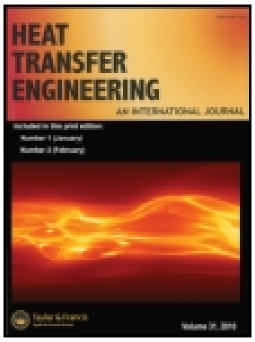

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Two-Phase Flow Modeling in Microchannels and Minichannels

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In this article, three different methods for two-phase flow modeling in microchannels and minichannels are presented. They are effective property models for homogeneous two-phase flows, an asymptotic modeling approach for separated two-phase flow, and bounds on two-phase frictional pressure gradient. In the first method, new definitions for two-phase viscosity are proposed using a one-dimensional transport analogy between thermal conductivity of porous media and viscosity in two-phase flow. These new definitions can be used to compute the two-phase frictional pressure gradient using the homogeneous modeling approach. In the second method, a simple semitheoretical method for calculating two-phase frictional pressure gradient using asymptotic analysis is presented. Two-phase frictional pressure gradient is expressed in terms of the asymptotic single-phase frictional pressure gradients for liquid and gas flowing alone. In the final method, simple rules are developed for obtaining rational bounds for two-phase frictional pressure gradient in minichannels and microchannels. In all cases, the proposed modeling approaches are validated using the published experimental data.

INTRODUCTION

The pressure drop in two-phase flow through microchannels and minichannels constitutes an important parameter because pumping costs could be a significant portion of the total operating cost. As a result, expressions are needed to predict the pressure drop in two-phase flow through microchannels and minichannels accurately.

Total pressure drop for two-phase flow in microchannels and minichannels has three different components. They are frictional, acceleration, and gravitational components. It is necessary to know the void fraction (the ratio of gas flow area to total flow area) to compute the acceleration and gravitational components. To compute the frictional component of pressure drop, either the two-phase friction factor or the two-phase frictional multiplier must be known [1].

There are two principal types of frictional pressure drop models in two-phase flow: the homogeneous model and the separated flow model. In the first, both liquid and vapor phases move at the same velocity (slip ratio = 1). Consequently, the homogeneous model has also been called the zero-slip model. The homogeneous model considers the two-phase flow as a single-phase flow having average fluid properties depending on mass quality. Thus, the frictional pressure drop is calculated by assuming a constant friction coefficient between the inlet and outlet sections. The prediction of the frictional pressure drop using the homogeneous model is reasonably accurate only for bubble and mist flows since the entrained phase travels at nearly the same velocity as the continuous phase. In the second, two-phase flow is considered to be divided into liquid and gas streams. Hence, the separated flow model has been referred to as the slip flow model. The separated model is popular in the power plant industry. Also, the separated model is relevant for the prediction of pressure drop in heat pump systems and evaporators in refrigeration. The success of the separated model is due to the basic assumptions in the model, which are closely met by the flow patterns observed in the major portion of the evaporators.

In this study, three different methods for two-phase flow modeling in microchannels and minichannels are presented. They are effective property models for homogeneous two-phase flows, an asymptotic modeling approach for separated two-phase flow, and bounds on two-phase frictional pressure gradient.

The literature review on two-phase flow modeling in microchannels and minichannels can be found in tabular form in a number of textbooks [2–6].

PROPOSED METHODOLOGIES

Homogeneous Property Modeling

In the first method, new definitions for two-phase viscosity are proposed [7] using a one-dimensional transport analogy

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between thermal conductivity of porous media [8] and viscosity in two-phase flow. The series and parallel combination rules for thermal conductivity of porous media are analogous to existing rules proposed by McAdams et al. [9], and Cicchitti et al. [10]. McAdams et al. [9], introduced the definition of two-phase viscosity (μ_m) based on the mass averaged value of reciprocals as follows:

$$\mu_m = \left(\frac{x}{\mu_g} + \frac{1-x}{\mu_l} \right)^{-1} \tag{1}$$

They proposed their viscosity expression by analogy to the expression for the homogeneous density (ρ_m). Equation (1) leads to the homogeneous Reynolds number (Re_m) being equal to the sum of the liquid Reynolds number (Re_l) and the gas Reynolds number (Re_g). In the realm of two-phase flow viscosity models, Collier and Thome [11] mentioned that the definition of μ_m proposed by McAdams et al. [9], Eq. (1), is the most common definition of μ_m. Cicchitti et al. [10] introduced the definition of two-phase viscosity (μ_m) based on the mass averaged value as follows:

$$\mu_m = x\mu_g + (1-x)\mu_l \tag{2}$$

They used the preceding definition of μ_m in place of the definition proposed by McAdams et al. [9]. The only reason for doing this, in addition to simplicity, was a reasonable agreement with experimental data.

Definitions for two-phase viscosity were generated by analogy to the effective thermal conductivity using the Maxwell–Eucken I and II models [12]. Maxwell–Eucken I [12] is suitable for materials in which the thermal conductivity of the continuous phase is higher than the thermal conductivity of the dispersed phase ($k_{cont} > k_{disp}$), like foam or sponge. In this case, the heat flow essentially avoids the dispersed phase. In the case of momentum transport, this is akin to a bubbly flow, where the dominant phase is the liquid. This definition for two-phase viscosity is:

$$\mu_m = \mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} \tag{3}$$

Maxwell–Eucken II [12] is suitable for materials in which the thermal conductivity of the continuous phase is lower than the thermal conductivity of the dispersed phase ($k_{cont} < k_{disp}$), like particulate materials surrounded by a lower conductivity phase. In this case, the heat flow involves the dispersed phase as much as possible. In the case of momentum transport, this is akin to droplet flow, where the dominant phase is the gas. This definition for two-phase viscosity is:

$$\mu_m = \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)} \tag{4}$$

A definition for two-phase viscosity was also generated by analogy to the effective thermal conductivity using the effective medium theory (EMT [13, 14]). The effective medium theory (EMT [13, 14]) is suitable for the structure that represents a heterogeneous material in which the two components are distributed randomly, with neither phase being necessarily continuous or dispersed. In the case of momentum transport, this averaging scheme seems reasonable given the unstable and random distribution of phases in a liquid/gas flow. This definition for two-phase viscosity is:

$$(1-x)\frac{\mu_l - \mu_m}{\mu_l + 2\,\mu_m} + x\frac{\mu_g - \mu_m}{\mu_g + 2\,\mu_m} = 0 \tag{5}$$

which may be rewritten to be explicit for μ_m:

$$\mu_m = 1/4\left(\begin{array}{l} (3x-1)\mu_g + [3(1-x)-1]\mu_l \\ + \sqrt{[(3x-1)\mu_g + (3\{1-x\}-1)\mu_l]^2 + 8\mu_l\mu_g} \end{array} \right) \tag{6}$$

