$\ln y = \sin x \ \ln x = \frac{\sin x}{\frac{1}{\ln x}}$

$\lim \ln y = \lim\limits_{x \to 0} \frac{\cos x}{-\frac{1}{\ln x^2} \cdot \frac{1}{x}}$

10.03.24. Интегралы.

7.1. $\int 2x^7 = 2\int x^7 = 2\frac{x^8}{8} = \frac{x^8}{4} + C$

7.2. $4\int \sqrt[3]{x} = 4\int x^{\frac{1}{3}} = 4 \cancel{\frac{3}{4\sqrt[3]{x^2}}} = \cancel{\frac{3}{\sqrt[3]{x^2}} + C}$ $\quad 4 \frac{x^{\frac{4}{3}}}{\frac{4}{3}} = 3\sqrt[3]{x^4} = 3x\sqrt[3]{x} + C$

7.3. $\int \frac{3}{x} + \frac{5}{x^2} = 3\ln|x| + 5 \cancel{\ln|x|} \cancel{\frac{x^3}{3}}$ $\quad \int 5x^{-2} = \frac{5x^{-1}}{-1} = 3\ln|x| - \frac{5}{x} + C$

7.4. $\int \frac{x^3 + 5x^2 - 1}{x} = \cancel{\left(\frac{x^4}{4} + \frac{5x^3}{3} - x\right) \ln|x|}$ $\quad \int (x^3 + 5x^2 - 1) \cdot \frac{1}{x} =$

$= \int \frac{x^3}{x} + \frac{5x^2}{x} - \frac{1}{x} = \int x^2 + 5x - \frac{1}{x} = \frac{x^3}{3} + \frac{5x^2}{2} - \ln|x| + C$

7.5. $\frac{(\sqrt{x}+1)^3}{x\sqrt{x}} = \frac{x\sqrt{x}}{x\sqrt{x}} + \frac{3x}{x\sqrt{x}} + \frac{3\sqrt{x}}{x\sqrt{x}} + \frac{1}{x\sqrt{x}} = \int 1 + \frac{3}{\sqrt{x}} + \frac{3}{x} + \frac{1}{x\sqrt{x}} =$

$= x + 3\sqrt{x} \cdot 2 + 3\ln|x| + \cancel{\frac{2\sqrt{x^5}}{5}} + C \ \cancel{+} - \frac{2}{\sqrt{x}}$

7.9. $\int$

7.10. $\int$

7.11. $\int x$

=

$x^2 = 0$

$3x\sqrt[3]{x} + c$

$\ln|x| - \frac{5}{x} + c$

c

7.6. $\int 1 - 2\sin^2 \frac{x}{2} = \cancel{x} - \cancel{x} \quad \int \cos x = \sin x + c$

7.7. $\int \frac{1}{\sqrt{a+bx}} = \int \frac{1}{(a+bx)^{\frac{1}{2}}} = 2\sqrt{(a+bx)} \cdot \frac{bx^2}{2}$

$t = a + bx \qquad dx = \frac{1}{t}\, dt \qquad t = b$

$\int \frac{1}{\sqrt{a+bx}} \cdot \frac{1}{b}\, dt = \int \frac{1}{\sqrt{t} \cdot b} = \frac{1}{b} \int \frac{1}{\sqrt{t}} = \frac{1}{b}\, \sqrt{t} \cdot 2 =$

$= \frac{2}{b} \cdot \sqrt{a+bx} + c$

7.8. $\int e^{2-3x} = \int \frac{e^2}{e^{3x}} \qquad dx = \frac{1}{t}\, dt, \quad t = 2-3x$ и $t' = -3$

$\int e^{2-3x} \cdot \left(-\frac{1}{3}\right) dt = \int -\frac{e^{2-3x}}{3}\, dt = \int -\frac{e^t}{3}\, dt = -\frac{1}{3} e^t =$

$= -\frac{1}{3} e^{2-3x} + c$

7.9. $\int \frac{1}{\sqrt[3]{5^x}} = \int \frac{1}{5^{\frac{x}{3}}}\, dx = \int 5^{-\frac{x}{3}}\, dx \qquad dx = \frac{1}{t}\, dt \qquad t = -\frac{x}{3} \qquad t' = -\frac{1}{3}$

$\cancel{\int \frac{1}{5^{\frac{x}{3}}} \cdot \frac{1}{3}\, dt = \int \frac{1}{5^t \cdot 3}\, dt} = \frac{1}{3} \frac{5^t}{\ln 5} =$

