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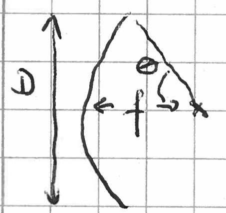
Some useful optical formulas

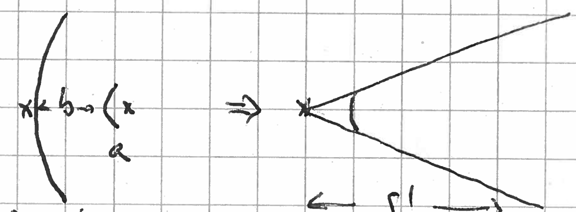


$F = f/D$ $\cos\theta = \dfrac{(4F)^2 - 1}{(4F)^2 + 1}$

e.g. for $F = 0.4$, $2\theta = 128°$ for $F = .6$, $2\theta = 90°$
$F = 0.8$, $2\theta = 69°$ for $F = .5$, $2\theta = 106°$

$\Omega \sim \dfrac{4\pi}{g}$ $\left(\dfrac{128}{69}\right)^2 \to +5.3\,\text{db}$; $\left(\dfrac{90}{69}\right)^2 \to 2\,\text{db}$

Casagrain magnification



$f' = f\,b/a = Mf$ and $F' = MF$

Defocus curve for $s = 0.8\lambda$, -13db taper for $s = 1.0\lambda$

F	Σ	A	F	Σ	A
0.4	0.8	0.6	0.4	1	.48
0.5	0.52	0.84	0.5	.65	.73
0.6	0.36	0.90	0.6	.45	.85
0.8	0.20	0.96	0.8	.25	.95
1.0	0.13	1.0	1.0	.16	.98

Ruze Losses $\exp - \left\{2\pi \frac{\sigma}{\lambda}\right\}^2$ ~~(8)~~ $\lambda = 3$cm. $\Rightarrow e^{-\left(\frac{4\pi\sigma}{\lambda}\right)^2}$

$\lambda/16 \to 0.85$ $\sigma \sim 0.2$cm (2mm) (80 mils)
$\lambda/7.5 \to 0.50$ σ .4 cm (4 mm) (160)

Seti Workshop

At what level of antenna gain will the star dominate RCVR temperature?

Fermi's paradox for robots, who may be ageless.

→ Purchase and measure some dishes for RMS and also check the mounts

Tony Stark - cheap 7-10m for 0.3m - 20GHz

1. Industrial technology (a) boat hull fiberglass
(b) support structure - steel bridgework
custom I beam

(c) bearings and gears - excavator
(d) stepper motors: 3000:1 - 5000:1 speed reduction
guess that cost ≤ 30-50K $

2. Off-axis design
a) Keep elevation axis low
b) Keep receiver ~~low~~ - easy to fix
c) reduced interference
d) Keep costs low by large numbers

Ibeam egg-crate for backing the reflector
note: advantage of no encoders for opt. fiber through hole.
wide band dynamic range problem.

Matt Fleming -

List of many manufacturers
T. Howard - work with the cheap manufactures
preliminary mount design: $ 7000
expect $ 2000 for dish

Tony Howard: $D \lesssim 3-4$ m is a big break point in the handling and assembly

Rohde + Schwartz has a LP with reflector
Tony use the open LP type for higher gain
and space for equipment

Sandy - Measurements show ≥ 7k for commercial items

Doug - narrow band fo the Laser BW

Rick - Analog filters are needed at the input to counter on terference

Look at the ridge guide horns - there may be more cooling capacity

Focal plane arrays? Problems with wide bandwidth

What are the upgrade paths for the hardware

Tony - how do we squeeze it? "supply management" Greg

Can the ~~parts~~ components be commercialized

Mike - Do a more staged development -

Tay Howard - high performance feeds on small dishes
wide bandwidth is new - but there are many problems -
likes the offset feed e.g. large F for LP feed
also cryo is important and it is out of the way
good for future performance trade-offs

L.J. Bregmann says the feed match is more difficult.

GP questions the separation of R+D and manufacturers + suppliers

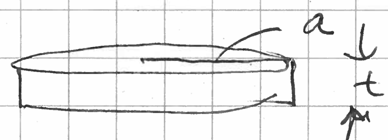
→ The pyramid LP feed (offset fed) needs to be studied -

Next meeting $\boxed{\text{Feb. 17, 18, 9}}$

Ron's summary.

Some LP feed options   12/20/98

First: Reflector stiffness as a function of dimensions



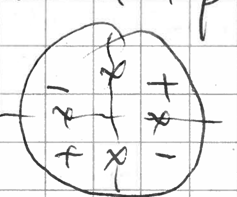
$k\Delta x = mg$, $\Delta x = g\frac{m}{k} = \frac{g}{\omega^2}$

$\omega \propto \frac{t}{a^2}\sqrt{\nu_j \frac{E}{\rho}}$

Gravitational Deflexions

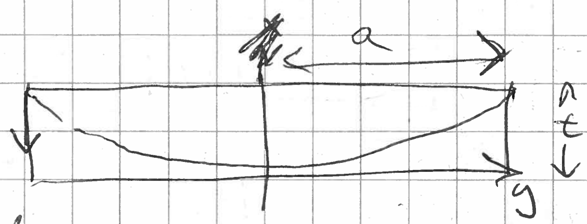
$$\boxed{\Delta x \propto \frac{a^4}{t^2}\,\frac{(\rho/E)}{\nu_j}}$$

ρ = density E = modulus of elasticity
ν_j is the mode number.
e.g. (a) disk clamped in the center: $\nu_0 = 13.8$
(b) with a 4 point rigid suspension:



$\nu_0 = 30.4$

Dimensions



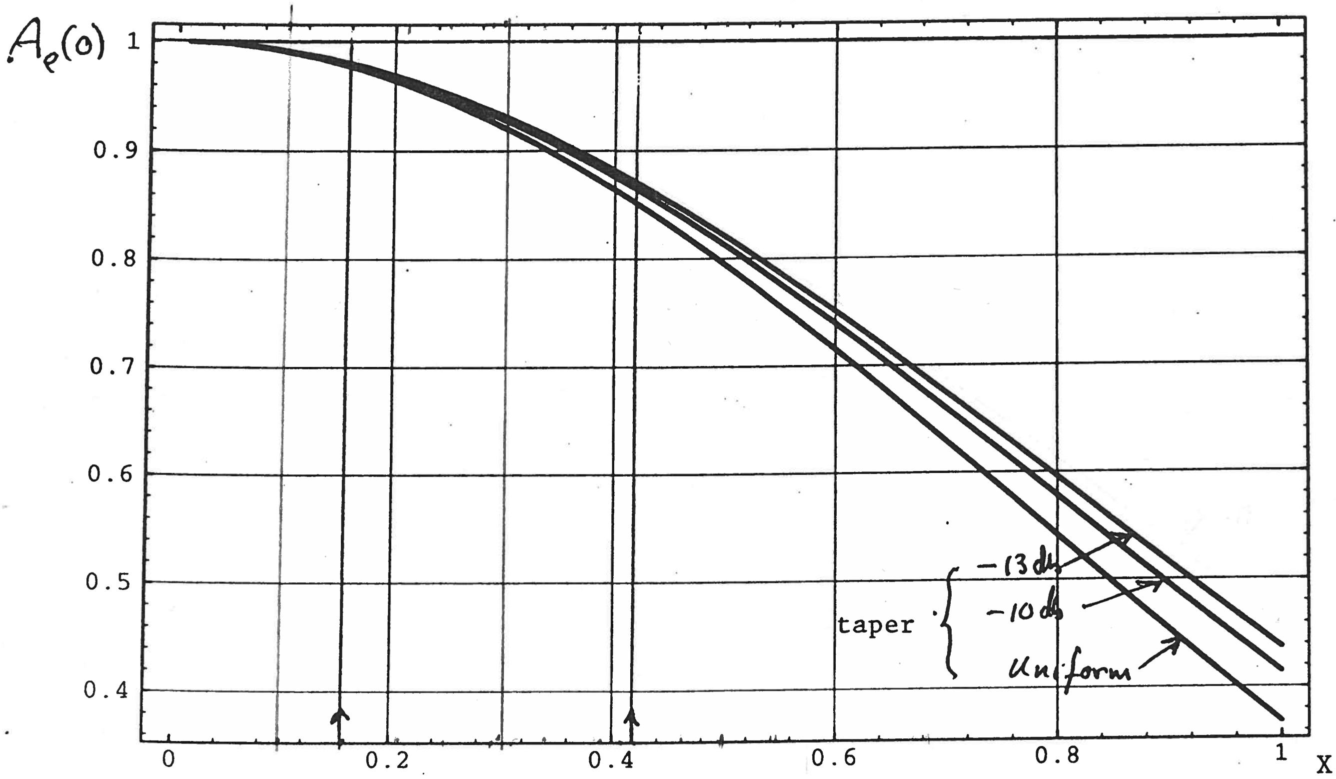
For a paraboloid: $y^2 = 4fx$ $a^2 = 4f\,t$; $a = D/2$

$t^2 = a^4/(4f)^2$

$$\Delta x \propto \frac{(D/2)^4}{(D/2)^4/(4f)^2} = \underline{(4f)^2 = 16F^2D^2}$$

(see p. 23)

Gain loss due to phase error from being out-of-focus



Gain loss due to axial focus error s, where $X = (17)(s/\lambda)/(1 + 16(F/.41)^2)$

Figure 2

Gain change with frequency depending on F.

I. $400\ \text{MHz} < \nu < 1600\ \text{MHz}$ Focal position is 0.8λ

take $F = 0.42$ and set correct focus at 1000 MHz

Focal position

ν	pos.	η_D
400	60 cm	0.84
500	48	0.88
1000	24	1.00
1500	16	0.88
1600	15	0.84
300		0.78
1700		0.78

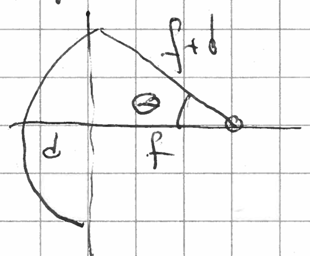
For 3:1 BW $\eta_D \gtrsim 0.88$
4:1 BW $\eta_D \geq 0.84$
7:1 BW $\eta_D \geq 0.78$

II. take $F = 0.33$

ν	η_D
400	0.70
500	0.78
1000	1.00
1500	0.78
1600	0.70

For 3:1 BW $\eta_D \gtrsim 0.78$
4:1 BW $\eta_D \gtrsim 0.70$

III. Spillover estimates

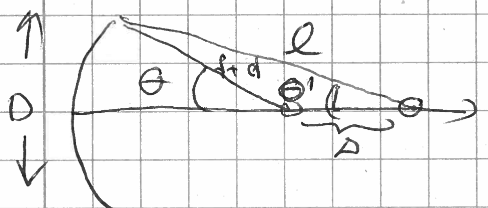


$$\cos\Theta = \frac{f-d}{f+d} = \frac{f - \frac{D^2}{16f^2}}{f + \frac{D^2}{16f^2}} = \frac{(4F)^2 - 1}{(4F)^2 + 1}$$

$$d = \frac{D^2}{16f}$$

F	Θ
0.33	74.°3
0.42	61.°5
0.50	53.°1
0.60	45.°2
0.70	39.°3

IV. Correct the illumination for the actual phase center.



$$l^2 = \Delta^2 + (f+d)^2 - 2\Delta(f+d)\cos(\pi - \theta)$$
$$l^2 = \Delta^2 + (f+d^2) + 2\Delta(f+d)\cos\theta$$

For the case $D = 5m$, $F = 0.42$, $\nu = 4000$, $\Delta = 60$ cm.

$f = 0.42(5m) = 2.1$ m $\quad d = D^2/16f = (25)/16(2.1) = 0.74$ m

$f+d = 2.1 + .74 = 2.84$ m

$l^2 = (0.6)^2 + (2.84)^2 + (2)(0.6)(2.84)\cos 61.5^\circ$; $\underline{l = 3.17 m}$

$$\frac{l}{\sin(\pi-\theta)} = \frac{f+d}{\sin\theta'}, \quad \sin\theta' = \left(\frac{f+d}{l}\right)\sin(180-61.5) = .7873$$

$\underline{\theta' = 51.93^\circ}$

V. $F = 0.42$ centered at 4000 MHz.

ν	focal pos.	η_D
1000	24 cm	0.75
2000	12	0.87
3000	8	0.96
4000	6	1.00
5000	4.8	0.96
6000	4.0	0.88
7000	3.43	0.75
8000		0.00

phase only correction

For $\eta_D \geq 0.70$
$\underline{BW = 7:1}$

Board Meeting 12/21/98

1. Lew report on the MOU
Note: Jack ~~must not participate in SE~~ abstain from SETI board votes related to 1HT. J must report this to the SETI board -
Note also the overhead waiver document -

⇒ 2. Jack Keep track of Sandy and his student

3. Greg discusses donor list - Some individuals interested in single antennas - Location of the telescope needs to be settled - talking to the Barney Group - G. needs a place to take pple to -
⇒ naming issues need to be understood -
$\boxed{\text{FD}}$ Issues: Someone may wish to give land
⇒ What about ~~insurance~~ interference. [do the measurements]
Hat Creek is inconveniently distant.

Other comments: "partnership th UC is an ~~ad~~ advantage" will be important to potential donors

[Is the Tektronix spectrum analyzer sensitive enough?]
Also look at the satellite foot prints

G. Minimize the unknowns - fix the site
TP. Note Bob Krekorian's proposal to Paul Allen from the MIRA institute for the Greenfield Site
Frank: both Greenfields and Barney's ranch are bad below 1GHz
Jack went out for MMA phone conversation 1:00 - 1:50

Jill: We need a well developed plan to report to the Working Group - e.g. get John L for the Cryo ASAP - 12th + 13th of May for the meeting -

Second te. Is t needed? F. Doesn't multibeams do it.
Isn't it overkill?
Good idea - leave it up in the air -

What process to decide configuration?

○

Transmission Line impedances and losses

Two wire line

Capacitance/unit length: $C = \dfrac{\pi\epsilon}{\cosh^{-1}(s/D)}$

$Z_0 = \sqrt{\dfrac{L}{C}}$, $v = \dfrac{1}{\sqrt{LC}} = C_0 = 3 \times 10^8$ m/sec in free space.

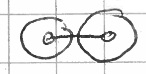
$C_0^2 = \dfrac{1}{LC}$, $L = \dfrac{1}{C_0^2 C}$; $Z_0 = \sqrt{\dfrac{1}{C_0^2 C} \cdot \dfrac{1}{C}} = \dfrac{1}{C_0 C}$

$Z_0 = \dfrac{\cosh^{-1}(s/D)}{\pi\epsilon\, C_0}$ or $s/D = \cosh\left[Z_0 \pi \epsilon C_0\right]$

For $Z_0 = 100\,\Omega$, $100 = \dfrac{\cosh^{-1}(s/D)}{C_0 \pi \epsilon}$

$\cosh^{-1}(s/D) = (100)(3\times10^8)\cancel{\pi}\,\dfrac{10^{-9}}{36\cancel{\pi}} = 0.8333$, with $\epsilon = \epsilon_0$

$s/D = \cosh(0.8333) = 1.37$

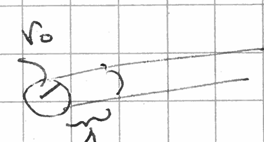


$s = 1.37\,D$

For the connection to the plug, use teflon to hold the wires in position $k_e = 2.0$

Then for $Z_0 = 100\,\Omega$, $\cosh^{-1}(s/D) = 1.666$
and $s/D = \cosh 1.666 = 2.73$ $\underline{s = 2.73\,D}$
(the same for $200\,\Omega$ in air)

Wire Losses



R/unit length $= \dfrac{R_s}{2\pi r_0}$ $R_s = 2.52 \times 10^{-7}\sqrt{f}$ for silver.
At 10 GHz, $R_s = .0252$

For a 2 wire line, resistance/cm $= \dfrac{2R_s}{2\pi r_0} = \dfrac{R_s}{\pi r_0}$
(approximate, assumes uniform current)

For $D = 0.12$ cm $= 1.2$ mm $\cong 50$ mils, $\dfrac{R_s}{\pi r_0} = \dfrac{.0252}{\pi \cdot \frac{.12}{2}} = 0.134\,\Omega$/cm.