Finally, a definition for two-phase viscosity was also proposed based on the arithmetic mean of Maxwell–Eucken I and II models [12]. This is proposed here as a simple alternative to the effective medium theory:

$$\mu_m = \frac{1}{2}\left[\mu_l \frac{2\mu_l + \mu_g - 2(\mu_l - \mu_g)x}{2\mu_l + \mu_g + (\mu_l - \mu_g)x} + \mu_g \frac{2\mu_g + \mu_l - 2(\mu_g - \mu_l)(1-x)}{2\mu_g + \mu_l + (\mu_g - \mu_l)(1-x)} \right] \tag{7}$$

It is clear that these new definitions satisfy the following two conditions: namely, (i) the two-phase viscosity is equal to the liquid viscosity at mass quality = 0% and (ii) the two-phase viscosity is equal to the gas viscosity at mass quality = 100%. These new definitions overcome the disadvantages of some definitions of two-phase viscosity such as the Davidson et al. definition [15], Owens's definition [16], and the García et al. definition [17, 18] that do not satisfy the condition at $x = 1$, $\mu_m = \mu_g$. For example, García et al. [17, 18] defined the Reynolds number of two-phase gas–liquid flow using the kinematic viscosity of liquid flow (ν_l) instead of the kinematic viscosity of two-phase gas–liquid flow (ν_m). They used this definition because the frictional resistance of the mixture was due mainly to the liquid. This was equivalent to defining μ_m as

$$\mu_m = \mu_l \left(\frac{\rho_m}{\rho_l} \right) = \frac{\mu_l \rho_g}{x\rho_l + (1-x)\rho_g} \tag{8}$$

From Eq. (8), it is clear that $\mu_m = \mu_l\rho_g/\rho_l$ at $x = 1$. This result leads to $\mu_m = 0.067\mu_g$ at $x = 1$ for an air–water mixture at atmospheric conditions.

These new definitions of two-phase viscosity can be used to compute the two-phase frictional pressure gradient using the homogeneous modeling approach.

It is desirable to express the two-phase frictional pressure gradient, $(dp/dz)_f$, versus the total mass flux (G) in a dimensionless form like the Fanning friction factor (f_m) versus the Reynolds number (Re_m). The Fanning friction factor (f_m) based on the homogeneous model (f_m) can be expressed as follows:

$$f_m = \frac{\rho_m (dp/dz)_f d}{2G^2} \tag{9}$$

$$\rho_m = \left(\frac{x}{\rho_g} + \frac{1-x}{\rho_l} \right)^{-1} \tag{10}$$

The Reynolds number based on the homogeneous model (Re_m) can be expressed as follows:

$$Re_m = \frac{Gd}{\mu_m} \tag{11}$$

Equations (10) and (11) represent the two-phase density based on the homogeneous model (ρ_m) and Reynolds number based on the homogeneous model (Re_m).

To satisfy a good agreement between the experimental data and well-known friction factor models, assessment of the best definition of two-phase viscosity among the different definitions (old and new) is based on the definition that corresponds to the minimum root mean square (RMS) error.

The fractional error (e) in applying the model to each available data point is defined as:

$$e = \left| \frac{Predicted - Available}{Available} \right| \tag{12}$$

For groups of data, the root mean square error, e_{RMS}, is defined as:

$$e_{RMS} = \left[\frac{1}{N} \sum_{K=1}^{N} e_K^2 \right]^{1/2} \tag{13}$$

For the case of microchannels and minichannels, the friction factor is calculated using the Churchill model [19], which allows for prediction over the full range of laminar–transition–turbulent regions. The Fanning friction factor (f_m) can be predicted using the Churchill model [19] as follows:

$$f_m = 2 \left[\left(\frac{8}{Re_m} \right)^{12} + \frac{1}{(a_m + b_m)^{3/2}} \right]^{1/12} \tag{14}$$

$$a_m = \left[2.457 \ln \left(\frac{1}{(7/Re_m)^{0.9} + (0.27\varepsilon/d)} \right) \right]^{16} \tag{15}$$

$$b_m = \left(\frac{37530}{Re_m} \right)^{16} \tag{16}$$

The Churchill model [19] is preferable since it encompasses all Reynolds numbers and includes roughness effects in the turbulent regime.

Asymptotic Modeling

In the second method, new two-phase flow modeling in microchannels and minichannels was proposed [20], based upon an asymptotic modeling method. Two-phase frictional pressure gradient is expressed in terms of the asymptotic single-phase frictional pressure gradients for liquid and gas flowing alone. Asymptotes appear in many engineering problems, such as steady and unsteady internal and external conduction, free and forced internal and external convection, fluid flow, and mass transfer. Often, there exists a smooth transition between two asymptotic solutions [21–24]. This smooth transition indicates that there is no sudden change in slope and no discontinuity within the transition region.

The asymptotic analysis method was first introduced by Churchill and Usagi [21] in 1972. After this time, this method of combining asymptotic solutions proved quite successful in developing models in many applications [24]. Recently, it has been applied to two-phase flow in circular pipes, minichannels, and microchannels [20]. Moreover, Awad and Butt have shown that the asymptotic method works well for petroleum industry applications for flows through porous media [25], liquid–liquid flows [26], and flows through fractured media [27].

The main advantage of the asymptotic modeling method in two-phase flow is taking into account the important frictional interactions that occur at the interface between liquid and gas because the liquid and gas phases are assumed to flow in the same channel. This overcomes the main disadvantage of the separate cylinders model [28] for two-phase flow.

