An Improved Method For Applying The Lockhart–Martinelli Correlation To Three-Phase Gas–Liquid–Solid Horizontal Pipeline Flows

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Three-phase (G/L/S) horizontal pipe flow data collected from the literature are used to evaluate the performance of a number of correlations designed to predict the pipeline pressure gradient. In the present study, a number of popular two-phase gas–liquid pressure loss correlations were modified for three-phase flow predictions. The primary modification is to assume that the slurry (L/S) mixture behaves as a singlephase. The modified Dukler and the Beggs and Brill correlations did not provide accurate estimates of the three-phase pressure gradients. When the classical Lockhart–Martinelli (L–M) correlation was used, along with a kinematic friction loss model to calculate the slurry (L/S) superficial flow pressure gradient, accurate predictions of the three-phase (G/L/S) pressure gradient were obtained provided the slurry did not exhibit non-Newtonian behaviour and that Coulombic (sliding bed) friction was negligible. Additional experiments should be conducted before the improved version of the L–M correlation is applied to commercial installations with pipe diameters greater than 100 mm.

Keywords: three-phase flow, pressure gradient, horizontal pipe, slurry, kinematic friction, Lockhart–Martinelli correlation

INTRODUCTION

While applications involving pipeline transport of three-phase (G/L/S) mixtures have become quite common, research in this area is in many ways poorly represented in the public domain. Examples of three-phase (G/L/S) pipeline transport that have garnered commercial interest include, for example biomass transport,[1] horizontal oil production wells,[2] nuclear waste decommissioning,[3] pulp and paper production, the use of air injection to (i) reduce pumping costs in slurry pipeline flows[4] and (ii) improve bitumen recovery from mineable oil sands.[5] It is in this area where the interests of the authors of the present work fall. Specifically, if gas is injected into a pipeline carrying a highly concentrated slurry, we wish to know how the presence of the gas phase affects the pipeline's operating envelope and overall energy consumption.

From an engineering perspective, the most critical parameters required for pipeline design (i.e. scale-up) of three-phase G/L/S flows are:

- The frictional pressure loss (in, say, Pa/m) over the expected range of operating conditions.
- The flow regime, that is the spatial distribution of the components over the flow domain.
- The minimum operating velocity required to ensure that particles do not accumulate in the pipeline to form a stationary bed (the 'deposition velocity').

Unfortunately, a review of the literature suggests that, for G/L/S flows, these design criteria are especially poorly understood. Gravity and buoyancy forces act normal to pipe axis, promoting slug and stratified flows at the expense of dispersed bubble flow—which, in the oil sands industry, is the desired flow regime.[5] If the flow regime is such that bubbles are present in the flow, the effect of the presence of the dispersed solids phase on bubble break-up is not well understood, particularly at the average liquid velocities at which pipelines typically operate. As Orell[4] points out, there are very few experimental studies published in the literature. The few that are published represent 'snapshots', with very narrow ranges of operating conditions tested, with different studies focusing on wildly disparate operating envelopes. Much of the test work has been done in small-diameter pipes (10–100 mm) and nearly every study is 'incomplete' in the sense that a limited number of measurements are made: for example pressure drop and minimum operating velocity data are collected, but flow regime information is not; or, in situ gas and solids concentrations ('hold-up') are measured but frictional pressure losses are not. One can quickly see why so few experimental studies have been done. Pipeline flow experiments require large, complicated set-ups, and it is difficult to make reliable measurements in high-velocity, opaque flows.

Often, when experiments are costly, time-consuming or involve hazardous materials, computational fluid mechanics (CFD) is considered as a viable option, allowing one to conduct 'numerical experiments'. In the literature, three-phase computational models have been limited to vertical flows with a very dilute solid phase (e.g. Ref.[6]) and limited to very specific assumptions regarding the G/L and S/G flow regimes.[7] Ultimately, though, reliable computer simulations of three-phase G/L/S horizontal pipeline flows are not yet possible unless two of the three phases can be considered to form a single-phase, homogeneous flow.[8] The primary issue, which is problematic even in the much more highly developed (and more intensely studied) fields of G/L and L/S (slurry) flows, is the empiricism inherent in the so-called 'closure relations' needed to solve combined continuity and momentum equations for each phase.[9] Many computational fluid dynamics (CFD) applications for G/L pipeline flows, for example require one to specify the flow

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Can. J. Chem. Eng. 91:1372–1382, 2013

DOI 10.1002/cjce.21843
Published online 31 May 2013 in Wiley Online Library (wileyonlinelibrary.com).

regime before the model can solve for the local, averaged concentrations and/or velocities of each phase.[9,10] Only recently have flow-regime independent G/L pipeline flow models been developed, but even these have very specific limitations.[11] For these models to be extended, experiments are required to formulate and/or validate the required closure relations. The situation is not much better in slurry flow simulations, where the choice of turbulence model, interphase closure relations, and boundary conditions[12,13] can have a significant impact on the veracity of the predictions. In short, accurate and reliable CFD simulations of three-phase (G/L/S) horizontal pipeline flows will only be possible once comprehensive experiments have been conducted and the appropriate closure relations have been developed.

While it is a longer-term objective of the University of Alberta's *Pipeline Transport Processes Research Group* to produce 'complete' data sets for horizontal three-phase (G/L/S) pipeline flows, we focus in the present study on the existing methods used to provide one of the most critical pipeline design parameters: that is, the calculation of frictional pressure loss as a function of operating conditions. A standard approach is to adapt the well-known Lockhart–Martinelli (L–M) correlation (see, e.g. Ref.[14]). Use of the conventional two-phase L–M correlation requires one to determine the frictional pressure gradients (dP/dz) of the gas and liquid phases as if each was flowing alone in the pipe under the conditions of interest. In adapting the L–M correlation for three-phase (G/L/S) flows, the slurry (L/S) flow is generally taken as the 'liquid' phase and thus a calculation of a slurry flow frictional pressure gradient is required. Please note that the specifics of the method are described subsequently in greater detail in a subsequent section of the paper.

In the present study, we evaluate the accuracy of pressure loss predictions for three-phase (G/L/S) flows obtained by adapting other well-known two-phase (G/L) models (in addition to the L–M correlation), and show that the modified L–M correlation appears to be superior. We then illustrate that rather inaccurate and/or outdated methods have been used for the calculation of the slurry frictional pressure loss, and that improved three-phase frictional pressure loss predictions are obtained when better slurry flow calculations are used. Finally, we use the results of this analysis to recommend a comprehensive experimental program that will significantly improve the understanding and modelling of three-phase (G/L/S) pipeline flows.