~~$s = 2.37$~~

For small losses $\alpha \cong \frac{R}{2Z_0} = \frac{0.13}{2(100)} = .00065$

where $Z_0 = \sqrt{\frac{L}{C}}$ [voltage attenuation factor]

For 1" length, 2.5 cm, $2\alpha L = 2(.0016)$; this should probably be doubled, since most of the current will be on one side of the wire. (not quite)

$\boxed{2\alpha L \Rightarrow .0064}$

s/D vs. Z_0 in air

approximate:

$Z_0(\Omega)$	s/D
100	1.37
200	2.73
300	6.13
400	13.9
500	32.40
240	3.69

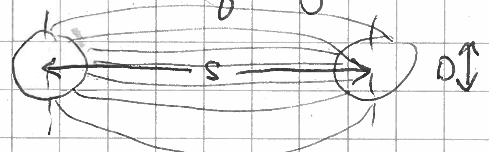
from $Z_0 = \frac{\cosh^{-1}(s/D)}{\pi \epsilon_0 c_0}$

$Z_0 = 120 \cosh^{-1}(s/D)$

$\cosh(Z_0/120) = s/D$

For D = 1.2 mm, 2.37D = 2.8 mm

Use the assumption that all the E-field lines land uniformly on one half of each wire to calculate losses.



too big a loss see p.31

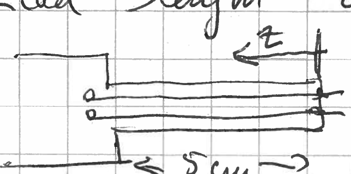
R = Res/unit length $\cong \frac{2R_s}{\pi r_0}$, including both wires.

R_s $2.52 \times 10^{-7}\sqrt{f}$ for silver.

For a fixed wire diameter

Losses into the cryostat, based on Sandy's Drawing.

Lead length cm, silver plated stainless steel.



wire diameter = 0.12 cm. f GHz

$R_s = .0138\ \Omega$

Two wire line, $Z_0 = 100\ \Omega$, res $= \frac{2R_s}{\pi D/2} = \frac{4R_s}{\pi D}$

Assume a constant temp. gradient from 300K to 80K.

Since $\sigma \propto T$ $\quad R_s \propto \sqrt{T}$ $\qquad R_s = 0.0138\sqrt{\dfrac{T(z)}{300}}$

$$T(z) = 300 - \frac{z}{5}(220)$$

small optical depth : $2\alpha(z) = \dfrac{\cancel{2}R}{\cancel{2}Z_0}$, $Z_0 = 100\ \Omega$

$$T_B = \int_0^{5\text{cm}} T(z)\, 2\alpha(z)\, dz \quad ; \quad 2\alpha(z) = \left(\frac{4R_s}{\pi D}\right)\frac{1}{Z_0} = \frac{4}{Z_0\pi D}(.0138)\sqrt{\frac{T}{300}}$$

$$T_B = \int_0^{5\text{cm}} \left[300 - \frac{z}{5}220\right]\frac{4}{Z_0\pi D}(.0138)\left[\frac{300 - \frac{z}{5}220}{300}\right]^{1/2} dz$$

$$= \frac{4(.0138)}{Z_0\pi D\sqrt{300}}\int_0^5 \left[300 - \frac{z}{5}220\right]^{3/2} dz$$

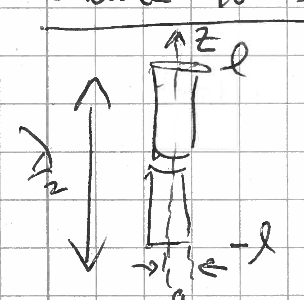
$$\int_0^5 (300 - 44z)^{3/2} dz = \int_0^5 (300-44z)^{3/2}(-44dz)\left(\frac{-1}{44}\right)$$

$$= -\frac{1}{44}(300-44z)^{5/2}\frac{2}{5}\Bigg|_0^5 = -\frac{2}{220}\left\{(80)^{5/2} - (300)^{5/2}\right\} = \frac{1.558\times10^6 - 5.7\times10^4}{110}$$

$$= 1.36\times10^4$$

$$T_B = \frac{(1.36\times10^4)(4)(.0138)}{100\,\pi\,(.12)\sqrt{300}} = \underline{\underline{1.15\,^\circ K}}$$

Ohmic losses in a dipole antenna



$$I(z) = I(0)\cos kz \qquad 2\ell = \frac{\lambda}{2}, \quad \frac{2\pi}{\lambda}\ell = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \frac{\pi}{2}$$

For a symmetric current distribution $\quad R = \dfrac{R_s}{2\pi a}$

$$W = \frac{1}{2}\,\mathrm{Re}\int_{-\ell}^{\ell} I^2(z)R\,dz = \frac{1}{2}\left(\frac{R_s}{2\pi a}\right)\int_{-\ell}^{\ell} I^2(0)\cos^2 kz\,dz$$

$$= \frac{1}{2}\left(\frac{R_s}{2\pi a}\right)I^2(0)\int_{-\ell}^{\ell}\frac{1}{2}(1+\cos 2kz)\,dz$$

$$\frac{1}{2}\int_{-\ell}^{\ell}(1+\cos 2kz)\,dz = \frac{1}{2}\,z\Big|_{-\ell}^{\ell} + \frac{1}{2}\,\frac{\sin 2kz}{kz}\Big|_{-\ell}^{\ell} = \ell \checkmark$$

$$W = \frac{R_s\,\ell}{4\pi a}\,I^2(0) = \frac{I^2(0)}{2}\left\{\frac{R_s\,\ell}{2\pi a}\right\}$$, Note: ℓ/a is fixed here.

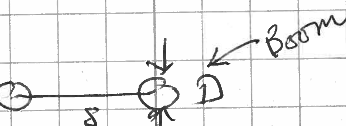
The power radiated is $\frac{I^2(0)}{2}R_r$ with $R_r = 72\,\Omega$

At 3 GHz, $R_s = .0138$, $\ell = \lambda/4 = 10\,\text{cm}/4 = 2.5$ cm. [$\lambda = 10$ cm]
The trace width is 0.82 cm for the 300 Ω case so take $a = 0.41$ cm, $a/\ell = 0.16$

$$\frac{R_s\,\ell}{2\pi a} = \frac{(.0138)(2.5)}{2\pi(.41)} = 0.0134\,\Omega \quad ; \quad \frac{.0134}{72} = 2\times10^{-4} = \frac{W_{res}}{W_{rad}}$$

2×10^{-4} of input power goes into resistive losses.

The contribution to T_{sys} is $(2\times10^{-4})(300) = \underline{.06\,K}$



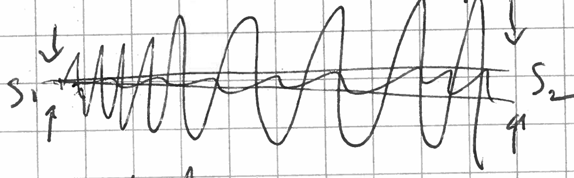
Losses along the antenna

$R_s = 2.5\times10^{-7}\sqrt{f}$ for Cu.

For fixed D, $$\alpha = \frac{RS}{60\pi D\cosh^{-1}(S/D)} = \frac{R}{2Z_0} = \frac{4R_s/D\pi}{2\cdot 120\cosh^{-1}(S/D)}$$

For fixed S/D $$\alpha(s) = \frac{1}{S}\left\{\frac{R_s\,S}{60\pi D\cosh^{-1}(S/D)}\right\} \propto \frac{1}{S}$$

S/D should be kept constant along the antenna for self similarity



The boom is the main transmission line.

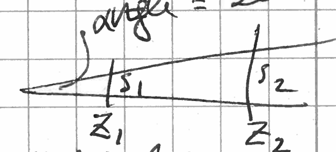
Start at $S_1 = 0.5$ cm, go to $S_2 = 5$ cm $= \lambda/2$ at 3 GHz.

1. S/D = 2.73 gives $\underline{200\,\Omega}$ in free space. eep ratio constant along the boom by expanding trace

$$\alpha_R(s) = \frac{1}{s}\left\{\frac{R_s(S/D)}{60\pi \cosh^{-1}(S/D)}\right\} = \frac{1}{s}\left\{\frac{(.0138)(2.73)(2)}{\pi[120 \cosh^{-1} 2.73]}\right\}$$

$$= \frac{1}{s}\left\{\frac{(.0138)(2.73)(2)}{\pi \; 200}\right\} = \frac{1.2\times10^{-4}}{s} \quad cm^{-1}$$

Loss from S_1 to S_2 or Z_1 to Z_2



$S = \alpha_0 Z$

$\alpha = \frac{1}{3}, 20°$

Power loss: $2\alpha_T = 2\int_{Z_1}^{Z_2} \frac{1.2\times10^{-4}}{s}\,dZ =$ ~~[illegible]~~

$$2\alpha_T = 2\int_{Z_1}^{Z_2} 1.2\times10^{-4}\frac{dZ}{\alpha Z} = (2.4\times10^{-4})\,3\,\ln(Z_2/Z_1)$$

$S_1 = 0.5\,cm$, $Z_1 = 1.5\,cm$; $S_2 = 5\,cm$, $Z_2 = 15\,cm$.

$$2\alpha_T = 7.2\times10^{-4}\ln(15/1.5) = 7.2\times10^{-4}(2.306) = \underline{\underline{.0017}}$$

$\ln 10 = 2.3$; $\ln 5 = 1.6$

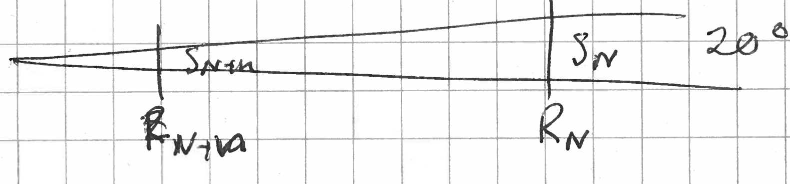
2. <u>For the 300 Ω case</u> $\quad S/D = 6.13$

$$2\alpha_R(s) = \frac{2}{s}\left\{\frac{(.0138)(6.13)(2)}{\pi \; 300}\right\} = \frac{.00036}{s} = \frac{3.6\times10^{-4}}{s}$$

$$2\alpha_T = \int_{Z_1}^{Z_2} \frac{3.6\times10^{-4}}{\alpha Z}\,dZ = (3.6\times10^{-4})\,3\,\ln(Z_2/Z_1) = \underline{\underline{.0025}}$$

(small losses.)

<u>Period sizes and trace width</u>



$S_N = 5\,cm$
$S_{N+n} = 0.5\,cm.$
$(\tau = 0.95)$

$$\frac{R_{N+1}}{R_N} = \tau, \quad \frac{R_{N+2}}{R_{N+1}} = \tau; \qquad R_{N+2} = \tau^2 R_N \qquad R_{N+n} = \tau^n R_N$$

$$S_N = \alpha R_N \qquad \frac{S_{N+n}}{S_N} = \tau^n = \frac{0.5}{5} = \frac{1}{10}$$

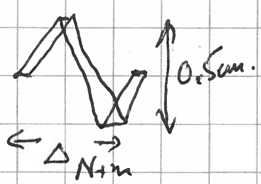
$n \ln\tau = \ln(1/10)$; $n = \dfrac{\ln(1/10)}{\ln(.975)} = \dfrac{-2.3026}{-.0253} = 91$

There are 91 cells or log periods.

Period size: $\Delta_N = R_{N+1} - R_N = \tau R_N - R_N = (\tau-1) R_N$

At $z_i = z_1$, the input, $\Delta_{N+n} = (\tau-1) R_{N+n} = (\tau-1) S_{N/\alpha_0}$

$\Delta_{N+n} = 3(\tau-1)0.5 = 3(.0250)(0.5) = .0375 \text{ cm} = .015''$



The trace width has to be $\simeq \Delta_{N+n}/5 = .003''$ for small overlap.

For $s/D = 6.13$, the boom trace must be $0.5/6.13 = .0816 \text{ cm} = .032''$ wide

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The simulation field for the $\tau = .975$, $\alpha = 20°$, antenna (zig-zag)

The electric field pattern is very nearly the same in both the E-plane and the H-plane, so we'll use it's average.

Use $F = 0.42$ and $D = 5$ m for the reflector

The angle at the edge. $\cos\Theta = \dfrac{(4F)^2-1}{(4F)^2+1}$ $\qquad \Theta_{edge} = 61.5°$

Assume that the focus of the feed and that of the paraboloid are the same point.

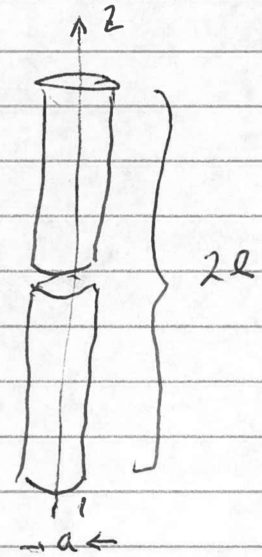
First, find the spillover

$$K = \frac{2\pi}{\lambda} \qquad \ell = \lambda/4$$

$$2K\ell = 2\cdot\left(\frac{2\pi}{\lambda}\right)\cdot\frac{\lambda}{4} = \pi$$

$$\frac{1}{2}\int_{-\ell}^{\ell}(1+\cos 2KZ)\,dZ = \frac{1}{2}\left\{ Z\Big|_{-\ell}^{\ell} + \frac{\sin 2KZ}{2K}\Big|_{-\ell}^{\ell}\right\}$$

$$= \frac{1}{2}\left\{2\ell + \frac{\lambda}{4\pi}(0-0)\right\} = \ell.$$



$$I(z) = I(0)\cos KZ$$

$$K = \frac{2\pi}{\lambda}, \quad \text{for } 2\ell = \frac{\lambda}{2}, \quad K = \frac{2\pi}{4\ell} = \frac{\pi}{2\ell}$$

The power radiated is: $P_r = \frac{I^2(0)}{2} R_r$

$R_r = 72\,\Omega$ for $2\ell = \frac{\lambda}{2}$

The loss in the wire is: $P_\ell = \frac{1}{2}\left(\frac{R_s}{2\pi a}\right)\int_{-\ell}^{\ell} I^2(0)\cos^2 KZ\,dZ$

$$P_\ell = \frac{I^2(0)}{2}\left(\frac{R_s\ell}{2\pi a}\right) = \frac{I^2(0)}{2}\left\{\frac{R_s\lambda}{8\pi a}\right\} \quad \text{for } 2\ell = \frac{\lambda}{2}$$

$$\frac{P_\ell}{P_r} = \frac{1}{R_r}\left\{\frac{R_s\lambda}{8\pi a}\right\} \text{ or } \frac{1}{R_r}\left\{\frac{R_s\ell}{2\pi a}\right\}$$