$\int 5^{-\frac{x}{3}} \cdot \left(-\frac{1}{3}\right) dt \quad -\frac{1}{3} \int 5^t\, dt = -\frac{1}{3} \cdot 5^t\, \frac{5^t}{\ln 5} = -\frac{3 e^{-\frac{x}{3}}}{\ln 5} + c$

7.10. $\int \frac{1}{\cos^2 4x}\, dx = \qquad dx = \frac{1}{t}\, dt \qquad t = 4x \qquad t' = 4$

$\int \frac{1}{\cos^2 4x} \cdot \frac{1}{4} \cdot dt = \frac{1}{4} \int \frac{1}{\cos^2 t}\, dt = \frac{\operatorname{tg} t}{4} + c = \frac{\operatorname{tg} 4x}{4} + c$

7.11. $\int \frac{x^3+1}{x-1} = \int \cancel{x^3-1} \quad \cancel{(x+1)}\, (x+1)(x-1) \quad \int x^2 + x + 1 + \frac{2}{x-1}\, dx$

$= \frac{x^3}{3} + \frac{x^2}{2} + x + 2\ln|x-1| + c$

7.15. $\int (3x^2+2x+\frac{1}{x})\,dx = x^3 + x^2 + \ln|x| + c$

7.16. $\int \frac{2x+3}{x^4}\,dx = \int\left(\frac{2}{x^3}+\frac{3}{x^4}\right)dx = -\frac{\cancel{2}\,1}{\cancel{2}\,x^2} + -\frac{1}{x^3} + c$

7.17. $\int \sqrt{mx}\,dx = \int (mx^{\frac{1}{2}})\,dx$ $\quad dx = \frac{1}{t}\,dt \quad t = mx \quad t' = m$

$\int \sqrt{mx}\,\frac{1}{m}\,dt = \frac{1}{m}\int \sqrt{t}\,dt = \cancel{\frac{1}{m}\frac{t^2}{2} = \frac{1}{2m}}$

$= \frac{1}{m}\frac{\sqrt{t^3}\,2}{3} = \frac{2}{3m}\sqrt{(mx)^3} = \frac{2mx\sqrt{mx}}{3} = \frac{2x\sqrt{mx}}{3} + c$

7.18. $\int \frac{dx}{\sqrt[n]{x}} = \int x^{-\frac{1}{n}}\,dx$

$n^{-1} = \frac{1}{n}$ $\quad dx = \frac{1}{t}\,dt \quad \sqrt[n]{x} = t \quad t' = x^{\frac{1}{n}} = \left(\frac{1}{n}\right)x^{\frac{1}{n}}$

$= \frac{x^{\frac{1}{n}-1}}{n} = \frac{x^{\frac{1-n}{n}}}{n}$

$\int \frac{1}{\sqrt[n]{x}}\,\frac{n}{x^{\frac{1-n}{n}}}\,dt = \int \frac{1}{\sqrt[n]{x}}\,\frac{n}{\frac{\sqrt[n]{x}}{x}}\,dt = \int \frac{1}{t}\cdot\frac{nx}{t} = n\int \frac{x}{t^2}\,dt =$

$= nx\int \frac{1}{t^2}\,dt = -nx\,\frac{1}{t}$

7.19. $\int \frac{1}{\sqrt[3]{x^2}} - \frac{x+1}{\sqrt[4]{x^3}} = \int\left(\frac{1}{1}x^{-\frac{2}{3}} - \frac{x}{x^{\frac{3}{4}}} + \frac{1}{x^{\frac{3}{4}}}\right)dx = 3\sqrt[3]{x} - \frac{4x\sqrt[4]{x}}{5} + 4\sqrt[4]{x} + c$

$\frac{x}{x^{\frac{3}{4}}} = x^{\frac{1}{4}}$

7.20. $\int \frac{(\sqrt{a}+\sqrt{x})^2}{\sqrt{ax}}\,dx \quad \int \frac{a+2\sqrt{ax}+x}{\sqrt{ax}} = \int\left(\frac{a}{\sqrt{ax}} + 2 + \frac{x}{\sqrt{ax}}\right)dx =$

$= \int \frac{\sqrt{ax}}{x} + \int 2 + \int \frac{\sqrt{ax}}{a}$ $\quad dx = \frac{1}{t}\,dt \quad t = \sqrt{ax} \quad t' = \frac{1}{2\sqrt{ax}}\,a$

$\int \frac{\sqrt{ax}}{x}\cdot\frac{2\sqrt{ax}}{a}\,dt = \int \frac{t}{x}\,\frac{2t}{a}\,dt = \frac{2}{xa}\int t^2\,dt = \frac{2t^3}{3xa} = \frac{2}{3}\,\frac{ax\sqrt{ax}}{ax}$

