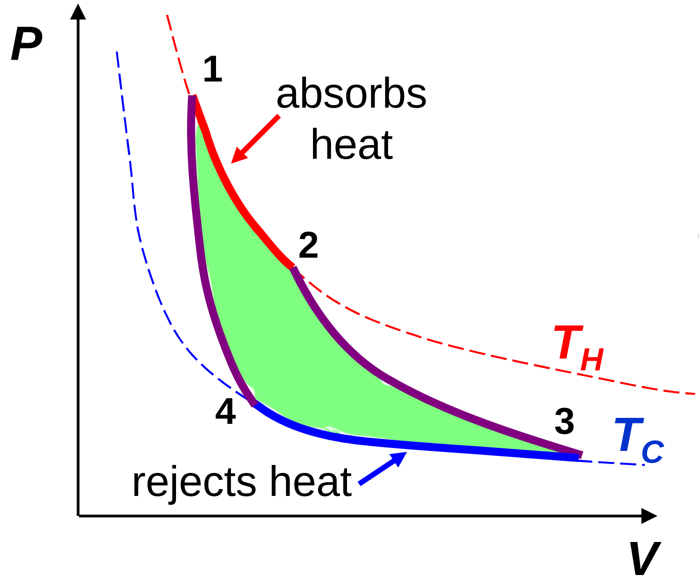
Lecture 11. Real Heat Engines and refrigerators (Ch. 4)

- Stirling heat engine
- Internal combustion engine (Otto cycle)
- Diesel engine
- Steam engine (Rankine cycle)
- Kitchen Refrigerator

Carnot Cycle

- is not very practical (too slow), but operates at the ***maximum*** efficiency allowed by the Second Law.



1 – 2 isothermal expansion (in contact with T_H)

2 – 3 isentropic expansion to T_C

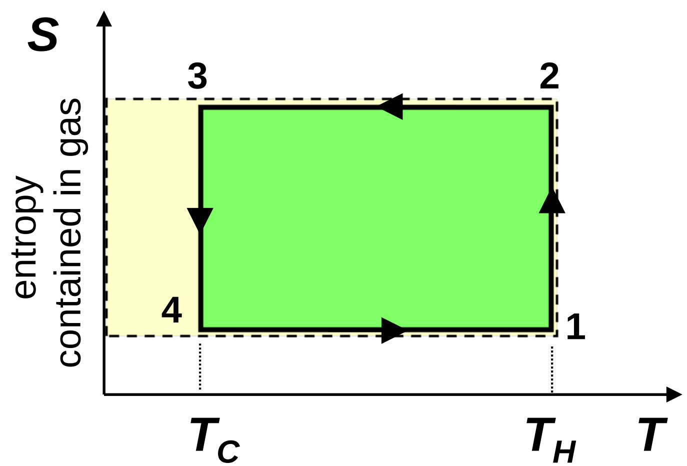
3 – 4 isothermal compression (in contact with T_C)

4 – 1 isentropic compression to T_H

(isentropic ≡ adiabatic+quasistatic)

Efficiency of Carnot cycle for an ideal gas: (*Pr*. 4.5)

$$\boxed{e_{\max} = 1 - \frac{T_C}{T_H}}$$



On the ***S -T*** diagram, the work done is the area of the loop:

$$\oint dU = 0 = \oint TdS - \oint PdV$$

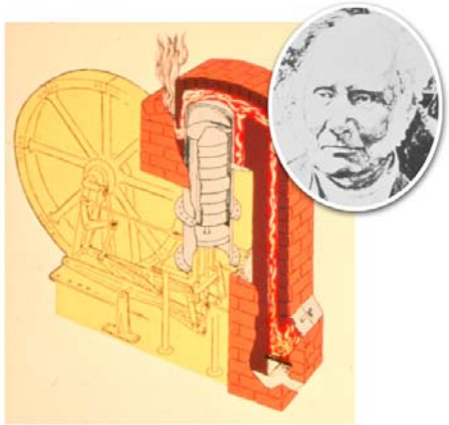
The heat consumed at T_H (**1 – 2**) is the area surrounded by the broken line:

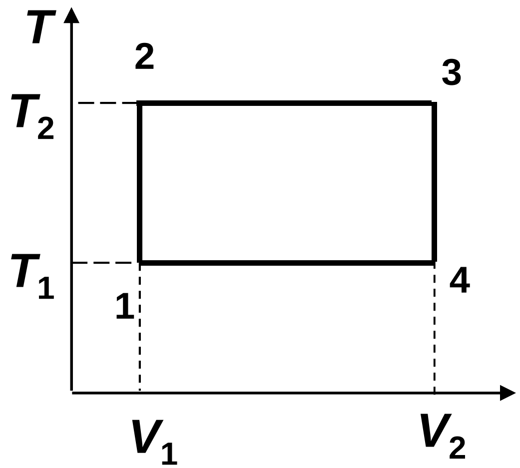
$$Q_H = T_H\left(S_H - S_C\right)$$

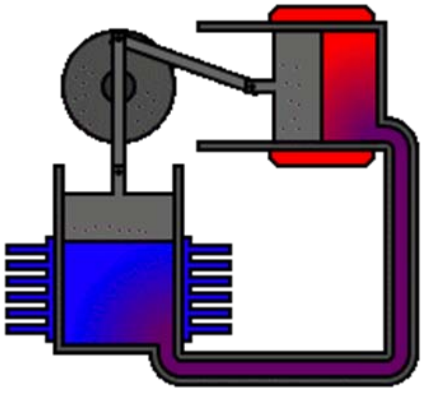
S - entropy contained in gas

Stirling heat engine

Stirling engine – a simple, practical heat engine using a gas as working substance. It’s more practical than Carnot, though its efficiency is pretty close to the Carnot maximum efficiency. The Stirling engine contains a fixed amount of gas which is transferred back and forth between a "cold" and and a "hot" end of a long cylinder. The "displacer piston" moves the gas between the two ends and the "power piston" changes the internal volume as the gas expands and contracts.







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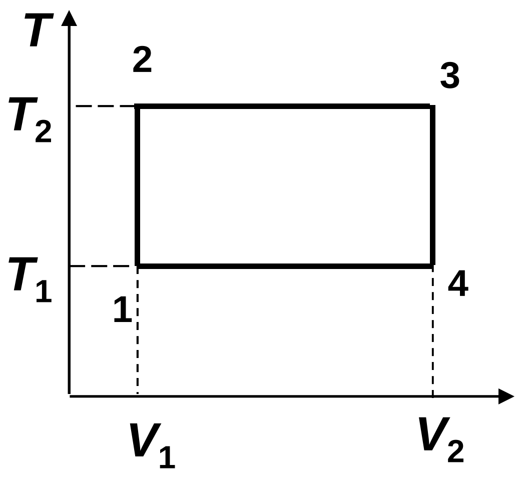
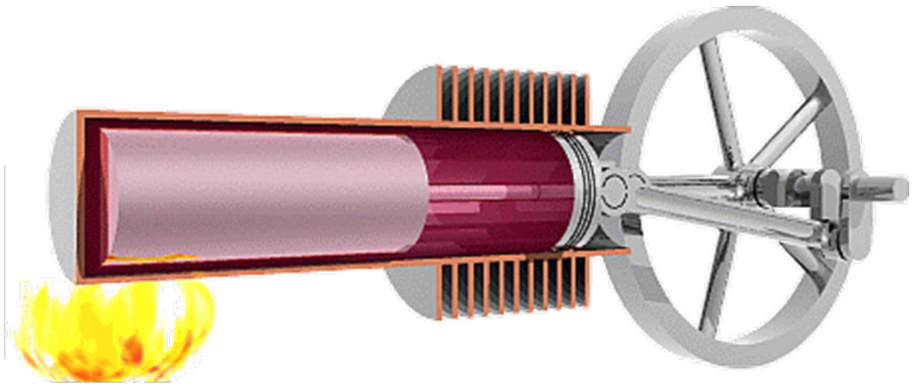
Stirling heat engine

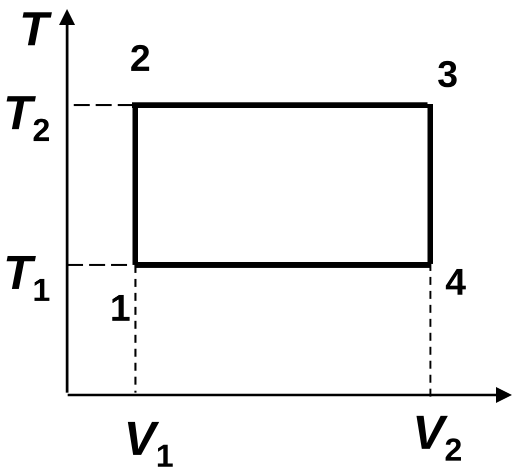
The gases used inside a Stirling engine never leave the engine. There are no exhaust valves that vent high-pressure gasses, as in a gasoline or diesel engine, and there are no explosions taking place. Because of this, Stirling engines are very quiet. The Stirling cycle uses an external heat source, which could be anything from gasoline to solar energy to the heat produced by decaying plants. Today, Stirling engines are used in some very specialized applications, like in submarines or auxiliary power generators, where quiet operation is important.





