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An improved formulation for speed of sound in two-phase systems and development of 1D model for supersonic nozzle

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ABSTRACT

The Speed of sound is one of the most prominent parameters in the fluid dynamics and aerodynamic studies. A major challenge lies in the fact that two-phase region properties are dramatically different from the single phase condition and the simple models do not satisfy the adequate accuracy required for the sound speed prediction. The present research explores various attempts for calculating the speed of sound and suggests a new practical equation which is correlated with fluid properties and it is developed by genetic Algorithm. The obtained results are encouraging and the final derived model has the most accuracy for the two-phase region covering different types of thermodynamic processes. In addition, the new model and the classic 1D model are combined to perform an analysis of the role of the two-phase sound speed consideration. The temperature and the pressure profiles inside a converging - diverging nozzle are investigated, representing the phase-change phenomena inside the supersonic nozzle. The related Mach number and the nozzle performance are studied using the proposed model.

1. Introduction

During recent years, research on the topic of the speed of sound has been focused on investigating different gases and liquids including both the pure and the mixture ones. The speed of sound in non-ideal condition is highly dependent on the thermodynamic conditions. Therefore, in addition to the type of the fluid, the effect of temperature and pressure on the speed of sound in different fluids is studied by the researchers as Tat et al. [1] who analized isentropic bulk modulus and sound speed of mixtures in a wide range of pressure.

The issue is raised when the number of components or phases is higher than one. In other words, either multicomponent or multi-phase flow is a new area of research for the speed of sound investigation. Based on the literature, there are lots of studies over the past years for sound speed prediction through different media, representing the fact that two-phase region has a dramatically different speed of sound than either pure fluids or mixtures in single phase. The sound speed in mixtures depends on the gas fraction, the pressure, temperature, the frequency of the sound wave, and, for small bubbles, on bubble radius (Werner, 1977 [2]).

As application of this topic, the simulation, the basic design and the construction of devices working with compressible flows extremely rely on the speed of sound. Nozzles, jets, wind tunnels and other devices dealing with shockwaves or aerodynamics are the best-known examples of such devices. A novel application of the supersonic nozzle is moisture separation and the performance highly depends on the Mach number of the gas flow (Niknam et al., 2016 [3]).

Also, the issue of sound speed in two-phase region for the special case of supersonic nozzle working with natural gas was raised by Secchi et al. [4] who developed a modified 1D model for prediction of the dynamic behavior of the nozzle. The supersonic nozzle involves the multiphase flow condition, as the condensation happens when the gas crosses the phase envelope of the mixture due to the expansion in the supersonic condition, which cause a large temperature drop. Thus, both single and two-phase conditions are found in a supersonic nozzle and it is an appropriate case study to explore the importance of the speed of sound and evaluate the associated models in two-phase region.

The speed of sound waves in a medium depends on the compressibility and the density. If the medium is a liquid or a gas having a bulk modulus β and density ρ, the speed of sound waves, c, in that medium using general equation of state and classical

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Nomenclature

x	Vapor quality (mass fraction)
f_v	vapor fraction (mole fraction)
f_i	fraction of component i in fluid
a, b	parameter of Peng-Robinson equation of state
c	Speed of sound (m/s)
d	axial distance form the nozzle entrance
e	Entrained droplets in vapor phase fraction
m, n	Correlated coefficients
k	Polytropic index
K_l	Coefficient of compressibility (1/Pa)
M	Mach number (–)
M_w	Molecular weight (g/mol)
R	Ideal gas constant, J mol^{-1} K^{-1}
$\widehat{R}$	Specific ideal gas constant, J kg^{-1} K^{-1}
V	Volume
P	Pressure, Pa
T	Temperature, K
Z	Compressibility factor
$c_{i,j}$	calculated sound speed of ith data in jth set
$c'_{i,j}$	experimental sound speed of ith data in jth set

Greeks

α	void fraction
Φ	Mole/volume fraction
γ	Adiabatic index
β	Bulk modulus
ρ	Molar density

Subscripts

0	stagnation condition
g	Gas phase
l	Liquid phase
in	nozzle inlet
out	nozzle outet
x	upstream of shockwave
y	downstream of shockwave
s	Constant Entropy
P	Constant pressure
T	Constant temperature
m	Mixture
$i = 1, 2\ldots n$	Data index
$j = 1, 2\ldots m$	Set of data index

mechanics is given by:

$$c = \sqrt{\frac{\beta_s}{\rho}} \tag{1}$$

and β_s is the bulk modulus, which is the change in pressure divided by the relative change in volume, having the same dimension as the pressure and derived as Eq. (2):

$$\beta_s = \rho\left(\frac{\partial P}{\partial \rho}\right)_s \tag{2}$$

It can be calculated using isothermal approach by the pressure derivative as:

$$\beta_s = \gamma\beta_T = \gamma\rho\left(\frac{\partial P}{\partial \rho}\right)_T \tag{3}$$

