Пересчет всех подмножеств множеств и n элем.

- Фактор. ф. $f(n) = !n$

Опред. все перестановки элем.

$n! \gg 2^n \gg n^3 \gg n^2 \gg n\log n \gg n \gg \log n \gg 1$

01.02.24

5.231) $\lim\limits_{n\to\infty} \frac{n-1}{3n} = \frac{1-\frac{1}{n}}{3} = \frac{1}{3}$

5.232) $\lim\limits_{n\to\infty} \frac{5n+1}{7-9n} = \frac{5+\frac{1}{n}}{\frac{7}{n}-9} = -\frac{5}{9}$

5.233.) $\lim\limits_{n\to\infty} \frac{(n+1)^2}{2n^3} = \lim\limits_{n\to\infty} \frac{n^2+2n+1}{2n^3} = \frac{\frac{1}{n}+\frac{2}{n^2}+\frac{1}{n^3}}{2} = 0$

5.234) $\lim\limits_{n\to\infty} \frac{3n^2-7n+1}{2-5n-6n^2} = \frac{3-\frac{7}{n}+\frac{1}{n^2}}{\frac{2}{n^2}-\frac{5}{n}-6} = -\frac{1}{2}$

5.235) $\lim\limits_{n\to\infty} \frac{(n+2)^3-(n-2)^3}{95n^3+39n} = \frac{(n+2)^2 - n^3 + 6n^2 - 8n + 8}{95n^3+39n} = 0$

5.236) $\lim\limits_{n\to\infty} \left(\frac{2n-1}{5n+7} - \frac{1+2n^3}{2+5n^3}\right) = \frac{(2n-1)(2+5n^3)-(1+2n^3)(5n+7)}{(5n+7)(2+5n^3)} =$

$= \frac{4n + 10n^4 - 2 - 5n^3 - 5n - 7 - 10n^4 - 14n^3}{\infty} = 0$

5.237.) $\lim\limits_{n\to\infty} \frac{\sqrt[3]{n^4+3n+1}}{n-1} = \frac{(n^4+3n+1)^{\frac{1}{3}}}{n-1} = \frac{\sqrt[3]{n^3\left(n+\frac{3}{n^2}+\frac{1}{n^3}\right)}}{n\left(1-\frac{1}{n}\right)} = \frac{\sqrt[3]{n+\frac{3}{n^2}+\frac{1}{n^3}}}{1-\frac{1}{n}} =$

$= \frac{\infty}{1} = +\infty$

5.238) $\lim\limits_{n\to\infty} (\sqrt{n+2} - \sqrt{n}) = \cancel{\sqrt{n^2\left(\frac{1}{n}+\frac{2}{n^2}\right)} - \sqrt{n^2\cdot\frac{1}{n}} = n\cdot\sqrt{\frac{1}{n}+\frac{2}{n^2}} - n\sqrt{\frac{1}{n}} =}$

$\cancel{= n\left(\sqrt{\frac{1}{n}+\frac{2}{n^2}} - \sqrt{\frac{1}{n}}\right) =}$

$= \frac{(\sqrt{n+2}-\sqrt{n})(\sqrt{n+2}+\sqrt{n})}{\sqrt{n+2}+\sqrt{n}} = \frac{\cancel{n}+2-\cancel{n}}{\sqrt{n+2}+\sqrt{n}} = \frac{2}{\underbrace{\sqrt{n+2}+\sqrt{n}}_{\to\infty}} = 0$

5.239) $\lim\limits_{n\to\infty} n^{3/2}(\sqrt{n^3+1} - \sqrt{n^3-2})$ $\cancel{= n^{\frac{3}{2}}\left(\frac{\cancel{n^3}+1-\cancel{n^3}+2}{\sqrt{n^3+1}+\sqrt{n^3-2}}\right) = n^{\frac{3}{2}}\left(\frac{3}{\sqrt{n^3+1}+\sqrt{n^3-2}}\right) =}$

$\cancel{= \frac{3n^{\frac{3}{2}}}{n\left(\sqrt{n+\frac{1}{n^2}}+\sqrt{n-\frac{2}{n^2}}\right)} = \frac{3\cdot\sqrt{n}}{\sqrt{n+\frac{1}{n^2}}+\sqrt{n-\frac{2}{n^2}}}}$

$\lim\limits_{n\to\infty} n^{3/2}(\sqrt{n^3+1} - \sqrt{n^3-2}) = \sqrt{n^3}\cdot\sqrt{n^3+1} - \sqrt{n^3-2}\cdot\sqrt{n^3} =$

