PROPOSED CORRELATION OF DATA FOR ISOTHERMAL TWO-PHASE, TWO-COMPONENT FLOW IN PIPES*

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Data are presented for the simultaneous flow of air and liquids including benzene, kerosene, water and various oils in pipes varying in diameter from .0586 in. to 1.017 in. Four types of isothermal two-phase, two-component flow are shown to exist depending upon whether each phase is flowing viscously or turbulently. The pressure drop resulting from these various flow mechanisms is correlated by means of parameter X equal to the square root of the ratio of the pressure drop in the pipe if the liquid flowed alone to the pressure drop if the gas flowed alone. Tentative criteria are proposed for the transition of the flow from one type to another. The per cent of pipe filled with liquid under any flow conditions is also shown to be correlated for all four flow types by means of the parameter X.

IN two previous papers (4,5) on the subject of two-phase, two-component flow, it was demonstrated that four types of flow mechanisms can exist during the simultaneous flow of a liquid and a gas or vapor. These flow mechanisms are:

1. Flow of both the liquid and the gas may be turbulent (turbulent-turbulent flow)
2. Flow of the liquid may be viscous and flow of gas may be turbulent (viscous-turbulent flow)
3. Flow of the liquid may be turbulent and flow of the gas viscous (turbulent-viscous flow)
4. Flow of both the liquid and the gas may be viscous (viscous-viscous flow)

In the papers just referred to, the pressure drop per unit length during two-phase flow,

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP}$$

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* Appendix B of this article, 28 pages of experimental data, are on file (Document 2533) with the American Documentation Institute 1719 N Street, N.W., Washington, D. C. Can be obtained by remitting $.50 for microfilm or $2.80 for photocopies.
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‡ In this paper a new parameter ϕ_l is introduced. The two-phase pressure drop,

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = \phi_l\left(\frac{\Delta P}{\Delta L}\right)_l , \text{ where } \left(\frac{\Delta P}{\Delta L}\right)_l$$

is the pressure drop per unit length that would exist if the liquid phase is assumed to flow alone.

is expressed as

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = \phi_g^2\left(\frac{\Delta P}{\Delta L}\right)_g$$

where

$$\left(\frac{\Delta P}{\Delta L}\right)_g$$

is the pressure drop per unit length which would exist if the gaseous phase is assumed to flow alone, and ϕ_g is a parameter ‡ which is a function of a dimensionless variable, χ. The variable χ is a function of the ratio of weight rates of the liquid and gas, the ratio of densities of the liquid and gas, the ratio of viscosities of the liquid and gas, and tube diameter. In these previous publications, each type of flow mechanism was associated with a different form of χ. Table 1 summarizes these various parameters as presented in (4,5).

Further, in (5) it was suggested that the fraction of pipe filled with liquid, R_l, was also a function of the same parameter χ. Data for R_l vs. χ for viscous-viscous flow were presented in (5). Thomsen (9) and Taylor (8) in their research measured the quantity R_l, which, when plotted against the parameter χ for each type of flow, indicated reasonably good correlation.

Inspection of the curves of ϕ_g vs. χ, and R_l vs. χ, for the various flow mechanisms, revealed a similarity of

curve shape and numerical magnitude which seemed to indicate that a more generalized type of correlation should exist, which would allow the data for all four mechanisms of flow to be compared directly. It is the purpose of this paper to develop such a generalized correlation.

The main variables to be correlated are the experimental variables ϕ_g, and R_l; in addition, a number of derived variables will be discussed. All these variables will be correlated by means of a new parameter, similar to the variable χ, but of a more generalized form. The new parameter will be designated by X to distinguish it from the old parameter χ.

As was discussed in (4), the basic postulates upon which is based the analysis of the pressure drop resulting from the simultaneous flow of a liquid and a gas in a tube, are:

1. Static pressure drop for the liquid phase must equal the static pressure drop for the gaseous phase regardless of the flow pattern, as long as an appreciable radial static pressure difference does not exist
2. The volume occupied by the liquid plus the volume occupied by the gas at any instant must equal the total volume of the pipe

These postulates infer that the flow pattern does not change along the tube length, and thus "slug" flow, in which alternate slugs of liquid and gas move down the tube, is eliminated from consideration.

As will be seen below, the algebraic formulation of these two postulates involves a number of steps which may be considered somewhat oversimplified. The analysis, however, leads to dimensionless groups with definite physical significance, which are found to correlate experimental data in which all physical variables were varied over wide limits. As such, the analysis is felt to be of much more worth and significance than an analysis, utilizing that tool of dubious

TABLE 1.—SUMMARY OF TWO-PHASE FLOW PARAMETERS

$$X_{tt} = \frac{W_l}{W_g}\left(\frac{\rho_g}{\rho_l}\right)^{0.555}\left(\frac{\mu_l}{\mu_g}\right)^{0.111} = X_{tt}^{1.11}$$

$$X_{vt} = Re_{gp}^{-0.2}\frac{C_l}{C_g}\frac{W_l}{W_g}\frac{\rho_g}{\rho_l}\frac{\mu_l}{\mu_g} = X_{vt}^2$$

$$X_{tv} = Re_{lp}^{0.2}\frac{C_l}{C_g}\frac{W_l}{W_g}\frac{\rho_g}{\rho_l}\frac{\mu_l}{\mu_g} = X_{tv}^2$$

$$X_{vv} = \frac{W_l}{W_g}\frac{\rho_g}{\rho_l}\frac{\mu_l}{\mu_g} = X_{vv}^2$$

TABLE 2.—VALUES OF EXPONENTS m, n, AND CONSTANTS C_l AND C_g FOR VARIOUS FLOW TYPES

	t-t	v-t	t-v	v-v
n	0.2	1.0	0.2	1.0
m	0.2	0.2	1.0	1.0
C_l	0.046*	16.	0.046*	16.
C_g	0.046*	0.046*	16.	16.
Re_{lp}	>2000	<1000	>2000	<1000
Re_{gp}	>2000	>2000	<1000	<1000

* For smooth pipes.

value, dimensional analysis, which, in this case, would yield the trivial result that the dimensionless variables must involve the ratios of all the physical properties of the liquid and gas. The main function of the analysis presented here is, therefore, not to analyze the details of the flow phenomena but to lead to physically significant dimensionless variables for generalized correlations of experimental data.

Development of Generalized Parameters. The static pressure drop due to the liquid flow may be written by the usual Fanning equation as:

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = 2f_l\frac{\rho_l V_l^2}{D_l g} \qquad (1)$$

where D_l is known as the "hydraulic diameter" of the liquid flow pattern. Similarly, for the gas flow

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = 2f_g\frac{\rho_g V_g^2}{D_g g} \qquad (2)$$

D_l and D_g are unknown, but will be related to the tube diameter and the flow conditions in the analysis which follows.

The pressure drop in the two-phase flow is greater than that for the flow of either single phase alone for various reasons, among which are the irreversible work done by the gas on the liquid and the fact that the presence of the second fluid reduces the cross-sectional area of flow for the first fluid. Thus, during two-phase flow, the hydraulic diameters D_l and D_g are always less than the pipe diameter D_p, and, as noted in Equations (1) and (2), this reduction of hydraulic diameter will increase the pressure drop greatly.

The hydraulic diameter of the flow pattern is related to the cross-sectional area through which the fluid is flowing at any instant. For a cylindrical flow pattern, the simple relation

$$A = \frac{\pi}{4}D_h^2 \qquad (3)$$

applies. For a more complex cross-sectional area, such as that possessed by the fluids in the pipe, the following relations may be written

$$A_l = a\left(\frac{\pi}{4}D_l^2\right) \qquad (4)$$

$$A_g = \beta\left(\frac{\pi}{4}D_g^2\right) \qquad (5)$$

where a and β are, in effect, the ratio of the actual cross-section area of flow to the area of a circle of diameters D_l and D_g respectively. The parameters a and β are introduced in order to allow a definition of the "hydraulic diameters," D_l and D_g, by the simple Equations (4) and (5). The secondary parameters a and β can be determined from the experimental data, as will be described.