PREVIOUS STUDIES

Two-Phase (G/L) Pressure Loss Correlations

Three common gas–liquid pressure loss correlations are described here; namely, the correlations of Lockhart and Martinelli,[15] Dukler et al.[16] and Beggs and Brill.[17] In this section, each correlation is described. Subsequently, each correlation will be adapted to try to predict the frictional pressure loss for three-phase (G/L/S) horizontal pipe flows. While numerous G/L pipe flow correlations exist, these three were chosen because their utility for pressure loss predictions have been demonstrated hundreds of times over many decades of use. In the case of the L–M correlation, numerous authors have previously attempted to adapt it to G/L/S flows, and so it represents the 'base-case' for three-phase flow pressure loss prediction.

The L–M correlation[15] is still widely used to predict pressure losses in two-phase (G/L) pipeline flows.[18] One of the main advantages of the L–M method is its simplicity and generally accurate predictions even when the flow regime is not known. However, it is limited to cases where the pipe is horizontal. The L–M correlation is expressed as follows:

$$X^2 = \frac{(dP/dz)_L}{(dP/dz)_G} \tag{1}$$

$$\left(\frac{dP}{dz}\right)_{MP} = \phi_L^2 \left(\frac{dP}{dz}\right)_L \tag{2}$$

In these equations, $(dP/dz)_L$ refers to the frictional pressure gradient calculated using the liquid superficial velocity, that is Q_L/A. Similarly, $(dP/dz)_G$ refers to the pressure gradient calculated for the gas flowing at its superficial mixture velocity, that is Q_G/A.

The pressure gradient for the 'multiphase' mixture (in this case, a gas–liquid mixture) is given by $(dP/dz)_{MP}$. Lockhart and Martinelli[15] correlated the two dimensionless parameters, X and ϕ, through the Reynolds number of each phase and an analysis of the relationship between friction loss and hydraulic diameter for single-phase flows of each fluid. Later, Chisholm[19] proposed a convenient curve-fitting correlation, which relates the dimensionless coefficient X with the L–M coefficient ϕ and is presented as follows:

$$\phi_L^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \tag{3}$$

The coefficient, C, is determined from the Reynolds numbers of the liquid and gas phases using the appropriate superficial velocity; that is $U_L = Q_L/A$ and $U_G = Q_G/A$, respectively. The coefficient C values are given in Table 1. These values are restricted to two-phase mixtures with gas–liquid density ratios corresponding roughly to those of air–water mixtures at atmospheric pressure.

Pressure gradients for the constituent phases can be calculated in the standard way, using, for example:

$$\left(\frac{dP}{dz}\right)_i = \frac{f_i (U_i)^2 \rho_i}{2D} \tag{4}$$

where f_i is the Darcy–Weisbach friction factor.

Dukler et al.[16] analysed the performance of the LM correlation against a data set of approximately 2400 data points. Generally, they found that the standard deviation (σ) of the predictions, that is the measure of the scatter about the mean, was more than 35% if all flow regimes were included, with pressure loss predictions for slug flow being the most accurate ($\sigma = 17.5\%$) and predictions for wavy flow being the worst ($\sigma = 86\%$). They also noted that the performance of the LM correlation was best for low-viscosity (water-like) liquids and pipes of smaller diameter. Wallis[20]

Table 1. The idealised L–M flow regimes[19]

Idealised L–M regime	C
Liquid phase: turbulent Gas phase: turbulent	20
Liquid phase: laminar Gas phase: turbulent	12
Liquid phase: turbulent Gas phase: laminar	10
Liquid phase: laminar Gas phase: laminar	5

provides a similar but more comprehensive comparison of pressure loss predictions obtained using the LM correlation with experimental data, showing scatter of the error in the predictions of approximately the same magnitude as Dukler et al.[16] Since then, others have also commented on the overall accuracy of the LM correlation for specific conditions, for example at high pressures[21] or in mini-channels.[22]

Dukler et al.[16] also developed a pressure gradient correlation using the principles of dynamic similarity for two-phase (G/L) flows. The assumption was that with the proper definition of mixture density and viscosity, dictated by the requirements of dynamic similarity, one could then calculate an effective Reynolds number and two-phase friction factor. Once a friction factor is obtained, the calculation of a two-phase pressure gradient is straightforward. The equations comprising the Dukler correlation are:

$$\rho_{ns} = \rho_L \kappa + \rho_G (1-\kappa) \tag{5}$$

$$\mu_{ns} = \mu_L \kappa + \mu_G (1-\kappa) \tag{6}$$

where

$$\kappa = \frac{Q_L}{Q_L + Q_G} \tag{7}$$

The Reynolds number is defined as:

$$Re_{GL} = \frac{4W_T}{\pi D \mu_{ns}} \beta \tag{8}$$

where W_T is the total mass flow rate of liquid and gas and β is defined as:

$$\beta = \frac{\rho_L}{\rho_{ns}} \frac{\kappa^2}{\alpha_L} + \frac{\rho_G}{\rho_{ns}} \frac{(1-\kappa)^2}{\alpha_G} \tag{9}$$

The frictional pressure gradient is obtained using:

$$\left(\frac{dP}{dz}\right)_{GL} = \frac{\rho_{ns} U_{GL}^2 f_0}{2D} y(\kappa) \beta \tag{10}$$

where

$$f_0 = 0.00140 + \frac{0.125}{(Re_{GL})^{0.32}} \tag{11}$$

and

$$y(\kappa) = \frac{f_{GL}}{f_0} = 1.0 + \frac{\kappa'}{1.281 - 0.478(\kappa') + 0.444(\kappa')^2 - 0.094(\kappa')^3 + 0.00843(\kappa')^4} \tag{12}$$

In Equation (12), $\kappa' = -\ln(\kappa)$.

