V. Two-Phase Flow and Heat Transfer

Va. Two-Phase Flow Fundamentals

At the first glance, two-phase or, in general, multi-phase flow seems an exotic topic used only in scientific experiments. In reality however, we may encounter two-phase flow in everyday activities. Flow of carbonated water pouring out of a bottle, ocean waves carrying oxygen, or even the action of the windshield wiper to remove rain involves two-phase flow. These are examples of isothermal flow. Of special interest is the flow of water and steam in heated channels such as in a BWR core or the tube-bundle of a PWR steam generator. Although continuous efforts are being made to formulate two-phase flow aspects by analytical means, most two-phase flow formulations are based on experimental data and hence are in the form of correlations. In this chapter, following the definition of pertinent terms, we discuss such important topics as calculation of two-phase flow pressure drop and critical flow.

1. Definition of Two-Phase Flow Terms

Two-phase flow generally refers to the flow of a liquid and a gas or vapor such as the flow of water and steam, water and air, etc.

Two-phase mixture refers to the mathematical analysis of two-phase flow where the two-phase mixture is treated as a pseudo single-phase.

Two-fluid model refers to the mathematical analysis of two-phase flow where phases are treated separately. Such treatment requires consideration of mass, momentum, and energy transfer between the phases. This model provides more information but also requires more experimentally based constitutive equations than a two-phase mixture model.

Multifluid flow refers to such cases as the flow of water droplets in bulk steam, surrounded by a film of flowing water.

Multi-phase flow refers to the flow of several phases such as steam, ice, and water.

Multicomponent flow refers to the flow of several phases having different chemical composition such as the flow of water, steam and air.

Thermodynamic equilibrium exists between phases when the liquid (l) and vapor (v) phases are at equal temperature, $T_l = T_v$.

Homogeneous is applied to two phases that flow at the same speed in the same direction.

Homogenous Equilibrium Model (HEM) is a means of mathematically describing two-phase flow, where $\vec{V}_l = \vec{V}_v$ (same flow direction at the same velocity) and also $T_l = T_v$ (thermodynamic equilibrium). If phase velocities are not equal ($\left|\vec{V}_l\right| \neq \left|\vec{V}_v\right|$) but temperature of the phases are, then the mathematical model for analysis of the two-phase flow is referred to as the *Separated Homogeneous Model* or SEM.

Quality is defined in various ways depending on the type of application. For example, considering steam and water, in Chapter II, we defined quality as $x = m_g/m$, referred to as the static quality, and may also be written as x_s. The thermodynamic quality is defined as $x = (h - h_f)/h_{fg}$, also written as x_e for equilibrium quality. The flow quality for a mixture of water and steam is defined as the ratio of mass flow rate of steam to mass flow rate of the mixture:

$$X = \frac{\dot{m}_g}{\dot{m}}$$

The flow quality, X, and thermodynamic quality, x, become equal only when thermal equilibrium conditions exist. Thus, $X = x$ only if $T_f = T_g$.

Void fraction in a control volume made up of liquid and gas mixture is the volume fraction of the gas phase. Hence, void fraction (α_g or simply α) is given by $\alpha = V_g/V$. Similarly, $1 - \alpha = V_f/V$. Note that void fraction is a space and time averaged quantity. The static quality, as defined above, can be expressed in terms of void fraction by noting that $x = m_g/(m_f + m_g) = \rho_g V_g/(\rho_f V_f + \rho_g V_g) = \rho_g \alpha V/[\rho_f(1 - \alpha) + \rho_g \alpha]V$. Hence,

$$x = \frac{\rho_g \alpha}{\rho_f(1-\alpha) + \rho_g \alpha}$$

Mixture density is given by $\rho = (m_f + m_g)/V$. Substituting for $m_f = \rho_f V_f$ and $m_g = \rho_g V_g$, we find $\rho = \rho_f V_f/V + \rho_g V_g/V$. Since $V_f/V = 1 - \alpha$ and $V_g/V = \alpha$, the mixture density in terms of void fraction becomes:

$$\rho = (1 - \alpha)\rho_f + \alpha\rho_g$$

Phasic mass flux, is the mass flow of a given phase per mixture area. Thus, for a mixture of water and steam for example, $G_g = \dot{m}_g/A$. Using the definition of

flow quality, $G_g = X\dot{m}/A = XG$. Similarly, for water we have $G_f = \dot{m}_f/A$. Substituting, $G_f = (1 - X)\dot{m}/A = (1 - X)G$ where G is the mixture mass flux.

Mixing cup density is similar to the mixture density but is averaged with respect to the phasic mass flux; $v' = 1/\rho' = [\rho_f(1-\alpha)V_f^2 + \rho_g\alpha V_g^2]/G^2$. Similar to the mixing cup density, a mixing cup enthalpy is defined as; $h' = [\rho_f(1-\alpha)V_f h_f + \rho_g\alpha V_g h_g]/G$

Phasic volumetric flow rate is defined similar to the single-phase flow hence, for the gas component of a mixture, $\dot{V}_g = \dot{m}_g/\rho_g = XGA/\rho_g$ and for the liquid component $\dot{V}_f = \dot{m}_f/\rho_f = (1 - X)GA/\rho_f$.

Superficial velocity is the velocity a phase would have if it were flowing alone in a channel. As such, the superficial velocity is obtained by dividing the related volumetric flow rate by the mixture area. For example, for the flow of water and steam in a channel, while water velocity is given by $V_f = \dot{V}_f/A_f$, where A_f is the water flow area, the superficial velocity for water is defined as $J_f = \dot{V}_f/A$ where A is total flow area of the channel. Similarly, the superficial velocity of steam is found as $J_g = \dot{V}_g/A$. To relate the superficial velocities to flow quality, we write:

$$J_g = \dot{V}_g/A = \dot{m}_g/(\rho_g A) = G_g/\rho_g = XG/\rho_g$$

Similarly, for J_f we find

$$J_f = (1 - X)G/\rho_f$$

We now define $J = J_f + J_g$. Substituting for J_f and J_g, we find J to be given by $J = [(1 - X)/\rho_f + X/\rho_g]G$. We also note that $J_g = \dot{V}_g/A = V_g A_g/A = aV_g$. Similarly, for the liquid phase we have $J_f = (1 - a)V_f$.

Slip ratio is defined as the ratio of the gas velocity to liquid velocity, $S = V_g/V_f$. Substituting, we find;

$$S = \frac{V_g}{V_f} = \frac{J_g/\alpha}{J_f/(1-\alpha)} = \frac{1-\alpha}{\alpha}\frac{XG/\rho_g}{(1-X)G/\rho_f} = \left(\frac{1-\alpha}{\alpha}\right)\left(\frac{X}{1-X}\right)\left(\frac{\rho_f}{\rho_g}\right)$$

$$\text{Va.1.1}$$

Thus, the slip ratio relates X and α. If for simplicity, we represent the quality and the density ratios with y:

$$y = \frac{1 - X}{X} \frac{\rho_g}{\rho_f}$$

Equation Va.1.1 simplifies to:

$$S = \frac{(1 - \alpha)}{y\alpha} \qquad\qquad Va.1.2$$

from which we can find void fraction as:

$$\alpha = (1 + yS)^{-1} \qquad\qquad Va.1.3$$

Volumetric flow ratio as defined for the gas phase is given as $\beta = \dot{V}_g / \dot{V} = J_g/J$, which may be written as:

$$\beta = \frac{J_g}{J} = \frac{J - J_f}{J} = 1 - \frac{J_f}{J}$$

Substituting for the superficial velocities in terms of flow quality X and mass flux G, we find:

$$\beta = (1 + y)^{-1} \qquad\qquad Va.1.4$$

Alternatively, by substituting for y from its definition above, we find:

$$\beta = \frac{Xv_g}{v_f + Xv_{fg}} \qquad\qquad Va.1.5$$

Wallis number is the ratio of inertial force to hydrostatic force on a bubble or drop of diameter D. Hence, the Wallis number (Wa) can be defined for both gas and liquid. For example,

$$Wa_g = [\rho_g/gD(\rho_f - \rho_g)]^{0.5} J_g.$$

Kutateladze number is defined similarly to the Wa number except for the length scale D, which is replaced by the Laplace constant $[\sigma/g(\rho_f - \rho_g)]^{0.5}$. Hence for a gas the Ku number becomes;

$$Ku_g = \left[\rho_g / \left(g\sigma(\rho_f - \rho_g)\right)\right]^{0.5} J_g$$

Flooding refers to the condition in which the upward flow of a gas stalls the downward flow of a liquid. This is accomplished through the momentum transfer at the liquid-gas interface. According to Wallis for flooding in vertical tubes,

$J_g^{0.5} + J_f^{0.5} = C$ where for round tubes C = 0.9 and for sharp-edged tubes $C = 0.75$.

Flow reversal refers to condition in which the upward flow of two phases is interrupted by a reduction in gas velocity. The lack of sufficient momentum transfer at the interface results in the gravity and frictional forces eventually stopping and finally reversing the flow of liquid. For flow reversal, $Ku_g = 3.2$.

Flow patterns of gas-liquid flow in an unheated pipe depend on such factors as pipe orientation, diameter, mass flux, flow quality, and phasic densities. Patterns of gas-liquid flow in a horizontal unheated tube and in upflow of a vertical unheated tube are shown in Figure Va.1.1.
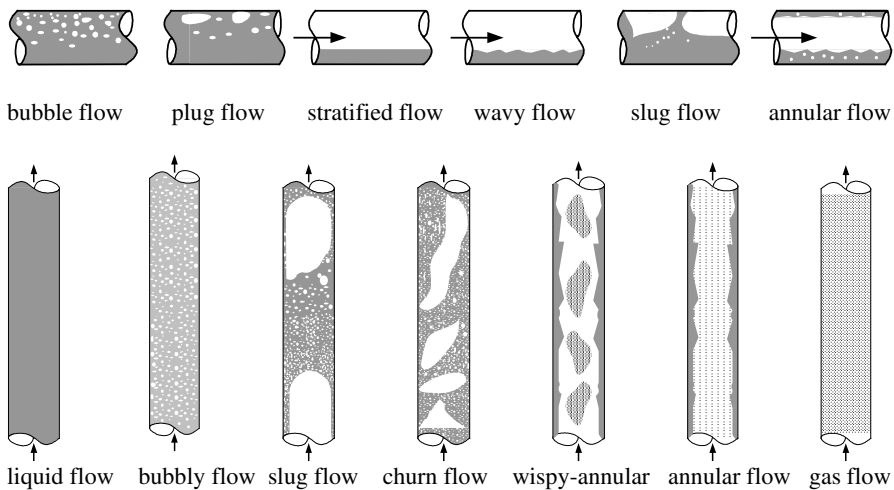

bubble flow   plug flow   stratified flow   wavy flow   slug flow   annular flow

liquid flow   bubbly flow   slug flow   churn flow   wispy-annular   annular flow   gas flow

Figure Va.1.1. Flow patterns in horizontal and vertical tubes

Flow pattern map reduces various flow regimes to identifiable patterns. Such maps associate the key flow parameters to a specific pattern. For a given set of such parameters, the flow pattern map determines the corresponding flow regime. Conversely, by knowing the flow regime, we can find a specific range for the key parameters. An example of such maps is shown in Figure Va.1.2. Hewitt has suggested the left side map for upflow and the right side map is used in the RELAP-5 thermalhydraulic computer code.
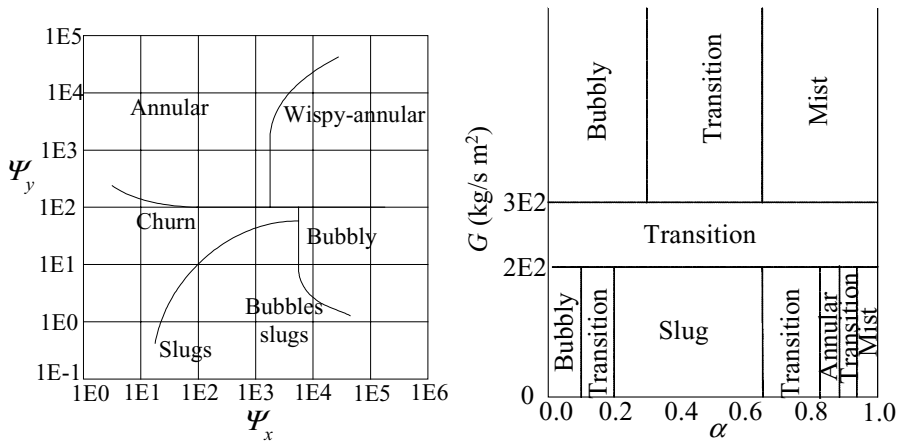
Figure Va.1.2. Flow pattern maps for vertical flow (low pressure air-water and high pressure steam-water)

The coordinates of the Hewitt map (left figure) are $\Psi_x = \rho_f J_f^2$ (kg/s²·m) and $\Psi_y = \rho_g J_g^2$ (kg/s²·m).

Example Va.1.1.   Water and steam flow at 1000 psia (~7 MPa) and 2 lbm/s (~1 kg/s) in a 1 in (2.54 cm) diameter tube. Find the flow regime at a location where $X = 0.2$.

Solution: At 1000 psia, $\rho_f = 46.32$ lbm/ft³ and $\rho_g = 2.24$ lbm/ft³. Since $A = 3.14 \times (1/12)^2/4 = 5.45E\text{-}3$ ft², then $G = 2/5.45E\text{-}3 = 366.7$ lbm/ft² s (1790 kg/s m²). Using Hewitt's map, we find:

$\rho_f J_f^2 = G^2(1-X)^2 / \rho_f = 366.7^2(1-0.2)^2/46.32 = 1858$ lbm/s² ft (2765 kg/s²·m)

$\rho_g J_g^2 = G^2 X^2 / \rho_g = 366.7^2(0.2)^2/2.24 = 2400$ lbm/s² ft (3589 kg/s² m)

Thus, the flow regime is Wispy – annular.

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2. Two-Phase Flow Relation

For two-phase flow in a conduit, there are two methods for solving for such state parameters as pressure, temperature, and velocity. In the first method, we assign a control volume to each phase. We then write the three conservation equations of mass, momentum, and energy for each control volume and solve them simultaneously. These control volumes exchange mass, momentum, and energy with each

other and exchange momentum and energy with the surface of the conduit. This is called the two-fluid model. In the second method, being basically a pseudo single-phase flow model, we use such parameters as void fraction, slip ratio, and two-phase friction multiplier to solve for only the three conservation equations written for the mixture. In this section, we discuss the two-phase flow parameters used in the pseudo single-phase analysis such as void fraction, flow quality, and slip ratio as well as pressure differential terms for two-phase flow.

2.1. One Dimensional Relation for Void Fraction

Determination of void fraction is essential in several aspects of two-phase flow analysis such as calculation of pressure difference terms. Equation Va.1.3 shows that void fraction varies inversely with the slip ratio. Hence, for given P and X, as S increases, the void fraction decreases. For example, for the flow of water and steam at $P = 1000$ psia and $X = 12\%$, α drops from 75% to 40% when S increases from 1 to 4.

Example Va.2.1. Express the slip ratio only in terms of α and β.

Solution: We use the definition of β given by $\beta = \{1 + [(1 - X)/X] \, (\rho_g/\rho_f)\}^{-1}$ to find $1 - \beta$. We then divide these to get $(1 - \beta)/\beta = [(1 - X)/X] \, (\rho_f/\rho_g)$. Substituting in Equation Va.1.1, we obtain:

$$S = \frac{1-\alpha}{\alpha} \frac{\beta}{1-\beta}$$

The slip ratio in general is a function of pressure (P), quality (X), and mass flux (G).

Example Va.2.2. Compare X for the flow of water and steam at 1000 psia for $\alpha = 50\%$ and $S = 1$, 2, and 3.

Solution: We solve Equation Va.1.1 for X to get:

$$X = \frac{\alpha \rho_g S}{(1-\alpha)\rho_f + \alpha \rho_g S}$$

At 1000 psia, $\rho_f = 46.32$ lbm/ft^3 and $\rho_g = 2.24$ lbm/ft^3. Substituting values, we find:

$X = 4.6\%$, 9%, 12.5% for $S = 1$, 2, and 3, respectively.

Example Va.2.3. For the flow of steam - water, find α, β, ρ, and x. Use $T_{sat} = 270$ C, $X = 0.15$, and $S = 3$.

Solution: At 270 C, $\rho_f = 767.9$ kg/m^3 and $\rho_g = 28.06$ kg/m^3. Substitute values in y:

$$y = \frac{1-X}{X}\frac{\rho_g}{\rho_f} = \frac{1-0.15}{0.15}\frac{28.06}{767.9} = 0.207$$

Next, we find α, β, the mixture density, and quality:

$\alpha = 1/(1 + yS) = 1/(1 + 0.207 \times 3) = 0.62$
$\beta = 1/(1 + y) = 0.83$
$\rho = (1 - \alpha)\rho_f + \alpha\rho_g = (1 - 0.62) \times 767.9 + 0.62 \times 28.06 = 309.2$ kg/m^3
$x = \rho_g\alpha/[\rho_f(1 - \alpha) + \rho_g\alpha] = 28.06 \times 0.62/[767.9(1 - 0.62) + 0.62 \times 28.06] = 0.056.$

As specified in Example Va.2.1, slip ratio itself is a function of pressure, mass flux, density, and void fraction distribution at a given cross section. There are several correlations for the calculation of slip ratio. An analytical method is offered by Zivi. In this method, the flow kinetic energy is set to a minimum (i.e., $K.E. = \sum\left(\rho_i V_i^2 \dot{V}_i\right) = 0$ where subscript i refers to liquid and vapor). If we substitute for $\dot{V}_f$ and $\dot{V}_g$ from the definition of the phasic volumetric flow rate, we find:

$$K.E. = \left[\frac{X^3}{\alpha^2\rho_g^2} + \frac{(1-X)^3}{(1-\alpha)^2\rho_f^2}\right]\frac{AG^3}{2}$$

Taking the derivative with respect to α and setting it equal to zero, we obtain $\alpha/(1 - \alpha) = [X/(1 - X)](\rho_f/\rho_g)^{2/3}$. By comparing this result with Equation Va.1.1, we find that $S = (\rho_f/\rho_g)^{1/3}$. Since Zivi's method expresses the slip ratio only in terms of densities, Zivi's model does not compare well with experimental data. In Example Va.2.2, according to Zivi's method, S is always $S = (46.32/2.24)^{1/3} = 2.75$ for any mass flux. By definition, the homogenous model gives $S = 1$. Thom, recognizing the dependency of S on X, developed a relation for S based on best fit to data for various system pressures. Winterton collected these data in a single equation in terms of the saturated specific volumes:

$$S = 0.93(v_g/v_f)^{0.11} + 0.07(v_g/v_f)^{0.561} \qquad\qquad\qquad \text{Va.2.1}$$

This correlation fits Thom's data well within 1% and can be used for pressures ranging from atmospheric up to the critical point. To estimate S from Equation Va.2.1, a thermal equilibrium condition must exist.

Example Va.2.4. Water enters a heated channel at rate of 20 kg/s with a degree of subcooling of 15 C. Use Equation Va.2.1 and a reference pressure of 7 MPa to find the rate of heat transfer to this channel to ensure the exit void fraction equals 75%.

Solution: To ensure the void fraction at the exit of the heated channel remains at the specified limit, we need to fix the value of the exit quality from Equation Va.1.1, with S given by Equation Va.2.1.

Next, having quality at the exit of the channel, we can find the flow enthalpy at the exit. The rate of heat transfer is subsequently found from a steady-state energy balance.

At 7 MPa, $v_f = 0.001351$ m^3/kg, $v_g = 0.02737$ m^3/kg, and $v_g/v_f = 20.26$

Substituting in Equation Va.2.1:

$S = 0.93(20.26)^{0.11} + 0.07(20.26)^{0.561} = 1.67$

Having S and α_e, we find $y \approx 0.2$ from Equation Va.1.2.
Having y, ρ_f, and ρ_g, we find x_e from $y = [(1 - x_e)/x_e](\rho_g/\rho_f)$
$(1 - x_e)/x_e = y(v_g/v_f) = 0.2 \times 20.26 = 4$ resulting in $x_e = 0.198$

We now find the inlet and exit enthalpies. At 7 MPa, $T_{sat} = 285.88$ C. To find h_i, we need to find the enthalpy of subcooled liquid at $P = 7$ MPa and $T = 285.88 - 15 = 273.85$ C resulting in $h_i \approx 1204$ kJ/kg

The exit enthalpy is: $h_e = h_f + x_e\, h_{fg} = 1266.97 + 0.198 \times 1505.1 = 1565$ kJ/kg

Therefore, $\dot{Q} = \dot{m}(h_e - h_i) = 20(1565 - 1204) = 7.22$ MW.

2.2. Drift Flux Model for Void Fraction

This method, introduced by Zuber-Findlay, and also described by Wallis, is based on the relative motion of the phases and accounts for the void fraction dependency on mass flux and void distribution at a given cross section in the flow. The notable approach in this method is the introduction of a relative motion. In general, the liquid and gas in a mixture travel at different velocities for which we define the relative velocity between the phases as:

$V_{gf} = V_g - V_f$

Expressing the phasic velocities in terms of their corresponding superficial velocities, we find:

$V_{gf} = J_g/\alpha - J_f/(1 - \alpha)$

Multiplying both sides of this relation by $\alpha(1 - \alpha)$ we obtain;

$\alpha(1 - \alpha)V_{gf} = (1 - \alpha)J_g - \alpha J_f.$

The left side term has units of velocity known as the drift velocity or *drift flux*, J_{gf}. The right side term can be rearranged to get $J_g - \alpha(J_f + J_g) = J_g - \alpha J$. Thus; $J_{gf} = J_g - \alpha J$.

To get a physical interpretation of drift flux, we may say that drift flux is the gas volumetric rate passing through a unit area of a plane, normal to the channel axis and traveling at velocity αj.

While the above relation was derived for one-dimensional flow, the usefulness of the drift flux model is in the fact that it accounts for the void fraction distribution at a cross section. We now find the average value of variables over a flow cross section. For example:

$$\overline{\alpha} = \iint_A \alpha \frac{dA}{A}$$

Va.2.2

By so doing, the drift flux can be written as $\overline{J}_{gf} = \overline{J}_g - \overline{\alpha J}$. Dividing this relation by $\overline{\alpha}$ and noting that $\overline{\alpha J} \neq \overline{\alpha}\,\overline{J}$, we obtain:

$$\frac{\overline{J}_{gf}}{\overline{\alpha}} = \overline{V}_g - \frac{\overline{\alpha J}}{\overline{\alpha}}$$

We simplify this relation by defining $\overline{V}_{gJ}$ such that $\overline{J}_{gf} = \overline{\alpha}\ \overline{V}_{gJ}$ and a parameter C_o such that:

$$C_o = \frac{\overline{\alpha J}}{\overline{\alpha}\,\overline{J}}$$

Substituting, we find $\overline{V}_{gJ} = \overline{V}_g - C_o \overline{J}$. Dividing both sides of this relation by $\overline{J}$ and replacing $\overline{V}_g / \overline{J} = \overline{\beta} / \overline{\alpha}$ we obtain:

$$\overline{\alpha} = \frac{\overline{\beta}}{C_o + (\overline{V}_{gJ} / \overline{J})}$$

Va.2.3

Equation Va.2.3 is the Zuber-Findlay drift flux model for the calculation of void fraction. This equation is important for the fact that it also accounts for mass flux, G. The parameter C_o, as introduced by Zuber-Findlay, is the key in this model. This parameter helps to distinguish between the concentration profile at a cross section from the velocity profile. For example, for one-dimensional homogenous flow, we know that $\alpha = \beta$. From Equation Va.2.3, this is possible when $\overline{V}_{gJ} = 0$ and $C_o = 1$.

To write an alternative expression for Equation Va.2.3 we first substitute for $j_g = \beta j$ in Equation Va.2.3 to get $\overline{\alpha} = \overline{J}_g / (C_o \overline{J} + V_{gJ})$. We then substitute for

$\bar{J}_g = XG/\rho_g$ and $\bar{J} = [(1 - X)/\rho_f + X/\rho_g]G$. Dividing both numerator and denominator by XG/ρ_g and using $y = (1 - X)\rho_g/X\rho_f$, as defined earlier, the drift flux model for void fraction becomes:

$$\alpha = \frac{1}{C_0(1+y) + \dfrac{V_{gJ}\rho_g}{XG}} \qquad\qquad \text{Va.2.3}$$

where for simplicity, the volume-averaged symbol is now dropped. Substituting for void fraction from Equations Va.2.3 to Equation Va.1.3 and solving for the slip ratio, we find:

$$S = \left[C_o + \frac{C_o - 1}{y}\right] + \left[\frac{\rho_g V_{gJ}}{yXG}\right] \qquad\qquad \text{Va.2.4}$$

Equation Va.2.4 consists of two terms:

term 1: $C_o + [(C_o - 1)/y]$.
   This term pertains to nonuniform void distribution in a given flow cross section

term 2: $V_{gJ}\rho_g/(yXG)$.
   This term pertains to velocity differential between the liquid and the gas phase.

If there is no void, then $C_o = 0$. Depending on the void fraction distribution, C_o ranges from 1.0 to 1.3. If the ratio of void fraction at the tube surface to the void fraction at the tube center is unity, then C_o is a minimum. The value of C_o increases to a maximum as the above ratio decreases to zero.

The Zuber-Findlay model for void fraction (Equation Va.2.3) is applicable for vertical upflow. If the flow regime is bubbly flow, Zuber and Findlay suggest $C_o = 1.13$ and V_{gj} is found from:

$$V_{gJ} = 1.41\left[\frac{g(\rho_f - \rho_g)\sigma}{\rho_f^2}\right]^{1/4} \cong 1.41\left(\frac{g\sigma}{\rho_f}\right)^{1/4} \qquad\qquad \text{Va.2.5}$$

These values correlate well to round tube data.

Example Va.2.5. A mixture of water and steam flows up a 20 mm diameter tube at a rate of 4000 kg/m^2·s and temperature of 290 C. At a location where $X = 30\%$ find: a) void fraction, b) the mixture mixing cup density, c) mixture density using the HEM, d) mixture thermodynamic density.

Solution: For saturated mixture at $T = 290$ C, $\rho_f = 732$ kg/m^3, $\rho_g = 39$ kg/m^3, $\sigma = 0.0166$ N/m.

a) We find the parameter y from:

$y = [(1 - X)/X](\rho_g/\rho_f) = (0.7/0.3)(39/732) = 0.124$

$\beta = 1/(1 + y) = 1/(1 + 0.124) = 0.89$

Next, we calculate V_{gj} From Equation Va.2.5 and then find J, α, and S:

$V_{gJ} = 1.41[g(\rho_f - \rho_g)\sigma/\rho_f^2]^{0.25} = 1.41[9.81 \times (732 - 39) \times 0.0166/732^2]^{0.25} = 0.17$ m/s

$J = (1 + y)XG/\rho_g = (1 + 0.124) \times 0.3 \times 4000/39 = 34.58$ m/s

$\alpha = \beta/[C_o + V_{gJ}/J] = 0.89/[1.13 + 0.17/34.58] = 0.78$

(Compare with $\alpha_{HEM} = \beta = 0.89$)

$$S = \left[1.13 + \frac{1.13 - 1}{0.124}\right] + \left[\frac{39 \times 0.17}{0.124 \times 0.3 \times 4000}\right] = 2.2$$

b) For the mixing cup density we need phasic velocities. We find V_g and V_f from J_g and J_f:

$V_g = J_g/\alpha = G_g/\alpha\rho_g = XG/\alpha\rho_g = 0.3 \times 4000/(0.78 \times 39) = 39$ m/s

$V_f = (1 - X)G/(1 - \alpha)\rho_f = (1 - 0.3) \times 4000/[(1 - 0.78) \times 732] = 17.4$ m/s

(Note; $S = V_g/V_f = 2.2$)

$1/\rho' = [39 \times 0.78 \times 39.25^2 + 732 \times (1 - 0.78) \times 17.38^2]/4000^2 = 5.97\text{E-}3$ m³/kg

Thus, $\rho' = 167.5$ kg/m³. Find mixture density from:

$\rho = (1 - \alpha)\rho_f + \alpha\rho_g = (1 - 0.78) \times 732 + 0.78 \times 39 = 191.46$ kg/m³

c) The HEM density is obtained by substituting related values for specific volumes;

$v = (1 - X)v_f + Xv_g$. Thus, $\rho_{HEM} = (1 - X)/\rho_f + X/\rho_g = (1 - 0.3)/732 + 0.3/39 = 115.6$ kg/m³.

d) Since we have used saturation properties, we have implicitly assumed thermal equilibrium exists between the phases hence, $x = X = 0.3$.

Whalley uses the definition of slip velocity to express drift flux in terms of void fraction as:

$$J_{gf} = V_b\alpha(1 - \alpha)^2 \qquad\qquad\qquad \text{Va.2.6}$$

where V_b is the rising velocity of a single bubble as a function of pressure (Table Va.2.1).

Table Va.2.1. Values of V_b for water-steam flow (Whalley)

P (bar)	V_b (m/s)	P (psia)	V_b (ft/s)
1	0.22	14.5	0.722
3	0.21	43.5	0.689
10	0.20	145	0.656
30	0.19	435	0.623
100	0.16	1450	0.525
221.2*	0.0	3207.4*	0.00

*: At critical pressure $\rho_g \rightarrow \rho_f$, $\sigma \rightarrow 0$, and $V_b \rightarrow 0$

Example Va.2.6. A mixture of water and steam flows up a 10 mm diameter tube at a rate of 4000 kg/s·m^2 and 2 kg/s·m^2, respectively. Assume an adiabatic condition. Find void fraction in the tube.