Equations (4) and (5) may be utilized to evaluate the respective velocities V_l and V_g; thus

$$V_l = \frac{W_l}{a\left(\frac{\pi}{4}D_l^2\right)\rho_l} \qquad (6)$$

$$V_g = \frac{W_g}{\beta\left(\frac{\pi}{4}D_g^2\right)\rho_g} \qquad (7)$$

The velocities, V_l and V_g, in Equations (1) and (2) involve the relative velocities between the fluids, and thus substitution of the magnitudes of the velocities from Equations (6) and (7) into Equations (1) and (2) is not exact. This difficulty is not so serious as it first appears, for the ratios a and β are evaluated from experimental data and thus probably include the effect of relative motion as well as the fluid geometry.

The friction factors f_l and f_g may be expressed in the generalized Blasius form:

$$f_l = \frac{C_l}{Re_l^n} = \frac{C_l}{\left(\frac{4}{\pi}\frac{W_l}{aD_l\mu_l}\right)^n} \qquad (8)$$

$$f_g = \frac{C_g}{Re_g^m} = \frac{C_g}{\left(\frac{4}{\pi}\frac{W_g}{\beta D_g\mu_g}\right)^m} \qquad (9)$$

When Equations (6)-(9) are substituted into Equations (1) and (2), and Equations (1) and (2) equated, the result is that

$$\frac{D_l^{5-n}}{D_g^{5-m}} = \frac{C_l W_l^{2-n}\mu_l^n\rho_g\beta^{2-m}}{C_g W_g^{2-m}\mu_g^m\rho_l a^{2-n}}\left(\frac{\pi}{4}\right)^{n-m} \qquad (10)$$
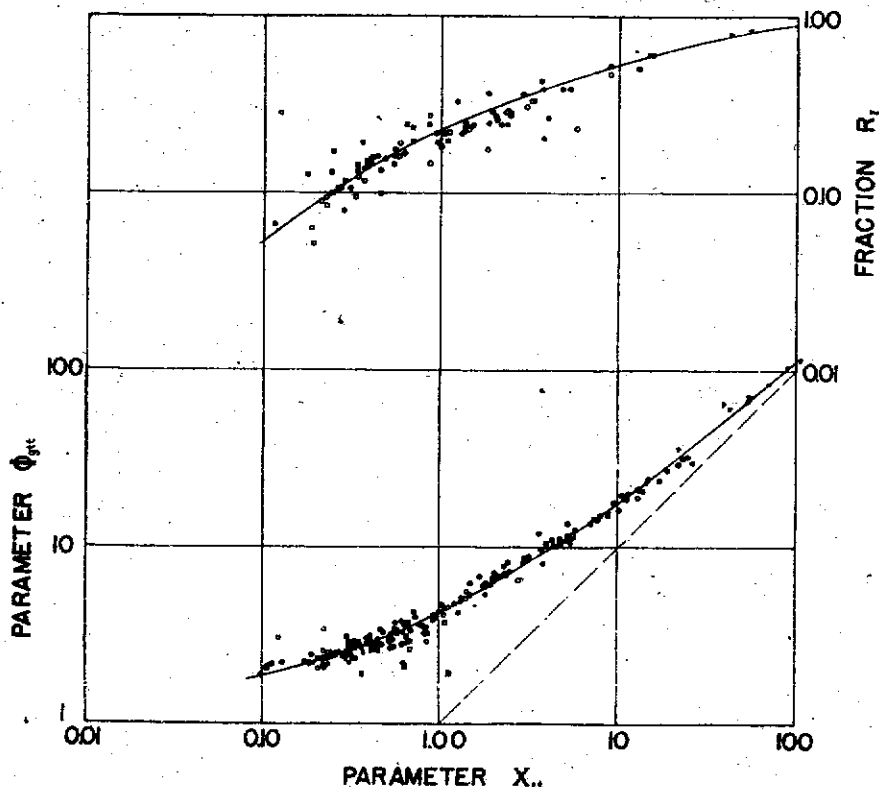

Fig. 1. Relation between R_l, ϕ_g, and parameter X for mechanism 1, turbulent-turbulent flow. Experimental groups 3, 5, 8, 10, 13-17 are included. A few points from experimental groups 4, 7, and 9 for flow mechanism 3, turbulent-viscous flow are also plotted

Returning to the generalized form of Equation (1)

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = \frac{2C_l W_l^2}{\left(\frac{4W_l}{\pi a D_l \mu_l}\right)^n D_l^5 a^2 \left(\frac{\pi}{4}\right)^2 g\rho_l}$$

$$= \left[\frac{2\left(\frac{4}{\pi}\right)^{2-n} C_l \mu_l^n W_l^{2-n}}{D_p^{5-n}\rho_l g}\right] a^{n-2}\left(\frac{D_p}{D_l}\right)^{5-n}$$

Since the bracketed term is the pressure drop per unit length for only the liquid flowing in the pipe, then,

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = \left(\frac{\Delta P}{\Delta L}\right)_l a^{n-2}\left(\frac{D_p}{D_l}\right)^{5-n} \quad (11)$$

For simplicity, let

$$\sqrt{\frac{\left(\frac{\Delta P}{\Delta L}\right)_{TP}}{\left(\frac{\Delta P}{\Delta L}\right)_l}} = \phi_l$$

$$= a^{\frac{n-2}{2}}\left(\frac{D_p}{D_l}\right)^{\frac{5-n}{2}} \quad (12)$$

Similarly, if Equation (2) is generalized, the result is that

$$\left(\frac{\Delta P}{\Delta L}\right)_{TP} = \left(\frac{\Delta P}{\Delta L}\right)_g \beta^{m-2}\left(\frac{D_p}{D_g}\right)^{5-m} \quad (13)$$

For simplicity, let

$$\sqrt{\frac{\left(\frac{\Delta P}{\Delta L}\right)_{TP}}{\left(\frac{\Delta P}{\Delta L}\right)_g}} = \phi_g$$

$$= \beta^{\frac{m-2}{2}}\left(\frac{D_p}{D_g}\right)^{\frac{5-m}{2}} \quad (14)$$

Application of the second postulate, equating the liquid and gas volumes to pipe volume, yields

$$a D_l^2 + \beta D_g^2 = D_p^2 \quad (15)$$

The fraction of tube filled with gas, R_g, is

$$R_g = \frac{\beta D_g^2}{D_p^2} = 1 - a\left(\frac{D_l}{D_p}\right)^2$$

or

$$a = (1 - R_g)\left(\frac{D_p}{D_l}\right)^2 = R_l\left(\frac{D_p}{D_l}\right)^2 \quad (16)$$

Likewise, the fraction of the tube filled with liquid, R_l, is then

$$R_l = \frac{a D_l^2}{D_p^2} = 1 - \beta\left(\frac{D_g}{D_p}\right)^2$$

or rearranging

$$\beta = (1 - R_l)\left(\frac{D_p}{D_g}\right)^2 = R_g\left(\frac{D_p}{D_g}\right)^2 \quad (17)$$

In all the equations thus far, four variables have appeared, namely

$$\frac{D_l}{D_p}, \frac{D_g}{D_p}, a, \text{ and } \beta \quad (18)$$

which can be expressed in terms of the four experimentally determined variables,

$$\phi_l, \phi_g, R_l \text{ and } R_g \quad (19)$$

by the relations in Equations (12), (14), (16), and (17), as shown in Appendix A on page 43.