The Dukler correlation was compared with 2400 experimental data points. Errors were approximately 19% and 61% for two-phase (G/L), two-component flows and two-phase, single-component, steam–water flow, respectively. The large error associated with one component steam–water mixture is primarily due to inadequate predictions of the acceleration pressure gradient terms in the total pressure gradient calculation. The acceleration component of the total pressure gradient is significant during heat transfer and in situ evaporation events.[23]

The Beggs and Brill[17] correlation accounts for the fact that the pressure gradient in two-phase (G/L) flow is complicated by flow phenomena such as slippage between the phases, change of flow pattern, mass transfer between the phases and a reduced area available for the flow of each phase. The Beggs and Brill[17] correlation for calculating the pressure gradient works for horizontal, inclined and vertical flows. The purpose of their study was to develop a correlation useful in designing pipelines for hilly terrain and tubing strings for inclined wells. The equation used to calculate the pressure gradient when both gas and liquid flow in a pipe is:

$$\left(\frac{dP}{dz}\right)_{GL} = \frac{\rho_m [g \sin(\theta) + ((f_m U^2)/2D)]}{1 - ((\rho_m U_{GL} U_G)/P)} \tag{13}$$

where

$$\rho_m = \rho_L \alpha_L + \rho_G (1-\alpha_L) \tag{14}$$

The correlation provides the total pressure gradient, that is the sum of the pressure gradient due to potential energy (elevation) changes, kinetic energy changes and friction losses. When the flow is horizontal, $\theta = 0$. The correlation was validated with experimental data collected for the following conditions: (1) gas flow rates: 0–98 m^3/s; (2) liquid flow rate: 0–1.9 L/s; (3) average system pressure: 241–655 kPa; (4) pipe diameter: 25.4–38.1 mm; (5) liquid holdup: 0–0.87; (6) pressure gradient: 0–2.63 kPa/m. It was found that the error for predicting the experimental pressure gradient using their model was 2.57%, 1.43% and 0.47% for horizontal pipes, uphill pipes and downhill pipes, respectively.

Three-Phase (G/L/S) Experimental Studies

Table 2 provides an overview of the three-phase (G/L/S) horizontal pipeline flow studies that have been reported in the literature. Much of the work has been done using relatively small-diameter pipes ($D \leq 50$ mm). Three-phase flow data are frustratingly incomplete; some studies report pressure losses but do not correlate them with specific flow regimes. Others describe flow regimes that occurred under different conditions but pressure losses were not measured. Conditions required to prevent the formation of a stationary bed of solids were considered in only a few studies (e.g. Ref.[24]).

In some studies, solids concentrations are not reported. Finally, it is interesting to note how few conditions are repeated: there is almost no 'overlap' of common test conditions among data sets. One of the obvious conclusions that results from an analysis of the G/L/S pipeline flow data available in the open literature is that some 'complete' data sets are sorely needed. The reference to 'complete' data sets is really a reference to experiments where the following are measured simultaneously for a wide range of operating conditions:

- Deposition velocity, V_c (i.e. minimum superficial mixture velocity required to prevent accumulation of solids in the pipeline).
- Frictional pressure gradient, dP/dz.
- Flow regime.
- In situ concentration of each phase.
- If possible, spatial distributions of each phase within the flow domain.

Table 2. Previous studies of three-phase (G/L/S) horizontal pipeline flow

Refs.	D (mm)	d_p (μm)	C_s (%)	Flow regime
Scott and Rao[24]	25, 50	100, 500	0–14	Bubble, plug, slug
Toda et al.[44]	14, 30	462, 985	22	Bubble, plug, slug
Barnea et al.[49]	50	462, 985	4–12	Slug moving bed
Hatate et al.[29]	15, 26	29–98	0–37	Homogeneous, moving bed
Fukuda and Shoji[25]	42	74	4–25	Plug, slug
Kago et al.[50]	52	59	22–40	Plug, slug
Toda and Konno[51]	14	462	7.5	Bubble, plug, slug moving bed
Angelsen et al.[41]	25–100	30–550	—	Stationary and moving bed
Oudeman[52]	70	150–690	0–2	Slug, moving and stationary bed
Takahashi et al.[43,53]	30	990–2900	0–16	Slug stationary, bubble/slug moving bed
Gillies et al.[14]	52	10–200	7–28	Slug
Mao et al.[3]	25	<15	9, 15	Homogeneous
Pironti et al.[54]	35	210	7, 32	Homogeneous and moving bed
Salama[55]	108	100–500	—	Moving bed
Stevenson et al.[56]	40, 70	150–1180	0.003	Slug and plug/elongated bubble
King et al.[57]	57, 152	255–1100	—	Homogeneous, slug moving bed
Bello et al.[2,33]	40, 114	600	<1	Plugs, elongated bubbles, slugs, stratified
Danielson[58]	69	280, 550	<1	Slug
Goharzadeh et al.[59]	25	230–2000	8–15	Slug (moving dune)

In order to analyse the performance of the adapted two-phase (G/L) correlations, we have extracted all the pressure loss data reported in the studies described in Table 2. This compiled data set was used to determine which two-phase pressure loss correlation, adapted for G/L/S flow, provided the most accurate pressure loss correlations overall. We then compare predictions obtained from the 'best' adapted model to data obtained in one of the more thorough experimental studies.[25]

Three-Phase (G/L/S) Pressure Loss Correlations

The data collected for three-phase (G/L/S) horizontal flows covers relatively narrow ranges of operating parameters. This fact, along with the complexity of these flows, has prevented the development of broadly applicable physics-based models. Attempts to predict frictional pressure losses have been limited to specific conditions and/or flow regimes. The most common approach is to treat the slurry (L/S) flow as the 'liquid' phase and then apply the L–M correlation. In this adaptation of the L–M correlation, $(dP/dz)_L$ in Equations (1) and (2) refers to the frictional pressure drop of the slurry flowing at a superficial velocity of Q_{LS}/A, while $(dP/dz)_{MP}$ refers to the three-phase (G/L/S) pressure gradient.

Scott and Rao[24] appear to be the first to propose that the L–M correlation be modified for three-phase (G/L/S) horizontal pipe flows. They used a form of the Durand and Condolios (D&C) model[26] to predict slurry pressure gradient. Scott and Rao's[24] implementation of the D&C model was:

$$i = i_w(1 + \varphi C_s) \tag{15a}$$

where

$$\varphi = 121\psi^{-1.5} \tag{15b}$$

with

$$\psi = \frac{U_{LS}^2\sqrt{C_D}}{gD(S-1)} \tag{15c}$$

In Equation (15a), the hydraulic gradient of the slurry, expressed in (meters of water/meter of pipe) is denoted using the symbol 'i', while the hydraulic gradient for the suspending liquid (water) is shown as 'i_w'. The frictional pressure gradient is equal to the product '$i\rho_w g$'. The two constants in Equation (15b) were found experimentally and reported prior to Scott and Rao's[24] work. Many different values for these constants were reported by slurry flow researchers in the 1960s and 1970s, and the reader is encouraged to see the summaries published by Shook and Roco[27] and Wilson et al.[28] The average slurry velocity (Q_{LS}/A) is again denoted as U_{LS}, while C_D represents the particle drag coefficient. The pipe diameter is 'D' and S represents the density ratio ρ_s/ρ_w.

