

Asymptotic Generalizations of the Lockhart–Martinelli Method for Two Phase Flows

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The Lockhart–Martinelli (1949, "Proposed Correlation of Data for Isothermal Two Phase Flow, Two Component Flow in Pipes," Chem. Eng. Prog., 45, pp. 39–48) method for predicting two phase flow pressure drop is examined from the point of view of asymptotic modeling. Comparisons are made with the Lockhart–Martinelli method, the Chisholm (1967, "A Theoretical Basis for the Lockhart–Martinelli Correlation for Two Phase Flow," Int. J. Heat Mass Transfer, 10, pp. 1767–1778) method, and the Turner–Wallis (1969, One Dimensional Two Phase Flow, McGraw-Hill, New York) method. An alternative approach for predicting two phase flow pressure drop is developed using superposition of three pressure gradients: single phase liquid, single phase gas, and interfacial pressure drop. This new approach allows for the interfacial pressure drop to be easily modeled for each type of flow regime such as bubbly, mist, churn, plug, stratified, and annular; or based on the classical laminar-laminar, turbulent-turbulent, laminar-turbulent, and turbulent-laminar flow regimes proposed by Lockhart and Martinelli. [DOI: 10.1115/1.4001157]

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1 Introduction

It has been 60 years since the publication of Lockhart and Martinelli's [1] seminal paper on two phase or two component flows. This paper has essentially defined the methodology for presenting two phase flow data in non-boiling and boiling flows. The mere fact that it has received nearly 1000 citations in journal papers alone is a testament to its contribution to the field of two phase flow. Since the publication of Lockhart and Martinelli's paper [1], a number of extensions, modifications, and closures have been proposed for simple two phase flow modeling assuming separated flows.

In the present work, analysis of two phase flows using asymptotically based correlation methods is examined. The proposed method is distinguished from the classic Lockhart–Martinelli approach, which is shown to be a special case in the present method since it includes a general means of accounting for interfacial pressure drop effects, i.e., those resulting from the interaction of the two phases. Closure is provided by means of two coefficients, which are determined by means of comparison with experimental data. The proposed models are compared with two phase and two component flow data found in a variety of gas-liquid applications including pipes and micro- and minichannels. However, the approach can also be applied to two phase flow in porous media, fractured media, microgravity, and with proper attention, liquid-liquid applications.

2 Review of Models

A general overview of a number of related methodologies is first given. We consider three classical approaches in detail, which are all somewhat related. These are the original Lockhart–Martinelli method, the Turner–Wallis method also known as the separate cylinders model, and the Chisholm method. Additional models from the recent literature are also examined.

2.1 Lockhart–Martinelli Method. In 1949, Lockhart and Martinelli [1] proposed a correlation scheme for two phase flows. Using a simple analysis founded on the premise that the static pressure drop for the liquid and gas phases flowing simultaneously is equal at any point along the duct, they developed expressions for predicting the two phase pressure drop, which contained four variables: D_l/D , D_g/D , α , and β . The first two represented the ratio of the hydraulic diameter to pipe diameter for each phase flowing in the pipe, while the latter two represented the ratio of the true area occupied by each phase to the hydraulic area determined from the hydraulic diameters of each flowing phase. Using the simple pressure drop calculations,

$$\left. \frac{dp}{dx} \right|_{tp} = \frac{2f_l}{D_l} \rho U_l^2 = \frac{2f_g}{D_g} \rho U_g^2 \quad (1)$$

based on simple friction laws, i.e., $f_l \sim C_l / \text{Re}_l^n$ and $f_g \sim C_g / \text{Re}_g^m$, Lockhart and Martinelli [1] deduced that the two phase pressure drop could be predicted using

$$\left. \frac{dp}{dx} \right|_{tp} \sim \left. \frac{dp}{dx} \right|_l \alpha^{n-2} \left(\frac{D}{D_l} \right)^{5-n} \quad (2)$$

or

$$\phi_l^2 \sim \left. \frac{dp}{dx} \right|_{tp} \sim \alpha^{n-2} \left(\frac{D}{D_l} \right)^{5-n} \quad (3)$$

Without a loss in generality, one can now stipulate that

$$\phi_l^2 \sim \text{const} \quad (4)$$

A similar analysis for ϕ_g^2 leads to

$$\left. \frac{dp}{dx} \right|_{tp} \sim \left. \frac{dp}{dx} \right|_g \beta^{m-2} \left(\frac{D}{D_g} \right)^{5-m} \quad (5)$$

or

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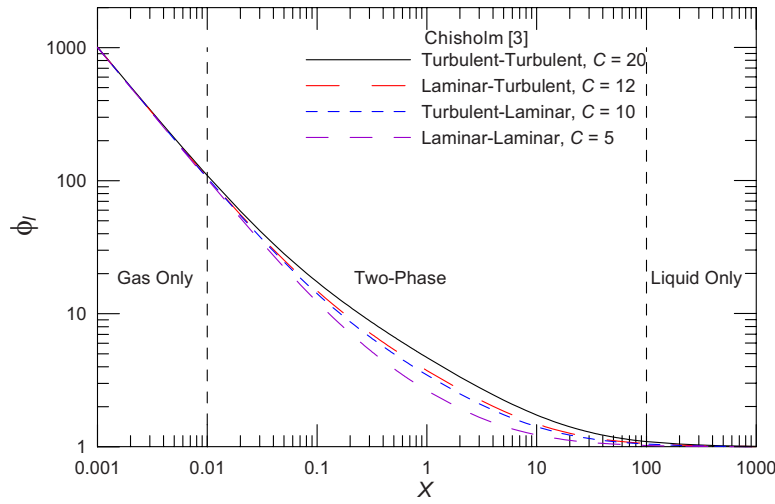


Fig. 1 Lockhart and Martinelli's [1] curves for the four flow regimes showing ϕ_l versus X

$$\phi_g^2 \sim - \left. \frac{dp}{dx} \right|_p \sim \beta^{m-2} \left(\frac{D}{D_g} \right)^{5-m} \quad (6)$$

and

$$\phi_g^2 \sim \text{const} \quad (7)$$

In Eqs. (3) and (6), the constants m and n represent the exponent on the Reynolds number in the simple friction models. Thus, given that for any particular combination of mass flow rates for the liquid and gas phases, a unique value for D_l and D_g would arise, and hence α and β also, giving rise to a constant value for ϕ_l^2 and ϕ_g^2 for the given flow conditions.

Lockhart and Martinelli [1] further showed through their analysis that the generalized variable

$$X^2 \sim - \left. \frac{dp}{dx} \right|_l \sim \frac{2f_l}{D_l} \rho U_l^2 \sim \frac{\text{Re}_g^m}{\text{Re}_l^n} \frac{C_l \rho_l \dot{m}_l^2}{C_g \rho_g \dot{m}_g^2} \quad (8)$$

is also a required correlating parameter. The parameter X has come to be known as the *Martinelli* parameter, and is sometimes also denoted as χ and Ma . The latter being discouraged to avoid confusion with the Mach number.