The obtained equation is applicable for both single phases, including gas and liquid. Also, sound speed in gas phase can be derived using the real gas equation as:

$$c_g = \sqrt{\frac{Z\gamma RT}{M_w}} \tag{4}$$

In which the adiabatic index (γ) is calculated using

$$\gamma = \frac{C_p}{C_p - R}. \tag{5}$$

Minnaert et al. [5] used the following mixing rule for speed of sound in the two-phase region:

$$c = \left(\rho_m \sum_{i=1}^{N} \frac{\alpha}{\rho_i {c_i}^2}\right)^{-\frac{1}{2}} \tag{6}$$

in which i is the phase index and the mix density ρ_m is calculated by:

$$\rho_m = \left[(1-\alpha)\rho_l + \alpha\rho_g\right] \tag{7}$$

Also, from the isentropic relationship of a real gas we have:

$$\frac{\partial P}{\partial \rho} = \gamma\frac{P}{\rho} \tag{8}$$

Combination and substitution of Eqns. (1), (2), (7) and (8) into Eq. (6) and assuming general polytrophic behavior of the gas ($\rho^k \propto P$) give Eq. (9), which was firstly presented by Minnaert et al. [5]. The model predict the sound speed in the two-phase region, which is one of the most remarkable features of gas/liquid or mixtures. The basic form of the equation includes two types of the average term including a linear average formulation for the density of phases and a thermodynamic-based average calculation for the pressure, which is based on the procedure reported in literature:

$$c = \left(\left[(1-\alpha)\rho_l + \alpha\rho_g\right]\left[\frac{\alpha}{kp} + \frac{(1-\alpha)}{\beta_s}\right]\right)^{-\frac{1}{2}} \tag{9}$$

Michaelides et al. [6] studied a computational method with thermodynamic basis for the calculation of the speed of sound in two-phase mixtures. Picard et al. [7] investigated the speed of sound problem using an iterative isentropic flash calculation method. Two more leading numerical investigations carried out by Firoozabadi [8]) and Firoozabadi and Pan [9], generalized sonic velocity problems using thermodynamic state functions and considering the capillary pressure on the phase behavior of multicomponent mixtures. They evaluated the proposed model with both air-water and hydrocarbon mixtures. This approach is similar to that of Castier [10], because in both it is necessary to find the thermodynamic properties derivatives. Also, there are simplified approaches which concluded to correlations. One of the well-known ones is the correlation introduced by Wood [11]. It is based on a modified mixing function of two phases sonic velocities including the liquid phase sound speed, c_l, and the gas phase sound speed, c_g.

This Wood's law is widely cited in the literature and is commonly used for the evaluation of new models in other research works.

The Wood's law uses mixture density as:

$$\rho_m = \varnothing_l \rho_l + (1-\varnothing_l)\rho_g \tag{10}$$

where $\varnothing_l$ is the mole fraction of the liquid phase and the speed of sound is calculated from:

$$c = \left(\rho_{mix}\left[\frac{\varnothing_l}{\rho_l {c_l}^2} + \frac{1-\varnothing_l}{\rho_g {c_g}^2}\right]\right)^{-\frac{1}{2}} \tag{11}$$

Nichita et al. [12] stated that Wood's law fails to predict the abrupt changes in sound speed close to phase boundaries and he took a different path to solve the problem by means of flash routine and total enthalpy expression.The Wood's framework was later modified by Zhu et al. [13], who proposed an adapted mixing rule for the consideration of the sound speed in the two-phase region:

$$c = \left(\frac{\alpha}{c_g^2} + \frac{(1-\alpha)^2}{c_l^2} + \frac{\alpha(1-\alpha)\rho_l}{\gamma p}\right)^{-\frac{1}{2}} \tag{12}$$

He recommended the combination function of c_g and c_l with different weighted void fraction, α, coefficients and correlated power values. An analogous correlation with non-symmetrical terms for gas and liquid phases was proposed by Lamarre [14].

$$c = \left(\frac{\alpha^2\gamma}{c_g^2} + \frac{(1-\alpha)^2}{c_l^2} + \alpha(1-\alpha)\left(\frac{\rho_l}{p} + \rho_l K_l\right)\right)^{-\frac{1}{2}} \tag{13}$$

where K_l is the Coefficient of compressibility and equals to reciprocal of bulk modulus. Furthermore, several investigations attempt to compare various equation of State (EoS) and find the most accurate one for a specific type of fluids (Salimi et al. [15], Botros [16]).

Most of the models for the sound speed use the void fraction as the independent variable and it is expected that the accuracy of the model highly depends on the input, which is calculated by some other associated models. In other words, sound speed correlation is as reliable as the input information that is adopted to determine the void fraction. In this regards, one of the most accurate models for the void fraction calculation is used for further evaluation steps.

The Zivi void fraction expression [17] has good agreement with the experimental data (Hashizume et al. [18] and Tandon et al. [19]) and it is expressed as:

$$\alpha = \frac{1}{1 + \frac{1-x}{x}\left(\frac{\rho_g}{\rho_l}\right)^{2/3}} \tag{14}$$

Zivi [17] also proposed a more complex model containing the fraction of the liquid entrained as droplets in vapor phase (e):

$$\alpha = \frac{1}{1 + e\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right) + (1-e)\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)^{\frac{2}{3}}\left[\frac{1+e\left(\frac{1-x}{x}\right)\left(\frac{\rho_g}{\rho_l}\right)}{1+e\left(\frac{1-x}{x}\right)}\right]^{1/3}} \tag{15}$$

If $e = 0$ then the above expression reduces to the prior expression of Zivi [17] and if $e = 1$, the expression reduces to the homogeneous void fraction equation. The sensitivity analysis for the value of e is reported in the next section.

Thus, substitution of the calculated void fraction by Zivi [17] equation for the related parameter in Minnaert [5] equation predicts the sound speed value. Also, the bulk modulus and phases' densities are calculated using the well-known Peng-Robinson EoS:

$$P = \frac{\rho RT}{1-b\rho} - \frac{a\rho^2}{1+2b\rho-b^2\rho^2} \tag{16}$$

where P is the pressure, ρ is the density, R is the gas constant, T is the absolute temperature, and a and b are coefficients for the Peng-Robinson equation of state. They can be calculated knowing the critical temperature T_c and pressure P_c of the gas under pressure, and are given by the following relationships.