Data: P = 1 atm, ρ_f = 1000 kg/m^3, ρ_g = 0.598 kg/m^3.

Solution: One way to find α is to set Equation Va.2.5 equal to the definition of J_{gf} = $(1 - \alpha)J_g - \alpha J_f$:

$$V_b\alpha(1 - \alpha)^2 = J_{gf} = (1 - \alpha)J_g - \alpha J_f$$

We now need J_f and J_g, which are calculated as:
$J_f = G_f/\rho_f = 4000/1000 = 4$ m/s
$J_g = G_g/\rho_g = 2/0.598 = 3.34$ m/s

Finding V_b from Table Va.2.1 as 0.22 m/s, one obtains the following non-linear equation for α:

$$(J_f + J_g)\alpha + V_b\alpha(1 - \alpha)^2 - J_g = 0$$

By iteration, we find $\alpha \approx 0.455$.

2.3. Conservation Equations

Derivation of conservation equations for multi-phase and multi-dimensional flow in a control volume with a deformable boundary is beyond the scope of this book. Here, we consider a one-dimensional two-phase flow in a fixed boundary control volume with constant flow area, as shown in Figure Va.2.1 (Myer). To write the
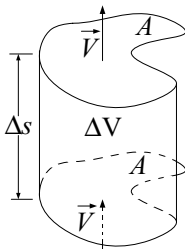
Figure Va.2.1. One-dimensional control volume for conservation equations

conservation equation for the mixture, we use volume-averaged quantities similar to Equation Va.2.2, defined for void fraction. Since the flow area is assumed to be constant, the area averaged value for any quantity such as ψ is simplified to $<\psi> = (\int \psi dA)/A$.

Beginning with the conservation equation of mass, we apply Equation IIIa.3.2 to the two-phase flow in the elemental control volume of Figure Va.2.1. We then divide each term by $A\Delta z$ and let Δz approach zero:

$$\frac{\partial}{\partial t} <\rho_g \alpha + \rho_f (1-\alpha)> + \frac{\partial}{\partial s} <\rho_f V_f (1-\alpha) + \rho_g V_g \alpha> = 0 \qquad \text{Va.2.7}$$

If we now substitute for the mixture density, $\rho = (1 - \alpha)\rho_f + \alpha\rho_g$ and the mixture mass flux, defined as; $G = <\rho_f(1 - \alpha)V_f + \rho_g a>$, the one-dimensional continuity equation for the flow of a two-phase mixture becomes:

$$\frac{\partial \rho}{\partial t} + \frac{\partial G}{\partial s} = 0 \qquad \text{Va.2.8}$$

If Equation Va.2.8 is integrated over a macroscopic control volume $V = sA$, we obtain Equation IIa.5.1.

Regarding the one-dimensional conservation equation of momentum for two-phase flow in a constant area channel, we use Equation IIIa.3.6. Assuming gravity to be the only body force and substituting for the shear stresses, such as surface force, we find the momentum equation for uniform flow at a cross section z:

$$\frac{\partial G}{\partial t} + \frac{\partial}{\partial s} <v'G^2> = -\frac{\partial P}{\partial s} - \frac{P_w \tau_s}{A} - \rho g \sin \gamma \qquad \text{Va.2.9}$$

where τ_s is the shear stress at the wall of the channel, P_w is the channel wetted perimeter, and $\rho' = 1/v'$ is the mixing cup density. Also note that γ is the angle between the flow velocity vector and the horizontal plane (see Figure Va.2.2) and ranges from $-\pi/2 \leq \gamma \leq \pi/2$. For horizontal channels $\gamma = 0$. For vertical channels, if flow is upward, $\gamma = \pi/2$ and if flow is downward, $\gamma = -\pi/2$.

The term representing shear stresses in Equation Va.2.9 can be substituted from Equation IIIb.2.5 for single-phase flow so that $P_w \tau_s /A = fv|G|G/2D_h$ where f is the friction factor and using the absolute value of G ensures opposing force in the case of flow reversal in the channel. To obtain the momentum equation for a macroscopic control volume, we integrate Equation Va.2.9 over a finite length s:

$$\frac{s}{A}\frac{d\dot{m}}{dt} + \Delta\left(\frac{v'\dot{m}^2}{A^2}\right) = -\Delta P - \frac{fv|\dot{m}|\dot{m}}{2A^2}\frac{s}{D_h} - \rho_m Vg \sin \gamma \qquad \text{Va.2.10}$$

The two-phase flow momentum equation is discussed in more detail in Section 2.4.

The conservation equation of energy for two-phase flow in a constant area channel can be derived from Equation IIIa.3.9 with work terms substituted from Equation IIIa.3.10. Ignoring the contribution by kinetic and potential energies and considering only pressure work, the energy equation simplifies to:

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial s}(Gh') = \frac{\partial P}{\partial s} + \frac{\dot{q}''P_h}{A} + \dot{q}''' \qquad \text{Va.2.11}$$

where the mixture density h represents $h = <\rho_f h_f(1 - \alpha) + \rho_g h_g \alpha>/\rho$ and h' is the mixing cup enthalpy. To obtain the energy equation for a macroscopic control volume, we integrate Equation Va.2.11 over control volume $V = sA$:

$$\frac{\partial}{\partial t}(\rho h V) + \Delta(\dot{m}h') = V\frac{\partial P}{\partial s} + \dot{q}''P_h s + \dot{q}'''V \qquad \text{Va.2.12}$$

2.4. Pressure Differential Terms

Equation Va.2.10 includes five pressure differential terms for two-phase flow which are similar to the pressure differential terms for single-phase flow defined in Equation IIIa.3.43 (with $\Delta P_{shaft} = 0$). At steady state, $\Delta P_{Inertia} = 0$ and Equation IIIa.3.43 simplifies to:

$$(\frac{dP}{ds})_{stat} + (\frac{dP}{ds})_{acc} + (\frac{dP}{ds})_{fric} + (\frac{dP}{ds})_{grav} = 0 \qquad \text{Va.2.13}$$

where the terms are differentiated with respect to s, the element of length as shown in Figure Va.2.2, not to be confused with the slip ratio, S. Let's now evaluate each term in Equation Va.2.13.
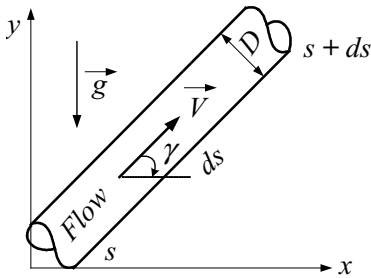

Figure Va.2.2. Steady-state flow of a two-phase mixture in a pipe

Static pressure gradient, $(dP/ds)_{stat}$ if integrated over a finite length in the flow path gives the total pressure drop from inlet (point i) to exit (point e):

$$\int_i^e \left(\frac{dP}{ds}\right)_{stat} ds = P_e - P_i \,.$$
Va.2.14

Pressure gradient due to acceleration is given by:

$$(\frac{dp}{ds})_{acc} = \frac{1}{A}\frac{d(\dot{m}V)}{ds} = \frac{1}{A}\frac{d[(\rho VA)V]}{ds} = \frac{d}{ds}(\frac{G^2}{\rho'})$$
Va.2.15

The derivative can be carried out based on the simplification made for the mixing cup density. For example, if velocities of both phases are uniform across the channel, we may substitute for V_g and V_f from $V_g = G_g/\alpha\rho_g = XG/\alpha\rho_g$ and $V_f = (1 - X)G/(1 - \alpha)\rho_f$ in the relation for mixing cup density to get;

$$\frac{1}{\rho'} = \frac{1-X^2}{(1-\alpha)\rho_f} + \frac{X^2}{\alpha\rho_g}$$
Va.2.16

Substituting for ρ' from Equation Va.2.16 into Va.2.15 and carrying out the derivative, we find:

$$(\frac{dP}{ds})_{acc} = G^2[-\frac{2(1-X)^2\mathrm{v}_f}{(1-\alpha)} + \frac{2X\mathrm{v}_g}{\alpha}](\frac{dX}{ds}) + G^2[\frac{(1-X)^2\mathrm{v}_f}{(1-\alpha)^2} + \frac{X^2\mathrm{v}_g}{\alpha^2}](\frac{d\alpha}{ds}) +$$
$$G^2[\frac{X^2}{\alpha}\frac{\partial\mathrm{v}_g}{\partial P}](\frac{dP}{ds})$$
Va.2.17

This derivation applies to separated flow. For HEM, the mixing cup density is calculated from $1/\rho' = \mathrm{v} = (1 - X)\mathrm{v}_f + X\mathrm{v}_g$. Substituting this relation in Equation Va.2.15 and using the equal phase velocity assumption, we get:

$$(\frac{dP}{ds})_{acc} = G^2\frac{d}{ds}[\mathrm{v}_f + X\mathrm{v}_{fg}] = G^2\frac{d}{ds}[X\frac{\partial\mathrm{v}_{fg}}{\partial P}\frac{dP}{ds} + \mathrm{v}_{fg}\frac{dX}{ds}]$$
Va.2.18

Neglecting the compressibility of liquid (i.e., setting $\partial\mathrm{v}_f/\partial P = 0$), we find $(dP/ds)_{acc}$ for HEM as:

$$(\frac{dP}{ds})_{acc} = G^2[X\frac{\partial\mathrm{v}_g}{\partial P}\frac{dP}{ds} + \mathrm{v}_{fg}\frac{dX}{ds}]$$
Va.2.19

Pressure gradient due to friction for two-phase flow is similar to friction pressure drop for single-phase flow. For example, suppose a mixture of water and steam is flowing in a heated pipe of diameter D and length L at the mass flow rate of $\dot{m}$. The friction pressure drop for the two-phase mixture is obtained from:

$$\Delta P_{tp} = f_{tp}\frac{L}{D}\frac{\dot{m}^2}{2\rho_{tp}A^2}$$
Va.2.20(a)

where subscript *tp* stands for two-phase. To find f_{tp}, we now assume a case that *only water* is flowing in the same pipe at the same mass flow rate as the mixture of water and steam:

$$\Delta P_{sp} = f_{sp} \frac{L}{D} \frac{\dot{m}^2}{2\rho_{sp} A^2}$$
Va.2.20(b)

where *sp* stands for single-phase, hence, $\rho_{sp} = \rho_f$. Dividing Equation Va.2.20(a) by Va.2.20(b) we get:

$$\frac{\Delta P_{tp}}{\Delta P_{sp}} = \phi = \frac{f_{tp}}{f_{sp}} \frac{\rho_{sp}}{\rho_{tp}}$$
Va.2.21

This ratio is referred to as the *two-phase friction multiplier*. Substituting for $f_{tp}/\rho_{tp} = \phi(f_{sp}/\rho_{sp})$ from Equation Va.2.21 in Equation Va.2.20(a), we find the two-phase friction pressure drop as:

$$\Delta P_{tp} = \phi f_{sp} \frac{L}{D} \frac{\dot{m}^2}{2\rho_{sp} A^2}$$
Va.2.22

For homogeneous equilibrium conditions assuming $f_{tp} = f_{sp}$, the two-phase friction multiplier becomes:

$$\phi = \frac{\rho_{sp}}{\rho_{tp}} = \frac{v_{tp}}{v_{sp}} = [1 + X \frac{v_{fg}}{v_f}]$$
Va.2.23

This derivation was for a heated pipe. If the mixture is flowing in a pipe that is being cooled resulting in steam condensation, the comparison is made with *only steam* flowing in the pipe (i.e., $\rho_{sp} = \rho_g$).

Example Va.2.7. Derive alternative relations for the two-phase friction multiplier.

Solution: Rather than assuming equal single-phase and two-phase friction factors, let's substitute for friction factors in turbulent flow from Equation IIIb.3.6. In this case, Equation Va.2.23 becomes:

$$\phi = \frac{f_{tp}}{f_{sp}} \frac{\rho_{sp}}{\rho_{tp}} = \frac{0.184/\operatorname{Re}_{tp}^{0.2}}{0.184/\operatorname{Re}_{sp}^{0.2}} \frac{v_{tp}}{v_{sp}} = (\frac{\mu_{tp}}{\mu_{sp}})^{0.2} \frac{v_{tp}}{v_{sp}}$$

The two-phase to single-phase viscosity ratio may be evaluated based on correlations by McAdams, Cichitti, or Duckler:

$$(\frac{\mu_{tp}}{\mu_{sp}})_{McAdam} = [1 + X(\frac{\mu_f}{\mu_g} - 1)]^{-1}; \quad (\frac{\mu_{tp}}{\mu_{sp}})_{Cichitti} = [1 + X(\frac{\mu_g}{\mu_f} - 1)];$$

$$(\frac{\mu_{tp}}{\mu_{sp}})_{Dukler} = [1 + \beta(\frac{\mu_g}{\mu_f} - 1)]$$

Reddy has recommended a relation similar to Equation Va.2.23:

$$\phi = 1 + X(v_{fg}/v_f)C \qquad\qquad \text{Va.2.24}$$

where $C = C'X^{-0.175}G^{-0.45}$. If $P > 600$ psia then $C' = 1.02$ otherwise, $C' = 0.357[1 + (P/P_{critical})]$. In this relation, mass flux G, is in Mlbm/ft²·h. This correlation is valid for vertical upflow in tubes of 0.2 to 0.6 inches in diameter and 5 to 100 inches in length. The range for mass flux is 0.35 to 3.3 Mlbm/ft²·h. The advantage of Reddy's correlation is that it also accounts for the effect of mass flux.

A more recent correlation based on a vast bank of data is suggested by Friedel:

$$\phi = C_1 + \frac{3.24C_2C_3}{Fr^{0.045}We^{0.035}}$$

where Fr and We are the Froude and Weber numbers. The Froude number, the ratio of inertial to gravity force (Table A.I.6) is given as:

$$Fr = \frac{V^2}{gL} = \frac{\dot{m}}{\rho^2 gD}$$

and the Weber number, the ratio of inertial to surface tension force, is given as:

$$We = \frac{\rho V D^2}{\sigma} = \frac{(G_f + G_g)^2 D}{\rho\sigma}$$

where $\rho = 1/v = v_f + Xv_{fg}$. Constants C_1, C_2, and C_3 are related to steam quality and two-phase properties:

$$C_1 = (1-X)^2 + X^2\frac{\rho_f}{\rho_g}\frac{f_{sp,g}}{f_{sp,f}}, \qquad\qquad C_2 = X^{0.78}(1-X)^{0.24}, \qquad\qquad \text{and}$$

$$C_3 = \left(\frac{\rho_f}{\rho_g}\right)^{0.91}\left(\frac{\mu_g}{\mu_f}\right)^{0.19}\left(1-\frac{\mu_g}{\mu_f}\right)^{0.7}$$

Calculating the two-phase friction multiplier from any of the above relations, the frictional pressure gradient, $(dP/ds)_{fric}$, from Equation Va.2.22 can therefore be expressed as:

$$(\frac{dP}{ds})_{fric} = \phi f_{sp}\frac{1}{D}\frac{G^2}{2\rho_{sp}} \qquad\qquad \text{Va.2.25}$$

Pressure gradient due to gravity is given by:

$$(\frac{dP}{ds})_{grav} = \rho g\sin\gamma = \frac{1}{v}\sin\gamma g = \frac{g\sin\gamma}{v_f[1+X(v_{fg}/v_f)]} \qquad\qquad \text{Va.2.26}$$

2.5. Static Pressure Gradient, HEM

We now can find $(dP/ds)_{stat,HEM}$ if we substitute for various pressure gradient terms into Equation Va.2.13. The result depends on whether we use Equation Va.2.17 or Va.2.19 to represent $(dP/ds)_{acc}$. If we use Equation Va.2.19, which is applicable to HEM, we find:

$$-\left(\frac{dP}{ds}\right)_{stat} = \frac{\phi f_{sp} \dfrac{1}{D_h} \dfrac{G^2}{2\rho_{sp}} + G^2 v_{fg} \dfrac{dX}{ds} + \rho g \cos\gamma}{1 + G^2 X \dfrac{\partial v_g}{\partial P}} \qquad \text{Va.2.27}$$

where ϕ for HEM is given by Equation Va.2.23. We also used D_h so that Equation Va.2.15 is applicable to channels other than pipes and tubes. We may now integrate Equation Va.2.27 for the special case of a uniformly heated channel of length L and hydraulic diameter D_h. At the channel inlet (i) we have $s_i = 0$ and at the channel exit (e) we have $s_e = L$. In this integration, we assume saturated single-phase liquid enters the channel (i.e., $X_i = 0$). Since the channel is heated uniformly, we can make a change of variable from s to X according to:

$$\frac{dX}{ds} = \frac{X_e - X_i}{L}$$

so that $ds = (L/X_e)dX$. Note that the gas compressibility is generally very small, $|v_g/P| \ll 1$, which greatly simplifies the integration of Equation Va.2.27. Replacing ρ with $\rho = [v_f + Xv_{fg}]^{-1}$ and assuming that f_{sp} and v_{fg}/v_f remain constant, we integrate Equation Va.2.15 from the inlet to any point along the channel:

$$(P_i - P) = f_{sp} \frac{L}{D_e} \frac{G^2 v_f}{2}[1 + (\frac{v_{fg}}{v_f})\frac{X}{2}] + G^2 v_{fg} X + \frac{gL\cos\gamma}{v_{fg}X}\ln[1 + X(\frac{v_{fg}}{v_f})]$$

$$\text{Va.2.28}$$

where the quality and thermodynamic properties are evaluated at system pressure and $D_e = D_h$ is the equivalent hydraulic diameter.

Example Va.2.8. Water at 70 bar, 210 C, and a mass flow rate of 0.1 kg/s enters a uniformly heated vertical tube of diameter 2 cm and length 4 m. The applied heat flux is 600 kW/m². Find a) length of the tube over which water remains subcooled, b) pressure drop for the subcooled section, and c) total pressure drop.

Solution: At $P = 70$ bar; $T_{sat} = 285.9$ C, $h_f = 1267$ kJ/kg, $h_{fg} = 1505$ kJ/kg. h_i(70 bar & 210 C) $\cong 900$ kJ/kg.

a) Since $T_i < T_{sat}$, subcooled water enters the heated channel. The length of the single-phase or pre-heating section is found from a heat balance:
$$\dot{q}''(\pi d L_f) = \dot{m}(h_f - h_i)$$

Substituting values: $600[\pi \times (2/100) \times L_f)] = 0.1[1267 - 900]$
Solving for L_f, we find: $L_f \cong 1$ m.

Water then boils over the remaining 3 m of the tube. To find the exit quality we write the energy balance over the boiling length:

$600[\pi \times (2/100) \times (L - L_f)] = 0.1[h_e - 1267]$

Substituting values we find: $h_e = 2398$ kJ/kg

$X_e = (h_e - h_f)/h_{fg} = (2398 - 1267)/1505 = 0.75$

b) Over the single-phase section, we find pressure drop due to friction, acceleration, and gravity:
At $T_{sat} = 285.9$ C; $v_f = 0.00135$ m^3/kg, $v_{fg} = 0.026$ m^3/kg, $\mu_f = 0.943$E-4 N·s/m^2

Tube flow area is: $A_f = \pi \times 0.02^2/4 = 3.14$E-4 m^2
Thus, the mass flux is: $G = 0.1/3.14$E-4 $= 318.3$ kg/m^2·s

$(P_i - P_e)_{fric} = f_{sp}(L_f / D_e)v_f \dot{m}^2 /(2A_f^2)$

To find f_{sp}, we must calculate the Reynolds number:
$Re_{sp} = \dot{m}D_e / \mu A = 0.1 \times 0.02/[0.943$E-4 $\times 3.14$E-4 $= 67{,}510$ (Flow is turbulent)
$f_{sp} = 0.184/Re^{0.2} \cong 0.0199$

The friction pressure drop over the single-phase section is found as:

$$(\Delta P)_{fric,sp} = \left(f_{sp}\frac{L_f}{D_e}\right)\frac{v_f \dot{m}^2}{2A_f^2} = \left(0.0199\frac{1}{0.02}\right)\frac{0.00135 \times 0.1^2}{2 \times 9.87\text{E}-8} = 0.068 \text{ kPa}$$

$(\Delta P)_{acc,sp} = G^2(v_f - v_i) = (318.3)^2 \times (0.00135 - 0.00117) = 0.018$ kPa
$(\Delta P)_{grav,sp} = gL_f\cos\beta/[(v_i + v_f)/2] = 9.81 \times 1/[(0.00117 + 0.00135)/2] = 7.8$ kPa.

c) We now find pressure drop due to friction, acceleration, and gravity over the two-phase section using the homogenous model:

$$(\Delta P)_{fric,tp} = f_{sp}\frac{L_b}{D_e}\frac{G^2 v_f}{2}[1 + (\frac{v_{fg}}{v_f})\frac{X}{2}] =$$

$$0.0199\frac{(4-1)}{0.02}\frac{(318.3)^2 \times 0.00135}{2}[1 + (\frac{0.026}{0.00135})\frac{0.75}{2}] = 1.68 \text{ kPa}$$

$$(\Delta P)_{acc,tp} = G^2 v_{fg} X = (318.3)^2 \times 0.026 \times 0.75 = 1.97 \text{ kPa}$$

$$(\Delta P)_{grav,tp} = gL_b \cos \gamma \ln[1 + X(\frac{v_{fg}}{v_f})]/(v_{fg}X) =$$

$$9.81 \times 3 \times \ln[1 + 0.75(\frac{0.026}{0.00135})]/(0.026 \times 0.75) = 4.1 \text{ kPa}$$

$$\Delta P_{total} = (0.068 + 1.68) + (0.018 + 1.97) + (7.8 + 4.1) = 15.6 \text{ kPa}$$

Comment: Calculation of the properties at the inlet pressure is a reasonable assumption in this problem since the pressure drop is a small percentage of the inlet pressure ($\approx 0.2\%$). In general, an iterative solution may be required to find properties at an average pressure.

2.6. Static Pressure Gradient, Separated Flow Model (SFM)

To obtain $(dP/ds)_{stat}$ for the separated flow model, we substitute for various pressure gradient terms into Equation Va.2.13. For $(dP/ds)_{acc,}$ we substitute from Equation Va.2.17. If we integrate the resulting equation between the channel inlet at $s_i = 0$ and any location along the channel, we find the gradient for static pressure as:

$$(P_i - P) = f_{sp} \frac{1}{D_h} \frac{G^2}{2\rho_{sp}} \int_{s_i}^{s} \phi ds + G^2 \left[\frac{(1-X)^2}{(1-\alpha)\rho_f} + \frac{X^2}{\alpha\rho_g} \right]_{X_i}^{X} + \int_{s_i}^{s} g \sin \gamma [(1-\alpha)\rho_f + \alpha\rho_g] ds$$

$$\text{Va.2.29}$$

The acceleration pressure drop, being a perfect differential, depends only on the end points and is independent of the flow path. Expressing mixture property variations as a function of s, we can integrate the first and the third terms of Equation Va.2.29. Similar to the case of HEM, for a special case of uniformly heated channel, a change of variable can be introduced by replacing the differential length, ds, with dX, the differential quality. Substituting the change of variable into Equation Va.2.29, for the special case of uniformly heated channels with $X_i = \alpha_i = 0$, we find:

$$(P_i - P_e) = f_{sp} \frac{1}{D_h} \frac{G^2 L}{2\rho_{sp}} \frac{\int_0^{X_e} \phi dX}{X_e} + \frac{G^2}{\rho_f} \left[\frac{(1-X_e)^2}{(1-\alpha_e)} + \frac{X_e^2 \rho_f}{\alpha_e \rho_g} - 1 \right]$$

$$+ \frac{gL \cos \gamma}{X_e} \int_0^{X_e} [(1-\alpha)\rho_f + \alpha\rho_g] dX$$

This equation can alternatively be written as:

$$(P_i - P_e) = f_{sp} \frac{1}{D_h} \frac{G^2 L}{2\rho_{sp}} (C_1) + \frac{G^2}{\rho_f} (C_2) + gL\rho_f \sin \beta (C_3) \qquad \text{Va.2.30}$$

Constants C_1, C_2, and C_3 may be obtained from the Martinelli-Nelson or the Thom correlation. Figure Va.2.3 gives the values of constants C_1, C_2, and C_3 according to Thom's correlation.

Example Va.2.9. Solve Example Va.2.8 based on the Separated Flow Model.

Solution: The same total pressure drop is applicable in the preheating section for both HEM and SFM. For the boiling section, we use Figure Va.2.3 for $(\Delta P)_{fric}$, $(\Delta P)_{acc}$ and $(\Delta P)_{grav}$ respectively. These result in: $C_1 \approx 8.5$, $C_2 \approx 14$, and $C_3 \approx 0.24$.

Substituting the constants in Equation Va.2.30, we find:

$$(\Delta P)_{fric,tp} = f_{sp} \frac{1}{D_h} \frac{G^2 L_b}{2 \rho_{sp}} (C_1) =$$

$$0.0199 \frac{1}{0.02} \frac{(318.3)^2 \times 3 \times 0.00135}{2} \times 8.5 = 1.74 \text{ kPa}$$

$$(\Delta P)_{acc,tp} = [G^2 / \rho_f](C_2) = (318.3)^2 \times 0.00135 \times 7.8 = 1.90 \text{ kPa}$$

$$(\Delta P)_{grav,tp} = g L_b \rho_f \cos\beta (C_3) = 9.81 \times (3/0.00135) \times 0.24 = 5.2 \text{ kPa}$$

Therefore, total pressure drop over the tube is found as:
$$\Delta P_{total} = (0.068 + 1.74) + (0.018 + 1.90) + (7.8 + 5.2) = 16.7 \text{ kPa}$$

This result is in reasonable agreement with the result obtained from the homogenous model in Example Va.2.8.

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3. Two Phase Critical Flow

Similar to the critical flow of compressible, single-phase fluid, as discussed in Chapter IIIc, flow of a two-phase mixture in a channel may also become critical. For cases where saturated water is contained under pressure, opening of a valve or sudden burst of a connecting pipe results in expulsion of the tank inventory. In such a case, the saturated water may partially flash to steam as it approaches the break area, which is at much lower pressure. We will seek an analytical solution for the two-phase critical flow of water and steam under the following conditions; flow is homogeneous ($V_f = V_g$), thermodynamic equilibrium exists between the phases ($T_f = T_g$), and the process is isentropic. These assumptions lead to the determination of critical flow for HEM. Maintaining the assumption of an isentropic process, analytical solutions are also extended to two equilibrium non-homogenous cases. The first case uses a slip ratio calculated from either the Moody or the Fauske model. The second case uses models from Burnell and Henry-Fauske. These cases are summarized in Table Va.3.1 and then discussed in detail next.
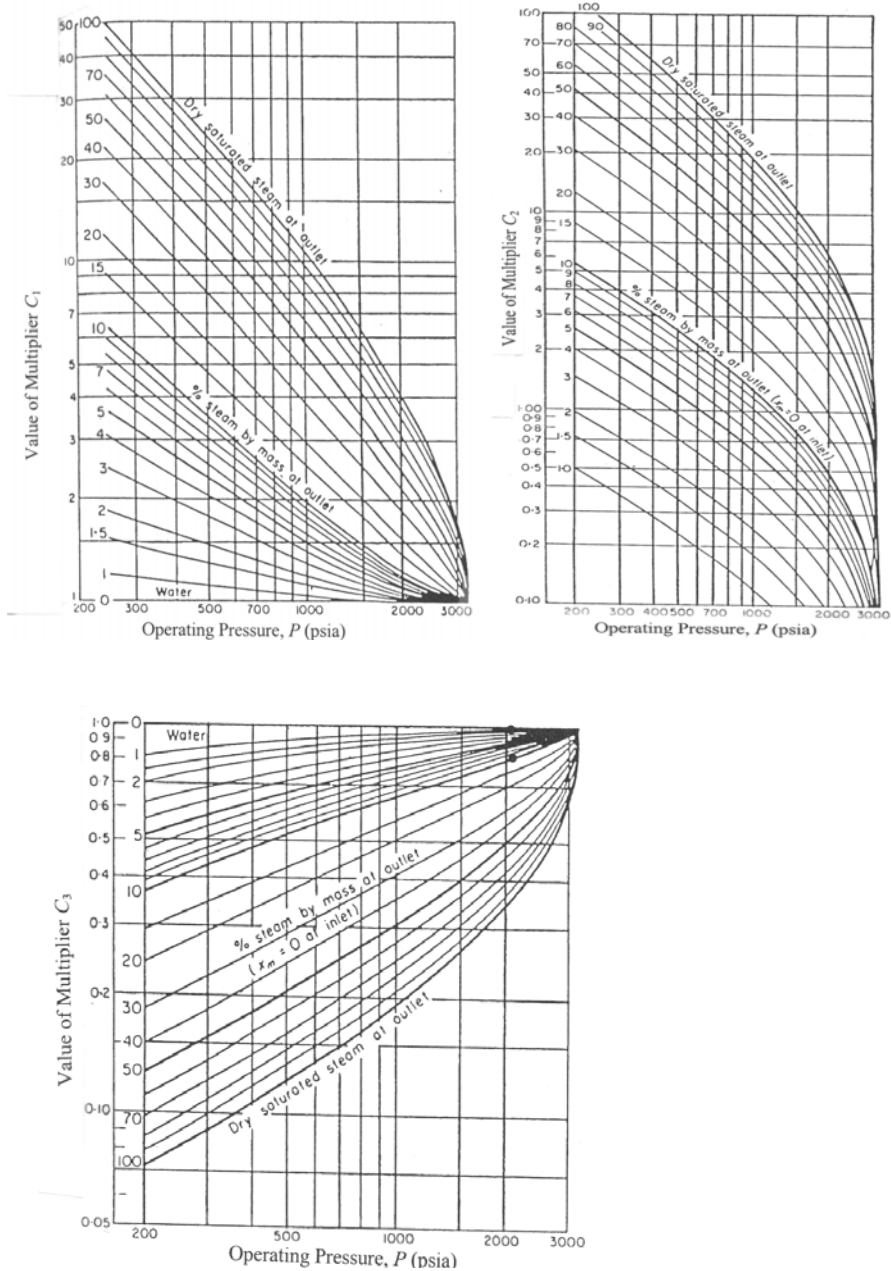
Figure Va.2.3. Coefficients for frictional, acceleration, and gravitational pressure drop (Thom 1964)

Table Va.3.1. Various critical flow models

	Homogenous Equilibrium	Equilibrium Non-homogenous		Homogenous Non-equilibrium
	(HEM)	(Moody Model)	(Fauske Model)	(Henry-Fauske Model)
$s_f = s_g$	Yes	Yes	Yes	Yes
$V_f = V_g$	Yes	No	No	Yes
$T_f = T_g$	Yes	Yes	Yes	No

3.1. Two-Phase Critical Flow (Homogeneous Equilibrium Flow)

This model is based on solving the conservation equations of mass and energy, in conjunction with the equations of state, under steady-state condition. The conservation equation of mass becomes:

$$\dot{m} = \rho V A = \text{constant} \qquad\qquad \text{IIa.5.2}$$

where ρ is the mixture density. Note that no distinction is made between V_f and V_g. The energy equation for the mixture, using the upstream stagnation condition (shown with subscript o), becomes:

$$h_o = h + V^2/2 \qquad\qquad \text{IIIc.2.1}$$

We may substitute for velocity from Equation IIIc.2.1 into IIa.5.2 and write the result in terms of mass flux:

$$G = \rho V = \frac{\left[2\left(h_o - (1-x)h_f - xh_g\right)\right]^{1/2}}{(1-x)v_f + xv_g} \qquad\qquad \text{Va.3.1}$$

where we also made use of the equation of state for $h = (1-x)h_f + xh_g$. If we substitute for quality calculated from the mixture entropy

$$x = \frac{s_o - s_f}{s_g - s_f}$$

into Equation Va.3.1, we obtain a relation that is solely a function of pressure. By iteration with the steam tables, we can then find a pressure that maximizes mass flux. Alternatively, we may express all thermodynamic properties and their derivatives as functions of pressure (examples of such functions are given in Appendix II, Table A.II.3). To find the pressure that maximizes mass flux, we then substitute these functions into Equation Va.3.1, take the derivative of G and set it equal to zero.