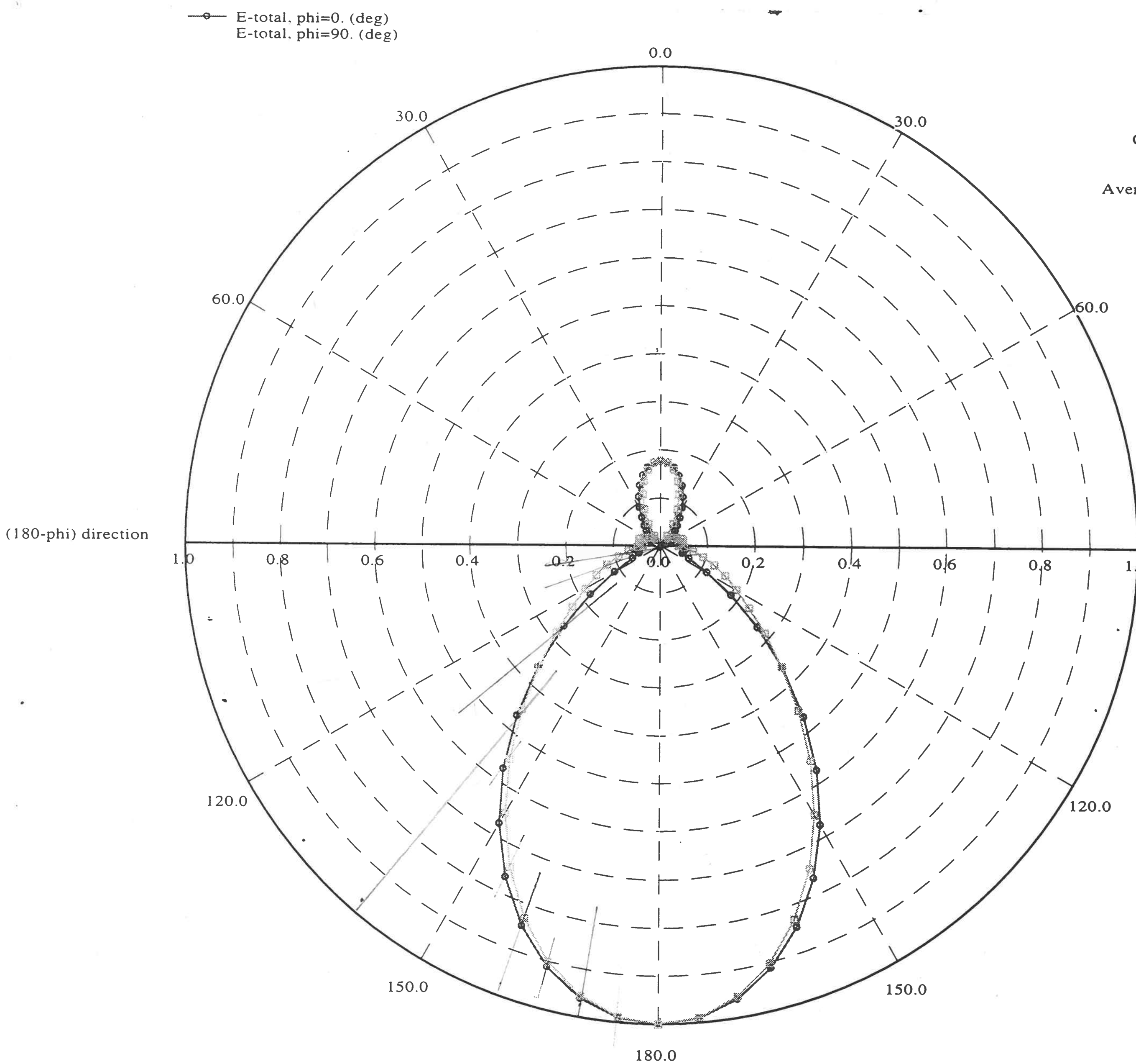
$R_r = 72$ for $2\ell = \frac{\lambda}{2}$; Also $R_s = 2.52\times10^{-7}\sqrt{f}$

At 3GHz, $R_s = .014\,\Omega$, $\lambda = 10$ cm., $2\ell = 5$ cm., take $a = .4$ cm.

[redacted]

Frequency: 2 (GHz)

Maximum at (180, 70) degrees
The Directivity: 11.1212 (dB)
Mismatch Loss: -4.70425 (dB)
Circular Polarization Loss: -2.96747 (dB)
Efficiency: 46.0473% (-3.36796 dB)
Total Radiated Power: 0.00155876 (W)
Average Radiated Power: 0.000124042 (W/s)
Input Power at Ports: 0.00338513 (W)

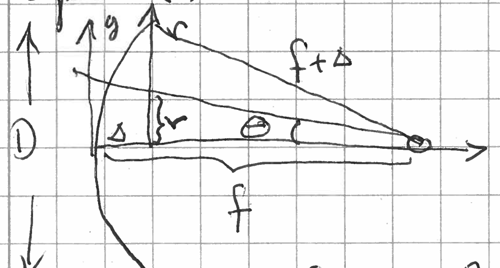


Polar Elevation Pattern

θ°	$E(\theta)$	$E^2(\theta)$	$\sin\theta$	$\sum E^2 \sin\theta$	%/100
0	1	1	0	0	0
5	.987	.974	.0872	.0849	.0514
10	.953	.908	.1736	.2426	.1467
15	.899	.808	.2588	.4517	.273
20	.831	.691	.3420	.6880	.416
25	.745	.555	.4226	.9225	.558
30	.653	.426	.5	1.1355	.687
35	.555	.308	.5736	1.3122	.794
40	.451	.203	.6428	1.4420	.873
45	.351	.123	.7071	1.5297	.9252
50	.269	.072	.7660	1.5848	.958
55	.192	.037	.8192	1.6151	.977
60	.146	.021	.8660	1.6333	.988
→ 65	.096	.0092	.9063	1.6417	.993
70	.079	.0062	.9397	1.6475	.997
75	.054	.0029	.9659	1.6503	
80	.042	.0018	.9848	1.6521	
85	.029	.0008	.9962	1.6529	
90	.020	.0004	1	1.6533	

The edge is at 61.5°   The spillover is about 1.0% !?
Note that there is about 2% in the back lobe, looking at the sky

Aperture efficiency for this feed with $F = 0.42$, ($D = 5$ m) including the $1/R$ effect of the field in the transformation to the aperture.



R varies from $R = f$ to $R = f + \Delta$

$y^2 = 4fx$ so $(D/2)^2 = 4f\Delta$; $f = FD = .1$m

$\Delta = D^2/16f = \frac{25}{16(2.1)}$ 0.74m

so $2.1 < R < 2.84$m

Correct the field by linearly interpolating this between center and edge.

$\theta \to r$ transformation: $\tan\theta \;/4\;) = \;/f(1 - \frac{1}{(4F)^2})$

In this case $\tan\theta = [0.738r]$
(approximation)

r (m)	θ°	$E(\theta)$	$E(r)$	$E^2(r)$	$r\,E^2(r)$	$r\,E(r)$
0	0	1.0	1.0	1.0	0	0
.25	10.5	.948	.92	.85	.2125	.23
.50	20.3	.826	.78	.61	.3050	.39
.75	29.0	.671	.62	.38	.285	.465
1.00	36.4	.526	.47	.22	.22	.47
1.25	42.7	.397	.34	.090	.1125	.425
1.50	47.9	.303	.26	.068	.102	.39
1.75	52.2	.235	.19	.036	.063	.333
2.00	55.9	.184	.15	.023	.046	.300
2.25	58.9	.156	.12	.014	.0315	.270
2.50	61.5	.131	.097	.009	.0225	.2425
					$\Sigma =$ 1.400	$\Sigma =$ 3.516

2.

The taper to the edge is ~ -20 db.

Note: $E(r) \cong (1 - r/2.5)$

$\Delta r = 0.25$

$$A_e(0) = \frac{\left|2\pi \int_0^{2.5} E(r)\, r\, dr\right|^2}{2\pi \int_0^{2.5} |E(r)|^2 r\, dr} = \frac{\left|2\pi \sum E(r)\, r\, \Delta r\right|^2}{2\pi \sum |E(r)|^2 r\, \Delta r}$$

$$= \frac{2\pi(\Delta r)\left[\sum_i E(r_i) r_i\right]^2}{\sum |E(r_i)|^2 r_i} = \frac{2\pi(0.25)(12.36)}{1.40} = 13.87$$

$$\text{Area} = \pi(2.5)^2 = 19.63\ m^2$$

$$\eta_A = \frac{13.87}{19.63} = \underline{\underline{70.7\%}} \qquad = \eta_{ill.}$$

The effect of capacitive loading by the input teeth

Near the input $S_{Nth} = 0.5$ cm. $= 0.200''$ Since the trace folds back it's effective width is $\sim 2(.003'') = .006$. It's length is effectively $S_{Nth} \sim 200$, including the teeth on both sides.

$S_{Nth}/w = .200/.006 \doteq 32.$

Γ a simple dipole from Balanis p. 452, and $l/d = 25$, the reactance is $-600\,\Omega$ when $l = 0.14\lambda = (0.3)(0.5\lambda)$.

At resonance, at 3GHz, $S_N = 5\text{cm}$.

At $S_{N+n} = 0.3\, S_N$, $R_{N+n} = 0.3\, R_N$

At this point the reactance of one pair of teeth is -600Ω.
-600 means capacitive

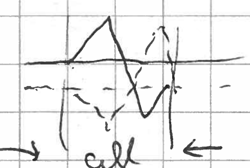
$$600 = \frac{1}{\omega C} \qquad C = \frac{1}{2\pi\, 3\times10^9 \times 600} = 8.85\times10^{-14}\ \text{fd.}$$

How many teeth/cm at this point?

$$-\Delta_{N+n} = R_{N+n+1} - R_{N+n} = -(1-\tau)\, R_{N+n}\text{, but } S_N = \alpha_0 R_N$$
where α_0 is the angle of the "boom", 20°

$$\text{So } \Delta_{N+n} = (1-\tau)\, S_{N+n}/\alpha_0 = 3(1-\tau)\ 0.3\ S_N = 3(1-\tau)(0.3)(5)$$
$$= 0.1125\,\text{cm}$$

So there are 9 teeth in one cm. The capacitance/cm of the teeth is therefore $9\times 8.85\times10^{-14} = 7.9\times10^{-13}$ fd/cm.



But there are really two teeth/cell.

Thus $C = 1.6\times10^{-12}$ fd/cm $=$ $\underline{\underline{1.6\,\text{pf/cm}}}$ $= 160\,\text{pf/m}$

This is ~~the~~ just the free space capacitance.
The boom capacitance for $Z_0 = 300\,\Omega$ in free space is:

with $(S/D) = 6.13$

$$C = \frac{\pi\epsilon_0}{\cosh^{-1}(6.13)} = \frac{\cancel{\pi}\ 10^{-9}}{6\cancel{\pi}\ (2\quad)} = 1.11\times10^{-11}\ \text{fd/m} = 11.1\times10^{-12}\ \text{fd/m}$$

The loading is 160 p/m whereas the boom cap. is 11.1 pf/m.

so the loading dominates Z_0! $160/11 = 14.5$

Assume there's no inductive contribution

$$Z_0 = \sqrt{\frac{L}{C}} \qquad \sqrt{14} \qquad 3.8 \qquad 300\,\Omega \Rightarrow \frac{300}{.8} - 80\,\Omega$$

This would also explain the observed slow wave.

$v = \frac{1}{\sqrt{LC}}$ just increasing c by 14 should decrease v to $\frac{v}{3.8}$.

In fact, it's not that slow. In Rumsey, p. 67, its quoted to be $\frac{2}{3}c$. We should be able to use this value to calculate Z_0. $v = \frac{1}{\sqrt{LC'}}$ $v^2 = \frac{1}{LC'} = \left(\frac{2}{3}c_0\right)^2 = \frac{4}{9}c_0^2$

$c_0 = \frac{1}{\sqrt{LC}}$, $c_0^2 = \frac{1}{LC}$ $\frac{1}{LC'} = \frac{4}{9}c_0^2$; $Lc_0^2 = \frac{1}{C}$; $\frac{1}{C'} = \frac{4}{9}Lc_0^2$

$\therefore C' = \frac{9}{4}C$

Then $Z_0' = \sqrt{\frac{L}{C'}}$ and $Z_0 = \sqrt{\frac{L}{C}}$

$$Z_0' = \sqrt{\frac{L}{9/4\,C}} = \sqrt{\frac{4}{9}\frac{L}{C}} = \frac{2}{3}Z_0$$

Thus a nominal line impedance of $300\,\Omega \to 200\,\Omega$.

$A_e(0)$ dependence on edge illumination.

The illumination was found to be close to a linear taper above.

I. If we use $E(r) = \left(1 - \frac{r}{2.5}\right)$ which goes $\to 0$ at the edge we find $\frac{A_e(0)}{\pi(2.5)^2} = \underline{0.67}$

II. If we use $E(r) = \left(1 - \frac{r}{3.57}\right)$ which goes to 0.3 at the edge, 2.5.
we find $\frac{A_e(0)}{\pi(2.5)^2} = \underline{0.91}$

This case would put the edge at $\theta \simeq 48°$ The spillover would be about 7% instead of 1%, dropping $\frac{A_e()}{\pi r^2}$ to about 85% and adding ~~4~~ 18K to Tsys.

[There may be an optimum in here somewhere]

Preliminary Summary of Noise Contributions ($f = 3\,GHz$)

Losses into cryostat ($Z_0 = 100\,\Omega$)		$1.1\,°K$
Losses at the connector ($Z_0 = 100\,\Omega$) ;	$(.0064) \times 300 =$	$1.9\,K$
Losses in the antenna	$\frac{3}{2}(.0025) \times 300$	$1.1\,K$
Spillover past the primary	$1\% \times 300$	$3\,K$
		$7.1\,K$

Re-evaluation:

into cryostat		$1.2\,°K$
connector	$.0064 \times 300 =$	1.9
antenna	$.0075 \times 300 =$	2.2
spillover	1% @ $300K$ =	3.0
		8.3

Atmosphere	$\gtrsim 3.0\,K$
Feed legs + mirror	~ 5.0 (perhaps high)
BB Background.	3.0
	$\sim 20\,K$

A little more about the reflector deflection (p. 6)

$\Delta x \quad \frac{g}{\omega^2} - \frac{980}{(2\pi)^2 \nu_0^2} - \frac{1 \text{ m}}{(\nu_0/157)^2}$ $\qquad g = \quad$ cm/sec

$\nu_j = \frac{\omega_j}{2\pi} = \frac{t}{\pi a^2}\left[\frac{\mu_j g E}{12(1-\sigma)\rho}\right]^{1/2}$

For ULE glass $\begin{cases} E - \ \times 10^5 \text{ kgm/cm}^2 \\ \sigma = \ .17 \end{cases}$ $\quad \rho = 2.2\times10^{-3}$ kgm/cm^3 $\quad \underline{E/\rho = 1.8\times10^8 \text{ cm}}$

For steel $\begin{cases} E = \cancel{\ } 2\times10^6 \text{ kgm/cm}^3 \\ \rho \quad 7.9\times1^{-3} \text{ kgm/cm}^2 \end{cases}$ $\quad \underline{E/\rho \quad 2.5\times10^8 \text{ cm}}$

For the dish supported in the center, $\mu_j = 13.8$

← (ULE glass)

$\left[\frac{(13.8)(980)\,7\times10^5}{(12)(2.2\times10^{-3})}\right]^{1/2} = 5.988\times10^5$

$\omega_j = 2\pi\nu_j = \frac{t}{a^2}\ 5.988\times10^5$

$\Delta x = \frac{g}{\omega^2} = \frac{a^4\ 980}{t^2(5.988\times10^5)^2}$ $\qquad y^2 = 4fx \rightarrow (D/2)^2 = 4ft = a^2$; $\quad a^4/t^2 = (4f)^2$

$\Delta x = \frac{(4f)^2\ 980}{(5.988\times10^5)^2} = 4.4\times10^{-8} f^2 \text{ cm}$ $\qquad \boxed{\Delta x = 4.4\times10^{-8} f^2}$ cm.

For $D = 5$m and $f/D = 0.42$, $f = 2.1$m $= 2100$ cm

$\Delta x = (4.4\times10^{-8})(2100)^2 = 0.19$ cm $= \underline{1.9 \text{ mm}}$

The RMS is $\sim \frac{\text{Peak}}{2.5} = \underline{0.8 \text{ mm}}$ $\qquad$ (not quite; see p. 6)

What if $f/D = 0.6$? Then $f = 3.0$m and $\Delta x = \underline{3.96\text{mm}}$
and RMS ~ 1.6 mm ; $\frac{\Delta}{3.5} = \frac{3.96}{3.5} = 1.13$ mm

---

Ruze losses: For $\lambda/20$ RMS $\quad G = 0.67$
For $\lambda/30$ RMS $\quad G = 0.84$
For 0.8mm $\quad \lambda/30$ is at 24mm $= 2.4$cm $\quad$ 12.5 GHz
$\lambda/40$ RMS $\quad G = \quad 0.92$

$(6/5)^4 = 2.0$ for same depth ; Going from 5m to 6m doubles the deflection
RMS: 0.8mm ⟶ 1.6 mm ; peak: 3.8mm = 0.″15

2. Resonant Frequencies of ~~the Mirror Segments~~ Disks

The vibrating modes of thin round plates are discussed in detail in Prescott's monograph: Applied Elasticity (Longmans, Green and Co., New York, 1924). The proposed mirror segments are essentially thin round plates of 10 cm thickness and 1.8 m diameter. The possible modes of vibration depend, in part, on the boundary conditions, how the plate is held or whether it is free.

