$$\lim_{n\to\infty} \frac{\sqrt{n}\sin n}{2n+6} \quad /n \quad \frac{\infty}{\infty} \text{ - неопределенность}$$

20.10

$$\lim_{n\to\infty} \frac{\frac{\sqrt{n}\cdot\sin n}{n}}{2+\frac{6}{n}} = \frac{\lim\limits_{n\to\infty}\overset{\overset{0}{\frown}}{\frac{1}{\sqrt{n}}}\cdot\lim\limits_{n\to\infty}\sin n}{\lim\limits_{n\to\infty}2+\lim\limits_{n\to\infty}\underset{\underset{0}{\smile}}{\frac{6}{n}}} =$$

$$= \lim_{n\to\infty} x_n = \frac{0}{2+0} = 0$$

$\infty - \infty \Rightarrow$ неопределенность

$$\lim_{n\to\infty}\left(\sqrt{n+1}-\sqrt{n}\right)$$

$$\lim_{n\to\infty} x_n = \lim_{n\to\infty}\frac{(\sqrt{n+1}-\sqrt{n})(\sqrt{n+1}+\sqrt{n})}{(\sqrt{n+1}+\sqrt{n})} =$$

$$= \lim_{n\to\infty}\frac{n+1-n}{\sqrt{n+1}+\sqrt{n}} = \frac{\lim\limits_{n\to\infty}1}{\lim\limits_{n\to\infty}(\sqrt{n+1}+\sqrt{n})} =$$

$$= \frac{1}{\infty} = 0$$

Бесконечно убывающая геометрическая прогрессия

$$\lim_{x\to\infty}\left(\frac{1}{4}+\frac{1}{4^2}+\ldots+\frac{1}{4^n}\right)$$

$$\boxed{S = \frac{b_1}{1-q}}$$

$b_1 = \frac{1}{4}$

$q = \frac{1}{4}$

$x_1 = \frac{1}{4}$

$x_2 = \frac{1}{4} + \frac{1}{4^2}$

$x_3 = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3}$

...

$x_n = \frac{1}{4} + \frac{1}{4^2} + \ldots + \frac{1}{4^n}$

$$\lim_{n \to \infty} x \frac{\frac{1}{4}}{1 - \frac{1}{4}} = \frac{\frac{1}{4}}{\frac{3}{4}} = \frac{1}{4} \cdot \frac{4}{3} = \frac{1}{3}$$

Арифметическая прогрессия

$$\boxed{S = \frac{(a_1 + a_n) \cdot n}{2}}$$

$a_1 = 1$, $a_n = 2n - 1$

$$\lim_{n \to \infty} \frac{1 + 3 + \ldots + (2n-1)}{3n^2}$$

$$x_1 = \frac{1^{\text{-слева}}}{3 \cdot 1^2} = \frac{1}{3}$$

$$x_2 = \frac{1+3^{\text{-слева 2 индукция}}}{3 \cdot 2^2} = \frac{4}{12} = \frac{1}{3}$$

По индукции?

$$x_3 = \frac{1+3+5}{3 \cdot 3^2} = \frac{9}{27} = \frac{1}{3}$$

$$x_4 = \frac{1+3+5+7}{3 \cdot 4^2} = \frac{16}{16 \cdot 3} = \frac{16}{48} = \frac{1}{3}$$

...

$$x_n = \frac{1+3+\ldots+(2n-1)}{3n^2}$$

$$\lim_{x \to \infty} \frac{\frac{(1+2n-1) \cdot n}{2}}{3n^2} = \lim_{x \to \infty} \frac{2n^2}{6n^2} = \frac{1}{3}$$

$$\lim_{n \to \infty} \frac{(2n-1)^3 + (1-3n)^3}{8n^3 - 2n} = \frac{\infty}{\infty}^{\text{неопр (1)}} =$$

$$= \lim_{n \to \infty} \frac{(2n-1+1-3n)\left((2n-1)^2 - (2n-1)(1-3n) + (1-3n)^2\right)}{8n^3 - 2n}$$

$$= \lim_{n \to \infty} \frac{-n\left((2n-1)^2 - 2n + 1 + 6n^2 - 3n + (1-3n)^2\right)}{8n^3 - 2n}$$

$$= \lim_{n\to\infty} \frac{8n^3 - 3\cdot 4n^2 + 3\cdot 2n - \cancel{1} + \cancel{1} - 3\cdot 3n + 3\cdot 9n^2 -}{8n^3 - 2n}$$

$$= \lim_{n\to\infty} \frac{8n^3 - 27n^3 - 12n^2 + 27n^2 + 6n - 9n}{8n^3 - 2n} \; (2) =$$

$$= \lim_{n\to\infty} \frac{-19n^3 + 15n^2 - 3n}{8n^3 - 2n} \begin{matrix} |:n^3 \\ |:n^3 \end{matrix} = \frac{-\infty}{\infty} \; (3) =$$

