Lecture 10. Heat Engines and refrigerators (Ch. 4)

A **heat engine** – any device that is capable of converting thermal energy (***heating***) into mechanical energy (***work***). We will consider an important class of such devices whose operation is ***cyclic***.

Heating – the transfer of energy to a system by thermal contact with a reservoir.
Work – the transfer of energy to a system by a change in the external parameters (***V***, el.-mag. and grav. fields, etc.).

The main question we want to address: what are the limitations imposed by thermodynamic on the performance of heat engines?

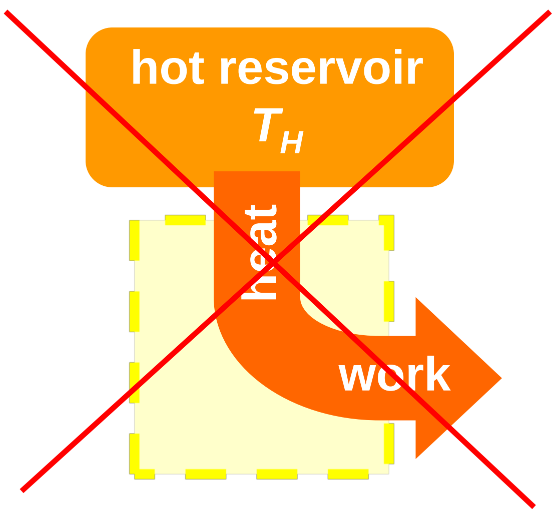
Perpetual Motion Machines are Impossible

- **Perpetual Motion Machines of the first type** – these designs seek to create the energy required for their operation out of nothing.

violation of the First Law (energy conservation)

- **Perpetual Motion Machines of the second type** - these designs extract the energy required for their operation in a manner that decreases the entropy of an isolated system.

violation of the Second Law



impossible cyclic heat engine

Word of caution: for ***non-cyclic*** processes, 100% of heat ***can*** be transformed into work without violating the Second Law.

Example: an ideal gas expands isothermally being in thermal contact with a hot reservoir. Since $\boldsymbol{U}$ = const at $\boldsymbol{T}$ = const, all heat has been transformed into work.

Fundamental Difference between Heating and Work

- is the difference in the ***entropy transfer!***

- **Transferring purely mechanical energy to or from a system does not (necessarily) change its entropy: ΔS = 0 for reversible processes**. For this reason, all forms of work are ***thermodynamically equivalent*** to each other - they are freely convertible into each other and, in particular, into mechanical work.

- **Work can be completely converted into heat, but the inverse is not true.** The transfer of energy by heating is accompanied with the entropy transfer

$$dS = \frac{\delta Q}{T}$$

Thus, entropy enters the system with heating, but does not leave the system with the work. On the other hand, for a continuous operation of a heat engine, **the net entropy change during a cycle must be zero!**

How is it possible???

The Price Should be Paid...

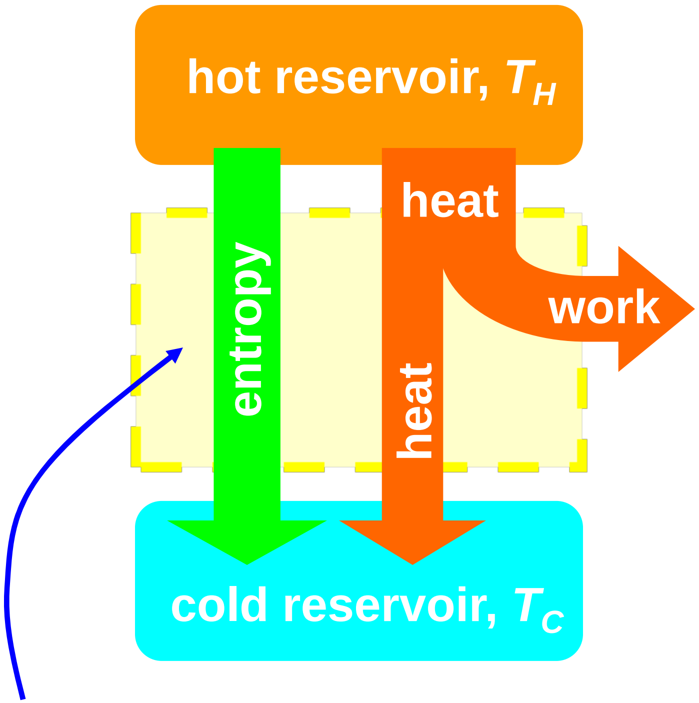
An engine can get rid of ***all*** the entropy received from the hot reservoir by transferring only ***part*** of the input thermal energy to the cold reservoir.