Efficiency of Stirling Engine

In the Stirling heat engine, a working substance, which may be assumed to be an ideal monatomic gas, at initial volume $\boldsymbol{V_1}$ and temperature $\boldsymbol{T_1}$ takes in “heat” at constant volume until its temperature is $\boldsymbol{T_2}$, and then expands isothermally until its volume is $\boldsymbol{V_2}$. It gives out “heat” at constant volume until its temperature is again $\boldsymbol{T_1}$ and then returns to its initial state by isothermal contraction. Calculate the efficiency and compare with a Carnot engine operating between the same two temperatures.



1-2 $$Q_{12} = C_V(T_2 - T_1) = \frac{3}{2} nR(T_2 - T_1) > 0 \qquad W_{12} = 0$$

2-3 $$Q_{23} = \int_{V_1}^{V_2} P\,dV = nRT_2 \int_{V_1}^{V_2} \frac{dV}{V} = nRT_2 \ln\left(\frac{V_2}{V_1}\right) > 0 \qquad Q_{23} = -W_{23}$$

3-4 $$Q_{34} = C_V(T_1 - T_2) = \frac{3}{2} nR(T_1 - T_2) < 0 \qquad W_{34} = 0$$

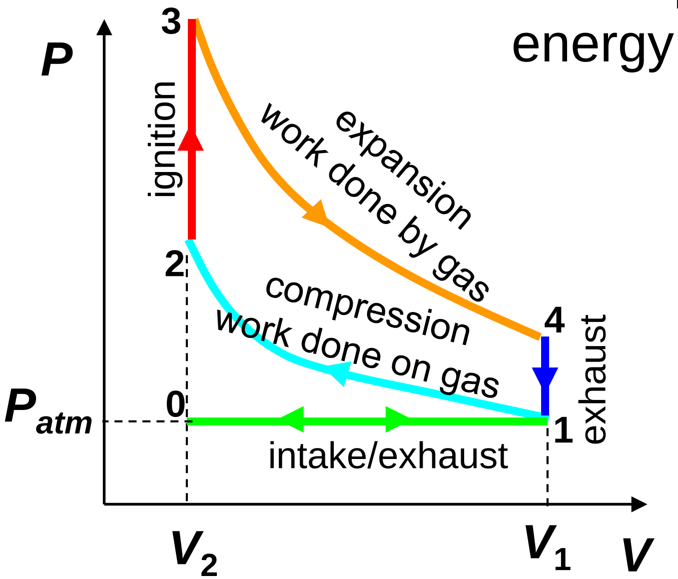
4-1 $$Q_{41} = \int_{V_2}^{V_1} P\,dV = nRT_1 \int_{V_2}^{V_1} \frac{dV}{V} = nRT_1 \ln\left(\frac{V_1}{V_2}\right) < 0 \qquad Q_{41} = -W_{41}$$

$$\frac{1}{e} = \frac{Q_H}{W} = \frac{Q_{12} + Q_{23}}{Q_{23} - Q_{41}} = \frac{\frac{3}{2}(T_2 - T_1) + T_2 \ln(V_2/V_1)}{(T_2 - T_1)\ln(V_2/V_1)} = \frac{T_2}{T_2 - T_1} + \frac{3}{2\ln(V_2/V_1)} > \frac{1}{e_{\max}}$$

Internal Combustion Engines (Otto cycle)

- engines where the fuel is burned ***inside*** the engine cylinder as opposed to that where the fuel is burned ***outside*** the cylinder (e.g., the Stirling engine). More economical than ideal-gas engines because a small engine can generate a considerable power.

Otto cycle. Working substance – a mixture of air and vaporized gasoline. No hot reservoir – thermal energy is produced by burning fuel.



0 → 1 intake (fuel+air is pulled into the cylinder by the retreating piston)

1 → 2 isentropic compression

2 → 3 isochoric heating

3 → 4 isentropic expansion

4 → 1 → 0 exhaust

Otto cycle (cont.)