3.2. Two-Phase Critical Flow (Equilibrium Non-homogeneous Flow)

This is similar to the homogenous flow, but we must account for $V_f \neq V_g$. The mass balance becomes:

$$G = \frac{W}{A} = \frac{W_g / x}{A_g / \alpha} = \left(\frac{\alpha}{x}\right)\frac{W_g}{A_g} = \left(\frac{\alpha}{x}\right)\frac{V_g}{\mathrm{v}_g} = \left(\frac{1-\alpha}{1-x}\right)\frac{V_f}{\mathrm{v}_f}$$

Va.3.2

The energy equation can be partitioned to account for the contribution of each phase as follows:

$$h_\mathrm{o} = (1-x)\left(h_f + \frac{V_f^2}{2}\right) + x\left(h_g + \frac{V_g^2}{2}\right)$$

Va.3.3

To find G, we first substitute for void fraction from Equation Va.1.3. We then find V_f and V_g in terms of G from Equation Va.3.1, substitute them into Equation Va.3.2, and solve for G to obtain:

$$G = \rho * \sqrt{2\left[h_\mathrm{o} - h_f - (s_e - s_f)\frac{h_{fg}}{s_{fg}}\right]}$$

Va.3.4

where in Equation Va.3.4, $\rho*$ is given by:

$$\frac{1}{\rho *} = [S(1-x)\mathrm{v}_f + x\mathrm{v}_g]^2\left[\frac{1-x}{S^2} + x\right]$$

Va.3.5

Note that in Equation Va.3.5 the slip ratio S as given by Equation Va.1.1 should not be confused with s, the specific entropy. We now need to determine the slip ratio such that the mass flux is maximized. There are two models for this, as discussed next.

The Moody Model. In this model, the mass flux given by Equation Va.3.4 is maximized by setting the derivative of the kinetic energy with respect to slip ratio equal to zero, $\partial K.E./\partial S = 0$:

$$\frac{\partial}{\partial S}\left[\frac{(1-x)V_f^2}{2} + \frac{xV_g^2}{2}\right] = 0$$

Taking the derivative and setting it equal to zero, we obtain:

$$S_{Moody} = (\mathrm{v}_g/\mathrm{v}_f)^{1/3}$$

The Moody model compares well with data in the range of 14.7 – 400 psia.

The Fauske Model. In this model, the mass flux given by Equation Va.3.4 is maximized by setting the derivative of the flow momentum, with respect to slip ratio, equal to zero:

$$\frac{\partial}{\partial S}\left[(1-x)V_f + xV_g\right] = 0$$

This maximizes the axial pressure gradient for a given flow rate and steam quality. Substituting from Equation Va.3.1 for V_f and V_g, introducing Equation Va.1.3 for void fraction, and applying the chain rule for differentiation results in:

$$S_{Fauske} = (v_g/v_f)^{1/2}$$

Example Va.3.1. Derive the critical condition for annular flow using the conservation equation for mass and momentum.

Solution: We use the thermodynamic equilibrium and non-homogenous assumption, $V_f \neq V_g$. The combined continuity and momentum equations result in.

$$d(\dot{m}_f V_f + \dot{m}_g V_g) + A dP = 0$$

Substituting from Equation V.3.2 for V_f and V_g in terms of G, we find:

$$dP + d\left[\frac{(1-x)^2}{1-\alpha}v_f G^2 + \frac{x^2}{\alpha}v_g G^2\right] = 0$$

Solving for G, we obtain:

$$G = \sqrt{-(dP/dv)_s} \qquad\qquad \text{Va.3.6}$$

where specific volume in Equation Va.3.6 is given by:

$$v = \frac{(1-x)^2}{1-\alpha}v_f + \frac{x^2}{\alpha}v_g \qquad\qquad \text{Va.3.7}$$

Having found the slip ratio by the Moody or the Fauske method, we express the thermodynamic properties and their derivatives in terms of pressure and substitute into Equation IIIa.3.6-2. For example, for the Fauske model, we use Equations IIIa.3.6-2, Va.1.3 (with $S = \sqrt{v_g / v_f}$), and Va.3.6 to obtain (Nahavandi):

$$\frac{-1}{G^2} = \left\{ [(1-x+xS)x]\frac{dv_g}{dP} + [(2xS - 2S - 2S^2 x + S^2)v_f + (1 + 2xS - 2x)v_g]\frac{dx}{dP} \right\}_s$$

$$\text{Va.3.8}$$

Nahavandi then plotted Equation Va.3.8 for mass flux as a function of the stagnation enthalpy and pressure, as shown in Figure Va.3.1.
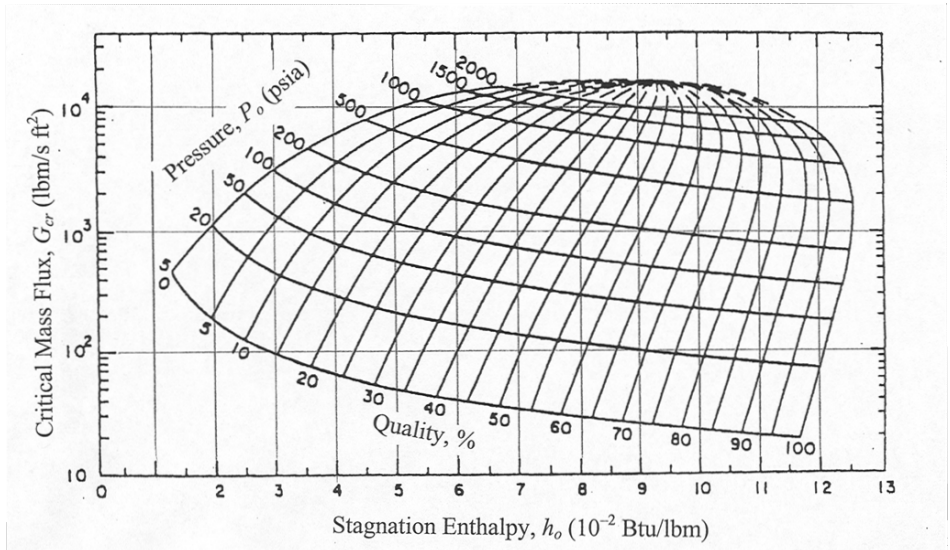
Figure Va.3.1. Critical mass flux versus stagnation enthalpy and pressure (Fauske)

Two observations can be made from Figure Va.3.1. First, the higher the source stagnation pressure, the higher the mass flux at the same steam quality. Second, for a given pressure, the critical mass flux increases with decreasing quality. Hence, as expected, the higher the liquid content, the higher the critical mass flux. For example, a stagnation pressure of 200 psia and $x = 75\%$ corresponds to the same mass flux of 1000 lbm/s·ft^2 as a stagnation pressure of only 50 psia but steam quality of $x = 11\%$.

Example Va.3.2. Calculate the maximum mass flux for the flow of saturated water and steam at 2000 psia and enthalpy of 800 Btu/lbm, according to the Fauske model.

Solution: From Figure Va.3.1 for P_o = 2000 psia and h_o = 800 Btu/lbm, we find $G_{max} \approx 11,000$ lbm/s·ft^2.

Example Va.3.3. A discharge line is connected to a pressurized tank, which contains saturated water at 1000 psia. The discharge line is equipped with a safety valve, having a flow area of 1.4 in^2. We now open the valve. Find the maximum flow rate that leaves the tank. Assume $C_d = 1$.

Solution: Since the discharge valve opens to atmosphere, the flow is definitely choked. The maximum flow rate occurs at the moment that pressure is still at 1000 psia and just begins to drop. Thus from Figure Va.3.1 we find the mass flux as G_{cr} = 10,100 lbm/s·ft^2 and the mass flow rate as:

$\dot{m} = 0.8(1.4/144) \times 10,100 = 78.5$ lbm/s

As for the Moody model, if we substitute $S = (v_g/v_f)^{1/3}$ into Equation Va.3.5 and then use the result in Equation Va.3.4, we find the following relation for the critical mass flux:

$$G^2 = \frac{2[h_o - xh_g - (1-x)h_f]}{v_g^2[x + (1-x)(v_f/v_g)^{2/3}]^3} \qquad \text{Va.3.9}$$

Similar to the Fauske mode, mass flux from the Moody model is maximized and plotted for various values of the source stagnation pressure and enthalpy, as shown in Figure Va.3.2.
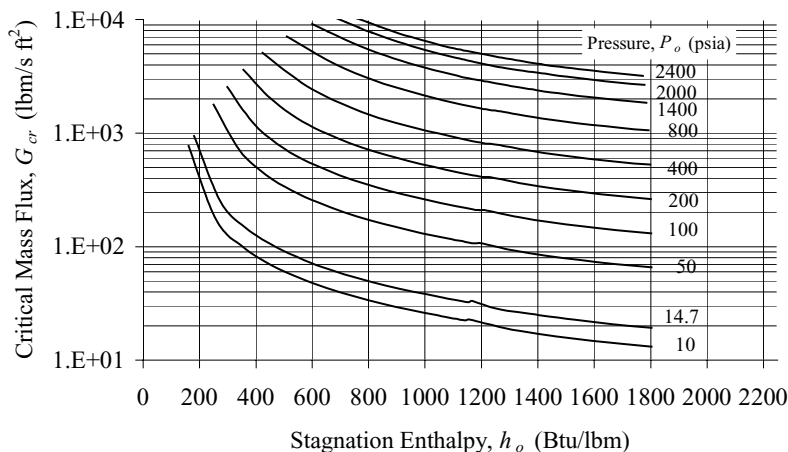

Figure Va.3.2. Critical mass flux versus stagnation enthalpy and pressure (Moody)

3.3. Two-Phase Critical Flow (Homogeneous Non-equilibrium Flow)

Earlier we showed that the existence of analytical solutions was primarily due to the isentropic process assumption. This assumption implies that the length and diameter of the flow path should be such that the frictional effects are minimized. Thermodynamic equilibrium in turn requires a reasonably long flow path to allow the phases to reach equilibrium. As such, the shorter the flow path, the higher the flow rate since less liquid would flash to steam. Fauske has identified three ranges for the L/D; 0–3, 3–12, and 12–40 (Figure Va.3.3). For $0 < L/D < 3$, the flow path is too short for the phases to reach equilibrium while for $12 < L/D < 40$, the flow path allows the phases to reach equilibrium. Therefore, for the L/D range of $0 – 3$, the critical flow can be estimated from such equations as IIIa.3.46, IIIb.3.14, IIIb.4.3, and IIIb.4.4:

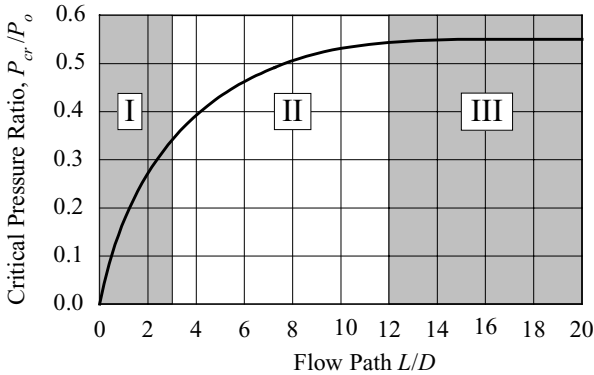$$G = 0.61\sqrt{2\rho_f(P_o - P_{cr})} \qquad \text{Va.3.10}$$

Figure Va.3.3. Critical pressure ratio versus flow path length over diameter (Fauske)

where in Equation Va.3.10, a discharge coefficient of 0.61 is accounted for. In this equation, P_{cr} stands for the critical pressure. According to Fauske, the value of P_{cr} depends on the value of the L/D ratio as shown in Figure Va.3.2. Note that, for an orifice, ($L/D = 0$), P_{cr} is the actual back pressure.

In conclusion, Region I in Figure Va.3.3 is applicable to non-equilibrium flow regimes and Region III of this figure is well suited for the HEM, since the sufficient flow path length allows the phases to reach thermal equilibrium. Note that Figure Va.3.1 would under-predict flow in Regions I and II of Figure Va.3.3.

Example Va.3.4. A pressurized tank containing saturated liquid at 2000 psia is connected to atmosphere by a 0.5 in diameter pipe. A frictionless valve on the pipe is suddenly opened. Find the maximum mass flux for three different pipe lengths of $L_1 = 0.1$ ft, $L_2 = 0.25$ ft, and $L_3 = 1$ ft at this pressure.
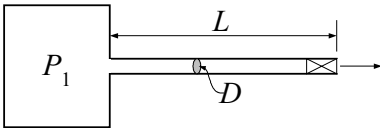
Solution:

Find $G = 0.61\sqrt{2 \times 32.2 \times 38.98 \times 144 (P_o - P_{cr})} = 366.78\sqrt{(P_o - P_{cr})}$

Next, we need to find $P_o - P_{cr}$, having L/D. Since $D = 0.5/12 = 0.042$ ft
$L_1/D = 0.1/0.042 = 2.4$, $L_2/D = 6$, and $L_3/D = 24$.

We now use Figure Va.3.3:
For $L_1/D = 2.4$, we find $P_{cr}/P_o = 0.30$. Thus,

$G = 366.78\sqrt{(2000 - 600)} = 13,720$ lbm/s·ft².

This is in good agreement with G from Figure Va.3.1 at
$P_o = 600$ psia and $h_o = h_f(2000) = 672$ Btu/lbm.
For $L_2/D = 6.0$, we find $P_{cr}/P_o = 0.48$. Thus,

$$G = 366.78\sqrt{(2000 - 960)} = 11{,}830 \ \text{lbm/s·ft}^2$$

For $L_3/D = 24$, we find $P_{cr}/P_o = 0.55$.

Thus, $G = 366.78\sqrt{(2000 - 1100)} = 11{,}000 \ \text{lbm/s·ft}^2$

As expected, G_{cr} is over-predicted compared to $G_{cr} \approx 8500 \ \text{lbm/s·ft}^2$ read from Figure Va.3.1 at $P_o = 1100$ psia and $h_o = h_f(2000 \ \text{psia}) = 672$ Btu/lbm.

Henry (1970) and later Henry-Fauske (1971) analytically derived the relation for two-phase critical mass flux for homogenous non-equilibrium flow, based on the isentropic assumption. This is a reasonable assumption for short flow paths for which the frictional pressure drop due to the wall shear forces is negligible compared to the momentum and pressure gradient terms. The RELAP4 (Moore) and GOTHIC (George) computer codes have tabulated the Henry correlation for various stagnation pressure and enthalpy. These are plotted in Figure Va.3.4.
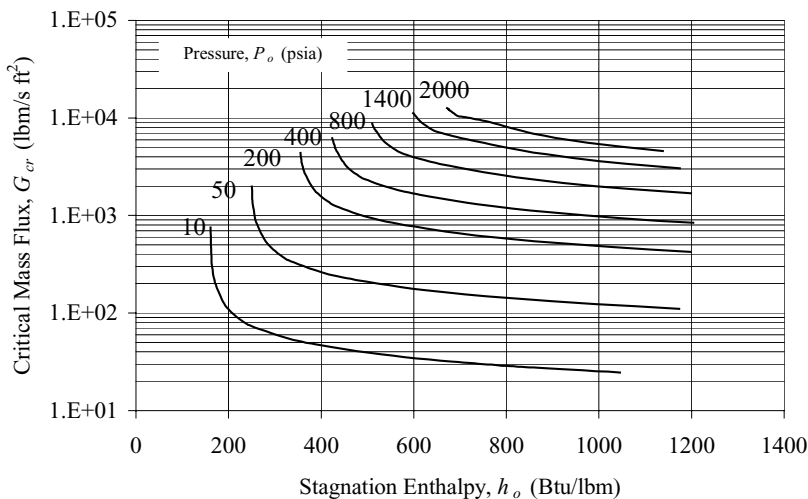

Figure Va.3.4. Critical mass flux versus stagnation enthalpy and pressure (Henry non-equilibrium model)

Example Va.3.5. In Example Va.3.5, find the maximum mass flux for $L_1 = 0.1$ ft.

Solution: From Figure Va.3.4 for $P_o = 2000$ psia and saturated water, we find $G_{max} \approx 10{,}500 \ \text{lbm/s ft}^2$.

3.4. Two-Phase Critical Flow
(Homogeneous Non-equilibrium, Subcooled Fluid)

So far we dealt with saturated water and two-phase mixture. If the pressurized water is subcooled, then the percentage of flashing decreases, resulting in higher mass flow rate. Indeed test data indicates that the higher the degree of subcooling, the higher the critical mass flow rate. The Henry-Fauske correlation is extended to cover subcooled liquid. The extended Henry-Fauske correlation for maximum mass flux is plotted as a function of stagnation enthalpy and pressure in Figure Va.3.5.
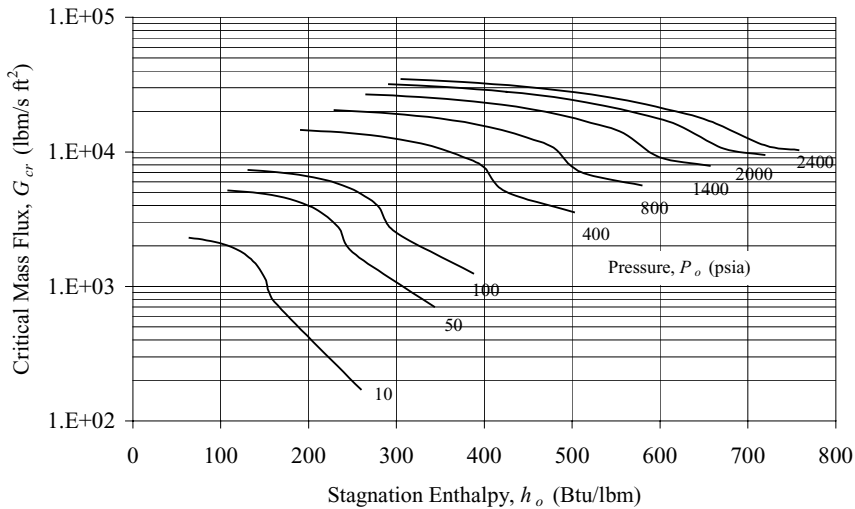

Figure Va.3.5. Critical mass flux versus stagnation enthalpy and pressure (Henry-Fauske model)

Example Va.3.6. A pressurized tank containing subcooled liquid at 2000 psia is connected to atmosphere by a frictionless valve. Find the maximum mass flux for two cases of $h_1 = 400$ Btu/lbm and $h_2 = 600$ Btu/lbm.

Solution: We expect to obtain higher mass flux at lower upstream enthalpy. According to Figure Va.3.5, at a upstream pressure of 2000 psia and enthalpy of 400, the critical mass flux is about 12,000 lbm/s. At the same pressure but enthalpy of 600 Btu/lbm, the critical mass flux is about 10,800 lbm/s.

QUESTIONS

– When does flow quality become equal to the thermodynamic quality?
– What is the slip ratio?
– What is superficial velocity?
– What does the slip ratio signify? What is its value for homogenous flow?
– Does the Zivi model predict data well? Why or why not?
– What is the physical significance of the drift flux model?
– Comparing drift flux and the relative velocity between phases, which one is always larger than the other?
– What is the advantage of the drift flux model compared to other void fraction models?
– Is the drift flux model suited for the annular flow regime?
– What is the purpose of defining a two-phase friction multiplier?
– What is the advantage of Reddy's two-phase friction multiplier?
– Regarding friction pressure drop, what is the distinction between two-phase flow in a heated pipe and in a pipe being cooled down?
– The acceleration pressure gradient, per Equation Va.2.5, is a perfect differential. Hence, it depends only on the end points. Why, then, do we carry out the derivative per Equations Va.2.5 and Va.2.6?
– Why does the critical mass flow rate increase with decreasing steam quality?
– For the exact same conditions, which model predicts higher mass flow rate, the Moody or the Fauske model?
– What is the major difference between the HEM, the Moody model, and the Fasuke model?
– Are water and steam more likely to reach equilibrium in a shorter flow path or a longer flow path?
– How do we know if a two-phase flow is choked in a given flow path?

PROBLEMS

1. Consider the annular flow regime in a tube. If the thickness of the liquid film on the tube wall is much smaller than the tube diameter ($\delta \ll D$), show that the void fraction is given by $\alpha = 1 - (4d/D)$.

2. In this problem you are asked to investigate the effect of system pressure and steam quality on void fraction. For this purpose, use Equation Va.1.1, where $S = 1$, and produce a graph of $\alpha = f(x)$. You need to first choose a pressure and plot void fraction as a function of quality, ranging from 0% to 20%. Repeat this for another value for pressure. Choose $P = 14.7$ psia, 500 psia, 1000 psia, 2000 psia, and 3,206 psia.
Outline your observations.

3. In this problem you are asked to investigate the effect of slip ratio and steam quality on void fraction. For this purpose, use Equation Va.1.1 where $P = 1000$

psia, and produce a graph of $\alpha = f(x)$. You need to first choose a value for void fraction. Try $S = 1, 2, 3,$ and 4. Outline your observations.

4. Plot the density ratio ρ_f / ρ_g for the two-phase flow of water - air and water - steam versus pressure, ranging from 1 psia to 3200 psia. For water - air flow, use a temperature of 80 F (27 C). Outline your observation.

5. A tank is filled with a homogenous mixture of water and steam at thermal equilibrium. The tank pressure is 6.89 MPa. The liquid volume fraction in the tank is 40%. Find the mixture density and static quality. [Ans.: 318 kg/m^3 and 0.07].

6. Use the definition of the mixture density and show that for homogenous flow $(S = 1)$, $v = (1 - x)v_f + xv_g$.

7. Liquid enters a heated tube at velocity V_f. Derive a relation for the superficial velocity in terms of V_f, X, v_f, and v_g. [Hint: Use $j = [(1 - X)/\rho_f + X/\rho_g]G$ and substitute for ρ_f, ρ_g, and G]. [Ans.: $j = \{1 + (v_g/v_f - 1)X\}V_f$].

8. A BWR plant is operating at steady state, producing 1260 kg/s (1E7 lbm/h) dry, saturated steam for the turbine. Pressure in the vessel is 7 MPa (1000 psia). Feedwater enters the vessel at 150 C (~ 300 F). The average void fraction at the exit of the core is 40%. Assuming a uniform slip ratio of 2 throughout the core, find the recirculation ratio and total rate of heat transfer in the core. [Ans.: 15.476, 2703 MWth].

9. Two-phase mixture flows at a rate of 750 kg/m^2 s in a vertical tube of diameter 3 cm at 70 bar. Find the flow pattern at a cross section where $X = 10\%$.

10. Use Figure Va.1.1 and find the most likely flow pattern for the following six cases:

| Case	G (kg/s m^2)	P (bar)	x	Case	G (kg/s m^2)	P (bar)	x
1	600	35	0.01	4	2600	35	0.01
2	600	75	0.10	5	2600	75	0.10
3	600	180	0.50	6	2600	180	0.50

11. Estimate the slip ratio for a two-phase mixture flowing in a channel. The channel pressure is maintained at 1200 psia and at thermal equilibrium.
[Ans.: 1.6].

12. A heated channel is operating at 7.5 MPa. Water enters the channel at a rate of 17 kg/s and an inlet subcooling of 18 C. Find the maximum rate of heat transfer that can be transferred to this channel while the void fraction at the channel exit is maintained at 0.8. Ignore pressure drop in the channel.

13. A mixture of water and steam is flowing up a 25 mm diameter channel at a rate of 4500 kg/s m^2 and temperature of 295 C. Assume thermal equilibrium and find the following items at an elevation where $x = 25\%$: a) void fraction, b) the mixture mixing cup density, c) mixture density using the HEM, d) mixture thermodynamic density.

14. Steam and water flow in a 25 mm tube at 300 C, 3500 kg/m^2 s, and $X = 0.4$. Use the drift flux model and find α and S. [Ans. 0.68 and 4.7]

15. An element of volume for the separated up-flow of water and steam in a channel is shown in the figure. Use the conservation equation of momentum at steady-state and directly derive Equation Va.2.4.
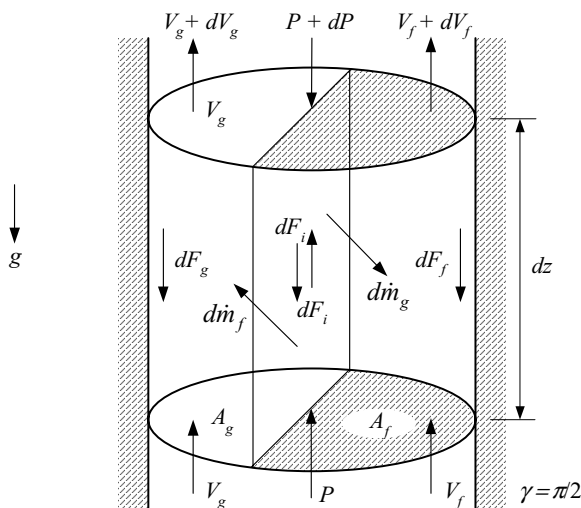
16. Use $(dP/ds)_{acc}$ according to the HEM and derive ΔP_{acc} for two-phase flow in a uniformly heated channel of length L. [Ans.: $(gL\cos)\ln(1 + Xv_{fg}/v_f)/(Xv_{fg})$].

17. Derive ΔP_{acc} by applying the momentum equation between inlet and outlet. [Hint: $F = \Delta$(momentum flux). Use $F = A\Delta P$ and substitute for momentum out = $\dot{m}_f V_f + \dot{m}_g V_g$ and for momentum in = $\dot{m}_i V_i$. Then replace each velocity by mass flow rate divided by ρA and introduce α and x to replace area and mass ratios].

18. In order to integrate the differential pressure drop, we ignored the compressibility of the gaseous phase, (v_g/ P). Show the reasonableness of this assumption. Find the magnitude of this term at an operating pressure of 1000 psia (82.75 bar). [Ans.: $-7.57E-8$ ft$^4 \cdot$s^2/lbm^2].

19. Consider two-phase flow in a tube of diameter 0.5 in and length 3 ft. For $P = 1000$ psia, $\dot{m} = 0.35$ lbm/s and $X = 50\%$ compare ϕ_{HEM} with ϕ_{Reddy}. [Ans.: $\phi_{HEM} = 10.83$. $\phi_{Reddy} = 12.73$].

20. Consider flow of water in two identical pipes of $D_e = 5$ cm and $L = 3$ m. One pipe is insulated while the other is heated so that $X_o = 0.2$. In both pipes, $P = 100$ bar and $G = 0.3$ kg/s. Compare $(\Delta P_1)_{fric}$ with $(\Delta P_2)_{fric}$.

21. Use the data of the above problem and compare $(\Delta P_1)_{grav}$ with $(\Delta P_2)_{grav}$ if $\beta_1 = 0$ and $\beta_2 = 30°$.