$\int 2 = 2x$

$\int \frac{\sqrt{ax}}{a}\cdot\frac{2\sqrt{ax}}{a}\,dt = \frac{2}{a^2}\int t^2\,dt = \frac{2}{a^2}\,\frac{t^3}{3} = \frac{2}{3}\,\frac{ax\sqrt{ax}}{a^2} = \frac{2}{3}x\sqrt{ax}$

7.21. $\int \frac{x^3}{x} + \frac{2}{x}\, dx = \int x^2 + \frac{2}{x}\, dx = \frac{x^3}{3} + 2\ln|x| + C$

7.22. $\int 2^x e^x\, dx$

$(2^x e^x)' = \ln 2\, 2^x e^x + 2^x e^x = e^x \cdots e^x 2^x (\ln 2 + 1)$

$\int \cancel{t}\ \cancel{1}$ $\quad \frac{1}{\ln 2 + 1} \int e^x \frac{1}{e^x 2^x}\, dt$

$\int (2e)^x\, dx = \frac{(2e)^x}{\ln 2e} = \frac{2^x e^x}{\ln + 1} + C$

7.23 $\int 2^x (1 + 3x^2 \cdot 2^{-x})\, dx = \int \cancel{\frac{1 \cdot 2^x + 2^x}{2^x (1 + 3x^2 2^{-x})}}\, dx = \int (2^x + 3x^2)\, dx =$

$= \frac{2^x}{\ln 2} + \cancel{3}x^3 + C$

7.24. $\int (2x + 3\cos x\, dx = x^2 + 3\sin x + C$

7.25. $\int \frac{2 - \sin x}{\sin^2 x}\, dx = \int \frac{2}{\sin^2 x} - \frac{1}{\sin x}\, dx = -2\,\mathrm{ctg}\, x - \ln\left(\mathrm{tg}\frac{x}{2}\right) + C$

7.26. $\int \frac{3 - 2\,\mathrm{ctg}^2 x}{\cos^2 x} = \int \frac{3}{\cos^2 x} - \frac{2\,\mathrm{ctg}^2 x \cos^2 x}{\sin^2 x \cos^2 x}\, dx = 3\,\mathrm{tg}\, x + 2\,\mathrm{ctg}\, x + C$

7.27. $\int \frac{\cos 2x}{\cos^2 x \sin^2 x}\, dx = \int \frac{\cos^2 x - \sin^2 x}{\cos^2 x \sin^2 x}\, dx = \int \frac{1}{\sin^2 x} - \frac{1}{\cos^2 x}\, dx =$

$= -\mathrm{ctg}\, x - \mathrm{tg}\, x + C$

7.28. $\int \sin^2 \frac{x}{2}\, dx$ $\quad \left(\frac{x}{2}\right)' = \frac{1}{2}$

$2\int \sin^2 t\, dt = \cancel{-2\,\mathrm{ctg}\frac{x}{2}} + C$ $\quad \sin^2 t$ $\quad \frac{\sin t \cdot \sin t}{\cancel{1 - \sin t + \sin t}}$

$= \int -\frac{\cos x + 1}{2}\, dx = \int -\frac{\cos x}{2} + \frac{1}{2}\, dx = -\frac{\sin x}{2} + \frac{1}{2}x + C$

7.29. a) $\int tg^2 x \, dx = \cos^2 \int \frac{\sin^2 x}{\cos^2 x} dx$

24.03.24.

7.44. $\int \sqrt{3+x} \, dx$ $\qquad dx = \frac{1}{t'} dt \qquad t = \sqrt{3+x} \neq \qquad t' = 1$

$\int \sqrt{t} \, dt = \frac{2\sqrt{t^3}}{3} = \frac{2t\sqrt{t}}{3} = \frac{2(3+x)\sqrt{3+x}}{3} + C$

7.45. $\int (3-4\sin x)^{\frac{1}{3}} dx$ $\qquad t = 3 - 4\sin x \neq \qquad t' = -4\cos x$

$\int -\frac{1}{4\cos x} \cos x \, t^{\frac{1}{3}} dx = \frac{3\sqrt[3]{t^4}}{4 \cdot 4\cos x} = \frac{3\sqrt[3]{(3-4\sin x)^4}}{16\cos x} = -\frac{1}{4}\frac{3}{4}(3-4\sin x)^{\frac{3}{4}} + C$

7.84.