As a result of this simple analysis, Lockhart and Martinelli [1] established four flow regimes based on the values of m and n in the simple friction laws. These correspond to the laminar-laminar (ll), turbulent-turbulent (tt), laminar liquid-turbulent gas (lt), and turbulent liquid-laminar gas (tl) flow combinations of the liquid and gas. While both convenient and simple, these four flow regimes say nothing with regard to the type of flow pattern. Lockhart and Martinelli [1] devised four separate correlations, one for each flow regime, as shown in Fig. 1. These were presented both graphically and in tabular form in the original paper in the form of a family of curves:

$$\phi^2 \sim \phi_l^2(X), \quad \phi_g^2(X) \quad (9)$$

As a correlating scheme, the Lockhart-Martinelli [1] approach represents an asymptotic normalization of two phase flow data, given the Eqs. (3), (6), and (8). The scheme can be summarized as follows:

$$\left. \begin{aligned} \phi_l^2 &\rightarrow \frac{1}{X^2}, & X &\rightarrow 0 & \text{gas flow} \\ \phi_l^2 &\sim f(X), & 0.01 < X < 100 & \text{two phase flow} \\ \phi_l^2 &\rightarrow 1, & X &\rightarrow \infty & \text{liquid flow} \end{aligned} \right\} \quad (10)$$

In the region of $0.01 < X < 100$, the results are strongly dependent on the nature of the flow, i.e., the four flow regimes of Lockhart and Martinelli [1], in addition to the type of flow, i.e., bubbly, stratified, churn, slug, annular, and mist. In this region, the interfacial effects are strongest and hence most dependent on the type of flow or flow pattern. Addressing these issues phenomenologically is best dealt with using empirical models utilizing one or more closure constants. However, as we shall see shortly, a simple theoretical closure can be found, which is independent of the type of flow and only depends on which of the four flow regimes is assumed. The Lockhart-Martinelli curves are plotted in Fig. 1.

The Lockhart-Martinelli [1] scheme can be viewed as universal for any flow regime. The influence of the flow pattern or regime generally only manifests itself in the transition region due to interfacial effects. In the asymptotic region, the plot is universal in that it returns the single phase flow pressure drop regardless of flow regime, i.e., laminar or turbulent. Thus two flows having the same X value are only distinguishable as a result of interfacial effects. This is clearly seen in Fig. 1. In the original analysis of Lockhart and Martinelli, these interfacial effects are stronger for turbulent flows than for laminar flows, as expected. But within a given flow regime such as turbulent-turbulent, the type of flow pattern should also lead to distinguishable interfacial effects, for example, annular flow versus churn flow.

2.2 Turner-Wallis Method. More than a decade had passed since Lockhart and Martinelli's [1] paper had been published when Turner and Wallis [2] had developed a theoretical approach that allowed for an analytical closure of the Lockhart-Martinelli [1] equations. Turner and Wallis [2] proposed that the two phase flow could be analyzed by modeling the flow as two parallel single phase streams flowing in separate pipes, each with a cross-sectional area equal to its actual flow area. This approach is also referred to as the "separate cylinders" model [2]. By assuming that the pressure gradient across the parallel streams was equal to that in the two phase flow, and that the cross-sectional areas of the two smaller pipes equaled the area of the actual pipe carrying the combined stream, they were able to develop simple analytical

Table 1 Turner–Wallis closure coefficients

Liquid	Gas	p
$m=0$	$n=0$	2.5
$m=0.2$	$n=0.2$	2.4
$m=0.25$	$n=0.25$	2.375
$m=1$	$n=1$	2
$m=1$	$n=0.25$	2.10
$m=0.25$	$n=1$	2.05

relationships for the laminar-laminar, turbulent-turbulent, and constant friction factor flow regimes. For the mixed flow regimes, the approach leads to implicit relationships for ϕ_l^2 and ϕ_g^2 , which can easily be solved numerically.

Assuming that the simple friction models are stated as before, i.e., $f_l \sim C_l / \text{Re}_l^n$ and $f_g \sim C_g / \text{Re}_g^m$, it is easy to show that the following expression can be obtained:

$$\left(\frac{1}{\phi_l^2}\right)^{2/(5-n)} + \left(\frac{1}{\phi_g^2}\right)^{2/(5-m)} = 1 \quad (11)$$

Introducing $X^2 = \phi_g^2 / \phi_l^2$, we can solve for a number of special cases, which yield both explicit and implicit forms for ϕ_l^2 . Turner and Wallis [2] suggested that for any two phase flow, the “separate cylinders” analysis, Eq. (11), could be generalized in the following way:

$$\left(\frac{1}{\phi_l^2}\right)^{1/p} + \left(\frac{1}{\phi_g^2}\right)^{1/p} = 1 \quad (12)$$

such that any value of p could be chosen to fit a particular data set. This leads to a one parameter empirical closure model. This generalization was proposed in Ref. [2], but no physical basis was given. Given that $X^2 = \phi_g^2 / \phi_l^2$, one can rearrange the above expression to obtain:

$$\phi_l^2 = \left[1 + \left(\frac{1}{X^2} \right)^{1/p} \right]^p \quad (13)$$

The value of p has been found using the separate cylinders approach to equal $p=2$ for laminar-laminar flow, $2.375 < p < 2.5$ for turbulent-turbulent flows based on friction factor, and $2.5 < p < 3.5$ turbulent-turbulent flows calculated on a mixing length basis [2]. These are summarized in Table 1 and in Fig. 2.

In the case of the two mixed flow regimes, the Turner–Wallis [2] method leads to the following implicit expressions:

$$\phi_l^2 = \left[1 + (\phi_l^2)^{3/38} \left(\frac{1}{X^2} \right)^{1/2.375} \right]^2 \quad (14)$$

for the laminar-turbulent case and

$$\phi_l^2 = \left[1 + (\phi_l^2)^{-3/38} \left(\frac{1}{X^2} \right)^{1/2} \right]^{2.375} \quad (15)$$

for the turbulent-laminar case. These can be solved numerically and fit to simple relationships, such as Eq. (13).

Using the data of Lockhart and Martinelli [1], Turner and Wallis [2] found that $p=2.75$ fit the laminar-laminar flow data better, while $p=4$ fit the turbulent-turbulent flow data better. Irrespective of the four Lockhart–Martinelli [1] flow regimes, Turner and Wallis [2] found that $p=3.5$ fit all data in a best fit sense, i.e., equal scatter about Eq. (13). Equation (13) also has the advantage of overcoming the implicit formulation, which results from Eq. (11) in the mixed flow regimes. However, in these cases, it now becomes an *empirical* closure rather than an *analytical* closure. In any event, the fact that the results of the analytical closure are not in full agreement with the better empirical closure suggests that additional attention is required to better predict and understand the interfacial interactions. As stated by Wallis [2], there is no rationale for the modest agreement between their analytical results and the empirical results of Lockhart and Martinelli [1]. Despite this, the method is still widely accepted due in part to its simplicity, and in the opinion of the present authors, its analytical elegance.

