1) $\lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{3\operatorname{arctg} x} = \lim\limits_{x\to 0} \dfrac{\sqrt{4+x}-2}{3\cdot\dfrac{\sin x}{\cos x}} =$

$= \lim\limits_{x\to 0} \dfrac{(\sqrt{4+x}-2)(\sqrt{4+x}+2)}{3\cdot\dfrac{\sin x}{\cos x}\cdot(\sqrt{4+x}+2)} =$

$= \lim\limits_{x\to 0} \dfrac{4+x-4}{3\dfrac{\sin x}{\cos x}(\sqrt{4+x}+2)} =$

учтем, что $\operatorname{arctg} x = x \Rightarrow$

$\Rightarrow \lim\limits_{x\to 0} \dfrac{4+x-4}{3x(\sqrt{4+x}+2)} =$

$= \dfrac{1}{3\cdot\sqrt{4}+2} = \dfrac{1}{12}$

2) $\cancel{\lim_{x\to 4} \frac{2^x-16 = \sin}{\sin \pi x + \sin \pi x - \sin 3\pi x}}$

$=$

2) $\lim_{x\to 4} \frac{2^x - 16}{\sin \pi x} = \lim_{x\to 4} \frac{\ln(2)\cdot 2^x}{\cos(x(\pi))\cdot \pi} =$

$= \frac{\ln(2)\, 2^4}{\cos 4\pi \cdot \pi} = \frac{\ln(2)\cdot 16}{\pi}$

3) $\lim_{x\to \pi} \frac{\cos\frac{x}{2}}{e^{\sin x} - e^{\sin 4x}} =$

$= \lim_{x\to \pi} \frac{-\sin\left(\frac{x}{2}\right)\cdot\frac{1}{2}}{\underset{1}{e^{\sin x \to 0}}\cdot\cos x - e^{\sin 4x}\cos(4x)\,4} =$

$= \frac{-1\cdot\frac{1}{2}}{-1-1\cdot 4} = \frac{1}{10}$

4) $$\lim_{x \to 0} \frac{e^{2x} - e^{x}}{\sin 3x - \sin 5x} =$$

$$= \lim_{x \to 0} \frac{e^{2x} \cdot 2 - e^{x}}{\cos 3x \cdot 3 - \cos 5x \cdot 5} =$$

$$= \frac{2 - 1}{1 \cdot 3 - 5} = -\frac{1}{2}$$

5) $$\lim_{x \to 1} \frac{1 - x}{\log_2 x} =$$

$$= \lim_{x \to 1} \frac{-1}{\frac{1}{x \cdot \ln 2}} = \lim_{x \to 1} -x \ln 2 = -\ln 2$$

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б) $\lim\limits_{x\to 0} \cos x^{\frac{1}{\ln(1+\sin^2 x)}} =$

Исходя из ф-лы $u(x)^{v(x)} = e^{\lim\limits_{x\to a}[(u(x)-1)\cdot v(x)]}$

$$= e^{\lim\limits_{x\to 0}\frac{\cos x - 1}{\ln(1+\sin^2 x)}} = e^{\lim\limits_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{\underbrace{\frac{\ln(1+\sin^2 x)}{\sin^2 x}}_{=1}\cdot\sin^2 x}} =$$

$$= e^{\lim\limits_{x\to 0}\frac{-2\sin^2\frac{x}{2}}{\sin^2 x}} =$$ т.к. $x \to 0$, то $\sin\frac{x}{2} = \frac{x}{2} \Rightarrow$

$$\Rightarrow e^{\lim\limits_{x\to 0}\frac{-2\left(\frac{x}{2}\right)^2}{x^2}} = e^{\frac{-2x^2}{4x^2}} = e^{-\frac{1}{2}} = \frac{1}{\sqrt{e}}$$

7) $$\lim_{x \to 0} \left[\sin x + 2\right]^{\frac{3}{3+x}} =$$

$$= \sin 2^{\frac{3}{3}} = \sin 2$$

8) $\lim\limits_{x\to\frac{\pi}{2}} (\sin x)^{6tg x \cdot tg 3x} =$

$= e^{\lim\limits_{x\to\frac{\pi}{2}} (\sin x - 1)\cdot(6tg x \cdot tg 3x)} =$

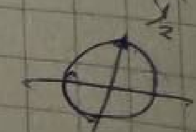
$= e^{\lim\limits_{x\to\frac{\pi}{2}} (\sin x - 1)\cdot \frac{6\sin x \sin 3x}{\cos x \cos 3x}} =$

$= e^{\lim\limits_{x\to\frac{\pi}{2}} (\sin x - 1)\cdot\frac{6\sin x\cdot\sin 3x}{\cos x(4\cos^3 x - 3\cos x)}} =$

$= e^{\lim\limits_{x\to\frac{\pi}{2}} \frac{(\sin x - 1)6\sin x\cdot \sin 3x}{\cos^2 x(4\cos^2 x - 3)}} =$

$= e^{\lim\limits_{x\to\frac{\pi}{2}} \frac{(\sin x - 1)6\sin x\sin 3x}{(1-\sin^2 x)(4\cos^2 x - 3)}} =$

$= e^{\lim\limits_{x\to\frac{\pi}{2}} \frac{(\sin x - 1)6\sin x\sin 3x}{(1-\sin x)(1+\sin x)(4\cos^2 x - 3)}} =$



$= e^{\lim\limits_{x\to\frac{\pi}{2}} \frac{-\cancel{(1-\sin x)}6\sin x\sin 3x}{\cancel{(1-\sin x)}(1+\sin x)(4\cos^2 x - 3)}} =$

$= e^{\frac{-1\cdot 6\cdot 1\cdot(-1)}{(1+1)(-3)}} = e^{-\frac{6}{2\cdot 3}} = e^{-1} = \frac{1}{e}$

9)

$$\lim_{x\to 2}\left(\frac{\sqrt{x+2}-2}{x^2-4}\right)^{\frac{1}{x}} =$$

$$= \lim_{x\to 2}\left(\frac{x+2-4}{(x^2-4)(\sqrt{x+2}+2)}\right)^{\frac{1}{x}} =$$

$$= \lim_{x\to 2}\left(\frac{\cancel{(x-2)}\,1}{(x+2)(\sqrt{x+2}+2)}\right)^{\frac{1}{x}} =$$

$$= \left(\frac{1}{4\cdot 4}\right)^{\frac{1}{2}} = \frac{1}{4}$$

10) $$\lim_{x\to 0}\frac{\sqrt[3]{\underbrace{tg\,x}_{\to 0}}\;arctg\,\frac{1}{x}+3}{2-tg(1+\sin x)} =$$

$$= \frac{3}{2-tg\,1}$$