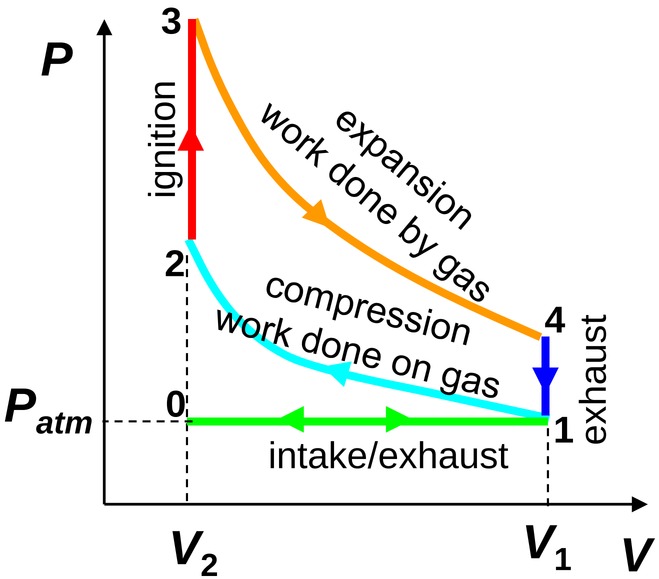
The efficiency: (Pr. 4.18)

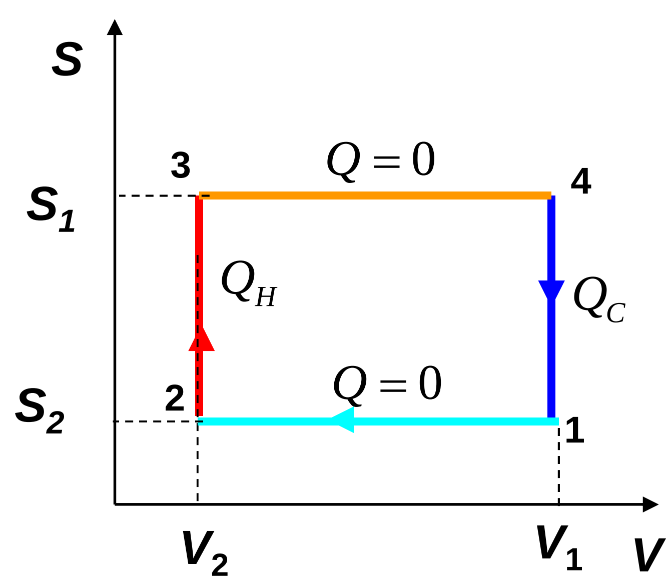
$$\boxed{e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} = 1 - \frac{T_1}{T_2}}$$

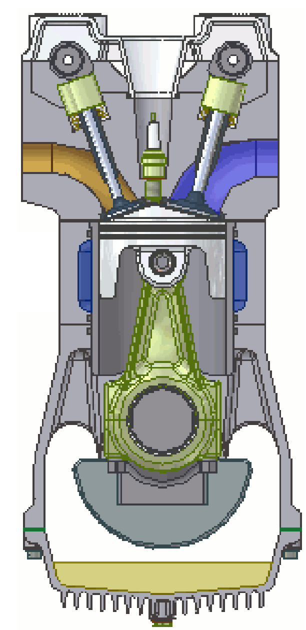
- V_2 - maximum cylinder volume
- V_1 - minimum cylinder volume
- $\dfrac{V_2}{V_1}$ - the compression ratio
- $\gamma = 1+2/f$ - the adiabatic exponent

For typical numbers $\boldsymbol{V_1/V_2}$ ~8 , γ ~ 7/5 $\rightarrow$ ***e*** = 0.56, (in reality, ***e*** = 0.2 – 0.3)

(even an "ideal" efficiency is *smaller* than the second law limit $\mathbf{1\text{-}}\boldsymbol{T_1/T_3}$)