While the agreement between the predictions made using the modified L–M correlation and their experimental data was found to be satisfactory, the Durand correlation is known to provide reasonable predictions of slurry flow friction losses only when the solids volume concentration is less than 15%.[27] Note that Scott & Rao's experiments were conducted using $C_s \le 0.14$.

Hatate et al.[29] proposed an empirical correlation containing hydrodynamic parameters historically used for slurry (L/S) flow pressure loss calculations, that is it has a form similar to the D&C model. Their correlation, to estimate the pressure gradient of horizontal three-phase flow (G/L/S) flow is:

$$\frac{(dP/dz)_{GLS} - (dP/dz)_{GL}}{C_s(dP/dz)_L} = K\psi^n \tag{16}$$

where the terms on the left-hand side of the equation have their usual meanings and ψ is defined by Equation (15c). Hatate et al.[29] found that the parameters listed in Table 3 best described their experimental data.

In the above equation $(dP/dz)_{GL}$ was evaluated following the method prescribed in the classical L–M method. In this correlation the challenge is to calculate properly the in situ gas holdup (α_G) in the flow. The experimental data were correlated within 30% error by this correlation. However, their experiments were conducted using very relatively small particles ($d < 100$ μm). Because of the

Table 3. Empirical values for K and n[29]

Range	K	n
$\psi < 20$	70	−1
$\psi \geq 20$	3.5	0

empirical nature of their slurry (L/S) friction loss correlation, the method proposed by Hatate et al.[29] should not be expected to perform well for pipe diameters larger than those tested during that study; in other words, this empirical correlation is unlikely to provide good estimates for three-phase pressure losses in pipes much larger than 25 mm in diameter.

Gillies et al.[14] conducted a study of three-phase (G/L/S) flows, focusing on superficial velocities, particle sizes, solids concentrations and in situ gas holdups in the ranges likely encountered in the flow in underground horizontal oil wells. They considered flows over a stationary sand bed as well as flows where all the solids in the pipe were fully suspended. For cases where the flow was known to be turbulent, they applied a modified L–M correlation, where prediction of the slurry flow pressure drop was used in the correlation instead of the liquid-phase pressure drop. However, they assumed that the slurry flow friction losses could be adequately represented using the so-called 'equivalent fluid' model. The basis of this approach is the assumption that the slurry can be treated as a single-phase liquid, whose density and viscosity are altered by the presence of the solids phase. Gillies et al.[14] calculated the slurry density and viscosity of their 'equivalent fluids' using the in situ solids volume concentration of the slurry, C_s:

$$\rho_{LS} = C_s\rho_s + (1 - C_s)\rho_L \tag{17}$$

$$\mu_{LS} = \mu_L[1 + 2.5C_s + 10.06C_s^2 + 0.00273\,e^{16.6C_s}] \tag{18}$$

Equation (18) was developed by Thomas[30] for spherical particles. Improved mixture viscosity correlations, which are more suited for somewhat angular sand particles, were subsequently developed by Gillies et al.[31] and Schaan et al.[32] Again, it should be noted that the solids volume concentration in the L/S mixture, C_s, should be used, rather than the solids concentration in the three-phase mixture. Since the experiments conducted by Gillies et al.[14] sometimes involved three-phase (G/L/S) flow over a stationary bed of solids, it was necessary for them to calculate the cross-sectional area available of flow before the modified L–M correlation could be applied. Gillies et al.[14] found that, for their experiments, the modified L–M correlation provided reasonable pressure gradient predictions for three-phase flows containing narrowly graded particles that were 0.01, 0.1 or 0.2 mm diameter, for slurry superficial velocities $(Q_{LS}/A) \leq 0.5$ m/s and gas volume fractions of 0.5 or less, as long as the flow was turbulent. The authors suggest that this result should apply to other pipe diameters although tests were done using a single pipe diameter, that is $D = 0.05$ m. As the results of the present study show, the results are unlikely to apply to larger pipe diameters.

Orell[4] developed a model for predicting the average pressure gradient in a fully suspended three-phase slug flow using a series of mass balances for each phase, a correlation relating the hold up of the slurry (L/S) mixture within an aerated slug and momentum balances on the liquid film and the gas bubble in the film zone. The equations are applied over a slug 'unit' (i.e. comprised of the 'slug' and 'film' zones) and require one to assume that the thickness of the liquid film is uniform and that the solid particles are suspended uniformly throughout the slug and film cross-sections. Orell's[4] model yields a set of five simultaneous equations that must be solved.

According to Orell,[4] accurate predictions should be obtained provided that the three-phase flow is in the slug flow regime and that accurate values of the model's input parameters are provided. Additionally, three intrinsic properties of the particles must be specified.[4] These values cannot be calculated theoretically and must therefore be determined experimentally. The main issue, in fact, is in almost any commercial application of three-phase pipeline technology, the flow regime is not known. Ultimately, this model is based on a more representative treatment of the physics, and its broader applicability should be further evaluated.

It worth mentioning that Orell[4] also proposed different models for other flow regimes such as moving bed and flow over a stationary bed. These models also possess the same shortcomings as fully suspended slug model described above; and, in addition, they are more complex.

Bello et al.[33] proposed a slightly different approach for predicting the pressure gradient for the three-phase flow:

$$\left(\frac{dP}{dz}\right)_{GLS} = \left(\frac{dP}{dz}\right)_{GL} + \left(\frac{dP}{dz}\right)_{LS} - \left(\frac{dP}{dz}\right)_{L} \tag{19}$$

The liquid phase pressure gradient is calculated in the standard way, using the liquid superficial velocity and a Darcy–Weisbach friction factor based on the superficial liquid Reynolds number and wellbore roughness, k. The horizontal gas–liquid pressure gradient is calculated assuming the gas–liquid mixture forms an equivalent fluid with averaged properties obtained as a function of the in situ volume fractions of gas and liquid in the two-phase mixture. An empirical relationship using the single-phase liquid friction factor and the Reynolds number of the equivalent liquid as inputs provides the two-phase (G/L) friction factor. Although the authors state that $(dP/dz)_{LS}$ is based on a 1D momentum balance, the reality is that the force components included in the model, along with the empirical and/or outdated expressions used for some of the force terms, make the model rather untrustworthy. This is especially true in any situation where the friction loss attributable to the presence of the solids is expected to be important, that is coarse particles, high solids concentrations and/or higher superficial slurry velocities. It is notable also that Bello et al.[33] did not compare their model against any experimental data, choosing instead to compare it to the predictions obtained using the Scott and Rao[24] modified L–M correlation for a specific case where no data exist.