2. Methods

2.1. Proposed model for speed of sound

There are different experimental data and different numerical results in the literature for the determination of sound speed in the two-phase region. After reviewing the available models, this research attempts to perform a modification or even a new type of semi-empirical equations to have a general model covering different cases. In this effort, all the available literature data are employed and the optimization routine will beneficiary of a worthy database to find an all-purpose function which calculates an accurate speed of sound for different process types.

Both Lamarre [14] and Zhu et al. [13] tried for generalizing the speed of sound for their own experimental cases. These two pieces of research used dissimilar coefficients of two phases' properties in their final model. This fact allows us to use correlated coefficients with no resemblance in appearance.

It is proposed to consider power value to give sufficient weight to the density and the pressure, looking for finding a well-correlated model. Therefore, the speed of sound in a two-phase mixture fluid is represented by the modified form of:

$$c = \left(\left[(1-\alpha)^{m_1}\rho_l + \alpha^{m_2}\rho_g\right]\left[\frac{\alpha^{m_3}}{kp} + \frac{(1-\alpha)^{m_4}}{\beta_s}\right]\right)^{-\frac{1}{2}} \tag{17}$$

It's also valid that use of volume fraction instead of the void fraction with the same procedure gives the following equation (Flåtten et al. [20]); in the present research, mole/volume fraction is found preferable to that in void fractions.

$$c = \left(\left[(1-\varnothing)^{n_1}\rho_l + \varnothing^{n_2}\rho_g\right]\left[\frac{\varnothing^{(n_3)}}{kp} + \frac{(1-\varnothing)^{n_4}}{\beta_s}\right]\right)^{-\frac{1}{2}} \tag{18}$$

The same as Minnaert model [5], the proposed one for $\varnothing = 1$ becomes simple ideal gas relationship for vapor, as $c = \left(\frac{\rho_g}{\gamma p}\right)^{-\frac{1}{2}}$ and considering $\varnothing = 0$ it becomes the simple form for liquid as $c = \left(\frac{\rho_l}{\beta_s}\right)^{-\frac{1}{2}}$.

The new correlation form needs to be fitted by the optimization algorithm based, which used sufficient reliable data.

In the present case, an objective function which should be minimized is defined as Eq. (19):

$$Min\ : \sum_{j=1}^{m}\sum_{i}^{n}\left(\left(c_{i,j} - c'_{i,j}\right)^2\right) \tag{19}$$

Many kinds of methods can be considered such as calculus-based methods and random search algorithms to optimize objective

Table 1
The fitted coefficients of proposed model for Two-phase region sound speed found by GA.

coefficient	value
n_1	2.49
n_2	−1.19
n_3	0.39
n_4	9.51

functions. Algebraic methods have been studied heavily, but not effectively, for the objective function of Eq. (19), because of lacking information on the gradient. Although random search approaches require a high computational cost to search and save the best global solution, in recent years much effort has been contributed to use random search algorithms. Genetic algorithm (GA) is a search algorithm based on the mechanisms of natural selection and natural genetics. In the same way, prominent from the GA features is that it works with the objective function and doesn't require derivatives. It searches from a population instead of a single point and uses probabilistic transition rules rather than deterministic rules.

Following of the analogy of biological mechanisms, an individual in GA defined as a chromosome with as many genes as those representing all unknown parameters, which are a = (a1, a2, a3, a4) for the fitness function of Eq. (19). The individuals are then evaluated and ranked and, in every generation, a new set of artificial creatures is created using genes of the old generation by reproduction, crossover, and mutation. The possibility of survival of a gene depends on its fitness value and there is a mutation operator that will randomly make one bit of a gene changed from 0 to 1 or vice versa. GA is a modified random walk which uses the existing historical information efficiently to produce a better generation with expected improved performance.

It is concluded that GA is an effective technique to find out an optimized correlation following the structure of the previous speed of sound models but with an improved accuracy. The coefficients found by GA are listed in Table 1.

Note that single phase calculation is required for any gas mixture before the optimization takes place. It is performed using conventional routine with the EoS in an isothermal-equilibrium flash calculation. However, the pseudo code of the entire algorithm, from the phase separation step to final optimization, is shown in Fig. 1.

2.2. Speed of sound and the application of supersonic nozzle

Temperature, pressure and the Mach number are calculated for the supersonic nozzle using 1D uniform model. From the entering section of the nozzle up to the possible shockwave, Eq. (20) and Eq. (21) are used, which are valid for isentropic conditions. The same are used for the shock wave downstream to the exit side. For the shockwave, Equations (22)–(24) are used. The Eq. (25) stands for the relation between cross sectional area and the Mach number, which should be numerically solved to find the corresponding parameter.

$$\frac{P_0}{P} = \left[1 + \left(\frac{k-1}{2}\right)M^2\right]^{\frac{k}{k-1}} \quad (20)$$

$$\frac{T_0}{T} = 1 + \left(\frac{k-1}{2}\right)M^2 \quad (21)$$

$$\frac{T_y}{T_x} = \frac{1 + \frac{k-1}{2}M_x^2}{1 + \frac{k-1}{2}M_y^2} \quad (22)$$

