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Research Paper

Frictional pressure drop correlation for two-phase flows in mini and micro multi-channels

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HIGHLIGHTS

- A new correlation was developed to predict pressure drop in mini/micro multi-channels.
- A database of frictional pressure drop in two-phase flow in mini/micro multi-channels was collected.
- Homogenous and separated flow models of two-phase flow frictional pressure drop in multi-channels were reviewed.

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ABSTRACT

1029 frictional pressure drop data of two-phase flow in mini/micro multi-channels were collected from 11 literatures. This database included 8 working fluids: R134a, R22, R404a, FC-72, water, CO_2, R236fa and R245fa. The channel dimension ranged from 0.109 to 2.13 mm. Frictional pressure drop range of the database was from 5 to 150 kPa. The applicability of 11 existing correlations developed for mini/micro single channels to mini/micro multi-channels were evaluated with the multi-channel database. Among the existing correlations, the correlation of Lee-Mudawar predicted the database with the mean absolute percentage error (MAPE) of 35.3%, the correlations of Lee-Garimella and Sun-Mishima had MAPEs within 45.0%. In the process of new correlation development, the database was divided into three categories (namely, 1: gas laminar-liquid laminar, 2: gas turbulent-liquid laminar, 3: gas turbulent-liquid turbulent) in terms of liquid Reynolds number and gas Reynolds number, since gas laminar-liquid turbulent data did not exist in the literature. The newly developed correlation was formulated by a function of the two-phase Reynolds number, Re_{tp}, the two-phase viscosity number, $N_{\mu_{tp}}$, and the vapor quality, x. The correlation could predict the measured frictional pressure drop with the MAPE of 18.9%. The correlation demonstrated an excellent performance of the two-phase flow frictional pressure drop prediction in mini/micro multi-channels.

1. Introduction

The demand of high speed and high power of electronic applications growing increasingly is a driving force to develop more compact and lightweight packaging as well as better thermal management devices. In view of demanded high heat transfer performance, boiling heat transfer has been widely adopted in many cooling devices for computer chips, fuel cells and spacecraft electronics. Compared with the traditional cooling methods using pool boiling, spray and flow boiling in the conventional channels, cooling with flow boiling in mini/micro multi-channel heat sinks demonstrates better performances than the others [1–3]. Many researchers studied the heat transfer characteristics of flow boiling in mini/micro multi-channel heat sinks. Qu and Mudawar [4] measured the flow boiling heat transfer in micro multi-channel heat sink with the dimensions of width 231 μm and depth 713 μm. They proposed the heat transfer correlation based on the annular flow model. Ong and Thome [5] investigated the effects of multi-channel dimension, heat flux, flow pattern and fluid properties on two-phase heat transfer characteristics. Lee and Mudawar [6] measured the heat transfer coefficients of micro multi-channel heat sink with varied ranges of heat flux and vapor quality. Nucleate boiling and convective boiling were dominant at low and high quality regions, respectively. Chen and Garimella [7] conducted flow boiling experiments in a silicon multi-channel heat sink with

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Nomenclature

Bo	bond number
C	Chisholm parameter
C_p	specific heat capacity
D	channel hydraulic diameter
e	error between experimental value and predicted value
f	friction factor
G	mass flow rate
h_{fg}	latent heat
$\dot{m}$	volume flow rate
MAPE	mean absolute percentage error
ME	mean error
MPE	mean percentage error
N_μ	viscosity number
P	pressure
Q	heat power
PE	percentage error
Re	Reynolds number
RMSE	root mean square error
RMSPE	root mean square percentage error
We	Weber number
x	vapor quality
X	Martinelli parameter
z	length

Subscripts

crit	critical condition
exp	experimental
f	fluid
g	gas
in	inlet
l	liquid
pre	predicted
sat	saturated temperature
tp	two-phase flow

Greek symbols

ε	roughness
μ	viscosity
ρ	density
σ	surface tension coefficient
ϕ^2	two-phase multiplier

24 micro-channels of 389 μm wide and 389 μm deep. The heat transfer coefficient increased with increasing heat flux, while the heat transfer coefficient was independent of flow rate in fully developing boiling. Kandlikar [8] demonstrated the heat transfer rate in multi-channel evaporators was different from single channel evaporator under the same operating conditions. The flow instability was considered as the potential reason for the difference.

In addition to the heat transfer coefficient of two-phase flow in mini/micro multi-channels, the frictional pressure drop is one of key parameters for designing two-phase flow mini/micro exchanger. To study two-phase flow frictional pressure drop characteristics and mechanisms, numerous researchers carried out experiments on two-phase frictional pressure drop in mini/micro single channels. Hwang and Kim [9] used R134a as working fluid to test the two-phase frictional pressure drop in several single stainless steel channels with inner diameters of 0.244, 0.430, and 0.792 mm. The increasing quality, mass flux, and decreasing tube diameters increased the frictional pressure drop. Hun and Kim [10] measured the frictional pressure drop of water in a single horizontal rectangular micro-channel with diameter of 100 μm. Tibirica and Ribatski [11] studied the two-phase flow frictional pressure drop of R245fa in a single-channel with diameter of 2.32 mm under adiabatic and diabatic conditions. Wu and Koettig [12] investigated CO_2 two-phase flow frictional pressure drop in a circular stainless steel tube with inner diameter of 1.42 mm. Measurements were performed by varying the mass flux from 300 to 600 kg/m^{2}s, heat flux from 7.5 to 29.8 kW/m^2. Maqbool and Palm [13] carried out ammonia two-phase flow frictional pressure drop in single mini-channels with diameter of 1.70 and 1.224 mm. The heat flux and mass flux ranged from 15 to 355 kW/m^2 and 100 to 500 kg/m^{2}s, respectively. All experiments in mini/micro single-channels demonstrated the heat flux, mass flux and vapor quality affected the frictional pressure drop.

Knowledge of frictional pressure drop in the mini/micro single-channels is useful to obtain some insights in calculating the frictional pressure drop in mini/micro multi-channel heat sinks. As shown in Fig. 1, the multi-channel construction is widely used in cooling high power electronic devices [8]. The total frictional pressure drop in multi-channels is one of important factors to determine the pump power. Because of the manifolds at the inlet and outlet, the two-phase flow may not be evenly distributed in each channel. It's impractical to predict the flow distribution of each channel in the multi-channels.

The present work is focused on the development of a correlation for the frictional pressure drop of two-phase flow in mini/micro multi-channels. In the present work, existing frictional pressure drop correlations for two-phase flow in multi-channels have been reviewed. With the collected database (8 working fluids, channel dimension ranging from 0.109 to 2.13 mm), a new frictional pressure drop correlation is proposed for two-phase flow in mini/micro multi-channels.

2. Existing frictional drop correlations

2.1. Homogenous flow model

The homogenous flow model is a simplified method to calculate the frictional pressure drop of two-phase flow. The homogenous flow model assumes two-phase flow mixture with no relative velocity between two phases. The functional form of the homogeneous flow model is identical with the form of single-phase flow frictional pressure drop. The properties of the two-phase mixture are calculated with the vapor quality and the properties of liquid and gas fluids. The frictional pressure drop based on the homogenous flow model [14] is expressed as:

$$\left(\frac{dp}{dz}\right)_{tp} = f_{tp}\frac{2G^2}{D\rho_{tp}} \tag{1}$$

G is the mass flow rate, D is the hydraulic diameter, ρ_{tp} is the density of two-phase mixture fluid. f_{tp} is the two-phase friction factor. In the present work, the two-phase frictional factor is calculated by using the Churchill correlation [15]:

$$f_{tp} = 8\left[\left(\frac{8}{Re_{tp}}\right)^{12} + \frac{1}{(A+B)^{3/2}}\right]^{1/12} \tag{2}$$