Using the asymptotic analysis method, two-phase frictional pressure gradient $(dp/dz)_f$ can be expressed in terms of single-phase frictional pressure gradient for liquid flowing alone $(dp/dz)_{f,l}$ and single-phase frictional pressure gradient for gas flowing alone $(dp/dz)_{f,g}$ as follows:

$$\left(\frac{dp}{dz} \right)_f = \left[\left(\frac{dp}{dz} \right)_{f,l}^p + \left(\frac{dp}{dz} \right)_{f,g}^p \right]^{l/p} \tag{17}$$

Equation (17) reduces to $(dp/dz)_{f,l}$ and $(dp/dz)_{f,g}$ as $x = 0$ and 1, respectively.

If the two-phase frictional pressure gradient $(dp/dz)_f$ is presented in terms of the single-phase frictional pressure gradient for liquid flowing alone $(dp/dz)_{f,l}$, then the model can be expressed using the Lockhart–Martinelli parameter (X) as follows:

$$\left(\frac{dp}{dz} \right)_f = \left(\frac{dp}{dz} \right)_{f,l} \left[1 + \left(\frac{1}{X^2} \right)^p \right]^{l/p} \tag{18}$$

Equation (18) can be expressed in terms of a two-phase frictional multiplier liquid flowing alone (ϕ_l^2) as follows:

$$\phi_l^2 = \left[1 + \left(\frac{1}{X^2} \right)^p \right]^{l/p} \tag{19}$$

On the other hand, if the two-phase frictional pressure gradient $(dp/dz)_f$ is presented in terms of the single-phase frictional pressure gradient for gas flowing alone $(dp/dz)_{f,g}$, then the model can be expressed using the Lockhart–Martinelli parameter (X) as follows:

$$\left(\frac{dp}{dz}\right)_f = \left(\frac{dp}{dz}\right)_{f,g} [1 + (X^2)^p]^{1/p} \tag{20}$$

Equation (20) can be expressed in terms of a two-phase frictional multiplier for gas flowing alone (ϕ_g^2) as follows:

$$\phi_g^2 = [1 + (X^2)^p]^{1/p} \tag{21}$$

In this method, p is chosen as the value, which minimizes the root mean square (RMS) error, e_{RMS} (Eq. (13)), between the model predictions and the available data.

Bounds

In the third method, simple rules were developed for obtaining rational bounds for two-phase frictional pressure gradient in minichannels and microchannels [29]. This approach is very useful in design and analysis, as engineers can then use the resulting average and bounding values in predictions of system performance. The approach is also useful when conducting new experiments, since it provides a reasonable envelope for the data to fall within. The bounds are intended to provide the most realistic range of data and not firm absolute limits. Statistically, this is unreasonable as the upper and lower bounds would be far apart. The bounds are not fit to capture all data but rather a majority of data points, as some outlying points are due to experimental error. If a vast majority of data is within the bounds, then a reasonable expectation is realistically assured.

These bounds may be used to determine the maximum and minimum values that may reasonably be expected in a two-phase flow. Further, by averaging these limiting values an acceptable prediction for the pressure gradient is obtained, which is then bracketed by the bounding values:

$$\left(\frac{dp}{dz}\right)_{f,lower} \leq \left(\frac{dp}{dz}\right)_f \leq \left(\frac{dp}{dz}\right)_{f,upper} \tag{22}$$

The bounds model can be in the form of two-phase frictional pressure gradient versus mass flux at constant mass quality; they may also be presented in the form of a two-phase frictional multiplier, which is often useful for calculation and comparison needs. For this reason, development of lower and upper bounds in terms of a two-phase frictional multiplier (ϕ_l and ϕ_g) versus the Lockhart–Martinelli parameter (X) will also be presented.

Awad and Muzychka [30, 31] applied the bounds method for the case of turbulent/turbulent flow in large circular pipes because, in practice, both Re_l and Re_g are most often greater than 2,000. Faghri and Zhang [32] further commented that the use of bounds alleviates the uncertainty in the separated flow models. In the present study, the method is applied for the case of laminar/laminar flow in minichannels and microchannels because, in practice, both Re_l and Re_g are most often lower than 2,000.

The lower bound is based on the Ali et al. correlation [33] for laminar–laminar flow. This correlation is based on modification of a simplified stratified flow model derived from the theoretical approach of Taitel and Dukler [34] for the case of two-phase flow in a narrow channel. The equations of the lower bound are

$$\phi_l^2 = 1 + \frac{1}{X^2} \tag{23}$$

or

$$\phi_g^2 = 1 + X^2 \tag{24}$$

and

$$\left(\frac{dp}{dz}\right)_{f,lower} = \frac{2(fRe)G(1-x)\mu_l}{d_h^2\rho_l} * \left[1 + \left(\frac{x}{1-x}\right)\left(\frac{\rho_l}{\rho_g}\right)\left(\frac{\mu_g}{\mu_l}\right)\right] \tag{25}$$

The equations of the lower bound are equivalent to the Chisholm correlation [35] with $C = 0$. The physical meaning of the lower bound ($C = 0$) is that the two-phase frictional pressure gradient is the sum of the frictional pressure of liquid phase alone and the frictional pressure of gas phase alone. This means no pressure gradient caused by the phase interaction.