IMPROVING THE MODIFIED L–M CORRELATION

One of the great advantages of the modified L–M model is that reasonable predictions of three-phase (G/L/S) pressure gradient can be obtained even when the flow regime is not known. Additionally, nearly all of the experimental studies that have been done to date confirm the fact that, from the perspective of applying the modified L–M correlation, it makes sense to consider the gas as one phase and the slurry as the other. We thus argue that:

- The approach taken by Scott and Rao,[24] Hatate et al.[29] and Gillies et al.,[14] wherein they assume the superficial slurry slow pressure drop can be used in place of the liquid-phase pressure drop in the original LM correlation, is appropriate.

- The specific improvement that needs to be made is in the estimation of the slurry flow pressure gradient, $(dP/dz)_{LS}$.

The approach taken by the Saskatchewan Research Council's (SRC) Pipe Flow Technology Centre to model slurry flows (see, e.g. Refs.[34–36]) is followed here. The SRC Pipe Flow model is used for the design and trouble-shooting of slurry pipelines in the oil sands industry[37] and in mining and mineral processing operations around the world.[38] Slurry flow friction losses are obtained by considering the solids as a three-component system: the 'fine' particles (typically <44 μm), which are assumed to form, with the suspending liquid, a carrier fluid whose density and viscosity are augmented by the presence of the fines; coarse particles that are effectively suspended by fluid turbulence; and particles that are not effectively suspended and thus transfer their immersed weight to the pipe wall. The latter component is responsible for the Coulombic or 'sliding bed' friction. In this analysis, it is assumed that slurries are comprised only of particles from the second category; that is, coarse particles that are effectively suspended by fluid turbulence. For a slurry of this type, that consists of narrowly graded particles in water, the wall shear stress can be said to be 'kinematic' or velocity dependent, and is calculated using[35,36]:

$$\tau_w = \frac{U_{LS}^2}{8}(\rho_w f_w + \rho_s f_s) \tag{20}$$

where U_{LS} represents the slurry velocity (Q_{LS}/A); ρ_w and ρ_s represent the water and particle density, respectively; f_w is the Darcy–Weisbach friction factor for water flowing at the same conditions (velocity, pipe diameter, hydrodynamic roughness) as the slurry; and f_s represents the solids friction factor, which is calculated using the empirical expression:

$$f_s = 0.00132\lambda^{1.25}[0.15 + e^{-0.1d^+}] \tag{21}$$

In Equation (21), λ is the linear concentration and d^+ is a dimensionless particle diameter, defined as follows:

$$\lambda = \left[\left(\frac{C_{max}}{C_s}\right)^{1/3} - 1\right]^{-1} \tag{22a}$$

and

$$d^+ = \frac{dU_{LS}\sqrt{f_w/8}}{\mu_w/\rho_w} \tag{22b}$$

where C_{max} represents the maximum (limiting) coarse solids volume fraction for the coarse particles in question. The other variables have their usual meanings.

The so-called 'kinematic' friction accounts for the friction caused by the flow of the suspending liquid (in this case, water), as well as that portion related to particle–particle and particle–wall collisions. The friction is tempered by the lubrication effect of so-called near-wall lift[39,40] where appropriate. The friction factor correlation was developed using narrowly graded sands of different diameters, whose psd's are similar to the sand found in Alberta's oil sands.[36]

Figure 1 shows data collected at the SRC Pipe Flow Technology Centre[32] for a slurry of 90 μm particles tested in a 158 mm pipeline loop. The solids concentration, C_s, was 0.31. Predictions obtained using the equivalent fluid model, the Rao and Scott version of the Durand & Condolios model, and the SRC kinematic friction model are also shown on this figure. The pressure gradient data and predictions are presented in the form of an effective friction factor:

$$f_m = \frac{dP}{dz}\frac{2D}{\rho_m U_{LS}^2} \tag{23}$$

to better illustrate the trends of the different models/correlations. For these particles, the SRC kinematic friction model provides the most accurate predictions. Numerous additional slurry flow experimental results, for conditions where Coulombic friction was negligible, were tested. Although the results are not reported here, the reader is referred to Gillies et al.[36] to see other favourable comparisons between slurry flow friction loss data and the SRC kinematic friction model.

In the next section, performance of the L–M correlation, improved for three-phase (G/L/S) horizontal pipeline flow predictions by assuming $(dP/dz)_{LS}$ can be calculated using the SRC kinematic friction model, is analysed. First, the performance of the modified L–M correlation is compared with that of other modified two-phase (G/L) flow models adapted for three-phase (G/L/S) flows. Predictions of the improved L–M correlation (subsequently referred to as LMKF) are then compared to other three-phase flow models, including the modified L–M correlations implemented by Scott and Rao,[24] Hatate et al.[29] and Gillies et al.[14] and the more recent phenomenological models of Orell[4] and Bello et al.[33]

ANALYSIS AND DISCUSSION

Performance of Other Modified Two-phase Flow Models

In this section, we compare the performance of the improved L–M (LMKF) correlation with that of a modified Dukler et al.[16] model and a modified Beggs and Brill[17] model. Experimental results reported by Fukuda and Shoji[25] were used for the comparison. As reported in Table 1, the Fukuda and Shoji[25] data were collected for

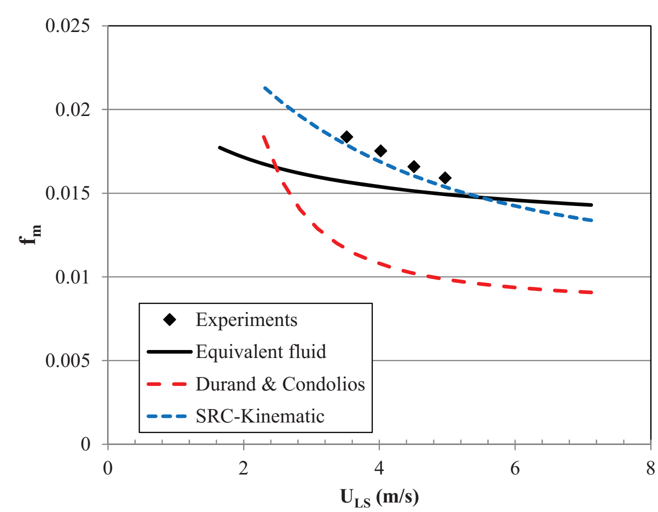

Figure 1. Slurry (L/S) pipeline flow friction loss measurements[32]: $d = 90$ μm; $C_s = 0.31$; $D = 158$ mm; $T = 20°C$; $k = 15$ μm; $C_{max} = 0.5$). Also shown are the predictions obtained using the equivalent fluid model, Scott and Rao's[24] version of the Durand & Condolios model, and the SRC kinematic friction model.

the following conditions: $D = 41.6$ mm; sand particles with $d_{50} = 74$ μm and $\rho_s = 2650$ kg/m^3; $C_s = 3.8\%$ to 24.7%; $U_{LS} = 1$–3.2 m/s; $U_G = 0.25$–4.6 m/s.