$= \sqrt{n^6+n^3} - \sqrt{n^6-2n^3} = \frac{\cancel{n^6}+n^3-\cancel{n^6}+2n^3}{\sqrt{n^6+n^3}+\sqrt{n^6-2n^3}} = \frac{3n^3}{\sqrt{n^6+n^3}+\sqrt{n^6-2n^3}} =$

$= \frac{3\cancel{n^3}}{\cancel{n^3}\left(\sqrt{1+\underbrace{\frac{1}{n^3}}_{\to 0}}+\sqrt{1-\underbrace{\frac{2}{n^3}}_{\to 0}}\right)} = \frac{3}{2} = 1{,}5$

5.240) $\lim\limits_{n\to\infty} \frac{2^n+3^n}{2^n-3^n} \; {\scriptstyle |:3^n} = \frac{\frac{2^n}{3^n}+1}{\frac{2^n}{3^n}-1} = \frac{\left(\frac{2}{3}\right)^n+1}{\left(\frac{2}{3}\right)^n-1} = -1$

5.241) $\lim\limits_{n\to\infty} \left(\frac{1}{n^2}+\frac{2}{n^2}+\dots+\frac{n-1}{n^2}\right) = \cancel{\frac{\frac{1}{n^2}}{1-2} = \frac{1}{n^2}\cdot\frac{1}{1}}$

$\cancel{q = \frac{2}{n^2}\cdot\frac{n^2}{1} = 2}$ $= \frac{1}{n^2}(1+2+\dots+n-1)$

$q = 2-1 = 1 \quad \frac{2}{1} = 2 \quad S = \cancel{\frac{1}{1-2}} \quad \frac{1+(n-1)}{2}n =$

$= \frac{n^2-n}{2} = \frac{1-\frac{1}{n}}{\frac{2}{n^2}}$

5.242) $\lim\limits_{n\to\infty} \frac{1^2+2^2+\ldots n^2}{n^3} =$

$S_n = \frac{\overset{n^2+n}{n(n+1)}(n+2)}{6} = \frac{1}{6}\lim\limits_{n\to\infty} \frac{n^3+2n^2+n^2+2n}{n^3} = \frac{1+\frac{3}{n}+\frac{2}{n^2}}{1}$

$\underline{\underline{1^2+2^2+3^2+\ldots n^2}} = \boxed{S_n = \frac{\overset{n^2+n}{n(n+1)}(2n+1)}{6}} = \frac{1}{6}\lim \frac{2n^3+n^2+2n^2+n}{n^3} = \frac{1}{6}\cdot 2 = \frac{1}{3}$

5.243) $\lim\limits_{n\to\infty} \frac{\sqrt[3]{n^2}\cdot\sin(n^2)}{n^1-1} = \frac{\sqrt[3]{n^2}\cdot\sin(n^2)}{n^2\left(\frac{1}{n}-\frac{1}{n^2}\right)} =$ т.к. $\frac{\sin a}{a} = 1$

$= \frac{n^{\frac{2}{3}}}{\frac{1}{n}-\frac{1}{n^2}} = \frac{n^{\frac{2}{3}}}{\frac{1}{n}\left(1-\frac{1}{n}\right)} = \frac{n^{\frac{2}{3}}}{n^{-1}\left(1-\frac{1}{n}\right)} = \frac{n^{\frac{5}{3}}}{1-\frac{1}{n}} = 0$

5.244) $\lim\limits_{n\to\infty}\left(\frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\ldots+\frac{1}{n(n+1)}\right) = \left(1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\ldots\frac{1}{n}-\left(\frac{1}{n+1}\right)\right) =$

$= \lim\limits_{n\to\infty}\left(1-\frac{1}{n+1}\right) = 1$

5.245) $\lim\limits_{n\to\infty}\left(\frac{1}{5}-\frac{1}{25}+\ldots+(-1)^{n-1}\frac{1}{5^n}\right) = \frac{-1^n}{-1}\frac{1}{5^n} = -\frac{1^n}{5^n} = -\frac{\frac{1^n}{5^n}}{1} = 0$

$q = \frac{1}{25} \cdot 5 = -\frac{1}{5}$ $\quad S = \frac{0+1\frac{1}{5}}{1\frac{1}{5}-1} = \frac{\frac{1}{5}}{\frac{6}{5}} = \frac{1}{5}\cdot\frac{5}{6} = \frac{1}{6}$