7.81. $\int \frac{x-1}{(x+2)^2} dx = \int \frac{x-1}{x^2+4x+4} = \int \frac{1}{x+2} dx - \int \frac{3}{(x+2)^2} dx$

$t = x+2 \qquad t' = 1$

$\frac{1}{t} \int \frac{1}{t} dt - \int \frac{3}{t^2} dt = \ln|t| + \frac{3}{t} = \ln|x+2| + \frac{3}{x+2} + C$

7.82. $\int \frac{x^2}{3+x^2} dx = = \int 1 - \frac{3}{3+x^2}$ $\qquad t = 3+x^2 \qquad t' = 2x$

7.114.

$\int x - \int \frac{3}{t}\frac{1}{2x} dt = x - \frac{1}{2x}3 \ln|t| = x - \frac{3\ln|3+x^2|}{2}$

$\frac{x^2+3}{3+x^2} \qquad \frac{-3}{3+x^2} = 1 - \frac{3}{3+x^2} = \frac{3+x^2-3}{3+x^2}$

$x - 3\int \frac{dx}{x^2+(\sqrt{3})^2} = x - \frac{3}{\sqrt{3}} \operatorname{arctg} \frac{x}{\sqrt{3}} + C$

7.83. $\int \frac{x^2-2x+3}{x^2-4} dx = \int 1dx + \int \frac{3}{4(x-2)} dx - \int \frac{11}{4(x+2)} dx$ $\qquad t = \cancel{x+2} \qquad t' = 1$

$x + \frac{3}{4}\int \frac{dx}{x-2} - \frac{11}{4}\int \frac{1}{x+2} dx = x + \frac{3}{4}\ln|x-2| - \frac{11}{4}\ln|x+2| + C$

$t = x^2 - 4 = (x+2)(x-2) \Rightarrow x+2 = \frac{t}{x-2}$

$t' = 2x$

$\frac{3}{4} \int \frac{dx}{t} \frac{x-2}{2x} =$

$= \frac{3}{4} \frac{x-2}{2x} \int \ln|t| =$

$= x + \frac{3}{4} \ln|x-2| - \frac{11}{4} \ln|x+2| + C$

$= \frac{3}{4} \frac{x-2}{2x} \ln|x^2-4|$

$(\ln x)^{\frac{3}{4}} + C$

$= 1$

7.84. $\int \frac{x\,dx}{a^2x^4 - b^2} = \int x(a^2x^3 - b^2) = \int x + \frac{(a^2x^4 - b^2) - (a^2x^4 - b^2)}{a^2x^4 - b^2}\,dx =$

$\int$

$x \cdot c\,x$

$a^2x^4 - b^2 = (ax^2 - b)(ax^2 + b)$

$x^4 = t$

$x =$

$t = x^2 \quad t' = 2x$

$x = \sqrt{t}$

$ax^2 = t$

$t' = 2ax$

$\int \frac{x}{2b(ax^2 - b)} \frac{1}{2x}\,dt$

$\int \frac{x}{(t-b)(t+b)} \frac{1}{2ax}\,dt = \frac{1}{2a} \int \frac{dt}{t^2 - b^2} =$

$\int$

$= \frac{1}{2a} \frac{1}{2b} \ln\left|\frac{t-b}{t+b}\right| = \frac{1}{4ab} \ln\left|\frac{ax^2 - b}{ax^2 + b}\right| + C$

$x^3 = 1 - t^2$

$t^2 = 1 - x^3$

$t = \sqrt{1-x^3} \quad x = \sqrt[3]{(1-t^2)}$

$t = 1 - x^3 \quad x = \sqrt[3]{t-1}$

7.114. $\int \frac{dx}{x\sqrt{1-x^3}}$

$t' = -3x^2$

$dx = -\frac{1 \cdot 2t}{3\sqrt[3]{(1-t^2)^2}}\,dt$

$\left((1-t^2)^{\frac{1}{3}}\right)' = -\frac{2}{3} \frac{2t}{\sqrt[3]{(1-t^2)^2}} = -\frac{4t}{3\sqrt[3]{(1-t^2)^2}}$

$+\int \frac{dt}{x\,2x\sqrt{3}}$

$\int \frac{dx}{x\sqrt{t}} = \int \frac{dt}{x\sqrt{t}\,(-3x^2)} =$

$\int \sqrt[3]{1-t}$

$= \int \frac{dt}{x(\sqrt{t} - 3x)} = \frac{1}{x} \int \frac{dt}{\sqrt{t} - 3x}$

$\int \frac{2t}{\sqrt[3]{(1-t^2)^2}\,\sqrt[3]{(1-t^2)}\,\sqrt{1-1+t^2}}\,dt = \frac{4}{3} \int \frac{dt}{1-t^2} = \frac{2}{3} \ln\left|\frac{1+t}{1-t}\right| =$