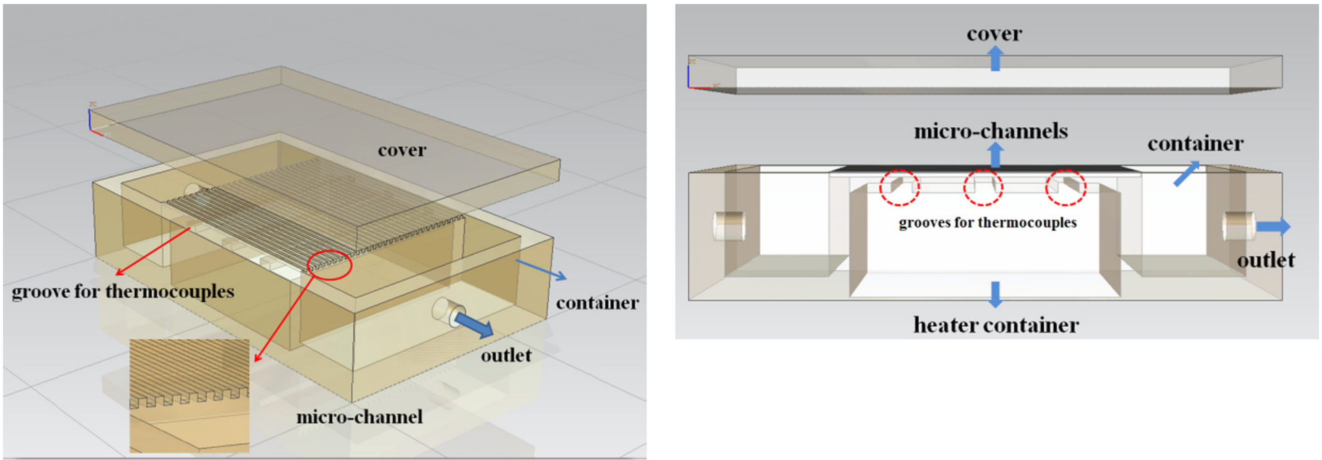

Fig. 1. Structure of the multichannel heat sink.

where

$$A = \left\{2.457\ln\left[\frac{1}{\left(\frac{7}{Re_{tp}}\right)^{0.9} + 0.27\frac{\varepsilon}{D}}\right]\right\}^{16} \tag{3}$$

$$B = \left(\frac{37530}{Re_{tp}}\right)^{16} \tag{4}$$

D is the hydraulic diameter, ε is the roughness, Re_{tp} is the two-phase Reynolds number. The two-phase Reynolds number Re_{tp} used in the homogeneous model is defined as:

$$Re_{tp} = \frac{Gd}{\mu_{tp}} \tag{5}$$

In order to calculate the two-phase Reynolds number, Re_{tp}, two-phase viscosity μ_{tp} should be properly given. Davidson and Hardie et al. [16] proposed the homogenous flow model with the two-phase viscosity given by a function of liquid viscosity, vapor quality, liquid and vapor densities. The widely accepted forms of the homogenous flow model were proposed by McAdams and Woods et al. [17] and Cicchitti and Lombardi et al. [18]. Their model used vapor quality to express the vapor fraction in tubes. Awad and Muzychka [19] analogized the thermal conductivity of porous media and viscosity of two-phase flow and proposed a new two-phase viscosity. Their new model was verified with various kinds of coolant in multi-channels.

The homogenous flow models discussed above have been verified for two-phase-flow in mini/micro multi-channels. Two-phase viscosity μ_{tp} used in homogeneous models and applicable conditions are listed in Table 1.

Table 1
Summary of the two-phase viscosity μ_{tp} used in homogeneous models for multi-channels.

Reference	Definition of two-phase viscosity μ_{tp}	Conditions
[17]	$\frac{1}{\mu_{tp}} = \frac{x}{\mu_g} + \frac{1-x}{\mu_l}$	Benzene-oil mixturesConventional multi-channelsAdiabatic
[16]	$\mu_{tp} = \mu_l\left[1 + x\left(\frac{\rho_l}{\rho_g} - 1\right)\right]$	WaterConventional multi-channelsAdiabatic
[18]	$\mu_{tp} = x\mu_g + (1-x)\mu_l$	Conventional multi-channels Adiabatic
[19]	$\mu_{tp} = \mu_g \frac{2\mu_g+\mu_l-2(\mu_g-\mu_l)(1-x)}{2\mu_g+\mu_l+(\mu_g-\mu_l)(1-x)}$	RefrigerantsMulti-channelsDiabatic

2.2. Separated flow model

The separated flow model considers the effect of the separated phases on the frictional pressure drop. Lockhart and Martinelli [20] correlation is widely used and expressed as:

$$\left(\frac{dp}{dz}\right)_{tp} = \phi_l^2\left(\frac{dp}{dz}\right)_l \tag{6}$$

$\left(\frac{dp}{dz}\right)_l$ is the frictional pressure drop of liquid single-phase. ϕ_l^2 is the two-phase multiplier which is expressed by Chisholm [21]:

$$\phi_l^2 = 1 + \frac{C}{X} + \frac{1}{X^2} \tag{7}$$

C is the Chisholm parameter, X is the Martinelli parameter defined as:

$$X^2 = \frac{\left(\frac{dp}{dz}\right)_l}{\left(\frac{dp}{dz}\right)_g}, \left(\frac{dp}{dz}\right)_l = f_l\frac{2G^2(1-x)^2}{d\rho_l} \text{and} \left(\frac{dp}{dz}\right)_g = f_g\frac{2G^2x^2}{D\rho_g} \tag{8}$$

where f_l and f_g are, respectively, the liquid and gas friction factors. G, ρ, D and x are the mass flow rate, density, channel hydraulic diameter and vapor quality, respectively. The Chisholm's parameter C in Eq. (7) is a function of fluid properties and experimental conditions. Researchers proposed numerous forms of the parameter C to modify the separated flow model for predicting the frictional pressure drop in mini/micro multi-channels. Qu and Mudawar [22] studied the frictional pressure drop in heat sink with micro multi-channels composed of 21 single channels with 231 μm width and 713 μm length. Lee and Mudawar [23] measured the frictional pressure drop in a micro multi-channel heat sink used in a refrigeration system. The dimensions of the channel were 231 μm wide and 713 μm deep. By comparing their experimental data with existing homogenous flow models and separated flow models, they proposed a new correlation to provide better prediction accuracy. The Reynolds number was employed to represent the effects of the velocity and the viscosity on the parameter C. The Weber number was also used to consider the effect of surface tension on the parameter C. Besides the viscosity and physical properties, flow conditions were also taken into consideration in these correlations. Lee and Garimella [24] investigated flow boiling in a silicon parallel channels heat sink. The dimensions of the microchannel were width from 102 μm to 997 μm, 400 μm in depth. Their correlation predicted the experiments results with mean absolute error of 11.4%. Sun and Mishima [25] collected 2092 frictional pressure drop data including 11 fluids. The range of the component single-channel diameter in multi-channels was from 0.506 to 12 mm. By analyzing with statistical method, they found that the Chisholm's parameter had some correlation with the ratio of liquid Reynolds number and gas Reynolds number, particularly when they were both higher than 2000. In order to avoid any discontinuity in the parameter, C, the ratio of x and $1\text{-}x$ was also introduced in the correlation. Lee and Liu et al. [26] investigated the frictional pressure drop characteristics in mini/micro multi-channels, and found the surface tension and the vapor quality had significant effects on the frictional

Table 2
Summary of previous separated flow model for multi-channel.

Reference	Correlations	Conditions
[22]	$C = 21(1 - e^{319D})\,(0.00418\,G + 0.0613)$	Water Multichannel 0.231 mm × 0.713 mm Heat
[23]	For laminar liquid and laminar vapor flows $C = 2.16 Re_l^{0.047} We_l^{0.23}$ For laminar liquid and turbulent vapor flows $C = 1.45 Re_l^{0.25} We_l^{0.23}\ Re_l = \frac{GD}{\mu_l}, We_l = \frac{v_l G^2 D}{\sigma}$	R134a Multichannel 0.231 mm × 0.713 mm Heat
[24]	$C = 2566 G^{0.5466} D^{0.8819}(1 - e^{-319D})$	Microchannel heat sink
[25]	$C = 1.79\left(\frac{Re_g}{Re_l}\right)^{0.4}\left(\frac{1-x}{x}\right)^{0.5}$	2092 data points R123, R134a, R22, R236ea, R245fa, R404a, R407C, R410A, R507, CO2 0.506–12 mm Multichannel Heat
[26]	$C = 121.1(1 - e^{-23.8Bo})x^{1.8}$	Multichannel Heat
[27]	$\varnothing_l^2 = (1-x)^2 + 2.87x^2\left(\frac{p}{p_{crit}}\right)^{-1} + 1.68x^{0.25}(1-x)^2\left(\frac{p}{p_{crit}}\right)^{-1.64}$	R134a, R22, R404A Multichannel 2.13 mm, 6.25 mm, 3.25 mm Insulated

Table 3
Collected database of two-phase frictional pressure drop in multi-channels.