$$\frac{P_y}{P_x} = \frac{M_x}{M_y}\sqrt{\frac{1 + \frac{k-1}{2}M_x^2}{1 + \frac{k-1}{2}M_y^2}} \quad (23)$$

```
1.  procedure optimization
2.  For all T or P Range do
3.  P_in ← //set the pressure
4.  T_in ← // set the Temperature
5.  f_i ← // set the composition i = [1 … number of component]
6.  HYSYS flash calculation to find vapor fraction (f_v) // based on selected EOS
7.  if f_v=1 or f_v=0
8.  Flash calculation for single phase // based on selected EOS
9.  Goto 22 // speed of sound calculation for single phase //thermodynamic
10. Save single phases properties
11. else
12. Goto 22 //speed of sound calculation for vapor phase
13. Goto 22 //speed of sound calculation for liquid phase
14. save separated phases properties
15. endfor
16. For all T or P Range do
17. F(x) //define Fitness function as root mean square error of calculated and expected data
18. Min(Fitness function) ← iterative procedure by Genetic algorithm
19. n1...n4 ← // found coefficient
20. calculate RMSE
21. Endfor
22. Subprocedure speed of sound derivation
23. P_in ← P_in + ΔP //set the temporarily pressure
24. HYSYS property calculation for single phase // based on selected EoS
25. speed of sound using g(ΔP/Δρ) //fitness function defined as Eq. 19
26. Endsubprocedure
27. Endprocedure
```

Fig. 1. Algorithm for finding the coefficients.

procedure profile
$P_0 \leftarrow$ //set the stagnation pressure
$T_0 \leftarrow$ // set the stagnation Temperature
$d_i, A_i, A^* \leftarrow$ // Defined Geometry
$P_i, T_i M_i$ form isentropic equations Eq.20, Eq.21
if $P_{out} > P_{design}$ // P_{design} is the outlet pressure for the chocked supersonic condition
Assume an d_{shock} in diffuser section for shock wave
endif
if $d_i < d_{two\text{-}phase}$
  a. P_x, T_x form isentropic relations Eq.20, Eq.21
  b. M_x form $\min(f(M))$ of relations Eq.25, Eq.26
else
P_x, T_x form isentropic relations Eq.20, Eq.21
M_x form $\min(f_{new}(M))$ of relations Eq.25, Eq.26
endif
P_y, T_y, M_y form shock relations Eq.22...24
Calculate new $P_0\, T_0\, A^*$
P_i, T_j downstream of shock to the exit side by isentropic relations Eq.20, Eq.21
Endprocedure

Fig. 2. Algorithm of finding the effect of two-phase region speed of sound.

$$M_y^2 = \frac{M_x^2 + \frac{2}{(k-1)}}{\frac{2k}{(k-1)}M_x^2 - 1} \tag{24}$$

$$f(M) = \left[\frac{A}{A^*} - \frac{1}{M}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2}M^2\right)\right]^{\frac{k+1}{2(k-1)}}\right]^2 \tag{25}$$

In the case of shockwave inside the diffuser or chocked supersonic condition, the temperature decrease continuously and partially respectively and, if it crosses the dew point curve of the fluid, it means that the fluid entered the two-phase zone, which has a different speed of sound than the single phase condition.

For the reduced value of the speed of sound, a mixture Mach number replaces the Mach value in the base equation (Eq. (25)), and therefore Eq. (26) is derived as:

$$f_{new}(M) = \left[\frac{A}{A^*} - \frac{1}{M_m}\left[\frac{2}{k+1}\left(1 + \frac{k-1}{2}{M_m}^2\right)\right]^{\frac{k+1}{2(k-1)}}\right]^2 \tag{26}$$

Assuming a range for reducing the level of Mach number, conventional 1D-model with a partial modification described in Fig. 2 has been utilized to study how it affect the final temperature and pressure profile through the convergent-divergent nozzle.

3. Results and discussion

The simple form and second form the Zivi equation are compared to find out the sensitivity of the model to the entrained parameter identified by e in the second form of the Zivi model. Both of them are evaluated coupling with the Minnaert model and the result is illustrated in Fig. 3. It is found that it has less than 2% alteration of the final predicted speed of sound with different assumptions of the entrained value (e). As there is no explicit definition for this factor in the literature, the simple form of the Zivi is used for further calculations.

Evaluation of previous model and the proposed one performed in two types of isobaric and isothermal processes. Both Wood's law and the combination of Zivi with Minnaert approaches are evaluated besides the proposed model. In an isobaric process, as shown in Fig. 5, it is found that although the Wood's law model calculates

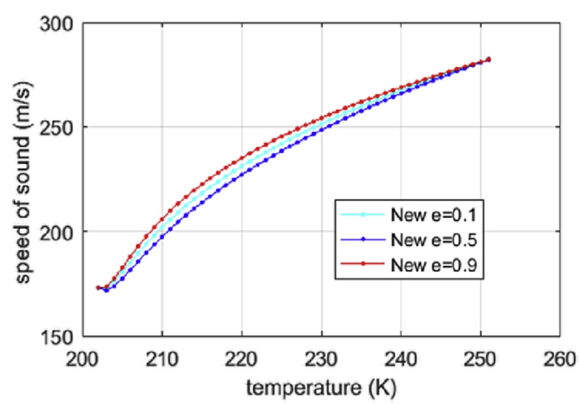

Fig. 3. Influence of entrained liquid fraction on sound speed using Zivi equation [17], EoS: PR, mixture: 14 components (Prudhoe Bay mixture (Picard et al. [7]]).

an almost reasonable response to sound speed for an isobaric process, it has a little over calculation. The Minnaert model performs better than the Wood's Law. In the case of an isothermal process, both the Minnaert and the Wood's Law deviate from the expected results and predict nonreasonable behavior, which is shown in Fig. 6. It seems that the effect of the pressure is neglected in both Minnaert and Wood's Law and the consequence of this is the deviation of the sound speed in the two-phase region, which is highlighted in the boundary of the liquid phase and the two-phase region; and the more the liquid fraction, the higher the deviation of the sound speed. The statistical details are listed in Table 2 for all models.