Figure 2 shows a comparison of Fukuda and Shoji's experimental data against the predictions of the three modified correlations. It is evident that the modified L–M correlation is able to predict the pressure gradient for all flow conditions tested by Fukuda and Shoji.[25] The modified Dukler et al.[16] and Beggs and Brill[17] models do not provide good predictions, as they both yield frictional pressure losses that significantly underpredict the experimental measurements. It should be noted, however, that for the particle size and solids concentrations tested by Fukuda and Shoji,[25] the L–M correlation modified to incorporate the equivalent fluid model (following the procedure of Ref.[14])also provides excellent agreement between the measured and predicted pressure gradients. However, because the equivalent fluid model is not appropriate for coarser particles, higher concentrations and larger pipe diameters, it is not presented here.

For the remainder of the analysis, we therefore consider only the L–M correlation, modified for three-phase flows using the SRC kinematic friction model.

Performance of other Three-Phase (G/L/S) Pressure Drop Correlations

In Figure 3, the performance of a number of different models for predicting pressure gradients in three-phase (G/L/S) pipeline flows is compared, again using the data of Fukuda and Shoji.[25] Again, the improved L–M correlation (LMKF) provides some of the best overall predictions. The adaptations of the L–M correlation implemented by Scott and Rao[24] and Hatate et al.[29] provide reasonable predictions, especially at lower values of pressure gradient, which correspond to lower solids concentrations and lower slurry superficial velocities. The Orell[4] model provides very accurate predictions; however, as mentioned previously, it is difficult to use and requires that the flow regime be known. This significantly limits its overall utility. The model of Bello et al.[33] does not provide accurate predictions even though its basis is a 1D force balance. The deficiency of the Bello et al.[33] model is in the force terms that are implemented in the balance.

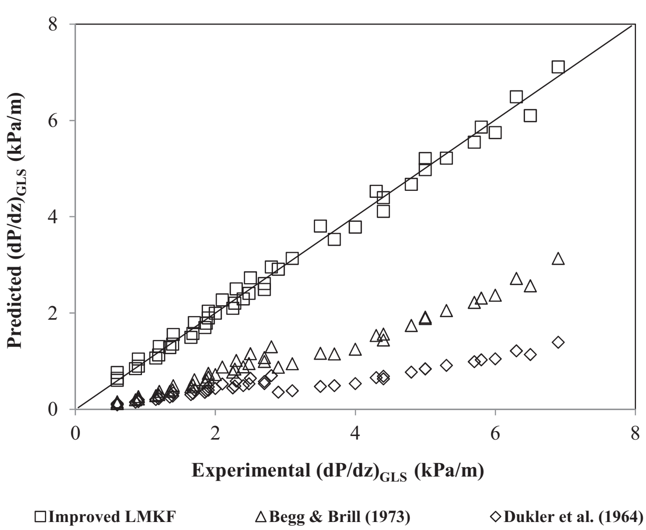

Figure 2. Comparison of the predictions of the improved Lockhart–Martinelli correlation, incorporating the SRC kinematic friction model with modified versions of the Dukler et al.[16] and Beggs and Brill[17] two-phase pressure drop models. Comparisons made against the data of Fukuda and Shoji.[25]

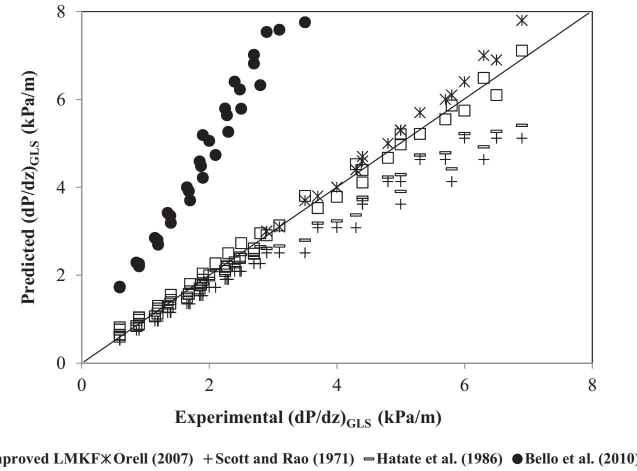

Figure 3. Comparison of the predictions of the improved Lockhart–Martinelli correlation, incorporating the SRC kinematic friction model, with other three-phase pressure drop models. Comparisons made against the data of Fukuda and Shoji.[25]

Performance of the Improved Lockhart–Martinelli Correlation

The performance of the improved L–M correlation, incorporating the SRC kinematic friction model, is analysed below in greater detail. Figure 4 shows pressure gradient predictions for the three-phase (G/L/S) horizontal pipeline flow data reported in the literature. Note that experimental data reported previously are *excluded* from Figure 4 if:

- All relevant operating parameters were not provided. For example the measurements of Angelsen et al.[41] are not included because they did not report a particle size for their studies.
- The slurry (L/S) 'phase' behaved as a homogeneous, non-Newtonian single-phase fluid (e.g. Refs.[42,43]).

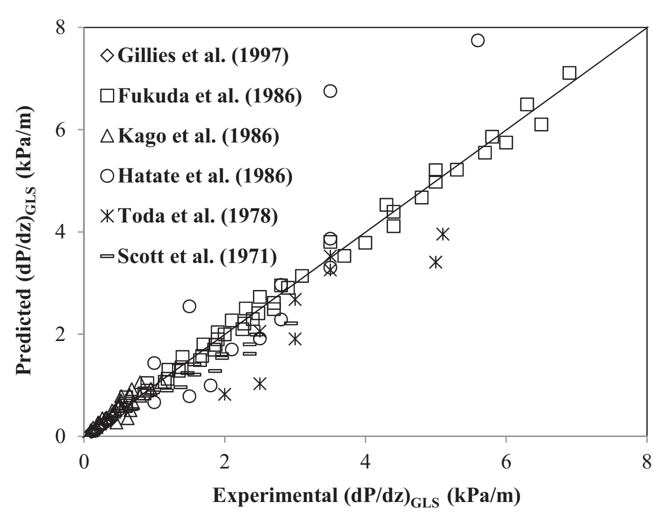

Figure 4. Comparison of the predictions of the improved Lockhart–Martinelli correlation, incorporating the SRC kinematic friction model, with experimental data available in the literature for horizontal, three-phase (G/L/S) pipeline flows.