Reference	Geometry	Working fluid and operating condition	Data points
[23]	0.231 mm × 0.713 mm	R134a Mass rate: 127–654 kg/m^2 s Heat flux: 31.6–93.8 W/cm^2	80
[27]	2.13 mm	R134a, R22 R404a Mass rate range 400–1000 kg/m^2 s	119
[28]	1.36 mm 1.44 mm	R410A Heat flux range 10–20 kW/m^2 Mass rate range 200–400 kg/m^2 s Saturation temperature range:0,5,10 °C	19
[29]	2 mm 4 mm	FC-72 Heat flux range 0.5–3 W/cm^2 Mass rate range 265.3, 442.1, 663.1 kg/m^2 s Saturation temperature range: 45, 55, 65, 75 °C	100
[30]	0.276 mm × 0.225 mm	FC-72 Heat flux range 60.4–130.6 kW/m^2 Mass flux range 341–531 kg/m^2 s	45
[31]	2.01 mm	R134a Heat flux range 6–31.6 kW/m^2 Mass flux range 90–295 kg/m^2 s	62
[32]	0.6 mm	Water Heat flux range 10–20 W/cm^2 Mass flux range 183–420 kg/m^2 s Saturation temperature range: 30, 60, 90 °C	300
[33]	0.2 mm	Water Heat flux range 9–614 W/cm^2 Mass flux range 115–389 kg/m^2 s	85
[34]	0.2 mm	R236fa, R245fa Heat flux range 130–1400 k W/m^2 Mass flux range 499-1100 kg/m^2 s Saturation temperature range: 30.5 °C	200
[35]	0.109 mm	Water Heat flux range 0–30 W/cm^2 Mass flux range 44–1114 kg/m^2 s	19

pressure drop. To express the surface tension effect on the frictional pressure drop, they used the Bond number in their correlation, which contained the fluid density, channel diameter and surface tension. Except the Chisholm's parameter *C*, Zhang and Webb [27] used vapor quality, pressure and critical pressure at the saturated temperature in their multiplier. All separated flow models tested for predicting frictional pressure drop in multi-channels are listed in Table 2.

3. Existing databases

1029 data were collected from 11 literatures including the adiabatic and diabatic systems and summarized in Table 3. The database includes 8 working fluids. The range of the channel diameters is from 0.109 to 2.13 mm.

Zhang and Webb [27] used R134a, R22 and R404A as working fluids to measure the frictional pressure drop in adiabatic two-phase flow system. The test section was manufactured on aluminum bars. Eight channels with diameter of 3.25 and 6.25 mm were extruded on these aluminum blocks respectively. The two-phase flow pressure drops were measured with different vapor quality, mass velocity and saturation temperatures. 119 data were collected from their experiments. Lee and Mudawar [23] and Agostini and Bontemps [31] also used R134a as working fluid in their experiments. Lee and Mudawar [23] conducted experiments with a micro-channel heat sink which was applied as an evaporator in a refrigeration system. The dimensions of micro-channels were 231 μm in width and 713 μm in depth. 53 micro-channels were cut on the copper plate. The inlet pressure ranged from 1.44 to 6.60 bar. The experimental results suggested that the frictional pressure drop was dominant in the total pressure drop. The pressure drop range in their experiments was from 0.05 to 0.25 bar. Agostini and Bontemps [31] measured the pressure drop of R134a in an aluminum test section with 11 mini-channels on it. The hydraulic diameter of the mini-channel was 2.01 mm. With different saturation temperatures, mass velocities and heat fluxes, the pressure drop in the experiments reached to 40 kPa/m. FC-72 were used as working fluid by Jang and Park et al. [29] and Megahed and Hassan [30] in their experiments. Jang and Park et al. [29] carried out the experiments for cooling electronic devices with small channel heat sinks. 9 small channels with diameters of 2 and 4 mm were machined in the test section. The effects of the inlet temperature, mass flux, channel diameter and inlet pressure on the frictional pressure drop were studied in experiments. The frictional pressure drop varied from 0.1 kPa/m to 30 kPa/m under different operation conditions. They found the dimension of small channel had significant effect on the pressure drop. The frictional pressure drops in 2 mm small channels were about 100% higher than 4 mm channels. Megahed and Hassan [30] conducted experiments with FC-72 in a silicon micro-channel heat sink. 45 rectangular micro-channels were etched on the silicon wafer. Each channel had dimension of 276 μm depth and 225 μm width. The frictional pressure drop in the experiments ranged from 400 kPa/m to 1100 kPa/m. According to the experimental results, the frictional pressure drop increased with the increasing of vapor quality and the mass flux. At a constant mass flux condition, the frictional pressure drop varied nearly linearly with the change of vapor quality. R410A, R236fa and R245fa were used by Yun and Heo et al. [28] and Costa-Party and Olivier et al. [34] for their experiment studies. Yun and Heo et al. [28] studied the frictional pressure drop of R410A in rectangular channels with hydraulic diameters of 1.36 and 1.44 mm. Measured frictional pressure drops were from 5 kPa/m to 60 kPa/m. They found that the saturated temperature affected the frictional pressure drop due to the viscosity and the density of the working fluid. They also found that frictional pressure drop in multi-channels were larger than single channel at the same operation conditions. Costa-Party and Olivier et al. [34] investigated the frictional pressure drop characteristics of R236fa and R245fa in a silicon micro multi-channels test section. The channels were 85 μm in width and 560 μm in height. There were 135 micro-channels etched on the silicon test section. The frictional pressure drops varied from 5 kPa to 80 kPa. In addition to the refrigerants, water was also a working fluid for measuring the frictional pressure drop in the multi-channels. Qu and Siu-Ho [32] studied the frictional pressure drop characteristics of water in micro heat sinks. The frictional pressure drops in their experiments ranged from 2 kPa to 70 kPa. Kosar and Kuo et al. [33] presented the frictional pressure drop of water in multi-micro-channels with 200 μm in width and 264 μm in depth. Singh and Bhide et al. [35] presented the frictional pressure drop characteristics of water in micro multi-channels with the hydraulic diameter of 109 μm. Their experiments results showed that the frictional pressure drops were affected by various mass velocity conditions.

4. Performance evaluation of available correlations using existing databases

4.1. Statistical parameters for evaluation of existing correlations

To evaluate the performance of the existing correlations using the available database, 5 statistical parameters including scale (or value) dependent and independent measures are adopted.

The scale dependent measures involve the Mean Error (ME) and the Root Mean Square Error (RMSE). The definition of the error between each experimental value, Exp_i, and the corresponding predicted value, Pre_i, is defined as e_i:

$$e_i = Pre_i - Exp_i \tag{9}$$

The mean error (ME) and the root mean square error (RMSE) are defined by using e_i as Eqs. (15) and (16), respectively:

$$ME = \frac{1}{n}\sum_{i=1}^{n} e_i \tag{10}$$

Table 4
Comparison between the existing correlations and the database.

No.	Correlations	ME (kPa)	RMSE (kPa)	MAPE (%)	MPE (%)	RMSPE (%)
1	Davidson and Hardie [16]	−3.83	46.6	98.3	−104	170
2	McAdams and Woods [17]	2.03	26.9	80.4	−43.7	157
3	Cicchitti and Lombardi [18]	−6.70	21.7	71.8	9.59	104
4	Qu and Mudawar [22]	−0.64	20.1	47.2	1.20	66.3
5	Lee and Mudawar [23]	−2.33	9.36	33.1	4.14	46.1
6	Lee and Garimella [24]	−0.20	27.2	45.9	2.76	62.9
7	Sun and Mishima [25]	−4.72	14.1	37.6	37.6	46.4
8	Lee and Liu [26]	−2.09	13.7	44.0	8.71	51.7
9	Zhang and Webb [27]	−9.93	15.4	50.4	32.6	55.7

$$\mathrm{RMSE} = \left(\frac{1}{n}\sum_{i=1}^{n} e_i^2\right)^{0.5} \tag{11}$$

The scale independent measures (or relative errors) are the Mean Percentage Error (MPE), the Root Mean Square Percentage Error (RMSPE), and the Mean Absolute Percentage Error (MAPE). With the definition of percentage error (PE) given in Eq. (12), the mean percentage error (MPE), the root mean square percentage error (RMSPE) and the mean absolute percentage error (MAPE) are defined as Eqs. (13), (14), and (15), respectively.