2.3 Chisholm Method. Not long after Turner and Wallis [2] proposed the separate cylinder model, Chisholm [3] proposed a more rigorous analysis, which was an extension of the Lockhart–Martinelli [1] approach, except that a *semi-empirical* closure was adopted. Chisholm’s [3] rationale for his study was the fact that the Lockhart–Martinelli [1] approach failed to produce suitable equations for predicting the two phase pressure gradient, given that the empirical curves were only presented in graphical and tabular forms. Despite Chisholm’s [3] claims, he developed his approach in much the same manner as Lockhart and Martinelli [1]. After several algebraic manipulations, Chisholm [3] proposed that the two phase flow pressure gradient could be predicted using only

$$A = A_g + A_l \quad (16)$$

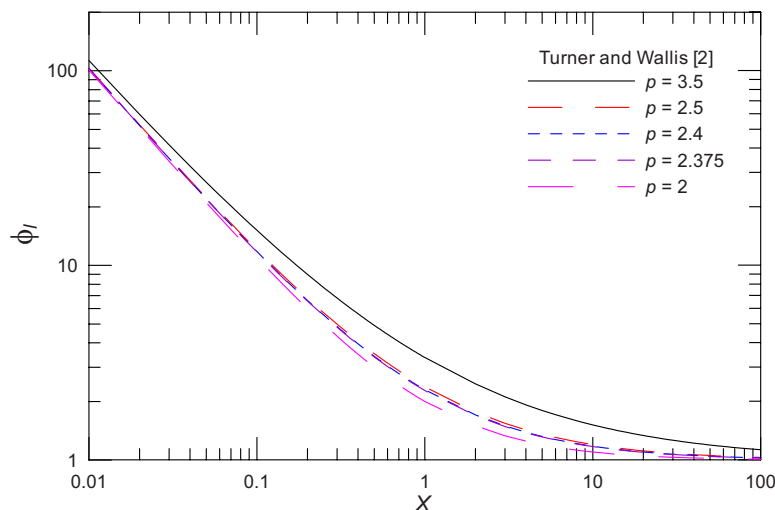


Fig. 2 Turner and Wallis’s [2] method for ϕ_l versus X for various derived and empirical values of p

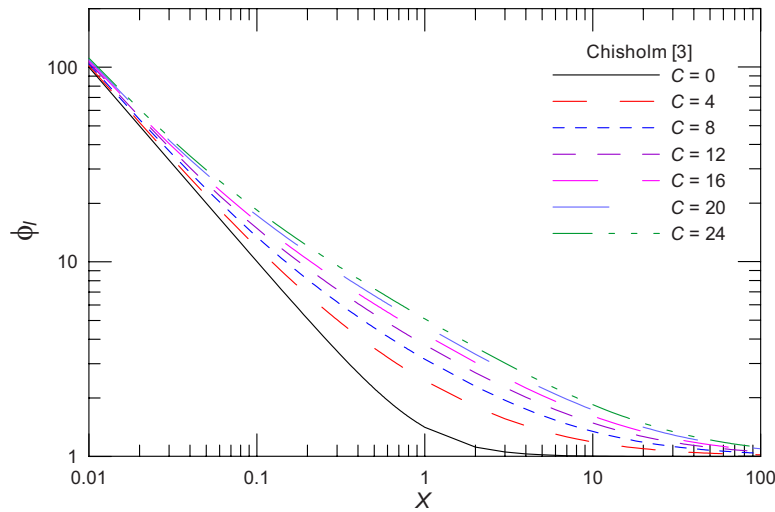


Fig. 3 Chisholm's [3] equation for ϕ_l versus X for various values of C

$$\frac{1}{\beta} = \frac{A}{A_g} - \frac{A_l}{\alpha A_g} \quad (17)$$

$$\left(\frac{A_l}{A}\right)^{(2-n)/2} \left(\frac{1}{1-A_l/A}\right)^{(2-m)/2} = \frac{X\beta^{(1+m)/2}}{Z\alpha^{(1+n)/2}} \quad (18)$$

$$\phi_l^2 = \frac{1}{\alpha^{1+n}} \left(1 + \frac{A_g}{A_l}\right)^{1-n} \left(\frac{A_g}{A_l Z^2} + 1\right) \quad (19)$$

where Z is physical parameter related to the interfacial stress referred to as a "shear force function" by Chisholm [3]. In general, Z can only be derived analytically for special cases such as annular flow and homogeneous flow. With Z known analytically or empirically, along with A_l/A and X , the above equations could be solved for α , β , A_g/A_l , and ϕ_l^2 . Chisholm [3] solved the above equations using a value $Z \approx 14$, which he determined by "trial and error," and values of A_l/A , i.e., the liquid volume fraction, as suggested by Lockhart and Martinelli [1]. Chisholm's [3] semitheoretical analysis predicted the tabulated values of the Lockhart-Martinelli [1] curves in the realm of -21% and $+16\%$ depending the flow regime.

Chisholm [3] felt that above equations were "unnecessarily complicated as far as the design engineer is concerned," and as such, in the same study he proposed the now widely adopted equations:

$$\phi_l^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \quad (20)$$

and

$$\phi_g^2 = 1 + CX + X^2 \quad (21)$$

where C was chosen to be equal to $C_{ll}=5$ for laminar-laminar flow, $C_{tt}=20$ for turbulent-turbulent flow, $C_{lt}=10$ for turbulent-laminar flow, and $C_{tl}=12$ for laminar-turbulent flow to provide agreement with the curves presented graphically in Lockhart and Martinelli's original paper [1]. These curves are shown in Fig. 1 to represent the original Lockhart-Martinelli [1] tabulated data. It should also be pointed out that the Chisholm constants have remain unchanged when Eqs. (20) and (21) are reported in the literature. They are based only on the reported data of Lockhart and Martinelli [1], as they are fits to the original tabulations presented in Ref. [1] for the four flow regimes.

However, as a one parameter model, C can be chosen to fit any one data set with greater accuracy. This is frequently done by

researchers in the open literature. The meaning of the coefficient C can be easily seen if one expands the Chisholm equations to obtain

$$\frac{dp}{dx} \Big|_{tp} = \frac{dp}{dx} \Big|_l + C \left(\frac{dp}{dx} \Big|_l \frac{dp}{dx} \Big|_g \right)^{1/2} + \frac{dp}{dx} \Big|_g \quad (22)$$

where we may now write that

$$\frac{dp}{dx} \Big|_i = C \left(\frac{dp}{dx} \Big|_l \frac{dp}{dx} \Big|_g \right)^{1/2} \quad (23)$$

is the *interfacial* contribution to the total two phase flow pressure drop. Thus, the constant C in Chisholm's model can be viewed as a weighting factor for the geometric mean of the single phase gas and liquid only pressure gradients (Fig. 3).