- Measurements were taken with stationary beds or moving dunes present.
- Pipelines were inclined or vertically oriented.

The improved L–M correlation provides reasonable predictions for most of the relevant experimental data reported previously in the literature. Recall that the data sources are provided in Table 1. Relatively poor agreement between predictions of the LMKF correlation and the experimental data is observed for the data of Toda et al.,[44] who tested very coarse particles (~0.5 and 1 mm) for which Coulombic friction would be dominant. Note that the LMKF correlation significantly underpredicts the three-phase pressure gradient for these cases, as should be expected. Some of the coarse-particle ($d \sim 0.5$ mm) experimental results reported by Scott and Rao[24] are also not well-predicted by the LMKF correlation.

The other data shown in Figure 4, for which relatively poor performance of the LMKF correlation is observed, were those collected by Hatate et al.[29] using a very small diameter (15.5 mm) pipeline loop. The experience of the authors, the research engineers of the SRC (see, e.g. Ref.[35]) and others[45] is that very small diameter pipe loops ($D < 25$ mm) do not typically yield results that scale with larger pipe diameters.

In the subsequent paragraphs, the ability of the improved L–M (LMKF) correlation to predict the effects of superficial gas phase velocity (U_G), solids concentration (C_s) and in situ gas volume fraction (α_G) on pipeline pressure gradient is illustrated.

Figure 5 shows the variation of three-phase pressure drop in horizontal pipe flow as a function of superficial gas velocity (U_G) for two different solids concentrations. The experimental data were published by Fukuda and Shoji.[25] The data were collected at $U_{SL} = 3$ m/s in a pipe loop having a diameter of 0.0416 m. Slurries were prepared using silica sand ($d_{50} = 0.074$ mm, $\rho_s = 2650$ kg/m^3). Solids concentrations $C_s = 8.8\%$ and $C_s = 24.7\%$ were tested. The solid lines represent the predictions obtained using the improved L–M correlation developed in the present study. It is evident that the LMKF correlation provides accurate predictions for these conditions.

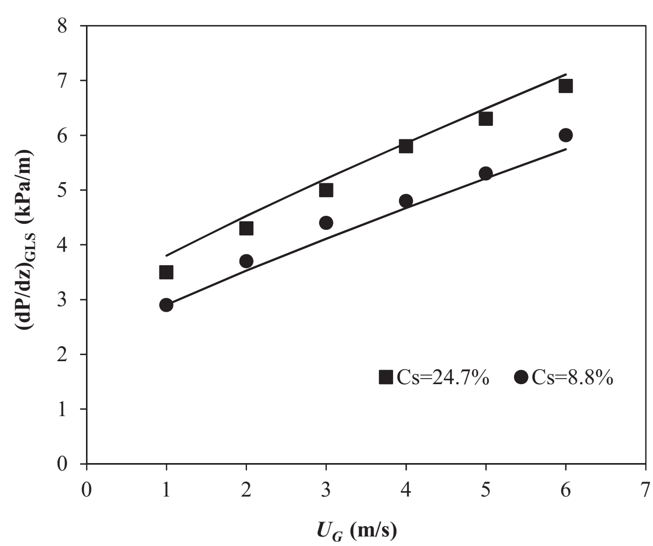

Figure 5. Ability of the improved Lockhart–Martinelli correlation, incorporating the SRC kinematic friction model, to predict the effects of gas phase superficial velocity (U_G) and slurry solids volume fraction (C_s) on the pipeline pressure gradient. Shown here are the data of Fukuda and Shoji[25]: $D = 0.0416$ m; $d_{50} = 0.074$ mm; $U_{SL} = 3$ m/s.

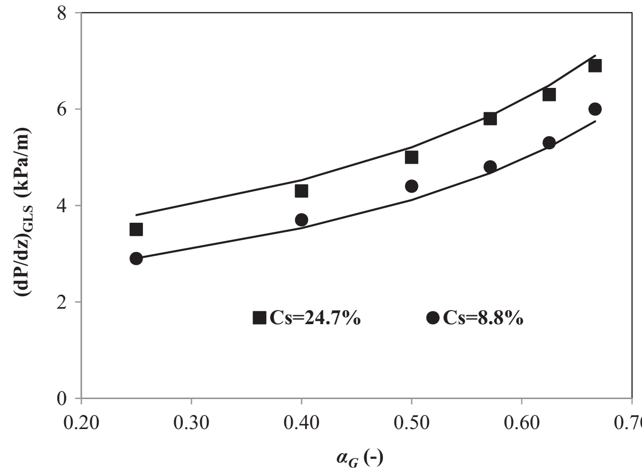

Figure 6. Ability of the improved Lockhart–Martinelli correlation, incorporating the SRC kinematic friction model, to predict the effects of in situ gas phase volume fraction (α_G) and slurry solids volume fraction (C_s) on the pipeline pressure gradient. Shown here are the data of Fukuda and Shoji[25]: $D = 0.0416$ m; $d_{50} = 0.074$ mm; $U_{SL} = 3$ m/s.

In Figure 6, data obtained from Fukuda and Shoji[25] showing the effects of in situ gas phase volume fraction on pipeline pressure gradient for two different slurry solids concentrations (again, 8.8% and 24.7% solids by volume). The slurry superficial velocity was $U_{SL} = 3$ m/s. Again, good agreement between the experimental data and the predictions obtained using the LMKF correlation is observed. This is in apparent contradiction to the experimental results obtained by Gillies et al.,[14] who noted that the L–M correlation modified using the equivalent fluid model failed to provide good predictions at $\alpha_G > 0.5$. However, the slurry superficial velocities used in the tests of Gillies et al.[14] were significantly lower ($U_{SL} = 0.5$ m/s). One of the reasons for the more accurate predictions of the LMKF correlation is that the velocity-dependence of the slurry pipeline pressure gradient, $(dP/dz)_{LS}$, is more accurately accounted for using the SRC kinematic friction model than when the equivalent fluid model is used to estimate the slurry pipeline pressure gradient.

Figure 7 shows the variation of three-phase (G/L/S) pipeline pressure gradient for two different values of slurry superficial velocity (U_{LS}) and at different solids concentrations. Again, the data are from Fukuda and Shoji.[25] Good agreement between the experimental data and predictions obtained using the LMKF correlation is observed, primarily for the following reasons:

- The slurry 'phase' friction losses are dominant in the calculation of the three-phase pipeline pressure gradient.
- The slurries tested by Fukuda and Shoji[25] do not demonstrate Coulombic (sliding bed) friction.
- The pipe diameter is similar to that tested by Lockhart and Martinelli. Moriyama et al.[46] and Chen et al.[47] both showed that the Chisholm constants decrease when the pipe diameter increases. In other words, the multiphase pressure gradient appears to be somewhat less than that predicted by the standard L–M correlation for large pipes.[48] This observation is expected to hold for three-phase flows in larger diameter pipes as well.