The convergence trend of the GA with the proceeding of the generation and the statistical analysis of the optimization are shown in Fig. 4 and Table 3, respectively.

Fig. 7 shows the speed of sound profile as the result of the proposed model, which is valid for all process types and the related

Table 2
Error criteria of different models.

index	Approach detail	Isobaric RMSE (m^2/s^2)	Isothermal RMSE (m^2/s^2)
1	Wood's Law	0.85	0.63
2	Minnaert	0.91	0.72
3	Proposed model	0.96	0.95

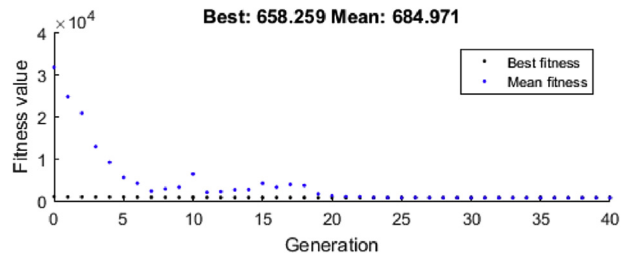

Fig. 4. GA optimization convergence versus generation.

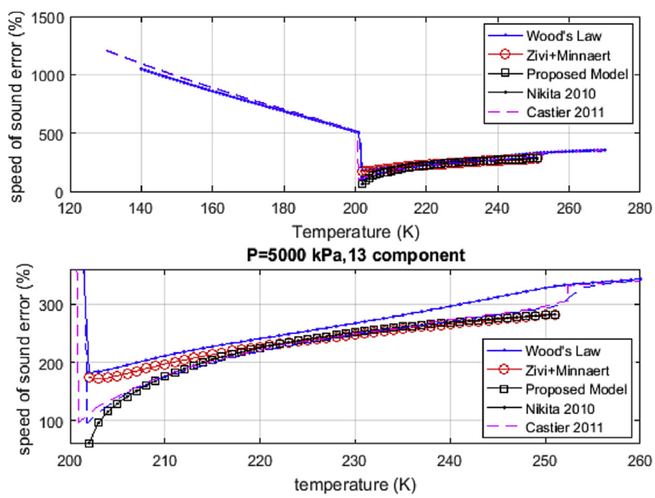

Fig. 5. Isobaric trend of mixture sound speed estimation, EOS: PR, mixture: 14 components, P = 5000 kPa.

volume fraction expressed in terms of temperature. Also, the volume fraction trend is depicted in the same axis to clarify the multiphase zone.

The proposed model is applied to a number of the speed of sound test cases available in the literature. The reference cases for optimization comprised a wide range of process conditions, including the temperature and the pressure in the form of one

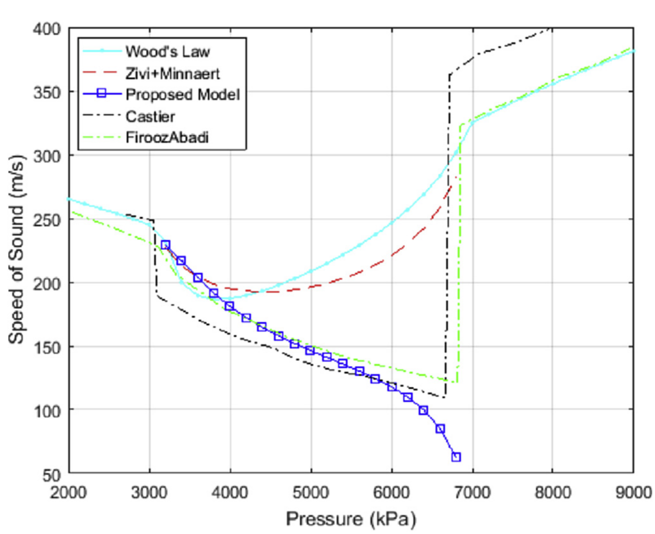

Fig. 6. Isothermal trend of mixture sound speed estimation, EOS: PR, mixture: 2 components, T = 327 K.

Table 3
Other error criteria analysis of the GA optimization.

parameter	value
R-Square	0.97
MSE	75.10
MSE	205.86
MAE	6.23
MAS	23.60

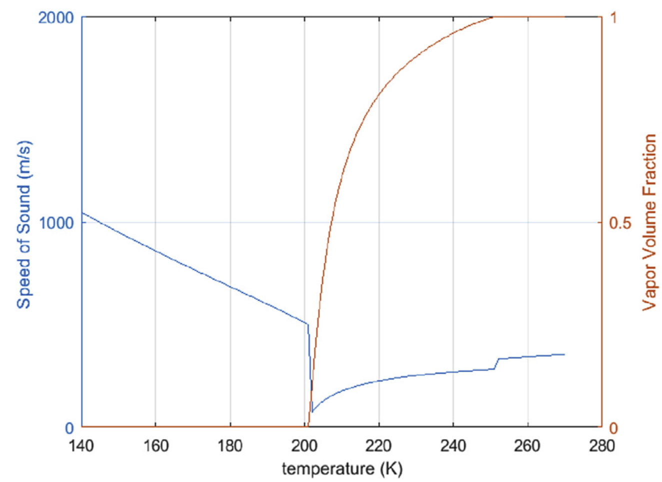

Fig. 7. Speed of sound profile using proposed model including single phase and two-phase regions, EoS:PR, mixture: 14 components.

isothermal (Firoozabadi et al. [9]) for a binary mixture of methane and propane ($z_{C1} = 0.3$, $z_{C2} = 0.7$) and three (Nikita et al., [12]) isobaric cases for a natural gas stream with 14 components, listed on Table 4, which are illustrated in Fig. 8. Moreover, the respective vapor fraction for each of them is shown on this figure.