The constant C can be derived analytically for a number of special cases. For example, Whalley [4] obtained for a homogeneous flow having constant friction factor:

$$C = \left(\frac{\rho_l}{\rho_g} \right)^{1/2} + \left(\frac{\rho_g}{\rho_l} \right)^{1/2} \quad (24)$$

which for an air/water combination gives $C \approx 28.6$, which is in good agreement with Chisholm's value for turbulent-turbulent flows. Whalley [4] also showed that for laminar and turbulent flows with no interaction between phases the values of $C \approx 2$ and $C \approx 3.66$ are obtained, respectively. Furthermore, if a laminar plug flow is assumed, a value of $C=0$ can be easily derived, which implies that the total pressure gradient is just the sum of the component pressure gradients based on plug length and component flow rate. This is a reasonable approximation provided that plug lengths are longer than 15 diameters [5]. This observation is also in agreement with recent models [6,7] in microchannel flows, which have yielded

$$C = 21[1 - \exp(-319D_h)] \quad (25)$$

where the hydraulic diameter D_h must be specified in m. In the case of microchannels, it can be seen that $C \rightarrow 0$ as hydraulic diameter becomes small, $D_h \rightarrow 0$. English and Kandlikar [7] extended the Mishima and Hibiki [6] model to laminar-laminar flow by replacing their constant, 21, with the Chisholm value for laminar-laminar flow of 5. In general, a correlation such as Eq. (25) should be discouraged due to the "dimensional" specification of D_h , as it is easy to miscalculate C if the proper dimensions are not used for D_h , which is occasionally reported erroneously in the

open literature.

Zhang [8] found that C was dependent on the Laplace number La because of the large effect of surface tension and gap size for mini- and microchannels. As a result, the Laplace number was used instead of the hydraulic diameter D_h in the Mishima and Hibiki [6] correlation as follows:

$$C = 21 \left[1 - \exp\left(-\frac{0.358}{La}\right) \right] \quad (26)$$

where

$$La = \frac{\sqrt{\sigma/g(\rho_l - \rho_g)}}{D} \quad (27)$$

is the Laplace number. The above equation has the advantage of being in a dimensionless form.

Additional formulations and correlations for the closure parameter C have functions of properties, mass flux, and quality that can be found in the open literature [9]. For example, Sun and Mishima [9] found that C was strongly affected not only by La but also by Re_l in the laminar flow region. As a result, the following expression was obtained for C in the laminar flow region:

$$C = 26 \left(1 + \frac{Re_l}{1000} \right) \left[1 - \exp\left(-\frac{0.153}{0.27 La + 0.8}\right) \right] \quad (28)$$

Their statistical analysis showed that C changed with Reynolds number. Moreover, the term C/X in the Chisholm correlation strongly depended on the ratio Re_g/Re_l , especially when Re_l or Re_g is over 2000. In addition, it can be found that the data points become more scattered with the increase in the ratio Re_g/Re_l . Based on the statistical analysis, they modified the Chisholm correlation as follows:

$$\phi_l^2 = 1 + \frac{1.79 \left(\frac{Re_g}{Re_l} \right)^{0.4} \left(\frac{1-x}{x} \right)^{0.5}}{X^{1.19}} + \frac{1}{X^2} \quad (29)$$

They pointed out that the C value calculated by Eq. (29) would not sharply change while $x \rightarrow 1$ or $x \rightarrow 0$ because Re_l or Re_g changed inversely with x and $1-x$, respectively. Particularly, for their database, x ranged from 1.5×10^{-5} to 0.98, while the calculated value of C was found to vary from 29.9 to 3.2, accordingly. Despite this extensive correlation of C , Sun and Mishima [9] concluded that their model is only marginally better than other models in many cases, and only slightly less accurate than the widely adopted Mueller–Stainhagen and Heck model [9]. Thus, correlating C as a function of other variables appears to offer little added value.

Additional extended Chisholm type models are reviewed by Sun and Mishima [9] and Awad [10].

2.4 Modified Turner–Wallis Model. Awad and Muzychka [11] arrived at the same simple form as the empirical Turner and Wallis [2] model, but with a different physical approach. Rather than model the fluid as two distinct fluid streams flowing in separate pipes, they proposed that the two phase pressure drop could be predicted using a nonlinear superposition of the component pressure drops that would arise from each stream flowing alone in the *same* pipe, through application of the Churchill–Usagi asymptotic correlation method [12]. This form was asymptotically correct for either phase as the mass quality varied from $0 < x < 1$. Furthermore, rather than approach the Martinelli parameter from the point of view of the four flow regimes using simple friction models, they proposed using the Churchill [13] model for the friction factor in smooth and rough pipes for all values of the Reynolds number. In this way, the proposed model was more general and contained only one empirical coefficient, the Churchill–Usagi [12] blending parameter. The resulting model takes the form

$$\frac{dp}{dx} \bigg|_{tp} = \left[\left(\frac{dp}{dx} \bigg|_l \right)^q + \left(\frac{dp}{dx} \bigg|_g \right)^q \right]^{1/q} \quad (30)$$

or when written as a two phase liquid multiplier:

$$\phi_l^2 = \left[1 + \left(\frac{1}{X^2} \right)^q \right]^{1/q} \quad (31)$$

which is the same as Eq. (13) from the Turner–Wallis approach, when $q=1/p$. The main exception is that the authors [11,14–16] developed values of q for different flow regimes using the following friction model to calculate X^2 :

$$f = 2 \left[\left(\frac{8}{Re_D} \right)^{12} + \left(\frac{1}{(A_1 + A_2)^{3/2}} \right) \right]^{1/12} \quad (32)$$

where

$$A_1 = \left\{ 2.457 \ln \left[\frac{1}{(7/Re_D)^{0.9} + (0.27k/D)} \right] \right\}^{16} \quad (33)$$

and

$$A_2 = \left(\frac{37530}{Re_D} \right)^{16} \quad (34)$$

The principal advantages of the above approach over the Turner–Wallis [2] method are twofold. First, all four Lockhart–Martinelli flow regimes can be handled with ease since the Turner–Wallis [2] method leads to implicit relationships for the two mixed regimes. Second, since the friction model used is only a function of Reynolds number and roughness, broader applications involving rough pipes can be easily modeled. Using the above equations, Awad [10] found that $q \approx 0.307$ for large tubes and $q \approx 0.5$ for microchannels, minichannels, and capillaries. The general characteristics of Eq. (31) are shown in Fig. 4 for a range of values of q . Thus, the extended Turner–Wallis approach is also a one parameter correlating scheme. Awad and Butt [14–16] showed that the method works well for liquid–liquid flows and flows through porous and fractured media for petroleum industry applications.

Approximate equivalence between Eq. (20) and Eq. (31) can be found when $q=0.36, 0.3, 0.285$, and 0.245 when $C=5, 10, 12$, and 20 , respectively. This yields differences of 3–9% rms. The special case of $q=1$ leads to a linear superposition of the component pressure drops, which corresponds to $C=0$. This limiting case is only valid for plug flows when plug length to diameter ratios exceed 15 [5].