Percentage error (PE):

$$PE_i = \frac{e_i}{Exp_i} \tag{12}$$

Mean percentage error (MPE):

$$\mathrm{MPE} = \frac{1}{n}\sum_{i=1}^{n} PE_i \tag{13}$$

Root mean square percentage error (RMSPE):

$$\mathrm{RMSPE} = \left(\frac{1}{n}\sum_{i=1}^{n} PE^2\right)^{0.5} \tag{14}$$

Mean absolute percentage error (MAPE):

$$\mathrm{MAPE} = \frac{1}{n}\sum_{i=1}^{n} |PE| \tag{15}$$

4.2. Performance of existing correlations

The present section gives a comparison between the database and correlations referred in the Section 2. Among the correlations listed in Section 2, 9 of them predict the data with MAPEs less than 100%. The comparison results expressed by 5 statistical parameters are listed in Table 4.

From Table 4, the MAPEs of the homogenous flow models are higher than those of the separated flow models. The homogenous flow models predicted the data with MAPEs higher than 71.8%. The separated flow models perform better and have MAPEs lower than 47.2%. Lee-Mudawar correlation predicted the data with MAPE of 33.1%, the lowest among all correlations considered in the present evaluation. The correlations of Qu-Mudawar, Lee-Garimella and Lee-Liu have similar MAPEs being about 45.0%. Fig. 2 shows the comparison between the evaluated correlations and the frictional pressure drop data in multi-channels.

As shown in Fig. 2 and Table 4, the homogenous flow models have RMSPEs higher than the separated flow models. Compared with the separated flow model correlations, the homogenous flow

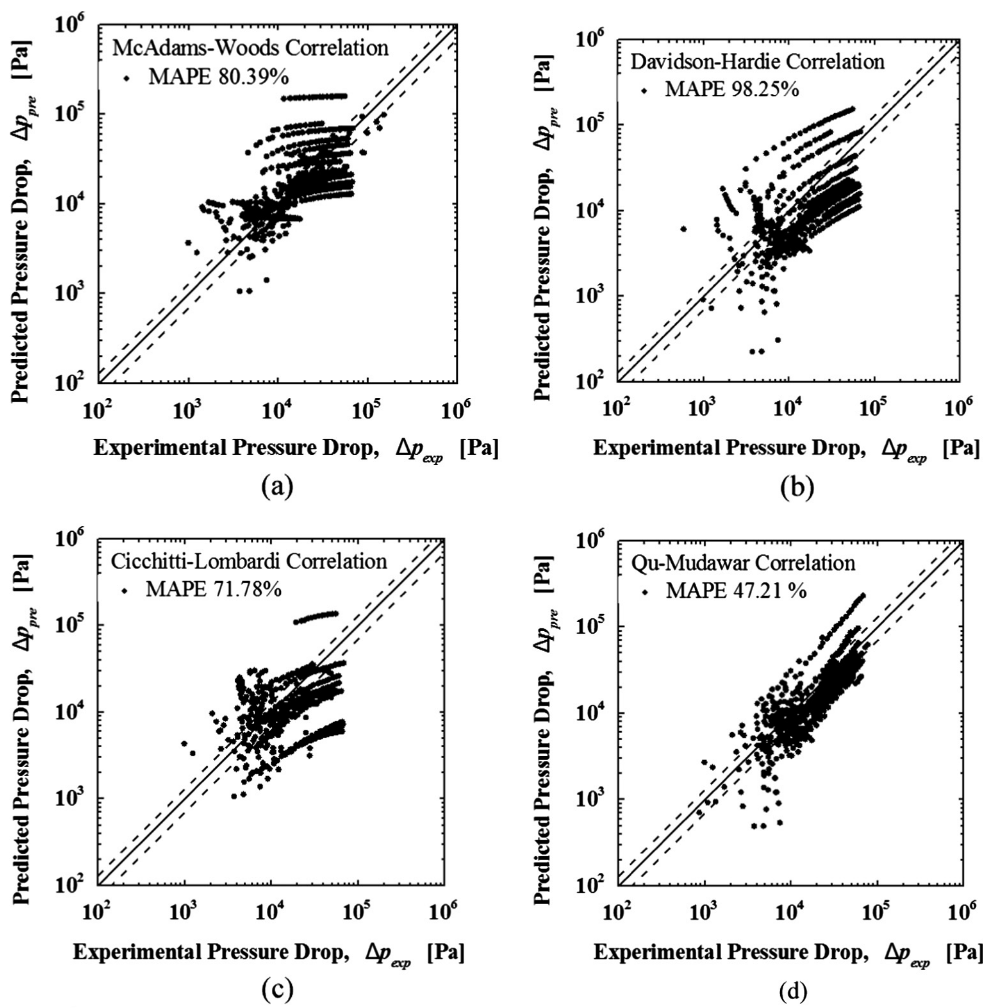

Fig. 2. Comparison of frictional pressure drop data in multi-channels with evaluated correlations: (a) McAdams and Woods et al. correlation [17]; (b) Davidson and Hardie et al. correlation [16]; (c) Cicchitti and Lombardi et al. correlation [18]; (d) Qu and Mudawar correlation [22]; (e) Lee and Mudawar correlation [23]; (f) Lee and Garimella correlation [24]; (g) Sun and Mishima correlation [25]; (h) Lee and Liu et al. [26]; (i) Zhang and Webb correlation [27].

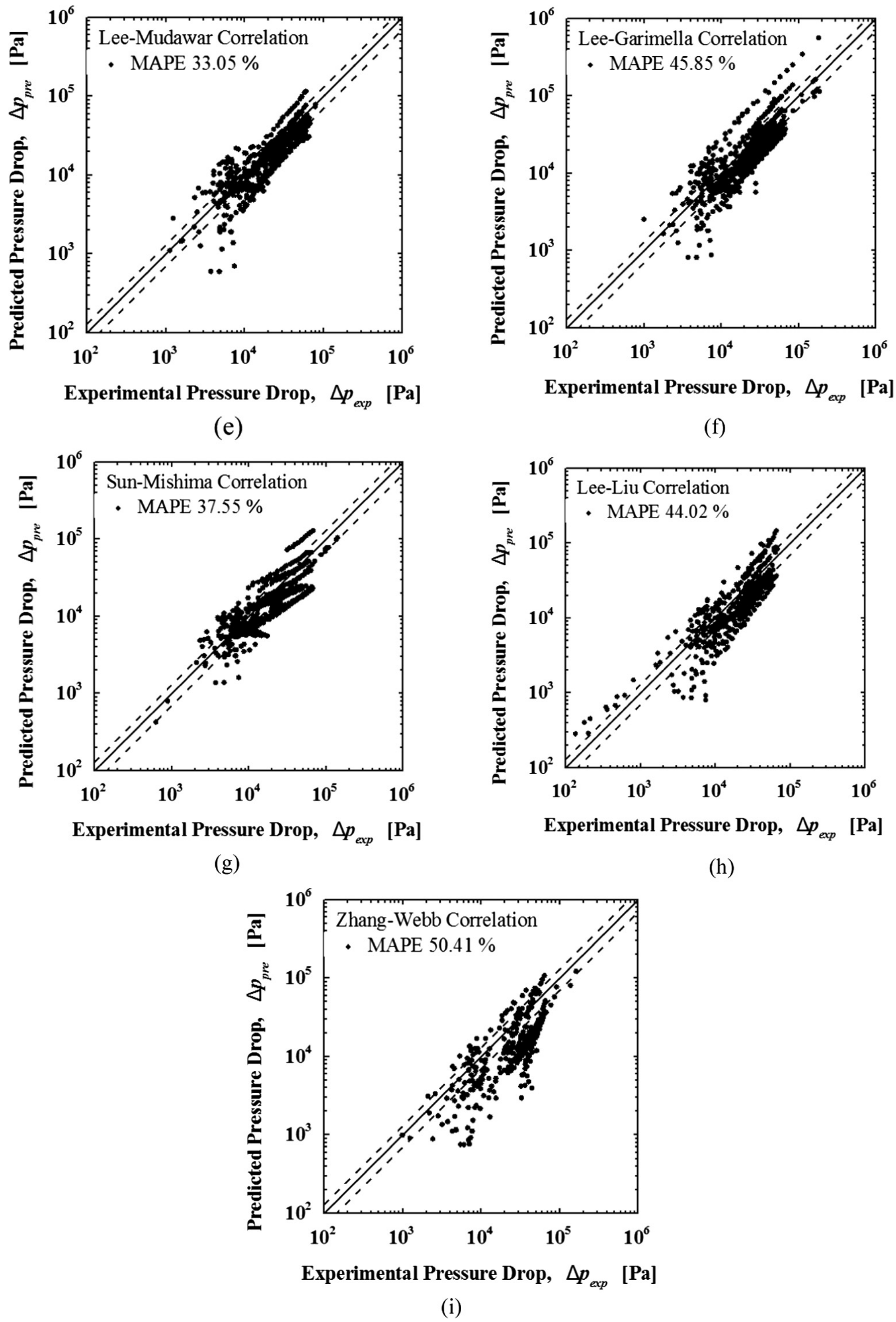

Fig. 2 (*continued*)

Table 5
Thresholds of Reynolds number between laminar and turbulent flows.