A more in-depth investigation is carried out by evaluating the final model with different EoS to check whether it is EoS dependent or not. As shown in Fig. 9, the proposed model is able to account for the calculation of the two-phase speed of sound using different EoS. Soave-Redlich-Kwong (SRK) [21] Peng-Robinson (PR) [22], Peng-Robinson-Stryjek-Vera (PRSV) [23] and Kabadi-Danner (KD) [24] equations of state are selected for sound speed model assessment. A comprehensive agreement is found between the selected EoS and the literature data.

One of the identified weaknesses of previous models was the pressure effect. One of the potential of the proposed model is well

Table 4
Composition of natural gas (Prudhoe Bay mixture, Picard et al. [7]).

Item	Component	Mol fraction (%)
1	C1	83.3310
2	C2	9.6155
3	C3	3.5998
4	i-C4	0.3417
5	n-C4	0.4585
6	i-C5	0.0403
7	n-C5	0.0342
8	C6	0.0046
9	C7	0,0003
10	C8	0.0001
11	C7H8	0.0002
12	N2	1.4992
13	O2	0.0008
14	CO2	1.0738

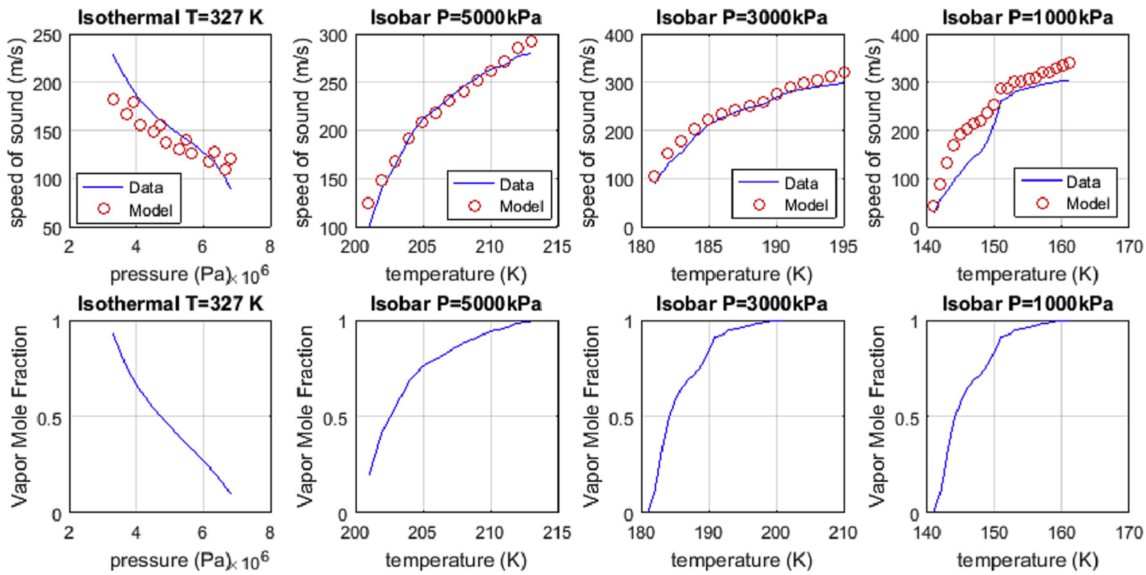

Fig. 8. Correlating the proposed mixing rule with different data including Nikita et al. [12] and Firoozabadi et al. [9].

considering the pressure effect. It is with different pressure ranges to assure about the generalization of the model. The speed of sound results of the model coupled with different EoS is depicted in Fig. 10.

The advantages of the proposed rule to the Nikita [12] and Wood's law [11] are that it doesn't need to calculate phase mixing properties of the EOS, such as van der Waals mixing rules for attraction parameter of PR (Kwak et al. [25]). Also, as shown in Fig. 11, it's evaluated for both isobaric and isothermal processes and it can perform an accurate prediction in both conditions, while the other correlations just cover the isobaric condition. Moreover, it is found that the proposed model has the capability of using various EoS while maintaining the accuracy of prediction results. The model correlation is a simple function of the phase fraction, the pressure, the bulk modulus, and phase densities. It gives a better prediction of the phase boundaries than Wood's model. It has the capability to be used in different models to account for the effects of pressure and phase fraction.

Special attention is paid to the case of the supersonic nozzle. The model results here presented show a dramatic improvement over traditional models, without the need for vapor fraction correlations and without any limitation regarding types of process. While no simple modeling methodology had ever yield perfect predictive capability, the present results revealed that the proposed approach which is adopted by the genetic algorithm provides an improvement in predictive performance of the two-phase region speed of sound model. Also, the accuracy of a model output depends on the accuracy of the input which are the properties of two phases.