2.5 Modified Chisholm Models. Finally, in a recent series of studies by Saisorn and Wongwises [17–19], a new correlation was proposed having the form

$$\phi_l^2 = 1 + \frac{6.627}{X^{0.761}} \quad (35)$$

for experimental data for slug flow, throat-annular flow, churn flow, and annular-rivulet flow [17], and

$$\phi_l^2 = 1 + \frac{2.844}{X^{1.666}} \quad (36)$$

for experimental data for annular flow, liquid unstable annular alternating flow (LUAFA), and liquid/annular alternating flow (LAAF) [18]. These correlations neglect the $1/X^2$ term, which represents the limit of primarily gas flow in the Lockhart–Martinelli [1] formulation. Neglecting this term ignores this important limiting case, which is an essential contribution in the Lockhart–Martinelli modeling approach. As a result, at low values of X , the proposed correlations undershoot the trend of the data, limiting their use in the low X range. Thus, a more appropriate and generalized form of the above correlations should be

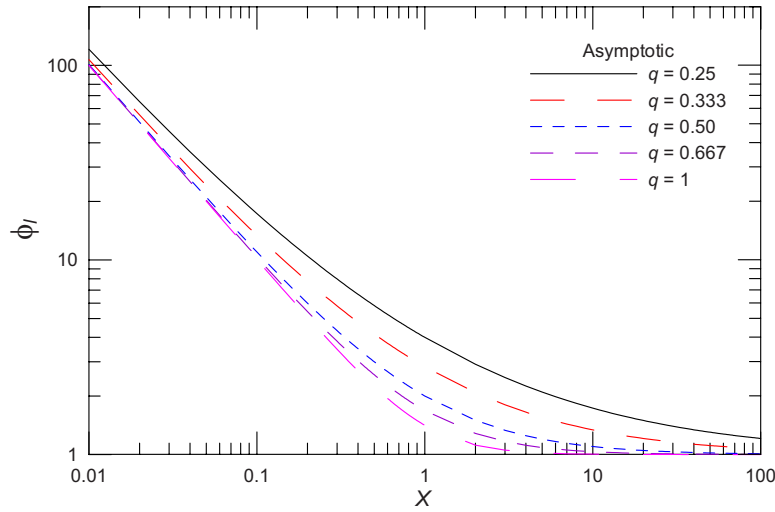


Fig. 4 Comparison of Awad and Muzychka [8] equation for various q

$$\phi_l^2 = 1 + \frac{A}{X^m} + \frac{1}{X^2} \quad (37)$$

or

$$\phi_g^2 = 1 + AX^m + X^2 \quad (38)$$

which are the focus of Sec. 3 for decomposing two phase flow pressure drop into three basic components.

3 Asymptotic Decomposition of Two Phase Flow

Gas-liquid two phase flow will be examined from the point of view of interfacial pressure drop. Recognizing that in a Lockhart–Martinelli [1] reduction scheme single phase flow characteristics must be exhibited in a limiting sense, they will be subtracted from the experimental data being considered to illustrate some benefits of using the one and two parameter models. For this purpose, we utilize some recent data for microtubes and the original data presented in the paper of Lockhart and Martinelli [1]. While not exhaustive, the general idea of this modeling approach is illustrated.

We can define the two phase flow pressure drop as a linear combination of three simple pressure drops or pressure gradients. These are the single phase liquid, single phase gas, and interfacial pressure drop. The rationale for such a choice lies in the definition of the Lockhart–Martinelli approach, whereby, for small and large values of the Martinelli parameter X , one obtains single phase flow, while in the transitional region between $0.01 < X < 100$, the interfacial effects result in an increase in ϕ^2 over the simple linear superposition of the single phase flow contributions. This is where a large scatter in data occurs depending upon flow regime or pattern.

Beginning with

$$\left. \frac{dp}{dx} \right|_{tp} = \left. \frac{dp}{dx} \right|_l + \left. \frac{dp}{dx} \right|_i + \left. \frac{dp}{dx} \right|_g \quad (39)$$

Introducing the two phase liquid multiplier as proposed by Lockhart and Martinelli [1], we obtain

$$\phi_l^2 = 1 + \phi_{l,i}^2 + \frac{1}{X^2} \quad (40)$$

where

$$\phi_{l,i}^2 = - \frac{\left. \frac{dp}{dx} \right|_i}{\left. \frac{dp}{dx} \right|_l} \quad (41)$$

is a two phase multiplier for the interfacial pressure drop. This can be viewed as an extended form of the Chisholm model, where the interfacial contribution is what is to be modeled.

3.1 One Parameter Models. Comparison with the Chisholm [3] formulation gives

$$\phi_{l,i}^2 = \frac{C}{X} \quad (42)$$

for the liquid multiplier formulation or

$$\phi_{g,i}^2 = CX \quad (43)$$

for the gas multiplier formulation.

This represents a simple one parameter model, whereby closure can be found with comparison with experiment. In addition, the simple asymptotic form of Eq. (31) also represents a one parameter model. If the interfacial effects can be modeled by Chisholm's proposed model or Eq. (31), then all of the reduced data should show trends indicated by Eq. (42) or Eq. (43). However, if data do not scale according to Eq. (42) or Eq. (43), i.e., a slope of -1 or $+1$, then a two parameter model is clearly required. This is shown in Fig. 5, which shows that all interfacial effects have the same slope but their intensity is governed by the value of C .

3.2 Two Parameter Models. The above equations may be extended to develop a simple two parameter power law model such that

$$\phi_{l,i}^2 = \frac{A}{X^m} \quad (44)$$

or

$$\phi_{g,i}^2 = AX^m \quad (45)$$

leading to Eq. (37) or Eq. (38).

These forms have the advantage that experimental data for a particular flow regime can be fitted to the simple power law after the removal of the single phase pressure gradients from the experimental data. Additional modifications may be introduced if the coefficients A and m can be shown to depend on other variables

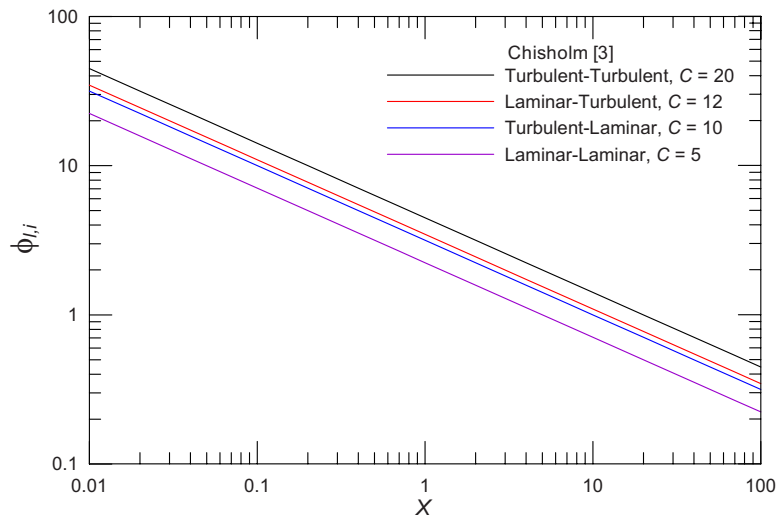


Fig. 5 Interfacial two phase multiplier for different flow regimes for the Chisholm model

