$$= \int \sin 2x \cdot x \, dx$$

$u_1 = x \qquad dv_1 = \sin 2x \, dx$

$du_1 = 1$

$$v_1 = \int \sin 2x \, dx = -\frac{\cos 2x}{2}$$

$$= -\frac{x \cdot \cos 2x}{2} + \int -\frac{\cos 2x}{2} \, dx = -\left(\frac{x \cos 2x}{2} + \frac{\sin 2x}{4} \right) \Bigg|_0^{\pi/4} =$$

$$= -\left(0 + \frac{1}{4} \right) = -\frac{1}{4}$$

$$\int_0^{\pi/4} x^2 \cos 2x \, dx = \frac{\pi^2}{32} \, 1 - \frac{1}{4} = \frac{\pi^2 - 8}{32}$$

7.407 $\int_0^{\pi/4} x^2 \cos 2x \, dx$

$u = x^2$ $\quad dv = \cos 2x \, dx$

$du = 2x \, dx$ $\quad v = \int \cos 2x \, dx = \int (\cos^2 x - \sin^2 x) \, dx =$

$= \int 2\cos^2 x - 1 \, dx = 2\int \cos^2 x \, dx - x$

$\cos^2 x$

$\cos x = t$

$t' = -\sin x$

$\int \cos^2 x \, dx = \int \frac{1 + \cos 2x}{2} = \int \frac{1}{2} dx + \int \cos 2x \, dx = \frac{x}{2} +$

$2x = t$

$t' = 2$

$\int \frac{\cos 2x}{2} dx = \int \frac{\cos t}{4} dt = \frac{\sin t}{4} = \frac{\sin 2x}{4}$

$\left(2\left(\frac{x}{2} + \frac{\sin 2x}{4}\right) - x\right)\Big|_0^{\pi/4} =$ $\quad \int \cos 2x \, dx$

$\left(\frac{\pi}{4}\right) = 2\left(\frac{\pi}{8} + \frac{1}{4}\right) - \frac{\pi}{4} = \frac{\pi}{4} + \frac{1}{2} - \frac{\pi}{4} =$

$dv = \cos 2x \, dx$

$v = \int \cos 2x \, dx = \int \frac{\cos t}{2} dt = \frac{\sin t}{2} = \frac{\sin 2x}{2}$

$t = 2x$

$t' = 2$

$uv = x^2 \cdot \frac{\sin 2x}{2}\Big|_0^{\pi/4} =$

$\int = \frac{\pi^2}{16} \cdot \frac{1}{2} = \frac{\pi^2}{32}$

$\int v\,du = \int \frac{\sin 2x}{2} 2x \, dx =$

=

28.03.

7.411

$t = \frac{\sin x}{\cos x} \Rightarrow \cos x = \frac{\sin x}{t} \quad \int t \quad \cos^2 x \; dt =$

$= \int t \; \frac{\sin^2 x}{t^2} \; dt$

$\int tg\, x = \int \frac{\sin x}{\cos x} \; dx =$

$t = \cos x \qquad t' = -\sin x$

$\int \sin x \; \frac{1}{-\sin x \; t} \; dt = -\int \frac{1}{t} \, dt = -\ln|t| = -\ln|\cos x|$

$x\, tg\, x \Big|_{\pi/6}^{\pi/3} = \left(\frac{\pi}{3}\sqrt{3} - \frac{\pi}{6} \cdot \frac{\sqrt{3}}{3}\right) = \frac{\pi 5\sqrt{3}}{18}$

$\ln|\cos x| \Big|_{\pi/6}^{\pi/3} = \ln\frac{1}{2} - \ln\frac{\sqrt{3}}{2} = \ln \frac{1 \cdot \cancel{2}}{\cancel{2}\sqrt{3}} = \ln\frac{1}{\sqrt{3}}$

$= \frac{\pi 5\sqrt{3}}{18} + \ln\frac{1}{\sqrt{3}}$

$\Big|_{1}^{3} =$

$=$

$e^0 = 1$

7.391 $\int_1^5 \frac{dx}{x+\sqrt{2x-1}}$

$t = \sqrt{2x-1}$

$t^2 = 2x-1$

$x = \frac{t^2+1}{2}$

$dx = \frac{2t}{2} = t\,dt$

$t_1 = 3 \quad t_2 = 1$

$$\int_1^3 \frac{t\,dt}{\frac{t^2+1}{2}+t} = \int \frac{2t\,dt}{t^2+2t+1} = 2\int \frac{t}{(t+1)^2}dt = 2(\cancel{t - \ln|t+1|})\Big|_1^3 =$$

$$= \cancel{2(3-\ln 4 - 1 + \ln 2} = \cancel{2(2+\ln\frac{1}{2})} = 2\left(\ln|t+1| + \frac{1}{t+1}\right)\Big|_1^3 =$$

$$= 2\left(\ln 4 + \frac{1}{4} - \ln 2 - \frac{1}{2}\right) = 2\left(\ln 2 - \frac{1}{4}\right)$$

7.399 $\int_0^1 x e^x dx$

$u = x \qquad dv = e^x dx$

$du = dx \qquad v = \int e^x dx = e^x$

$$\int_0^1 x e^x dx = xe^x\Big|_0^1 - \int_0^1 e^x dx = (e-0) - e^x\Big|_0^1 = e - e + e^0 = 1$$