The general equation for the two-phase speed of sound has been derived in the previous section and it has been revealed that the two-phase region has up to somehow a lower value than the single phase. Assuming two lowering levels of two-phase region speed of sound including −1%, −5% and −10% comparing to the single phase value, Fig. 12 and 13 show the hydrodynamic profile of the pressure and temperature in the nozzle. The effective zone in the nozzle is the place where the multiphase region is found and it is assumed that the transition from single phase to multiphase is located in the

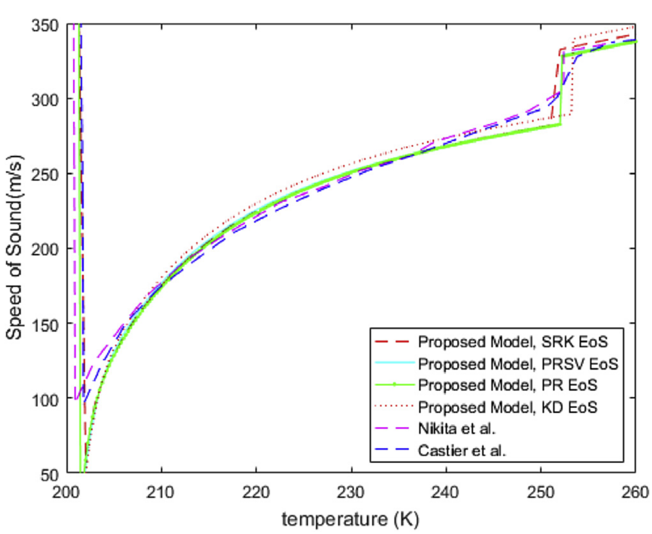

Fig. 9. Evaluation of different EOS with the proposed mixing rule for sound speed in two-phase region.

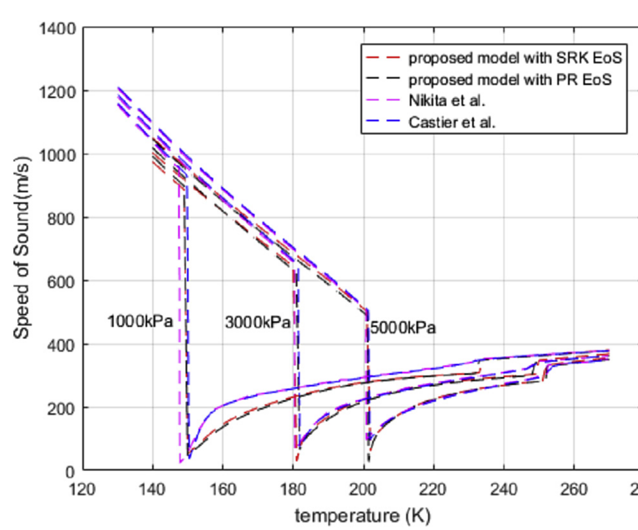

Fig. 10. Applying proposed model on different EOS in different pressure.

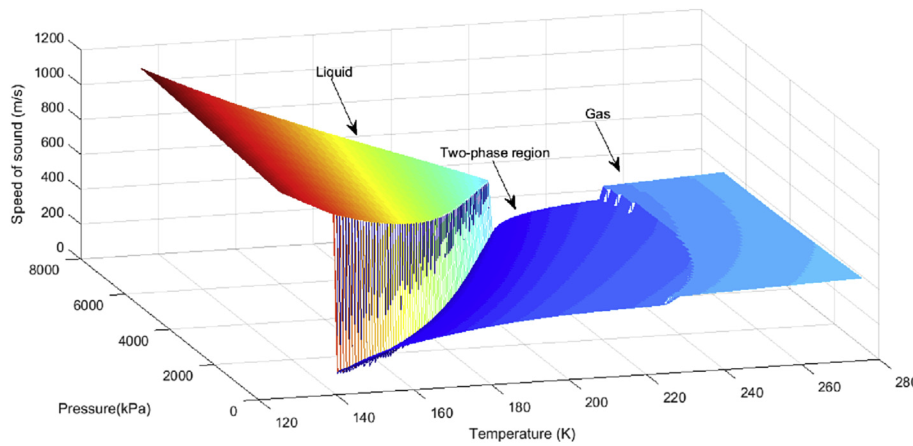

Fig. 11. Proposed model profile versus pressure and temperature.

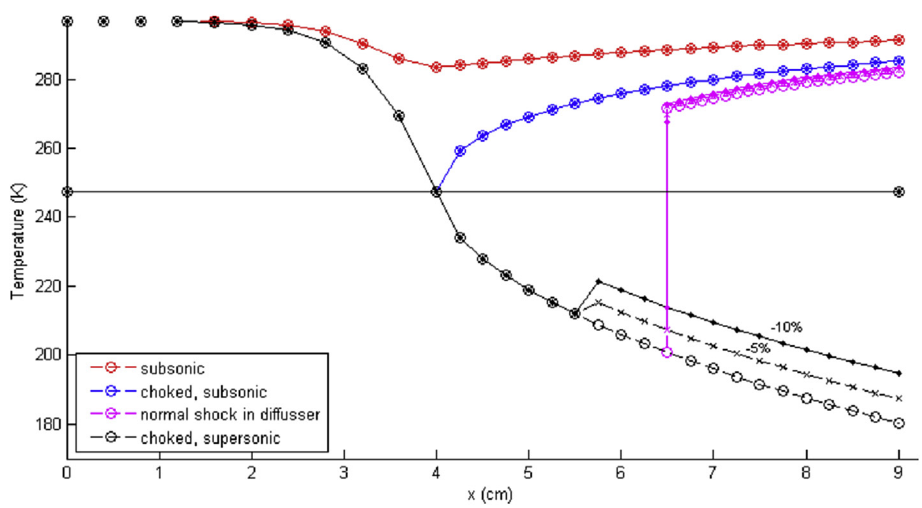

Fig. 12. Temperature profile along the nozzle for different speed of sound alteration in two-phase region.