Although the data points are in laminar–laminar flow, they cover different flow patterns such as bubble, stratified, and annular. As the mass flow rate of the gas in two-phase flow increases, the flow pattern changes from bubble until it reaches annular at a high mass flow rate of gas. As mentioned in the literature, the Chisholm correlation [35] has a good accuracy for annular flow pattern. This is why the upper bound is based on Chisholm correlation [35] for laminar–laminar flow. The equations of the upper bound are

$$\phi_l^2 = 1 + \frac{5}{X} + \frac{1}{X^2} \tag{26}$$

or

$$\phi_g^2 = 1 + 5X + X^2 \tag{27}$$

and

$$\left(\frac{dp}{dz}\right)_{f,upper} = \frac{2(fRe)G(1-x)\mu_l}{d_h^2\rho_l} * \left[1 + 5\left(\frac{x}{1-x}\right)^{0.5}\left(\frac{\rho_l}{\rho_g}\right)^{0.5}\left(\frac{\mu_g}{\mu_l}\right)^{0.5} + \left(\frac{x}{1-x}\right)\left(\frac{\rho_l}{\rho_g}\right)\left(\frac{\mu_g}{\mu_l}\right)\right] \tag{28}$$

A simple model may be developed by averaging the two bounds. This is defined as follows:

$$\phi_l^2 = 1 + \frac{2.5}{X} + \frac{1}{X^2} \tag{29}$$

or

$$\phi_g^2 = 1 + 2.5X + X^2 \tag{30}$$

and

$$\left(\frac{dp}{dz}\right)_{f,av} = \frac{2(fRe)G(1-x)\mu_l}{d_h^2\rho_l} * \left[1 + 2.5\left(\frac{x}{1-x}\right)^{0.5}\left(\frac{\rho_l}{\rho_g}\right)^{0.5}\left(\frac{\mu_g}{\mu_l}\right)^{0.5} + \left(\frac{x}{1-x}\right)\left(\frac{\rho_l}{\rho_g}\right)\left(\frac{\mu_g}{\mu_l}\right)\right] \tag{31}$$

The equations of the mean model are equivalent to the Chisholm correlation [35] with $C = 2.5$.

This model can be applied for circular shapes using tube diameter d, as well as using hydraulic diameter d_h for noncircular shapes. For noncircular shapes, the Hagen–Poiseuille constant $(fRe) = 16$ will be changed. For example, for a rectangular channel with the aspect ratio of 0, the Hagen–Poiseuille constant $(fRe) = 24$, while for a rectangular channel with the aspect ratio of 1 (square channel), the Hagen–Poiseuille constant $(fRe) = 14.23$.

RESULTS AND DISCUSSION

Comparisons of the two-phase frictional pressure gradient versus mass flux from published experimental studies in minichannels and microchannels are undertaken using the old and new definitions of two-phase viscosity, after expressing the data in dimensionless form as Fanning friction factor versus Reynolds number. The published data include different working fluids such as R717, R134a, R410A, and propane (R290) at different diameters and different saturation temperatures. Also, examples of two-phase frictional multiplier (ϕ_l and ϕ_g) versus Lockhart–Martinelli parameter (X) using published data of different working fluids, such as air–water mixture and nitrogen–water mixture in laminar–laminar flow, from other experimental work are presented to validate the asymptotic model and the bounds model in dimensionless form.

Figures 1 and 2 show the Fanning friction factor (f_m) versus Reynolds number (Re_m) in minichannels and microchannels using one of the old definitions (McAdams et al. [9]) and one of the new definitions (Maxwell–Eucken II [12]) of two-phase viscosity on log-log scale. The sample of the published data includes Ungar and Cornwell's data [36] for R 717 flow at $T_s \approx$ 74°F (165.2°C) in a smooth horizontal tube at $d = 0.1017$ inches (2.583 mm), the Tran et al. data [37] for R134a flow at saturation pressure of 365 kPa and $x \approx 0.73$ in a smooth horizontal pipe at $d = 2.46$ mm, the Cavallini et al. data [38] for 410A flow at $T_s = 40$°C and $x = 0.74$ in smooth multi-port minichannels at hydraulic diameter of 1.4 mm, and Field and Hrnjak data [39] for propane (R 290) flow at reduced pressure of 0.23 and $G \approx$ 330 kg/m^2-s in a smooth horizontal pipe at hydraulic diameter of 0.148 mm. The literature data represented a wide range of fluid properties, across R717, R134a, R410A, and propane (R290). Equation (9) defines the measured Fanning friction factor, while Eqs. (14)–(16) represent the predicted Fanning friction factor. Table 1 presents e_{RMS}% values based on measured Fanning friction factor and predicted Fanning friction factor using the six different definitions of two-phase viscosity for this sample of the published data. It can be seen that two-phase viscosity based on the Maxwell–Eucken II model [12] gives the best agreement between the published data and the Churchill model [19] with a root mean square error (e_{RMS}) of 16.47%.

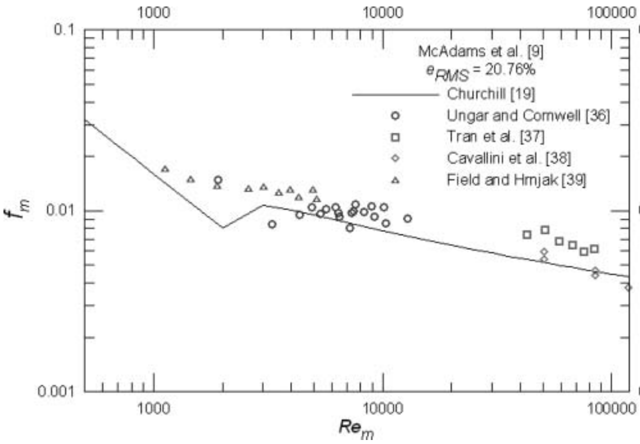

Figure 1 f_m versus Re_m in microchannels and minichannels using McAdams et al. [9] definition.

In Figure 2, it is interesting to observe that the fluids with the higher vapor–liquid density ratios, which were supposed to be more appropriate for the Maxwell–Eucken II model of homogeneous viscosity definition [12], might be thought to have better agreement. It can be seen from Figure 2 and Table 1 that the definition of effective viscosity based on the Maxwell–Eucken II model [12] appears to be more appropriate for defining two-phase flow viscosity in microchannels and minichannels. On

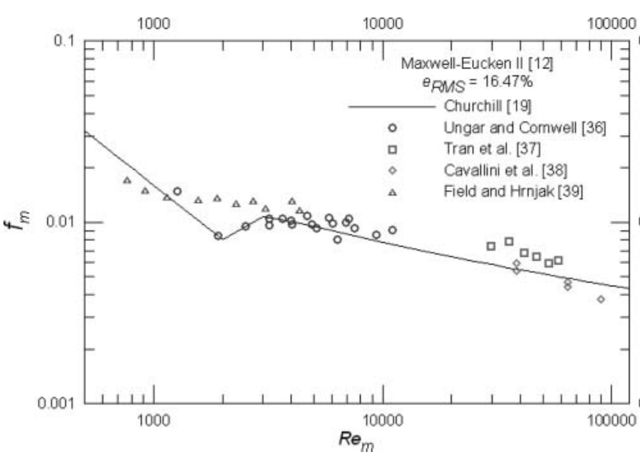

Figure 2 f_m versus Re_m in microchannels and minichannels using Maxwell–Eucken II [12] definition.