Diesel engine

$$e = 1 - \frac{\delta Q_c}{\delta Q_h} \qquad \text{(Pr. 4.20)}$$

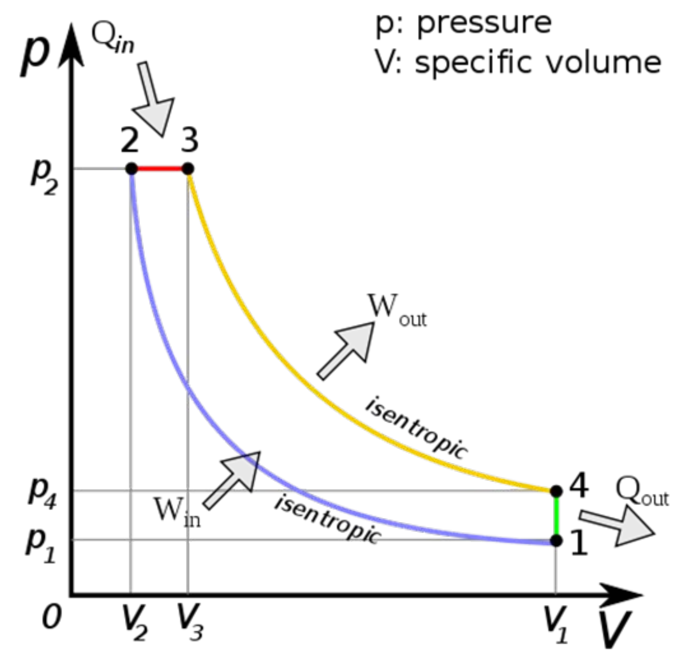
$$1 \to 2: \text{ adiabatic, } T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

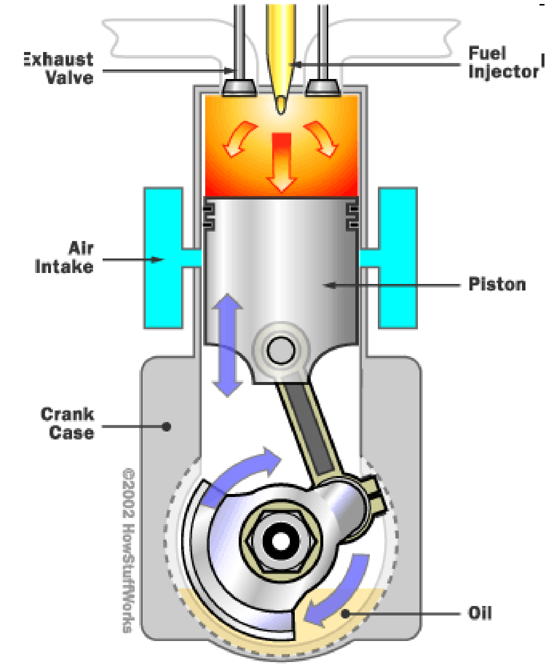
$$2 \to 3: \text{ isobaric, } \delta Q_h = \frac{f+2}{2} nR\left(T_3 - T_2\right)$$

$$3 \to 4: \text{ adiabatic, } T_3 V_3^{\gamma-1} = T_4 V_4^{\gamma-1}$$

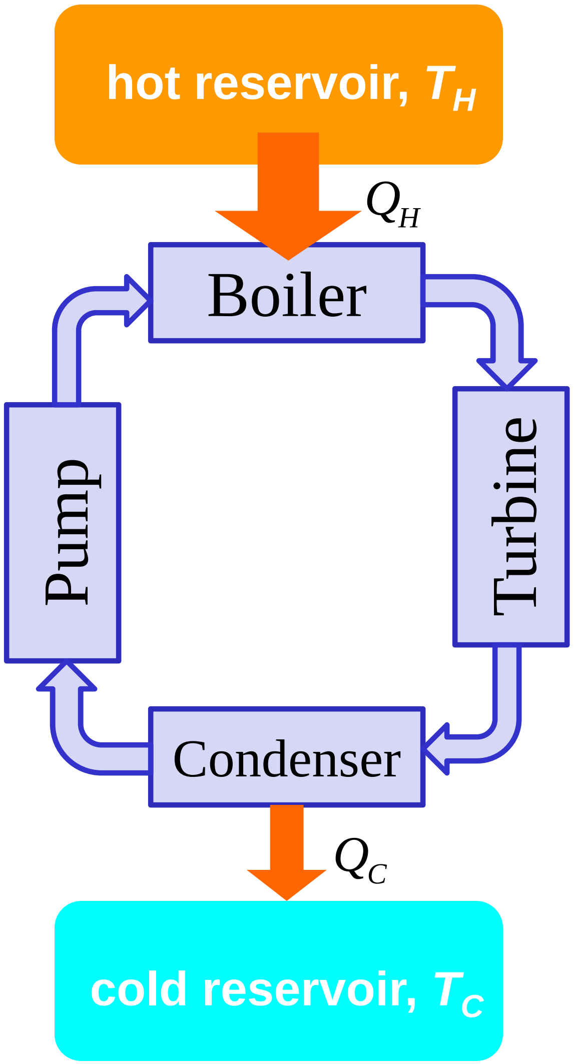
$$4 \to 1: \text{ isochoric, } V_4 = V_1,\ \delta Q_c = \frac{f}{2} nR\left(T_4 - T_1\right)$$