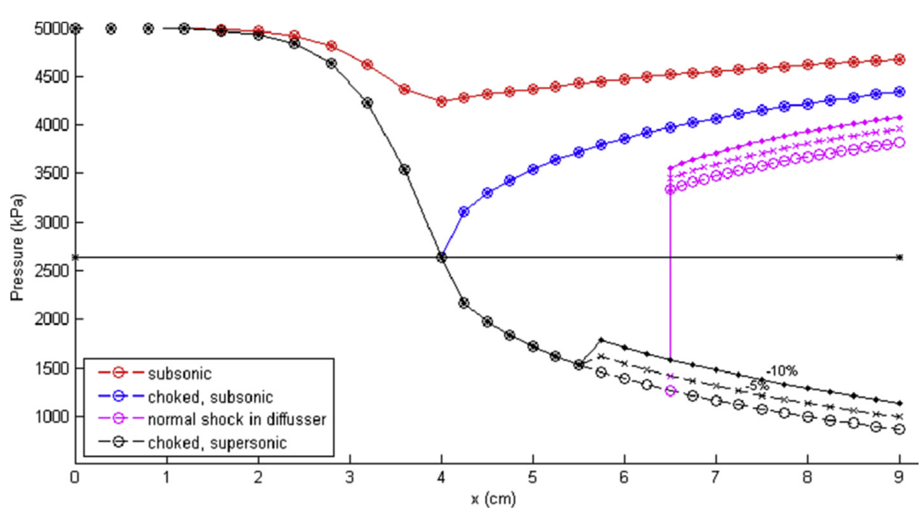

Fig. 13. Pressure profile along the nozzle for different speed of sound alteration in two-phase region.

middle of the throat to the nozzle outlet distance. The minimum temperature, which is one of the most important design parameters of the supersonic nozzle for condensation, separation, and removal of heavy natural gas fractions, has been selected for final comparison criteria, which is illustrated in Fig. 14 for all the assumed lowering levels. It is found that the absolute deviation of

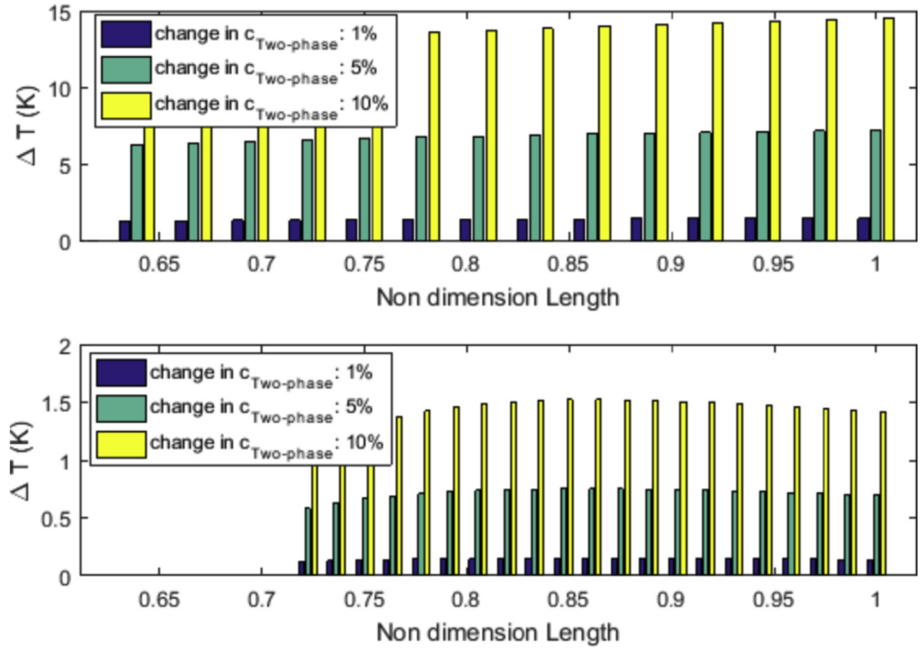

Fig. 14. Temperature drop along the nozzle for different speed of sound alteration in two-phase region (a) with and (b) without shockwave.

the modified model from the basic 1D model is up to 14 K for the design pressure condition and up to 1.5 K for the shockwave condition. As a result, the conventional 1D model based on the classic thermodynamic is not reliable enough to consider the multiphase conditions, unless it is modified towards the proposed one.

4. Conclusions

In the last few years, as a result of growing interest in the subject of two-phase fluid properties, many theoretical and experimental studies were carried out on the specific issue of sound speed. The present study aimed to find a new model by modifying the most popular and reliable approaches. Reliable and well established experimental and numerical data from the literature are used to validate the model. This research discusses the best fit achieved by applying the genetic algorithm to the correlations parameters used in the proposed model and demonstrated that it has the potential to predict the speed of sound in the two-phase region within satisfactory uncertainty margin.

We compared, presented and discussed the proposed model's results with those of conventional ones. The inability of Wood's law to predict two-phase region speed of sound for both isothermal and isobaric processes is shown. In the same manner, it was concluded that the Minnaert model couldn't predict reliable results under isothermal condition. Thus, in order to assess the speed of sound in the two-phase region, a more comprehensive model is investigated, which is derived taking into account both isothermal and isobaric data. The results indicate that the proposed model achieves significantly better results when compared with the others and the difference is even more evident when considering different types of processes. It is concluded that the proposed model is valid for both isobaric and isothermal conditions. The proposed method is a function of separated phases' properties and, with four tuned coefficients determined by a genetic algorithm, directly calculates the reliable value of sound speed in the two-phase region. A good agreement is achieved between the literature validated data and the simulations. For the application of supersonic nozzles, the proposed model is assessed and it is revealed that the consideration of the two-phase speed of sound extremely changed the results of the common 1D simplified approach.

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