It will be observed that in Equation (10), in addition to the variables in (18), there is one more variable, equal to

$$\left(\frac{4}{\pi D_p}\right)^{m-n} \frac{C_l}{C_g} \frac{W_l^{2-n}}{W_g^{2-m}} \frac{\rho_g}{\rho_l} \frac{\mu_l^n}{\mu_g^m} \quad (20)$$

which can also be written as

$$\frac{Re_{gp}^m}{Re_{lp}^n} \frac{C_l}{C_g}\left(\frac{W_l}{W_g}\right)^2 \frac{\rho_g}{\rho_l} \quad (21)$$

This variable, which shall be termed X^2, can be readily shown to equal the ratio of the liquid pressure drop to the gas pressure drop, assuming each phase to flow separately, i.e.,

$$X^2 = \left(\frac{\Delta P}{\Delta L}\right)_l \Big/ \left(\frac{\Delta P}{\Delta L}\right)_g$$

It is now postulated, subject to experimental verification, that all the parameters, a, β, $\frac{D_p}{D_l}$, $\frac{D_p}{D_g}$, or their equivalents, ϕ_l, ϕ_g, R_l and R_g are functions of the parameter X. Experimental verification would consist of a correlation of the experimentally determined variables ϕ_l, ϕ_g, R_l and R_g vs. X for a large range of all the variables which make up X. This correlation has already been partially verified (4, 5) for several of the various flow types, considering each flow type separately.

In this paper, it is proposed to correlate the various flow types simultaneously by plotting ϕ_l, ϕ_g, R_l and R_g vs. X, and evaluating the correct form of X for each flow type by substitution of the appropriate exponents n and m and the proper constants C_l and C_g. Combinations of n, m, C_l and C_g corresponding to each flow type, are given in Table 2.
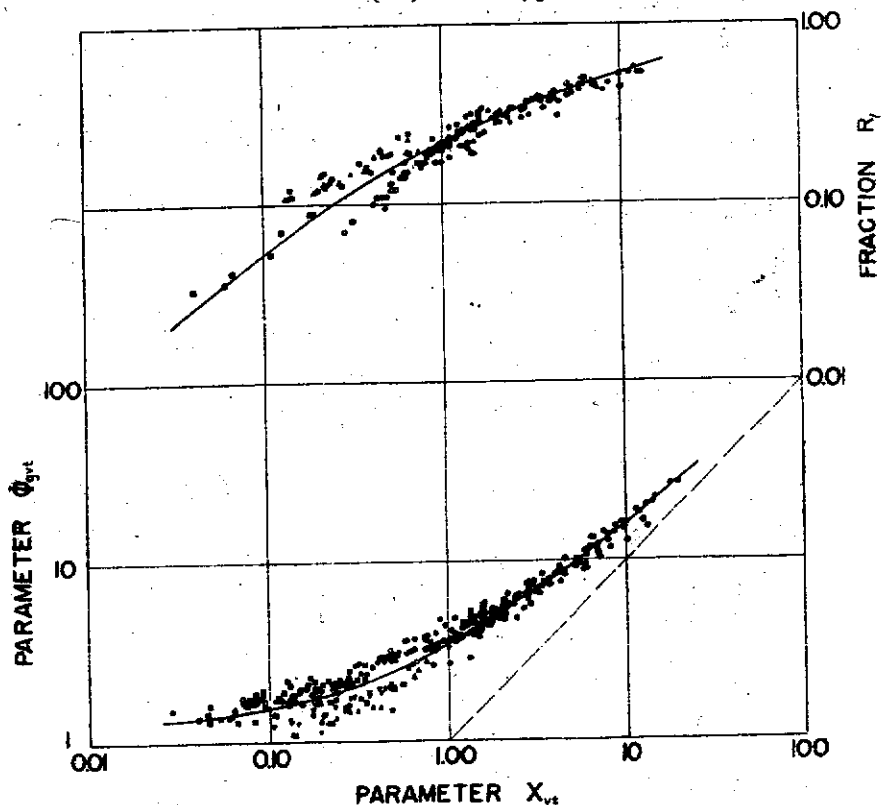

Fig. 2. Relation between R_l, ϕ_g, and parameter X for mechanism 2, viscous-turbulent flow. Experimental groups 1, 2, 3, 5, 6, 11-21 are included

TABLE 3.—RANGE OF VARIABLES [*]

Investigator	Liquid	Gas	Pipe I.D. (in.)	Dist. Between Pressure Taps ft.	Exper. Group	Symbol	Liquid Viscosity lb./(sec.)(ft.) 50° F.	70° F.	90° F.	Liquid Sp.Gr. 60/60° F.	Av. Pressure lb./sq. in.abs.	Av. Temperature °F.	Liquid Re_{lp} max.	min.	Gas Re_{gp} max.	min.
T. H. M. Taylor	Oil Blend 1	Air	1.017		1		.088600	.043000	.023200	.901	18	83	570	25	82,000	6,700
	Oil Blend 2	Air			2		.387000	.155000	.071000	.913	18	85	208	8.5	72,500	2,780
	Diesel Fuel Oil	Air	1.017		3		.005050	.003720	.002820	.867	18	80	12,200	179	86,000	1,050
	Kerosene	Air	glass	21.6	4,5		.002350	.002050	.001240	.830	18	78	39,000	316	83,500	715
	S.A.E. 40 Oil	Air			6		.999000	.363000	.147000	.912	18	85	69	4.0	65,500	1,100
	Water	Air			7,8		.000880	.000661	.000517	1.000	18	60	122,000	382	76,000	880
	Water (with Nekal)	Air			9,10		.000880	.000661	.000517	1.000	18	80	124,000	1310	83,200	750
R. W. Ravenscroft	Oil A	Air	0.622		11		.063000	.028600	.016900	.818	40	70	810	4.4	58,200	2,960
	Oil C	Air	galv.	50	12		.005300	.003380	.002280	.862	40	70	635	184	73,500	32,400
	Water	Air	iron		13		.000880	.000661	.000517	1.000	50	70	3,120	268	72,500	22,500
E. G. Thomsen	Benzene	Air	0.622		14		.000922	.000418	.000163	.881	52	72	4,500	830	71,000	9,350
	Kerosene	Air	galv.		15		.002350	.002050	.001240	.834	52	72	1,740	390	54,000	10,000
	Water	Air	iron	50	16		.000880	.000661	.000517	1.000	52	72	3,100	527	72,000	6,850
	Water (with Kemenol)	Air			17		.000880	.000661	.000517	1.000	52	70	3,260	845	62,700	12,900
R. W. Lockhart	Diesel Fuel Oil	Air	0.0816	2.19	18		.005500	.004250	.003220	.906	16	70	68.5	1.5	5,420	288
	Kerosene	Air	glass		19		.001590	.001350	.001160	.815	16	70	659.0	4.2	2,400	8
	Diesel Fuel Oil	Air	0.0586	2.34	20		.005500	.004250	.003220	.906	16	70	44.0	1.1	2,940	22
	Kerosene	Air	glass		21		.001590	.001350	.001160	.815	16	70	94.0	3.3	3,500	7

[*] For complete experimental data see (1).

Transition Criteria. The proper choice of n, m, C_l and C_g cannot be made until the type of flow mechanism is known. The question then arises as to the criterion to be used to establish the flow mechanism. At the present time, no precise criterion is known and, for the purposes of this paper, a tentative criterion is established on the basis of the following argument.

Assume that in single phase flow, transition occurs between laminar and turbulent flow at $Re = 2000$. One may then state that if gas is flowing as a single phase with $Re_{gp} = 2000$, the introduction of liquid increases the effective Reynolds modulus for the gas phase and insures turbulence in the gas phase, since the hydraulic diameter for the gas is decreased. One can state, therefore, that for $Re_{gp} > 2000$, the gas phase is turbulent. The same argument applies to the liquid phase. It may well be that even for $Re_{gp} < 2000$, the gas phase becomes turbulent as more liquid is added, but the exact point of transition is not known. The criterion of $Re = 2000$ for turbulence is therefore a conservative one.