Analytic solutions are often possible because of the simple shape. The basic differential equation that must be solved for mode shapes and resonant frequencies is:

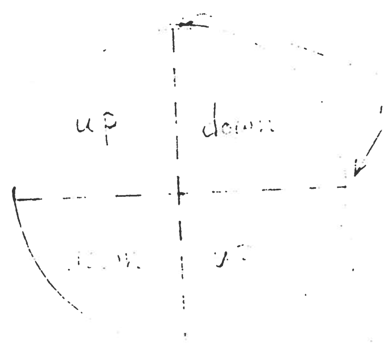
$$\nabla^4 w = k^4 w,$$

$$\text{where} \quad k^4 = \frac{12(1 - \sigma^2)\rho\omega^2}{gEt^2} .$$

w is the normal displacement, ρ is the density, E is the elastic modulus, ω is the resonant angular frequency, σ is the Poison ratio, g is the acceleration of gravity, t is the thickness of the plate. For a disk of radius a, solving the equation gives characteristic values of $\mu_j = (ka)^4{}_j$, and the corresponding resonant frequencies are given by

$$\nu_j = \frac{w_j}{2\pi} = \frac{t}{2\pi a^2}\left[\frac{\mu_j gE}{12(1-\sigma^2)\rho}\right]^{1/2}$$

The lowest frequency mode is the most important since it is the most likely to be excited either by the external forces discussed above or by the control system itself. In the model discussed above, the control frequency is 125 Hz. The lowest mode of a freely vibrating disk has two nodal diameters. It is therefore also a mode of a four point support system.



In fact, a three point support is planned, but the corresponding lowest ("potato chip") vibration with this constraint does not lead to a simple analytic solution. The lowest axisymmetric free vibration with one nodal circle at 0.7 the disk radius does fit a three point support, but its frequency is higher; it's not the lowest mode. We use the above pictured vibration with two nodal diameters.

$$\mu_o = (ka)_o^4 = 30.4$$

For ULE: $E \cong 7 \times 10^5$ kgm/cm^2, $\rho = 2.2 \times 10^{-3}$ kgm/cm^3, $\sigma = 0.17$. Then with $t =$ 10 cm and $a = 90$ cm, we find

$$\nu_o = 177 \text{ H}_z$$

The frequency for the lowest axisymmetric vibration is 285 H_z. These frequencies are well above the control frequency and the frequencies at which external forces have significant energy. Therefore, the segments may be regarded as perfectly rigid.

Calculate the RMS a little more correctly

Approximate the lowest mode, clamped in the center, by $y(r) = (r/a)^2\Delta$, where Δ is the edge deflection.

$$\langle y(r)\rangle = \int_0^a y(r)\,r\,dr \Big/ \int_0^a r\,dr$$

$$= \int_0^a (r/a)^2 \Delta\, r\,dr \Big/ \int_0^a r\,dr = \frac{2}{a^2}\Delta \cdot \frac{1}{a^2}\frac{r^4}{4}\Big|_0^a = \frac{\Delta}{2}$$

$$MS = \int_0^a (y - \langle y\rangle)^2\, r\,dr \Big/ \int_0^a r\,dr$$

$$= \frac{2}{a^2}\int_0^a \left[(r/a)^2\Delta - \frac{\Delta}{2}\right]^2 r\,dr = \frac{2\Delta^2}{a^2}\int_0^a \left[\frac{r^4}{a^4} - \frac{r^2}{a^2} + \frac{1}{4}\right] r\,dr$$

$$= \frac{2\Delta^2}{a^2}\left[\frac{r^6}{6a^4} - \frac{r^4}{a^2 4} + \frac{1}{4}\frac{r^2}{2}\right]_0^a = 2\Delta^2\left[\frac{1}{6} - \frac{1}{4} + \frac{1}{8}\right] = \frac{2\Delta^2}{24}$$

$$RMS = \sqrt{\frac{\Delta^2}{12}} = \frac{\Delta}{3.5}$$

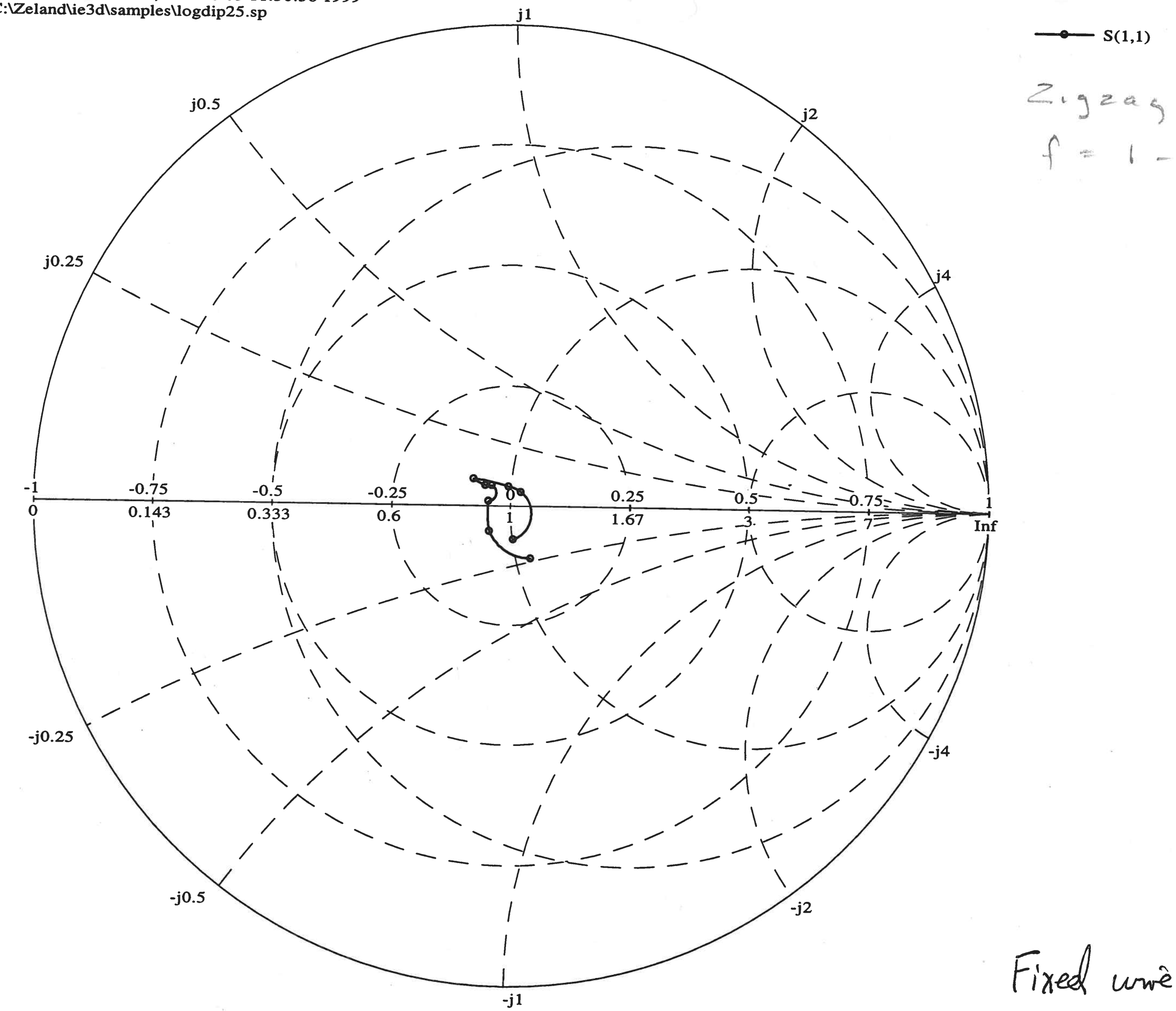
For $\Delta = 4.4\times10^{-8} f^2(cm)$, $RMS = 1.27\times10^{-8} f^2(cm)$

For $D = 5m$, $F = f/D = 0.42$, $f = 2.1m = 2100\ cm$,
<u>$RMS = 0.56mm$.</u>

$30 \times 0.56\,mm = 16.8\,mm$; for $\lambda = 1.68\,cm$, $\nu = 18\,GHz$

Zc(1)=(350, -50) Ohms. Impedance in actual Zc

Zeland Software, Inc., IE3D 5.0, Fri Feb 05 11:50:36 1999
Data File: C:\Zeland\ie3d\samples\logdip25.sp



Zigzag, single pol.,
f = 1 - 5 GHz

Fixed wire diameter
$Z_0 \sim 200\ \Omega$ for line
Reference = $350\ \Omega$

Feed Options – Summary

1. Input losses + Sandy's, with ~ 1-2% spillover losses, gives Tsys ~ 30 35K

2. Higher practical gain with LP feed ~ 11.5 db, dual pol. ok.

3. Movable focus with F/0.4 works best, has 4:1 BW with $\eta_A \geq 0.84$ at any given position. disadvantage is mechanical

4. Operation at F/0.6 fixed gives same gain but 10% spillover. Fix the spillover with either
   a) edge screen
   or (b) overall ground screen, or (c) lens system
   This is the most attractive solution: but disk must be stiffened

5. First cut at input impedance shows $\Gamma \leq 0.1$ (-20 db) at $Z_0 = 350\Omega$. better @ ~ 160Ω

6. Remaining Simulation Tasks
   (a) Vary the wire diameter to study Z_{in}.
   (b) Add the amplifier and scatter cone. Study the patterns and Z_{in}

7. Build a model and test it in Orbitron dish
8. We may wish to take over Sandy's effort, with him as consultant.

Mount Investigations

1. Matt looking at Orbitron motions.

2. Needed: optical guider for tracking accuracy
   drive arrangements
   coordinate conversions

RFI: engineer identified by JD.

Personnel:
a) Hire an EE to work for JW.
b) ~~another~~ Technical assistant for Matt
c) a high level cryo/vacuum/Telescope person (kaytem)
d) Research @MIMIC mounting (b) single mode options. Talk to Greg P.

AASC Radio Panel Martha Haynes chair

Jim Moran
Alan Marscher
John Carlstrom
Don Campbell
Mark Reid
Ken Kellermann
Steve Myers
Jackie Hewitt
Neal Evans
Jack Welch

Feb 19, 20 meeting Wash.
June 5, 6 Chicago Chicago
July 16, 17 Berkeley (RAL Hosts)

JW presentation

points SKA prototyping: SKA consortium.
1HT as an example of what's going on - no comments
Everyone had already seen the publicity. - Martha's comment.

Russ Taylor presentation: Everything's there, but not much punch

Feb. 24th Board Meeting

Leo: documents looks good; { portable hardware → SETI
{ fixed " → UC

Meeting with industry experts 3/15/99 at SETI Inst.

GTE Sylvania people worked on log-periodic feeds for reflectors - earth station applications - multi-mode -

Ridged Guide feed - 5- db ok for gain ~ 3 octaves done. The ridges can be put there by themselves, no waveguide. They can be nested -

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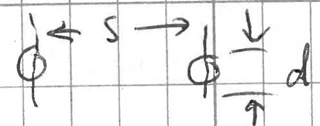
coplanar wg. or micro-strip { Kyocera
{ strategy ←

Fred

Two wire line losses revisited



From R+W, $R = \frac{2R_s}{\pi d}\left[\frac{s/d}{\sqrt{(s/d)^2 - 1}}\right]$

The second factor accounts for the non-uniform current distribution on the wires.

For $Z_0 = 200\,\Omega$ in air, $s/d = 2.73$ (p.12 of notes)

$\frac{s/d}{\sqrt{(s/d)^2-1}} = 1.07$, a small effect, not a factor of two.

For Z_0 $150\,\Omega$, $s/d = \cosh\left(\frac{150}{120}\right) = 1.89$

$\frac{1.89}{\sqrt{(1.89)^2-1}} = 1.18$, still a small factor of correction.

The lowest TE modes in coax line have:

$\lambda_c \sim \frac{2\pi}{n}\left(\frac{r_0 + r_c}{2}\right)$ or $2\pi\langle r\rangle = n\lambda_c$

For the lowest mode, $n=1$, $\lambda_c \sim 2\pi\langle r\rangle \sim \pi\langle d\rangle$, mean circumference

This suggests for the two wire line, $\lambda_c \approx 2\pi\frac{\left(\frac{d}{2}+\frac{s}{2}\right)}{2} = \pi\frac{(d+s)}{2}$

$(d+s) \lesssim \frac{2}{\pi}\lambda_c$

(a) With $\lambda = 3\,cm$ $d\ s \lesssim \frac{2}{\pi}3\,cm \lesssim 2\,cm$.
Then with $1.89\,d$, $d + 1.89d \lesssim 2\,cm$; $\underline{d \le 0.7\,cm}$
$d \le 0.7\,cm$, $s \le 1.3\,cm$, pretty large (?)

(b) With $\lambda = 1\,cm$, $d \le 2.3\,mm$ and $s \le 4.3\,mm$.

Power Losses $2\alpha_c = R/Z_0$

Take $Z_0 = 150\,\Omega$ and $s/d = 1.89$.

case ⓐ losses; $d = 0.7$ cm, $s = 1.3$ cm, $f = 10$ GHz

$$R = \frac{2R_s}{\pi d}(1.18) = \frac{2(1.18)\,2.52\times10^{-7}\sqrt{10^{10}}}{\pi\,0.7} = .027\ \Omega/\text{cm}$$

$2\alpha_{10} = .027/150 = .00018$, very small, but large wires

case ⓑ $f = 30$ GHz $d \to d/3$, $R \to 3R$; $3R =$

$2\alpha_{30} = .0009$/cm.

Here $d = 2.3$ mm, $s = 4.3$ mm max for $\lambda = 1$ cm

Select an aceptible size $d = .05''$, 50 mils, 1.2 mm,
smaller than $d = 2.3$ mm.
$s = 1.89 \times .05 = .095''$.

$$R = \frac{2\,(2.52\,10^{-7}\sqrt{10^{10}})(1.18)}{(.095)} = 0.1993 \doteq 0.2\ \Omega/\text{cm}.$$

$\underline{2\alpha_{10}} = 0.1993/150 = .0013$ neper/cm, $\underline{-.0058\ \text{db/cm}}$

Loss in 10 cm lengths (4") $t = 0.868$, 1.3 %

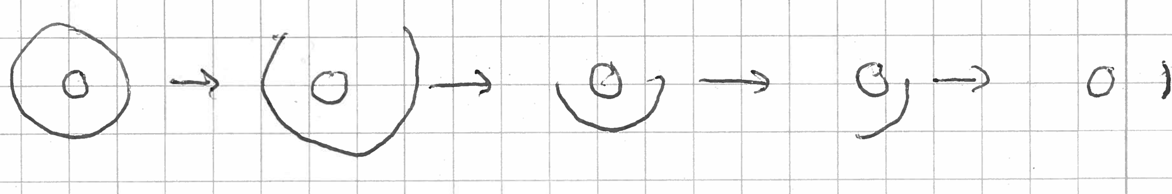
---

balun by slowly splitting an air coax line.

$Z_0 = 60 \ln(r_o/r) = 50$; $r_o/r_i = 3.32$

With $d_i = 50$ mils, $r_i = 25$ mils, $r_o = 0.083''$, 2.1 mm.

Shortest operating wavelength $\lambda_c \approx \left(\frac{r_o + r}{2}\right) 2\pi \to (.6 t_i) = \underline{0.86}$ cm.