Flow regimes	Liquid Reynolds number	Gas Reynolds number
Liquid turbulent and vapor turbulent	$Re_l > 2000$	$Re_g > 2000$
Liquid laminar and vapor turbulent	$Re_l < 1000$	$Re_g > 2000$
Liquid turbulent and vapor laminar	$Re_l > 2000$	$Re_g < 1000$
Liquid laminar and vapor laminar	$Re_l < 1000$	$Re_g < 1000$

models may not predict the effects of fluid properties on the frictional pressure drops. Among the separated flow models, the correlations of Qu-Mudawar [22], Lee-Garimella [24] and Lee-Liu [26] show similar prediction tendency and MAPEs. Qu and Mudawar's [22] correlation and Lee and Garimella's [24] correlation both consider the dependence of the hydraulic diameter *D* and mass velocity *G* on the Chisholm's parameter, *C*. The correlation of Lee and Liu [26] considers the dependence of the Bond number and vapor quality on the Chisholm's parameter, *C*. This correlation performs better at the high pressure drop range than the low pressure drop range. Although Sun-Mishima's [25] correlation predicts the data with MAPE of 37.6%, the trend of the frictional pressure drop prediction shows some defects. Among all of the correlations, the correlation of Lee and Mudawar [23] has the lowest MAPE. They proposed the Chisholm's parameter, C, for different flow conditions, and adopted the Reynolds number and Weber number as major parameters in their correlation. The Weber number considers the fluid density, fluid velocity, hydraulic diameter and surface tension. The excellent prediction accuracy of Lee and Mudawar's correlation implies that the surface tension and the hydraulic diameter are important to be included in the Chisholm's parameter C for predicting the frictional pressure drop in mini/micro multi-channels.

5. New correlation development and performance

5.1. New correlation development

The performance analysis of the existing correlations indicates a promising performance of the separated flow model correlations with the Chisholm's parameter, C, expressed as a function of two-phase Reynolds number, vapor quality, fluid properties and flow conditions. In the process of the correlation development, the database is divided into three categories based on gas and liquid Reynolds number first proposed by Chisholm [21] (namely, 1: gas laminar-liquid laminar, 2: gas turbulent-liquid laminar, 3: gas turbulent-liquid turbulent) in terms of liquid Reynolds number, Re_l, and gas Reynolds number, Re_g. It should be noted here that gas laminar-liquid turbulent data did not exist in the literature. The flow conditions are given in Table 5.

Several parameters are tested to correlate the Chisholm's parameter and the preliminary analysis suggests that the two-phase Reynolds number Re_{tp}, vapor quality x and two-phase viscosity number $N_{\mu_{tp}}$ are key parameters to develop the correlation of the Chisholm's parameter, C. The two-phase Reynolds number, Re_{tp}, is defined as:

$$Re_{tp} = \frac{GD}{\mu_{tp}} \tag{16}$$

The two-phase viscosity μ_{tp} used in the two-phase Reynolds number is given by:

$$\frac{1}{\mu_{tp}} = \frac{1-x}{\mu_l} + \frac{x}{\mu_g} \tag{17}$$

The average vapor quality x over the channel or simply vapor quality is used in the correlation development and is calculated by

$$x = \frac{1}{h_{fg}}\left(0.5 * \frac{Q}{\dot{m}} - C_p(T_{sat} - T_{in})\right) \tag{18}$$

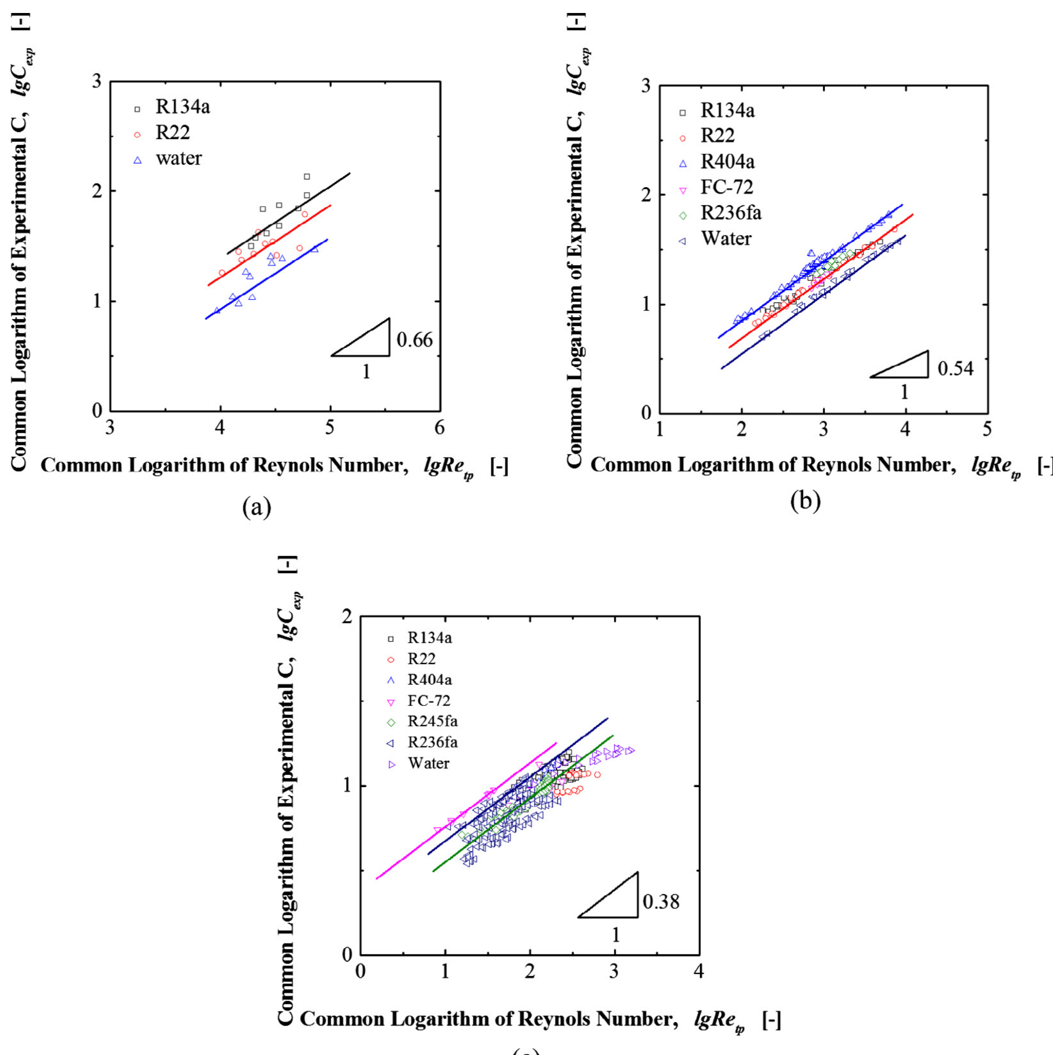

Fig. 3. Relationship between the Chisholm's parameter C and the two-phase Reynolds number Re_{tp}. (a) the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region.