22. For the flow of water and steam in a vertical insulated channel, calculate total pressure drop in the channel.
Data: channel diameter: 1 cm, channel length: 3 m, mass flow rate: 0.3 kg/s, pressure: 7.4 MPa, steam quality at inlet: 0.025. [Ans.: 77.7 kPa].

23. Find the total pressure drop for flow of water in a heated vertical tube. The diameter and the length of the tube are 12.7 mm and 1.55 m, respectively. The tube is heated uniformly at a rate of 39 kW. Water enters the tube at a rate of 0.0882 kg/s, temperature of 277 C and pressure of 70 bar. [Ans.: 10.6 kPa].

24. Water enters a uniformly heated vertical tube of diameter 2.5 cm and length 4.5 m. Total heat applied to the tube is 650 kW. Inlet pressure, temperature, and mass flow rate of water are given as 100 bar, 285 C, and 1.5 kg/s, respectively. Find the pre-heating length, exit quality, and various pressure differentials terms. [Ans.: $L_f = 1.52$ m, $X_o = 0.22$, $(\Delta P)_{fric,sp} = 5$ kPa, $(\Delta P)_{fric,tp} = 21.73$ kPa, $(\Delta P)_{acc,sp} = 1.05$ kPa, $(\Delta P)_{acc,tp} = 34.17$ kPa, $(\Delta P)_{grav,sp} = 10.7$ kPa, $(\Delta P)_{grav,tp} = 10.05$ kPa, $(\Delta P)_{total} = 82.7$ kPa. All based on HEM]

25. Water enters a uniformly heated vertical tube of diameter 3 cm and length 5 m. Total heat applied to the tube is 1250 kW. Inlet pressure, temperature, and mass flow rate of water are given as 100 bar, 300 C, and 3 kg/s, respectively. Find the pre-heating length, exit quality, and various pressure differential terms. [Ans.: $L_f = 0.72$ m, $X_o = 0.29$, $(\Delta P)_{fric,sp} = 3.38$ kPa, $(\Delta P)_{fric,tp} = 54.61$ kPa, $(\Delta P)_{acc,sp} = 0.905$ kPa, $(\Delta P)_{acc,tp} = 89.72$ kPa, $(\Delta P)_{grav,sp} = 4.93$ kPa, $(\Delta P)_{grav,tp} = 12.57$ kPa, $(\Delta P)_{total} = 166.12$ kPa. All based on HEM]

26. In a certain adiabatic air – water flow experiment, it is desired to create the same pressure gradient in two vertical channels having hydraulic diameters D_1 and D_2, respectively. Given the conditions for channel 1 (i.e. the total mass flow rate $\dot{m}_1$ and steam quality x_1), determine the corresponding values in channel 2 so that
a) the two channels have almost the same void fraction, $\alpha_1 = \alpha_2$
b) the two channels have also the same liquid mass flow rate, $\dot{m}_1 = \dot{m}_2$ (in this case $\alpha_1 \neq \alpha_2$)
c) same as b but for horizontal channels.
   Pressure, temperature, and surface roughness are the same for the both channels. Assume single-phase friction factor is only a function of the Reynolds number, $f_{sp} = f(Re)$.

27. Water enters a uniformly heated vertical tube of diameter 2 in and length 15 ft. Total heat applied to the tube is 100 kW. Inlet pressure, temperature, and mass flow rate of water are given as 1000 psia, 500 F, and 0.5 lbm/s, respectively. Find the pre-heating length, exit quality, and various pressure differential terms.

[Ans.: L_f = 4.32 ft, X_o = 0.21, $(\Delta P)_{fric,sp}$ = 0.001 psia, $(\Delta P)_{fric,tp}$ = 0.005 psia, $(\Delta P)_{acc,sp} \approx 0$ psia, $(\Delta P)_{acc,tp}$ = 0.01 psia, $(\Delta P)_{grav,sp}$ = 1.43 psia, $(\Delta P)_{grav,tp}$ = 1.36 psia, $(\Delta P)_{total}$ = 2.81 psia. All based on HEM]

28. A mixture of water and steam flowing out of a vent at 2000 psia and 798 Btu/lbm. Find quality and the void fraction at critical flow condition according to Moody and Fauske models. [Ans. x = 0.27, α_{Moody} = 0.42, and α_{Fauske} = 0.50].

29. A mixture of water and steam flowing out of a vent at 200 psia and quality of 0.27. Find the void fraction at critical flow condition according to Moody and Fauske models. [Ans. α_{Moody} = 0.65 and α_{Fauske} = 0.81].

30. In Example Va.3.1, show that mass flux can be expressed as:

$$\frac{-1}{G^2} = \left\{ \left[\frac{2(1-x)v_f}{1-\alpha} - \frac{2xv_g}{\alpha} \right] \frac{dx}{dP} + \left[\frac{(1-x)^2}{1-\alpha} \frac{dv_f}{dP} + \frac{x^2}{\alpha} \frac{dv_g}{dP} \right] + \left[\frac{(1-x)^2 v_f}{(1-\alpha)^2} - \frac{x^2 v_g}{\alpha^2} \right] \frac{d\alpha}{dP} \right\}_s$$

31. Water flows in a vertical channel. Use the given data to find the frictional pressure drop from the inlet to the exit of the channel. Compare the calculated values for $\Delta P_{friction}$ by using the homogenous, the Reddy, the Friedel, and the Thom model. Data: H = 6 ft, D_h = 0.145 ft, P = 1000 psia, X_e = 8%, V_i = 3 ft/s, and T_i = 522 F.

32. Find the maximum flow rate for a mixture of water and steam flowing at 500 psia and x = 50%. What is the stagnation enthalpy for this mixture?

33. Derive an exact formulation for dP_{acc} for homogeneous, one-dimensional, steady, two-phase flow in a constant area channel assuming thermodynamic equilibrium. [Hint: Start with Equation Va.2.5 but written as $dP_{acc} = G^2 dv$, substitute for v = $v_f + x\, v_{fg}$, for x from $h = h_f + xh_{fg}$, and for h from the energy equation:

$$\dot{q}''(\pi Dds) = \dot{m}(dh + dV^2/2)$$

[Ans.: $(dP/ds)_{acc} = 4\,(G/D)\dot{q}''(v_{fg}/h_{fg})$].

34. A pressurized tank contains saturated water. A frictionless valve on the pipe is now opened to discharge the tank inventory to the atmosphere. Find if at the given pressure the flow rate is choked.
Data: P_1 = 200 psia, L = 4 ft, D = 2 in.
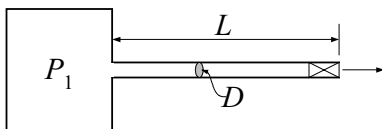
Vb. Boiling

Bubbles may be induced in a liquid in two ways. First, by sudden depressurization of the liquid referred to as flashing. For example, bubbles appear in a pressurized water tank if the sudden opening of a pressure relief valve results in the tank pressure dropping below the liquid saturation pressure. Similarly, if the flow of a liquid is accelerated or the flow encounters valve or fittings, vapor bubbles may appear as the result of the associated pressure drop. The second way is by addition of heat to a liquid at constant pressure. Boiling generally refers to the evaporation from the interface between a liquid and a heated surface. In boiling, liquid temperature remains practically constant and slightly higher than the saturation temperature at system pressure.

Knowledge about boiling heat transfer is essential in cases where we need a liquid to boil to enhance the rate of heat transfer and in such other cases in which preventing boiling is required by design. Boiling is the fundamental mode of heat transfer in the steam generator of a PWR, in the core of a BWR, and in the boiler of a fossil power plant. On the other hand PWRs must be operated to prevent boiling in the core. Boiling is also important to the cooling of rocket combustion chamber and electronic cooling. An interesting aspect of boiling heat transfer is the method that heat from a heated surface is transferred to the liquid (i.e., whether by carefully controlling the heated surface temperature or by controlling heat flux). The latter could be associated with a phenomenon referred to as boiling crises or critical heat flux. Boiling is the most desirable mode of heat transfer when a very high heat transfer coefficient (on the order of 1 MBtu/ft^2·h·F or higher) is required. Boiling heat transfer consists of various modes as are discussed in this chapter.

1. Definition of Boiling Heat Transfer Terms

Nucleation refers to the inception of the embryonic bubble.

Homogeneous nucleation (also known as spontaneous vaporization) refers to the appearance of bubbles in the bulk of a liquid without any need for a heated surface. Such nucleation occurs only if the liquid is contained in a mirror-finished container and very high degree of superheat is provided. That is to say, the liquid temperature should be increased by hundreds of degrees beyond the saturation temperature for homogeneous boiling to take place. Sufficiently superheated liquids in containers with very smooth surfaces would vigorously produce bubbles if nucleation sites (rough surfaces) are introduced into the liquid.

Heterogeneous nucleation refers to the appearance of boiling bubbles, as a result of liquid vaporization at the interface with a heated surface. This is the most common form of boiling. In Section 2, we demonstrate that boiling in a superheated liquid subject to the introduction of a rough surface is distinctly different than boiling in a liquid with a heated rough surface.

Local or subcooled boiling refers to heterogeneous nucleation occurring at a specific region in a liquid with the bulk liquid being subcooled. In local boiling, if bubbles grow and detach from the surface, they would soon collapse upon reaching the colder liquid. The energy contained in the collapsed bubble is transferred to the liquid, raising the liquid average temperature.

Bulk or saturation boiling starts following subcooled boiling after a sufficient number of bubbles have burst in the liquid, transferring their energy to and raising the subcooled liquid temperature to saturation.

Nucleate boiling refers to a specific boiling mode where bubbles appear as a result of the nucleation process on the heated surface. Initiation of nucleate boiling strongly depends on surface roughness.

Film boiling refers to vaporization of liquid while the heated surface is covered by a film of vapor. Film boiling takes place at elevated surface temperatures. The heat for liquid vaporization is transferred to the liquid by conduction through the film of vapor. Hence, surface roughness has no effect on film boiling.

Pool boiling refers to boiling in a quiescent liquid. The only liquid flow in pool boiling is due to free convection and mixing as a result of bubble departure from the heated surface.

Flow boiling, as is the case in the core of BWRs and PWR steam generators takes place with bulk liquid in motion. Liquid flow is due to external forces as well as free convection and bubble-induced mixing.

Bubble equilibrium is a result of three types of equilibrium. Consider an isolated bubble in the bulk of a liquid, Figure Vb.1.1(a). For this bubble to remain intact (i.e., neither grow nor collapse) three conditions must be met. These are mechanical equilibrium, thermal equilibrium, and equal chemical potentials.

Mechanical equilibrium requires the algebraic summation of all the forces applied to the bubble to be zero, as shown in Figures Vb.1.1(a).

ΣF = internal pressure force + external pressure force + surface tension force = 0

Substituting for the three forces, we find:

$$(\pi r_e^2 P_v) + (\pi r_e^2 P_l) + (2\pi r_e \sigma) = 0$$

where r_e is the radius of the bubble at equilibrium. This equation simplifies to $P_v - P_l = 2\sigma/r_e$. Since $P_v > P_l$ but $T_v = T_l$, as shown in Figure Vb.1.1(b), liquid surrounding the bubble must be superheated. If there are gases dissolved in the liquid, then r_e is given by $r_e = 2\sigma/[(P_v + P_a) - P_l]$ where P_a is the gas pressure.

Thermal equilibrium requires the temperature of the vapor and liquid to be equal, $T_v = T_l$, Figure Vb.1.1(b). Otherwise, a combination of heat and mass transfer processes would occur to establish thermal equilibrium at a larger bubble size or cause the bubble to collapse, depending on the magnitude of T_v and T_l.
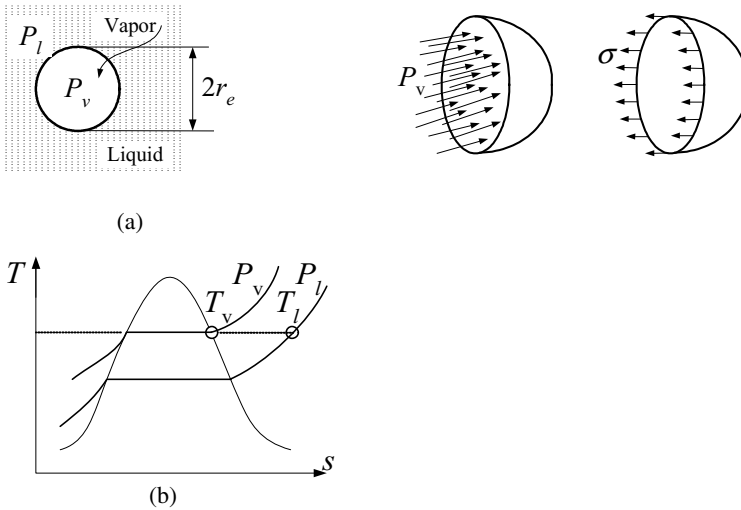
(a)

(b)

Figure Vb.1.1. Applied forces on a vapor bubble in equilibrium

Equal chemical potential $(g_v = g_l)$ is the third condition for the bubble to be in equilibrium. Substituting for the Gibbs function we get $(h - Ts)_v = (h - Ts)_l$. For discussion on equal chemical potential see Section 2.2.

Surface roughness has a profound effect on nucleate boiling initiation. Unless a surface is mirror finished, surfaces consist of tiny pits and scratches also referred to as cavities. Most cavities on metal surfaces may be considered conical as shown in Figure Vb.1.2. Bubble nucleation generally begins at the cavities, referred to as the nucleation sites. When a wall is first wetted, nuclei of size 2.5–7.5 μm are generally present.
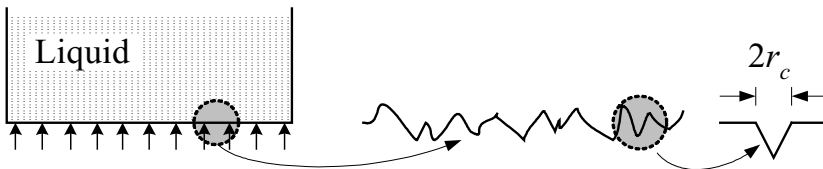

Figure Vb.1.2. Surface roughness acting as cavities or nucleation sites for bubble nucleation

Contact angle is a measure of the wet-ability of a liquid. Wetting itself is defined as the ability of liquids to form a boundary surface with solids. Figure Vb.1.3 shows a liquid in a capillary tube and in a surface cavity. Due to the action of surface tension, a wetting liquid in a capillary tube has a surface with a contact angle of $\phi < 90^o$ where ϕ is measured in the liquid. Non-wetting liquids
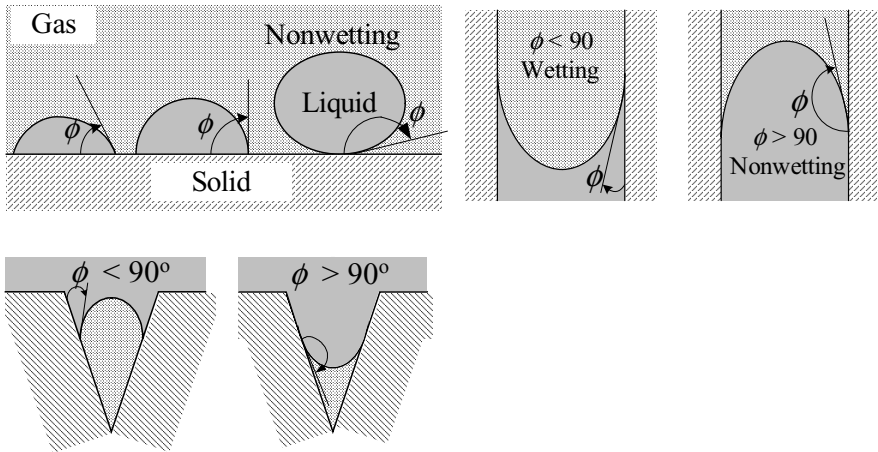
Figure Vb.1.3. Contact angle for liquid on horizontal surfaces, in capillary tubes, and in cavities

have convex surfaces and an angle of contact $\phi > 90^\circ$. Surface treatment affects liquid wet-ability. Wetting liquids fill surface cavities, preventing nucleation.

Bubble growth is the appearance and growth of a vapor bubble from cavities in a surface. Figure Vb.1.4(a) shows a surface cavity or nucleation site. Figure Vb.1.4(b) shows the inverse of the bubble radius ($1/r_b$) versus the bubble volume (V_b). The energy transferred from the heated surface to the trapped gas or vapor in this cavity (stage *A*) causes it to grow. As vapor volume increases, its radius (r_b) decreases. Stage B shows the moment that the bubble has reached at the mouth of the cavity. At this point, the bubble volume keeps increasing while the radius keeps decreasing. The minimum radius is reached when the bubble radius becomes equal to the radius of the cavity $r_b = r_c$ (stage *C*). Growth of the bubble beyond this point depends on the degree of superheat of the liquid. If sufficient superheat exists, bubbles eventually leave the nucleation sites towards the bulk. These stages are known as waiting period, growth period, agitation or displacement of liquid in the thermal sub-layer period, and departure (or collapse) period, as shown in Figure Vb.1.4(c) through Vb.1.4(f). As shown in Section 2, $1/r_b$ is proportional to the liquid superheat, $T_l - T_{sat}$.

Returning to Figure Vb.1.4(a), this discussion shows that the radius of the mouth of the cavity (r_c) determines the amount of superheat required for the vapor bubble to nucleate at that site (see Section 2.2).
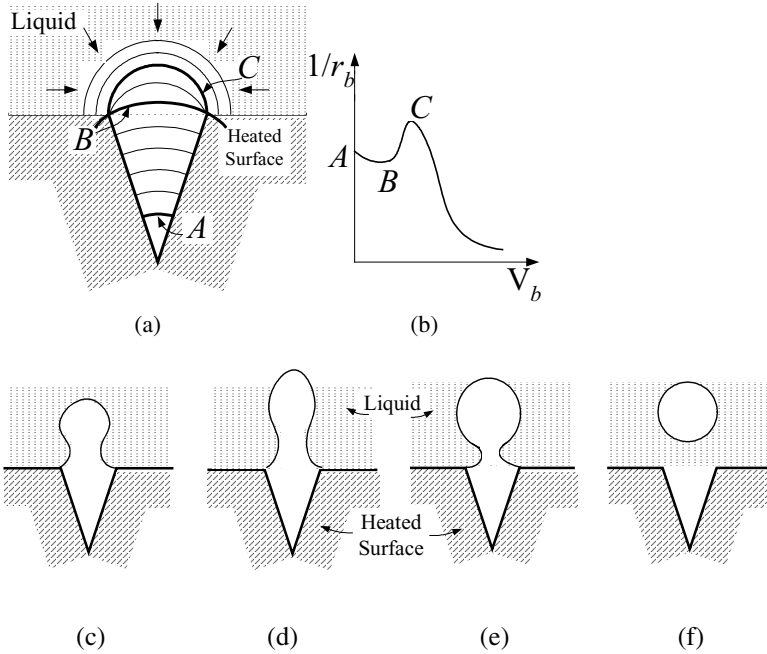
Figure Vb.1.4. Appearance and growth of a vapor bubble on a rough surface from conical cavities

Departure diameter, refers to the diameter of a bubble at the moment the bubble leaves the heated surface. We may estimate the bubble departure diameter from a force balance between buoyancy and surface tension:

$$\pi D \sigma = \frac{\pi D^3}{3}(\rho_f - \rho_g)g$$

Solving for the bubble departure diameter, we find:

$$D_d = [3\frac{\sigma}{g(\rho_f - \rho_g)}]^{1/2} = 1.73[\frac{\sigma}{g(\rho_f - \rho_g)}]^{1/2} \qquad \text{Vb.1.1}$$

2. Convective Boiling, Analytical Solutions

Certain aspects of boiling heat transfer are amenable to analytical solutions. However, due to the inherent complications associated with the boiling mechanisms, there is no general analytical solution for derivation of such an important parameter as the heat transfer coefficient, for example. Here we discuss few aspects of the boiling mechanism.

2.1. Dimensionless Groups

Practical aspects of boiling heat transfer are based on experimental data. To correlate such data we need to find the dominant factors in heat transfer associated with phase change. Such factors include the involved forces, key fluid properties, and the operational conditions. Viscous and buoyancy forces play a major role in heat transfer with phase change. Pertinent fluid properties include latent and specific heat (h_{fg}, c_p), density and thermal conductivity (ρ, k), and viscosity and surface tension (μ, σ). Finally, operational conditions include pressure, fluid and surface temperature, and the surface geometry (L). Since the effect of pressure appears in fluid properties, we can reduce the number of variables to 10. These are ρ_l, ρ_v, h_{fg}, c_p, k, μ, σ, ΔT, L and h. Incropera finds five dimensionless groups for these parameters; Nu = $hL/k = f[\rho g(\rho_l - \rho_v)L^3/\mu^2$, Ja, Pr, Bo] where the Bond number (Bo) is similar to the Gr number (Table A.I.6).

2.2. Determination of Degree of Superheat
for Equilibrium Bubble

Homogeneous nucleation: Earlier we noticed that for a bubble to be in equilibrium in the bulk of a liquid, three conditions must be met. To maintain equilibrium, we can use these conditions to find the minimum degree of superheat for a liquid; i.e. $T_l - T_{sat}$. From the requirement for equal chemical potential, we obtain the Clausius-Clapeyron equation $dP/dT = h_{fg}/(T_{sat}v_{fg})$. We integrate this equation assuming $T_{sat}v_{fg}/h_{fg}$ remains constant. We then substitute for $P_v - P_l = 2\sigma/r_e$ from the second requirement to find:

$$\Delta T = T_l - T_{sat} = \frac{T_{sat}v_{fg}}{h_{fg}}\Delta P = \frac{T_{sat}v_{fg}}{h_{fg}}\left(\frac{2\sigma}{r_e}\right) \cong \frac{2\sigma T_{sat}}{h_{fg}\rho_g r_e} \qquad \text{Vb.2.1}$$

where r_e is the radius of the equilibrium bubble. Equation Vb.2.1 shows that the degree of superheat is inversely proportional to the bubble radius. Thus, the smaller the bubble, the higher the required degree of superheat. That is why the homogenous nucleation requires very high degrees of superheat.

Heterogeneous nucleation: Regarding nucleation from a heated surface, we noticed that the minimum radius of a growing bubble is when $r_b = r_c$, where r_c is the radius of the cavity. Substituting into Equation Vb.2.1, we conclude that bubbles that have made it to the mouth of the cavity will grow if the degree of superheat in the liquid is at least equal to ΔT as given by Equation Vb.2.1. This is indeed the case if the bulk liquid is superheated. Superheating is achieved by heating the liquid in a pressurized vessel until liquid becomes saturated. When the heating process is terminated and the vessel is perfectly insulated, we reduce the liquid pressure. As the pressure drops, bubbles begin to form on the surface of the vessel when the liquid superheat becomes at least equal to that given by Equation Vb.2.1.
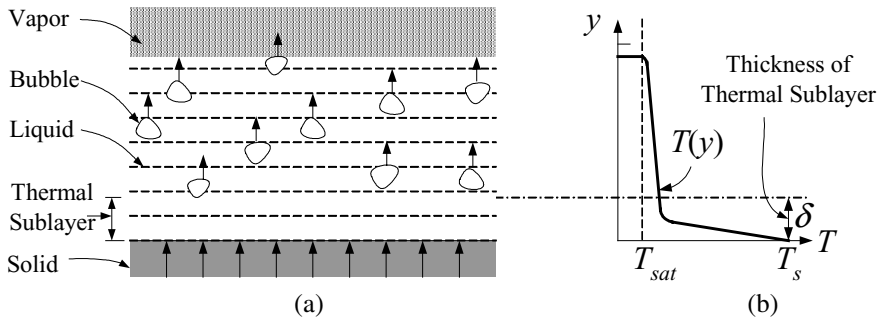
Figure Vb.2.1. Heterogeneous boiling and temperature gradient in liquid

It is fascinating to note that if boiling is induced solely by continuing to heat the vessel, the degree of superheat required for boiling is much higher than that predicted by Equation Vb.2.1. Indeed, some data have shown that the required degree of superheat for bubble growth is three times as much as that predicted by Equation Vb.2.1 (Hsu). The reason turns out to be the existence of a region (Figure Vb.2.1) referred to as the *thermal sub-layer*. Liquid temperature increases markedly in this region from T_{sat} to T_s, Figure Vb.2.1(b). It is in this thermal sublayer near the heated surface that liquid becomes superheated to provide sufficient heat for the bubble to grow and depart.

We may estimate the thickness of the thermal sub-layer by using the definition of heat transfer coefficient (h). Thus, the thickness of the thermal sub-layer is related to the liquid thermal conductivity as $\delta = k_f/h$ where h can be estimated from a correlation such as that of Fishenden for turbulent natural convection from a horizontal flat surface:

$$\mathrm{Nu} = 0.14(\mathrm{Gr}\,\mathrm{Pr})^{1/3}$$

where the Gr and Pr numbers are calculated for the liquid phase. Next, we focus on the heat transfer mechanism taking place in the thermal sub-layer.

Let's investigate the relation between the bubble equilibrium temperature (Equation Vb.2.1) and water temperature in the thermal sub-layer. Shown in Figure Vb.2.2 is a cavity of radius r_c on the heated surface.

Originally, both liquid and surface have the same temperature as the bulk liquid T_f (line *ZO*). We add heat to the surface and bring its temperature to T_A. The temperature in the thermal sub-layer is shown by line *ZA* where we have assumed a linear temperature profile in the thermal sub-layer. Since we do not observe any bubble in the liquid we increase the surface temperature to T_B with the liquid temperature shown by line *ZB*. As we heat up the surface, the bubble in the cavity begins to grow. We keep increasing the surface temperature until eventually the line representing temperature in the thermal sub-layer (line *ZC*) becomes tangential to the curve representing Equation Vb.2.1 for an equilibrium bubble. At this point, the bubble in the cavity has reached the mouth of the cavity and has the smallest radius of curvature. Hsu's condition for the bubble to grow is that the
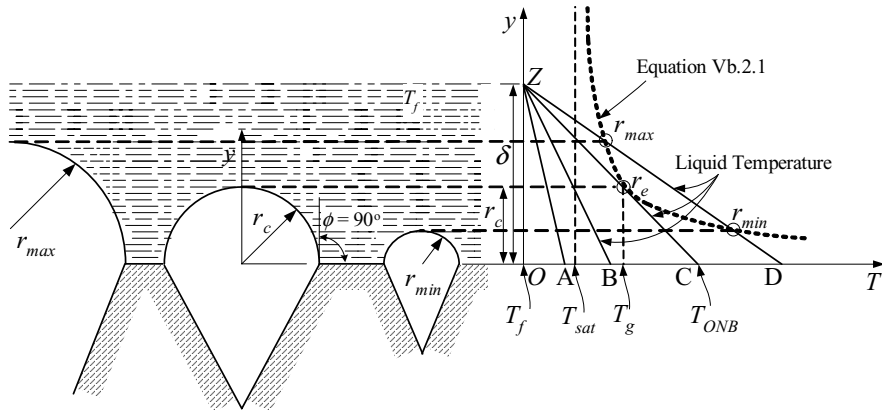
Figure Vb.2.2. Depiction of the onset of nucleate boiling (Hsu)

liquid temperature at $y = r_c$ must be at least equal to the bubble interior temperature for the bubble to grow. Thus, when the line for liquid temperature is tangent to the curve representing the equilibrium bubble (Equation Vb.2.1), cavities of radius r_c are active sites for nucleation. The corresponding temperature, is known as onset of nucleate boiling temperature (T_{ONB}).

If the surface temperature is further increased (say to T_D corresponding to line ZD for the thermal sub-layer), then at lower superheats the larger radii cavities and at higher superheats smaller radii cavities become active. But if a cavity of size r_c does not exist on the surface, the increasing surface temperature results in higher degrees of liquid superheat. This will continue until the temperature profile in liquid becomes tangent to the vapor temperature calculated from Equation Vb.2.1 for the cavity size that is present in the surface. When various cavity sizes exist and the bulk liquid is at saturation, we may approximate r_c as $r_c = \delta/2$. Using similar triangles, we find that $(\delta - r_c)/r_c = \Delta T_{sat}/\Delta T_{ONB}$ resulting in $\Delta T_{ONB} = 2\Delta T_{sat}$.

Example Vb.2.1. Find the degree of superheat ($T_s - T_{sat}$) for a horizontal flat plate in water at atmospheric pressure necessary to cause nucleation at all active sites. Data: At 1 atm, $\rho = 958$ kg/m^3, $\rho_g = 0.593$ kg/m^3, $h_{fg} = 2.257$E3 kJ/kg, $k_f = 0.68$ W/m·C, $\sigma = 0.059$ N/m, $\beta = 0.75$E-3 C^{-1}, $v = 0.292$E-6 m^2/s, Pr = 1.73.