$$dS = \frac{\delta Q}{T}$$

Thus, the only way to get rid of the accumulating entropy is to absorb more internal energy in the heating process than the amount converted to work, and to "flush" the entropy with the flow of the waste heat out of the system.

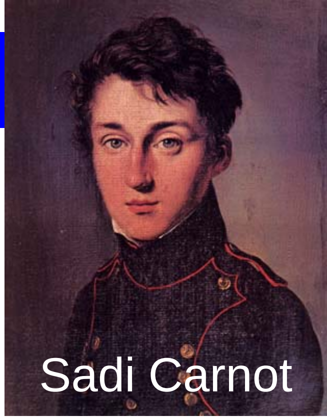
An essential ingredient: a temperature difference between hot and cold reservoirs.

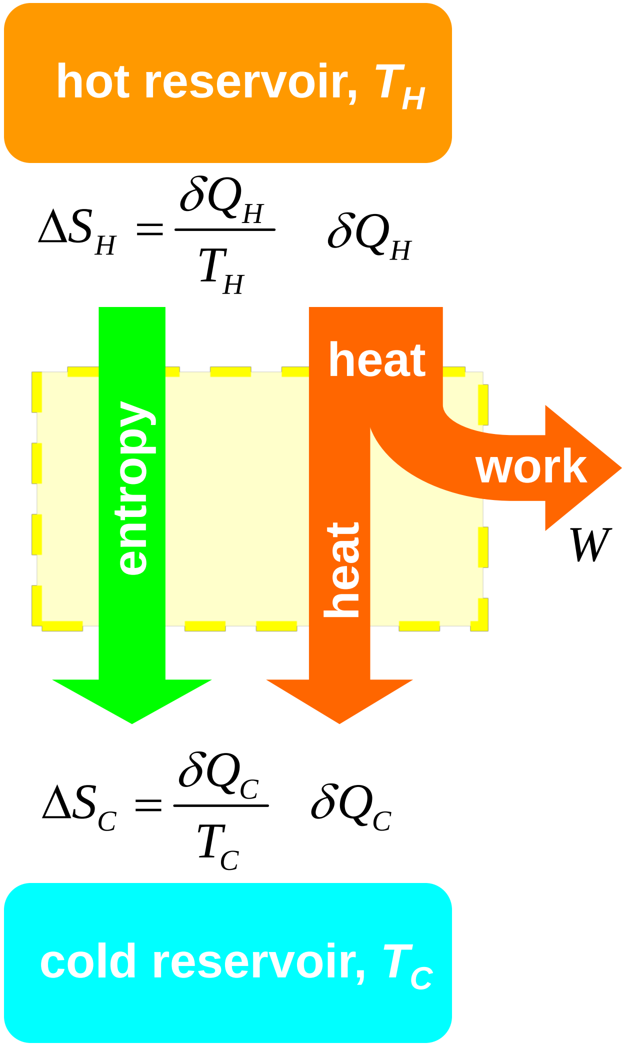
Essential parts of a heat engine (any continuously operating reversible device generating work from "heat")



"Working substance" – the system that absorbs heat, expels waste energy, and does work (e.g., water vapor in the steam engine)

Perfect Engines (no extra S generated)





$$\Delta S_H = \frac{\delta Q_H}{T_H} \qquad \delta Q_H$$

$$\Delta S_C = \frac{\delta Q_C}{T_C} \qquad \delta Q_C$$

The condition of continuous operation:

$$\Delta S_H = \Delta S_C \qquad \frac{\delta Q_H}{T_H} = \frac{\delta Q_C}{T_C}$$

$$\boxed{\delta Q_C = \frac{T_C}{T_H}\delta Q_H}$$

The work generated during one cycle of a reversible process:

$$\boxed{\delta W = \delta Q_H - \delta Q_C = \frac{T_H - T_C}{T_H}\delta Q_H}$$

Carnot efficiency:

the highest possible value of the energy conversion efficiency

$$\boxed{e_{\max} \equiv \frac{\delta W}{\delta Q_H} = 1 - \frac{T_C}{T_H} < 1}$$