Table 1 e_{RMS}% Values based on measured Fanning friction factor and predicted Fanning friction factor in microchannels and minichannels using different definitions of two-phase viscosity

Definition	e_{RMS}
McAdams et al. [9]	20.76%
Cicchitti et al. [10]	31.06%
Maxwell–Eucken I [12]	24.78%
Maxwell–Eucken II [12]	16.47%
Effective medium theory (EMT [13,14])	23.60%
Arithmetic mean of Maxwell–Eucken I and II [12]	17.98%

Table 2 Values of the asymptotic parameter (p) in microchannels and minichannels at different conditions

Author	d (mm)	p	e_{RMS}	e_{RMS} $p = 1/2$
Lee and Lee [40]	0.78*	1/1.75	11.7%	14.07%
Chung and Kawaji [41]	0.1	1/1.7	13.44%	16.09%
Kawaji et al. [42]+	0.1	1/2.15	10.39%	11.34%
Kawaji et al. [42]++	0.1	1/2.55	11.65%	17.36%
Ohtake et al. [43]	0.32*	1/1.55	19.56%	24.16%
	0.42*		16.08%**	18.24%**
	0.49*			

*Hydraulic diameter.
**The two lower points are not taken into account.
+Gas in the main channel and liquid in the branch.
++ Liquid in the main channel and gas in the branch.

the basis of the data considered, a nominal 5–6% gain in accuracy can be achieved using the homogeneous flow modeling approach. When one considers the nature of the Maxwell–Eucken II definition, whereby the dominant phase is the lower viscosity phase, i.e., the gas, it is clear that this definition is most appropriate for liquid/gas mixtures that have very high density ratios. Thus, even for small mixture qualities, a significant portion of the flow volume is occupied by gas, making the Maxwell–Eucken II definition most appropriate.

Figure 3 shows ϕ_l versus Lockhart–Martinelli parameter (X) for laminar–laminar flow for different working fluids in smooth microchannels and minichannels of different diameters at different conditions using the present asymptotic model and the bounds model with the first three data sets in Table 2. Equation (18) represents the present asymptotic model with different values of p as shown in Table 2. Equation (23) represents the lower bound and Eq. (26) represents the upper bound, while Eq. (29) represents the average.

Figure 4 shows ϕ_g versus Lockhart–Martinelli parameter (X) for laminar–laminar flow for different working fluids in smooth microchannels and minichannels at different conditions using the present asymptotic model and the bounds model with the last two data sets in Table 2. Equation (21) represents the present asymptotic model with different values of p as shown in Table 2. Equation (24) represents the lower bound and Eq. (27) represents the upper bound, while Eq. (30) represents the average.

To have a robust model, one value of the fitting parameter (p) is chosen as $p = 1/2$. Choosing $p = 1/2$ is physically meaningful. In fact, $p = 1/2$ in the asymptotic model corresponds to $C = 2$ in the bound model. When $p = 1/2$, the root mean square (RMS) error e_{RMS} is 17.14%, or 15.69% if the two lower points of Ohtake et al. data [43] are not taken into account. Figure 3 shows ϕ_l versus X for the first three data sets in Table 2, while Figure 4 shows ϕ_g versus X for the last two data sets in Table 2 with $p = 1/2$. On the basis of the experimental data shown in Figures 3 and 4, it is clear that the experimental points set in a form, when $X \to 0$, $\phi_l \to \infty$, and $\phi_g \to 1$ and when $X \to \infty$, $\phi_l \to 1$, and $\phi_g \to \infty$ in line with the expected asymptotic behavior of the Lockhart-Martinelli correlation [44]. It can be seen that there is a good agreement between the present asymptotic model and the different data sets in Figures 3 and 4.

The mean model predicts the first three data sets in Table 2 of ϕ_l with the root mean square (RMS) error of 17.91%, 19.29%, and 10.49%, respectively while the asymptotic model gives the root mean square (RMS) error of 14.07%, 16.09%, and 11.34%, respectively. The mean model predicts the last two data sets in Table 2 of ϕ_g with the root mean square (RMS) error of 14.87%, and 28.04%, respectively while the asymptotic model gives the root mean square (RMS) error of 17.36% and 24.16%,

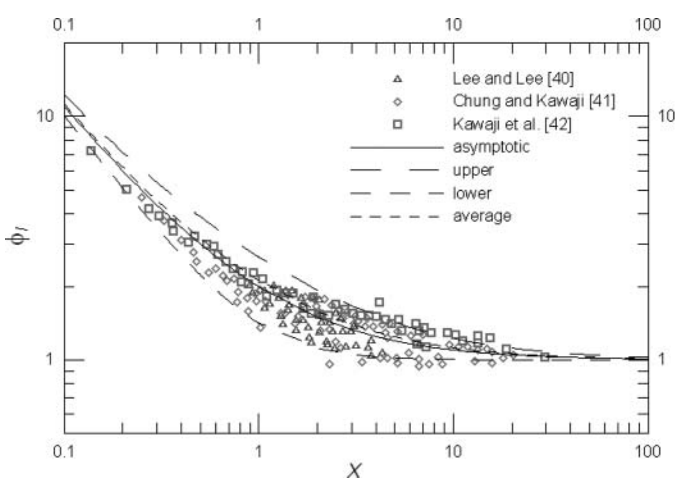

Figure 3 ϕ_l versus X for different sets of data.