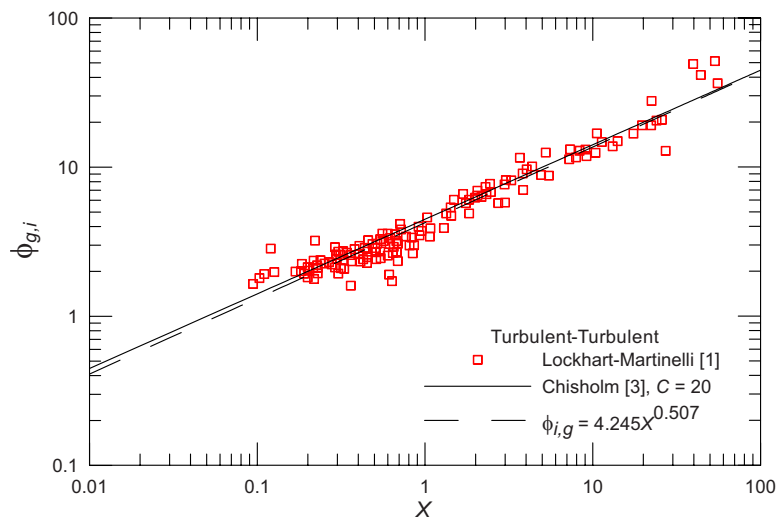


Fig. 6 Interfacial two phase multiplier for different flow regimes

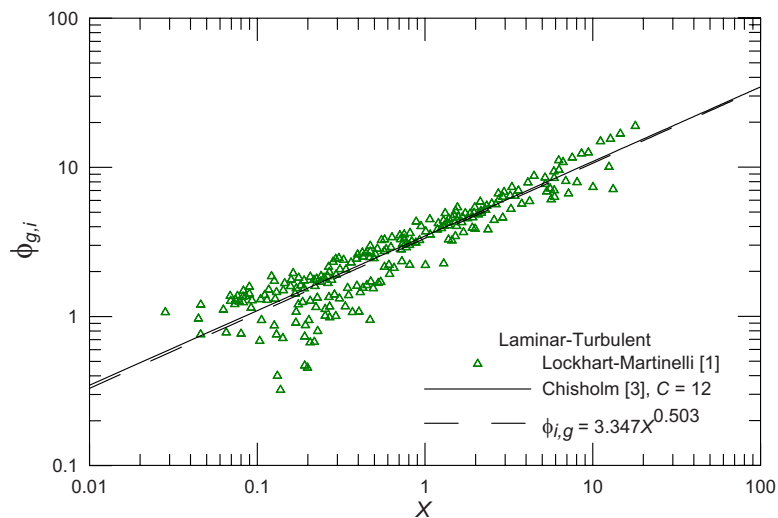


Fig. 7 Interfacial two phase multiplier for different flow regimes

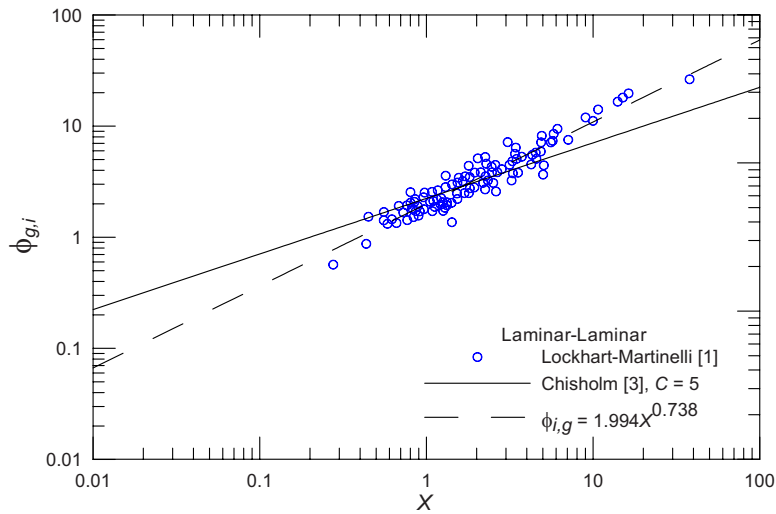


Fig. 8 Interfacial two phase multiplier for different flow regimes

such as mass flow rate, fluid properties, and mass quality or void fraction. These issues will be examined next considering a few data sets from the open literature.

4 Comparisons With Data

In this section, we will apply the approach outlined above to a few selected data sets. Given that two phase flow data are widely reported using the Lockhart–Martinelli parameters ϕ_l and X , we can present two phase flow data simply as an interfacial two phase multiplier using

$$\phi_{l,i}^2 = \phi_l^2 - 1 - \frac{1}{X^2} \quad (46)$$

Beginning with the data originally reported in Lockhart and Martinelli's [1] original paper, we consider the interfacial two phase multiplier for the flow regimes defined by Lockhart and Martinelli. Figures 6–8 present the data for turbulent-turbulent,

laminar-turbulent, and laminar-laminar flows. In the first two cases, one can see that the Chisholm one parameter model, which is proportional to X , gives good results. In the case of laminar-laminar flow, better agreement can be found with the two parameter modeling approach since the data are not directly proportional to X , as Eq. (43) implies. The results are summarized in Table 2 along with comparisons to the original Chisholm and Turner–Wallis one parameter models. Additionally, we present the best values of the Chisholm coefficient for each data set, given that the original Chisholm constants were not based on a more substantial data set.

All of the data in Figs. 6–8 are plotted with the alternate two parameter model for the full two phase flow multiplier in Fig. 9. Additional data in the Appendix of Lockhart and Martinelli's paper [1] was provided by Jenkins [20], which highlight the fact that even within the turbulent-turbulent flow regime, trends can be seen if the data are grouped by liquid mass flow rate. These data

Table 2 Comparison of one and two parameter models for ϕ

Flow	Two parameter			One parameter ^a		One parameter ^b		One parameter	
	A	m	%rms	C	%rms	C	%rms	q	%rms
Lockhart and Martinelli [1] for ϕ_g									
Laminar/laminar	3.98	1.476	13.49	5	16.35	5	16.35	0.370	15.53
Laminar/turbulent	11.20	1.060	20.01	12	20.87	10	19.51	0.290	19.62
Turbulent/turbulent	18.02	1.014	15.76	20	17.67	16	15.08	0.244	14.21
All data	10.97	1.000	25.61	8	23.86	8	23.86	0.299	23.12
Jenkins [20] for ϕ_g									
$\dot{m}=75.75$ kg/h	4.24	0.300	5.13	20	17.94	15	15.36	0.260	10.65
$\dot{m}=333.39$ kg/h	11.53	0.558	7.87	20	22.37	14	16.22	0.253	14.19
$\dot{m}=726.65$ kg/h	19.56	0.832	4.24	20	7.42	19	7.15	0.227	7.36
All data	15.26	0.811	13.50	20	17.34	16	14.64	0.247	12.99
Saisorn and Wongwises [17–19] for ϕ_l									
Slug	2.06	0.453	13.69	5	24.35	2	15.13	0.465	14.44
Churn	6.27	0.278	8.11	5	17.40	8	10.01	0.303	9.34
Throat-annular	2.69	1.740	11.71	5	26.38	2	11.93	0.476	11.45
Annular-rivulet	0.0795	1.075	22.50	5	26.14	1	6.28	0.606	6.08
Annular flow	2.45	0.308	24.45	5	19.41	3	10.73	0.465	10.30
LUAAF	0.908	0.870	18.43	5	47.50	1	7.53	0.645	7.36
LA AF	1.49	0.678	9.82	5	37.08	2	6.65	0.541	4.59
All data	3.21	1.042	18.14	5	27.16	2	14.26	0.488	14.13

^aActual Chisholm constant.