Consequences

- Any difference $T_H - T_C \neq 0$ can be exploited to generate mechanical energy.
- The greater the $T_H - T_C$ difference, the more efficient the engine.
- Energy waste is ***inevitable***.

Example: In a typical nuclear power plant, T_H = 300^{0}C (~570K), T_C = 40^{0}C (~310K), and the maximum efficiency e_{max}=0.45. If the plant generates 1000 MW of "work", its waste heat production is at a rate

$$\delta Q_C = \delta Q_H - \delta W = \delta W \left(\frac{1}{e} - 1 \right) \approx 1220 \text{ MW}$$

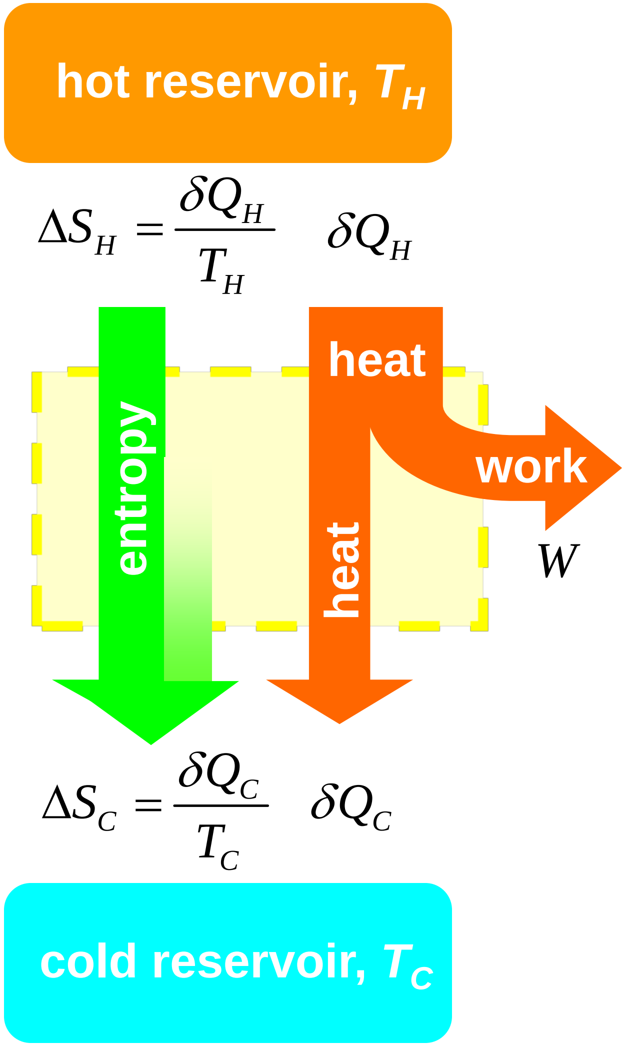
- more fuel is needed to get rid of the entropy then to generate useful power.

definition of efficiency

$$\text{General definition: efficiency} = \frac{\text{benefit}}{\text{cost}}$$

	benefit	cost	efficiency
heat engine	W	Q_h	W/Q_h
refrigerator	Q_c	W	Q_c/W
heat pump	Q_h	W	Q_h/W

Real Engines



Real heat engines have lower efficiencies because the processes within the devices are not perfectly reversible – the entropy will be generated by ***irreversible*** processes:

$$e = \frac{\delta W}{\delta Q_H} \le 1 - \frac{T_C}{T_H} = e_{\max}$$

e = ***e***$_{\mathbf{max}}$ only in the limit of ***reversible operation***.

<u>Some sources of irreversibility</u>:

- heat may flow directly between reservoirs;
- not all temperature difference $\boldsymbol{T_H - T_C}$ may be available (temperature drop across thermal resistances in the path of the heat flow);
- part of the work generated may be converted to heat by friction;
- gas may expand irreversibly without doing work (as gas flow into vacuum).