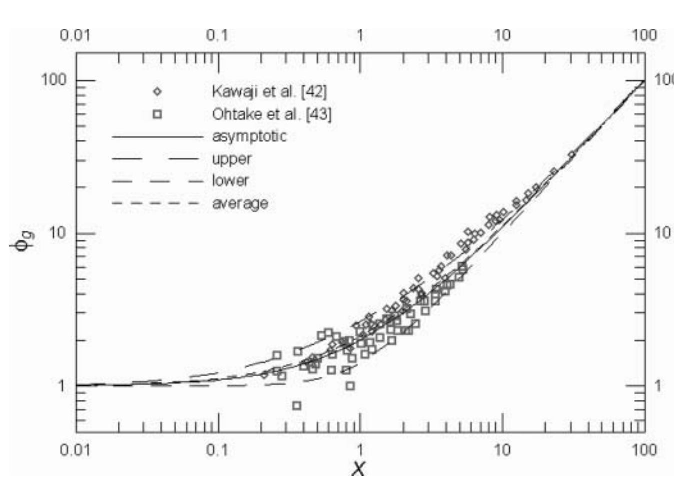

Figure 4 ϕ_g versus X for different sets of data.

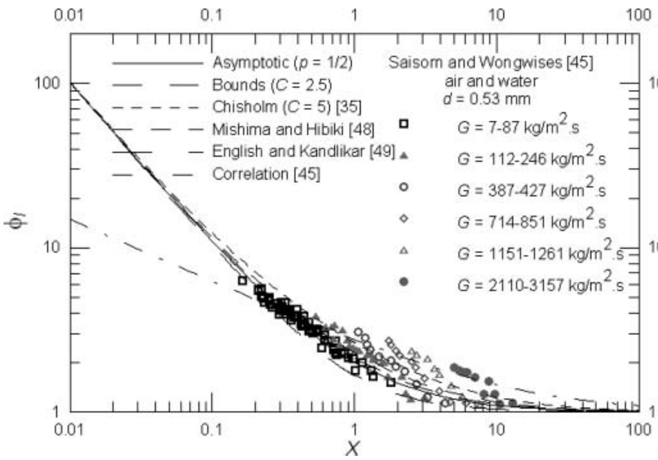

Figure 5 ϕ_l versus X for Saisorn and Wongwises's data [45] with various mass flux values at $d = 0.53$ mm.

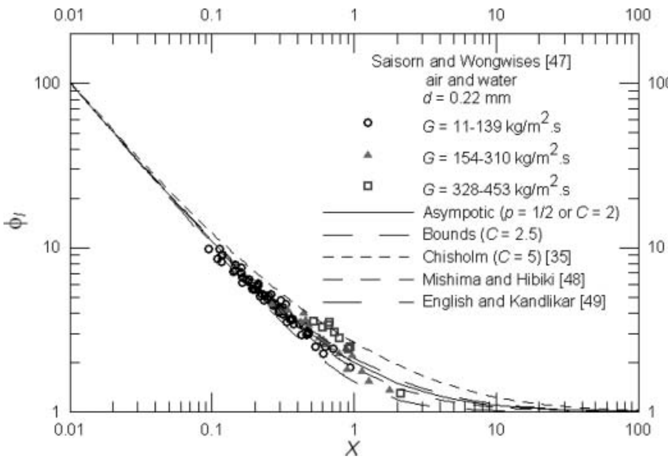

Figure 7 ϕ_l versus X for Saisorn and Wongwises's data [47] with various mass flux values at $d = 0.22$ mm.

respectively. In Figure 4, if the the two lower points of Ohtake et al. data [43] are not considered, the root mean square (RMS) error will be 21.77% instead of 28.04%, while the asymptotic model gives the root mean square (RMS) error of 18.24% instead of 24.16%. These outlying points are likely affected by experimental error.

The second method (the asymptotic model ($p = 1/2$ or $C = 2$)) and the third method (the bounds model ($C = 2.5$)) are also validated against the recent data sets of Saisorn and Wongwises [45–47] that were published in 2008 and 2009 for two-phase air–water flow in circular microchannels of $d = 0.53$, 0.15, and 0.22 mm, respectivley. Figures 5–7 show ϕ_l versus Lockhart–Martinelli parameter (X) for Saisorn and Wongwises's data [45–47] with various mass flux values for laminar–laminar two-phase air-water flow in circular microchannels of $d = 0.53$, 0.15, and 0.22 mm, respectivley. Figures 8–10 show ϕ_l versus Lockhart–Martinelli parameter (X) for Saisorn and Wongwises's data [45–47] with various flow patterns for laminar–laminar two-phase air–water flow in circular microchannels of $d = 0.53$, 0.15, and 0.22 mm, respectivley. The observed flow patterns in a 0.53-mm-diameter channel include slug flow, throat-annular flow, churn flow, and annular–rivulet flow. The observed flow patterns in a 0.15-mm-diameter channel include liquid unstable annular alternating flow (LUAAF), liquid/annular alternating flow (LAAF), and annular flow. The observed flow patterns in a 0.22-mm-diameter channel include throat-annular flow, annular flow, and annular–rivulet flow. Equation (18) represents the present asymptotic model with $p = 1/2$. Equation (23) represents the lower bound and Eq. (26) represents the upper bound, while Eq. (29) represents the average. In Figures 5–10, Saisorn and Wongwises's data [45–47] are also compared with the Mishima and Hibiki correlation [48] ($C = 21(1 - e^{-319d})$) and the English and Kandlikar correlation [49] ($C = 5(1 - e^{-319d})$). It should be noted that the Saisorn and Wongwises correlations ($\phi_i^2 = 1 + (6.627/X^{0.761})$) [45] for $d = 0.53$ mm and ($\phi_i^2 = 1 + (2.844/X^{1.666})$) [46] for $d = 0.15$ mm neglect the $1/X^2$ term, which represents the limit of primarily gas flow in the Lockhart–Martinelli [44] formulation. Neglecting this term ignores this important limiting case, which is an essential contribution. As a result, at low values of X, the proposed correlations