$= \frac{1}{3} \ln\left|\frac{\sqrt{1-x^3} - 1}{\sqrt{1-x^3} + 1}\right| + C$

7.115. $\int \frac{dx}{x\sqrt{4-x^2}}$ $\frac{1}{2}$ ^{2} $x = \frac{2}{t}$ $dx = -\frac{2}{t^2} dt$

$-\int \frac{2}{t^2 \frac{2}{t} \sqrt{4-\frac{4}{t^2}}} = \int \frac{2}{t} \frac{dt}{\sqrt{(2-\frac{2}{t})(2+\frac{2}{t})}}$

$t = \sqrt{4-x^2}$ $t' = \frac{-2x}{2\sqrt{4-x^2}} = -\frac{x}{\sqrt{4-x^2}}$ $=$

$\int \frac{dt\sqrt{4-x^2}}{x\ t\ x}$

$x = \frac{2}{t}$ $dx = -\frac{2}{t^2} dt$ $t = \frac{2}{x}$

$\frac{1}{2} \int \frac{t\ dt\ t^2}{} \frac{-2}{t^2\sqrt{4-x^2}} = \frac{1}{2}\left(\int -\frac{2}{t} dt - \int \frac{2}{\sqrt{4-x^2}} dt\right) =$

$= \frac{1}{2}\left(-2 \ln|t| - \frac{2t}{\sqrt{4-x^2}}\right)$

$= \int \frac{1}{x\sqrt{4-x^2}} \cdot \frac{1}{\frac{x}{\sqrt{4-x^2}}} dt = -\int \frac{1}{x^2} dt = \int \frac{1}{-x^2+4-4} dt = \int \frac{1}{(\sqrt{4-x^2})^2 - 4} dt =$

$= \int \frac{1}{t^2-4} dt = \frac{1}{2 \cdot 2} \ln\left|\frac{t-2}{t+2}\right| = \frac{1}{4} \ln\left|\frac{\sqrt{4-x^2}-2}{\sqrt{4-x^2}+2}\right| + C$

7.124 $\int \arccos x\ dx = x \arccos x + \int \frac{1}{\sqrt{1-x^2}} du \frac{x}{\sqrt{1-x^2}} dx =$

$u = \arccos x$ $dv = dx$

$du = -\frac{1}{\sqrt{1-x^2}} dx$ $v = \int dx = x$

$=$

$t = 1-x^2 \Rightarrow x = \sqrt{1-t}$

$t' = -2x$

$\int \frac{x}{t} \frac{dt}{-2x} = -\frac{1}{2}\int \frac{1}{t} dt = -\frac{1}{2} \ln|t| = -\frac{1}{2} \ln|1$

7.125.

7.126

7.127.

$$\int \frac{\cancel{x}}{\sqrt{t}\,\cdot(-2\cancel{x})}\,dt = -\frac{1}{2}\int \frac{dt}{\sqrt{t}} = -\frac{1}{2}\,\sqrt{t}\cdot 2 = -\sqrt{t} = -\sqrt{1-x^2}$$

$$\int \arccos x\,dx = x\arccos x - \sqrt{1-x^2} + C$$

7.125. $\int x\cos x\,dx$

$u = x \qquad dv = \cos x\,dx$

$du = dx \qquad v = \int \cos x\,dx = \sin x$

$$\int x\cos x\,dx = x\sin x - \int \sin x\,dx = x\sin x + \cos x + C$$

7.126. $\int x\ln x\,dx = \ln x\cdot\frac{x^2}{2} - \int \frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx = \frac{x^2\ln x}{2} - \frac{x^2}{4}\cancel{d} + C$

$u = \ln x \qquad dv = x\,dx$

$du = \frac{1}{x}\,dx \qquad v = \int x\,dx = \frac{x^2}{2}$

7.127. $\int \frac{\ln x}{\sqrt[3]{x}}\,dx = \int \ln x\cdot\frac{1}{\sqrt[3]{x}}\,dx =$

$u = \ln x \qquad dv = \frac{dx}{\sqrt[3]{x}}$

$du = \frac{1}{x}\,dx \qquad v = \int x^{-\frac{1}{3}}\,dx = \frac{3x^{\frac{2}{3}}}{2}$

$$= \frac{\ln x\cdot 3\sqrt[3]{x^2}}{2} - \int \frac{3}{2}x^{\frac{2}{3}}\frac{1}{x}\,dx = \frac{3}{2}\int x^{-\frac{1}{3}}\,dx = \frac{3x^{\frac{2}{3}}}{2}\cdot\frac{3}{2}$$

$$= \frac{3\sqrt[3]{x^2}}{2}\ln x - \frac{9}{4}\sqrt[3]{x^2} + C$$