Problem

The temperature inside the engine of a helicopter is 2000^{0}C, the temperature of the exhaust gases is 900^{0}C. The mass of the helicopter is $\boldsymbol{M}$ = 2·10^3 kg, the heat of combustion of gasoline is $\boldsymbol{Q_{comb}}$ = 47·10^3 kJ/kg, and the density of gasoline is ρ = 0.8 g/cm^3. What is the maximum height that the helicopter can reach by burning $\boldsymbol{V}$ = 1 liter of gasoline?

The work done on lifting the helicopter: $W = MgH$

For the ideal Carnot cycle (the maximum efficiency): $e \equiv \frac{W}{Q_H} = 1 - \frac{T_C}{T_H}$

Thus, $W = \left(1 - \frac{T_C}{T_H}\right) Q_H$

The heat released in the gasoline combustion: $Q_H = q_{comb} \cdot \rho \cdot V$

$$MgH = \left(1 - \frac{T_C}{T_H}\right) q_{comb} \cdot \rho \cdot V$$

$$H = \frac{q_{comb} \cdot \rho \cdot V}{Mg}\left(1 - \frac{T_C}{T_H}\right) = \frac{47 \cdot 10^3 \,\text{kJ/kg} \times 0.8\,\text{kg/liter} \times 1\,\text{liter}}{2000\,\text{kg} \times 9.8\,\text{m/s}^2}\left(1 - \frac{1173\,\text{K}}{2273\,\text{K}}\right) = 928\,\text{m}$$

Note: if T_H and/or T_C vary in the process, we still can introduce an "instanteneous" efficiency:

$$e(T) = \frac{\delta Q_H - \delta Q_C}{\delta Q_H} = \frac{\delta W}{\delta Q_H}$$

Problem [$T_H = f(t)$]

A reversible heat engine operates between two reservoirs, T_C and T_H. The cold reservoir can be considered to have infinite mass, i.e., $T_C = T_1$ remains constant. However the hot reservoir consists of a finite amount of gas at constant volume (**1** mole with a specific heat capacity c_V), thus T_H decreases with time (initially, $T_H = T_2$, $T_2 > T_1$). After the heat engine has operated for some long period of time, the temperature T_H is lowered to $T_C = T_1$

(a) Calculate the heat extracted from the hot reservoir during this period.
(b) What is the change of entropy of the hot reservoir during this period?
(c) How much work did the engine do during this period?

(a) $$|Q_H| = c_V\left(T_2 - T_1\right)$$

(b) $$dS = \frac{\delta Q_H}{T_H} = \frac{c_V dT_H}{T_H} \to \Delta S = \int_{T_2}^{T_1} \frac{c_V dT_H}{T_H} = c_V \ln\frac{T_1}{T_2}$$

(c) $$e(T) = \frac{\delta Q_H - \delta Q_C}{\delta Q_H} = \frac{\delta W}{\delta Q_H} = 1 - \frac{T_1}{T_H}$$

$$\delta Q_H = -c_V dT_H$$

$$\delta W = \delta Q_H\left(1 - \frac{T_1}{T_H}\right) = -c_V dT_H\left(1 - \frac{T_1}{T_H}\right)$$

$$W = -\int_{T_2}^{T_1}\left(1 - \frac{T_1}{T_H}\right)c_V dT_H = c_V\left(T_2 - T_1\right) - c_V T_1 \ln\frac{T_2}{T_1}$$

Problem Given 1 kg of water at 100^0C and a very large block of ice at 0^0C. A reversible heat engine absorbs heat from the water and expels heat to the ice until work can no longer be extracted from the system. The heat capacity of water is 4.2 J/g·K. At the completion of the process:

a) What is the temperature of the water?

b) How much heat has been absorbed by the block of ice in the process?

c) How much ice has been melted (the heat of fusion of ice is 333 J/g)?

d) How much work has been done by the engine?