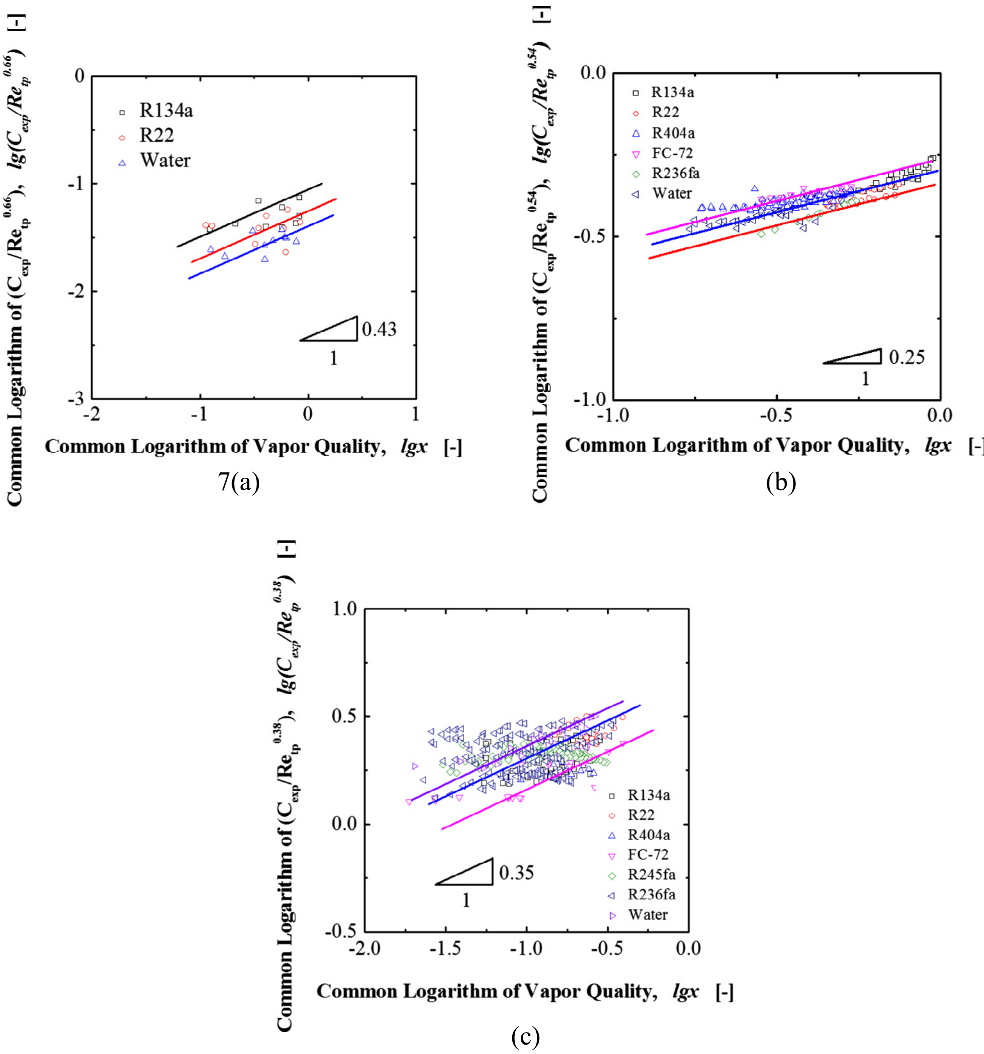

Fig. 4. Relationship between the common logarithm of (C_{exp}/Re_{tp}^{n}) and the vapor quality x (a) the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region.

h_{fg} is the latent heat, Q is the heat power, $\dot{m}$ is the volume flow rate, C_p is the specific heat capacity, T_{sat} is the saturated temperature of fluid, T_{in} is the temperature of inlet fluid.

The single-phase viscosity number is defined as $N_{\mu f} = \mu_f/(\rho_f\sigma\sqrt{\sigma/g\Delta\rho})^{1/2}$ [36]. Where μ_f and ρ_f are viscosity and density of liquid phase, respectively. A pre-analysis indicates that the single-phase viscosity number is not a suitable non-dimensional parameter in developing a correlation. This is because two-phase mixture properties are different from single-phase properties. The viscosity μ_{tp} and the density ρ_{tp} of two-phase mixture are affected by the vapor quality x significantly. Hence, the two-phase viscosity number $N_{\mu_{tp}}$ is proposed and defined as:

$$N_{\mu_{tp}} = \frac{\mu_{tp}}{\left(\rho_{tp}\sigma\sqrt{\frac{\sigma}{g\Delta\rho}}\right)^{0.5}} \tag{19}$$

The two-phase viscosity in Eq. (19) is given by Eq. (17). The two-phase mixture density ρ_{tp} is given by:

$$\rho_{tp} = x\rho_g + (1-x)\rho_l \tag{20}$$

To ascertain the dependence of the Chisholm's parameter on Re_{tp}, x and $N_{\mu_{tp}}$, the dependence of Chisholm's parameter on Re_{tp}, x and $N_{\mu_{tp}}$ are plotted in Figs. 3–5, respectively. In the figures, the values of the parameters are shown by their common logarithms. Slopes shown in Figs. 3–5 are exponents of Re_{tp}, x and $N_{\mu_{tp}}$ respectively.

Fig. 3 shows the relationship between the Chisholm's parameter, C, and the two-phase Reynolds number, Re_{tp}. The abscissa is the common logarithm of the two-phase Reynolds number, lgRe_{tp}. The ordinate is the common logarithm (a base-10 logarithm) of the experimentally determined Chisholm's parameter, C, lg$C$. According to Fig. 3, the exponents of Re_{tp}, n, are 0.66, 0.54 and 0.38 for (a) the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region, respectively.

Fig. 4 shows the relationship between (C_{exp}/Re_{tp}^{n}) and the vapor quality x. The abscissa is the common logarithm of the vapor quality x. The ordinate is the common logarithm of (C_{exp}/Re_{tp}^{n}). The slopes, m, shown in Fig. 4 represent the exponent of x. They are 0.43, 0.25 and 0.35 for (a) the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region, respectively.

Fig. 5 shows the relationship between ($C_{exp}/Re_{tp}^{n}\,x^{m}$) and two-phase viscosity number $N_{\mu_{tp}}$. The abscissa is the common logarithm of the two-phase viscosity number. The ordinate is the common logarithm of ($C_{exp}/Re_{tp}^{n}\,x^{m}$). Exponents of $N_{\mu_{tp}}$ are 1.2, 0.51 and 0.21 for (a)