^bBest Chisholm constant.

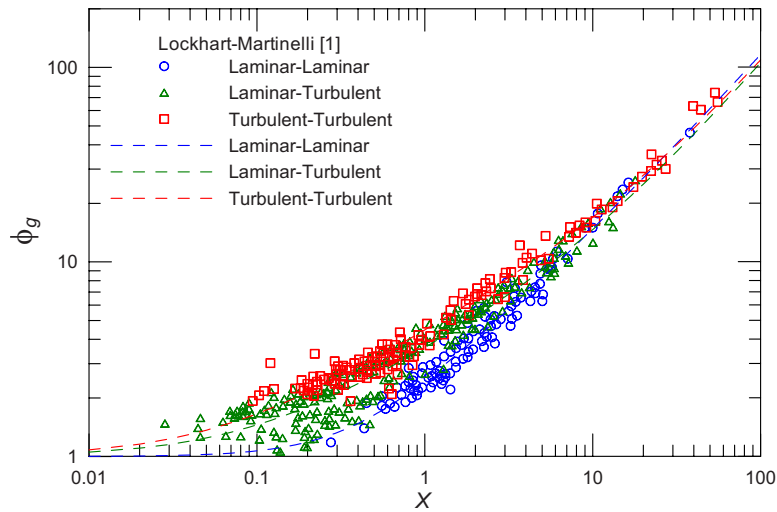


Fig. 9 Two phase multiplier for all data based on different flow regimes

are re-analyzed using Eq. (38) and the two parameter interfacial two phase multiplier is given in Fig. 10 along with the single parameter Chisholm model.

It is easily seen that as mass flow rate increases, the interfacial pressure drop increases with an observed increase in the slope of the curve. All of the data are plotted in Fig. 11 as a full two phase flow multiplier. The accuracy of the various approaches are given in Table 2 for all of the data reported in Ref. [1]. It is seen that the two parameter empirical approach is somewhat better for correlating data.

Finally, we present some data for capillary tubes obtained from Refs. [17,18]. These data are reported based on the flow pattern, i.e., slug, annular, and churn. Data are plotted in Figs. 12–14 for the two parameter model, where it can be seen that the various annular flow patterns have similar interfacial behavior. The greatest variation is observed with the drastically different flow patterns of slug, churn, and annular, as shown in Fig. 12. The various accuracies for both one and two parameter models are given in Table 2. It can be seen as the flow rate increases and hence a tendency toward turbulent flow, the interfacial effects increase the two phase flow multiplier systematically, hence an increase in the Chisholm parameter C and increases in the A and m . Similarly, a decrease in q is observed in the asymptotic based one parameter model. This is not as evident with the subjective flow pattern

analysis using the data of Saisorn and Wongwises [17,18]. Also shown in Figs. 12–14 is the interfacial parameter derived from the correlations given in Refs. [17,18]. The correlation departs from the trends of the given data and fits due to the fact that the authors [17,18] did not include the single phase gas flow limit in their development. When presented in this manner, one can see that large errors will occur in the region of $0.01 < X < 1$. It can also be seen that the correlation does not adequately account for the interfacial effects at higher values of X as the trend in the data do not follow the correlation properly. Thus, any two phase flow modeling using empirical or semi-empirical models must include the asymptotic limits of single phase flow for both the gas and liquid phases.

5 Summary and Conclusions

Two phase flows in pipes, minichannels, and microchannels was considered. A review of the classic Lockhart–Martinelli method and other asymptotic representations of two phase flow was examined. These approaches can all be considered one and two parameter models. The two parameter models offer more flexibility for developing empirical models for specific flow regimes when the simple Chisholm model fails to capture the slope of the interfacial two phase multiplier. The above approaches were com-