7.401 $\int_{\pi/6}^{\pi/3} \frac{x\,dx}{\cos^2 x} = \int x \frac{1}{\cos^2 x} dx = x\,tg\,x\Big|_{\pi/6}^{\pi/3} - \int_{\pi/6}^{\pi/3} tg\,x\,dx$

$u = x \qquad dv = \frac{1}{\cos^2 x} dx \qquad t = tg\,x$

$du = dx \qquad t' = \frac{1}{\cos^2 x}$

$v = \int \frac{1}{\cos^2 x} dx = tg\,x$

$$\int tg\,x\,dx = \int t \cos^2 x\,dt = \cos^2 x \frac{t^2}{2} = \frac{\cos^2 x\,tg^2 x}{2} = \frac{\sin^2 x \cos^2 x}{\cos^2 x \cdot 2} =$$

$$= \frac{\sin^2 x}{2}$$

7.379. $\varphi(x) = \int_0 \frac{\sin t}{t} dt = \int_0 \sin t \cdot \frac{1}{t} dt = \int_0^1 \left(\frac{\cos t}{t} - \frac{\sin t}{t^2}\right) dt =$

$=$

7.375. $\int_{\frac{1}{x}}^{\sqrt{x}} \sin t^2 \, dt = \sin t^2 \left(+\frac{\sin x}{2\sqrt{x}} + \sin\left(\frac{1}{x^2}\right) \cdot \frac{1}{x^2} \cdot \sin\left(\frac{1}{x^2}\right)\right)$

7.376. $\varphi(x) = \int_x^0 \frac{dt}{\sqrt{1+t^3}} = -\frac{1}{\sqrt{1+x^3}}$

7.380. $\int_1^6 \frac{dx}{1+\sqrt{3x-2}}$

$t^2 = 3x-2$

$t = \sqrt{3x-2}$ $\quad t_1 = \sqrt{16} = 4$ $\quad t_2 = \sqrt{1} = 1$

$dt = -\frac{3}{2\sqrt{3x-2}}$

$\int_1^6 \frac{dx}{1+t^2} =$

$x = \frac{t^2+2}{3}$

$dx = \frac{2t}{3} dt$

$(16-1) =$

$= \int_1 \frac{dt}{1+t} \cancel{\cdot -\frac{3}{2\sqrt{3x-2}}} = \int_1^6 \frac{dt}{1+t} \cdot \frac{(-3)}{2t} = -\frac{3}{2}\left(\int \frac{1}{t} dt + \int \frac{dt}{1+t}\right.$

$= -\frac{3}{2} \cancel{\left(\ln|t| + \operatorname{arctg} t\right.}$

$\int_1^6 \frac{dt}{1+t} \frac{2t}{3} = \frac{2}{3}\left(\int_1^6 \frac{t+1-1 \, dt}{1+t}\right) = \frac{2}{3}\left(\int dt + \int \frac{dt}{1+t}\right) =$

$= \frac{2}{3}\left(t - \ln|t+1|\right)\Big|_1^4 = \frac{2}{3}\left(4 - \ln 5 - 1 + \ln 2\right) =$

$= \frac{2}{3}\left(3 + \ln\frac{2}{5}\right)$

27.02 Определен интегр.

$$\int_1^3 x^2 dx$$

7.324 $\int_{-1}^{2} x^3 dx = \frac{x^4}{4}\Big|_{-1}^{2} = \frac{2^4}{4} - \frac{(-1)^4}{4} = 4 - \frac{1}{4} = 3,75$

7.325 $\int_1^8 \frac{dx}{\sqrt[3]{x}} = 3\frac{\sqrt[3]{x^2}}{2}\Big|_1^8 = \frac{3}{2}\left(\frac{\sqrt[3]{64}}{x} - \sqrt[3]{1}\right) = \frac{9}{2}$

7.330 $\int_{\frac{\pi}{2}}^{\pi} \sin x\, dx = -\cos x\Big|_{\frac{\pi}{2}}^{\pi} = -(-1 - 0) = 1$

7.329 $\int_2^9 \sqrt[3]{x-1}\, dx = \int_2^9 \sqrt[3]{t}\, dt = 3\frac{\sqrt[3]{t^3}}{4} = \frac{3}{4}\left(\sqrt[3]{(x-1)^4} - \sqrt[3]{(x-1)^4}\right)\Big|_2^9 = \frac{3}{4}(16 - 1) =$

$= \frac{3 \cdot 15}{4} = \frac{45}{4}$

$dx = \frac{1}{t'}dt \quad t = x - 1 \quad t' = 1$

7.331 $\int_{-\frac{\pi}{4}}^{0} \frac{dx}{\cos^2 x} = tg\,x\Big|_{-\frac{\pi}{4}}^{0} = (0+1) = 1$

7.334 $\int_2^5 \frac{dx}{x} = \ln|x|\Big|_2^5 = \ln|5| - \ln|2| = \ln\left|\frac{5}{2}\right| = \ln|2,5|$

7.342 $\int_3^4 \frac{x^2+3}{x-2}dx = \int x + 2 + \frac{7}{x-2} = \left(\frac{x^2}{2} + 2x + 7\ln|x-2|\right)\Big|_3^4 =$

$t = x-2 \quad t' = 1$ $\quad = (8 + 8 + 7\ln 2) - (4,5 + 6 + 7\ln|x-2|) =$

$= 16 - 10,5 + (7\ln 2 - 7\ln) = 5,5 + 7(\ln 2$