At the other limit, if the pipe is filled with liquid, flowing at, say $Re_{lp} = 10$, the introduction of gas will increase the Reynolds modulus for the liquid, but Re_{lp} will never exceed an effective value of 2000 due to the small initial magnitude of Re_{gp}. The highest value of Re_{gp} which will keep the liquid flow laminar even in the presence of turbulent gas will be the criterion of viscous two-phase flow. This value is, of course, not known, but inspection of the data available indicates that a tentative value of 1000 is not unreasonable. Then if Re_{lp} is equal to or less than 1000, the flow of the liquid is postulated to be viscous. The same argument applies to the gas. A summary of these remarks is shown in Table 2.

Values of Re_{lp} or Re_{gp} between 1000 and 2000 can be considered in the "transition" region. Obviously, experimental investigation will change and refine these rough preliminary criteria.

Data (2, 3, 7, 8, 9) have been classified according to the criteria in Table 2, and are summarized in (1).

Discussion of Correlation. Having divided the available data into the four flow mechanisms, plots of the two main variables, ϕ_g and R_l, vs. X were made for each flow type.[*] These are shown in Figures 1-3. The variables ϕ_l and R_g are directly related to ϕ_g and R_l, as shown in Appendix A, and are therefore not plotted in these figures.

Table 3 summarizes the range of variables covered in these plots. The only quantity not varied is the viscosity of the gas, since air was used in all tests. Since the range of other variables is so great, however, one may conclude that the postulate of the dependence of ϕ_l, ϕ_g, R_l and R_g on

[*] All runs for which the pipe was not horizontal were corrected for static head of fluid in the pipe as suggested (5).

the parameter X has been established.[†]

For comparison, the average curves for all flow types are shown in Figure 4. It is interesting to note that, as one would expect, the curves for the various flow types converge at both high and low values of X. At high values of X, ϕ_l approaches unity, while at low values of X, ϕ_g approaches unity. The ϕ curves for viscous-viscous flow are lowest, the curve for turbulent-turbulent flow highest, and the viscous-turbulent flow curve is intermediate. This may be well explained by the fact that the interface between liquid and gas is smoothest for the viscous-viscous flow and roughest for the turbulent-turbulent flow. This "roughness" effect casts some doubt on the validity of the 0.2 exponent in the Blasius equation for f_g used in the analysis. It may be that a re-evaluation of this exponent may be necessary in a more refined analysis.

An interesting and surprising fact is the almost perfect symmetry of the ϕ_l and ϕ_g curves for tt and vv flow. The curve for turbulent-viscous flow was predicted from that for viscous-turbulent flow, assuming symmetry; only eleven points are available to substantiate this curve. It will be interesting to see whether further experimental data will justify this extrapolation.

[† Possibility of at least another parameter entering the correlation is indicated, however, by consideration of the limiting condition of two-phase flow, namely flow at the critical point. This question is not considered in this paper, but has been discussed (6).]

For all practical purposes, a single correlation has been obtained for R_l vs. X. This may well be fortuitous. The values of R_l were obtained by Thomsen and Taylor by trapping a representative sample of the flow mixture between quick closing valves and then blowing down the pipe and measuring the liquid content. Thus, the experimental values may be low, due to liquid remaining on the tube walls. Lockhart, however, measured R_l by trapping the liquid between quick closing valves, washing out the tube with a large quantity of volatile solvent and evaporating the excess solvent. His values should, therefore, probably be high. The fact that the correlation between the two methods is so close may mean that the errors are not so high as one might expect. Experimental work using a more refined technique is necessary, however.

For convenience, a numerical tabulation of the coordinates of the curves in Figure 4 is given in Table 4. Although reasonably good correlation has been obtained insofar as ϕ_l, ϕ_g, R_l and R_g are concerned, when the magnitudes of α and β are calculated using the algebraic relations tabulated in Appendix A, the surprising fact is discovered that α is less than unity for most of the experimental data. From a simple geometrical interpretation of α, values above unity should be expected. At least two explanations for this discrepancy come to mind. The first is that, due to the macroscopic nature of the analysis, the correct dimensionless groups may have been established but all the factors which go into explaining the significance of α have not been considered. The second is that depending on the type of flow, α depends on either the square or fourth power of R_l so that inaccuracies in R_l are greatly magnified in α. Further work should yield an explanation either through a better analysis or the accumulation of more exact data.

Conclusions

This paper presents a summary of the two-phase, two-component research conducted at the University of California to date. The following conclusions may be drawn:

1. The presence of four flow mechanisms, namely turbulent-turbulent, viscous-turbulent, turbulent-viscous and viscous-viscous flow has been established
2. Criteria for the transition between these flow mechanisms are not known; tentative criteria have been suggested in this paper
3. The four flow types have been correlated by means of the parameter X, which is equal to the square root of the ratio of the pressure drop of the liquid to the pressure drop of the gas, assuming each phase flowing separately
4. Curves have been presented for the prediction of pressure drop and for the prediction of the fraction of pipe filled by liquid or gas, during two-phase two-component flow in horizontal tubes
5. More data are needed to establish definitely the validity of the correlations suggested in this paper, particularly at very high and very low values of the parameter X, and for two-phase mixtures near the critical point

It is suggested that the correlation of two-phase data presented in this paper can also be utilized for:

APPENDIX A.—SUMMARY OF ALGEBRAIC RELATIONS BETWEEN THE VARIOUS PARAMETERS

NOTE: For any flow mechanism $R_l + R_g = 1$ and $\phi_g = X\phi_l$.

