Exploring why Trcvr shows changes of a few x 10 K between:

(a) different trials for a single antenna and polarization,
(b) different polarization on the same antenna for different trials
(c) different antenna but same polarization and same trials.

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Step 1: Exploring changes between trials for a single antenna and polarization:

Consider two trials of an on-off observation with a perfect system.

$$
\begin{aligned}
T_{sys,on,1} &= G_{on,1} \cdot P_{on,1} = T_{Rcvr,on,1} + T_{Common} + T_A \\
T_{sys,off,1} &= G_{off,1} \cdot P_{off,1} = T_{Rcvr,off,1} + T_{Common} \\
\\
T_{sys,on,2} &= G_{on,2} \cdot P_{on,2} = T_{Rcvr,on,2} + T_{Common} + T_A \\
T_{sys,off,2} &= G_{off,2} \cdot P_{off,2} = T_{Rcvr,off,2} + T_{Common}
\end{aligned}
\tag{1}
$$

Above assumes linearity between values measured by the backend (in green) and the **actual** system and receiver temperature (in black). In blue are values assumed to be known perfectly for the contributions to Tsys common to both phases and trials (e.g., Milky Way, CMB, atmosphere) and from the observed source (T_A).

Since we have 4 equations, 8 unknowns, we cannot directly solve.
For now, assume the system is 100% stable between on and off:

- Gains do not change: $G_{on} = G_{off}$, but $G_1 \neq G_2$
- Receiver's contribution to Tsys doesn't change between trials: $T_{Rcvr,on} = T_{rcvr,off}$

Now, four equations, four unknowns; we can solve for the desired entities

$$T_{sys,off,1}^{Measured} = T_{Rcvr,1}^{Measured} + T_{Common} = \frac{P_{off,1}}{P_{on,1} - P_{off,1}} \cdot T_A$$

$$T_{sys,off,2}^{Measured} = T_{Rcvr,2}^{Measured} + T_{Common} = \frac{P_{off,2}}{P_{on,2} - P_{off,2}} \cdot T_A \qquad (2)$$

System doesn't follow this as the measured T_{Rcvr} changes between trials.

What if we break assumption that $G_{on} = G_{off.}$. Instead, assume one of the 4 gains has changed slightly. **G = $G_{on,1}$ = $G_{off,1}$ = $G_{off,2}$, but $G_{off,2}$ = G·(1 + ΔG/G). That is, the gain for one phase of one trial has changed.**

$$T_{sys,off,1}^{Measured} = T_{Rcvr,1}^{Measured} + T_{Common} = \frac{P_{off,1}}{P_{on,1} - P_{off,1}} \cdot T_A$$

$$T_{sys,off,2}^{Measured} = T_{Rcvr,2}^{Measured} + T_{Common} = \frac{\left(1 + \frac{\Delta G}{G}\right) \cdot P_{off,2}}{P_{on,2} - \left(1 + \frac{\Delta G}{G}\right) \cdot P_{off,2}} \cdot T_A \qquad (3)$$

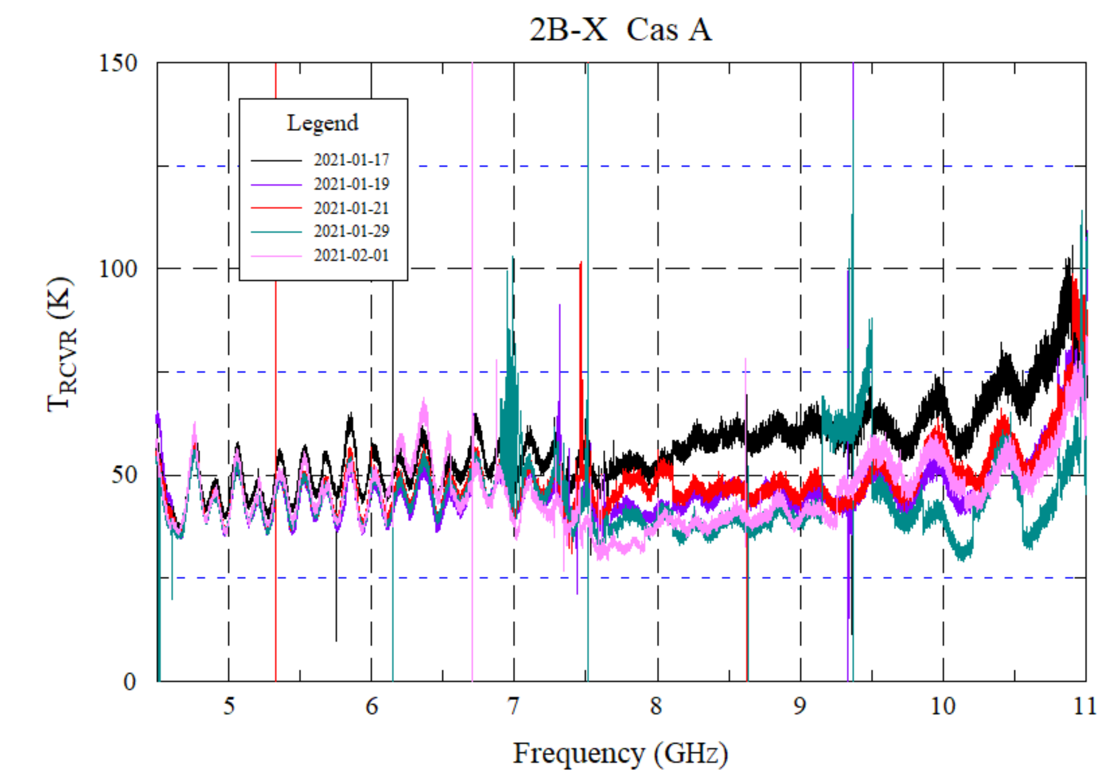
Take the difference to see how the measured T_{rcvr} changes with a single change in gain.