(Lance 860 752-79 24)

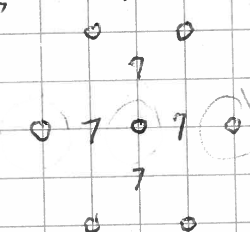
May 5 meeting at SETI

to do: model of feed

Both Al Bagley + Don Hammond are not persuaded that SETI is worthwhile —
Leo: get the "primordial film" - Ron Ekers has it -
Get Geoff Marcy - a distraction? no, "planets mean life"

Build an array at Sun -
get Dave deBoer
How many antennas? - What mounts?
What to test? Beam Forming and RFI?
Work at low frequencies only - < 5 plus HI
Single or dual polarization?
→ Jack call Greg about where to put it - which side of bldg - 7, 9, or 10
At Sun? at SETI?
Who to manage? - D. Back, R. Simegal (?)
how about his VISA?

← 21m → ~75 feet



Orbitron dishes 3 m - to 7 m

or a liner array with d = 7m
with N = 7, need ~50m trip (150')

next meeting July 26 -

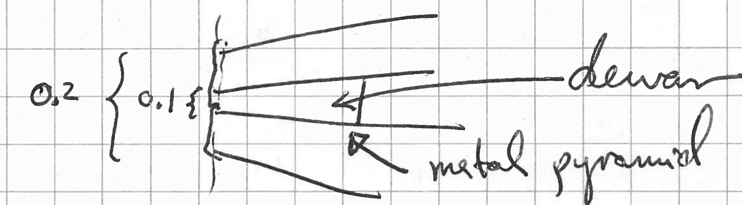
Greg P gregp@Corp.Sun.com
~~(415) 786 - 7378~~
(650) 960 - 1300 ( )

Use 3.6m Orbitron
study
ⓐ beam forming
ⓑ RF excursion || in parallel with the antenna and
ⓒ GP calibration || receiver prot typing
ⓓ drive control w/ a tuotis

Greg is enthusiastic - How about using a roof on one of the Sun buildings? How heavy would the antennas be What would be the cabling req
Visitor Areas etc?

5/23/99 Greg's latest simulation

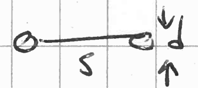
Input dimensions



① Upper frequency is here 11 GHz

② Antenna losses are about 1% only. , $Z_0 = 240 + j10$

③ Recalculate the losses into the dewar, using the necessary small wires here.



$s/d = 3.7$ for $Z_0 = 240\,\Omega$

Since the inner pyramid is only ~100 mils across, take d=15 mils and s= 55 mils. 15 mils = ~~.0066cm~~ .038 cm

$$R = \frac{2R_s}{\pi d}\frac{s/d}{\sqrt{(s/d)^2-1}} = \frac{(2.52\times10^{-7})(11\times10^{9})^{1/2}}{\pi\,(.038)}(1.04) = 0.44\,\Omega/\text{cm}$$

$$2\alpha(z) = \frac{\cancel{2}R}{\cancel{2}Z_0} = \frac{0.44}{240} = .0018\ \text{cm}^{-1}$$

$$T_B = \int_0^{5\text{cm}} T(z)\,2\alpha(z,T)\,dz = \int_0^{5\text{cm}}\left[300-\frac{z}{5}220\right](.0018)\left[\frac{300-\frac{z}{5}220}{300}\right]^{1/2}dz$$

$$= \frac{.0018}{\sqrt{300}}\int_0^5\left[300-\frac{z}{5}220\right]^{3/2}dz = \frac{.0018}{\sqrt{300}}\left(\frac{5}{220}\right)\int_0^5\left[300-\frac{z}{5}220\right]^{3/2}\frac{220}{5}dz$$

$$= \frac{0.0\;8\,(5)}{\sqrt{300}\,(220)}\left[300-\frac{z}{5}\cdot220\right]^{5/2}\frac{2}{5}\Bigg|_0^5 = \frac{(.0018)}{\sqrt{300}\;110}\left[(300)^{5/2}-(80)^{5/2}\right]$$

$$= 9.68\times10^{-7}\,(1.56\times10^{6}) = \underline{\underline{1.5\,K}}$$ ok, seems very small.

The connecting wires/xmission lines are ~70 mil long, 0.18 cm.

$(0.18)(.0018) = .0003$ negligible

(4) Overall Noise budget, apart from the amplifier

		×1.3 (real Silver losses)	
antenna: 1%	3K	} 5.9	
dewar Xmission line	1.5K		5cm in the dewar
2.7 Backgnd.	2.7		
atmos.	3.0		
	10.2	11.6 ≅ 12K ;	losses at 11GHz

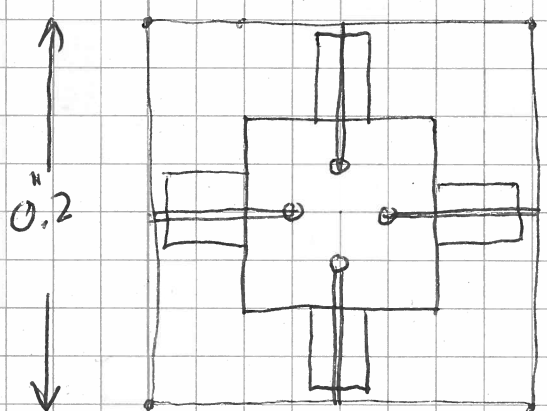
With Sandy's model 10K get 22K

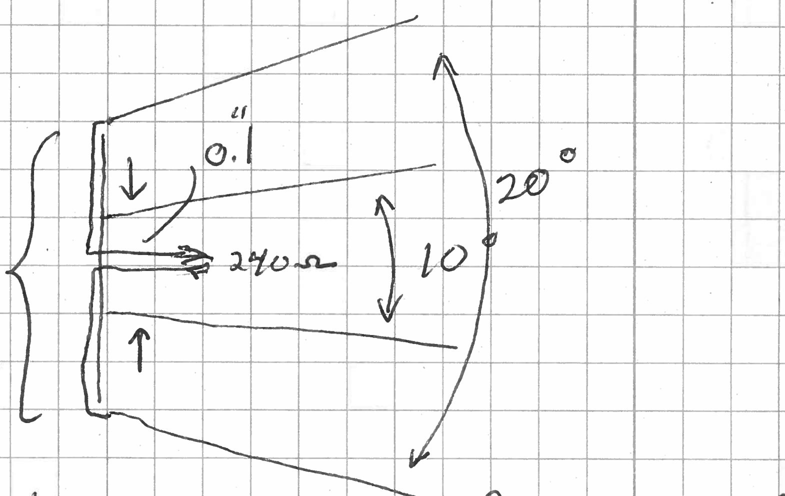
Hughes. rp. 25 get 36K

loss scaling $= \left[\frac{\ln f}{\sqrt{f}}\right]^{-1}$

At $T_0 = 3$ K, $T_{amp} \sim 45$K, and $T_{sys} \sim 60$K

The input region





Cover the end of the internal metal pyramid with conductor. Then 4 holes for the wires.
The cones are 15 mil silver platted stainless steel.

Antenna impedance: 240 − j10 Ω

A 2 wire line in free space with Z_0 240Ω has $d/s = 1/3.7$

The 2 0Ω 2 wire line has very small losses.

Antenna losses, using the simulator are ~1% only at 11GHz.

The input dimensions → 0.33" for an upper frequency of 11GHz

Pattern of the Feed with inner metal pyramid

Use the original coordinates.

Beware of poor sampling

Θ°	$E(\Theta)$	$E^2(\Theta)$	$\sin\Theta$	$\sum E^2 \sin\Theta$	Fraction
0	1	1	0	0	0
5	.983	.967	.0872	.0843	.0597
10	.934	.873	.1736	.2359	.1669
15	.858	.736	.2588	.4264	.3018
20	.762	.580	.3420	.6247	.4421 (.494)
~~20~~ 25	.653	.426	.4226	.8047	.5695
30	.541	.293	.500	.9512	.6732 (.752)
35	.431	.186	.5736	1.0579	.7487
40	.330	.109	.6428	1.1280	.7983 (.892)
45	.241	.058	.7071	1.1690	.8273 (.924)
50	.176	.031	.7660	1.1928	.8442 (.943)
55	.138	.019	.8192	1.2083	.8551
60	.127	.016	.8660	1.2222	.8650 (.966)
65	.123	.015	.9063	1.2358	.8746 (.977)
70	.105	.011	.9397	1.2461	.8819 (.985)
75	.082	.0067	.9659	1.2526	.8865
80	.062	.0039	.9848	1.2564	.8892 (.993)
85	.064	.0041	.9962	1.2665	.8963
90	.07	.0049	1.000	1.2654	.8955 (1)
95		.0043	.9962	1.2697	
100		.0033	.9848	1.2729	
105		.0051	.9659	1.2779	
110		.011	.9397	1.2882	.9117
115		.019	.9063	1.3054	
120		.025	.8660	1.3271	
125		.026	.8192	1.3484	
130		.024	.7660	1.3668	.9673
135		.018	.7071	1.3795	
140		.013	.6428	1.3878	
145		.009	.5736	1.3930	
150		.007	.5000	1.3965	.9883
155		.008	.4226	1.3999	
160		.011	.3420	1.4036	
165		.015	.2588	1.4075	
170		.020	.1736	1.4110	
175		.023	.0872	1.4130	
180		.025	0	1.4130	

$|E|^2$


```
<PolarPlot Title="Azimuth Pattern" Width="300" Height="300">
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<Axis Type="Y" Title="" From="0" To="1" Step="0.1"/>
<Line Legend="f=11(GHz), E-total, theta=90 (deg)" Color="#ff0000">
    -180 0.00418513
    -175 0.00374264
    -170 0.00389562
    -165 0.00660884
    -160 0.0112587
    -155 0.0151591
    -150 0.0173152
    -145 0.0211555
    -140 0.0335157
    -135 0.0616123
    -130 0.111238
    -125 0.186789
    -120 0.291066
    -115 0.423182
    -110 0.575455
    -105 0.731806
    -100 0.86963
    -95 0.965103
    -90 1
    -85 0.967327
    -80 0.873538
    -75 0.736473
    -70 0.579835
    -65 0.426381
    -60 0.292537
    -55 0.186415
    -50 0.109314
    -45 0.0587759
    -40 0.0306092
    -35 0.018977
    -30 0.0162759
    -25 0.015065
    -20 0.0114994
    -15 0.00671138
    -10 0.00392701
    -5 0.00408493
(90) 0 0.00492047
    5 0.00431119
    10 0.00333526
    15 0.00511555
    20 0.0110452
    25 0.0189515
    30 0.0249323
    35 0.026444
    40 0.0236432
    45 0.0184973
    50 0.0131978
    55 0.00925904
    60 0.00746017
    65 0.00806234
    70 0.0108731
    75 0.0151818
    80 0.0198049
    85 0.0233739
    90 0.0247837
    95 0.0235941
    100 0.0201949
    105 0.015662
    110 0.0113726
    115 0.00855665
    120 0.00797816
    125 0.00983351
```

Try a couple of different focal ratios (with $D = 5\,m$)

$$\cos\theta_{edge} = \frac{(4F)^2 - 1}{(4F)^2 + 1} \qquad \text{also/or} \qquad (4F)^2 = \frac{1+\cos\theta_e}{1-\cos\theta_e}$$

a) $40°$ $E^2 = .109$ at edge $(-9.6\,db)$
spillover between $40°$ and $90°$ is 11%
$40°$ and $180°$ is 20%

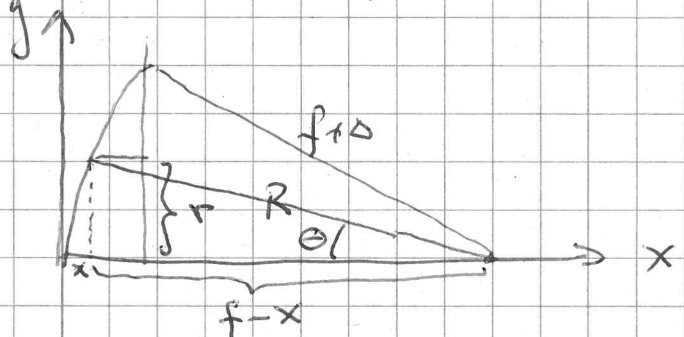
$$(4F)^2 = \frac{1 + .7660}{1 - .7660} = 7.5470 \qquad \underline{F = 0.69}$$

b) $45°$, $E^2 = .058$ at edge $(-12.4\,db)$
spillover between $45°$ and $90°$ is 7.6%
$\underline{F = 0.60}$

I. $\theta_e = 40°$; $F = 0.69$, ω $D = 5\,m$, $f = FD = 3.45\,m$
$\Delta = D^2/16f = 25/(16)(3.45) = \underline{0.45\,m}$

aperture radius is r $\qquad f < R < f + \Delta$; $\underline{3.45 < R < 3.90\,m}$

Do the transform to the aperture more accurately.



$y^2 = 4fx$ or $r^2 = 4fx$

$$R^2 = r^2 + (f-x)^2 = r^2 + \left(f - \frac{r^2}{4f}\right)^2$$

$$\tan\theta = \frac{r}{f-x} = \frac{r}{f - r^2/4f}$$

$$R^2 = r^2 + \frac{r^2}{\tan^2\theta} = r^2\left(1 + \frac{1}{\tan^2\theta}\right) = r^2\,\frac{\tan^2\theta + 1}{\tan^2\theta} = r^2\,\frac{\sec^2\theta}{\tan^2\theta}$$

$$\underline{R = r/\sin\theta}$$

The signal diminishes as $1/R$ but no more after reflection to the aperture plane. Scale the e-field to $E(0) = 1$, where $R = f$. For other values of r, use $E[\theta(r)]\,f/R$

For the case $\Theta_e = 40°$ and $F = 0.69$, $f = FD = 3.45\,m$
edge taper ~ $-9.6\,db$ $\quad D = 5m$

$r(m)$	$\Theta°$	R	$E[\Theta(r)]\,f/R$	$rE(r)$	$E^2(r)$	$rE^2(r)$
0	0	3.45	1.00	0	1	0
.25	4.1	3.50	.986	.247	.972	.243
.50	8.3	3.46	.951	.476	.904	.452
.75	12.4	3.49	.898	.674	.8[illegible]6	.605
1.00	16.5	3.52	.813	.813	.66	.661
1.25	[illegible].5	3.57	.726	.908	.527	.6[illegible]9
1.50	24.5	[illegible].62	.633	.950	.401	.6[illegible]2
1.75	28.5	3.67	.5[illegible]0	.945	.292	.[illegible]1
2.00	32.3	3.74	.452	.904	.204	.408
2.25	36.1	3[illegible]	.369	.830	.136	.306
2.50	39.8	3.[illegible]1	.291	.728	.085	.213
				$\Sigma = 7.475$		$\Sigma = 4.66$

$$A_e(0) = \frac{2\pi(\Delta r)\left[\sum_i r_i E(r_i)\right]^2}{\sum_i |E(r_i)|^2 r_i} = \frac{2\pi(.25)\,(7.475)^2}{4.66} = 18.825\,m^2$$

$$A = \pi r^2 = \pi(2.5)^2 = 19.635\,m^2 \qquad \eta_I = 18.825/19.635 = \underline{0.96}$$

Total feed radiation caught by antenna with $\Theta_e = 40°$ is $\underline{0.80}$
Total fraction of forward spillover, $40° \le \Theta \le 90°$, is $\underline{0.11}$

Forward spillover past $\Theta = 60°$ is 3.[illegible]%
" " " $\Theta = 70$ is 1.5%
" " " $\Theta = 80°$ is 0.7%
$\Theta = 6$[illegible] is 2.0%

<u>Aperture Efficiency</u>

$\eta_I = 0.96$
$\eta_{spillover} = 0.80$
$\eta_{focus\ err.} = 0.90$ $\quad$ for $F/0.69$

$\eta_{total} = \underline{0.69}$

To catch all but 2% of the feed radiation requires $\Theta \to 65°$ for the catch screen