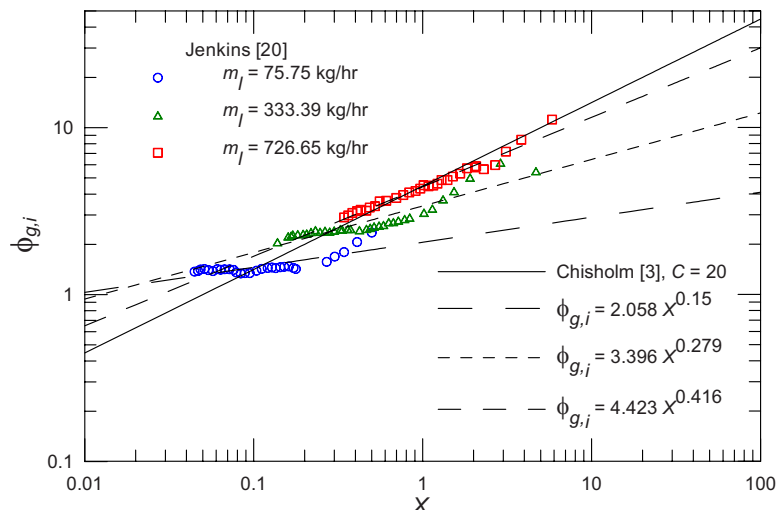


Fig. 10 Interfacial two phase multiplier for different flow rates

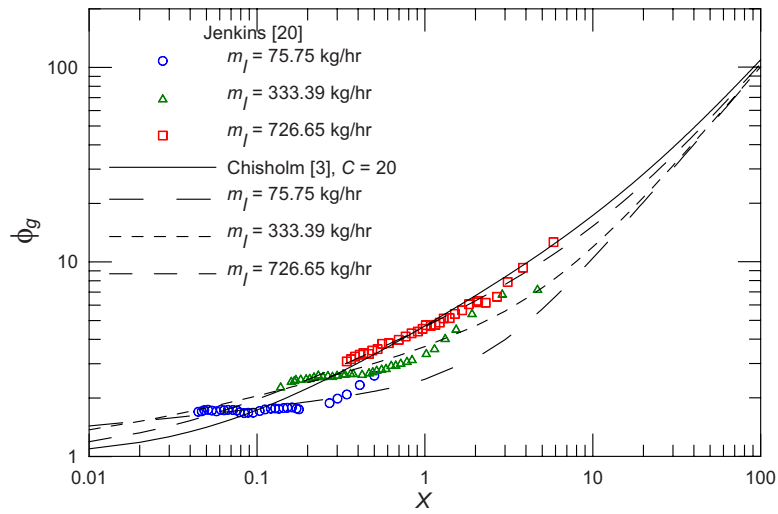


Fig. 11 Two phase multiplier for different flow rates

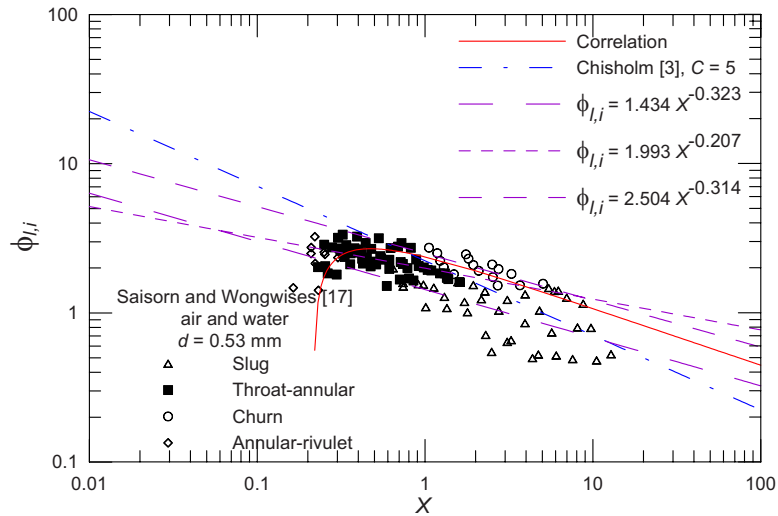


Fig. 12 Interfacial two phase multiplier for different flow patterns

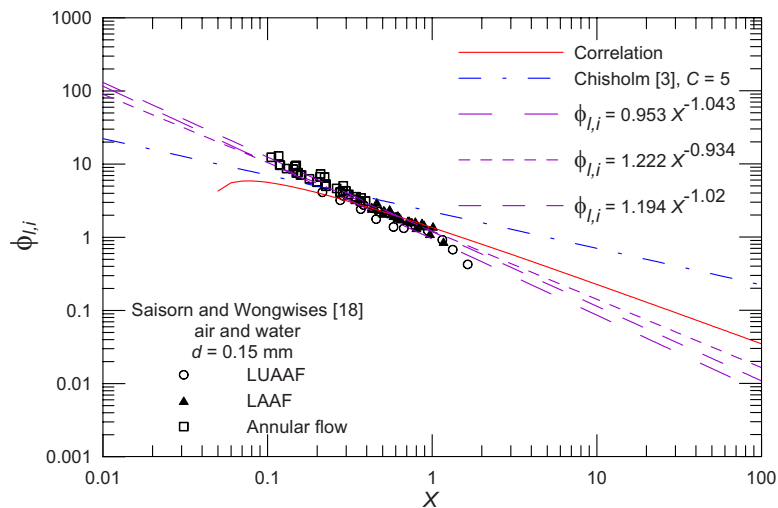


Fig. 13 Interfacial two phase multiplier for different flow patterns

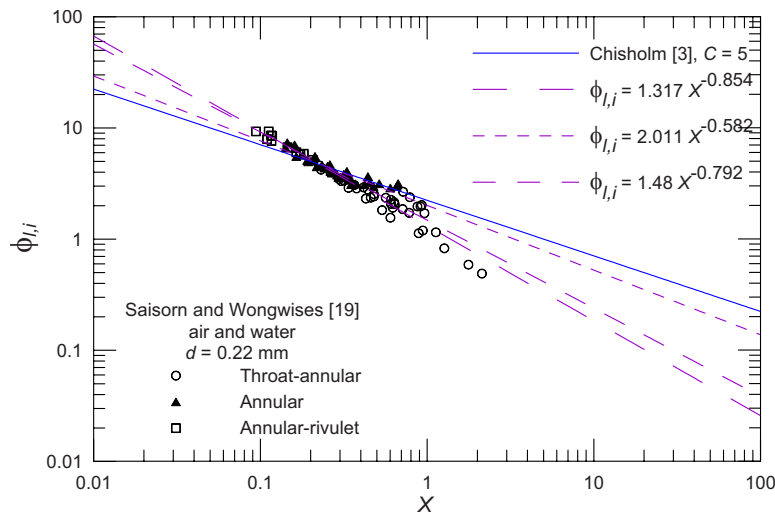


Fig. 14 Interfacial two phase multiplier for different flow patterns

pared for a few selected data sets and it was found that the proposed two parameter modeling approach is the best and offers the greatest flexibility. However, the modified Turner–Wallis approach of Awad and Muzychka [11] also provides comparable prediction of data sets.

Furthermore, as shown by Sun and Mishima [9], the return on investment in more complicated empirical modeling schemes, which include additional physical variables in the prediction of Chisholm constants, is not always positive, making the simple approaches reviewed in this paper a more viable approach. In fact, the approach used by Sun and Mishima [9] cannot guarantee better accuracy unless it is applied to a phenomenological case, such as a two phase plug flow, as shown in Ref. [5]. Sun and Mishima [9] even concluded their paper with the statement that their model only provides comparable accuracy to the popular Mueller–Steinhagen and Heck model despite having many more variables.

Two phase flow modeling from the point of view of interfacial phenomena is deemed to be a better approach from the standpoint of understanding what variables will affect the data in the interfacial region. The true impact of the two phase flow interfacial effects are more clearly seen, only after the removal of the single phase flow contributions from the Lockhart–Martinelli two phase multiplier. This is evident from Figs. 12–14, where the assumed Chisholm model clearly does not model the interfacial effects adequately. Decomposing the Lockhart–Martinelli approach into single phase and interfacial components would appear to provide better understanding of experimental data and lead to better model/correlation development.

Acknowledgment

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Nomenclature

A	= flow area, m^2
C	= Chisholm constant
C_g	= gas phase friction constant
C_l	= liquid phase friction constant
D_h	= hydraulic diameter, m
D	= diameter of circular duct, m
f	= fanning friction factor
k	= pipe roughness, m
L	= length of tube, m
La	= Laplace number

\dot{m}	= liquid mass flow rate, kg/s
m	= gas phase friction exponent
n	= liquid phase friction exponent
p	= pressure, Pa
p , Eq. (13)	= blending parameter, Eq. (13)
P	= perimeter, m
q	= blending parameter, Eq. (31)
Re_D	= Reynolds number, $\equiv UD/\nu$
U	= average liquid velocity, m/s
x	= mixture mass quality
x	= axial position in a duct
X	= Martinelli parameter

Greek Symbols

α	= liquid area ratio
β	= gas area ratio
ϕ	= two phase multiplier
ρ	= fluid density, kg/m^3
σ	= surface tension, N/m

Superscripts

*	= dimensionless
(\cdot)	= mean value

Subscripts

g	= gas
i	= interfacial
l	= liquid
tp	= two phase

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