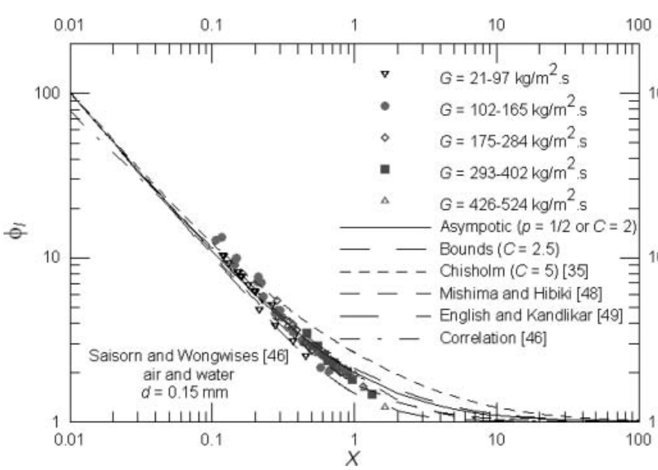

Figure 6 ϕ_l versus X for Saisorn and Wongwises's data [46] with various mass flux values at $d = 0.15$ mm.

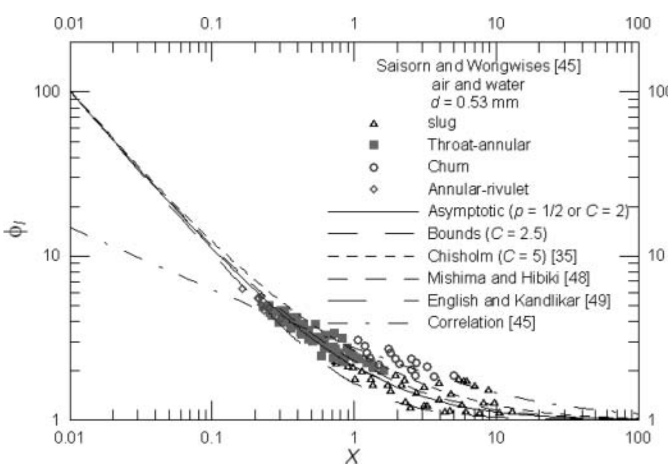

Figure 8 ϕ_l versus X for Saisorn and Wongwises's data [45] with various flow patterns at $d = 0.53$ mm.

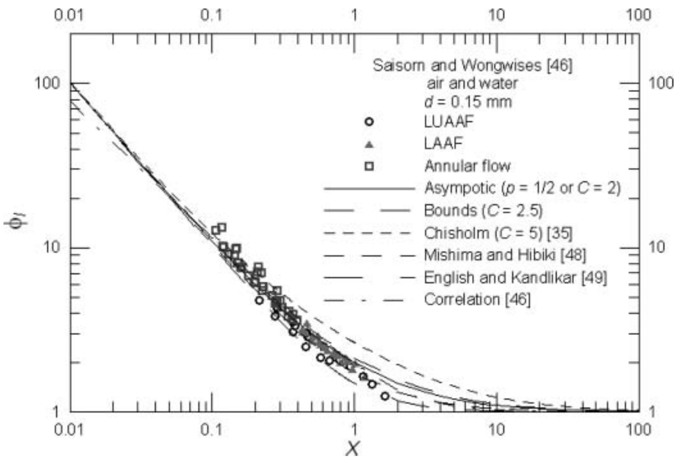

Figure 9 ϕ_l versus X for Saisorn and Wongwises's data [46] with various flow patterns at $d = 0.15$ mm.

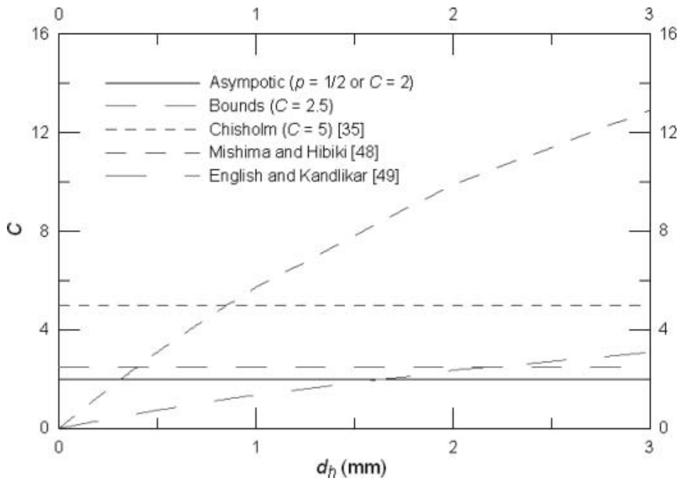

Figure 11 C parameter versus the channel diameter (d_h).

undershoot the trend of the data, limiting their use in the low X range [50].

From Figures 3–10, it clear that the greatest departure of bounds from the mean occurs at $X = 1$. From the relation that the Lockhart–Martinelli parameter (X) for laminar–laminar flow is equal to $((1-x)/x)^{0.5}(\rho_g/\rho_l)^{0.5}(\mu_l/\mu_g)^{0.5}$, this greatest departure of bounds from the mean corresponds to $x = 6.28\%$ for the air–water mixture at the atmospheric pressure while it corresponds to $x = 50\%$ at the critical state ($\rho_g = \rho_l$ and $\mu_g = \mu_l$) for any working fluid.

Figure 11 shows C parameter versus the channel diameter (d_h) for laminar–laminar flow using the asymptotic model ($p = 1/2$ or C = 2), the bounds model (C = 2.5), the Chisholm correlation [35] ($C = 5$), the Mishima and Hibiki correlation [48] ($C = 21(1 - e^{-319dh})$), and the English and Kandlikar correlation [49] ($C = 5(1 - e^{-319dh})$). It is found that the asymptotic model ($p = 1/2$ or $C = 2$) is equivalent to the Mishima and Hibiki correlation [48] at $d_h = 0.314$ mm and equivalent to the English and Kandlikar correlation [49] for laminar–laminar flow at $d_h = 1.601$ mm. Moreover, the bounds model ($C = 2.5$) is equivalent to the Mishima and Hibiki correlation [48] at $d_h = 0.397$ mm and equivalent to the English and Kandlikar correlation [49] for laminar–laminar flow at $d_h = 2.173$ mm.