Solution: We first find the thickness of the thermal sub-layer in terms of ΔT_s, the surface temperature minus the saturation temperature:

Gr $= \beta g \Delta T_s D^3/v^2 = [0.75\text{E-3} \times 9.81 \; \Delta T_s \; D^3/(0.292\text{E-6})^2] = 8.63\text{E10} \; \Delta T_s \; D^3$

$hD/k_f = 0.14\text{Gr}^{1/3}\text{Pr}^{1/3} = 0.14 \; [8.63\text{E10} \; \Delta T_s \; D^3]^{1/3}(1.73)^{1/3} = 743 \; (\Delta T_s)^{1/3} \; D$

Since $\delta = k_f/h$, we find:
$1/\delta = 743 \; (\Delta T_s)^{1/3}$. Therefore, $\delta = 1.35\text{E-3}(\Delta T_s)^{-1/3}$ m.   (1)

Next, we find the required surface superheat, $\Delta T_s = T_s - T_{sat}$ from Equation Vb.2.1:

$$\Delta T_s = 2\sigma T_{sat}/(h_{fg}\rho_g r_e) = 2 \times 0.059 \times (100 + 273)/(2.257\text{E}6 \times 0.593 \times r_b) = 3.3\text{E-}5/r_b \tag{2}$$

Substituting for $r_b = r_c = \delta/2$ into (2) yields: $\Delta T_s = 6.6\text{E-}5/\delta$. Substituting ΔT_s into (1) to find δ as:

$\delta = 1.35\text{E-}3[6.6\text{E-}5/\delta]^{-1/3}$. We, therefore, find $\delta = 6$ mm. Thus, $\Delta T_s = 0.019$ C.

Comment: It is seen when cavities of all sizes are present, the required degree of superheat is very small.
Let's assume only cavities of 8 μm exist. In this case, $r_c = 8\text{E-}6$ m and $(\Delta T_s)_{\text{required}} = 3.3\text{E-}3/8\text{E-}6 = 4$ C.

Heterogeneous nucleation formulation: We now want to quantify our qualitative argument regarding the vapor temperature and the thermal sub-layer temperature. For this purpose we find the equation for liquid temperature in the thermal sub-layer and set it equal to the vapor temperature in the bubble as given by Equation Vb.2.1. This method was originally suggested in 1962 by Hsu. Since the thermal sub-layer is thin, we use a linear temperature profile in this region which must satisfy the following boundary conditions:

At $y = 0$, $T(y = 0) = T_s$ and at $y = \delta$, $T(y = \delta) = T_f$

where T_s and T_f are the surface and the free stream temperatures of the bulk liquid, respectively. The profile is obtained as:

$$\frac{T_l - T_f}{T_s - T_f} = \frac{\delta - y}{\delta} \qquad\qquad \text{Vb.2.2}$$

We make a change of variable from y with r_c (See Figure Vb.2.3) to obtain:

$$y = c_1 r_c = (1 + \cos\phi)r_b \qquad\qquad \text{Vb.2.3}$$

We now set Equation Vb.2.1 equal to Equation Vb.2.2, while substituting for y from Equation Vb.2.3.
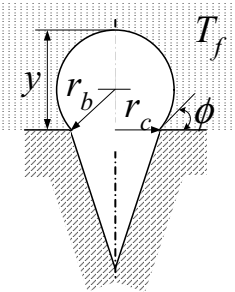

Figure Vb.2.3. Depiction of bubble height, radius, and cavity radius

Depending on the boiling condition, the curves representing the two temperature profiles may a) not meet, b) be tangent to each other, or c) intersect at two locations. These conditions are obtained from the solution to the following equation:

$$T_{sat} + \frac{2\sigma T_{sat}}{h_{fg}\rho_g c_2 r_c} = T_f + (T_s - T_f)\frac{\delta - c_1 r_c}{\delta}$$

This results in a second order algebraic equation for r_c. The solution is found as:

$$r_c = \frac{\delta(T_s - T_{sat})}{2c_1(T_s - T_f)}\left[1 \pm \sqrt{1 - \frac{8c_1}{c_2}\frac{(T_s - T_f)T_{sat}\sigma}{(T_s - T_{sat})^2\,\delta\rho_g h_{fg}}}\right] \qquad \text{Vb.2.4}$$
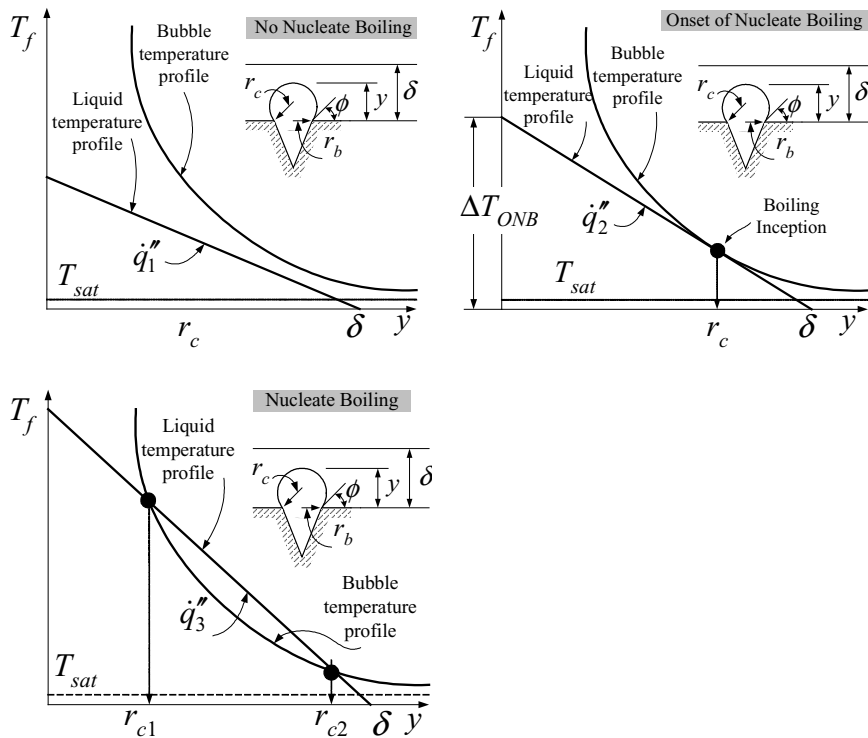
The results are plotted in Figure Vb.2.4.

Figure Vb.2.4. Comparison of the vapor bubble and liquid temperature profiles

These concepts are further developed in Chapter VIe.

2.3. Prediction of Bubble Growth

We can predict the growth rate of a vapor bubble rather accurately by treating the surrounding superheated liquid as a semi-infinite body. In this case, the specified boundary condition is heat flux at the interface between the liquid and the vapor bubble, as shown in Figure Vb.2.5. Since liquid is being cooled at the interface we can write the following energy balance:

Rate of increase in bubble internal energy = Rate of liquid cooldown at the interface

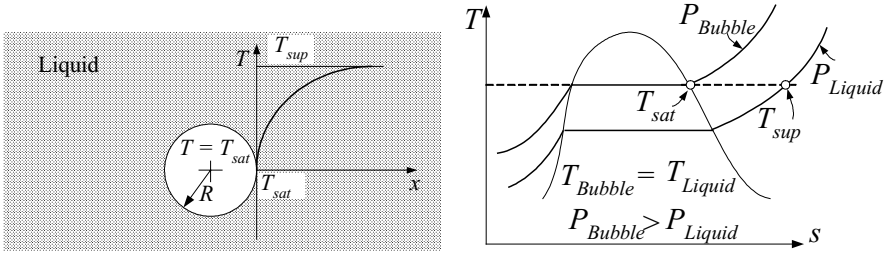$$\rho_g h_{fg} \frac{dV}{dt} = \left(4\pi R^2\right)\dot{q}''$$

Figure Vb.2.5. Growth of a vapor bubble in the pool of superheated liquid

Substituting for heat flux at the interface from Equation IVa.9.10 and for volume in terms of radius yields:

$$\rho_g h_{fg} \frac{d}{dt}\left(\frac{4}{3}\pi R^3\right) = \left(4\pi R^2\right)\frac{k(T_S - T_i)}{\sqrt{\pi\alpha\, t}}$$

Note in this case, the semi-infinite body is initially at $T_i = T_{sup}$ when the interface is suddenly cooled to $T_s = T_{sat}$. Subscripts *sup* and *sat* stand for superheated and saturated, respectively. Carrying out the derivative, canceling similar terms $(4\pi R^2)$ from both sides of the equation, and rearranging, we obtain:

$$\rho_g h_{fg}\, dR = \frac{k(T_S - T_i)}{\sqrt{\pi\alpha}}\frac{dt}{\sqrt{t}}$$

Using the initial condition of $R = 0$ at $t = 0$, we find:

$$R(t) = \frac{2}{\sqrt{\pi}}\frac{k\left(T_{sup} - T_{sat}\right)}{\rho_g h_{fg}\sqrt{\alpha}}t^{1/2} \qquad\qquad \text{Vb.2.5}$$

As Lienhard describes, Jakob initially suggested the method that led to the deriva-
tion of Equation Vb.2.5. As shown in Figure Vb.2.4, this equation under-predicts
the data obtained by Dergarabedian. Hence, Scriven used a more rigorous method
and found that $R_{bubble} = \sqrt{3}\, R_{Jakob}$, which closely matches the data.

Figure Vb.2.6 shows that the trend predicted by Jakob is as expected but the
absolute value under-predicts the data. The reason is that the bubble growth in-
creases the temperature gradient, which has been treated as constant in Jakob's
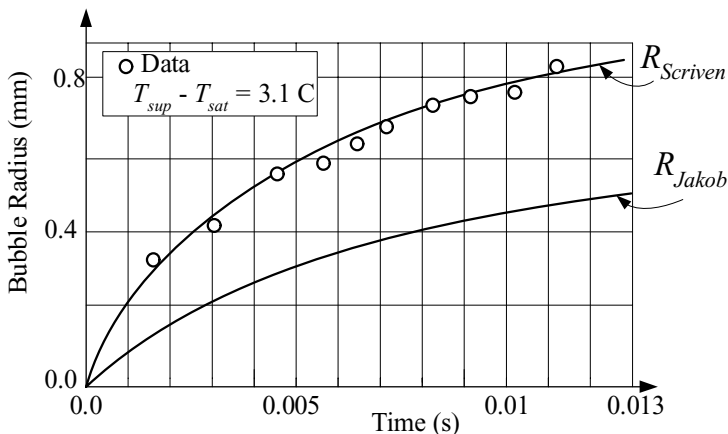model. Scriven accounts for this and practically matches the data.

Figure Vb.2.6. Comparison of R_{Jakob} and $R_{Scriven}$ with data

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Example Vb.2.2. Find the bubble diameter 0.01 s into the bubble growth for wa-
ter boiling at 1 atm and $\Delta T = 3$ C. Data: $k_f = 0.68$ W/m·K, $\rho_g = 0.593$ kg/m^3, $\alpha_f =$
1.68E-7 m^2/s, and $h_{fg} = 2.257$E6 J/kg.

Solution: From Equation Vb.2.5, we find $R(0.01$ s$)$ as:

$$R(t) = \frac{2}{\sqrt{\pi}} \frac{k\left(T_{sup} - T_{sat}\right)}{\rho_g h_{fg} \sqrt{\alpha}} t^{1/2} =$$

$$\frac{2}{\sqrt{\pi}} \frac{0.68 \times 3.1}{0.593 \times 2.257\text{E6}\sqrt{1.68\text{E}-7}} \sqrt{0.01} \times 1\text{E3} = 0.4336 \text{ mm}$$

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3. Convective, Boiling, Experimental Observation

Before discussing the two distinct modes of pool and flow boiling, we consider the
landmark experiment performed by Nukiyama in 1934, which led to the estab-

lishment of the boiling curve. The importance of this curve is in its clear depiction of various modes of heat transfer and demonstration of the effect of the method of heat addition to the liquid. This was the first experiment for the measurement of surface heat flux versus surface superheat ($\Delta T_{sl} = T_s - T_{sat}$). As shown in Figure Vb.3.1(a) the experiment consists of an electrically heated wire in a water container at atmospheric pressure. Nukiyama used a nichrome wire connected to an electric voltage. Data were obtained by varying the electric power measuring wire temperature after steady-state is achieved. This is referred to as power-controlled or heat flux controlled heating where $\dot{q}''_s$ is the independent variable and surface temperature (hence $\Delta T_{sl} = T_s - T_{sat}$) is the dependent variable. As power increased, there was a sudden jump in the wire temperature and eventual burnout. The heat up path is shown in Figure b with the arrows. The cool down path was obtained by reducing electric power to the wire as shown in Figure c by the arrows. As these figures indicate, on both heat up and cooldown paths, there is a jump from one side of the curve to the other. This is typical of power-controlled heat up and cooldown. Figure d shows how the entire boiling curve can be constructed if the process is temperature-controlled. In this case, there is a specific heat flux for a specific wall temperature. In practice, most processes such as production of heat in the core of nuclear reactors are power-controlled. As a results, in such applications, care must be exercised no to exceed the maximum heat flux as damage to the surface would follow.
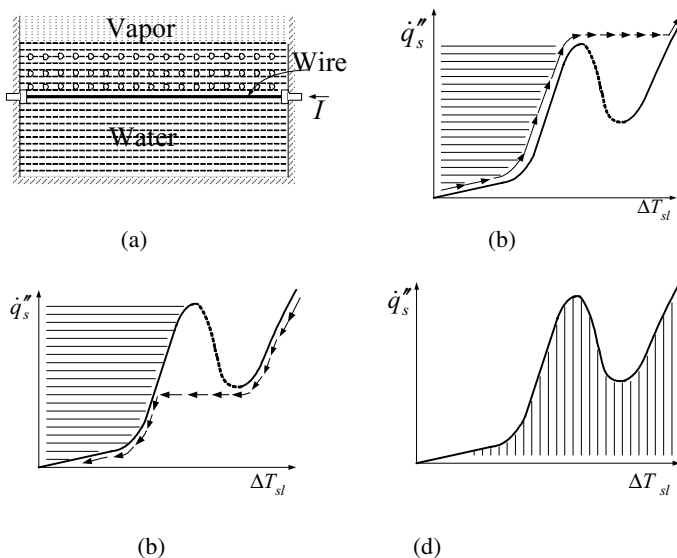

(a)

(b)

(b)

(d)

Figure Vb.3.1. Nukiyama experiment for developing the boiling curve

4. Pool Boiling Modes

The boiling curve for pool boiling heat transfer at atmospheric condition is shown in Figure Vb.4.1. Temperature is in degrees centigrade. Since the bulk liquid is quiescent, there is no heat flux when $\Delta T_{sl} = 0$. With an increasing degree of superheat, the surface heat flux increases solely due to free convection. At about $\Delta T_{sl} = 5$ C, the bubbles begin to grow and some may depart the surface. The buoyancy driven bubble causes agitation in the liquid. This mixing of liquid enhances heat flux. With increasing ΔT_{sl}, more bubbles are formed and the rate of carrying energy from the surface to the bulk liquid increases. Eventually, the rate of bubble production becomes so great that at ΔT_{sl} about 30 C, heat flux reaches its peak value. Beyond this point, the bubble population is so dense that it prevents liquid from reaching the surface. When this happens, heat transfer takes place only by conduction through the layer of vapor, which has blanketed the surface. With the surface being deprived of an efficient means of heat transfer by boiling bubbles, surface temperature jumps to elevated values. With heat flux maintained at its peak value, the jump in the surface temperature compensates for the sudden drop in the heat transfer coefficient. The heat transfer regime with vapor blanketing the surface is referred to as film boiling. The peak heat flux is referred to as the critical heat flux (CHF). A modest increase in heat flux beyond the CHF is due to both conduction through the vapor film and radiation due to the surface elevated temperature. On the cool down path, the reverse process occurs. When ΔT_{sl} reaches around 100 C, the vapor production is not vigorous enough to keep liquid away from the surface. With liquid in contact with the surface, the efficient heat transfer resumes. The point at which liquid contacts the surface again is known as the minimum stable film boiling (MSFB) or the Leidenfrost point. In 1756 Leidenfrost observed droplet boil off on hot surfaces. For surface temperature-controlled processes, the path between CHF and MSFB can be constructed. In this path, liquid and surface contact intermittently. This mode is known as *transition boiling*.
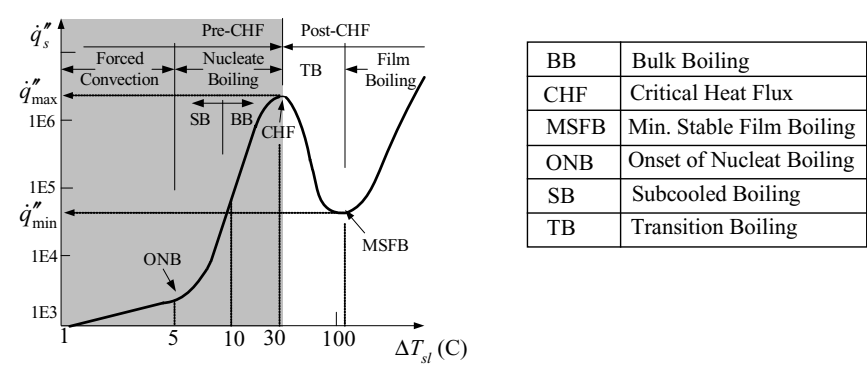

BB	Bulk Boiling
CHF	Critical Heat Flux
MSFB	Min. Stable Film Boiling
ONB	Onset of Nucleat Boiling
SB	Subcooled Boiling
TB	Transition Boiling

Figure Vb.4.1. The boiling curve for water at 1 atm and various heat transfer regimes

4.1. Nucleate Pool Boiling

Rohsenow, in 1952, obtained the nucleate pool boiling correlation in the form of Ja = f(Re, Pr). The Reynolds number was defined for the bubbles as Re = $G_b D_b / \mu_b$ where G_b is bubble mass flux, D_b is bubble diameter, and μ_b is liquid viscosity. Therefore, $c_{p,f} \Delta T / h_{fg} = C_{s,f}(G_b D_b / \mu_b)^r Pr^n$. Rohsenow introduced C_{s-f}, r, and n so that nucleation on a variety of heated surfaces and liquids can be represented by the same relation. Expressing the bubble diameter in terms of contact angle, surface tension, and fluid density as $D_b = 1.48\phi[2g_c\sigma/(g\Delta\rho)]^{0.5}$ and the bubble mass flux in terms of $G_b = \dot{q}'' / h_{fg}$, we find:

$$\frac{c_{p,f}(T_s - T_{sat})}{h_{fg}\,Pr_f^n} = C_{s-f}\left[\frac{\dot{Q}/A}{\mu_f h_{fg}}\sqrt{\frac{g_c\sigma}{g(\rho_f - \rho_g)}}\right]^{0.33} \qquad \text{Vb.4.1a}$$

Solving for heat flux:

$$\dot{q}'' = \mu_f h_{fg}\left[\frac{g(\rho_f - \rho_g)}{\sigma}\right]^{0.5}\left(\frac{c_{p,f}(T_s - T_{sat})}{C_{sf} h_{fg}\,Pr_f^n}\right)^3 \qquad \text{Vb.4.1b}$$

The values for coefficient C_{sf} and n for various surfaces and liquids are given in Table Vb.4.1.

Table Vb.4.1. Values for coefficients C_{sf} and n for various liquid and surfaces

Fluid	Surface	C_{sf}	n
Benzene	Chromium	0.1010	1.7
Carbon tetrachloride	Copper, polished	0.0070	1.7
Ethyl alcohol	Chromium	0.0027	1.7
Isopropyl alcohol	Copper	0.0025	1.7
n-Butyl alcohol	Copper	0.0030	1.7
n-Pentane	Copper, polished	0.0154	1.7
	Nickel, polished	0.0127	1.7
	Copper, emery-robbed	0.0074	1.7
	Chromium	0.0150	1.7
Water	Brass	0.0060	1.0
	Copper, polished	0.0128	1.0
	lapped	0.0147	1.0
	scored	0.0068	1.0
	Nickel	0.0060	1.0
	Stainless steel, ground & polished	0.0080	1.0
	, Teflon pitted	0.0058	1.0
	, chemically etched	0.0133	1.0
	, mechanically polished	0.0132	1.0
	Platinum	0.0130	1.0

Example Vb.4.1. Water is boiling in a container at atmospheric pressure. The heated surface area is 0.1 m^2 and is made of mechanically polished stainless steel. An electric heater is used to carefully maintain the heated surface superheat at 17 C. Find a) the required power, b) the heat transfer coefficient, and c) the rate of evaporation from the heated surface.

Solution: From Table AIV5(SI) for saturated water and steam at T_{sat} = 100 C, we find ρ_f = 958 kg/m^3, ρ_g = 0.595 kg/m^3, h_{fg} = 2257 kJ/kg, $c_{p.f}$ = 4.217 kJ/kg·K, μ_f = 0.279E-3 N·s/m^2, Pr$_f$ = 1.76, σ = 0.0589 N/m

a) Since the surface superheat, $T_s - T_{sat}$ = 17 C, thus T_s = 100 + 17 = 117 C. From Table Vb.4.1 for mechanically polished stainless steel and water, we find C_{sf} = 0.0132 and n = 1. Equation Vb.4.1b yields:

$$\dot{q}'' = 0.279\mathrm{E}-3\times2257\mathrm{E}3\times\left[\frac{9.8\times(958-0.595)}{0.0589}\right]^{0.5}\left(\frac{4.217\times17}{0.0132\times2257\times1.76}\right)^3$$

$$= 642 \text{ kW/m}^2$$

The required power for boiling at the specified condition is $\dot{Q} = \dot{q}'' \times A$ = 642 × 0.1 = 64.2 kW.

b) Having the surface superheat and heat flux, the related heat transfer coefficient is found as:

$$h = \frac{\dot{q}''}{T_s - T_{sat}} = \frac{642,000}{17} = 37.8 \text{ kW/m}^2\cdot\text{C}$$

c) At steady state, the power delivered to saturated water is converted to the latent heat of vaporization, Equation IIa.7.2:

$$\dot{m} = \frac{\dot{Q}}{h_{fg}} = \frac{64.2}{2257} = 0.0284 \text{ kg/s} = 102.4 \text{ kg/h}$$

Comment: By using the software included on the accompanying CD-ROM, it can be easily verified that for a specified surface and surface superheat, the heat flux increases with increasing system pressure.

4.2. Critical Heat Flux in Pool Boiling

Kutateladze and later Zuber devised the following CHF correlation, which is only a function of pressure:

$$\dot{q}''_{CHF} = 0.149 h_{fg}\rho_g\left[\frac{\sigma g(\rho_f - \rho_g)}{\rho_g^2}\right]^{0.25} \qquad \text{Vb.4.2}$$

To avoid damage to heated surfaces, a safety factor is applied in the design of the heating elements to maintain heat flux well below the value predicted by Equation Vb.4.2.

Example Vb.4.2. A PWR rod is operating at a linear heat generation rate of 7 kW/ft. Find the safety factor according to the pool boiling CHF.

Solution: We first find the maximum heat flux from Equation Vb.4.2.

Using properties at $P = 2250$ psia we find:

$\dot{q}''_{CHF} = 0.149(416.4 \times 6.4)[0.000335 \times 32.2^2 (37.1 - 6.4)/6.4^2] = 283.6$ Btu/ft^2·s

A typical PWR rod diameter is about 0.44 inch, thus:

$\dot{q}'' = \dot{q}'/\pi D = 7/(3.14 \times 0.44/12) = 60.8$ Btu/ft^2·s

The safety factor is 283.6/60.8 = 4.70.

4.3. Transition Pool Boiling

Transition boiling refers to the region between T_{CHF} and T_{MSFB}. This region is experienced in a temperature-controlled boiling process. In this region, both nucleate and film boiling mechanisms coexist as the surface temperature is not high enough for film boiling to dominate. Due to the complicated nature of the transition boiling mechanism, there is no correlation that can reliably predict the wall heat flux. Most correlations use a weighted average value between the heat flux corresponding to maximum heat flux ($\dot{q}''_{CHF}$) and the heat flux corresponding to the minimum stable film boiling ($\dot{q}''_{MSFB}$).

4.4. Minimum Stable Film Boiling

In temperature-controlled boiling, sufficiently high wall temperature in the transition boiling mode precludes nucleate boiling and covers the surface with a film of vapor. The temperature at which nucleation is completely ceased is the minimum stable film boiling (MSFB) temperature. To determine the heat flux at the point of minimum stable film boiling, we may use the definition of the Stanton number given as:

$$\text{St} = \frac{\dot{Q}}{\dot{m}\Delta h} = \frac{\dot{Q}}{(\rho V A)(c_p \Delta T)} = \frac{\dot{q}''}{\rho V c_p \Delta T}$$

when applied to conditions where a change of phase is involved, St becomes:

$$St = \frac{\dot{q}''_{min}}{\rho_g V_{min} h'_{fg}}$$

where V_{min} in the denominator is given by:

$$V_{min} = \left(\frac{g(\rho_l - \rho_v)L_c}{(\rho_l + \rho_v)} \right)^{1/2}$$

where L_c is a characteristic length given by Equation Vb.1.1. For large horizontal surfaces, Berenson suggested St = 0.09. Using Berenson's value and substituting for the characteristic length, the minimum heat flux is found as:

$$\dot{q}''_{MSFB} = 0.09\rho_v h_{fg} \left[\frac{\sigma g(\rho_l - \rho_v)}{(\rho_l - \rho_v)^2} \right]^{\frac{1}{4}} \qquad \text{Vb.4.3}$$

Example Vb.4.3. Find the heat flux at the Leidenfrost point. Use $P = 1$ atm.

Solution: Using the saturated water properties at 100 C, Equation Vb.4.3 gives

$$\dot{q}''_{MSFB} = 0.09 \times 0.59 \times (2257E3) \left[\frac{0.059 \times 9.81(958 - 0.59)}{(958 + 0.59)^2} \right]^{\frac{1}{4}} = 19 \text{ kW/m}^2$$

4.5. Film Pool Boiling

The resemblance of the vapor film in the film boiling heat transfer to the condensate film in the laminar film condensation heat transfer prompted Bromely to suggest a correlation similar to Equation Va.2.6 for boiling on cylinders and spheres:

$$\bar{h} \frac{D}{k_v} = C \left[\frac{g(\rho_f - \rho_v)\rho_v h'_{fg} D^3}{\mu_v k_v (T_s - T_{sat})} \right]^{1/4} \qquad \text{Vb.4.4}$$

In Equation Vb.4.4, $C = 0.62$ for horizontal cylinders and 0.67 for spheres. Also $h'_{fg} = h_{fg} + 0.4c_{p,v}(T_s - T_{sat})$. Vapor properties are evaluated at $T_{film} = (T_s + T_{sat})/2$. Since the contribution of radiation heat transfer becomes noticeable as the surface temperature approaches and exceeds 300 C, Bromley recommends the following h:

$$\bar{h} = \bar{h}_{convection} + 0.75\bar{h}_{radiation} \qquad \text{Vb.4.5}$$

where in Equation Vb.4.5, the heat transfer coefficient due to radiation is given as:

$$\overline{h}_{radiation} = \varepsilon\sigma(T_s^4 - T_{sat}^4)/(T_s - T_{sat})$$

In this relation ε and σ are surface emissivity and the Stefan-Boltzmann constant, respectively.

Example Vb.4.4. A horizontal cylinder, having a surface temperature of 300 C is submerged in saturated water at 100 C. Estimate the surface heat flux. Cylinder diameter is 4 cm and $\varepsilon = 0.85$.

Solution: Using saturation properties at 100 C we find $\rho_f = 958$ kg/m³, and $h_{fg} = 2257$ kJ/kg. For superheated properties at 1 atm and a film temperature of 200 C: $\rho_v = 0.46$ kg/cm³, $c_{p,v} = 1.98$ kJ/kg·K, $\mu_v = 0.16$E-4 Pa·s, and $k_v = 0.033$ W/m·K.

$$h'_{fg} = h_{fg} + 0.4c_{p,v}\Delta T$$

$$= 2257 + 0.4 \times 1.98 \times 200 = 2415 \text{ kJ/kg.}$$

Using Equation Vb.4.4:

$$\overline{h}_{convection} = 0.62\frac{0.033}{0.04}\left[\frac{9.81\times(958-0.46)\times0.46\times2415\text{E3}\times0.04^3}{(0.16\text{E}-4)\times0.033\times(300-100)}\right]^{1/4}$$

$$= 144 \text{ W/m}^2\text{·K}$$

$$\overline{h}_{radiation} = 0.85 \times 5.67\text{E-8} \ (573^4 - 373^4)/(573 - 373) = 21 \text{ W/m}^2\text{·K}$$

$$h = \overline{h}_{convection} + 0.75\,\overline{h}_{radiation} = 144 + 0.75 \times 21 = 160 \text{ W/m}^2\text{·K}$$

$$\dot{q}'' = h\Delta T = 160 \times 200 = 32 \text{ kW/m}^2.$$

For film boiling on horizontal plates, Berenson's correlation is given as:

$$\text{Nu} = h\delta_v / k_v = 0.425 \qquad\qquad\qquad\qquad \text{Vb.4.6}$$

where δ_v, an average vapor film thickness, used in the Nu number is given by:

$$\delta_v = \left[\frac{v_v^2 L_c}{g}\frac{\rho_v}{\rho_f - \rho_v}\frac{1}{\text{Pr}_v}\frac{\text{Ja}_v}{1 + 0.4\text{Ja}_v}\right]^{1/4} \qquad\qquad \text{Vb.4.7}$$

In Equation Vb.4.7, L_c is the characteristic length as given in Equation Vb.1.1 and vapor properties are evaluated at T_{film}.

Example Vb.4.5. Find the convection heat flux for film boiling of water at 1 atmosphere on top of a horizontal plate with $T_s = 900$ K.

Solution: Film temperature is $T_{film} = 0.5(373 + 900) = 636.5$ K. At atmospheric conditions, $\rho_f = 958$ kg/m³, $h_{fg} = 2.257\text{E}6$ J/kg and $\sigma = 0.06$ N/m.