$$= \lim_{n\to\infty} \frac{\frac{-19n^3 + 15n^2 - 3n}{n^3}}{\frac{8n^3 - 2n}{n^3}} = \lim_{n\to\infty} \frac{-19 + \underbrace{\frac{15}{n}}_{\to 0} - \underbrace{\frac{3}{n^2}}_{\to 0}}{8 - \underbrace{\frac{2}{n^2}}_{\to 0}} =$$

$$= -\frac{19}{8}$$

$$\lim_{n\to\infty} \frac{8n^3 - (1+2n)^3}{(1+2n)^2 + 4n^2} = \frac{\infty - \infty}{\infty} \text{ неопред.} =$$

$$= \lim_{n\to\infty} \frac{8n^3 - (1 + 3\cdot 2n + 3\cdot 4n^2 + 8n^3)}{1 + 4n + 4n^2 + 4n^2} =$$

$$= \lim_{n\to\infty} \frac{-12n^2 - 6n - 1}{8n^2 + 4n + 1} \begin{matrix} /:n^2 \\ /:n^2 \end{matrix} = \lim_{n\to\infty} \frac{-12 - \underbrace{\frac{6}{n}}_{\to 0} - \underbrace{\frac{1}{n^2}}_{\to 0}}{8 + \underbrace{\frac{4n}{n}} + \underbrace{\frac{1}{n^2}}_{\to 0}} =$$

$27n^3 =$ $= -\frac{12}{8} = -\frac{3}{2} = -1,5$

$$\lim_{n\to\infty} \frac{3^{n+1} + 2\cdot 4^n}{4^{n+1} - 5} = \lim_{n\to\infty} \frac{3^n\cdot 3 + 2\cdot 4^n}{4^n\cdot 4 - 5} \begin{matrix} /:4^n \\ /:4^n \end{matrix} = \frac{\infty}{\infty} =$$

$$= \lim_{n\to\infty} \frac{\frac{3^n\cdot 3 + 2\cdot 4^n}{4^n}}{\frac{4^n\cdot 4 - 5}{4^n}} = \lim_{n\to\infty} \frac{\left(\frac{3}{4}\right)^n\cdot 3 + 2}{4 - \underbrace{\frac{5}{4^n}}_{\to 0}} = \frac{1}{2}$$

$$\lim_{n\to\infty} \frac{7^n + 10\cdot 9^{n+1}}{5\cdot 10^{n-1} + 7^{n+2}} \begin{matrix} /:10^n \\ /:10^n \end{matrix} = \frac{\infty}{\infty} =$$

$$= \lim_{n\to\infty} \frac{\frac{7^n}{10^n} + \frac{90\cdot 9^n}{10^n}}{0,5 + \frac{49\cdot 7^n}{10^n}} = \lim_{n\to\infty} \frac{\overbrace{\left(\frac{7}{10}\right)^n}^{\to 0} + 90\cdot \overbrace{\left(\frac{9}{10}\right)^n}^{\to 0}}{0,5 + \underbrace{\left(\frac{7}{10}\right)^n}_{\to 0}\cdot 49} =$$

$$= \frac{0}{0,5} = 0$$

$$\lim_{n\to\infty} \frac{(2n+1)! + (2n+2)!}{(2n+3)!}$$

$(2n+3)! = 1\cdot 2\cdot 3 \ldots 2n(2n+1)\cdot(2n+2)(2n+3)$

$$\lim_{n\to\infty} \frac{(2n+1)!+(2n+2)!}{(2n+3)!} = \frac{\infty}{\infty} =$$

$$= \lim_{n\to\infty} \frac{1\cdot2\cdot3\ldots2n(2n+1)+1\cdot2\cdot3\ldots2n(2n+1)(2n+2)}{1\cdot2\cdot3\ldots2n(2n+1)(2n+2)(2n+3)} =$$

$$= \lim_{n\to\infty} \frac{\cancel{1\cdot2\cdot3\cdot2n(2n+1)}\left[1+(2n+2)\right]}{\cancel{1\cdot2\cdot3\ldots2n(2n+1)}(2n+2)(2n+3)} =$$

$$= \lim_{n\to\infty} \frac{1+2n+2}{(2n+2)(2n+3)} = \lim_{n\to\infty} \frac{\cancel{2n+3}}{(2n+2)\cancel{(2n+3)}} =$$

$$= \lim_{n\to\infty} \frac{1}{2n+2} = 0$$

$$\lim_{n\to\infty} \frac{(n+4)!-(n+2)!}{(n+3)!} = \frac{\infty-\infty}{\infty} =$$

$$= \lim_{n\to\infty} \frac{\cancel{1\cdot2\cdot3\ldots n(n+1)(n+2)}\cdot\left[(n+3)(n+4)-1\right]}{\cancel{1\cdot2\cdot3\ldots n(n+1)(n+2)}\cdot(n+3)} =$$

