Пример 10.

$$\lim_{x\to -2}\frac{2x^2+7x+6}{x^2+4x+4}=\left[\frac{0}{0}\right]=\frac{2\cancel{(x+2)}(x+1\frac{1}{2})}{(x+2)^2}=$$

$$=\frac{2\cdot(x+1\frac{1}{2})}{x+2}=\frac{2(x+1\frac{1}{2})}{2(\frac{x}{2}+1)}=\frac{2x+3}{x+2}=\frac{-1}{0}=$$

$$=\infty$$

$D=49-48=1$

$x_1=\frac{-7+1}{4}=-1\frac{1}{2}$

$x_2=\frac{-7-1}{4}=-2$

$D=\cancel{4}\ 16-16=0$

$x=\frac{-4}{4}=-2$

Пример 11.

$$\lim_{x\to -1}\frac{3x^4-2x^3-5}{x^3+1}=\left[\frac{0}{0}\right]=\frac{(x+1)(3x^3-5x^2+5x-5)}{(x+1)(x^2-x+1)}=$$

$$=\frac{3x^2-5x^2+5x-5}{x^2-x+1}=\frac{-18}{3}=-6$$

Пример 12.

$$\lim_{x\to 0}\frac{\sqrt{2+x}-\sqrt{2-x}}{5x}=\left[\frac{0}{0}\right]=\frac{(\sqrt{2+x}-\sqrt{2-x})(\sqrt{2+x}+\sqrt{2-x})}{5x(\sqrt{2+x}+\sqrt{2-x})}=$$

$$=\frac{2+x-2+x}{5x(\sqrt{2+x}+\sqrt{2-x})}=\frac{2x}{5x(\sqrt{2+x}+\sqrt{2-x})}=\frac{\sqrt{2}}{10}$$

Пример 13.

$$\lim_{x\to 2}\frac{x^2-4}{\sqrt{x+2}-\sqrt{6-x}}=\left[\frac{0}{0}\right]=\frac{(x^2-4)(\sqrt{x+2}+\sqrt{6-x})}{(\sqrt{x+2}-\sqrt{6-x})(\sqrt{x+2}+\sqrt{6-x})}=$$

$$=\frac{(x-2)(x+2)(\sqrt{x+2}+\sqrt{6-x})}{2(x-2)}=\frac{4(\sqrt{4}+\sqrt{4})}{2}=8$$

Пример 14.

$$\lim_{x\to 0}\frac{\sqrt{1+x}-\sqrt{1+x^2}}{\sqrt{1+x}-1}=\left[\frac{0}{0}\right]=\frac{(\sqrt{1+x}-\sqrt{1+x^2})(\sqrt{1+x}+\sqrt{1+x^2})(\sqrt{1+x}+1)}{(\sqrt{1+x}+\sqrt{1+x^2})(\sqrt{1+x}-1)(\sqrt{1+x}+1)}$$

$$=\frac{(x-x^2)(\sqrt{1+x}+1)}{x(\sqrt{1+x}+\sqrt{1+x^2})}=\frac{(1-x)(\sqrt{1+x}+1)}{(\sqrt{1+x}+\sqrt{1+x^2})}=\frac{2}{2}=1$$

Пример 15.

$$\lim_{x\to 64}\frac{\sqrt{x}-8}{\sqrt[3]{x}-4}=\left[\frac{0}{0}\right]$$

замени $\sqrt[6]{x}=t$

$\sqrt[3]{x}=t^2$, $\sqrt{x}=t^3$

при $x\to 64$ $t\to 2$

$$\lim_{t\to 2}\frac{t^3+8}{t^2-4}=\frac{(t-2)(t^2+2t+4)}{(t-2)(t+2)}=\frac{12}{4}=3$$

Пример 16.

$$\lim_{x\to 0} \frac{\sqrt[3]{x+1}-1}{\sqrt{x+1}-1} = \left[\frac{0}{0}\right] =$$ замена $x+1=t^6$

$$\lim_{t\to 1} \frac{t^2-1}{t^3-1} = \frac{(t-1)(t+1)}{(t-1)(t^2+2t+1)} = \frac{t+1}{t^2+2t+1} = \frac{2}{3}$$

Пример 17.

$$\lim_{x\to 0} \frac{1-\cos 4x}{3x^2} = \frac{2\sin^2 2x}{3x^2} = \frac{2\cdot 4}{3} \lim_{x\to 0} \left(\frac{\sin 2x}{2x}\right)^2 =$$

$$= \frac{8}{3}$$

Пример 18.