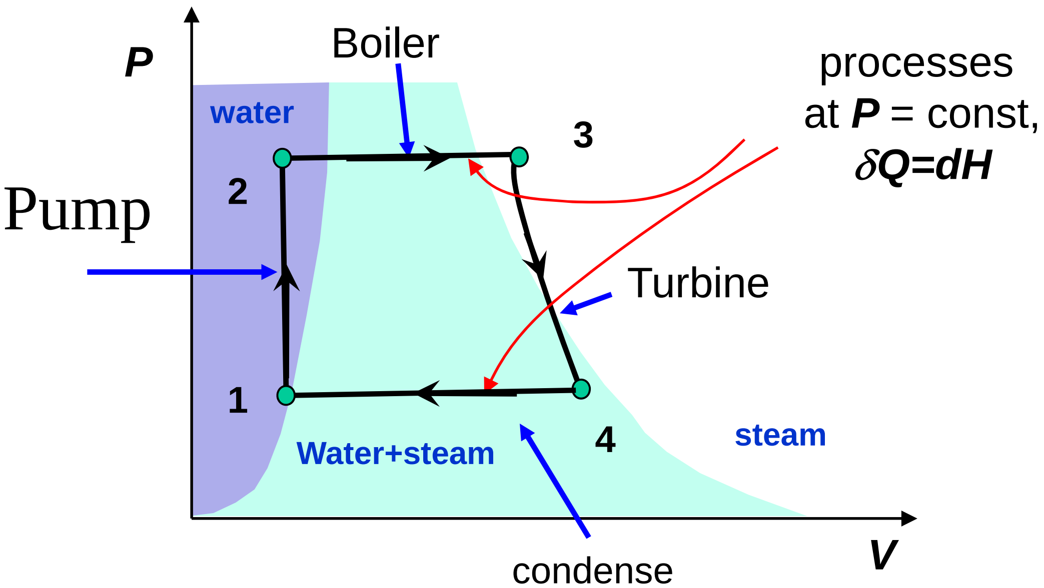
$$\boxed{e = 1 - \left(\frac{V_2}{V_1}\right)^{\gamma-1} \cdot \frac{1}{\gamma} \frac{\left(\frac{V_3}{V_2}\right)^{\gamma} - 1}{\frac{V_3}{V_2} - 1}}$$





Steam engine (Rankine cycle)





NOT an ideal gas!

$$H = U + PV, \Rightarrow (\delta Q)_P = (\Delta H)_P$$

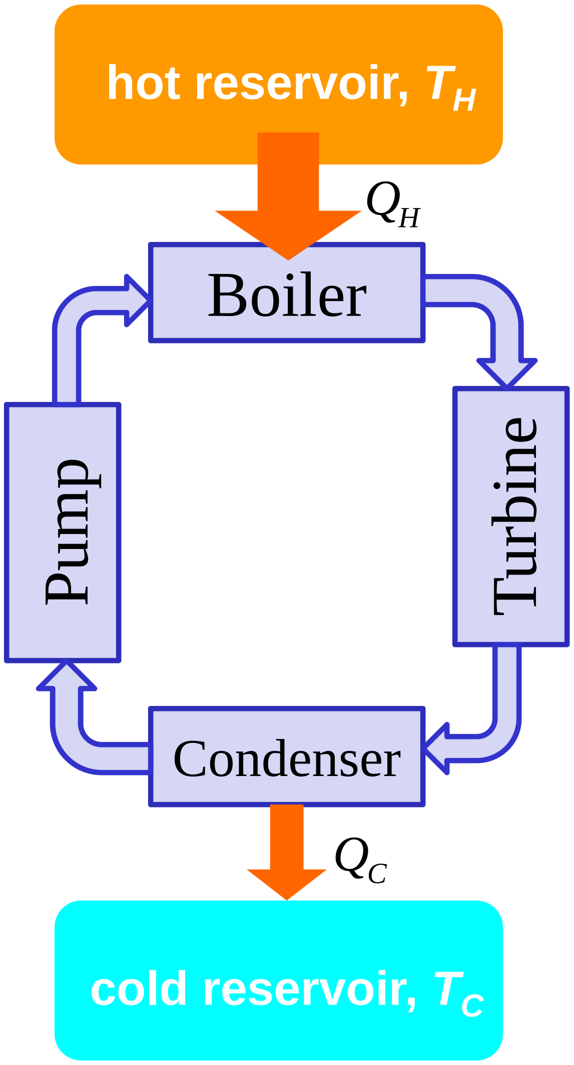
$$1 \rightarrow 2: \text{ isothermal} (\approx \text{adiabatic})$$