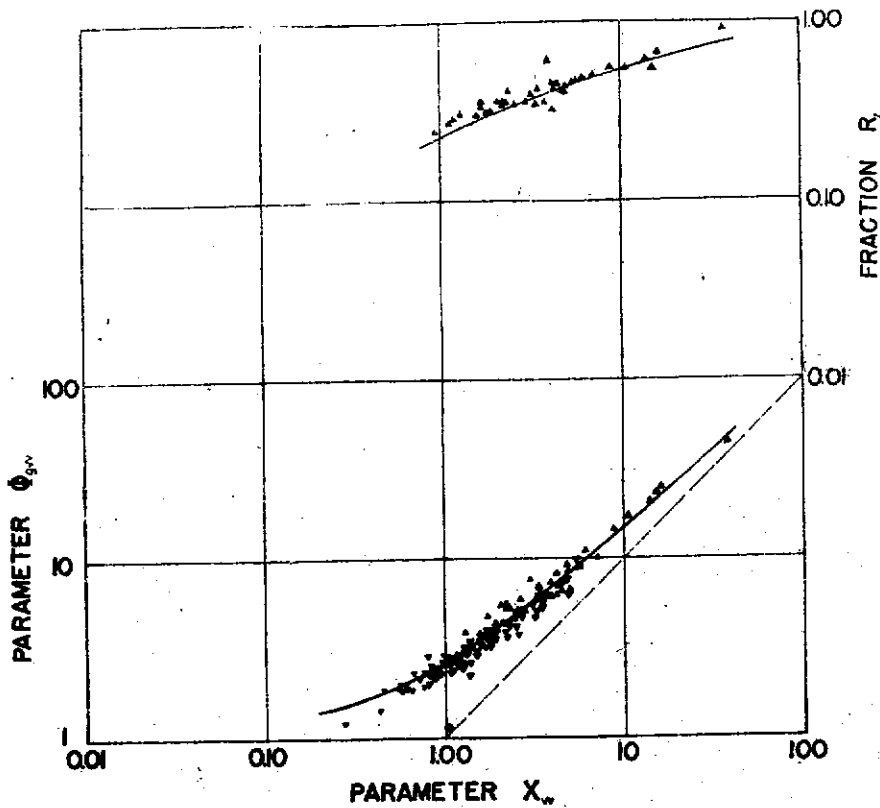| Parameter Flow	tt	vt	tv	vv
$X^2 = \dfrac{\left(\frac{\Delta P}{\Delta L}\right)_l}{\left(\frac{\Delta P}{\Delta L}\right)_g}$	$\left(\dfrac{W_l}{W_g}\right)^{1.8}\dfrac{\rho_g}{\rho_l}\left(\dfrac{\mu_l}{\mu_g}\right)^{0.2}$	$\dfrac{C_l}{C_g}Re_{g}{}^{-0.8}\dfrac{W_l}{W_g}\dfrac{\rho_g}{\rho_l}\dfrac{\mu_l}{\mu_g}$	$\dfrac{C_l}{C_g}Re_{l}{}^{0.8}\dfrac{W_l}{W_g}\dfrac{\rho_g}{\rho_l}\dfrac{\mu_l}{\mu_g}$	$\dfrac{W_l}{W_g}\dfrac{\rho_g}{\rho_l}\dfrac{\mu_l}{\mu_g}$
$\phi_l{}^2 = \dfrac{\left(\frac{\Delta P}{\Delta L}\right)_{TP}}{\left(\frac{\Delta P}{\Delta L}\right)_l}$	$\dfrac{\phi_{gtt}{}^2}{X_{tt}{}^2}$	$\dfrac{\phi_{gvt}{}^2}{X_{vt}{}^2}$	$\dfrac{\phi_{gtv}{}^2}{X_{tv}{}^2}$	$\dfrac{\phi_{gvv}{}^2}{X_{vv}{}^2}$
$\phi_g{}^2 = \dfrac{\left(\frac{\Delta P}{\Delta L}\right)_{TP}}{\left(\frac{\Delta P}{\Delta L}\right)_g}$	$\phi_{ltt}{}^2X_{tt}{}^2$	$\phi_{lvt}{}^2X_{vt}{}^2$	$\phi_{ltv}{}^2X_{tv}{}^2$	$\phi_{lvv}{}^2X_{vv}{}^2$
$\alpha = R_l\left(\dfrac{D_g}{D_l}\right)^2$	$R_l{}^4\phi_{ltt}{}^{3.56}$	$R_l{}^4\phi_{lvt}{}^2$	$R_l{}^4\phi_{ltv}{}^{3.56}$	$R_l{}^2\phi_{lvv}{}^2$
$\beta = R_g\left(\dfrac{D_g}{D_l}\right)^2$	$R_g{}^4\phi_{gtt}{}^{3.56}$	$R_g{}^4\phi_{gvt}{}^{3.56}$	$R_g{}^4\phi_{gtv}{}^2$	$R_g{}^2\phi_{gvv}{}^2$
$\left(\dfrac{D_g}{D_l}\right)^2 = \dfrac{\alpha}{R_l}$	$R_l{}^3\phi_{ltt}{}^{3.56}$	$R_l\phi_{lvt}{}^2$	$R_l{}^3\phi_{ltv}{}^{3.56}$	$R_l\phi_{lvv}{}^2$
$\left(\dfrac{D_g}{D_l}\right)^2 = \dfrac{\beta}{R_g}$	$R_g{}^3\phi_{gtt}{}^{3.56}$	$R_g{}^3\phi_{gvt}{}^{3.56}$	$R_g\phi_{gtv}{}^2$	$R_g\phi_{gvv}{}^2$

Fig. 3. Relation between R_l, ϕ_l and parameter X for mechanism 4, viscous-viscous flow. Experimental groups 18-21 are included

1. Prediction of pressure drop during evaporation or condensation by a step-wise solution
2. Prediction of the behavior of gas-lift pumps
3. Prediction of the heat-transfer conductance during forced convection boiling or condensation

The exact methods for the application of the isothermal data to these problems have not yet been established, but work is proceeding along these lines. Item (1) was discussed in (4, 6) and Item (2) in (3).

Acknowledgment

The authors wish to thank Dr. E. G. Thomsen, T. H. M. Taylor and R. W. Ravenscroft for permission to publish their data, upon which much

of the correlation was based. The two-phase flow research reported in this paper was initiated by Prof. L. M. K. Boelter, and at the present is being continued at the University of California, Berkeley, Calif.

Notation

A = cross-sectional area of pipe diameter D_p, sq.ft.

A_g = cross-sectional area of gas flow, sq.ft.

A_l = cross-sectional area of liquid flow, sq.ft.

D_g = hydraulic diameter of gas flow, ft.

D_h = hydraulic diameter of pipe, ft.

D_l = hydraulic diameter of liquid flow, ft.

D_p = inside diameter of pipe, ft.

g = gravitational force per unit mass = 32.2 (lb.)(ft.)/(lb.)(sec.)^2

L = length of pipe, ft.

m = exponent of Reynolds modulus in the Blasius expression for the friction factor for the gas phase

n = exponent of Reynolds modulus in the Blasius expression for the friction factor for the liquid phase

V_g = velocity of gas, ft./sec.

V_l = velocity of liquid, ft./sec.

W_g = weight rate of gas flow, lb./sec.

W_l = weight rate of liquid flow, lb./sec.

ρ_g = weight density of gas, lb./cu.ft.

ρ_l = weight density of liquid, lb./cu.ft.

μ_g = absolute viscosity of gas, lb./(sec.)(ft.)

μ_l = absolute viscosity of liquid, lb./(sec.)(ft.)

ΔL = finite difference of length, ft.

ΔP = finite difference of static pressure in length ΔL, lb./sq.ft.

$\left(\dfrac{\Delta P}{\Delta L}\right)_g$ = pressure drop per unit length due to gas flowing at a rate W_g with a density ρ_g, lb./(sq.ft.)(ft.)

$\left(\dfrac{\Delta P}{\Delta L}\right)_l$ = pressure drop per unit length due to liquid flowing at a rate W_l with a density ρ_l, lb./(sq.ft.)(ft.)

$\left(\dfrac{\Delta P}{\Delta L}\right)_{TP}$ = pressure drop per unit length for two-phase flow, lb./(sq.ft.)(ft.)

TABLE 4.—COORDINATES OF Φ AND R VS. PARAMETER X

X	All Mechanisms		Turbulent-Turbulent		Viscous-Turbulent		Turbulent-Viscous		Viscous-Viscous	
	R_l	R_g	ϕ_l	ϕ_g	ϕ_l	ϕ_g	ϕ_l	ϕ_g	ϕ_l	ϕ_g
0.01			128	1.28	120.	1.20	112.	1.12	105.	1.05
.02			68.4	1.37	64.0	1.28	58.0	1.16	53.5	1.07
.04			38.5	1.54	34.0	1.36	31.0	1.24	28.0	1.12
.07	0.04	0.96	24.4	1.71	20.7	1.45	19.3	1.35	17.0	1.19
.10	.05	.95	18.5	1.85	15.2	1.52	14.5	1.45	12.4	1.24
.2	.09	.91	11.2	2.23	8.90	1.78	8.70	1.74	7.00	1.40
.4	.14	.86	7.05	2.83	5.62	2.25	5.50	2.20	4.25	1.70
.7	.19	.81	5.04	3.53	4.07	2.85	4.07	2.85	3.08	2.16
1.0	.23	.77	4.20	4.20	3.48	3.48	3.48	3.48	2.61	2.61
2.0	.31	.69	3.10	6.20	2.62	5.25	2.62	5.24	2.06	4.12
4.0	.40	.60	2.38	9.50	2.05	8.20	2.15	8.60	1.76	7.00
7.0	.48	.52	1.96	13.7	1.73	12.1	1.83	12.8	1.60	11.2
10.	.53	.47	1.75	17.5	1.59	15.9	1.66	16.6	1.50	15.0
20.	.66	.34	1.48	29.5	1.40	28.0	1.44	28.8	1.36	27.3
40.	.76	.24	1.29	51.5	1.25	50.0	1.25	50.0	1.25	50.0
70.	.84	.16	1.17	82.0	1.17	82.0	1.17	82.0	1.17	82.0
100.	.90	.10	1.11	111.	1.11	111.	1.11	111.	1.11	111.