(a) Because the block of ice is very large, we can assume its temperature to be constant. When work can no longer be extracted from the system, the efficiency of the cycle is zero:

$$e = 1 - T_{ice} / T_{water} = 0$$

$$\rightarrow \quad T_{water} = T_{ice} = 0^0 C$$

(b) The heat absorbed by the block of ice:

$$e(T_W, T_I) = \frac{\delta Q_H - \delta Q_C}{\delta Q_H} \qquad \delta Q_C = [1 - e(T_W, T_I)]\delta Q_H = -[1 - e(T_W, T_I)]\, m_W c_W dT_W = -\frac{T_I}{T_W} m_W c_W dT_W$$

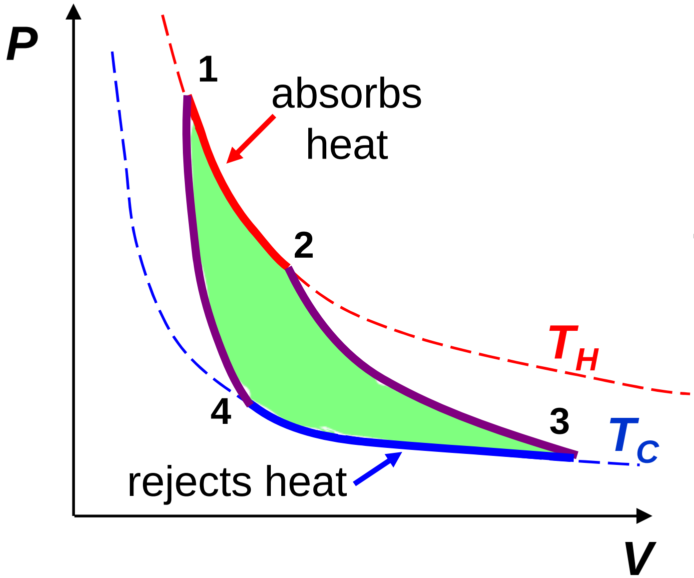
$$Q_C = -\int_{T_i}^{T_f} \frac{T_I}{T_W} m_W c_W dT_W = -T_I m_W c_W \int_{373}^{273} \frac{dT_W}{T_W} = 273\text{K} \times 1\text{kg} \times 4.2\text{kJ/kg} \times \ln\left(\frac{373}{273}\right) = 357.9\text{kJ}$$

(c) The amount of melted ice: $$M_I = \frac{Q_C}{L} = \frac{357.9\text{ kJ}}{333\text{ J/g}} = 1.07\text{ kg}$$

(d) The work : $$W = Q_H - Q_C = 1\text{kg} \times 4.2\text{kJ/kg} \cdot \text{K} \times 100\text{K} - 357.9\text{ kJ} = 62.1\text{ kJ}$$

Carnot Cycle

- is not very practical (too slow), but operates at the ***maximum*** efficiency allowed by the Second Law.



1 – 2 isothermal expansion (in contact with T_H)

2 – 3 isentropic expansion to T_C

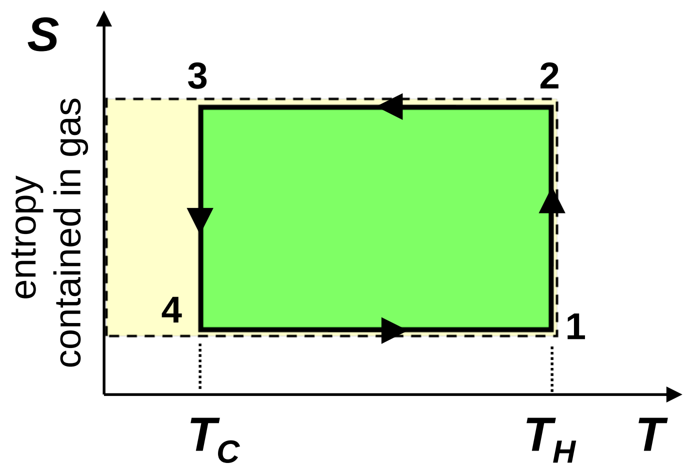
3 – 4 isothermal compression (in contact with T_C)

4 – 1 isentropic compression to T_H

(isentropic ≡ adiabatic+quasistatic)