$$\lim_{x\to 0} \frac{\sin 2x}{\sqrt{x+9}-3} = \frac{\sin 2x\,(\sqrt{x+9}+3)}{x} = 2\lim_{x\to 0}\frac{\sin 2x}{2x} \cdot$$

$$\lim_{x\to 0} (\sqrt{x+9}+3) = 2\cdot 1\cdot 6 = 12$$

Пример 19.

$$\lim_{x\to +\infty} \sqrt{x^2+6x+5}-x = \frac{(\sqrt{x^2+6x+5}-x)(\sqrt{x^2+6x+5}+x)}{\sqrt{x^2+6x+5}+x} =$$

$$= \frac{x^2+6x+5-x^2}{\sqrt{x^2+6x+5}+x} = \frac{6x+5}{\sqrt{x^2+6x+5}+x} = \left[\frac{\infty}{\infty}\right] \frac{6+\frac{5}{x}}{\sqrt{1+\frac{6}{x}+\frac{5}{x^2}}+1} = 3$$

Пример 20.

$$\lim_{x\to 3}\left(\frac{1}{x-3}-\frac{6}{x^2-9}\right)=[\infty-\infty]=\frac{x+3-6}{x^2-9}=$$

$$=\frac{x-3}{x^2-9}=\left[\frac{0}{0}\right]=\frac{1}{x+3}=\frac{1}{6}$$

Пример 21.

$$\lim_{x\to 0}\left(\frac{1}{4\sin^2 x}-\frac{1}{\sin^2 2x}\right)=[\infty-\infty]=$$

$$=\lim_{x\to 0}\left(\frac{1}{4\sin^2 x}-\frac{1}{4\sin^2 x\cos^2 x}\right)=\frac{\cos^2 x-1}{4\sin^2 x\cos^2 x}=\left[\frac{0}{0}\right]=$$

$$=\lim_{x\to 0}\frac{-\sin^2 x}{4\sin^2 x\cos^2 x}=\lim_{x\to 0}\frac{-1}{4\cos^2 x}=-\frac{1}{4}$$

Пример 22

$$\lim_{x\to\infty}\left(1+\frac{2}{x}\right)^{3x}=\left[\left(1+\frac{2}{x}\right)^{\frac{x}{2}}\right]^{6}=e^{6}$$

Пример 23

$$\lim_{x\to 0}\left(\frac{3+x}{3}\right)^{\frac{1}{x}}=[1^{\infty}]=\left[\left(1+\frac{x}{3}\right)^{\frac{3}{x}}\right]^{\frac{1}{3}}=e^{\frac{1}{3}}=\sqrt[3]{e}$$

Приклад 24.

$$\lim_{x\to\infty}\left(\frac{2x^2+3}{2x^2-1}\right)^{3x^2}=\left[1^{\infty}\right] \quad \Bigg| \quad \frac{2+\frac{3}{x^2}}{2-\frac{1}{x^2}}=\frac{2+0}{2-0}=1$$

$$\lim_{x\to\infty}3x^2=\infty$$

$$\frac{2x^2+3}{2x^2-1}=\frac{(2x^2-1)+4}{2x^2-1}=\frac{2x^2-1}{2x^2-1}+\frac{4}{2x^2-1}=$$

$$=1+\frac{4}{2x^2-1}$$

$$a(x)=\frac{4}{2x^2-1}$$

$$\lim_{x\to\infty}\frac{12x^2}{2x^2-1}=\left[\frac{\infty}{\infty}\right]=\lim_{x\to\infty}\frac{12}{2-\frac{1}{x^2}}=\frac{12}{2-0}=6$$

$$p=e^{\lim\limits_{x\to\infty}\frac{12x^2}{2x^2-1}}=e^6$$

Приклад 25.

$$\lim_{x\to\infty}\left[x\left(\ln(x+2)-\ln x\right)\right]=\infty\cdot 0=$$

$$=\lim_{x\to\infty}\left(x\ln\frac{x+2}{x}\right)=\lim_{x\to\infty}x\ln\left(1+\frac{2}{x}\right)=[\infty\cdot 0]=$$

$$=\lim_{x\to\infty}\ln\left(1+\frac{2}{x}\right)^x=\ln\left[\lim\left(1+\frac{2}{x}\right)^{x/2}\right]^2=$$

$\ln e^2 = 2$

Пример 26.

$$\lim_{x\to 0} \sqrt[x]{1+4x} = (1+4x)^{\frac{1}{4x}\cdot 4} = [1^\infty]$$

$$\left[\lim_{x\to 0} (1+4x)^{\frac{1}{4x}}\right]^4 = e^4$$

Пример 27

$$\lim_{x\to 0} \frac{e^{3x}-1}{5x} = \frac{3}{5} \lim_{x\to 0} \frac{e^{3x}-1}{3x} = \frac{3}{5}\cdot 1 = \frac{3}{5}$$