Dimensionless Moduli

a = flow-type modulus for liquid

β = flow-type modulus for gas

ϕ = function of X utilized in calculating two-phase pressure drop

χ = two-phase flow modulus defined in Table 1

X = two-phase flow modulus defined in Table 1

C_g = constant in Blasius equation for friction factor for the gas

C_l = constant in Blasius equation for friction factor for the liquid

f_g = friction factor for gas (see Eq. (2))

f_l = friction factor for liquid. (See Eq. (1))

R_g = fraction of tube filled with gas

R_l = fraction of tube filled with liquid

Re = Reynolds modulus, $\dfrac{VD\rho}{\mu}$

$= \dfrac{4W}{\pi D\mu}$

Re_g = Reynolds modulus of gas based on the hydraulic diameter of the gas $\dfrac{4W_g}{\pi D_g \beta \mu_g}$

Re_{gp} = Reynolds modulus for the gas based on the inside pipe diameter $\dfrac{4W_g}{\pi D_p \mu_g}$

Re_l = Reynolds modulus for liquid based on the hydraulic diameter of the liquid $\dfrac{4W_l}{\pi a D_l \mu_l}$

Re_{lp} = Reynolds modulus for the liquid based on the inside pipe diameter $\dfrac{4W_l}{\pi D_p \mu_l}$

Subscripts

c = cross-sectional

g = gas

l = liquid

p = pipe

TP = two phase

tt = turbulent liquid, turbulent gas flow

vt = viscous liquid, turbulent gas flow

tv = turbulent liquid, viscous gas flow

vv = viscous liquid, viscous gas flow

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Discussion

Carl Gazley, Jr., O. P. Bergelin (University of Delaware, Newark, Del.): The complex conditions of multiphase flow and the number of variables involved make it logical that the first attempt at correlation be based on analogy to single-phase flow. By using this approach a usable method for correlation has been obtained by Lockhart and Martinelli with relatively few data. This paper is a definite contribution in that it gives a generalized procedure for correlating the data which were presented previously at different times and in different journals (1,5,6). This method will, however, need verification and, if necessary, alteration at a later date when additional data are available and a more fundamental approach can be made.

In considering the accuracy of the proposed correlation, the curve of Lockhart and Martinelli is compared in Figure 1 with data taken at the University of Delaware during the simultaneous flow of air and water through a 1-in. copper tube. The data are seen to fall within +20% and −30% of the curve in a manner similar to the results of Lockhart and Martinelli. It might be noted at this point that plotting ϕ_g vs. X tends to
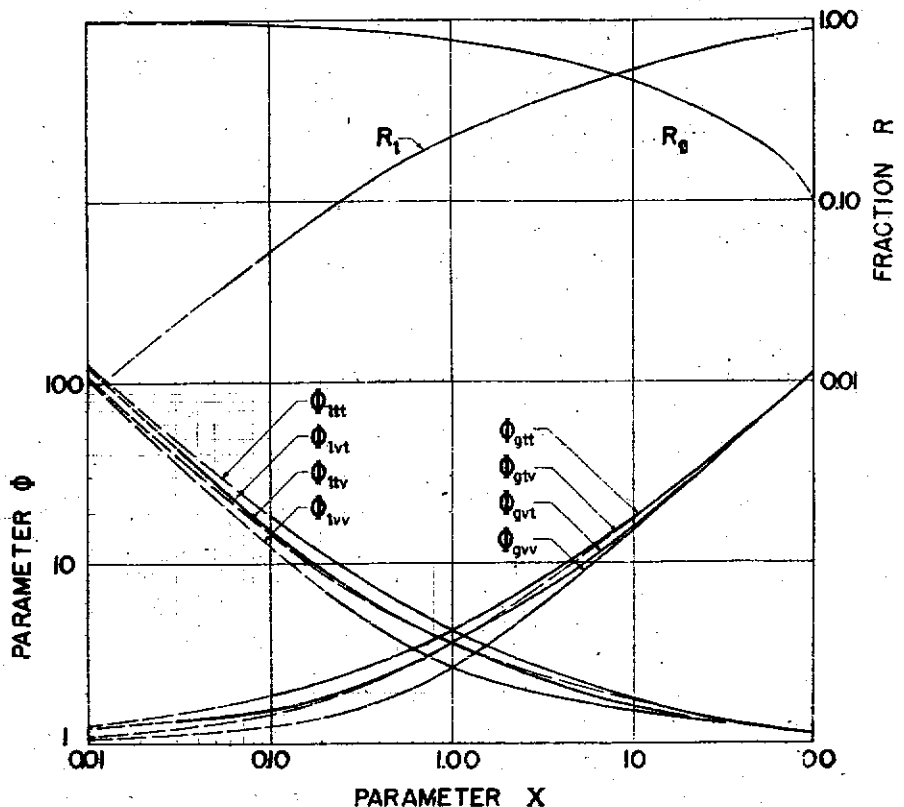

Fig. 4. Faired curves showing relation between ϕ_a, ϕ_g, R_l and R_g for all flow mechanisms.

reduce the spread of the data since ϕ_g is

$$\sqrt{\overline{\left(\frac{\Delta P}{\Delta L}\right)_{TP}} \Big/ \overline{\left(\frac{\Delta P}{\Delta L}\right)_{GP}}}$$

It would appear that a plot of ϕ^2 vs. X or ϕ^2 vs. X^2 would be a more suitable test of the correlation of the data. On such a basis the spread of data would be +44% and −50% from the proposed curve, and these are the important values since $\phi_g{}^2$ is directly proportional to the two-phase pressure drop.

When data from Figure 1 are plotted as separate lines for each liquid rate as shown in Figure 2 it appears that ϕ_g is not a function of X alone, for while the over-all correlation with Lockhart and Martinelli's line is still +20% and −30% the data now fall within ±5% of the best line through the points at a given liquid rate. Also, on examining the lines closely a definite change in slope can be noted in each line. By means of a different method of plotting which emphasizes any change in slope it was found that each visually observed transition between types of flow was accompanied by a change in slope. For example, a change in slope accompanies the transition from stratified to wave flow, from wave to slugging flow, and from slugging to annular flow. Data presented here are within the region defined by Lockhart and Martinelli as turbulent-turbulent, with the exception of the lowest water rate shown in Figure 2 which is in the range $1000 < NRe_{tp} < 2000$. It would appear then, that even within the turbulent-turbulent region (as defined by Lockhart and Martinelli) sharp changes in the rate of change of pressure drop occur which should be accounted for in a correlation. It may, therefore, be advisable to devise a correlation for each of the visually observed types of flow rather than for arbitrary viscous or turbulent regions.

As has been pointed out by Martinelli (4), the analysis of Lockhart and Martinelli is also limited in that it is not valid for stratified flow. For this case a hydraulic gradient exists and the pressure drops in the two phases are no longer equal. Recent tests in our laboratory (2, 3) show that different pressure drops are obtained in the two phases, thus verifying this contention.

It is felt that before any completely successful correlation of two-phase flow data can be obtained, the transition points between the various types of flow need to be determined. This, of necessity, will be an empirical procedure, and data with as wide a range of flow rates and physical properties as possible will be needed. For this reason it is strongly urged that the authors include descriptions of the appearance of the flow and notations of observed changes in flow pattern along with the complete original experimental data.

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R. W. Lockhart: I wish to thank the University of Delaware for its additional data on turbulent-turbulent flow.