II. For the case $\Theta_e = 45°$ and $F = 0.6$, $f = FD = \underline{3.0\ m}$

edge taper ~ −12.4db

r(m)	$\Theta°$	R	$E[\Theta(r)]f/R$	$rE(r)$	$E^2(r)$	$rE^2(r)$
0		3.00	1	0	1	0
0.25	4.8	3.00	.984	.246	.968	.242
.50	9.5	3.03	.930	.465	.865	.433
.75	14.3	3.04	.857	.643	.734	.551
1.00	18.9	3.09	.760	.760	.578	.578
1.25	23.3	3.16	.655	.819	.429	.536
1.50	28.1	3.18	.550	.825	.303	.455
1.75	32.5	3.26	.447	.782	.200	.350
2.00	36.9	3.33	.354	.708	.125	.250
2.25	41.1	3.42	.272	.612	.074	.167
2.50	45.2	3.52	.205	.513	.042	.105
				$\Sigma rE(r) = 6.373$		$\Sigma rE^2 = 3.667$

$$A_e(0) = \frac{2\pi(.25)(6.373)^2}{3.667} = 17.398 \ ; \ \eta_I = \frac{17.398}{19.635} = \underline{0.87}$$

Total feed radiation caught by the reflector: $\underline{0.83}$

Focus error phase loss at F/0.6 $\underline{0.85}$

total: $(0.87)(0.83)(0.85) = \underline{\underline{0.61}}$

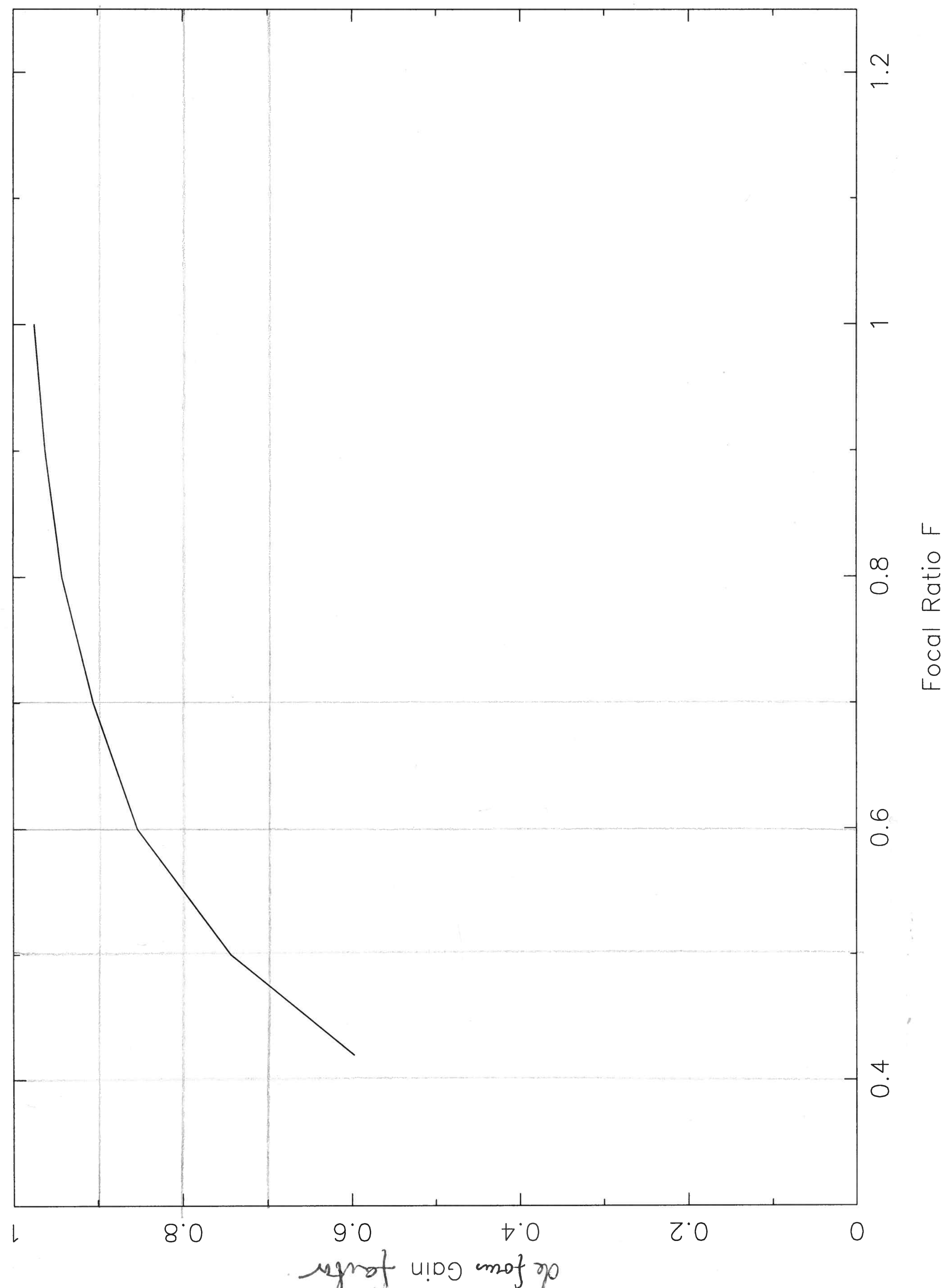
Forward spillover is now $\underline{7.5\%}$

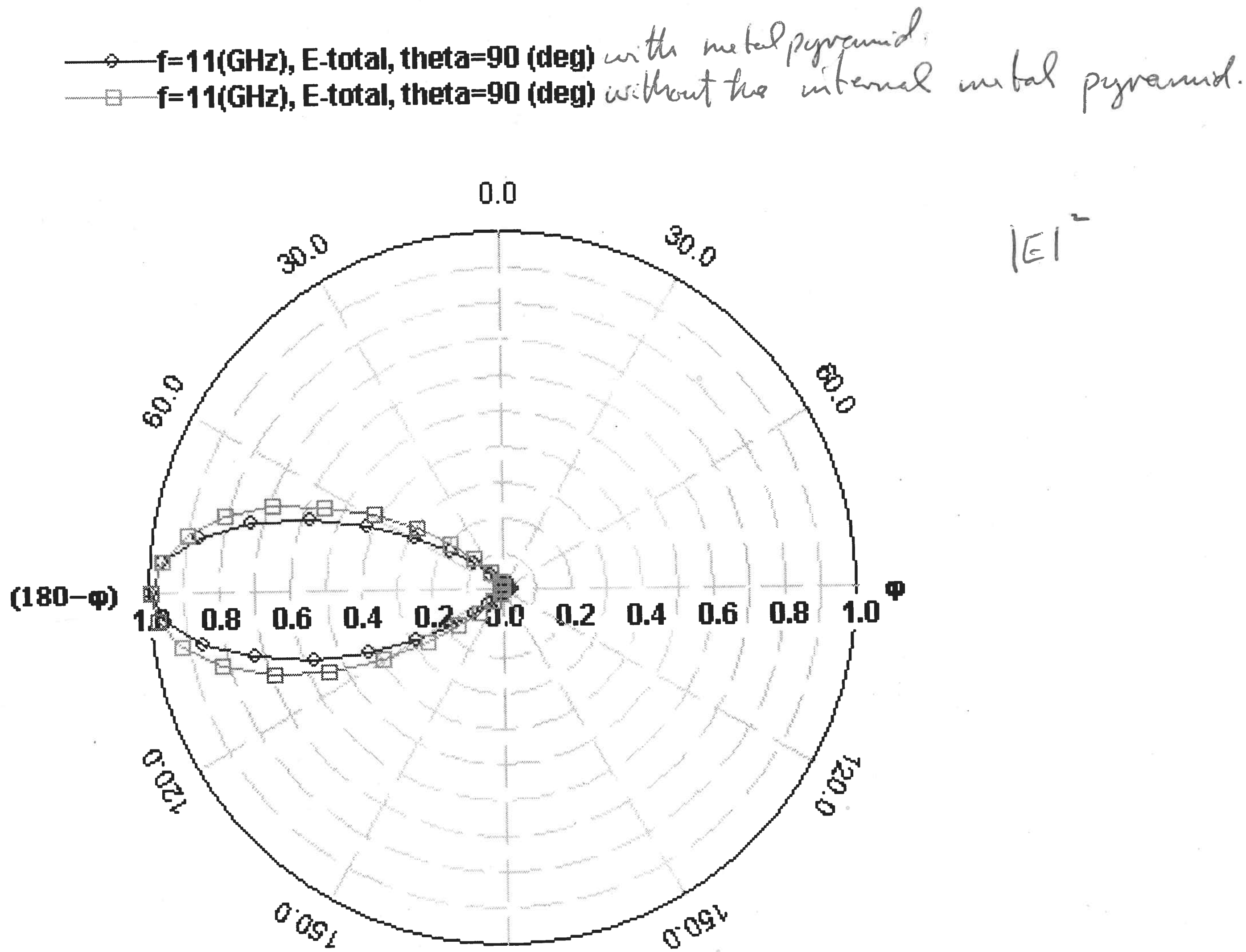
Back spillover is the same $\underline{10\%}$

---

defocus factor for a focus error of 0.8 λ as a function of F.

→

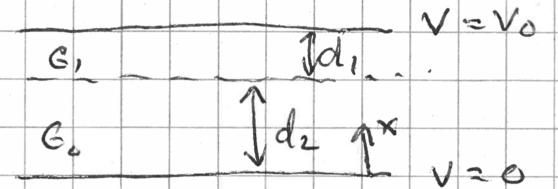

0.4
0.6
0.8
1
1.2
Focal Ratio F
0
0.2
0.4
0.6
0.8
1
de focus Gain factor


f=11(GHz), E-total, theta=90 (deg) with metal pyramid.
f=11(GHz), E-total, theta=90 (deg) without the internal metal pyramid.
$|E|^2$
0.0
30.0
30.0
60.0
60.0
(180−φ)
φ
1.0 0.8 0.6 0.4 0.2 0.0 0.2 0.4 0.6 0.8 1.0
120.0
120.0
150.0
150.0

Put Dale Gary, Stephen White, Gordon Hurford, and Tim Bastian on the 1AT / mailer- (?)

Use of thin pc board substate for the antenna

Model simply



$\hat{n}\cdot(\vec{D}_2 - \vec{D}_1) = 0$ $\quad \epsilon_1 E_1 = \epsilon_2 E_2 = \epsilon_0 E_0$, $\quad E_1 = \left(\frac{\epsilon_0}{\epsilon_1}\right) E_0$

There's less E-field in the dielectric $\quad V_0 = E_1 d_1 + E_0 d_2$

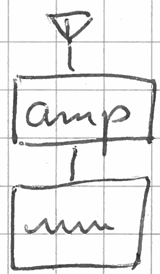
$V_0 = [(\epsilon_0/\epsilon_1) d_1 + d_2] E_0$

Losses are in ϵ_1 only: $\quad \sigma_1{}^2 E_1 d_1$

If $d_1 \ll d_2$, the volume losses in ϵ_1 due to σ_1 should be small. However, this is only the losses in the quasi TEM feeding mode. In the radiating mode the losses may be a problem with the PC Board substrate. Another way to build it would be to cut it out of brass shim stock. This might be mechanically weak, but it would be the lowest loss way to do it. We could even gold plate the brass.

RFI Measurements at Hat Creek 3/1/99 WJW/JD/MW

Located at station 2700N



LNA; Gain ~45db

spectrum analyzer: TeekTronics 494P

Scan with a resolution of 1 MHz

Noise level with ~~amp~~ LNA turned off −85dbm
Noise level with LNA turned on { −71dbm (single scan), −64dbm on max. hold

Noise level referred to LNA input: $-71-45$, −116dbm, 2.48×10^{-15} watts

Antenna effective area $A_e = \frac{g\lambda^2}{4\pi}$

For g = 1 at λ = 0.3m 1GHz) A_e = .007 /m², and the flux corresponding to the noise level is $3. \times 10^{-13}$ watts/m²

For an input noise of 2.48×10^{-15} watts the effective noise temperature of the system is 180k

Measurements

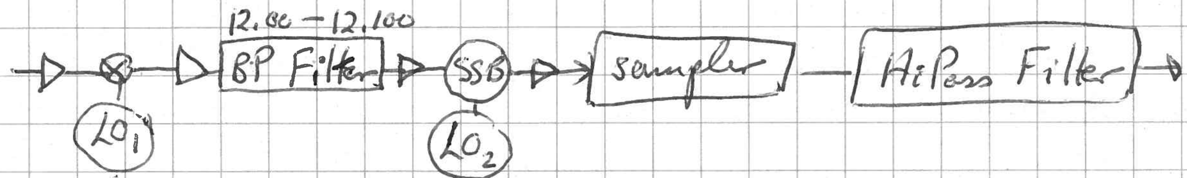
Antenna	Freq (GHz)	P(scan)	P(max hold)	Comments
HF Dis cone	1.1	−50	−50	sporadic (tacan)
f ≥ 1.0 GHz	1.1− 7	− 1	−64	noise level nothing)
	3.0-4.0	−71	−64	"
	4.0-5.0	−7	−64	
WJ out	1.1	−32	0	sporadic (tacan), jet
	1.553	−68		GPS ν 20 MHz
f ≥ c z	624	52	−42	IR DIUM Δν = 5MHz
	1. 7	−7	− 1	noise level (nothing)
	2 − 0	1	"	
	30 .0	"	"	

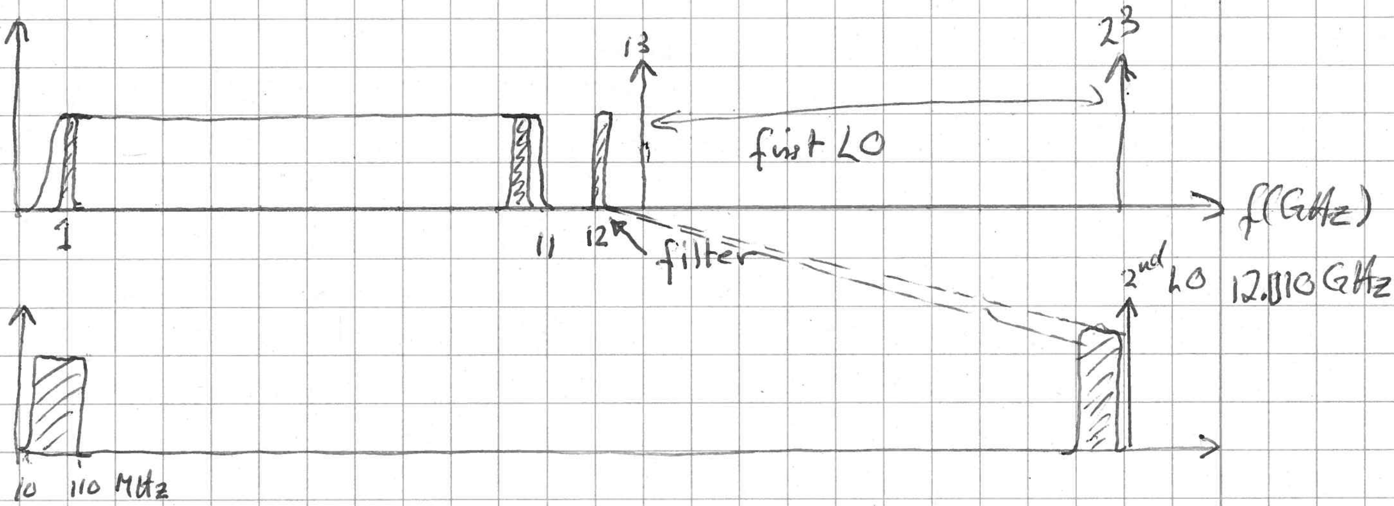
Measurements

Antenna	Freq. (MHz)	P(scan)	P(hold)		comments
LF Disc-Cone	942.5		−36	(−81?)	$\Delta\nu \sim 2$ MHz
	933.8, 932.2		−43		$\Delta\nu \sim$ few hundred kHz
	895.7		−40		variable
	896−87		−55		many signals
UHF TV	870−466	−70	~~(−70)~~		a few dozen (diffraction)
	462.8−462		−44		erratic
	454.5		−64		small complex $\Delta\nu \sim 1$ MHz
	192 − DC		−37		complex

Note: The Forest Service Communication Link is at 140 MHz.
(This is not certain)

Possible Double Conversion Base Band Converter





Filter Band 12.000 – 12.100 GHz 100 MHz

For n=10 Tchebyscheff with 0.1 db passband ripple
attenuation is 16db at 10% of the bandedge 5 MHz
31db at 20% 10 MHz

This may be as big a filter as it is practical to build.