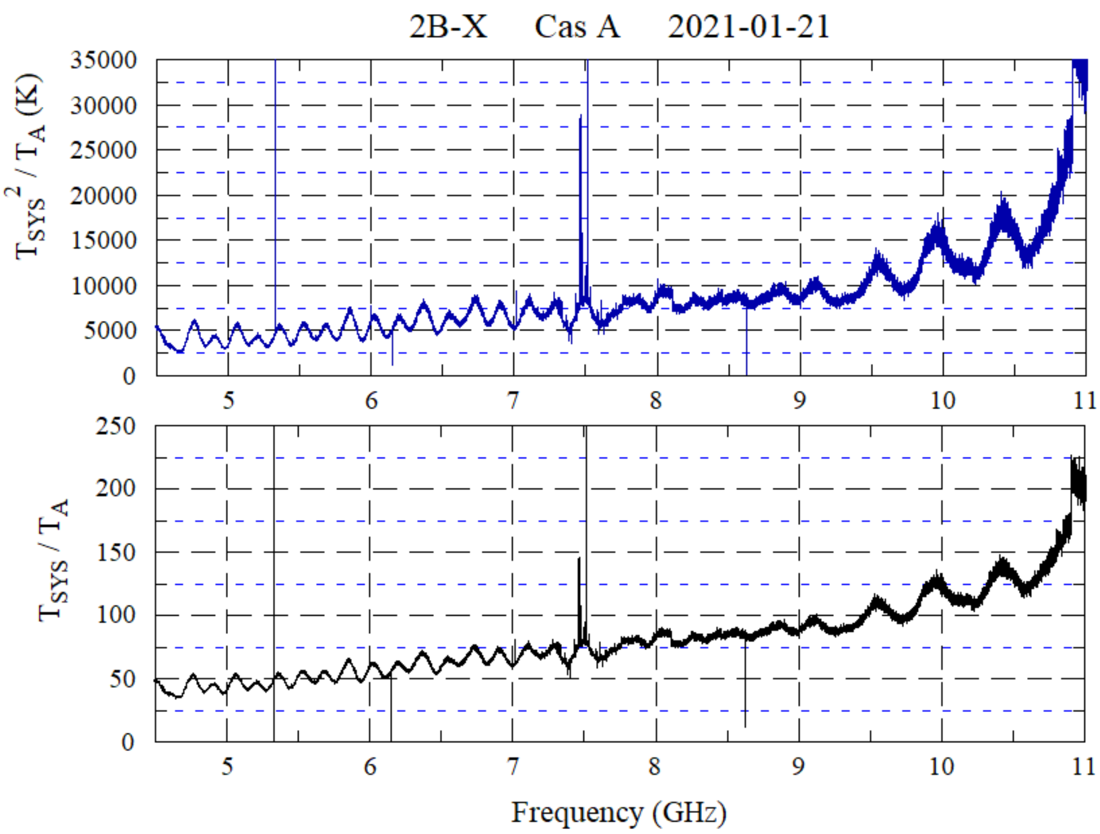
$$\Delta T_{sys}^{Measured} = \Delta T_{Rcvr}^{Measured} = T_A \cdot \left[\frac{P_{off,1}}{P_{on,1} - P_{off,1}} - \frac{\left(1 + \frac{\Delta G}{G}\right) \cdot P_{off,2}}{P_{on,2} - \left(1 + \frac{\Delta G}{G}\right) \cdot P_{off,2}} \right]$$