Efficiency of Carnot cycle for an ideal gas: (*Pr*. 4.5)

$$\boxed{e_{\max} = 1 - \frac{T_C}{T_H}}$$



On the ***S -T*** diagram, the work done is the area of the loop:

$$\oint dU = 0 = \oint TdS - \oint PdV$$

The heat consumed at T_H (**1 – 2**) is the area surrounded by the broken line:

$$Q_H = T_H\left(S_H - S_C\right)$$

S - entropy contained in gas

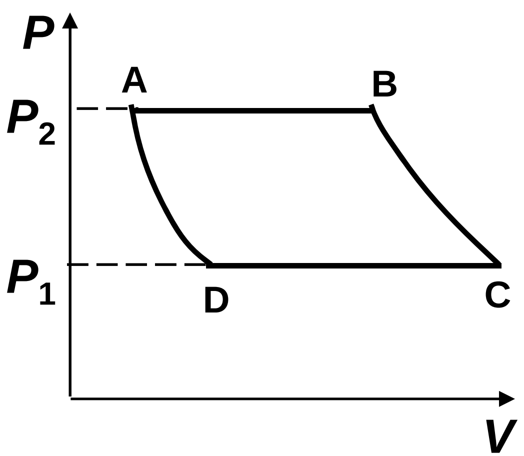
The Carnot heat engine operates at the ***maximum*** efficiency allowed by the Second Law. Other heat engines may have a lower efficiency even if the cycle is reversible (no friction, etc.)

Problem. Consider a heat engine working in a reversible cycle and using an ideal gas with constant heat capacity $\mathbf{c_P}$ as the working substance. The cycle consists of two processes at constant pressure, joined by two adiabatic processes.

(a) Which temperature of T_A, T_B, T_C, and T_D is highest, and which is lowest?

(b) Find the efficiency of this engine in terms of P_1 and P_2 .

(c) Show that a Carnot engine with the same gas working between the highest and lowest temperatures has greater efficiency than this engine.

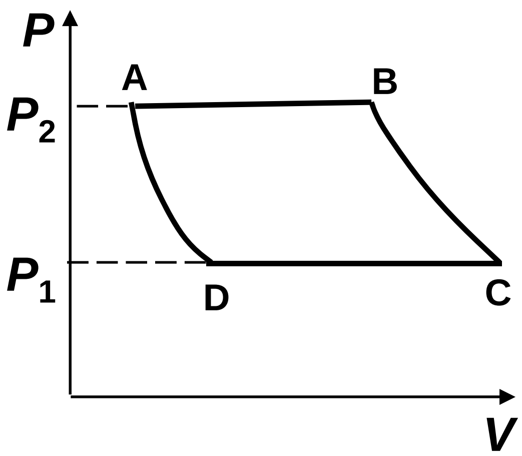


(a) From the equation of state for an ideal gas (***PV=RT***), we know that $T_B > T_A \qquad T_C > T_D$

From the adiabatic equation : $T_B > T_C \qquad T_A > T_D$

Thus $T_B = \max(T_A, T_B, T_C, T_D) \qquad T_D = \min(T_A, T_B, T_C, T_D)$

Problem (cont.)



(b) The heat absorbed from the hot reservoir $Q_{AB} = C_P\left(T_B - T_A\right)$

The heat released into the cold reservoir $Q_{CD} = C_P\left(T_C - T_D\right)$

Thus, the efficiency $$e = \frac{Q_{AB} - Q_{CD}}{Q_{AB}} = 1 - \frac{T_C - T_D}{T_B - T_A}$$

From the equation for an adiabatic process:

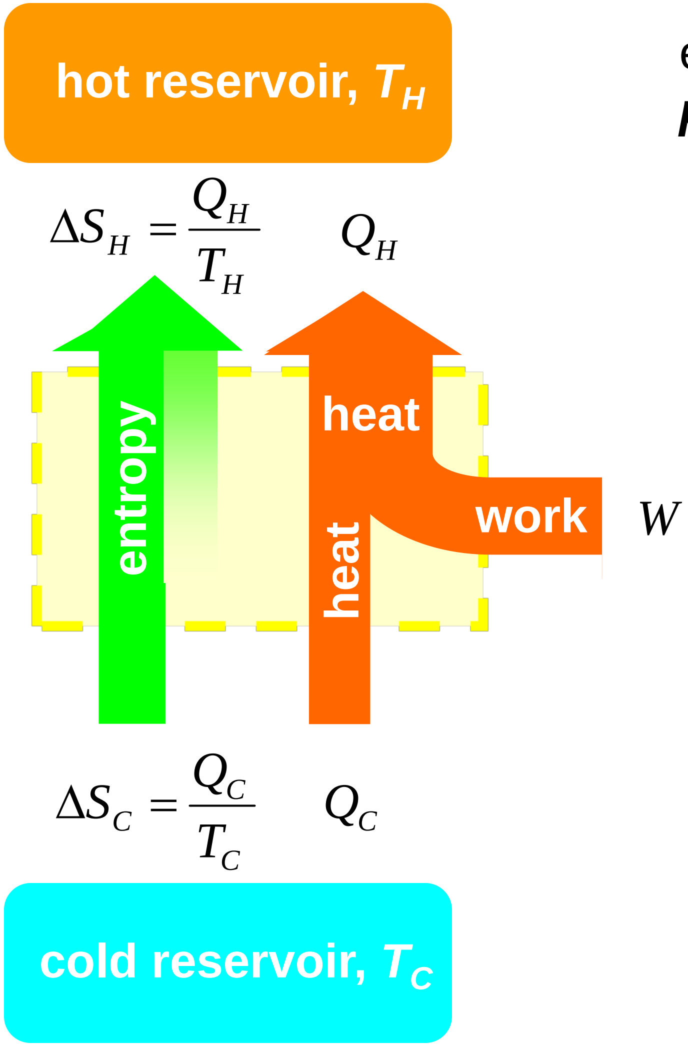
$$PV^{\gamma} = const \quad \rightarrow \quad T^{\gamma} / P^{\gamma-1} = const$$

$$\left.\begin{array}{l} P_A = P_B = P_2, \;\; P_C = P_D = P_1 \\ \dfrac{T_A{}^{\gamma}}{P_A{}^{\gamma-1}} = \dfrac{T_D{}^{\gamma}}{P_D{}^{\gamma-1}} \;\; \dfrac{T_B{}^{\gamma}}{P_B{}^{\gamma-1}} = \dfrac{T_C{}^{\gamma}}{P_C{}^{\gamma-1}} \end{array}\right\} \Rightarrow T_D = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} T_A, \;\; T_C = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} T_B$$

$$e = 1 - \frac{T_B\left(P_1 / P_2\right)^{(\gamma-1)/\gamma} - T_A\left(P_1 / P_2\right)^{(\gamma-1)/\gamma}}{T_B - T_A} = 1 - \left(P_1 / P_2\right)^{(\gamma-1)/\gamma}$$

(c) $$e = 1 - \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = 1 - \frac{T_D}{T_A} < 1 - \frac{T_D}{T_B} = e_{\max}$$

Refrigerators



The purpose of a refrigerator is to make thermal energy flow from cold to hot. The ***coefficient of performance*** for a fridge:

$$COP \equiv \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{1}{Q_H / Q_C - 1}$$