If we put the fixed second LO at 12.010 GHz,
10 MHz from the band edge, the output is the band
10-110 MHz. That has 31 db or more rejection.
The SSB mixer with fixed LO should give an
additional rejection of 30db.

Altogether we get ≥ 60 db out-of-band
rejection at the edge, and better within the band.

The total effective down converter frequency is

$$\nu_0 = \nu_{LO1} - \nu_{LO2}$$

1HT Board meeting 7/28/99

1. Staffing (JD)
Rick Smegal looking at spillover screen
Douglas Bock - RPA [will stay permanently]
Dave deBoer - RPA
Greg. Engargiola
4 pple at Sun are now involved.
~ 8 FTE now

Software(?) Jane Jordan + Kevin Daley (Jay Freeman)
may contribute; Jay Freeman looking at something
→ Do we advertise? define the position Leo, John, Jack
→ Jill talk to Jane and Kevin

How about a greenhouse?

[Rhode + Schwartz
a) feed $8000
b) Total is $171,000 w/ 3m dish]

→ start the patent process
get non-disclosure agreements

RPA
7 @ 3.6m dishes ordered from Orbitron
bearing + yoke improved
feed being built - look good
front-end package
1 Oertel fiber driver ordered 1.3-1.8 GHz
back-end in software under consideration [Sun]

Sun computer donations - also cash contributions
who at EECS? Does Greg P. have any ideas?

→ Jack talk to Greg re the Sun site problem.

Long discussion on the site

JD: pacing item is the fiber optics + maybe the cryo.

→ Site Selection Criteria: RFI, COST, SPEED, ... sky coverage, accessibility, weather (winds?)

Phase, frequency, and Delay control for the 1HT (9/5/99)

One antenna is the reference. The others are located at $\vec{S}_i$ wrt the reference antenna. Consider the formation of a beam for a finite bandwidth, $\Delta\nu_m$.

The voltage at the reference antenna is $v(t)$,

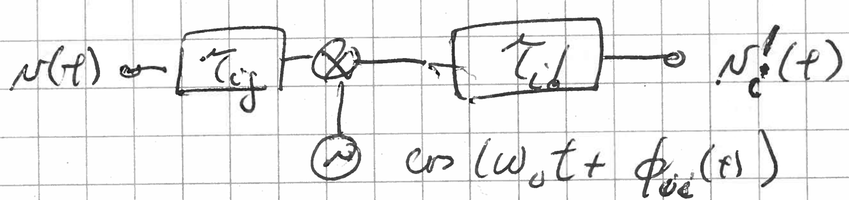
$$v(t) = \int_{-\infty}^{\infty} a(\nu)\, e^{i 2\pi\nu t}\, d\nu$$

Signals arriving at the other antennas are the same except for the delays.

$v_i(t) = v(t - \tau_{ig})$ where $\tau_{ig} = -\vec{S}_i \cdot \hat{n}(t)/c$

$\hat{n}(t)$ is for the earth rotation, the instantaneous direction of arrival.

$$v_i(t) = \int_{-\infty}^{\infty} a(\nu)\, e^{i 2\pi(t - \tau_{ig})\nu}\, d\nu \qquad \text{[bad choice]}$$



ω_0 translates the band to baseband.

$\nu - \nu_0 = \nu_\ell$ or $\boxed{\nu = \nu_0 + \nu_\ell}$ Also $\nu_0 = \nu_{LO1} - \nu_{LO2}$

$$\phi_{0i} = \int_0^t \omega_{0i}(x)\,dx + C_{0i} + \psi_{0i}(\;)$$

C_{0i} is the phase constant needed to zero all phases for the required beam direction on the sky at $t=0$. $\psi_{0i}(t)$ is our phase switching, e.g. a Walsh function, if it is needed for zero drifts.

The total output phase is now $2\pi\nu(t\ \tau_i \quad 2\pi\nu_0 t - \phi_{0i}$

$= 2\pi \quad \nu_0)t - 2\pi \quad {}_{ij} \quad \phi_{0i}$

$- \quad \nu - 2\pi\nu\tau_{ij} - \phi_i$

$\nu_\ell t \quad \nu\tau_i - 2\pi\nu_\ell\tau_{ij} - \phi_{0i}$

To correct the phase for frequencies other than ν_0, those near ν_0 that end up in the band $0<\nu_\ell<\nu_m$, we now introduce a ~~variable~~ delay τ_{id} in the video band. Then the total phase term is:

$$2\pi\nu_\ell t - 2\pi\nu_\ell \tau_{id} - 2\pi\nu_0\tau_{ig} - 2\pi\nu_\ell\tau_{ig} - \int_0^t \omega_{0i}(x)dx - C_{0i} - \varphi_{0i}(t)$$

$$= 2\pi\nu_\ell t - [2\pi\nu_\ell(\tau_{id}+\tau_{ig})] - \left[2\pi\nu_0\tau_{ig} + \int_0^t \omega_{0i}(x)dx + C_{0i}\right] - \varphi_{0i}(t)$$

Let τ_m be the maximum delay of the delay line. Then set

$$\tau_{id} = \frac{\tau_m}{2} + \frac{\vec{S}_i\cdot\hat{n}_0}{c},$$ where $\hat{n}_0$ is the center of the array beam.

W may be interested in some direction in the beam other than $\hat{n}_0$, call it $\hat{n}$. Then

$$\tau_{ig}\ \tau_{id} = -\frac{\vec{S}_i\cdot\hat{n}}{c} + \frac{\tau_m}{2} + \frac{\vec{S}_i\cdot\hat{n}_0}{c} = \tau\_ - \frac{\vec{S}_i}{c}\cdot(\hat{n}-\hat{n}_0)$$

This makes the second term nearly constant. It is constant when $\hat{n}=\hat{n}_0$.

Now zero the fringe rate wrt the reference antenna. This requires

$$2\pi\nu_0\tau_{ig} + \int_0^t \omega_{0i}(x)dx + C_{0i} \quad 0$$

W have to do this for the center of the array beam $\hat{n}_0$.

$$\tau_{g0} = -\frac{\vec{S}_i\ n_0}{c}$$

Take $\frac{d}{dt}\left\{2\pi\nu_0\tau_{ig0} + \int_0 \omega_{0i}(x)dx + C_{0i}\right\} = 0$

This cancels the third group of terms for the direction $\hat{n}_0$, but not C_{0i}.

$$2\pi\nu_0\frac{d}{dt}(\tau_{ig0}) + \omega_{0i}(t) = 0\ ;\quad \omega_{0i}(t) = 2\pi\nu_0\frac{d}{dt}\left[\frac{\vec{S}_i\cdot\hat{n}_0}{c}\right]$$

For other directions $\hat{n}$, we get (for this term)

$$-2\pi\nu_0\left[-\frac{\vec{S}_i\cdot\hat{n}}{c}\right] - \int_0^t\left[2\pi\nu_0\frac{d}{dt'}\left(\frac{\vec{S}_i\cdot\hat{n}_0}{c}\right)\right]dt' - C_{0i} - \varphi_{0i}(t)$$

$$= 2\pi\nu_0 \left\{ \frac{\vec{S}_i \cdot \hat{n}}{c} - \frac{\vec{S}_i \cdot \hat{n}_0}{c} \right\} - C_{0i} - \varphi_{0i}(t) = \frac{2\pi\nu_0}{c} \vec{S}_i \cdot (\hat{n} - \hat{n}_0) - C_{0i} - \varphi_{0i}$$

The total phase term is

$$2\pi\nu_l t - 2\pi\nu_l \frac{\tau_m}{2} + \frac{2\pi\nu_l}{c} \vec{S}_i \cdot (\hat{n} - \hat{n}_0) + \frac{2\pi\nu_0}{c} \vec{S}_i \cdot (\hat{n} - \hat{n}_0) - C_{0i} - \varphi_{0i}(t)$$

$$= 2\pi\nu_l t - 2\pi\nu_l \frac{\tau_m}{2} + \underbrace{\frac{2\pi\nu}{c} \vec{S}_i \cdot (\hat{n} - \hat{n}_0)} - C_{0i} - \varphi_{0i}(t)$$; since $\nu = \nu_0 + \nu_l$

This term is just the pattern, here relative to the beam center, where it is zero.

To get the beam, we add up the voltages, v_i with the weights W_i.

$$O(t) = \sum_i W_i \int_{-\infty}^{\infty} a(\nu) e^{i\left[2\pi\nu_l t - \ \ \frac{\tau_m}{2} + \frac{2\pi\nu}{c} \vec{S}_i \cdot (\hat{n} \ \hat{n}_0) - C_{0i} - \varphi_{0i}(t)\right]} d\nu$$

drop the constant error in delay

no, see next page

$$O(t) - \int_{-\infty}^{\infty} a(\nu) \left\{ \sum_{i=0}^{N} W\, e^{i\left[\frac{2\pi\nu}{c} S_i \cdot (\hat{n} \ \hat{n}) \ \ C_{0i}\right]} \right\} e^{i\left[2\pi\nu_l t \ \cancel{\ } - \varphi_{0i}(t)\right]} d\nu_l$$

$$\left\{ \sum_{i=0}^{N-1} W_i e^{i\left[\frac{2\pi\nu}{c} \vec{S}_i \cdot (\hat{n} - \hat{n}_0) - C_{0i}\right]} \right\} = E(\nu, \theta, \phi)$$, the e field

pattern of the array. θ, ϕ are polar angles measured from $\hat{n}_0$.

Thus $O(t) \int_{-\infty}^{\infty} a(\nu_0 + \nu_l)\, e^{i\left[2\pi\nu_l t - \varphi_{0i}(t)\right]} \cancel{\ } E(\nu_0 + \nu_l, \theta, \phi] \, d\nu_l$

$(L + \varphi_{0i} -)$

At beam center $E \sim$ ~~___~~, and $\underline{O(t) \sim N\hat{v}(t)}$, t quite noisy

In other directions $\hat{n}$, there is a weak frequency dependence to the array factor which will slightly modify $O(t)$. There also a small frequency dependence of the dishes. $g = 4\pi A_e \left(\frac{\nu^2}{c^2}\right)$ $A_e \cong$ constant

And for the e-field, the dependence is $\sqrt{g} \propto \nu$.

This modifies the voltage sum, even for $\hat{n} = \hat{n}_0$, but the effect should be small for $\nu_m/\nu_0 \ll 1$.

Summarize the control

a) $\tau_{id} = \frac{\tau_m}{2} + \frac{\vec{S}_i \cdot \hat{n}_0}{c}$ video band delay

b) $\omega_{oi}(t) = 2\pi\nu_0 \frac{d}{dt}\left[\frac{\vec{S}_i \cdot \hat{n}_0}{c}\right]$ fringe rotation

c) C_{oi} are set to whatever values are needed to make the signal phases $= 0$ at $t = 0$. This can be done with an observation of a phase calibrator

This is just what is done for the BIMA cross-correlator system and will work the same for the IHT cross correlator.

Using the complex Fourier transform in this way is not quite right without also writing $\cos\omega_0 t = (e^{i\omega_0 t} + e^{-i\omega_0 t})\frac{1}{2}$, because the voltages must be real.

Start with $v(t) = \int_0^\infty a(\nu)\cos(2\pi\nu t + \phi(\nu))\,d\nu$

to represent the incoming voltage.

Then multiplying by $\cos(2\pi\nu_0 t + \phi_{oi})$ results in just $\cos[2\pi\nu t + \phi(\nu) - 2\pi\nu_0 t - \phi_{oi}]$, the phase difference term, because the phase sum is at a higher frequency, $\nu + \nu_0$.

$v_i(t)\cos(2\pi\nu_0 t + \phi_{oi})$

$$= \int_0^\infty a(\nu)\cos\left\{\cancel{2\pi}\, 2\pi\nu_e\left(t - \frac{\tau_m}{2}\right) + \frac{2\pi\nu}{c}\vec{S}_i\cdot(\hat{n} - \hat{n}_0) - C_{oi} - \phi_{oi}(t)\right\}d\nu$$

Now, add up the voltages with the weights W_i, and drop the $\cos$ and $\phi_{oi}(t)$

$$O(t) = \sum_i W_i \int_0^\infty a(\nu) \cos\{\ \} d\nu$$

$$O(t) = \sum_i W_i \int_0^\infty a(\nu) \cos\left\{2\pi\nu_\ell\left(t - \frac{\tau_m}{2}\right) + \frac{2\pi\nu}{c} \vec{S}_i \cdot (\hat{n} - \hat{n}_0) + \phi(\nu)\right\} d\nu$$

At the peak of the beam, $\hat{n} = \hat{n}_0$, and

$$O_0(t) = \sum_i W_i \int_0^\infty a(\nu) \cos\left\{2\pi\nu_\ell\left(t - \frac{\tau_m}{2}\right) + \phi(\nu)\right\} d\nu$$

$\int_0^\infty a(\nu) \cos\left\{2\pi\nu_\ell\left(t - \frac{\tau_m}{2}\right) + \phi(\nu)\right\} d\nu = \hat{v}(t - \tau_m/2)$ is a distortion of the original $v(t)$ wherein it is $v(t)$ heterodyned to baseband, just as in a normal superhet receiver.

Since $\sum_i W_i \sim N$, $O_0(t) \approx N \hat{v}(t - \tau_m/2)$

Note that $O(t)$ is a sum of the voltages with real weights W_i, and this is a simple computation.

For other directions $\hat{n}$:

$$\cos\left\{2\pi\nu_\ell(t - \tau_m/2) + \frac{2\pi\nu}{c}\vec{S}_i \cdot (\hat{n} - \hat{n}_0) + \phi(\nu)\right\}$$

$$= \cos\left\{2\pi\nu_\ell(t - \tau_m/2) + \phi(\nu)\right\} \cos \frac{2\pi\nu}{c}\vec{S}_i \cdot (\hat{n} - \hat{n}_0)$$
$$- \sin\{\ \} \sin \frac{2\pi\nu}{c}\vec{S}_i \cdot (\hat{n} - \hat{n}_0)$$

Now, $\sum_i W_i e^{i \frac{2\pi\nu}{c}\vec{S}_i \cdot (\hat{n} - \hat{n}_0)} = E(\nu, \theta, \phi)$ the complex array pattern, relative to $\hat{n}_0$.

$$\text{so} \sum W_i \cos\{\ \} = \cos\left\{2\pi\nu_\ell\left(t - \frac{\tau_m}{2}\right) + \phi(\nu)\right\} R_e[E]$$
$$+ \sin\left\{2\pi\nu_\ell\left(t - \frac{\tau_m}{2}\right) + \phi(\nu)\right\} I\ [E]$$

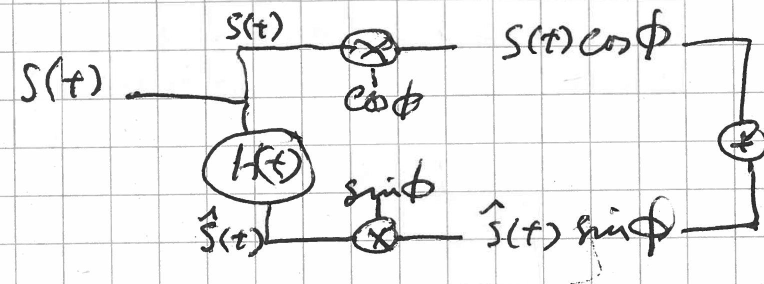
In this general case

$$a(t) = \int_0^\infty a(\nu)\, R_e\left[E(\nu,\theta,\phi)\right] \cos\left[2\pi\nu_l\left(t-\frac{\tau_m}{2}\right)+\phi(\nu)\right] d\nu$$
$$- \int_0^\infty a(\nu)\, I_m\left[E(\nu,\theta,\phi)\right] \sin\left[2\pi\nu_l\left(t-\frac{\tau_m}{2}\right)+\phi(\nu)\right] d\nu$$

Inserting the offset phase and delay

1. It can be entered in the analog part at the high frequency oscillator, as is done for BIMA.
2. Delay will be entered in the baseband, after the digitization.
3. Entering the phase digitally in the base band is probably better.