For superheated steam at 636.5 K:
$c_{pv} = 2048$ J/kg·K, $v_v = 66.4\text{E-6}$ m²/s, $\rho_v = 0.345$ kg/m³, and $k_v = 0.05$ W/m·K

$$\text{Ja}_v = c_{P,v}\,\Delta T / h_{fg} = 2048 \times (900 - 373)/2.257\text{E}6 = 0.48$$

$$L_c = (\sigma / g\Delta\rho)^{1/2} = [0.06/(9.81 \times 958)]^{0.5} = 2.53\text{E-3 m}$$

$$\delta_g = [\frac{(66.4\text{E}-6)^2 \times 2.53\text{E}-3}{9.81}\ \frac{0.345}{958 - 0.349}\ \frac{1}{0.93}\ \frac{0.48}{1 + 0.4 \times 0.48}]^{0.25} = 0.115 \text{ mm}$$

$h_{convection} = (k_v/\delta_v)\text{Nu} = (0.05/0.115\text{E-3}) \times 0.425 = 185$ W/m²·K

$\dot{q}''_{Convection} = h_{convection}\Delta T = 185 \times (900 - 373) = 98$ kW/m².

4.6. Minimum Stable Film Boiling Temperature

Temperature at the MSFB point may be found by using Newton's law of cooling and substituting for heat flux from Equation Vb.4.3 and for heat transfer coefficient from Equation Vb.4.4, for horizontal cylinders and spheres, or from Equation Vb.4.5 for horizontal plates. For horizontal cylinders and sphere we find:

$$T_{MSFB} - T_{sat} = 0.126 \frac{\rho_v h'_{fg}}{k_v} \left[\frac{g(\rho_l - \rho_v)\mu_v}{(\rho_l + \rho_v)^2} \right]^{1/3} \left[\frac{\sigma}{g(\rho_l - \rho_v)} \right]^{1/2} \qquad \text{Vb.4.8}$$

where the vapor physical properties are developed at the film temperature.

Example Vb.4.6. For heat treatment, a long steel cylindrical rod is immersed horizontally in a pool of water at atmospheric pressure. The rod diameter is 4 cm. Find T_{MSFB} and the heat flux when surface is at 400 C. Use $\varepsilon = 0.66$.

Solution: For sufficiently hot surface, the entire boiling curve is traversed when the rod is immersed. If the initial heat transfer regime is film boiling, surface temperature begins to drop by both convection and radiation mechanisms until the MSFB point is reached. When T_{rod} drops below T_{MSFB}, partial nucleation takes place in the transition boiling region, increasing heat flux. This trend continues until the maximum heat flux is reached, further cooling the rod. As the rod gets colder, the required superheat for nucleation diminishes and heat begins to transfer by single-phase natural convection. We then start by calculating T_{MSFB}:

To find T_{MSFB}, we need T_{film}. We assume $T_s = 200$ C and use Equation Vb.4.8:

$$\Delta T_{MSFB} = 0.126 \frac{0.52 \times 2336\text{E}3}{0.029} \left[\frac{9.81(958 - 0.52) \times (0.14\text{E} - 4)}{(958 + 0.52)^2} \right]^{1/3} \left[\frac{0.06}{9.81(957 - 0.52)} \right]^{1/2}$$

We find T_{MSFB} = 170 C. Next, we assume T_s = 180 C and find T_{MSFB} = 173 C. We continue iteration until the answer converges to $T_{MSFB} \approx$ 185 C. Thus initially heat transfer regime is film boiling. We find the initial heat flux from Equation Vb.4.4:

$$h = 0.62 \frac{0.038}{0.04} \left[\frac{9.81(958 - 0.42) \times 0.42 \times 2496\text{E}3 \times 0.04^3}{(0.18\text{E} - 4) \times 0.038 \times (400 - 100)} \right]^{0.25} = 140 \text{ W/m}^2 \cdot \text{K}$$

$$\dot{q}'' = h\Delta T + 0.75\varepsilon\sigma\left(T_s^4 - T_f^4\right)$$

$$= 140 \times 300 + 0.75 \times 0.66 \times 5.67\text{E} - 8 \times \left(673^4 - 373^4\right) = 47 \text{ kW/m}^2$$

4.7. Factors Affecting Pool Boiling

The pool boiling mode is influenced by several factors including gravity, pressure, surface roughness, aging, and the presence of noncondensable gases.

Effect of gravity. Among all the boiling modes, gravity primarily affects nucleation. In zero gravity, there is no nucleation as a large bubble would surround the heated surface. As gravity increases, heat transfer becomes more efficient due to enhanced free convection. If boiling is used as a cooling mechanism in the rotating machinery and in space vehicles, changes in gravity become a design consideration.

Effect of pressure. The rate of heat transfer in nucleate boiling is increased with pressure. This is apparent from the Clausius-Clapeyron equation. By treating vapor as an ideal gas, it can be shown that the degree of superheat is inversely proportional to pressure. Hence, at high pressures less ΔT_{sl} is needed for the same number of nucleation sites to become active. For example, if ΔT_{sl} = 12 F, when pressure is increased by about 6.5 times (from 383 psia to 2465 psia) heat flux increases by 12.5 times (from 8E4 Btu/ft^2·h to 1E6 Btu/ft^2·h).

Effect of surface roughness was extensively studied by Berenson who demonstrated that the effect of surface roughness on heat transfer depends on the boiling mode. Berenson showed that pre-CHF is affected strongly, transition boiling moderately, and film boiling is not affected by surface conditions.

Effect of aging. Aging adversely affects boiling heat transfer due to surface oxidation. The layer of oxide increases thermal resistance, but it also increases the population of cavities.

Effect of noncondensable gases on boiling is to enhance surface heat flux. The dissolved gases in the liquid are released near the heated surface, agitating the liquid and increasing mixing.

5. Flow Boiling Modes

Flow boiling is associated with the flow of liquids into a heated conduit. As shown in Figure Vb.5.1, flow at the entrance of the conduit is single-phase liquid and heat transfer from the heated wall is found from forced convection correlation (A). As the liquid travels in the conduit, the layer adjacent to the heated surface enters the surface cavities. If sufficient superheat is available, the site becomes active. Bubbles generated in such sites would migrate toward the bulk liquid, which is still subcooled (B). This constitutes the subcooled flow boiling regime. Expectedly, heat transfer in this regime is due to both subcooled boiling as discussed in Section 4 for subcooled pool boiling and forced convection for single-phase liquid. Collapse of a bubble increases liquid energy, more bubbles are produced, resulting in the related flow regime being referred to as *bubbly flow* (C). Wall temperature remains constant the moment subcooled boiling is initiated. On the other hand, liquid temperature keeps increasing until the bulk liquid eventually reaches saturation. Hence, the related heat transfer regime is called *saturated nucleate boiling* (D).

As flow travels further in the conduit, the nucleation process becomes so effective that bubble population grows to a point that bubbles eventually begin to coalesce to form a slug. The related flow regime is known as *slug flow* (D). When slugs coalesce, a central vapor core is formed. The flow pattern where the vapor core is surrounded by a film of liquid is known as annular flow (E). The related heat transfer regime remains saturated nucleate boiling. However, the process of nucleation is soon replaced by evaporation (E and F). In this regime, surface heat is transferred to the liquid film by forced convection, which is then transferred to the liquid-vapor interface where evaporation takes place. The corresponding heat transfer regime is often called *forced convection vaporization*. With continuous evaporation, the liquid film eventually dries out. Following *dryout*, surface temperature jumps to elevated values due to the lack of effective liquid cooling. There is a slight drop in the surface temperature due to the lingering droplets, which would randomly touch the surface (G). These drops soon vaporize, resulting in a continuous rise in surface temperature due to heat transfer to single-phase vapor.

5.1. Subcooled Flow Boiling

There are several correrlations for the calculation of the heat transfer coefficient in subcooled flow boiling (Delhaye, Ginoux, and Problem 9). However, the most widely used correlation, which is applied to both subcooled and saturation regions, is the Chen correlation as discussed next.

5.2. Saturated Flow Boiling

The Chen correlation accounts for both macro-convection due to flow and micro-convection due to boiling. As such, the Chen correlation is applicable over the en-
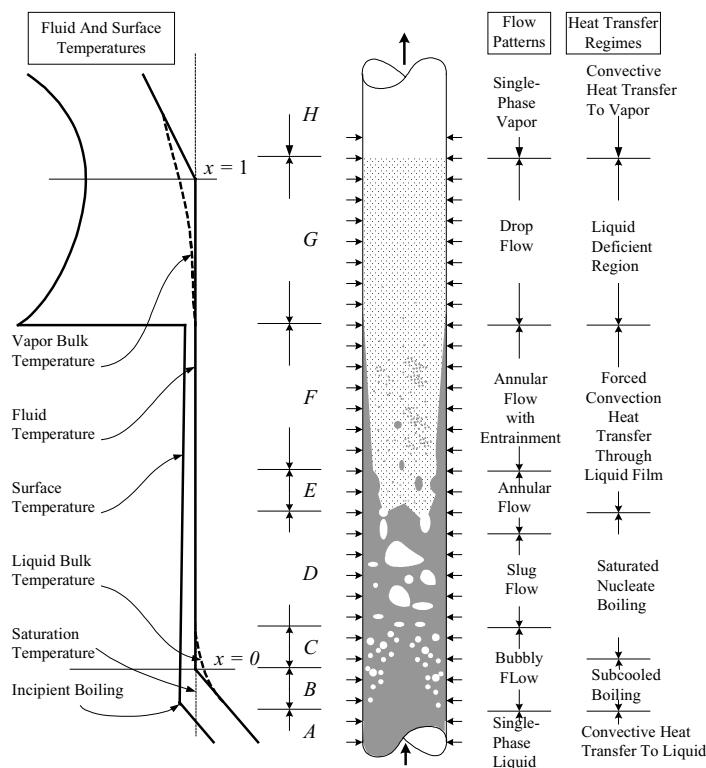
Figure Vb.5.1. Flow and heat transfer regimes in a sufficiently long heated conduit (Collier)

tire range of subcooled boiling, saturation boiling, and forced convection vaporization regions. The Chen correlation in SI units is given as:

$$h_{SI} = \frac{0.023k_f}{D_h}\left(\text{Re}_f\right)^{0.8}\text{Pr}_f^{0.4}\,F + 0.00122\,\frac{k_f^{0.79}c_{p,f}^{0.45}\rho_f^{0.49}}{\mu_f^{0.29}\sigma^{0.5}h_{fg}^{0.24}\rho_g^{0.24}}\,\Delta T_{sat}^{0.24}\Delta P^{0.75}S \qquad \text{Vb.5.1a}$$

substituting for ΔP from the Clapeyron equation, the Chen correlation in British units becomes:

$$h_{BU} = \frac{0.023k_f}{D_h}\left(\text{Re}_f\right)^{0.8}\text{Pr}_f^{0.4}\,F + 0.00122\,\frac{k_f^{0.79}c_{p,f}^{0.45}\rho_f^{0.49}g_c^{0.25}}{\mu_f^{0.29}\sigma^{0.5}h_{fg}^{0.24}\rho_g^{0.24}}\left[\frac{778h_{fg}}{v_{fg}(T_{sat}+460)}\right]^{0.75}\Delta T_{sat}^{0.99}S$$

$$\text{Vb.5.1b}$$

where $\text{Re}_f = G(1-x)D_h/\mu_f$ and the conversion factor g_c is defined in Chapter IIa. The first term in Equation Vb.5.1 applies to subcooled boiling and follows the Dittus–Boelter correlation with a modification factor F, which accounts for the en-

hanced flow and turbulence due to the presence of vapor. The second term in Equation Vb.5.1 applies to nucleation and follows the Forster-Zuber analysis, modified with the suppression factor S. The modification factors F is given in terms of X_M:

$$X_M = \left(\frac{x}{1-x}\right)^{0.9}\left(\frac{\rho_f}{\rho_g}\right)^{0.5}\left(\frac{\mu_g}{\mu_f}\right)^{0.1}$$

so that for the F factor, $F = 2.35[0.213 + X_M]^{0.736}$, which applies if $X_M > 0.1$.

Otherwise, $F = 1$. Also, the S factor is given as $S = [1 + (2.53E - 6)\,\mathrm{Re}^{1.17}]^{-1}$

where $\mathrm{Re} = \mathrm{Re}_f\, F^{1.25} = [G(1-x)D_h/\mu_f]F^{1.25}$.

Example Vb.5.1. Water flows in a vertical 1 in heated tube at a rate of 0.5 lbm/s. System pressure is 1000 psia. Find heat flux at a point where steam quality is 25% and surface superheat is $\Delta T_{sat} = T_s - T_{sat} = 8$ F.

Solution: First, we find properties at 1000 psia:
$\rho_f = 46.33$ lbm/ft^3, $\mu_f = 0.229$ lbm/ft·h, $c_{p,f} = 1.286$ Btu/lbm, $h_{fg} = 650.5$ Btu/lbm, $\sigma = 0.0012$ lbf/ft
$\rho_g = 2.23$ lbm/ft^3, $\mu_g = 0.046$ lbm/ft·h, $k_f = 0.33$ Btu/ft·h·F, $v_{fg} = 0.43$ ft^3/lbm, $\mathrm{Pr}_f = 0.899$, $T_{sat} = 544.33$ F
$\rho_f = 46.33$ lbm/ft^3, $\rho_g = 2.23$ lbm/ft^3, $h_{fg} = 650.5$ Btu/lbm, $c_{p,f} = 1.286$ Btu/lbm, $\mu_f = 0.229$ lbm/ft·h, $\mu_g = 0.046$ lbm/ft·h, $k_f = 0.33$ Btu/ft·h·F, $\mathrm{Pr}_f = 0.899$, $\sigma = 0.0012$ lbf/ft, $v_{fg} = 0.43$ ft^3/lbm and $T_{sat} = 544.33$ F.

We now find X_M, F, S, and finally h. First X_M is found from:
$$X_M = \left(\frac{0.25}{0.75}\right)^{0.9}\left(\frac{46.33}{2.23}\right)^{0.5}\left(\frac{0.046}{0.23}\right)^{0.1} = 1.44$$

The F factor becomes
$$F = 2.35[0.213 + X_M]^{0.736} = 2.35(0.213 + 1.44)^{0.736} = 3.41$$

Diameter and flow area are: $d = 1/12 = 0.0833$ ft and $A = \pi d^2/4 = 3.14 \times 0.0833^2/4 = 5.45\text{E-}3$ ft^2

$G = \dot{m}/A = 0.5/5.45\text{E-}3 = 92$ lbm/ft^2·s

To find S, we need to find:
$\mathrm{Re}_f = (0.5 \times 3600)(1 - 0.25) \times 0.0833/(0.229 \times 5.45\text{E-}3) = 90{,}072$
$\mathrm{Re} = \mathrm{Re}_f \times F^{1.25} = 90{,}072 \times 3.41^{1.25} = 417{,}381$
$S = (1 + 2.53\text{E-}6 \times 417{,}381^{1.17})^{-1} = 0.095$
The convection heat transfer coefficient (h_c) from Equation Vb.5.1b is:
$h_c = 0.023(0.33/0.0833)(90{,}072)^{0.8} \times 0.899^{0.4} \times 3.55 = 2737$ Btu/ft^2·h·F.

The nucleation heat transfer coefficient (h_b) from Equation Vb.5.1b is:

Numerator: $0.00122 \times 0.33^{.79} \times 1.286^{.45} \times 46.33^{.49} \times (32.2 \times 3600^2)^{.25} = 0.533$
Denominator: $0.229^{.29} \times 0.0012^{.5} \times 650.5^{.24} \times 2.23^{.24} = 0.13$
Bracket: $[778 \times 650.5/(0.43 \times 1004.33)]^{.75} = 200.3$

$h_n = (0.533/0.13) \times 200.3 \times 8^{.99} \times 0.095 = 611$ Btu/ft^2·h·F

$h = h_c + h_n = 2737 + 611 = 3348$ Btu/ft^2·h·F

$\dot{q}'' = h\Delta T = 3348 \times 8 = 26{,}786$ Btu/ft^2·h = 84 kW/m^2.

Example Vb.5.2. Water at a rate of 1200 kg/h flows in a tube having an inside diameter of 30 mm. Pressure is 10 MPa. Find the heat transfer coefficient and the surface heat flux at a location where surface superheat is 10 C and quality is 15%.

Solution: First, we find properties at 10 MPa:
$\rho_f = 688.7$ kg/m^3, $\mu_f = 0.86$E-4 N·s/m^2, $h_{fg} = 1320$ kJ/kg, $c_{p,f} = 6.159$ kJ/kg·K,
$\sigma = 0.012$ N/m,
$\rho_g = 55.14$ kg/m^3, $\mu_g = 0.21$E-4 N·s/m^2, $v_{fg} = 0.016$ m^3/kg, $k_f = 0.522$ W/m·K,
$\text{Pr}_f = 1.02$, $T_{sat} = 310.88$ C.

Similar to Example Vb.5.1, we find X_M, F and S factors, and h_c and h_n:

Find X_M from: $X_M = (0.15/0.85)^{0.9}(688.7/55.14)^{0.5}(0.21$E-4$/0.86$E-4$)^{0.1} = 0.644$

The F factor becomes:
$$F = 2.35\left[0.213 + X_M\right]^{0.736} = 2.35(0.213 + 0.644)^{0.736} = 2.1$$

$d = 0.03$ m, $A = \pi d^2/4 = 3.14 \times 0.03^2/4 = 7.068$E-4 m^2, and $G = 1200/7.068$E-4 $= 471.6$ kg/m^2·s

To find S, $\text{Re}_f = G(1 - x)D_h/\mu_f = 471.6 \times (1 - 0.15) \times 0.03/ 0.86$E-4$= 139{,}835$

$\text{Re} = \text{Re}_f \times F^{1.25} = 139{,}835 \times 2.1^{1.25} = 353{,}500$ and
$S = (1 + 2.53$E-6 $\times 353{,}500^{1.17})^{-1} = 0.11$

The convection heat transfer coefficient (h_c) from Equation Vb.5.1a is:

$h_c = 0.023(0.522/0.03)(139{,}835)^{0.8} \times 1.02^{0.4} \times 2.1 = 11{,}077$ W/m^2·K

The nucleation heat transfer coefficient (h_n) from Equation Vb.5.1a is:

Numerator: $0.00122 \times 0.522^{.79} \times (6.159$E3$)^{.45} \times 688.7^{.49} = 0.91$
Denominator: $(0.86$E-4$)^{.29} \times 0.012^{.5} \times (1320$E3$)^{.24} \times 55.14^{.24} = 0.559$

$\Delta T_{sat} = 10$ C, $T_{surface} = T_{sat} + 10 = 320.88$ C
$\Delta P = P_{sat}(320.88) - P_{sat}(310.88) = 11.42 - 10 = 1.42$E6 Pa
$h_n = (0.91/0.559) \times (1.42$E6$)^{0.75}(10)^{0.24} \times 0.11 = 12800$ W/m·K
$h = h_c + h_n = 11{,}077 + 12{,}800 = 23{,}878$ W/m·K
$\dot{q}'' = h\Delta T = 23{,}878 \times 10 = 238.7$ kW/m^2.

In the above examples, surface temperature was specified.   Otherwise, we should solve the problem by iteration. In an iterative solution, we assume a value for the surface temperature to find ΔT_{sat} and subsequently calculate h_{Chen} from the Chen correlation, Equation Vb.5.1b. Having h, we then recalculate ΔT_{sat} from a heat balance between the surface and the fluid; $\dot{q}'' = h\Delta T_{sat}$.   We continue this until we reach the intended convergence criterion, for example $\varepsilon \leq 1\%$. Kandlikar has introduced a more recent correlation, which, according to Lienhard, leads to mean deviations of 16% for water and 19% for refrigerants.

5.3. Critical Heat Flux in Flow Boiling

Due to the importance of flow boiling especially in the operation of nuclear reactors, extensive research has been performed in flow boiling CHF.   The CHF mechanism in flow boiling is a function of the flow regime and is either in the form of departure from nucleate boiling, DNB, or dryout. DNB is the mechanism of concern in the design and operation of PWRs. A PWR core contains pressurized subcooled water. In a high power channel, the rate of vaporization at the surface may become so vigorous that it may prevent liquid from reaching the surface, Figure Vb.5.2(a). Depriving the surface of liquid for nucleation results in elevated surface temperature, which may lead to fuel failure. On the other hand, the dryout mechanism is of concern in the design and operation of BWRs. This is because, in high power channels, the flow regime may become annular. With further increase in power, the liquid film may simply dryout as shown in Figure Vb.5.2(b). For this reason, the operational heat flux is maintained well below the CHF, through the application of a variety of safety factors.   As shown in Figure Vb.5.2(c), the magnitude of CHF is either a direct or an inverse function of the mass flux, depending on quality. In the DNB region (low x), CHF is a direct function of mass flux whereas in the dryout region (high x) CHF depends inversely on mass flux.

There are many CHF correlations for water in the literature, including Babcock & Wilcox, Combustion Engineering, EPRI, General Electric, Westinghouse,
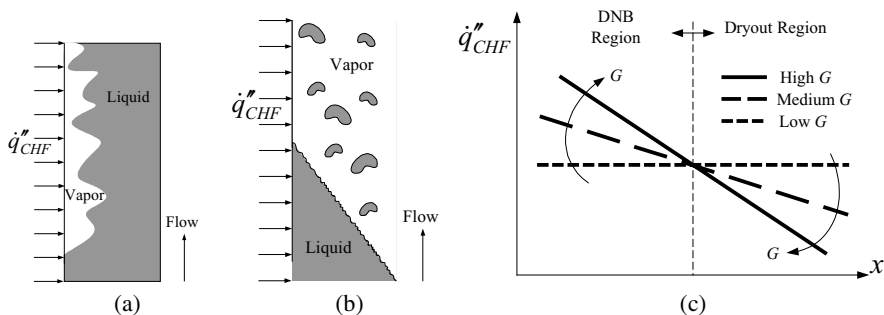

Figure Vb.5.2. Depiction of (a) DNB, (b) Dryout, and (c) Dependency of CHF on Mass Flux (Todreas)

Barnett, Biasi, Bernath, Bowring, Gaspari (CISE-4), and Katto. Next we discuss several of these correlations.

Barnett correlation. The critical heat flux in this correlation is expressed in terms of channel geometry (hydraulic diameter, heated diameter and heated length) as well as coolant mass flux and specific enthalpies. The Barnett correlation covers a narrow range for pressure (i.e., 4 – 10 MPa). The Barnett correlation in SI units is given as:

$$\dot{q}''_{CHF} = 3.1546 \times 10^6 \frac{3.584 C_1 h_{fg} + 4.3 \times 10^{-4} C_2 (h_f - h_i)}{C_3 + 39.37z} \qquad \text{Vb.5.2}$$

where constants C_1, C_2, and C_3 are expressed in terms of G and D as

$$C_1 = 230.7 D_h^{0.68} G^{0.192} \left[1 - 0.744 \exp(-0.3477 D_e G)\right],$$

$$C_2 = 0.1206 D_h^{1.415} G^{0.817} \text{, and } C_3 = 8249 D_e^{1.415} G^{0.212} .$$

Bernath correlation is $\dot{q}''_{CHF} = h_{CHF}(T_{s,CHF} - T_f)$ where h_{CHF} and $T_{s,CHF}$ are in turn obtained from the following relations:

$$h_{CHF} = 10,890 \frac{1}{1 + (P_h / \pi D_e)} + \frac{48V}{D_e^{0.6}}$$

$$T_{s,CHF} = 32 + 102.6 \ln P - \frac{97.2}{1 + (15/P)} - 0.45V$$

where P is system pressure in psia, V is coolant velocity in ft/s, T_f is bulk temperature of the coolant in F, D_e is the equivalent diameter in ft and P_h is the heated perimeter in ft. This correlation is valid for pressure ranging from 23 – 3000 psia, coolant velocity in the range of 4.0 – 54 ft/s, and equivalent diameter in the range of 0.143 – 0.66 in.

Biasi correlation expressed in terms of pressure, mass flux, quality, and diameter, has a much wider range of application for pressure compared to the Barnett correlation. Since the database covers both low and high steam quality, the Biasi correlation is applicable to both DNB and dryout. In SI units, the Biasi correlation for $G < 300$ kg/s·m^2 is given as:

$$\dot{q}''_{CHF} = S_1(1 - x) \qquad \text{Vb.5.3}$$

where S_1 = 15.048E7$(100D)^{-n}G^{-1/6}C_1$ and heat flux is in W/m^2. For mass fluxes higher than 300 kg/s·m^2, the heat flux in W/m^2 is obtained from:

$$\dot{q}''_{CHF} = S_2(S_3 - x) \qquad \text{Vb.5.4}$$

where $S_2 = 2.764 \times 10^7 (100D)^{-n} G^{-1/6}$ and $S_3 = 1.468 C_2 G^{-1/6}$. In Equations Vb.5.3 and Vb.5.4, constants C_1 and C_2 are only functions of pressure and are given as

$$C_1 = -1.159 + 1.49 P \exp(-0.19P) + 9P(1 + 10P^2)^{-1} \quad \text{and}$$
$$C_2 = 0.7249 + 0.99P \exp(-0.32P)$$

Exponent n is 0.4 if the $D_{\text{channel}} \geq 0.01$ m. Otherwise, its value is 0.6. Also P is in MPa.

If the channel is heated uniformly, an energy balance for a control volume extended from the inlet to height z (where CHF occurs) gives:

$$\dot{q}''(\pi D z) = (\pi D^2 / 4) G(h_l - h_i) \qquad \text{Vb..5.5}$$

Substituting for the local enthalpy from $h_l = h_f + x_l h_{fg}$ in Equation Vb.5.5, solving for x_l and substituting into Equations Vb.5.3 and Vb.5.4, the Biasi correlation for uniformly heated tubes becomes:

$$\dot{q}''_{CHF} = S_1(1 + \beta)/(1 + \alpha S_1) \qquad \text{Vb.5.3}$$
$$\dot{q}''_{CHF} = S_2(S_3 + \beta)/(1 + \alpha S_2) \qquad \text{Vb.5.4}$$

where $\alpha = 4z/(GDh_{fg})$ and $\beta = \Delta h_{sub,i}/h_{fg}$.

Bowring correlation has a wide range of applicability. It is based on the works of MacBeth and Barnett and in SI units is given as:

$$\dot{q}''_{CHF} = (C_1 - C_2 x h_{fg})/C_3 \qquad \text{Vb.5.6}$$

where coefficients C_1, C_2 and C_3 are functions of pressure, mass flux and channel diameter as follows:

$$C_1 = \frac{2.317 C_2 C_4 h_{fg}}{1 + 0.0143 C_5 D^{1/2} G}, \quad C_2 = \frac{DG}{4}, \quad C_3 = \frac{0.308 C_2 C_6}{1 + 0.347 C_7 (G/1356)^n}$$

where exponent n is a function of the reduced pressure (i.e., $n = 2 - 0.5 P_R$). The reduced pressure in turn is defined as $P_R = 0.145P$ MPa. Coefficients C_4 through C_7 are functions of reduced pressure. For $P_R < 1$ MPa;

$$C_4 = 0.478 + 0.52 P_R^{18.942} \exp[20.89(1 - P_R)],$$
$$C_5 = C_4 \{0.236 + 0.764 P_R^{1.316} \exp[2.444(1 - P_R)]\}^{-1},$$
$$C_6 = 0.4 + 0.6 P_R^{17.023} \exp[16.658(1 - P_R)], \text{ and } C_7 = C_6 P_R^{1.649}$$

and for $P_R > 1$ MPa, the above coefficients are given as:

$$C_4 = P_R^{-0.368} \exp[0.648(1 - P_R)], \quad C_5 = C_4 \left\{ P_R^{-0.448} \exp[2.445(1 - P_R)] \right\}^{-1},$$
$$C_6 = P_R^{0.219}, \quad C_7 = C_6 P_R^{1.649}.$$

For uniformly heated channels, we use Equation Vb.5.5 and substitute $h_l = h_f + x_l h_{fg}$. The same procedure was used for the Biasi correlation to write the Bowring correlation as:

$$\dot{q}''_{CHF} = \frac{C_1 + C_2 \Delta h_{sub}}{C_3 + z} \qquad\qquad \text{Vb.5.7}$$

CISE-4 correlation is applicable to BWRs and has a narrow range of application for both pressure and mass flux. This correlation is expressed in terms of CHF quality (x_{CHF}) whose value approaches unity when mass flux approaches 0.0. This correlation in SI units is given as:

$$x_{CHF} = \frac{D_h}{D_e} \left(\frac{C_1 L_{CHF}}{C_2 + L_{CHF}} \right) \qquad\qquad \text{Vb.5.8}$$

where C_1 and C_2 are functions of mass flux, pressure and critical pressure: The value of coefficient C_1 depends on mass flux as compared with a reference flux (G_R given by $G_R = 3375(1 - P/P_c)^3$). Hence;

$$C_1 = \left[1 + 1.481 \times 10^{-4} (1 - P/P_c)^{-3} G \right]^{-1}; \qquad\qquad G \leq G_R$$
$$C_1 = (1 - P/P_c)(G/1000)^{-1/3}; \qquad\qquad G > G_R$$

and coefficient C_2 is given by $C_2 = 0.199(P_c/P - 1)^{0.4} GD^{1.4}$. Also L_{CHF} is the boiling length to the point where CHF occurs. To find $\dot{q}''_{CHF}$ by the CISE–4 correlation, we need to find relations for x_{CHF} and L_{CHF}. To find L_{CHF}, we use an energy balance, Equation Vb.5.5. If the entire tube is uniformly heated at $\dot{q}''_{CHF}$, the portion of energy consumed to bring the subcooled water at the inlet of the tube to saturation, is found from:

$$\dot{q}''_{CHF} \left[\pi D (L - L_{CHF}) \right] = G \left(\pi D^2 / 4 \right) (h_f - h_{in})$$

Similarly, we can find x_{CHF} from Equation Vb.5.6 applied to the boiling section:

$$\dot{q}''_{CHF} \pi D L_{CHF} = G \left(\pi D^2 / 4 \right) (h - h_f) = G \left(\pi D^2 / 4 \right) x_{CHF} h_{fg}$$

Eliminating L_{CHF} between these equations and substituting, we obtain x_{CHF} as:

$$x_{CHF} = \left[4 L \dot{q}''_{CHF} / (GD h_{fg}) \right] - \left[(h_f - h_{in}) / h_{fg} \right] \qquad\qquad \text{Vb.5.9}$$

Substituting into Equation Vb.5.8, we find the following implicit equation for $\dot{q}''_{CHF}$:

$$y / h_{fg} = C_1 (D_h / D_e)\left[C_2 + L - (h_f - h_{in})/ y\right] \qquad \text{Vb.5.10}$$

where in Equation Vb.5.10, $y = 4\dot{q}''_{CHF} /(GD)$.