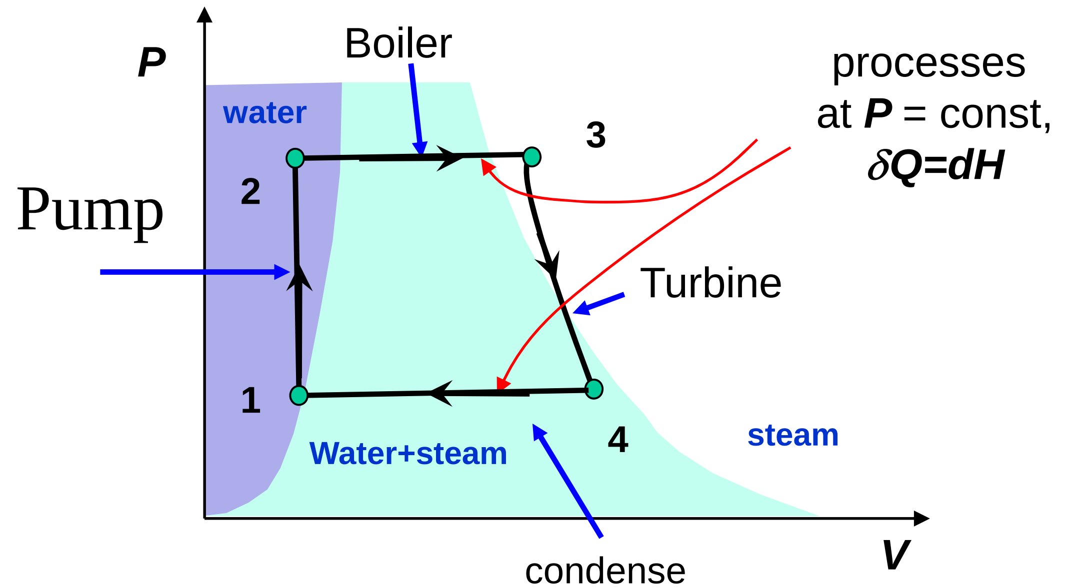
$$2 \rightarrow 3: \text{ isobaric, } \delta Q_h = H_3 - H_2$$

$$3 \rightarrow 4: \text{ adiabatic}$$

$$4 \rightarrow 1: \text{ isobaric, } \delta Q_h = H_4 - H_1$$

Steam engine (Rankine cycle)





$$\boxed{e = 1 - \frac{H_4 - H_1}{H_3 - H_2} \approx 1 - \frac{H_4 - H_1}{H_3 - H_1}} \qquad (4.12)$$

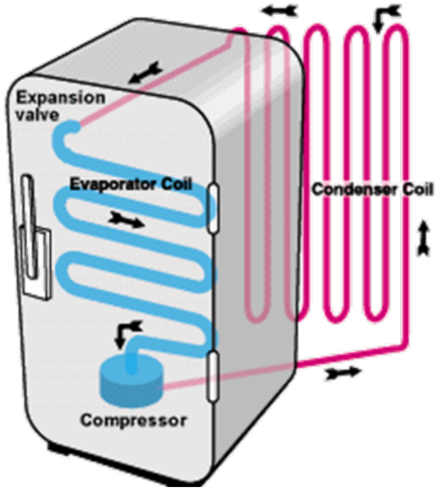
Here $H_2 \approx H_1$, water is almost incompressible.