$$\frac{Q_H}{Q_C} \geq \frac{T_H}{T_C}$$

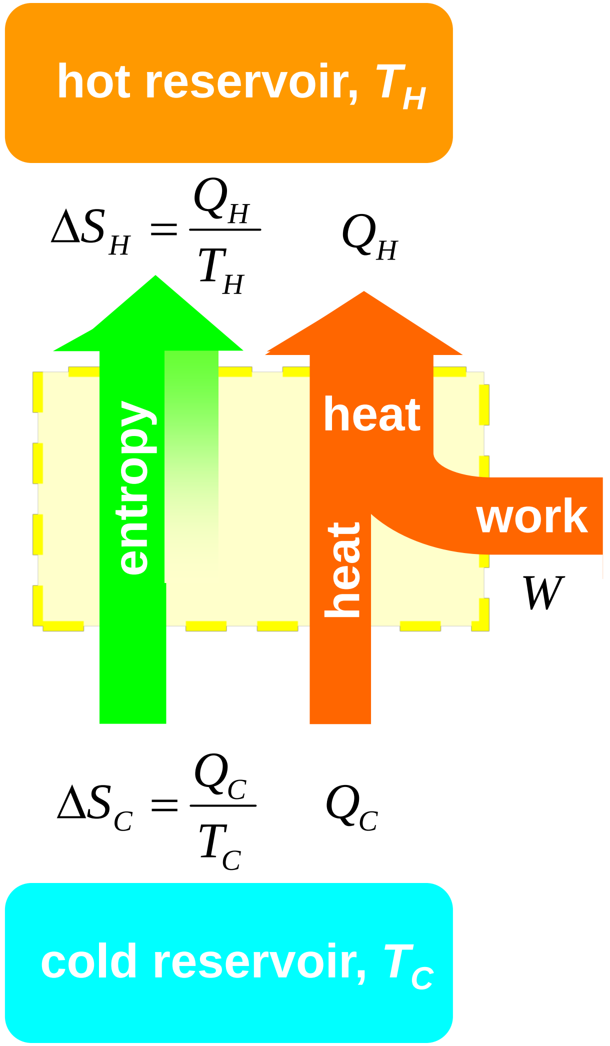
$$\boxed{COP \leq COP_{\max} = \frac{T_C}{T_H - T_C}}$$

COP is the largest when $\boldsymbol{T_H}$ and $\boldsymbol{T_C}$ are close to each other!

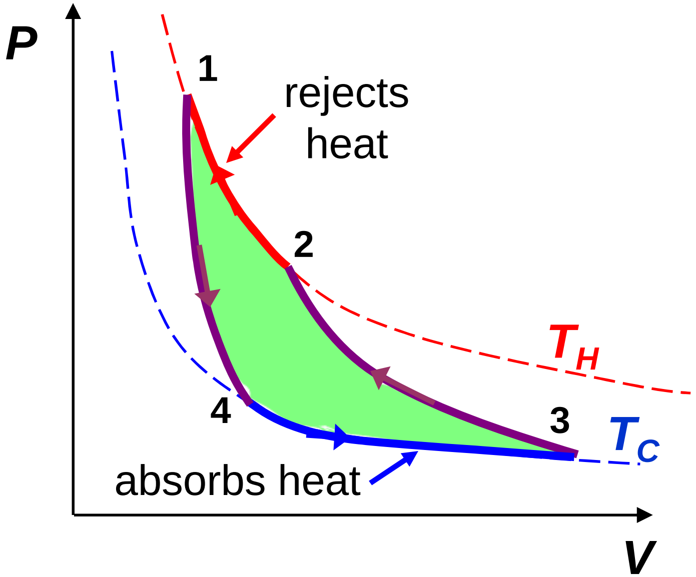
For a typical kitchen fridge $\boldsymbol{T_H}$ ~300K, $\boldsymbol{T_C}$~ 250K $\Rightarrow$ ***COP*** ~ 6 (for each J of el. energy, the coolant can suck as much as 6 J of heat from the inside of the freezer).

A fridge that cools something from *RT* to *LHe* temperature ($\boldsymbol{T_C}$~ 4K) would have to be much less efficient.

More on Refrigerators



We can create a refrigerator by running a Carnot engine backwards: the gas extracts heat from the cold reservoir and deposit it in the cold reservoir.



Example:

A “perfect” heat engine with ***e*** = 0.4 is used as a refrigerator (the heat reservoirs remain the same). How much heat $\boldsymbol{Q_C}$ can be transferred in one cycle from the cold reservoir to the hot one if the supplied in one cycle work is ***W*** =10 kJ?

$$e = \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} \qquad COP = \frac{Q_c}{W} = \frac{Q_c}{Q_H - Q_C} = \frac{\frac{Q_C}{Q_H}}{1 - \frac{Q_C}{Q_H}}$$

$$\frac{Q_C}{Q_H} = 1 - e \qquad COP == \frac{1-e}{1-1+e} = \frac{1-e}{e} = \frac{0.6}{0.4} = 1.5$$

$$Q_C \text{ (in one cycle)} = W \times COP = 15\,\text{kJ}$$