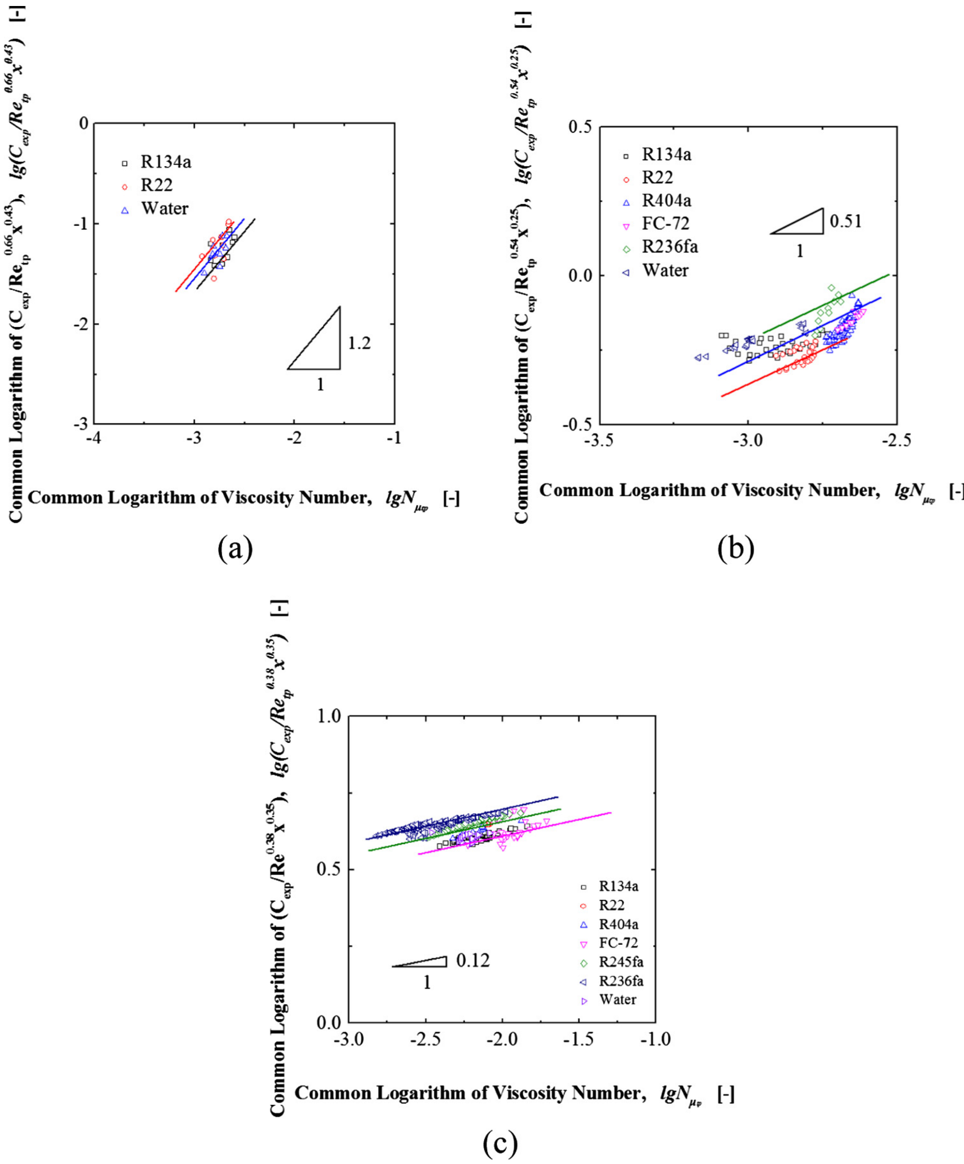

Fig. 5. Relationship between the common logarithm of $(C_{exp}/Re_{tp}^{n}x^{m})$ and the viscosity number $N\mu_{tp}$. (a) the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region.

Table 6
New correlations for different flow conditions.

Flow conditions	Parameter C
Liquid-turbulent and gas-turbulent	$C = 7.63Re_{tp}^{0.66}x^{0.43}N\mu_{tp}^{1.20}$
Liquid-laminar and gas-turbulent	$C = 2.23Re_{tp}^{0.54}x^{0.25}N\mu_{tp}^{0.51}$
Liquid-laminar and gas-laminar	$C = 1.87Re_{tp}^{0.38}x^{0.35}N\mu_{tp}^{0.12}$

the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region, respectively. Table 6 listed the correlation to calculate the Chisholm parameters for the three flow conditions.

The newly developed correlation is applicable for 8 working fluids such as R134a, R22, R404a, FC-72, water, CO_2, R236fa and R245fa. The diameter range is from 0.109 mm to 4 mm. The ranges of the two-phase Reynolds number, vapor quality and two-phase viscosity number are 85 to 63,095, 0.01 to 0.9 and 0.00063 to 0.016, respectively.

5.2. Performance analysis of the newly developed correlation

Fig. 6 compares the predicted two-phase multiplier, ϕ_{pre}^{2}, with the experimental two-phase multiplier, ϕ_{exp}^{2} determined based on the Churchill equation with single-phase liquid Reynolds number. Fig. 6 demonstrates an excellent performance of the newly developed correlation for the three flow conditions such as for (a) the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region. The MAPEs of the three conditions are 17.1%, 19.8% and 18.6%, respectively, which are all below 20%. The comparison results with the five statistical parameters are listed in Table 7.

Fig. 7 compares the predicted frictional pressure drop and the experimental frictional pressure drop in multi-channels. The new correlation predicts the frictional pressure drop data in mutli-channels with MAPE of 18.9% and the results indicate that the proposed correlations considerably improve the prediction accuracy of the frictional pressure drop in multi-channels.

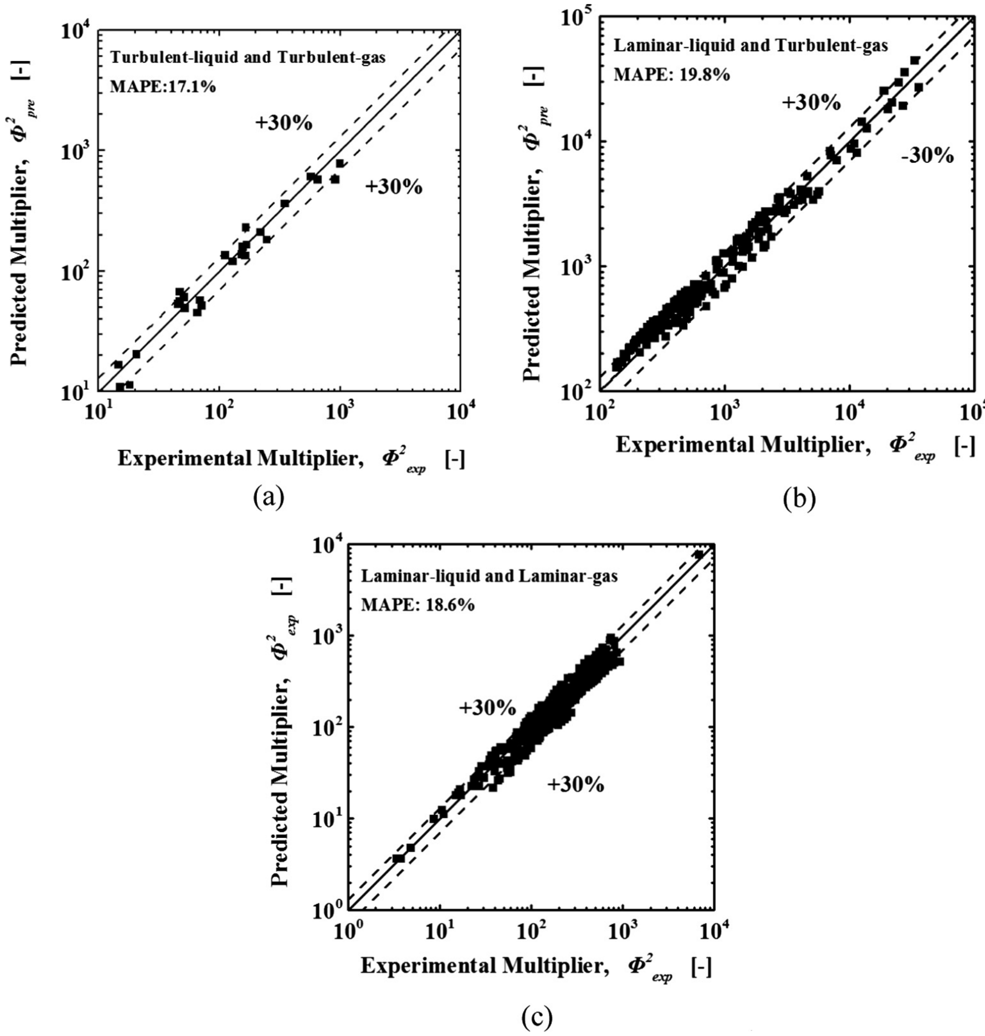

Fig. 6. Comparison between the predicted two-phase multiplier ϕ^2_{pre} and experimental two-phase multiplier ϕ^2_{exp} (a) Reynolds number: the liquid-turbulent and the gas-turbulent region; (b) the liquid-laminar and the gas-turbulent region; (c) the liquid-laminar and the gas-laminar region.

Table 7
Comparison between the newly developed correlation and the experimentally determined Chisholm's C parameter taken in multi-channels.

	ME (Pa)	RMSE (Pa)	MAPE (%)	MPE (%)	RMSPE (%)
Liquid-turbulent and gas-turbulent	−17.4	71.4	17.1%	−1.68%	21.2%
Liquid-laminar and gas-turbulent	−118	1539	19.8%	10.8%	19.4%
Liquid-laminar and gas-laminar	341	3200	18.6%	−0.07%	21.2%

6. Conclusions

In order to develop a correlation of the two-phase frictional pressure drop in mini/micro multi-channels, 1029 data were collected from 11 open literatures. This database includes 8 working fluids. The range of the channel dimension is from 0.2 mm to 2.13 mm. The frictional pressure drop range of the collected data is from 5 to 150 kPa.

