}+\frac{1}{2}=\frac{7}{6}$$

$$\sum_{n=1}^{\infty}\frac{1}{n^3+3n^2+2n}=\sum_{n=1}^{\infty}\frac{1}{n(n^2+3n+2)}$$

$$\Delta=9-4\cdot2=1$$

$$n=\frac{-3\pm1}{2}$$

$$n_1=-2$$

$$n_2=-1$$

$$\sum_{n=1}^{\infty}\frac{1}{n(n+2)(n+1)}$$

$$\frac{x}{n}+\frac{y}{n+2}+\frac{z}{n+1}=\frac{1}{n(n+1)(n+2)}$$

$$(n+1)(n+2)x+n(n+1)y+n(n+2)z=1$$

$$(n^2+n+2n+2)x+(n^2+n)y+(n^2+2n)z=1$$

$$\begin{cases}x+y+z=0\\3x+y+2z=0\\2x=1\end{cases}\qquad\begin{cases}y+z=-\frac{1}{2}\\y+2z=-\frac{3}{2}\end{cases}\qquad x=\frac{1}{2}$$

$$\begin{cases}y=-z-\frac{1}{2}\\y=-2z-\frac{3}{2}\end{cases}\qquad -z-\frac{1}{2}=-2z-\frac{3}{2}$$

$$\cancel{-3}z\quad z=-1$$

$$x = \frac{1}{2}$$

$$z = -1$$

$$y = +1 - \frac{1}{2} = \frac{1}{2}$$

$$\frac{\frac{1}{2}}{n} + \frac{\frac{1}{2}}{n+2} + \frac{-1}{n+1} = \frac{1}{2n} + \frac{1}{2n+4} - \frac{1}{n+1}$$

$$\sum_{n=1}^{\infty} \left(\frac{1}{2n} + \frac{1}{2n+4} - \frac{1}{n+1} \right) = \frac{1}{2} + \frac{1}{6} - \frac{1}{2} + \frac{1}{4} + \frac{1}{8} - \frac{1}{3} +$$

$$+ \frac{1}{6} + \frac{1}{10} - \frac{1}{4} + \frac{1}{8} + \frac{1}{12} - \frac{1}{5} \dots$$

$$= \frac{1}{2} \left(\frac{1}{n} + \frac{1}{n+2} - \frac{2}{n+1} \right)$$

$$\frac{1}{2} \sum_{n=1}^{\infty} \left(\frac{1}{n} + \frac{1}{n+2} - \frac{2}{n+1} \right) = \frac{1}{2} \left(\frac{1}{1} + \frac{1}{3} - \frac{2}{2} + \frac{1}{2} + \frac{1}{4} - \frac{2}{3} + \right.$$

$$\left. + \frac{1}{3} + \frac{1}{5} - \frac{2}{4} + \frac{1}{4} + \frac{1}{6} - \frac{2}{5} + \frac{1}{5} + \frac{1}{7} - \frac{2}{6} \dots \right)$$

$$= \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

$$\sum_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right) = \sum_{n=2}^{\infty} \ln\left(\frac{n^2-1}{n^2}\right) =$$

$$= \sum_{n=2}^{\infty} \ln\left(\frac{(n-1)(n+1)}{n^2}\right) =$$

т.к. $\log_a (b \cdot c) = \log_a b + \log_a c$

$\log_a \frac{b}{c} = \log_a b - \log_a c$

$$\sum_{n=2}^{\infty} \left(\ln(n-1) + \ln(n+1) - \ln(n^2)\right) =$$

т.к. $\log_a x^n = n \log_a x$

$$= \sum_{n=2}^{\infty} \left(\ln(n-1) + \ln(n+1) - 2\ln(n)\right) =$$

$$= \ln 1 + \cancel{\ln 3} - 2\ln 2 + \ln 2 + \cancel{\ln 4} - \cancel{2\ln 3} +$$

$$+ \cancel{\ln 3} + \cancel{\ln 5} - \cancel{2\ln 4} + \cancel{\ln 4} + \cancel{\ln 6} - \cancel{2\ln 5} \ldots =$$

$$= \ln 1 - \ln 2 = 0 - \ln 2 = -\ln 2$$