As for plotting ϕ^2 vs. X or X^2, my particular reason for not going into this method was the necessity of using many cycles of log-log paper.
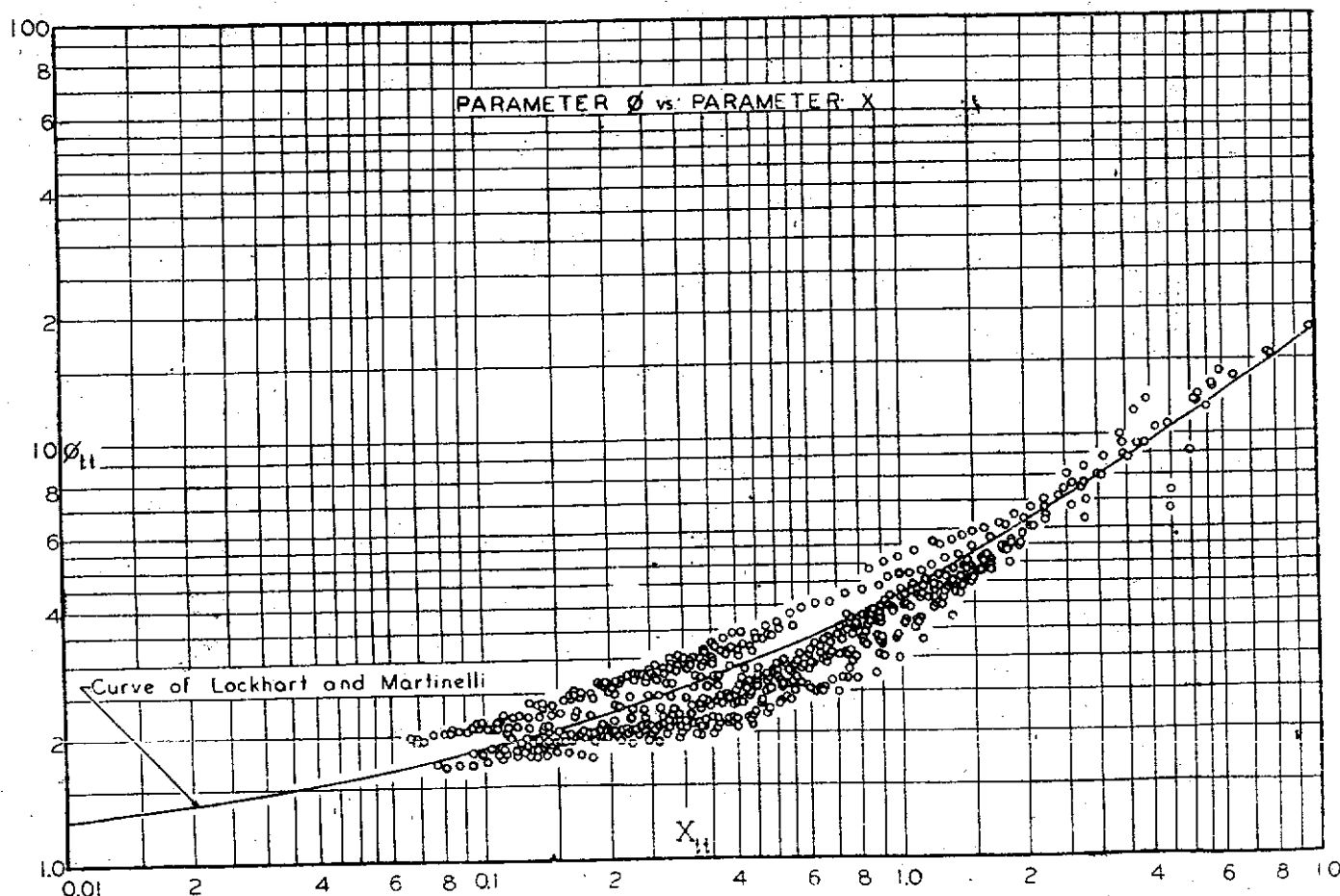

Fig. 1. (Gazley and Bergelin Discussion) ϕ_{tt} vs. X_{tt}. Data of Jenkins (3) Compared with Curve of Lockhart and Martinelli

It was very interesting to note that the change in slope of the constant liquid rate data appears in the region around X equal to 1. As I pointed out earlier, this is the region where we have observed the largest spread of data.

I wonder at this time why such a spread of data was obtained for one fluid. Data represented in our curves were for many fluids, and the variation occurred more for various fluids, than for any one fluid.

I hear that a different pressure drop was obtained for each fluid of a stratified flow. One of the basic postulates of our paper states that this type of flow, therefore, would not be applicable to our correlation.

Leo Friend (The M. W. Kellogg Co., New York): I would like some opinion on a phenomenon which may be taking place here, and which might account for some of the differences and also might make the extrapolation of the work done in your higher velocity region more valid.

As we all know, in this type of flow the relative pressure drops to that in single phase flow are large. As a result the exit volume of gas is large compared with the entering volume of gas, and considerable acceleration both of the gas and the liquid phase has

taken place. This produces the effect we call kinetic energy effect on the pressure drop.

I don't see how this was taken care of in your derivation, although in the final correlation it undoubtedly is taken care of, due to the fact that the friction factors are plotted as experimentally determined. In extrapolating the data to velocities other than those which were used for the basis of correlation it seems an error would be introduced, especially at high velocities, in the turbulent-turbulent flow pattern.

R. W. Lockhart: The experimental isothermal data that we have is for a limited range in velocities, naturally. Maximum velocities were about 100 ft./sec. Have you had any data that we may try to analyze.

Leo Friend: No, we have no experimental data, but I have been using your correlations, and I have applied them to some data on heat-transfer work published by Kelly Woods, of M.I.T., on the vaporization of water and benzene and benzene-oil mixtures. Using the correlations presented, we found that there was some spread in the data. However for the first time we were able to predict or calculate pressure drops within a reasonable amount of error.

However, in the development of these data and further study, I think that the kinetic energy effect should somehow be taken into account.

This is particularly true as the velocities go up, because the kinetic energy used in accelerating the liquid phase, which is rather large in mass as compared with the gas phase, would be considerable. It is important to industrial designers in vacuum furnaces where velocities go up to 400 or 500 ft./sec. and the liquid quantity is appreciable. We would like to have a more fundamental basis for design.

C. E. Lapple (E. I. du Pont de Nemours & Co., Wilmington, Del.): On the question that has been raised regarding kinetic energy at high velocities, I might add a few comments relative to the data of W. K. Woods.

At high pressure drops the velocity may be a controlling factor. In that case kinetic energy actually represents a limiting condition which, for a homogeneous fluid, corresponds to the attainment of some velocity. In general the limiting conditions would correspond to a velocity equal to the rate of propagation of a pressure wave through the fluid or mixture of fluids, which can be calculated from
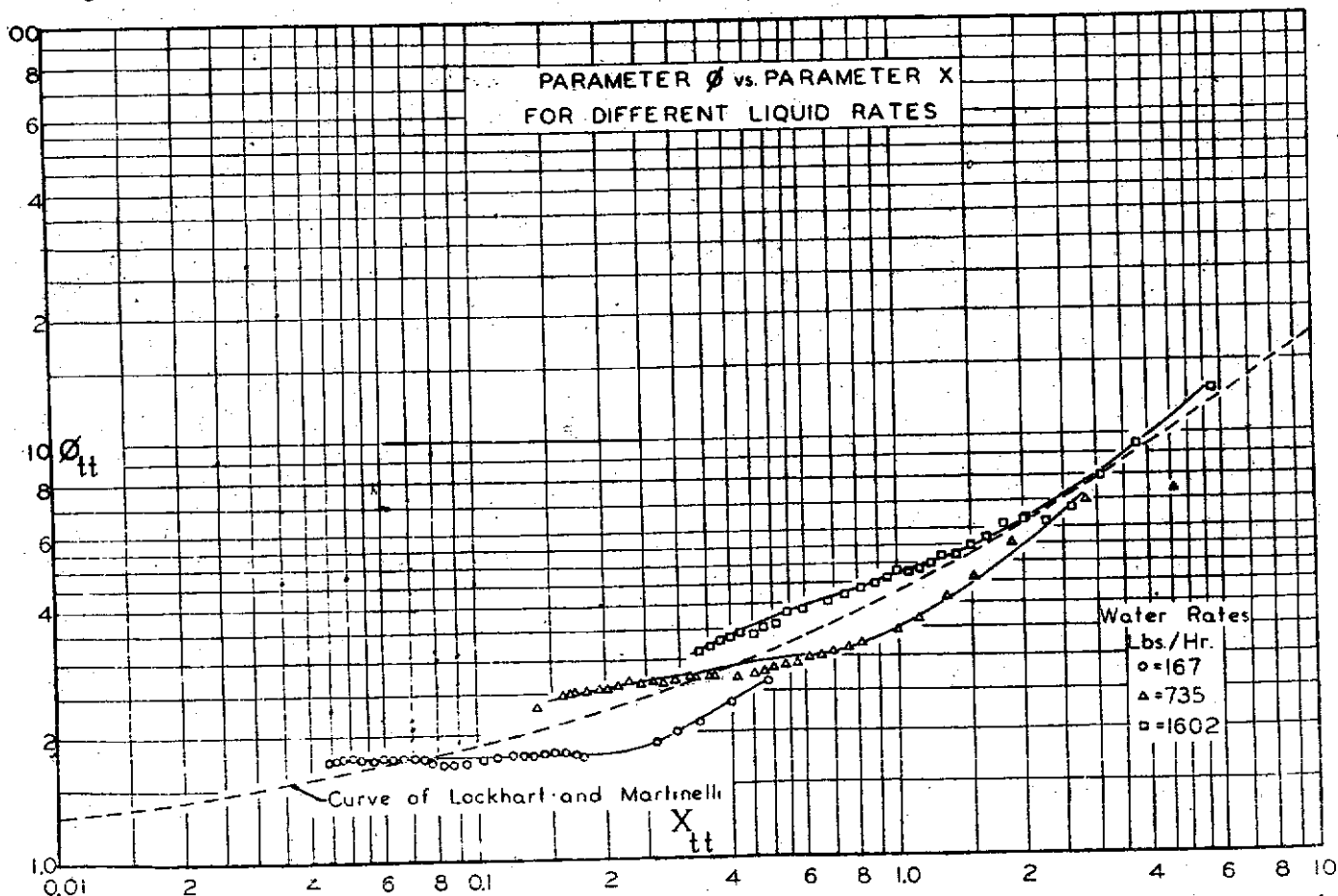