$$\Delta T_{sys}^{Measured} = \Delta T_{Rcvr}^{Measured} \approx T_A \cdot \frac{\Delta G}{G} \cdot \left[\frac{P_{off_1}}{P_{on_1} - P_{off_1}} - \frac{P_{off_1}^2}{\left(P_{on_1} - P_{off_1}\right)^2} \right] \qquad (4)$$

$$\Delta T_{sys}^{Measured} = \Delta T_{Rcvr}^{Measured} \approx \frac{\Delta G}{G} \cdot \left[T_{sys,off} - \frac{T_{sys,off}^2}{T_A} \right]$$

Using five trials of Cas A with antenna 2B-X:





At 10.75 GHz, $P^2_{off_1}/\left(P_{on_1}-P_{off_1}\right)^2 \approx T^2_{sys}/T_A \approx 20{,}000$ and $\Delta T_{Rcvr} \sim \pm 20$ K, which is consistent with **ΔG/G ~ 0.001.**

Very small gain changes between the On and Off in different trials are a possible cause of the large changes in measured values.

For example, at 11.75 GHz, FWHM beam width ~ 17’, a 20” **systematic** pointing offset between trials will be the equivalent of a $\Delta G/G$ **~** 0.001 and would produce the observed trial-to-trial change in T_{sys} and T_{rcvr}. Probably impossible to test and measure the pointing model for every antenna to better than 1’.

Other possibilities are slow changes in LNA gain due to, for example, slow changes in cryogenic temperatures.

Thus, trial-to-trial changes in measured values for Tsys and Trcvr are expected to be of this magnitude when using observations of Cas A.

Step 2, Exploring changes between polarizations for different trials for a single antenna:

- Substitute subscripts x and y for 1 and 2 in above equations.
- Return to the assumption of gain stability: Gon,x = Goff,x and Gony = Goff,y
- Allow $T_{sys,on,y} = T_{sys,on,x} + \Delta T$

$$\Delta T_{Rcvr}^{Measured} \approx \Delta T_{sys} \cdot \left[1 - \frac{T_{sys,off}}{T_A} \right] \quad (5)$$

At 10.75 GHz, $P_{off_1} / (P_{on_1} - P_{off_1}) \approx T_{sys} / T_A \approx 150$ and ΔT_{Rcvr} ~ ±20 K, which is consistent with **ΔT ~ 0.13 K. Again, a small, insignificant change in the system properties produce large changes in the measured quantities.**

There are a number of possible reasons why we can expect ΔT to slowly drift between trials for a single polarization on the same antenna.

- ○ Using a linearly-polarized source – as the parralactic angle changes as the source rises or sets, T_A for x and y will change slowly
- ○ There are slow but different drifts in the T_{rcvr} of the amplifier for x and y.

Step 3: Exploring changes between different antenna but same polarization and same trials

Combinations of the small changes discussed in Step 1 and 2 can explain the differences between antenna for the same trials. For example, each antenna has a different systematic error in its pointing model, and, thus, each antenna has a different systematic way in which pointing errors change with elevation, and, thus different ways in which the resulting ΔG/G drifts with elevation.

Conclusion and Recommendations:

Small drifts in either the system temperatures, receiver temperatures, or gains between the on and off observations for different observing sessions can produce the types of variations we have been seeing at high frequencies. This includes the seemingly random variations between different observing sessions for the same antennae and polarization, and between polarizations for the same antennae. These variations are a direct consequence of the low value of T_A for Cas A and Tau A at high frequencies and the algorithms.

- Antennae and polarizations with the worse gain stability should produce the largest variations in T_{Rcvr} between different observing sets.

- Antennae and polarizations with the worse T_{rcvr} or T_{sys} will have a tendency to exhibit large variations.
- Antennae and polarizations whose amplifiers have the least stable Tsys should also exhibit large variations.
- On-off observations using the Moon should show low variations at high frequencies but large variations at low frequencies, which is suggested by a few Moon observing sessions from October.

Cas A and Tau A have, respectively, T_A ~ 2 and 4 K at 11 GHz. If we used a source that was significantly brighter, like the Moon, we should expect variations in T_{Rcvr} from one observing session to another to be small. However, at 1 GHz, the Moon is very weak, ~ 2 K, when observed with the ATA. Thus, Cas A is the preferred source for determining performance below about 3 GHz.

The Moon also has the added advantage of being extended (therefore, consequences of systematic errors in the pointing models will have less affect). It is also unpolarized.