EPRI-1 correlation as reported by Pei, is based on a vast bank of data:

$$\dot{q}''_{CHF} = \frac{C_1 - x_{in}}{C_2 + (x_l - x_{in})/ \dot{q}''_l} \qquad \text{Vb.5.11}$$

where $\dot{q}''_l$ is the local heat flux and both heat flux terms in Equation Vb.5.11 are in MBtu/h·ft². Mass flux G is in Mlbm/h·ft². Coefficients C_1 and C_2 are given as:

$$C_1 = P_1 P_r^{P_2} G^{(P_5 + P_7 P_r)}$$
$$C_2 = P_3 P_r^{P_4} G^{(P_6 + P_8 P_r)}$$

Note the reduced pressure is $P_r = P/P_{critical}$. Constants P_1 through P_8 are given as $P_1 = 0.5328$, $P_2 = 0.1212$, $P_3 = 1.6151$, $P_4 = 1.4066$, $P_5 = -0.3040$, $P_6 = 0.4843$, $P_7 = -0.3285$, and $P_8 = -2.0749$.

For uniformly heated channels, we may substitute for the local quality from $x_l = (h_l - h_f)/h_{fg}$ and for the local heat flux from Equation Vb.5.5 to write the EPRI-1 correlation as:

$$\dot{q}''_{CHF} = \frac{C_1 - x_{in}}{C_2 + \left[4z /(GDh_{fg})\right]} \qquad \text{Vb.5.12}$$

Katto correlation (Collier) is expressed in terms of quality, mass flux, and enthalpy as:

$$\dot{q}''_{CHF} = XG\left[h_{fg} + K(h_f - h_i)\right] \qquad \text{Vb.5.13}$$

where h_i is the inlet enthalpy calculated at P and T_i. Coefficients X and K in Equation Vb.5.13 are functions of dimensionless numbers Z, R, and W defined as:

$$Z = \frac{z}{D}, \qquad\qquad R = \frac{\rho_f}{\rho_g}, \qquad\qquad W = \frac{\sigma \rho_f}{zG^2}$$

We need to calculate five values for X and three values for K as follows:

$$X_1 = CW^{0.043} / Z, \qquad X_2 = 0.1R^{0.133} W^{0.433} Z / y,$$
$$X_3 = 0.098R^{0.133} W^{0.433} Z^{0.27} / y,$$

$$X_4 = 0.0384 R^{0.6} W^{0.173} / \left(1 + 0.28 W^{0.233} Z\right),$$

$$X_5 = 0.234 R^{0.513} W^{0.433} Z^{0.27} / y$$

$$K_1 = 0.261 / \left(C W^{0.043}\right), \quad K_2 = 0.833 \left[0.0124 + Z^{-1}\right] / \left(R^{0.133} W^{0.333}\right),$$

$$K_3 = 1.12 \left[1.52 W^{0.233} + Z^{-1}\right] / \left(R^{0.6} W^{0.173}\right)$$

where $y = 1 + 0.0031Z$ and the value of C in these relations is found as:

$C = 0.25$	if $Z < 50$,
$C = 0.25 + 0.0009(Z - 50)$	if $50 < Z < 150$,
$C = 0.34$	if $Z > 150$.

The following logic should be used to find the applicable values of X and K:

$\underline{R < 0.15}$

If $X_1 < X_2$, $X = X_1$ If $X_1 > X_2$ and $X_2 < X_3$, $X = X_2$
If $X_1 > X_2$ and $X_2 > X_3$, $X = X$ If $K_1 > K_2$, $K = K_1$
If $K_1 < K_2$, $K = K_2$

$\underline{R > 0.15}$

If $X_1 < X_5$, $X = X_1$ If $X_1 > X_5$ and $X_4 < X_5$, $X = X_5$
If $X_1 > X_5$ and $X_4 > X_5$, $X = X_4$ If $K_1 > K_2$, $K = K_1$
If $K_1 < K_2$ and $K_2 < K_3$, $K = K_2$ If $K_1 < K_2$ and $K_2 > K_3$, $K = K_3$

General Electric correlation, devised for BWRs, expresses the lowest measured values of critical heat flux as a function of mass flux and quality:

$$\dot{q}''_{CHF} = 0.705 + 0.237G \qquad\qquad X < C_1$$

$$\dot{q}''_{CHF} = 1.634 - 0.27G - 4.71X \qquad\qquad C_1 < X < C_2$$

$$\dot{q}''_{CHF} = 0.605 - 0.164G - 0.653X \qquad\qquad C_2 < X$$

where constants $C1$ and $C2$ are given in terms of mass flux:

$C_1 = 0.197 - 0.108G$
$C_2 = 0.254 - 0.026G$

In these relations, X is the flow quality, mass flux G is in Mlbm/h·ft^2, and heat flux $\dot{q}''_{CHF}$ is in MBtu/h·ft^2. The above relations apply to a system at a pressure of 1000 psia. For other pressures, we find $\dot{q}''_{CHF}$ from:

$$\dot{q}''_{CHF}(P) = \dot{q}''_{CHF}(1000) + 440(1000 - P)$$

The GE correlation is valid for P in the range of 600 – 1450 spia, G in the range of 0.4 – 6 Mlbm/h·ft^2, quality in the rage of 0 – 0.45, channel length in the range of 29 – 108 in, and equivalent diameter in the range of 0.245 – 1.25 in. As reported

by Tong, the GE lower envelope correlation for low mass velocity CHF at pressures less than 1000 psia is obtained from:

$$\dot{q}''_{CHF} = 0.84 - x, \qquad\qquad G < 0.5 \text{ Mlbm/h·ft}^2$$
$$\dot{q}''_{CHF} = 0.80 - x, \qquad\qquad 0.5 < G < 0.75 \text{ Mlbm/h·ft}^2$$

where the critical heat flux is in MBtu/h·ft². The range of applicability of these correlations are shown in Table Vb.5.1.

Table Vb.5.1. Data base for various CHF correlations

Correlation	D (m)	L (m)	P (MPa)	G (kg/m² s)
Biasi	0.0030 – 0.3750	0.20 – 6.00	0.27 – 14	100 – 6000
Bowring	0.0020 – 0.0450	0.15 – 3.70	0.20 – 19	136–18,600
Barnett	0.0095 – 0.0960	0.61 – 2.74	6.9	190 – 8409
CISE-4	0.0102 – 0.0198	0.76 – 3.66	4.96 – 7.0	1085 - 4069
EPRI-1	0.0420 – 0.0139	0.76 – 4.27	1.38 – 17	271 – 5553
Katto	0.0010 – 0.038	0.01 – 8.80	00.1 – 21*	**
General Electric	0.0060 – 0.0320	0.74 – 2.74	4.14 – 10	550 – 8000

* Specified in terms of $0.0003 < \rho_g/\rho_f < 0.41$. ** Specified in term of $3E\text{-}9 < W < 2E\text{-}2$.

Predictions of several CHF correlations are compared in the following example.

Example Vb.5.3. Water at 288 C (550 F) enters a uniformly heated vertical tube of diameter 13.4 mm (0.528 in) and length 3.66 m (12 ft) at 3508 kg/s·m² (2.59 Mlb/s·ft²). The system pressure is 15.51 MPa (2250 psia). Find CHF from the Biasi, the Bowring, the EPRI-1, and the Katto correlations.

Solution: We first find water properties at 15.51 MPa: h_f = 1631 kJ/kg (701 Btu/lbm), h_g = 2600 kJ/kg (1118 Btu/lbm), h_{fg} = 969 kJ/kg (417 Btu/lbm), $h_i(P$ = 15.51 & T = 288) = 1273.7 kJ/kg (547 Btu/lbm).

Biasi correlation: Since G > 300 kg/s·m², we use Equation Vb.5.4:

$$C_1 = -1.159 + 1.49 P \exp(-0.19P) + 9P(1 + 10P^2)^{-1} = 0.1123$$
$$C_2 = 0.7249 + 0.99 P \exp(-0.32P) = 0.8322$$
$$S_2 = 2.764 \times 10^7 (100D)^{-n} G^{-1/6} = 0.5949E7,$$
$$S_3 = 1.468 C_2 G^{-1/6} = 0.3134$$
$$\alpha = 4z/(GDh_{fg}) = 4 \times 3.66/(3508 \times 0.0134 \times 969E3) = 3.2E{-}7$$
$$\beta = (h_f - h_{in})/h_{fg} = (1631 - 1273)/2600 = 0.3686$$
$$\dot{q}''_{CHF} = S_2(S_3 + \beta)/(1 + \alpha S_2) = 0.5949E7 \times (0.3134 + 0.3686)/(1 + 3.2E{-}7 \times 0.5949E7) = 1.39 \text{ MW/m}^2$$

Bowring correlation: We first obtain $C_4 = 0.478$, $C_5 = 0.4912$, $C_6 = 0.4$, and $C_7 = 0.0345$. We then find:

$$C_1 = 2.317 C_2 C_4 h_{fg} / \left(1 + 0.0143 C_5 D^{1/2} G\right) = 0.3274\text{E}7, \; C_2 = DG/4 = 11.75,$$

$$C_3 = 0.308 C_2 C_6 / \left[1 + 0.347 C_7 (G/1356)^n\right] = 1.3507$$

$\dot{q}''_{CHF} = (C_1 + C_2 \Delta h_{sub})/(C_3 + z)$ = (0.3274E7 + 11.75 × (1631 − 1273.7)E3/(1.3507 + 3.66) = 1.49 MW/m^2

EPRI-1 correlation: We use Equation Vb.5.12 for which we find coefficients C_1 and C_2 from:

$C_1 = P_1 P_r^{P_2} G^{(P_5 + P_7 P_r)}$ = 0.5328 × (2250/3205.6)$^{0.1212}$ × 2.56$^{[-0.3040 \; - \; 0.3285 \; \times \; (2250/3205.6)]}$ = 0.3069

$C_2 = P_3 P_r^{P_4} G^{(P_6 + P_8 P_r)}$ = 1.6151 × (2250/3205.6)$^{1.4066}$ × 2.56$^{[0.4843 - 2.0749 \times (2250/3205.6)]}$ = 0.3892

$$\dot{q}''_{CHF} = \frac{C_1 + \left(\Delta h_{sub,i} / h_{fg}\right)}{C_2 + \left[4z / \left(GD h_{fg}\right)\right]}$$

$$= \frac{0.3069 + [(701 - 547)/417]}{0.3892 + \left(4 \times 3.66 / (2.59 \times 0.0134 \times 417)\right)} \; 0.48 \text{ MBtu/h·ft}^2 = 1.52 \text{ MW/m}^2$$

Katto correlation: We first find $Z = z/D = 272.95$, $R = \rho_g/\rho_f = 0.17$, and $W = 0.65\text{E-7}$. Now find X_i & K_i:

$X_1 = 0.6114\text{E-3}$, $X_2 = 0.1732\text{E-3}$, $X_3 = 0.1475\text{E-3}$, $X_4 = 0.2881\text{E-3}$ $X_5 = 0.1803\text{E-3}$, $K_1 = 1.564$, $K_2 = 4.185$, $K_3 = 2.022$.

Using the selection logic, we find $X = 0.2881\text{E-3}$ and $K = 2.022$

$\dot{q}''_{CHF} = XG\left[h_{fg} + K(h_f - h_i)\right]$ = 0.2881E-3 × 3508[969E3 + 2.022(1631 − 1273.7)E3] = 1.71 MW/m^2

Example Vb.5.4. Water at 400 F and 1000 psia enters a uniformly heated channel at a rate of 1E6 lbm/h·ft^2. The channel heated and equivalent diameters are $D_h = D_e = 0.3$ in. Channel length is 1.5 ft. Find the critical heat flux and wall temperature at CHF.

Solution: We first use a CHF correlation, such as EPRI-1 for example to find the critical heat flux:

$$\dot{q}''_{CHF} = \frac{C_1 + \left(\Delta h_{sub,i} / h_{fg}\right)}{C_2 + \left(4z / GD h_{fg}\right)} = \frac{0.4627 + (166.5/650.5)}{0.3140 + \left(4 \times 1.5 / 1 \times 0.025 \times 650.5\right)}$$

= 1.05E6 Btu/h·ft^2

The solution to find T_{CHF} is basically iterative. However, we may use the Bernath correlation to find the onset of the boiling crisis. For this purpose, we need the local enthalpy:

$h_l = h_i + (4z\,\dot{q}''_{CHF}\,/GD) = 375.8 + [4 \times 1.5 \times 1.05/(1 \times 0.025)] = 627.8$ Btu/lbm

Density at the onset of CHF is $\rho = \rho(1000$ psia, 627.8 Btu/lbm). Hence, $\rho_l = 46.3$ lbm/ft^3:

$V = G/\rho = (1E6/3600)/46.3 = 6$ ft/s
$T_{s,CHF} = 32 + 102.6\ln 1000 - \left[97.2/(1+15/1000)\right] - 0.45 \times 6 \ = 642.3$ F.

5.4. Factors Affecting CHF

Critical heat flux correlations show the dependency of CHF on flow path diameter (d), flow path length (L), mass flux (G), inlet subcooling (Δh_{in}), and pressure (P). To determine the effect of each parameter, the rest of the parameters are kept constant. Let's first consider the case of upward flow of a liquid in a heated tube having diameter d and length L. Figure Vb.5.3 shows that CHF varies directly with Δh_{in}, G, and d and varies inversely with L. Pressure has a more interesting effect as CHF for water in both pool and flow boiling reaches a maximum at about 70 bar. To investigate the effect of heat flux on CHF, we try three cases of low, medium, and high heat fluxes, as shown by dashed lines 1, 2, and 3 in Figure Vb.5.4(A).
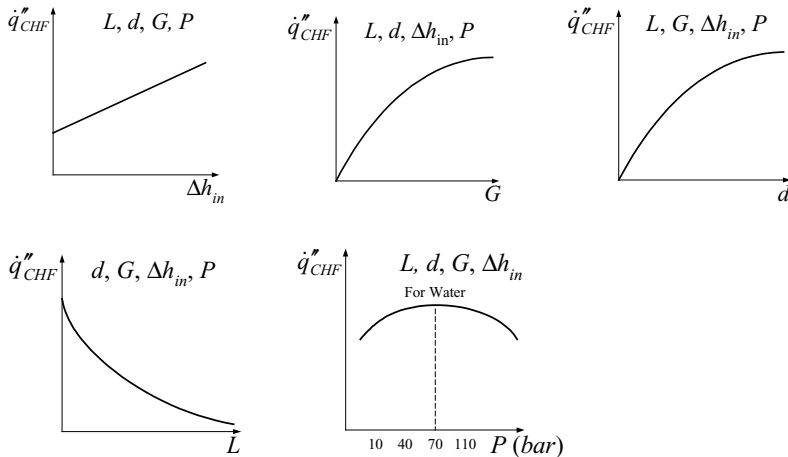

Figure Vb.5.3. Effect of various design parameters on CHF (Whalley)
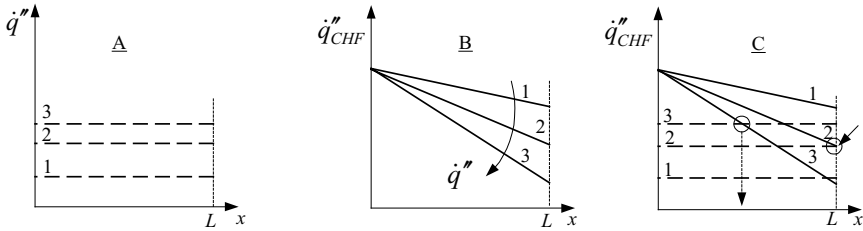
Figure Vb.5.4. Effect of uniform heat flux on critical heat flux

Expectedly, the critical heat flux correlations show that CHF inversely depends on heat flux. This is shown in figure B. Superimposing figures A and B, we obtain figure C. Let's examine this figure. We observe that at low heat flux (dashed and solid lines 1), CHF does not occur. When we increase heat flux, (dashed and solid lines 2), they intersect right at the tube exit. If we further increase heat flux, CHF occurs at a lower part of the tube (intersection of dashed and solid lines 3). The point at which CHF occurs moves towards the exit of the tube, as the heat flux is reduced. Hence, in uniformly heated channels, CHF always occurs first at the exit of the channel. Let's us now examine the case of non-uniform heat flux, which is the case in the core of nuclear plants. Since neutron flux has a sinusoidal distribution in the axial direction, heat flux has also a sinusoidal distribution, as shown in Figure Vb.5.5 (A). In this figure, two curves are shown for low heat flux (curve 1) and for high heat flux (curve 2). CHF versus tube length for the same two heat fluxes is shown in figure B. We obtain figure C by superimposing figures A and B. As seen in figure C, CHF occurs first in the upper part of the tube. As heat flux is increased, then CHF occurs at other locations along the tube.
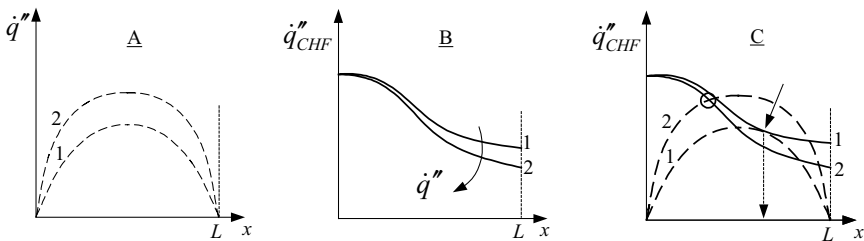

Figure Vb.5.5. Effect of non-uniform heat flux on critical heat flux

5.5. Transition Flow Boiling

In temperature controlled flow boiling, transition to film flow boiling occurs when the local heat flux exceeds the CHF. In this regime, heat transfer alternates between nucleate and film boiling regimes. The heat transfer coefficient for transition boiling may be calculated from the McDonough correlation:

$$\dot{q}'' = \dot{q}''_{CHF} - C_1(T_s - T_{s,CHF})$$ Vb.5.10

where T_s and $T_{s,CHF}$ are surface temperatures corresponding to $\dot{q}''$ and $\dot{q}''_{CHF}$, respectively. Coefficient C_1 is given as a function of pressure. If $P > 1200$ psia, then $C_1 = 1180.8 - 0.252(P - 1200)$. Otherwise, $C_1 = 1180.8 - 0.801(P - 1200)$. The heat transfer coefficient for transition boiling is obtained by dividing $\dot{q}''$ calculated from Equation Vb.5.10 by $(T_s - T_{sat})$. The transition boiling correlation is valid until the heat flux calculated from Equation Vb.5.10 becomes smaller than the heat flux corresponding to stable film flow boiling. More recently, Cheng suggested a similar correlation:

$$\dot{q}'' = \dot{q}''_{CHF} \left[(T_s - T_{sat}) / (T_{s,CHF} - T_{sat}) \right]^n$$ Vb.5.11

where for low-pressure $n = -1.25$. Bjornard tied the transition heat flux to CHF and MSFB:

$$\dot{q}'' = C_1 \dot{q}''_{CHF} + (1 - C_1)\dot{q}''_{MSFB}$$ Vb.5.12

where coefficient C_1 itself is tied to the T_{CHF} and T_{MSFB} as $C_1 = [(T_{MSFB} - T_s)/(T_{MSFB} - T_{CHF})]^2$.

5.6. Film Flow Boiling

The heat transfer coefficient in stable film flow boiling may be calculated from the correlation suggested by Dougal-Rohsenow. This correlation is a Reynolds number-modified Dittus-Boelter correlation given as:

$$h = (0.023k / D)\left[\text{Re}(v/v_g) \right]^{0.8} \text{Pr}^{0.4}$$ Vb.5.14

The appearance of v/v_g makes Equation Vb.5.14 also applicable to the flow of single-phase vapor.

QUESTIONS

– What are the three conditions for bubble equilibrium? Why should the liquid be superheated?
– What is the difference between boiling by a heated surface and boiling by reducing the pressure of a saturated liquid? Why is Equation Vb.2.1 not sufficient to predict the degree of superheat required for nucleation?
– A heated plate is immersed in water. The temperature profiles of the thermal sub-layer and bubble as given by the Clausius-Clapeyron relation do not intersect. What should be done to start nucleation?
– Can we construct the entire boiling curve in a power-controlled process?
– In which medium (liquid or vapor) is the contact angle measured?
– Is it correct to say that the heat flux in nucleate pool boiling is a function of surface conditions, surface superheat, and pressure?

– Consider two electrically heated stainless steel surfaces. One surface is mechanically polished and the other is ground and polished. Both surfaces are maintained at the same temperature and boiling with the same liquid is taking place at the same pressure. Which surface requires higher power?
– Since ρ, h_{fg}, T_{sat}, and σ are functions of pressure, can we conclude that $\dot{q}''$ in pool nucleate boiling increases as pressure increases?
– Why does surface aging decrease the rate of surface heat flux?
– Explain the effect of non-condensable gases on boiling and on condensation heat transfer.
– Since g appears in boiling correlations, can we conclude that heat flux is a function of gravity?
– Why is boiling heat flux not affected by surface roughness in the film boiling mode?
– What is the difference between dryout CHF and DNB CHF? In what type reactor is DNB a concern?
– What effect does mass flux have on critical heat flux?
– Can transition boiling be experienced in heat flux controlled boiling?
– Is the Chen correlation for heat transfer coefficient applicable to post-CHF heat transfer?
– In a uniformly heated tube, at what location is CHF most likely to occur?

PROBLEMS

1. We noticed that the required degree of superheat depends both on the surface condition (the size of the nucleation sites) and the type of liquid (wetting versus non-wetting). Regarding the type of liquid, we want to examine two extreme cases. Find the required degree of superheat for nucleation for liquids that completely wet the surface ($\phi = 0°$). Similarly, find the degree of superheat for completely non-wetting liquids ($\phi = 180°$). [Ans.: For $\phi = 0$, there is no nucleation. For $\phi = 180$, no superheat is required].

2. Similar to the bubble growth in a conical cavity, the bubble growth in a cylindrical cavity is shown in the figure. Verify the accuracy of the plot of the inverse of bubble radius ($1/r$) versus bubble volume.
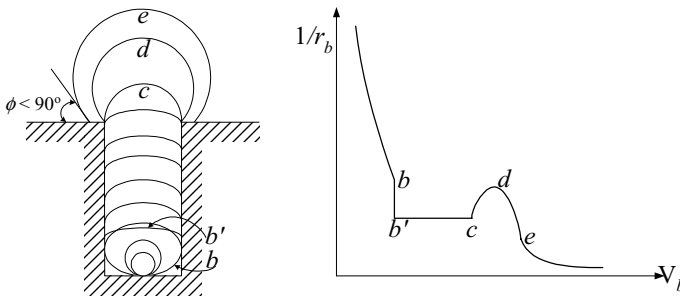
3. Find the degree of superheat $(T_s - T_{sat})$ for a horizontal flat plate in water at atmospheric pressure needed to cause nucleation if cavity sizes of 5 μm are present in the heated surface.

4. Use Equation Vb.2.4 to find the minimum degree of superheat for the onset of nucleate boiling.

5. Use Equation Vb.2.1 to compare the required degree of superheat for water and for sodium nucleation. [Hint: $\sigma_{sodium} > \sigma_{water}$].

6. A polished copper plate 0.05 m^2 in area is placed in water and electrically heated to 116 C. Find the rate of evaporation. [Ans.: $\dot{q}'' = 0.587\text{E}6$ W/m^2, $\dot{m} = 46.86$ kg/h, and $h = 36,688$ W/m^2·C].

7. A pan made of stainless steel contains water at atmospheric pressure. The pan diameter is 25 cm and its surface is mechanically polished. The pan is now heated while its surface is maintained at 116 C. Find the surface heat flux, the boil off (evaporation) rate, and the peak heat flux. [Ans.: $\dot{q}'' = 5.6\text{E}5$ W/m^2, $\dot{m} = 44$ kg/h, and $\dot{q}''_{CHF} = 1.27$ MW/m^2. Note that the operating heat flux is less than half of the peak heat flux hence, a safety factor of 1.27E6/0.56E6 = 2.26].

8. A platinum wire having a diameter of 1.27 mm is used to boil water at atmospheric pressure. The surface superheat is 650 C. Find h and $\dot{q}''$. [Ans.: $h_{conv} = 298$ W/m^2·C, $h_{total} = 368$ W/m^2·C, 240 kW/m^2]

9. Use the Rohsenow pool boiling correlation to find the heat flux at which incipient boiling occurs. The natural convection heat flux is given as $\dot{q}'' = 2.63\Delta T^{1.25}$ kW/m^2. Use water ($C_{sf} = 0.0132$) at $P = 3.5$ MPa. [Ans.: 4.2 kW/m^2].

10. A pool of liquid nitrogen at atmospheric pressure is used to cool an electronic device that generates a constant amount of heat. As the temperature of the device is unacceptably high, the following measures are proposed to lower the temperature:
a) substitute liquid hydrogen (a lower boiling point) for nitrogen
b) increase the heat transfer area by a factor of three
c) do both a and b.
Use the given data and recommend the course of action that should be followed.
Data: $(T_{Wall})_{Initial} = 1000$ R, $\dot{q}'' = 150,000$ Btu/h·ft^2, properties in British Units are:

	T_{sat}	h_{fg}	ρ_v	ρ_l	σ	k_v	k_l	μ_v	μ_l
H_2	37	190	0.084	4.50	1.45E-4	0.0080	0.067	0.0027	0.032
N_2	140	86	0.280	50.0	5.90E-4	0.0034	0.088	0.0130	0.440

where T_{sat} (R), h_{fg} (Btu/lbm), ρ (lbm/ft^3), σ (lbf/h), k (Btu/h·ft·F), μ (lbm/h·ft).

You may use the Rohsenow and Griffith correlation for critical heat flux: $\dot{q}''_{CHF} = 143\rho_v h_{fg} (\Delta\rho / \rho_v)^{0.6}$.

11. A tank of water at atmospheric pressure is heated by an electric resistance heater. The voltage to the heater is held constant at 1000 V. Over the range of interest, the resistance of the heater in British unites can be expressed as $R(T) = -21.07 + 0.11585T$ where T is in F and R is in ohms. The water is heated to saturation. At some location the heater temperature reaches 250 F, at which point CHF occurs, and the boiling regime changes to film boiling. Find the heat flux and the heater temperature at which the heater will be operating after this occurs. Data: $d_{Heater} = 0.25$ in, $A_{Heater} = 1$ ft^2. Neglect radiation effects.

12. Water at a rate of 1 lbm/s flows in a vertical heated tube ($d = 1.5$ in). System pressure is 1250 psia. Find heat flux at a point where steam quality is 15% and surface superheat is 12 F. [Ans.: $\dot{q}'' = 108$ kW/m^2]

13. Water flows at a rate of 0.1 kg/s in a tube with a diameter of 250 mm. The tube is heated uniformly at a rate of 135 kW/m^2. Find the wall temperature at a location where $T_{sat} = 180$ C and $x = 25\%$. [Ans.: $T_s = 188$ C]

14. Consider the case of liquid flow in a uniformly heated channel. Initially, heat flux is so low that it only increases the liquid sensible heat. We then start to increase heat flux until water starts to boil. We keep increasing heat flux until eventually we attain a specific value for heat flux at which the tube first experiences CHF. Under this condition at what point does CHF first occur?

15. A test tube for boiling water has a diameter of 25 mm. Water at a rate of 1000 kg/h enters the uniformly heated tube. If pressure is 7.5 MPa, find the heat transfer coefficient and heat flux at a location where mixture quality is 0.25. $\Delta T_{sat} = 10$ C. [$h_c = 18,193$ W/m·K, $h_n = 6,669$ W/m·K, $\dot{q}'' = 249$ kW/m^2]

16. Water at a rate of 0.25 lbm/s flows in a vertical heated tube having a diameter of 0.5 inches. Pressure in the tube is 900 psia. Find the heat flux at a point where the mixture enthalpy is 640 Btu/lbm and the surface superheat is 6 F. [Ans.: $\dot{q}'' = 97.6$ kW/m^2].