$$= \lim_{n\to\infty} \frac{(n+3)(n+4)-1}{n+3} = \lim_{n\to\infty} \frac{n^2+3n+4n+12-1}{n+3} =$$

$$= \lim_{n\to\infty} \frac{n^3+7n+11}{n+3} \begin{matrix} |:n^3 \\ |:n^3 \end{matrix} = \lim_{n\to\infty} \frac{1+\underbrace{\frac{7}{n^2}}_{\to 0}+\underbrace{\frac{11}{n^3}}_{\to 0}}{\underbrace{\frac{n}{n^3}}_{\to 0}+\underbrace{\frac{3}{n^3}}_{\to 0}} =$$

$$= \cancel{\lim_{n\to\infty}} \frac{1}{0} = \infty$$

Когда деление на 0 запрещено? В пределах никогда!

$$\lim_{n\to\infty} \frac{\sin(n!)}{n^2} = \lim_{n\to\infty} \Big(\underbrace{\frac{1}{n^2}}_{\to 0} \cdot \sin(n!)\Big) = 0$$

$$-1 \le \sin(n!) \le 1$$

А это имеет значение?

$$\lim_{n\to\infty} \left(2e^{-n} \cos^2\left(\frac{3n-5}{1+n^3}\right)\right) = 2\lim_{n\to\infty} \left(\overbrace{\frac{1}{e^n}}^{\to 0} \cdot \cos^2\left(\frac{3n-5}{1+n^3}\right)\right) =$$

Почему тут 2 вне предела вынесли?

$$= 2 \cdot 0 = 0$$

неопред?

$$\lim_{n\to\infty} \left(n^2 \operatorname{tg} \frac{1}{2n+1}\right) = \infty \cdot 0 =$$

неопределённость

$$= \lim_{n\to\infty} \frac{n^2 \cdot \sin\frac{1}{2n+1}}{\cos\underbrace{\frac{1}{2n+1}}_{\to 0} \Rightarrow 1} = \lim_{n\to\infty} n^2 \cdot \sin\frac{1}{2n+1} = \infty \cdot 0 =$$

$$= \lim_{n\to\infty} \frac{n^2 \cdot \overbrace{\sin\frac{1}{2n+1}}^{1}}{\frac{1}{2n+1}\cdot(2n+1)} = \lim_{n\to\infty} \frac{n^2}{2n+1} \; \begin{matrix} /:n^2 \\ \\ /:n^2 \end{matrix} = \frac{\infty}{\infty} =$$

замечат. предел

$$= \lim_{n\to\infty} \frac{1}{\underbrace{\frac{2}{n}}_{\to 0} + \underbrace{\frac{1}{n^2}}_{\to 0}} = \frac{1}{0} = \infty$$

Замечательные эквивалентности

$$\lim_{n\to\infty} \left((1-3n^2)\,\mathrm{arctg}^2 \frac{n}{3n^2-1} \right) =$$

$\mathrm{arctg}\, a \sim a$, если $a \to 0$

$$= \lim_{n\to\infty} \left((1-3n^2) \cdot \left(\frac{n}{3n^2-1} \right)^2 \right) =$$

$$= \lim_{n\to\infty} \frac{(1-3n^2)\cdot n^2}{(3n^2-1)^2} = \lim_{n\to\infty} \frac{n^2-3n^4}{9n^4-6n^2+1} \; \begin{matrix} /:n^4 \\ \\ /:n^4 \end{matrix} =$$

$$= \frac{-\infty}{\infty} = \lim_{n\to\infty} \frac{\underbrace{\frac{1}{n^2}}_{\to 0} - 3}{9 - \underbrace{\frac{6}{n^2}}_{\to 0} + \underbrace{\frac{1}{n^4}}_{\to 0}} = -\frac{1}{3}$$

$$x_n = \frac{(-1)^{n-1} \cdot (2-n)}{n^2+3}$$

$$\lim_{n\to\infty} |x_n| = |a|$$

$$\lim_{n\to\infty} |x_n| = \lim_{n\to\infty} \frac{2-n}{n^2+3}, \text{ т.к. } |x_n| = \frac{2-n}{n^2+3}$$

~~лим~~

?

$$\lim_{n\to\infty} \frac{2-n}{\underbrace{n^2+3}_{\to 0}} \Big/ :n^2 = \frac{-\infty}{\infty} \Big/ n^2 \to 0 = \lim_{n\to\infty} \frac{\frac{2-n}{n^2}}{\frac{n^2+3}{n^2}} =$$

$$= \lim_{n\to\infty} \frac{\frac{2}{n^2} - \frac{1}{n}}{1 + \underbrace{\frac{3}{n^2}}_{\to 0}} = 0$$

Време-нные написать $\frac{\infty}{\infty}$ фигуры?