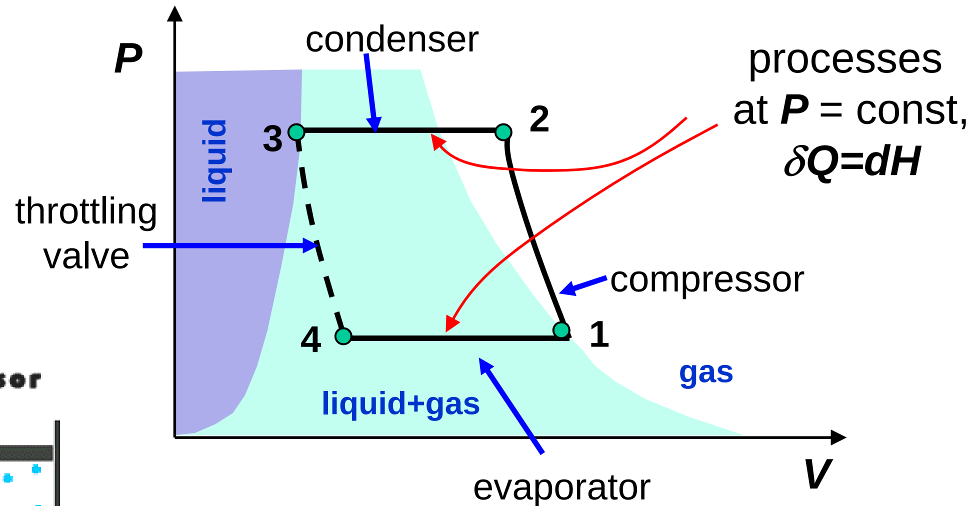
$$S_3 = S_4 \Rightarrow S_3^{\text{gas}} = x \cdot S_4^{\text{gas}} + (1 - x) \cdot S_4^{\text{liquid}}$$

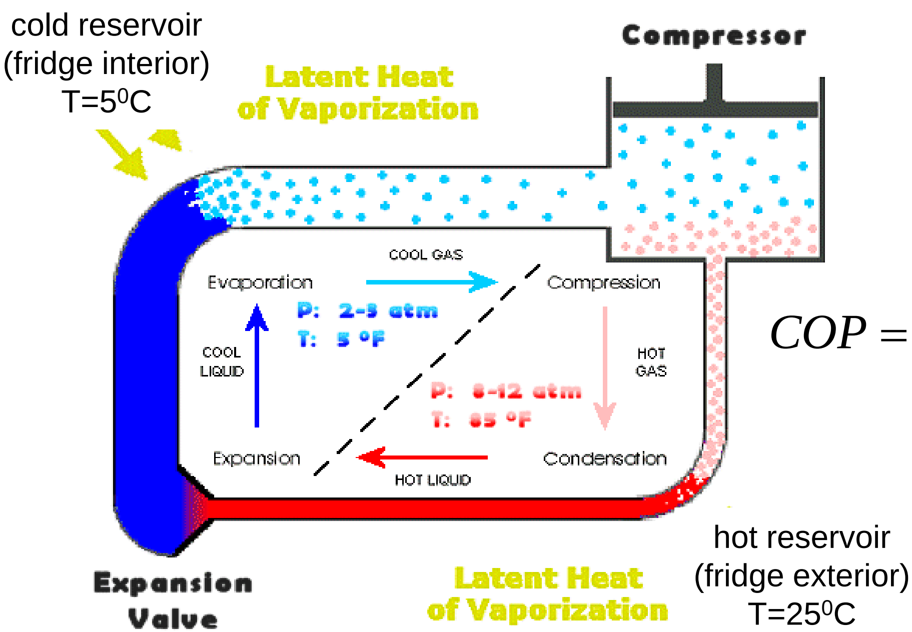
$$H_4 = x \cdot H_4^{\text{gas}} + (1 - x) \cdot H_4^{\text{liquid}}$$

Kitchen Refrigerator

A liquid with suitable characteristics (e.g., Freon) circulates through the system. The compressor pushes the liquid through the condenser coil at a high pressure (~ 10 atm). The liquid sprays through a throttling valve into the evaporation coil which is maintained by the compressor at a low pressure (~ 2 atm).







$$COP = \frac{Q_C}{Q_H - Q_C} = \frac{H_1 - H_4}{H_2 - H_3 - \left(H_1 - H_4\right)} = \frac{H_1 - H_4}{H_2 - H_1}$$

The enthalpies $\boldsymbol{H_i}$ can be found in tables.

$$H_3 = H_4, \Rightarrow H_3^{\text{liquid}} = x \cdot H_4^{\text{liquid}} + \left(1 - x\right) \cdot H_4^{\text{gas}}$$

$$S_2 = S_1 \rightarrow T_2 \rightarrow H_2\left(T_2, P_2\right)$$