RESEARCH NOTE: VOID FRACTION DURING TWO-PHASE FLOW

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From consideration of the homogeneous equation for frictional pressure gradient, and the relation between friction and void fraction, an equation for the ratio of the phase velocities during two-phase flow, is developed which is interesting in form, easy to use, and agrees with experiment.

Introduction

A CONVENIENT FORMULA is developed for the ratio of the vapour (or gas) velocity to that of the liquid during two-phase flow. The formula is compatible with a number of elementary methods of predicting two-phase frictional pressure drop and void fraction.

Notation

C_A Armand coefficient (equation (7)).
K Velocity ratio (equation (5)).
Δp_L Frictional-pressure change when liquid flows alone.
Δp_{LO} Frictional-pressure change when all mixture flows as liquid.
Δp_{TP} Frictional-pressure change during two-phase flow.
Re_L Reynolds number if liquid component flows alone.
U_G Velocity of gas component.
U_L Velocity of liquid component.
x Ratio of gas–mass flow rate to that of mixture.
α_L Ratio of liquid to total flow cross section.
β Ratio of gas flow rate by volume to that of mixture.
ρ_G Density of gas.
ρ_{HOM} Density of homogeneous mixture (equation (2)).
ρ_L Density of liquid.
ρ_v Density of vapour.

Friction during two-phase flow in pipes

For a considerable range of conditions the use of 'homogeneous' theory (1)† has been found to give good agreement with experiment. When the friction factors are independent of Reynolds number, homogeneous theory gives

$$\frac{\Delta p_{TP}}{\Delta p_{LO}} = \frac{\rho_L}{\rho_{HOM}} \quad . \quad . \quad . \quad . \quad (1)$$

where ρ_{HOM} is the homogeneous density (the density when the ratio of the phase velocities K is assumed to be unity).

The homogeneous density can be determined from

$$\frac{1}{\rho_{HOM}} = \frac{1-x}{\rho_L} + \frac{x}{\rho_G} \quad . \quad . \quad . \quad (2)$$

Another well-known approximate relationship for friction (2) is

$$\frac{\Delta p_{TP}}{\Delta p_L} = \frac{1}{\alpha_L^2} \quad . \quad . \quad . \quad . \quad (3)$$

where the liquid fraction is given by

$$\frac{1}{\alpha_L} = \frac{x/\rho_G + K(1-x)/\rho_L}{K(1-x)/\rho_L} \quad . \quad . \quad (4)$$

The velocity ratio K is

$$K = U_G/U_L \quad . \quad . \quad . \quad . \quad (5)$$

The liquid fraction can also be written

$$\frac{1}{\alpha_L} = \frac{x/\rho_G + (1-x)/\rho_L}{C_A K(1-x)/\rho_L} \quad . \quad . \quad . \quad (6)$$

where C_A is the Armand (3) coefficient

$$C_A = \frac{1-\alpha_L}{\beta} \quad . \quad . \quad . \quad . \quad (7)$$

and

$$\beta = \frac{x/\rho_G}{(1-x)/\rho_L + x/\rho_G} \quad . \quad . \quad (8)$$

For values of β between 0·4 and 0·9 the value of C_A is about 0·8 for a wide range of conditions, and approaches unity outside this range of β for highly turbulent conditions.

It should also be noted that

$$\frac{\Delta p_L}{\Delta p_{LO}} = (1-x)^2 \quad . \quad . \quad . \quad (9)$$

Velocity ratio

At this point, it will be assumed that C_A is unity in equation (6), an approximation that will be examined later. Hence, from equations (1)–(3), (6) and (9)

$$K = \left(\frac{\rho_L}{\rho_{HOM}}\right)^{1/2} \quad . \quad . \quad . \quad (10)$$

This equation is similar in form to equations of Fauske (4), Ryley (5), Zivi (6), and Smith (7).

The equation can also be expressed

$$K = \left(1-x+x\frac{\rho_L}{\rho_v}\right)^{1/2} \quad . \quad . \quad (11)$$

or as

$$K = \left(\frac{1-x}{1-\beta}\right)^{1/2} \quad . \quad . \quad . \quad (12)$$

From equations (4), (7), (8), and (11)

$$C_A = \frac{1+\dfrac{1-x}{x}\dfrac{\rho_G}{\rho_L}}{1+(1-x)^{1/2}\left\{1+\dfrac{x\rho_L}{(1-x)\rho_G}\right\}^{1/2}\dfrac{1-x}{x}\dfrac{\rho_G}{\rho_L}} \quad (13)$$

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Table 1. *Armand coefficient as a function of β and ρ_L/ρ_G*

ρ_L/ρ_G	β							
	0·3	0·4	0·5	0·6	0·7	0·8	0·9	1·0
800	0·88	0·84	0·81	0·81	0·80	0·80	0·82	1·0
20	0·89	0·86	0·84	0·83	0·82	0·83	0·86	1·0
2	0·94	0·93	0·93	0·93	0·93	0·95	0·97	1·0

Table 2. *Armand coefficient for steam–water mixtures as a function of steam pressure (for $\beta = 0{\cdot}7$)*

Pressure, bar	Armand coefficient		Pressure, bar	Armand coefficient	
	Experiment	Equation (13)		Experiment	Equation (13)
4·9	0·790	0·80	79	0·808	0·83
9·9	0·783	0·80	107	0·827	0·83
20	0·783	0·81	137	0·850	0·85
29	0·783	0·81	176	0·904	0·88
39	0·788	0·81	196	0·938	0·90
50	0·796	0·81	206	0·975	0·92

Table 1 gives values of C_A as a function of β and the density ratio of the phases. In conformity with experiment, C_A approaches a constant value in the range of β from 0·4 to 0·9. The magnitude of C_A compares favourably with Armand's experimental value obtained with air–water mixtures at atmospheric pressure in horizontal tubes ($\rho_L/\rho_G \approx 800$); the experimental value was 0·83 which agrees very well with the value from equation (13).

Comparison of equation (13) with Armand coefficients for steam–water mixtures is equally interesting. Table 2 compares predicted values (**8**) (**9**) at $\beta = 0{\cdot}7$ with experimental values. Apart from a considerable underprediction at the highest pressure the agreement is remarkable; despite the divergence between prediction and experiment as the critical pressure is reached, equation (13) gives C_A unity at the critical point as required.

It would be possible not to make the approximation that C_A is unity in equation (6), however the resulting equation for K is both more complex than equation (10) and gives much poorer agreement with experiment. No approximation would have been required if equation (3) were

$$\frac{\Delta p_{TP}}{\Delta p_L} = \frac{C_A^2}{\alpha_L^2} \quad . \quad . \quad . \quad . \quad (14)$$

In fact this equation will give better agreement with some of the available data (**10**) than equation (3).

Equation (10) can be expected to give reasonable agreement with experiment for $\beta < 0{\cdot}9$ provided that the fluid is no more viscous than water (**11**), that in horizontal flow the phases are not stratified, and that where the pipe is vertical the liquid component if flowing alone would be in well-developed turbulent flow ($Re_L > 10\,000$). For $\beta > 0{\cdot}9$ experiments show (**12**) a pronounced effect of the mass velocity not covered by equations of the form of equation (10).

Conclusion

The equation for the velocity ratio

$$K = \left(\frac{\rho_L}{\rho_{HOM}}\right)^{1/2} \quad . \quad . \quad . \quad (10)$$

has been shown to give close agreement with experiment for a wide range of conditions.

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