10 correlations proposed for two-phase frictional pressure drop are reviewed. These reviewed correlations include the homogenous flow model and the separated flow model with Chisholm's parameter, *C*. The comparison of the correlations with the two-phase flow mini/micro multi-channels frictional pressure drop

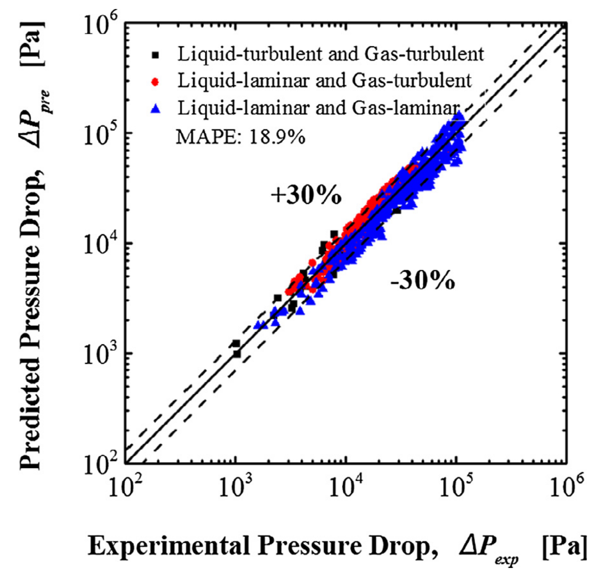

Fig. 7. The comparison between the experimental pressure drop and the predicted pressure drop.

data, 9 of the ten correlations have the mean absolute percentage errors (MAPEs) within 100%. The comparison results indicate that the Reynolds number and the vapor quality are two important parameters for correlating the frictional pressure drop in multi-channels. In addition, fluid properties also affect the frictional pressure drop in multi-channels.

In the process of new correlation development, the database was divided into three categories (namely, 1: gas laminar-liquid laminar, 2: gas turbulent-liquid laminar, 3: gas turbulent-liquid turbulent) in terms of liquid Reynolds number and gas Reynolds number, since gas laminar-liquid turbulent data did not exist in the literature. The newly developed correlation was formulated by a function of the two-phase Reynolds number, Re_{tp}, the two-phase viscosity number, $N_{\mu_{tp}}$, and the vapor quality, x. The correlation could predict the measured frictional pressure drop with the MAPE of 18.9%. The correlation demonstrated an excellent performance of the two-phase flow frictional pressure drop prediction in mini/micro multi-channels.

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Appendix A

A.1. Appendix A comparison between single-channel correlation and multi-channel correlation

Li and Hibiki [37] developed a two-phase frictional pressure drop correlation for mini/micro single-channels using an approach similar to the one adopted in the present work. The Chisholm's parameter for each flow condition is given in Table A1. The frictional pressure drop correlation for single-channels predicted the single-channel data with MAPE of 17.2%, and demonstrated an excellent performance. The correlation is applicable to 11 fluid systems: R22, R134a, R410A, R290, R744, ammonia, nitrogen, R245fa, Propane, water and CO_2, pipe diameter ranging from 0.1 mm to 3 mm, Reynolds number ranging from 100 to 35,000, vapor quality ranging from 0.01 to 0.9 and two-phase viscosity number ranging from 0.00063 to 0.01.

Table A1
Chisholm's parameters for correlations of single-channels for 4 flow conditions.

Flow conditions	Chisholm parameter C
Liquid-turbulent and gas-turbulent	$C = 6.28 N\mu_{tp}^{0.78} Re_{tp}^{0.67} x^{0.32}$
Liquid-turbulent and gas-laminar	$C = 1.54 N\mu_{tp}^{0.14} Re_{tp}^{0.52} x^{0.42}$
Liquid-laminar and gas-turbulent	$C = 245.5 N\mu_{tp}^{0.75} Re_{tp}^{0.35} x^{0.54}$
Liquid-laminar and gas-laminar	$C = 41.7 N\mu_{tp}^{0.66} Re_{tp}^{0.42} x^{0.21}$

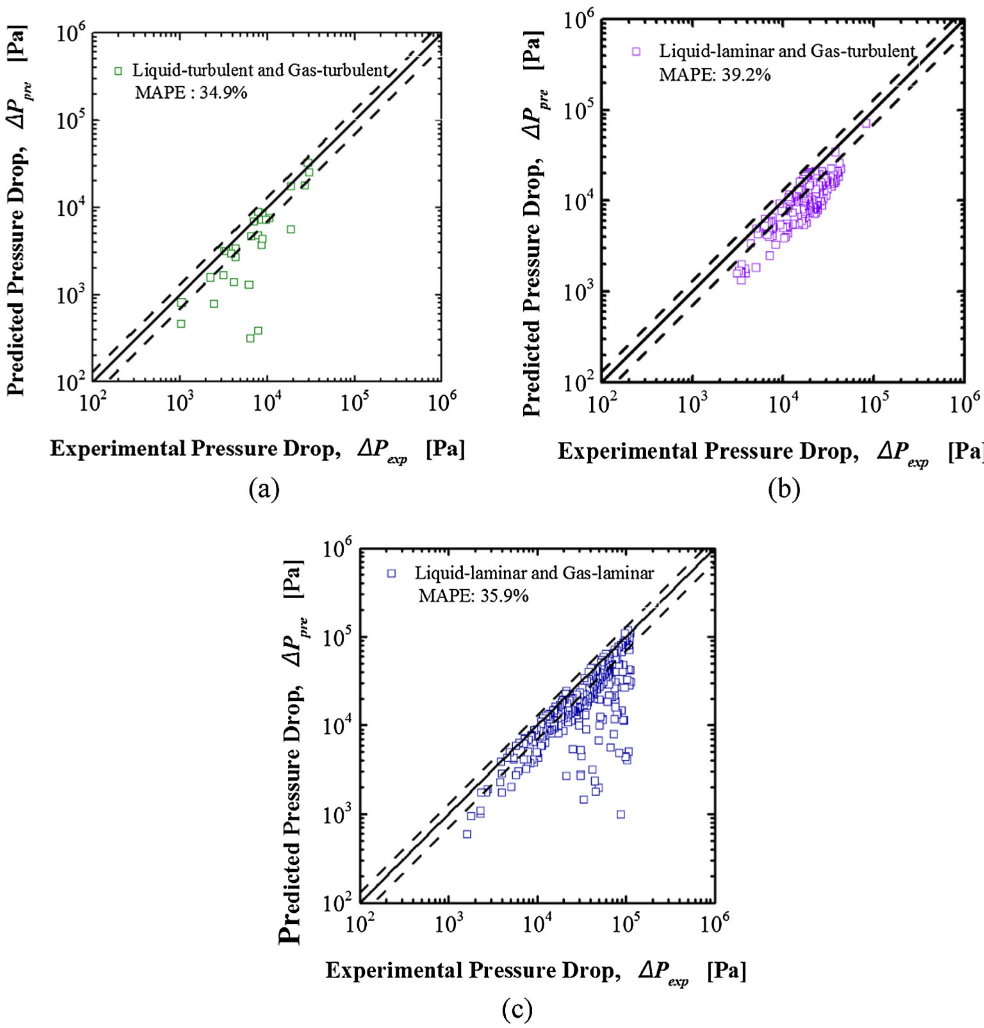

Fig. A1. Comparison between predicted multi-channel frictional pressure drop using a single-channel correlation and measured multi-channel frictional pressure drop.

The applicability of the single-channel correlation to multi-channel data is examined in Fig. A1. In the calculation using the single-channel correlation, the distribution of two-phase flow in each path of the multi-channels is assumed to be uniform. Fig. A1 indicates that the single-channel correlation with the uniform two-phase flow distribution in each channel underestimates the experimentally measured frictional pressure drop. This comparison clearly shows that non-uniform distribution of two-phase flow in each path of the multi-channel tends to increase the frictional pressure drop of the multi-channel. It is concluded that a single-channel frictional pressure drop correlation with the assumption of uniformly distributed two-phase flow in each path of a multi-channel may not be applicable for predicting the frictional pressure drop in the multi-channel unless the distributed flow conditions are known. A uniform distribution of two-phase flow in each path of a multi-channel is important in terms of frictional pressure drop and heat transfer performance of multi-channel heat exchangers.

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