17. Two simple correlations for nucleate flow boiling (subcooled and saturated) of water at 500 psia $\leq P \leq 1000$ psia are given by Jens-Lottes and by Thom-1966. These correlations in British units are:

Jens-Lottes: $\qquad\qquad\qquad\qquad \dot{q}'' / 1.\text{E6} = \text{Exp}(4P / 900)\Delta T_{sat}^4 / 60^4$

Thom: $\qquad\qquad\qquad\qquad\qquad \dot{q}'' / 1.\text{E6} = \text{Exp}(2P / 1260)\Delta T_{sat}^2 / 72^4$

where $\dot{q}''$ is in Btu/hr·ft^2, P is in psia, and T is in F. These correlations in SI units become:

Jens-Lottes: $\qquad\qquad\qquad\qquad \dot{q}'' / 1.\text{E6} = \text{Exp}(4P / 6.2)\Delta T_{sat}^4 / 25^4$

Thom: $\qquad\qquad\qquad\qquad\qquad \dot{q}'' / 1.\text{E6} = \text{Exp}(2P / 8.7)\Delta T_{sat}^2 / 22.7^4$

where $\dot{q}''$ is in W/m^2, P is in MPa, and T is in C. Use these correlations to compare the results with the Chen correlation for $P = 800$ psia, $\Delta T_{sat} = 10$ F, and steam quality equal to 0.1.

18. In flow boiling, we often need to find the surface temperature and its location at which subcooled boiling begins. Although such local temperature for the incipience of subcooled boiling is not a single fixed temperature, we can estimate its value from the following relation:

$$T_{SB} = T_{sat} + (\Delta T_{sat})_{\text{J-L}} - (\dot{q}'' / h)$$

where $(\Delta T_{sat})_{\text{J-L}}$ is found from the Jens-Lottes correlation (see Problem 9). Find the location and value of the surface temperature for the following case:

Water enters a heated pipe of 0.7 in diameter at $T = 525$ F, $P = 1000$ psia, and $V = 8$ ft/s. Surface heat flux is uniform at a rate of 1000 Btu/h·ft^2·F. [Ans.: $(T_s)_{\text{incipient boiling}} = 547.8$ F].

Vc. Condensation

Similar to boiling, condensation is another mode of heat transfer, which is associated with a phase change. Thus for constant system pressure, heat transfer takes place at constant fluid temperature. While boiling requires heat addition, in condensation, heat should be removed so that the process can take place. Such heat removal may be accomplished by employing a coolant or by transferring heat to a solid. Condensers are important components of steam power plants, refrigerators, and chemical plants. We begin this chapter with the definition of terms pertinent to condensation heat transfer.

1. Definition of Condensation Heat Transfer Terms

Sensible energy ($c_p\Delta T$) refers to the energy transfer due to the change in temperature.

Latent energy (h_{fg}) refers to the heat of vaporization, a process during which change of phase takes place at constant temperature. The latent energy is also known as latent heat.

Condensation is a process during which vapor changes phase and becomes liquid if vapor temperature is reduced to below the saturation temperature. If vapor also includes noncondensable gases, the saturation temperature corresponds to the condensable gas partial pressure. The condensable gas we consider in this chapter is steam. Modes of condensation are described below and shown in Figure Vc.1.1 (a) through (e).

Homogenous condensation is a mode of condensation, which occurs within the vapor field, where vapor forms tiny droplets of liquid suspended in the bulk of the vapor to form a fog (Figure a). At the formation, the drops are very small and fall so slowly that they can be considered suspended in the bulk vapor. As the concentration of these drops increases, they combine to form larger drops, falling as rain (*rainout*). If the vapor also contains gases, the fog is generated when the bulk vapor becomes supersaturated (relative humidity > 100%). That is to say that the vapor temperature drops below the saturation temperature at the steam partial pressure or the steam pressure is greater than the saturation pressure at the vapor temperature. A similar phenomenon, but for a liquid, is *flashing*, which occurs when the liquid temperature is above the saturation temperature at the total pressure.

Direct contact condensation is a mode of condensation where vapor is condensed directly on colder liquid. Examples for such mode of condensation include quench-tank of a PWR (Figure b) and the suppression pool of a BWR. Another example includes condensation of steam on the spray droplets.

Heterogeneous condensation occurs on a cooler surface (Figures c, d, and e). Heterogeneous condensation is the basis for the operation of condensers. During

the condensation process, the latent heat associated with the phase change is trans-
ferred to the cooler surface.

Dropwise condensation is a type of heterogeneous condensation (Figure c)
where drops randomly appear on a cooler surface placed in the bulk vapor. This
generally happens if the surface is not clean or the liquid does not wet the surface.
Rate of heat transfer in dropwise condensation is very high due to the high expo-
sure of surface area to the vapor. However, the tiny drops would eventually join,
reducing exposed surface area for condensation. Liquid wet-ability is discussed in
Chapter Vb.

Film condensation occurs when the liquid, which is formed from the conden-
sation of vapor, wets a clean and uncontaminated cooler surface, blanketing it with
a smooth film. In vertical plates, the thickness of the film increases as the conden-
sate flows downward (Figures d and e). Appearance of the film on the surface re-
duces the effectiveness of condensation heat transfer, due to the temperature gra-
dient across the film and the associated thermal resistance of the film. In this
chapter, we consider only film condensation.

Jakob number, after Maxim Jakob, is the ratio of sensible heat to the latent
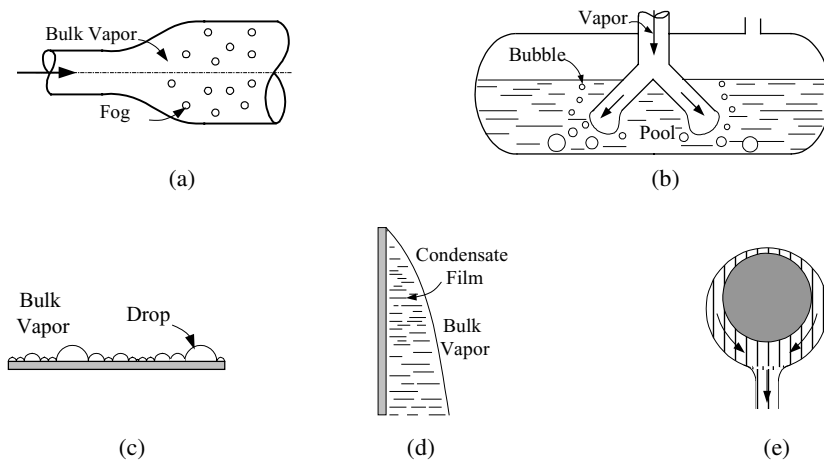heat, $Ja = c_p\Delta T/h_{fg}$.

Figure Vc.1.1. Various modes of condensation

2. Analytical Solution

To find an analytical solution for the heat transfer coefficient in condensation, we
consider the formation of a film of condensate on the cold surface of a vertical
plate placed in a vapor. The vapor generally includes noncondensable gases. As
shown in Figures Vc.2.1.(a and b) , the film thickness increases as liquid flows

down since more vapor condenses on the film. Liquid velocity is zero at the wall, increasing to its maximum value at the edge of the boundary layer. Liquid temperature approaches surface temperature near the wall and increases to saturation temperature at the edge of the boundary layer. Nusselt's derivation for film condensation now follows.
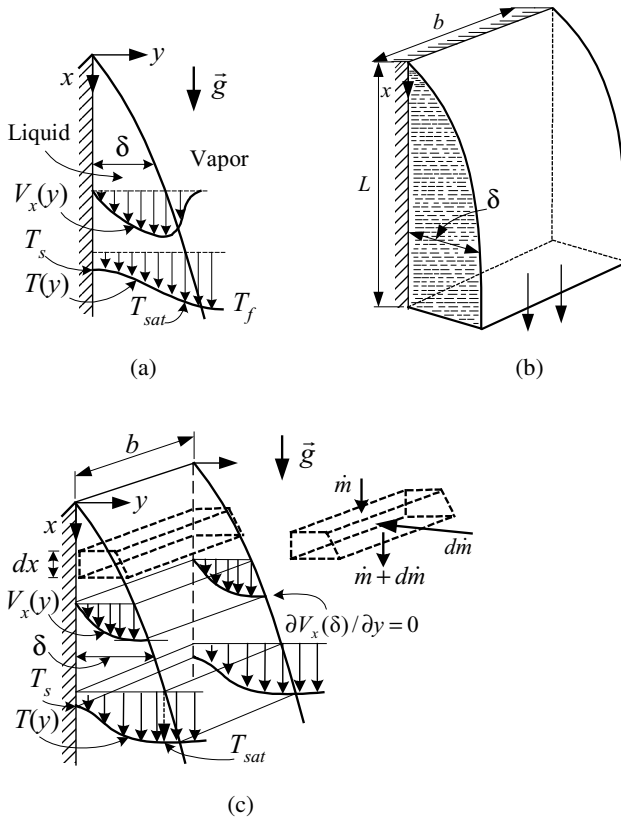

(a)   (b)

(c)

Figure Vc.2.1. (a) Film of condensate as boundary layer and (b) Nusselt model of the condesate film

2.1. Nusselt Derivation of Film Condensation

To be able to derive an analytical solution, several simplifying assumptions are made, per Nusselt. First we assume that vapor does not contain any noncondensable gas. Second, we assume that the flow of the film is laminar with thermal properties independent of temperature. Finally, we assume the shear stress at the edge of the boundary layer is negligible ($\partial u(\delta)/\partial y = 0$) and temperature profile in the film is linear. The governing equation for the hydrodynamic boundary layer is Equation IIIa.3.20-1, which reduces to:

$$0 = -\frac{1}{\rho}\frac{\partial P}{\partial x} + v\frac{\partial^2 V_x}{\partial y^2} + \frac{1}{\rho}X \qquad\qquad \text{Vc.2.1}$$

where the body force in the film is now $X = \rho_f g$ and the pressure gradient is $dP/dx = \rho_g g$. Substituting in Equation Vc.2.1, yields:

$$\frac{\partial^2 V_x}{\partial y^2} = -g(\rho_f - \rho_g)/\mu_f \qquad\qquad \text{Vc.2.2}$$

Integrating Equation Vc.2.2 and using the boundary conditions of $V_x(0) = 0$ and $\partial V_x(\delta)/\partial y = 0$ we find:

$$V_x(y) = \frac{g(\rho_f - \rho_g)\delta^2}{\mu_f}\left[\frac{y}{\delta} - \frac{1}{2}\left(\frac{y}{\delta}\right)^2\right] \qquad\qquad \text{Vc.2.3}$$

Note that V_x is also a function of x since $V_x = f[\delta(x)]$. Having the velocity profile, mass flow rate is:

$$\dot{m}(x) = \int_0^{\delta(x)} \rho V_x(y)bdy = bg\rho_f(\rho_f - \rho_g)\delta^3/(3\mu_f) \qquad\qquad \text{Vc.2.4}$$

Both V_x and $\dot{m}$ are functions of δ, itself an unknown. To find δ, we use an energy balance for the control volume of Figure Vc.2.1. At steady state, the energy entering the control volume ($d\dot{m}\,h_{fg}$) is equal to the energy leaving the control volume and entering the colder surface, $d\dot{Q} = k_f(bdx)(T_{sat} - T_s)/\delta$. Setting these equal and using Equation Vc.2.4, we find that $[bg\rho_f(\rho_f - \rho_g)\delta^2/\mu_f]h_{fg}\,d\delta/dx = k_f(bdx)(T_{sat} - T_s)/\delta$. So that δ becomes:

$$\delta(x) = \left(\frac{4k_f\mu_f(T_{sat} - T_s)x}{g\rho_f(\rho_f - \rho_g)h_{fg}}\right)^{0.25} \qquad\qquad \text{Vc.2.5}$$

Having δ, we can find both V_x and $\dot{m}$ as explicit functions of x. Since $h = k_f/\delta$ and $Nu = hL/k_f$, we then have both of these parameters also as functions of x. Integrating from x to L, we can find $\overline{h}$ and $\overline{Nu}$:

$$\overline{h}_L = 0.943\left(\frac{g\rho_f(\rho_f - \rho_g)k_f^3 h'_{fg}}{\mu_f(T_{sat} - T_s)L}\right)^{0.25} \qquad\qquad \text{Vc.2.6}$$

Note that in Equation Vc.2.6, we replaced h_{fg} by $h'_{fg} = h_{fg}(1 + 0.68Ja)$, per Rohsenow's recommendation. This is to account for two effects: cooling of the film below the saturation temperature and the non-linear temperature profile in the

film. Total rate of heat transfer to the plate is $\dot{Q} = \bar{h}_L A(T_{sat} - T_s)$ and total rate
of condensate produced is $\dot{m} = \dot{Q}/h'_{fg}$. To determine a condensation Reynolds
number, we use Equation III.6.3, $Re = 4\dot{m}/(\pi\mu D)$, which is appropriate for con-
densation on a vertical cylinder. For a flat plate we have $\pi D \equiv b$ hence,
$Re = 4\dot{m}/(\mu b)$. Substituting for $\dot{m}$ in term of $\dot{Q}$, we obtain the Reynolds num-
ber as $Re_L = 4\,\bar{h}_L L(T_{sat} - T_s)/(h_{fg}\mu_f)$ where L is the plate length. Flow is laminar if
$Re_L < 30$. For sufficiently large vertical plates the flow may become turbulent.
For fully turbulent flow $Re > 1800$ and in the range of $30 < Re < 1800$, the con-
densate film becomes wavy and hence, referred to as the wavy laminar region.

Example Vc.2.1. A vertical flat plate 1.2 ft long and 2 ft wide is maintained at
424.8 F and exposed to saturated steam at 450 psia. Find the total rate of heat
transfer to the plate and the condensate mass flow rate.

Solution: We first find T_{sat}(450 psia) = 456.4 F then T_{film} = (456.4 + 424.8)/2 =
440.6 F to find the following:
For steam at T_{sat} = 456.4 F: h_{fg} = 768.2 Btu/lbm and ρ_v = 0.968 lbm/ft^3
For water at T_{film} = 440.6 F: k_f = 0.37 Btu/ft·h·F, μ_f = 0.285 lbm/ft·h,
ρ_f = 52 lbm/ft^3, c_{pf} = 1.1 Btu/lbm·F

Since $T_s << T_{sat}$ we need to find $h'_{fg} = h_{fg}(1 + 0.68Ja)$

Ja = 1.1 × (456.4 – 424.8)/768.2 = 0.045. Thus, h'_{fg} = 791.7 Btu/lbm

$$\bar{h}_L = 0.943\left[\frac{g\rho_f(\rho_f - \rho_g)k_f^3 h'_{fg}}{(\mu_f(T_{sat} - T_s)L)}\right]^{0.25}$$

$$= 0.943\left[\frac{(32.2 \times 3600^2) \times 52(52 - 0.968) \times 0.37^3 \times 791.7}{0.285 \times (456.4 - 424.8) \times 1.2}\right]^{0.25}$$

$\bar{h}_L$ = 1342.6 Btu/ft^2·h·F

$\dot{Q} = \bar{h}_L A(T_{sat} - T_s)$ = 1342.6 × (1.2 × 2) × (456.4 - 424.8) = 101,823 Btu/h.

$\dot{m} = \dot{Q}/h'_{fg}$ = 101,823/791.7 = 0.036 lbm/s.

Example Vc.2.2. A steel plate 1/8 in. thick with L = 10 ft is placed in saturated
steam at 1 atm. At time zero, T_{s0} = 200 F. Find the time when $T_s = T_{sat}$. For steel,
ρ = 488 lbm/ft^3, c_p = 0.1 Btu/lbm·F, and k = 26.5 Btu/ft·h·F. For steam, ρ_f = 59.8
lbm/ft^3, ρ_g = 0.04 lbm/ft^3, and h_{fg} = 970 Btu/lbm.

Solution: For this transient problem, we may estimate the plate temperature, from an energy balance using the lumped capacitance approach:

$$\frac{d(\rho c V T_s)}{dt} = hA\left(T_{film} - T_s\right)$$

In this relation, $T_{film} \approx (T_{sat} + T_s)/2$ and h in Equation Vc.2.6 may be written as:

$h = \zeta(T_{sat} - T)^{-1/4}$ where $\zeta = 0.943\left(g\rho_f(\rho_f - \rho_g)k_f^3 h'_{fg}/(\mu_f L)\right)^{0.25}$

Assuming $\theta = T_{sat} - T_s$, the above energy balance simplifies to:

$\int d\theta/\theta^{3/4} = -(\zeta A/2\rho c V)\int dt$

Integrating and setting $\theta = 0$, we find $t = \theta_0^{1/4}/(2\zeta A/\rho c V)$

$\theta_0 = (T_{sat} - T_{s0}) = 212 - 200 = 12$ F, $A/V = 12/0.125$ ft, $\rho c = 48.8$ Btu/ft³·F
For the condensate layer, we find film properties at $T_{film} = 209$ F

$\zeta = 0.943\left(32.2 \times 3600^2 \times 59.89(59.89 - 0.035)0.39^3 \times 970/(0.69 \times 10)\right)^{0.25} = 1772.5$ Btu/ft²·h·F³/⁴

$t = (12)^{1/4}/[2 \times 1772.5 \times 12/(44.8 \times 0.125)] = 14$ s

3. Empirical Solution

Application of the empirical solutions depends on the value of the Reynolds number. If we substitute for mass flow rate, the Reynolds number can also be written as $Re_\delta = 4\dot{m}/(\mu b) = 4\rho \overline{V}_x \delta/\mu$ where the average film velocity is used and the plate width (b) cancels out from the numerator and the denominator.

3.1. Condensation on Vertical Plates and Cylinders

By defining a condensation Nusselt number, Nu_c also referred to as the *condensation number*, we may express the heat transfer coefficient in terms of the Reynolds number. These are shown in the table below. In the wavy laminar region, being the transition region between laminar and turbulent, the Kutateladze correlation and in the turbulent region the Labuntsov correlation are recommended.

Flow Regime	Range of Reynolds No.	Nusselt No.	Equation
Laminar	$Re_\delta \leq 30$	$Nu_c = 1.47\,Re_\delta^{-1/3}$	Vc.3.1
Wavy laminar	$30 \leq Re_\delta \leq 1800$	$Nu_c = Re_\delta/[1.08\,Re_\delta^{1.22} - 5.2]$	Vc.3.2
Turbulent	$1800 \leq Re_\delta$	$Nu_c = Re_\delta/[8750 + 58\,Pr^{-0.5}(Re_\delta^{0.75} - 253)]$	Vc.3.3

$Nu_c = \overline{h}_L(v_f^2/g)^{1/3}/k_f$ where $v = \mu/\rho$.

Example Vc.3.1. A vertical flat plate, 1.1 m long and 0.5 m wide maintained at 50 C is exposed to saturated steam at 0.5 bar. Find the total rate of heat transfer to the plate and the condensate mass flow rate.

Solution: We first find T_{sat}(0.5 bar) = 81.33 C then T_{film} = (81.33 + 50)/2 = 65.67 C:

For steam at T_{sat} = 81.33 C: h_{fg} = 2305.4 kJ/kg and ρ_v = 0.308 kg/m^3

For water at T_{film} = 65.67 C: k_f = 0.659 W/m·K, μ_f = 429E-6 N·s/m^2, ρ_f = 980 kgm/m^3, c_{pf} = 4.2 kJ/kg·K

Since $T_s \ll T_{sat}$, we calculate need to find h'_{fg} :

Ja = 4.2(81.33 − 50)/2305.4 = 0.057

h'_{fg} = 2305.4(1 + 0.68 × 0.057) = 2395 kJ/kg

$$\bar{h} = 0.943\left(\frac{9.8\times980(980-0.308)\times0.659^3\times2395E3}{429E-6\times(81.33-50)\times1.1}\right)^{0.25} = 4310 \text{ W/m}^2\text{·K}$$

$$\dot{Q} = \bar{h}A(T_{sat}-T_s) = 4310\,(1.1\times0.5)\times(81.33-50) = 74{,}268 \text{ W}$$

$$\dot{m} = \dot{Q}/h'_{fg} = 74{,}268/2.395E6 = 0.031 \text{ kg/s}$$

Finding $\text{Re}_\delta = 4\dot{m}/\mu_f b = 4 \times 0.031/(429E-6 \times 0.5) = 578$ shows the flow regime is actually wavy laminar. We should then use the heat transfer coefficient based on the Kutateladze correlation as shown below:

$$\bar{h}_L = \frac{\dot{m}h'_{fg}}{A(T_{sat}-T_s)} = \frac{\text{Re}_\delta(\mu_f b)h'_{fg}}{4A(T_{sat}-T_s)} = \frac{\text{Re}_\delta}{1.08\text{Re}_\delta^{1.22}-5.2}\frac{k_f}{(v_f^2/g)^{1/3}}$$

Solving for Re_δ, we find Re_δ = 716.5. Using this in the Kutateladze correlation, we find:

$\bar{h}_L$ = 716.5 × 0.659/{(1.08 × 716.5$^{1.22}$ −5.2)[(429E-6/980)^2/9.8]$^{1/3}$} = 5340 W/m^2·K

Revised values for $\dot{Q}$ and $\dot{m}$ are:

$$\dot{Q} = 5340\,(1.1\times0.5)\times(81.33-50) = 92{,}016 \text{ W}$$

$$\dot{m} = 92{,}016/2.395E6 = 0.0384 \text{ kg/s.}$$

The same procedure used for vertical flat plates is applicable to vertical cylinders if $\delta_L \ll 0.5D$.

3.2. Condensation on Spheres, Horizontal Cylinders and on Banks of Tubes

Correlations similar to Equation Vc.2.6 are obtained for condensation on radial systems (Dhir & Lienhard):

$$\overline{h}_D = C\left(\frac{g\rho_f(\rho_f - \rho_g)k_f^3 h'_{fg}}{\mu_f(T_{sat} - T_s)D}\right)^{0.25}$$ Vc.3.4

where C = 0.815 for condensation on spheres and C = 0.729 for condensation on horizontal tubes. Equation Vc.3.4 is also applicable to condensers, which consist of banks of horizontal tubes with cold fluid flows inside the tubes and vapor condenses on the tubes. For film condensation inside horizontal tubes, C = 0.555.

Example Vc.3.2. Find the condensate flow rate for the following data of a power plant condenser: P = 2 in Hg, D = 1¼ in, L = 28.5 ft, N_{tube} = 16500, T_s = 75 F.

Solution: We first find T_{sat}(2 in Hg) = 101 F then T_{film} = (101 + 75)/2 = 88 F

For steam at T_{sat} = 101 F: h_{fg} = 1036 Btu/lbm and ρ_g = 0.003 lbm/ft^3
For water at T_{film} = 88 F: k_f = 0.36 Btu/ft·h·F, μ_f = 1.9 lbm/ft·h, ρ_f = 62 lbm/ft^3, c_{pf} = 1 Btu/lbm·F

Since $T_s \ll T_{sat}$ we need to find $h'_{fg} = h_{fg}(1 + 0.68\text{Ja})$

Ja = $1 \times (101 - 75)/1036$ = 0.025 and h'_{fg} = 1054 Btu/lbm

e now use Equation Vc.3.4 to find the average heat transfer coefficient over a single tube:

$\overline{h}_D$ = 0.729 × {(32.2 × 3600^2) × 62^2 × 0.36^3 × 1054/[1.9 × (101 – 75) × (1/12)]}$^{0.25}$ = 1525 Btu/ft^2·h·F

$\dot{Q} = \overline{h}_D A(T_{sat} - T_s)$ = 1525 × (π × 28.5 × 1.25/12) × (101 - 75) = 369,800 Btu/h

$\dot{m} = N_{tube}\dot{Q}/h'_{fg}$ = 16,500 × 368,800/1054 = 5.8 Mlbm/h.

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4. Condensation Degradation

The presence of even a small amount of noncondensable gas significantly degrades the rate of condensation heat transfer. As shown in Figure Vc.4.1, these gases tend to migrate and accumulate near the colder surface, reducing the partial pressure of the vapor and subsequently the corresponding saturation temperature of the vapor. For containment response analysis, safety regulations require the use of the Tagami correlation for the case of a LOCA and the Uchida correlation during a MSLB analysis.

Tagami Correlation

Tagami, an empirical correlation, applies during the forced convection period following the blowdown phase of a LOCA. The key parameter in the Tagami corre-

lation is therefore time t_p, which marks the end of the blowdown-induced forced convection period. As such, the Tagami correlation is only applicable up to time t_p. The natural convection phase of a blowdown must be analyzed by using the Uchida or the turbulent natural convection correlation depending on the value of the Ra number. If $h_{maximum}$ is the heat transfer coefficient corresponding to time t_p, then the heat transfer coefficients at other times $(t < t_p)$ are obtained as $h = h_{max}(t/t_p)$. According to Tagami, $h_{maximum}$ itself is calculated from: $h_{max} = C_T\left[U/(Vt_p)\right]^{0.62}$, where U is the total blowdown energy released during t_p and V is containment free volume. In SI units, U is in J, t_p in s, V in m^3, and C_T is 0.607. Time t_p is not known beforehand rather it should be obtained by iteration. A typical value for t_p given a large dry containment is about 13 seconds.

Uchida Correlation

The Uchida correlation is given as a table of heat transfer coefficient versus the ratio of air to steam mass. The maximum value is 1590 $W/m^2 \cdot K$ and the minimum value is 11.4 $W/m^2 \cdot K$.

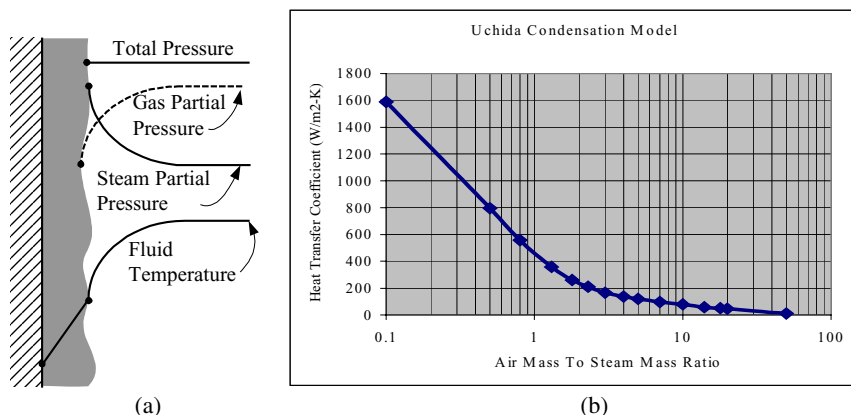(a)   (b)
Figure Vc.4.1. (a) Effect of noncondensable gas on pressure and (b) on h (Uchida model)

QUESTIONS

– Find the condensation mode for the following examples, a power plant condenser, steam condensation on spray droplets, steam condensation in a suppression pool, rain.
– Which condensation mode is more efficient, dropwise or film?
– Does dropwise condensation occur if a liquid wets the surface
– Can the heat transfer coefficient in condensation reach 10,000 $Btu/ft^2 \cdot h \cdot F$?
– Consider two condensers, one using horizontal tubes the other vertical tubes. Cold water flows inside the tubes. If all other parameters are identical which condenser is more efficient?

– Inside the condensate film, where can you find the minimum shear stress
– The flow of the condensate film beyond what Reynolds number becomes turbulent?

PROBLEMS

1. A vertical plate, 3 ft long and 5 ft wide is insulated from one side and the other side is exposed to steam at 15 psia. The plate temperature is maintained at 120 F. At what rate is condensate produced? [Ans.: Ja = 0.033, Re = 287.5, Nu = 7004, h = 878 Btu/ft^2·h·F, $\dot{Q}$ = 126 kW, and $\dot{m}$ = 0.116 lbm/s].

2. steam at 4 psia is condensing on a vertical plate 1 m long and 2 m wide. The plate is at 50 C. Find $\dot{m}$. [Ans.: Ja = 0.03, Re = 297, Nu 7621, h = 4968 W/m^2·K, $\dot{Q}$ = 278 kW, and $\dot{m}$ = 0.12 kgm/s].

3. A steam condenser consists of a square array of 529 horizontal tubes, 1 in diameter and 12 ft long. Tubes are maintained at 95 F to condense steam at 1 psia. Find the condensate production rate. [Ans.: 27 lbm/min.]

4. Find the condensation heat transfer coefficient for saturated Freon-12 at 50 C on a horizontal tube, having a diameter of 3 cm and maintained at 40 C. [Ans.: 1244 W/m^2·C].

5. Consider condensation of benzene vapor at 1 bar on a vertical flat plate of height 0.3 m. If the plate is kept at 60 C, find the condensation heat transfer coefficient. At 1 bar, T_{sat} = 80 C, ρ_l = 823 kg/m^3, ρ_g = 2.74 kg/m^3, h_{fg} = 398 kJ/kg, $c_{p,l}$ = 1.88 kJ/kg·K, μ_l = 321E-6 N·s/m^2, k_l = 0.131 W/m·K. [Ans. 1270 W/m^2·k].

6. Saturated steam at 1 atm is condensing on a horizontal tube at a rate of 300 kg/h. The tube is 2 m long and is maintained at 60 C. Find the tube diameter. [Ans.: 13 cm].

7. A steam power plant produces 2700 MWe at a thermal efficiency of 29%. Find the surface area and the number of tubes

8. A steel plate having L = 3 m, b = 1.5 m, and thickness of 0.5 cm is placed in steam at 1 atm. Initially, the plate is at 80 C. Plot the plate temperature versus time. Identify the simplifying assumptions made.

9. Water flows at a rate of 296 kg/h in a horizontal thin-walled tube (d = 0.025 m) at 25 C. Benzene vapor condenses at 1 bar on the tube. Find the rate of condensation per meter. [Ans.: By iteration 24 kg/m·h].