CONCLUSIONS

In this paper, we present an improved implementation of the modified L–M correlation, whereby the slurry 'phase' pressure

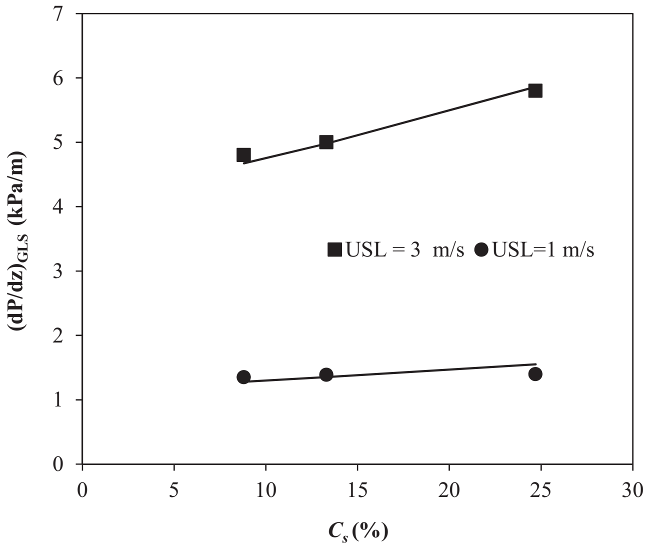

Figure 7. Ability of the improved Lockhart–Martinelli correlation, incorporating the SRC kinematic friction model, to predict the effects of slurry solids concentration (C_s) and slurry superficial velocity (U_{SL}) on the pipeline pressure gradient. Shown here are the data of Fukuda and Shoji[25]: $D = 0.0416$ m; $d_{50} = 0.074$ mm; $U_G = 4$ m/s.

gradient is calculated using the SRC kinematic friction loss model. Excellent agreement between the experimental pressure loss data available in the literature and the predictions obtained using the improved L–M (LMKF) correlation was observed provided that Coulombic (sliding bed) friction could be ignored. This is typically the case for smaller particles of the type most commonly tested in the literature. It should be noted that a criterion to evaluate the importance of sliding bed friction is available in the literature.[36] When the experiments involved coarse particles, such that Coulombic friction was important, the LMKF correlation did not provide good predictions, as was expected. Poor agreement between the predictions obtained using the LMKF correlation and the experimental measurements was also observed for very small diameter pipes (i.e. $D < 20$ mm).

Additionally, two other popular two-phase (G/L) pressure loss correlations, those of Dukler et al.[16] and Beggs and Brill,[17] were adapted for three-phase flow and tested against available experimental data and the improved L–M correlation. We found that the improved L–M provided much more accurate predictions of three-phase pressure gradients than were obtained using the other two correlations.

We note that caution should be taken when applying the improved L–M correlation to three-phase pipeline flows where the pipe diameter is greater than 100 mm, since no reliable three-phase data for larger pipes exist. Also, the classical L–M correlation is known to overpredict two-phase (G/L) pressure losses in larger diameter pipelines.[48] Coulombic friction also increases with the square of the pipe diameter.[28]

RECOMMENDATIONS

The following recommendations can be made based on the present study:

(i) In the oil sands industry, where the pipe diameters are large (typically $D > 0.6$ m) and the mass median particle diameter (d_{50}) of the particulate phase is typically between 110 and 300 μm,[37] engineers and designers should treat predictions obtained using the correlation presented here with caution.

(ii) Additional, comprehensive experimental data are required for horizontal three-phase (G/L/S) flows, including measurements of pressure gradient, deposition velocity, flow regime and in situ concentrations of each phase for a wide variety of operating conditions.

(iii) Experiments should be done in pipeline test loops where the pipe diameter is, at minimum, 50 mm and selected experiments conducted at the smaller scale should be repeated in a larger diameter pipeline loop (e.g. 250 mm).

(iv) Flow-regime dependent, three-phase (G/L/S) friction loss models should be developed.

NOMENCLATURE

A	cross-sectional area available for flow (m^2)
C	Chisholm coefficient in the L–M correlation
C_D	drag coefficient for a particle settling in the carrier fluid
C_{max}	volume fraction of particles in a settled bed
C_s	volume fraction of particles in the slurry (L/S) 'phase'
d	particle diameter (m)
d_{50}	mass median particle diameter (m)
D	pipe internal diameter (m)
f	friction factor
f_0	single-phase friction factor in the Dukler correlation
g	acceleration of gravity (9.81 m/s^2)
G	mass flux (kg/s/m^2)
i	hydraulic gradient (m water/m pipe)
k	pipe or wellbore hydrodynamic roughness (m)
P	pressure (Pa)
Q	volumetric flow rate (m^3/s)
S	density ratio (ρ_s/ρ_w)
U	superficial velocity (m/s)
V_c	deposition velocity (m/s)
W	mass flow rate (kg/s)
X	pressure gradient ratio in the L–M correlation
z	axial co-ordinate (m)

Greek Symbols

α	average in situ volume fraction
β	parameter in the Dukler two-phase (G/L) correlation
θ	angle of pipe inclination (°)
ϕ	L–M coefficient
κ	delivered (or input) volume fraction of liquid for a G/L system
λ	linear concentration
μ	viscosity (Pa s)
ρ	density (kg/m^3)
τ_w	wall shear stress (Pa)
ψ	Durand–Condolios coefficient

Subscripts

GL	gas–liquid
GLS	gas–liquid–solid
L	liquid
LS	liquid–solid (slurry)
m	mixture
MP	multiphase

ns no-slip conditions
f fluid
g gravity
i individual phase
T total
s solids
w water

ACKNOWLEDGEMENTS

The support of the Natural Sciences and Engineering Research Council of Canada (NSERC), Canadian Natural Resources Limited, Fort Hills LLP, Nexen Inc., Saskatchewan Research Council, Shell Canada Energy, Syncrude Canada Ltd. And Total E&P Canada through the *NSERC Industrial Research Chair in Pipeline Transport Processes* (RSS) is gratefully acknowledged.

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Manuscript received October 21, 2012; revised manuscript received December 14, 2012; accepted for publication December 17, 2012.