Fig. 2. (Gazley and Bergelin Discussion) ϕ_{tt} vs. X_{tt}. Data of Jenkins (3) at Three Liquid Rates Compared with Curve of Lockhart and Martinelli

the thermodynamic properties. In the flow of two phases, where one phase involves a flashing liquid, this limiting velocity can be fairly small. Some data of W. K. Woods corresponded to such conditions for which kinetic energy was a controlling factor rather than pressure drop due to pipe-line friction.

R. C. Martinelli: I would like to answer the question asked by Mr. Friend. It should be emphasized that the data presented in this paper were obtained during isothermal flow where the per cent of liquid and gas did not vary along the length of the pipe. Therefore, the acceleration problem did not exist. When these data are applied to conditions in which evaporation or condensation occurs, the added pressure drop due to acceleration must be considered.

We have attempted to correlate the present data with the experimental data of McAdams, Woods, and Heroman (2) and of Davidson, Hardie, Humphreys, Markson, Mumford, and Ravese (1). The pressure drop due to acceleration is negligible compared with the frictional pressure drop in the latter tests and is important in the low-pressure tests of McAdams, Woods, and Heroman.

A paper outlining the methods used for calculating the acceleration pressure drop was presented at the December, 1947, Annual Meeting of the Am. Soc. Mech. Engrs.

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John A. Putnam (University of California, Berkeley): Research at the University of California, Berkeley, since this paper was written, has been directed to an investigation of the transition between viscous- and turbulent-flow types. The apparatus developed for this purpose is not yet perfected in regard to accurate pressure-drop measurements. However, a much more satisfactory technique has been developed for measuring the per cent liquid saturation, R_i, in the flow tube.

In this technique, a glass tube, mounted in a vertical position, was calibrated for volume as a function of depth of liquid trapped in the tube. The tube bore had a measured variation in diameter of 1 part in 1000. A scale, graduated in per cent liquid saturation, was mounted directly behind the flow tube. After each experimental run was completed, the stop cocks in the pressure tap connections were closed followed by simultaneous closure of the inlet and exit valves at the extremities of the test section. After allowing sufficient time for all the trapped liquid to set-

tle to the bottom portion of the test section the liquid saturation was read directly from the scale. The pressure tap connections were short and of capillary size so that the volume of any liquid trapped in these connections was negligible.

Although the pressure-drop measurements obtained recently with this apparatus are not suitable for analysis, the liquid saturations represent the most accurate obtained to date and should therefore be of interest. The data, representing viscous-viscous flow, were obtained (1), and are presented in the accompanying curve (Fig. 1).

This figure also shows the average curve for viscous-viscous flow based on Lockhart's data (Fig. 3) and the generalized curve of Figure 4 of the paper under discussion. It is to be noted that the scatter of the points is considerably less than for the earlier work on which Figure 4 is based. In vestigation indicates that the scatter may be partly due to errors in measurement of flow rates. The fact that the data fall somewhat below the previous results bears out the conclusion of the authors that Lockhart's values for R_i might be high.

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R. W. Lockhart: About this more accurate method of measuring liquid saturation, three methods have been proposed so far: Investigators Taylor and Thompson used a blowing-out method, where they stopped the liquid flow with a quick shut-off valve, and blew out the pipe, collecting the liquid and measuring it. I used the method of dissolving various oils in a solvent, and removing the solvent by evaporation, and then measuring the residue. McElwee activated a quick shut-off valve on each end of the test section and then measured the liquid that collected in the pipe. For accurate measurement this method of McElwee's is limited to vertical flow. Data presented have been for horizontal flow only.

In the blow-out method some liquid may cling to the pipe walls. The same may be true of McElwee's method, as the oils undoubtedly wet the walls of the glass tube that he used.

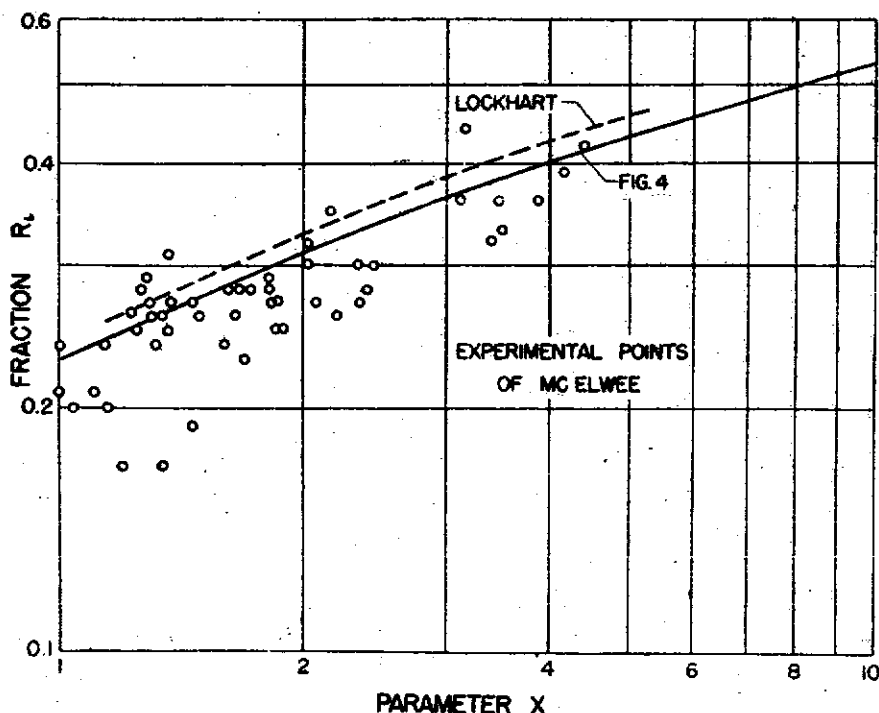*(Presented at Sixth Regional Meeting, Buffalo, N. Y.)*

Fig 1. (Putnam Discussion)