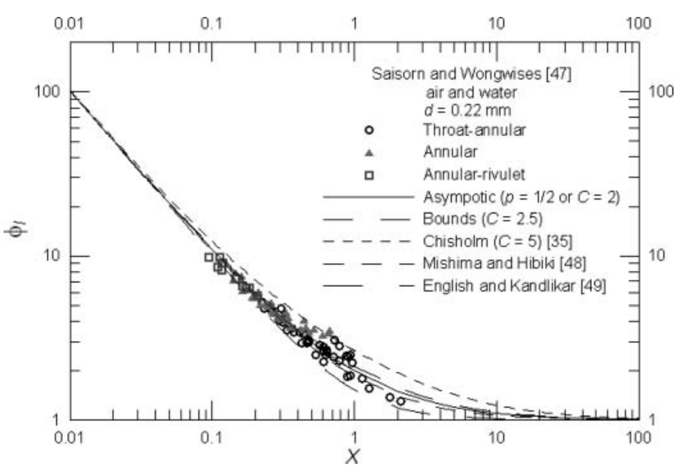

Figure 10 ϕ_l versus X for Saisorn and Wongwises's data [47] with various flow patterns at $d = 0.22$ mm.

SUMMARY AND CONCLUSIONS

First, using a one-dimensional transport analogy between thermal conductivity in porous media and viscosity in two-phase flow, new definitions for two-phase viscosity are examined. These new definitions for two-phase viscosity satisfy the following two conditions: (i) $\mu_m = \mu_l$ at $x = 0$ and (ii) $\mu_m = \mu_g$ at $x = 1$. These new definitions of two-phase viscosity can be used to compute the two-phase frictional pressure gradient using a homogeneous modeling approach. Expressing two-phase frictional pressure gradient in dimensionless form as Fanning friction factor versus Reynolds number is also desirable in many applications. The models are verified using published experimental data for two-phase frictional pressure gradient in microchannels and minichannels after expressing these in a dimensionless form as Fanning friction factor versus Reynolds number. The published data include different working fluids such as R717, R134a, R410A, and propane (R290) at different diameters and different saturation temperatures. To provide good agreement between the experimental data and well-known friction factor models such as the Churchill model [19], selection of the best definition of two-phase viscosity is based on the definition that corresponds to the minimization of the root mean square error (e_{RMS}). From $e_{RMS}\%$ values based on measured Fanning friction factor and predicted Fanning friction factor using the six different definitions of two-phase viscosity, it is shown that one of the new definitions of two-phase viscosity (Maxwell–Eucken II [12]) gives the best agreement between the experimental data and well-known friction factor models in microchannels and minichannels. These new definitions of two-phase viscosity can be used to analyze the experimental data of two-phase frictional pressure gradient in microchannels and minichannels using the homogeneous model.

Second, new two-phase flow modeling in microchannels and minichannels is proposed, based upon an asymptotic modeling method. The main advantage of the asymptotic modeling method in two-phase flow is taking into account the important frictional interactions that occur at the interface between liquid and gas, because the liquid and gas phases are assumed to flow in the same channel. This overcomes the main disadvantage of the separate cylinders model for two-phase flow. The only unknown parameter in the asymptotic modeling method in two-phase flow is the fitting parameter (p). The value of the fitting parameter (p) is chosen to correspond to the minimum root mean square (RMS) error e_{RMS} for any data set. To have a robust model, one value of the fitting parameter (p) is chosen as $p = 1/2$.

Third, simple expressions are presented for obtaining bounds for two-phase frictional pressure gradient in minichannels and microchannels. The lower bound is based on the Ali et al. correlation [33] for laminar–laminar flow. This correlation is based on modification of a simplified stratified flow model derived from the theoretical approach of Taitel and Dukler [34] for the case of two-phase flow in a narrow channel. The upper bound is based on Chisholm correlation [35] for laminar–laminar flow. The mean model is based on the arithmetic mean of lower bound and upper bound. The model is verified using published experimental data of two-phase frictional pressure gradient in circular and noncircular shapes. The bounds models are presented in a dimensionless form as two-phase frictional multiplier (ϕ_l and ϕ_g) versus Lockhart–Martinelli parameter (X) for different working fluids such as the air–water mixture and nitrogen–water mixture. The present model is very successful in bounding two-phase frictional multiplier (ϕ_l and ϕ_g) versus Lockhart–Martinelli parameter (X) well for different working fluids over a wide range of mass fluxes, mass qualities, and diameters. The proposed mean model provides a simple prediction of two-phase flow parameters.

Finally, the first method (homogeneous property modeling) is recommended in predicting the two-phase frictional pressure drop in microchannels and minichannels if we use the homogeneous model. It is reasonably accurate only for bubble and mist flows since the entrained phase travels at nearly the same velocity as the continuous phase. The second and third methods (asymptotic modeling and bounds) are recommended in predicting the two-phase frictional pressure drop in microchannels and minichannels if we use the separated model that originated from the classical work of Lockhart and Martinelli [44].

NOMENCLATURE

a	Churchill parameter
b	Churchill parameter
C	Chisholm constant
d	pipe diameter, m
e	error
f	Fanning friction factor
$f\,Re$	Hagen–Poiseuille constant
G	mass flux, kg/m^2.s
K	index for summation
k	thermal conductivity, W/m^2.K
N	number of data points
dp/dz	pressure gradient, Pa/m
Re	Reynolds number $= Gd/\mu$
T	temperature, °C
X	Lockhart–Martinelli parameter
x	mass quality

Greek Symbols

ε	pipe roughness, m
ρ	density, kg/m^3
ϕ_g^2	two-phase frictional multiplier for gas alone flow
ϕ_l^2	two-phase frictional multiplier for liquid alone flow
μ	dynamic viscosity, kg/m.s
ν	kinematic viscosity, m^2/s

Subscripts

av	average
$cont$	continuous phase
$disp$	dispersed (discontinuous) phase
f	frictional
g	gas
h	hydraulic
l	liquid
$lower$	lower bound
m	homogeneous mixture
RMS	root mean square
s	saturation
$upper$	upper bound

Superscript

p	fitting parameter

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