To do this requires an SSB mixer.



$\hat{s}(t)$ is $s(t)$ with ϕ ~~added~~ subtracted.

Write $s(t) = \int_0^\infty a(\nu)\cos(2\pi\nu t + \phi(\nu))\,d\nu$

$\hat{s}(t) = \int_0^\infty a(\nu)\sin(2\pi\nu t + \phi(\nu))\,d\nu$, the Hilbert Transform of $s(t)$.

$s(t)\cos\phi + \hat{s}(t)\sin\phi =$

$$\int_0^\infty a(\nu)\left\{\cos(2\pi\nu t + \phi(\nu))\cos\phi + \sin(2\pi\nu t + \phi(\nu))\sin\phi\right\} d\nu$$

$$= \int_0^\infty a(\nu)\cos(2\pi\nu t + \phi(\nu) - \phi)\,d\nu$$

This can be done digitally with good accuracy.
The Hilbert Transform has to be applied to the signal.
One way would be to used the Fourier Transform

i.e. $H[s(t)] = F^{-1}\{F\{s(t)\}\, H(\nu)\} = \hat{s}(t)$

where $H(\nu) = -i\,\mathrm{sgn}(\nu)$

To eliminate an interfering satellite, one way would be to form a beam on the satellite, wherever it is in the primary beam, and then subtract it from the main beam signal.
Exactly how much to subtract is not simple to determine. [Yes, it]

Better to form a null somewhere. [maybe]
If this is done by perturbing the delays, maybe it will be broadband.

The simple tree algorithm with first combining them all in pairs won't work unless the spacings are small, of order $\lambda/2$. With dishes, the spacings are many wavelengths, so the two element patterns have many nulls. These would wreck the main beam pattern.
e.g. 10m spacing of 5m dishes @ $\lambda = 20$ cm.
$2\pi \frac{D}{\lambda} \sin\Theta = \Phi$ For $\Theta \to \Theta + \delta\Theta$ and $\Phi \to \Phi + 2\pi$, $D/\lambda \cos\Theta\, \delta\Theta = 1$

so $\delta\Theta = \frac{1}{D/\lambda \cos\Theta} \sim \frac{2}{D/\lambda}$, $\delta\Theta \sim \frac{2}{50} \sim 2°$.

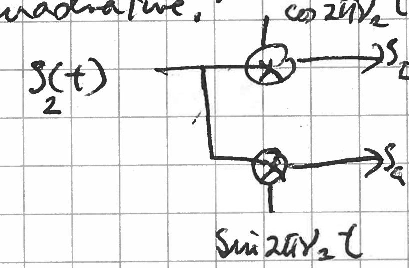
Realizing the Hilbert Transform directly may not be worth the effort. So (1) make quadrature mixers with an L.O. just above the baseband edge (or maybe in the middle), (2) digitize both the real and quadrature signals (3) then do the $\sin\phi$ and $\cos\phi$ multiplications digitally and the sum and/or difference digitally. (4) This should then followed by digital delays.

~~In this last step~~

More details on the baseband conversion

The first step, upconversion, is done as shown on p. 46. For the down conversion of the fixed band at ~12 GHz we use an LO (ν_2) just at the bottom edge of the fixed filter.

For this step we do only the two mixings to baseband in quadrature.



$$S_I(t) = \int_0^\infty a_2(\nu) \cos(2\pi\nu t + \phi_2(\nu))\, d\nu$$

$\nu > \nu_2$

$S_2(t)$ is the signal output from the fixed band filter.

$$S_I(t) = \int_0^\infty a_2(\nu) \cos[2\pi\nu t + \phi_2(\nu)]\, d\nu \cos 2\pi\nu_2 t$$

$$= \int_0^\infty a_2(\nu) \left\{ \cos[2\pi(\nu-\nu_2)t + \phi_2(\nu)] + \cos[2\pi(\nu+\nu_2)t + \phi_2(\nu)] \right\} d\nu$$

The double frequency term $\nu + \nu_2 \simeq 2\nu_2$ is filtered out

Hence $$S_I(t) = \int_0^\infty a_2(\nu) \cos[2\pi(\nu-\nu_2)t + \phi_2(\nu)]\, d\nu$$

From the other mixer

$$S_Q(t) = \int_0^\infty a_2(\nu) \cos[2\pi\nu t + \phi_2(\nu)]\, d\nu \; \sin 2\pi\nu_2 t$$

$$= \int_0^\infty a_2(\nu) \left\{ \sin[2\pi(\nu+\nu_2)t + \phi_2(\nu)] - \sin[2\pi(\nu-\nu_2)t + \phi_2(\nu)] \right\} d\nu$$

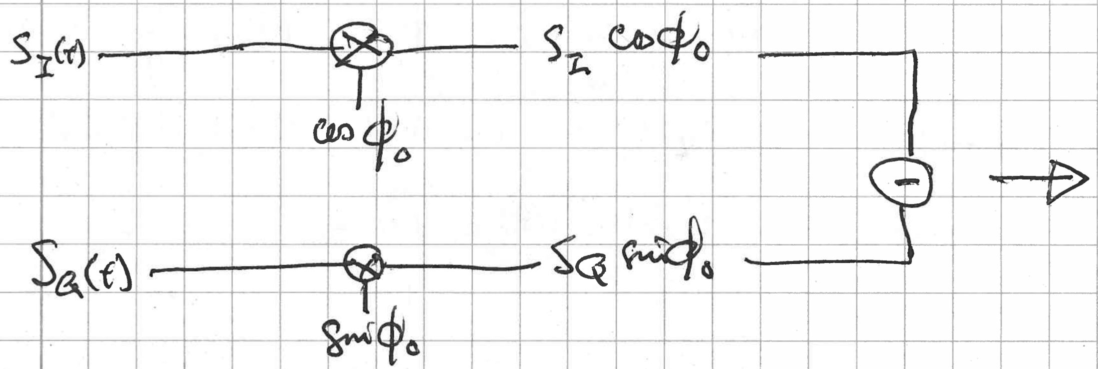
Again the double frequency term is filtered out

$$S_Q(t) = \hat{S}_I(t) = -\int_0^\infty a_2(\nu) \sin[2\pi(\nu-\nu_2)t + \phi_2(\nu)]\, d\nu$$

At this point both S_Q and S_I are digitized, and further operations are digital. 12 bit digitization here will produce 1:4000 accuracy.

The baseband signal may now have phase and delay added to it numerically.

For phase addition, we use a SSB mixer arrangement.



At the combining point: $S_I \cos\phi_0 - S_Q \sin\phi_0$

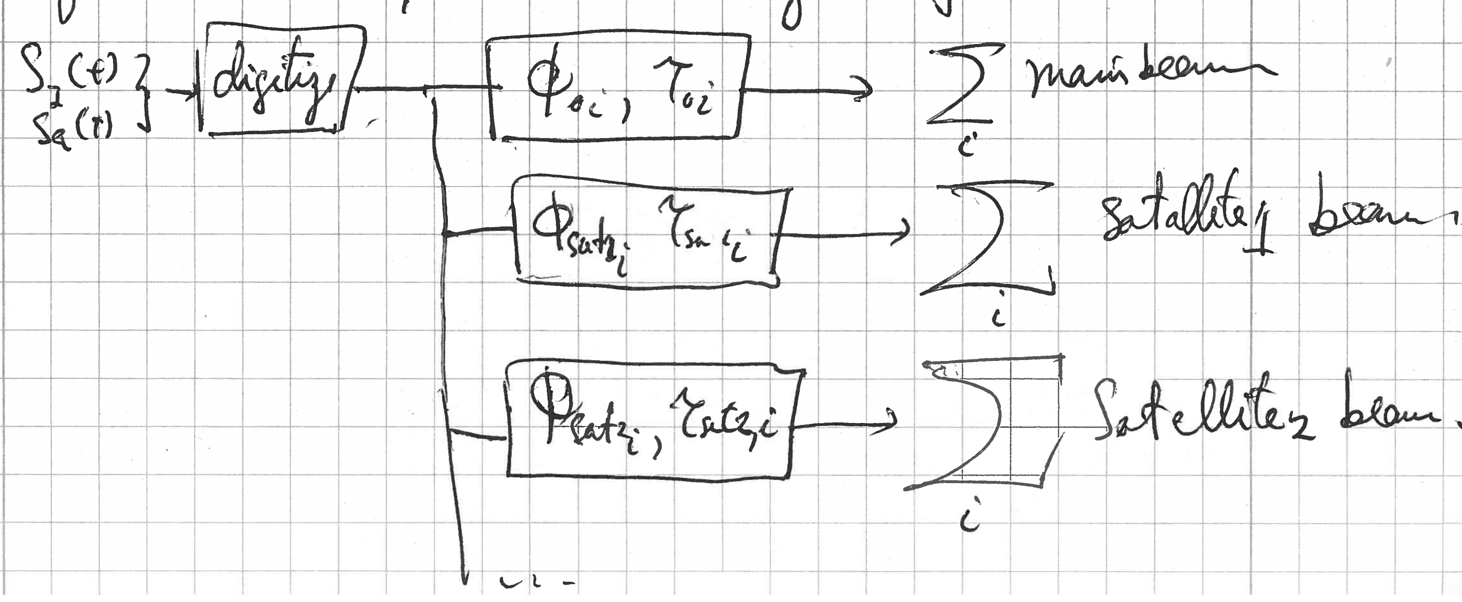
$$= \int_0^\infty a(\nu) \left\{ \cos\phi_0 \cos[2\pi(\nu-\nu_2)t + \phi_2(\nu)] + \sin\phi_0 \sin[2\pi(\nu-\nu_2)t + \phi_2(\nu)] \right\} d\nu$$

$$= \int_0^\infty a(\nu) \cos[2\pi(\nu-\nu_2)t + \phi_2(\nu) - \phi_0]\, d\nu$$

Phase $\phi_0(t)$ is thus subtracted from each frequency component of the signal.

Next, delay, τ_0, may be added to the signal.

For both a main beam formation and satellite beams for subtraction, the block diagram for one antenna is:



Each satellite beam thus formed is simply subtracted (in the regular time domain) from the main beam to eliminate it's interference in the main beam.

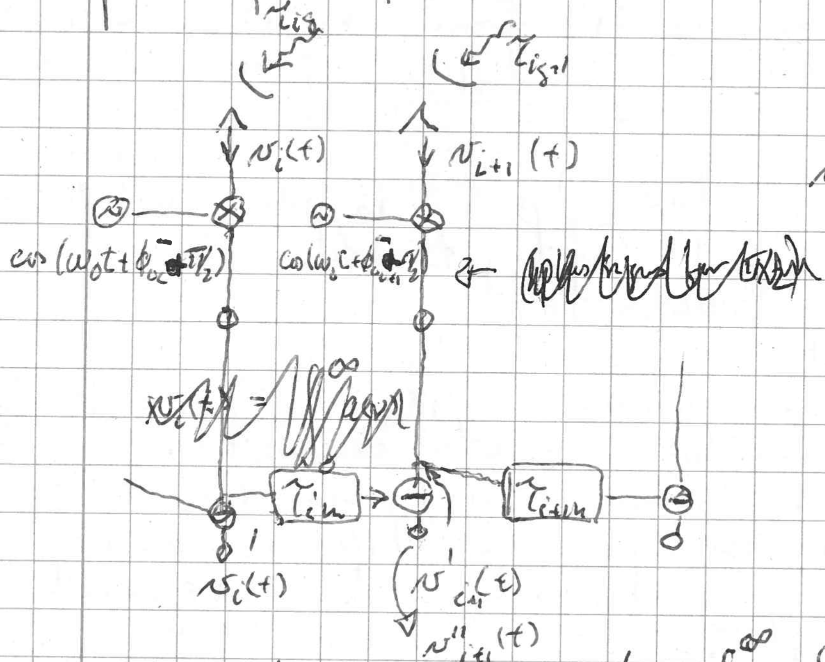
What's required is to obtain $\hat{a}_{sat}(\nu)$ from $V_{sat}(t)$, the result of the satellite beam summation. This requires a Fourier Transform.

Then the calculation at the top of p. 53 is performed to get the value of the satellite signal that gets into the main beam. This is simply subtracted.

Note that the Fourier transform (spectrum) of the main beam signal is often calculated for further data analysis. In this case the subtraction can be made in the Fourier Domain; the integral of p. 53 is unnecessary. The interference can be subtracted by ~~the~~ performing the scaling in the integrand of the expression atop p. 53.

Forming a broadband null

Use the tree scheme in which every adjacent antenna pair first forms a null in the chosen direction $\hat{n}_n$.



$$v_i(t) = \int_0^\infty a(\nu)\cos[2\pi\nu(t-\tau_{ig}) + \phi(\nu)]\,d\nu$$

After the mixer
$$v_i'(t) = \int_0^\infty a(\nu)\cos\left[2\pi(\nu-\nu_0)t - 2\pi\nu\tau_{ig} - \phi_{0i} - \frac{\pi}{2} + \phi(\nu)\right]d\nu$$

$$= \int_0^\infty a(\nu)\sin\left[2\pi(\nu-\nu_0)t - 2\pi\nu\tau_{ig} - \phi_{0i} + \phi(\nu)\right]d\nu$$

Now add τ_{in} (to produce the 2-element null) and subtract:

$$v'_{i+1}(t) - v_i'(t-\tau_{in}) = v''_{i+1}(t)$$

$$= \int_0^\infty a(\nu)\left\{\sin\left[2\pi(\nu-\nu_0)t - 2\pi\nu\tau^{i+1}_{ig} - \phi_{0i+1} + \phi(\nu)\right] - \sin\left[2\pi(\nu-\nu_0)(t-\tau_{in}) - 2\pi\nu\tau_i - \phi_{0i} + \phi(\nu)\right]\right\}d\nu$$

Use the identity $\sin A - \sin B = 2\sin\frac{A-B}{2}\cos\frac{(A+B)}{2}$

But recall that we can write:

$$2\pi(\nu-\nu_0)t - 2\pi\nu\tau_{ig} - \phi_{0i} + \phi(\nu)$$
$$= 2\pi(\nu-\nu_0)t - 2\pi(\nu-\nu_0)\tau_{ig} - \underline{2\pi\nu_0\tau_{ig} - \phi_{0i}} + \phi(\nu)$$

We set the terms in the bracket to cancel (rotate the fringe to zero), leaving

$$2\pi(\nu-\nu_0)(t-\tau_{ig}) + \phi(\nu)$$

ϕ_{oi} and ϕ_{oi+1} are: $2\pi\nu_0\tau_{ig} + \phi_{oi} = 0$ and $2\pi\nu_0\tau_{i+1g} + \phi_{o(i+1)} = 0$

Now $v''_{1+i}(t)$

$$= \int_0^\infty (-1)a(\nu)\left\{\sin\left[2\pi(\nu-\nu_0)(t-\tau_{ig}^{1+i}) + \phi(\nu)\right]\right.$$
$$\left. - \sin\left[2\pi(\nu-\nu_0)(t-\tau_{i+1g} + \tau_{in}) + \phi(\nu)\right]\right\}d\nu$$

$$v''_{1+i}(t) = \int_0^\infty a(\nu)\,2\sin\left[2\pi(\nu-\nu_0)\left(\frac{\tau_{1+ig}-\tau_{ig}\mp\tau_{in}}{2}\right)\right]\cos\left[2\pi(\nu-\nu_0)\left(t-\frac{\tau_{1+ig}+\tau_i+\tau_{in}}{2}\right)+\phi(\nu)\right]d\nu$$

Now $\tau_i = -\vec{S}_i\cdot\hat{n}/c$ Also write (choose) $\tau_{in} = -\hat{n}_n\cdot(\vec{S}_{i+1}-\vec{S}_i)/c$

Then $\tau_{1+ig} - \tau_{ig} - \tau_{in} = -\dfrac{(\vec{S}_{i+1}-\vec{S}_i)\cdot(\hat{n}_o-\hat{n}_n)}{c}$

So

$$v''_{1+i}(t) = \int_0^\infty a(\nu)\sin 2\pi(\nu-\nu_0)\left[\frac{(\vec{S}_{i+1}-\vec{S}_i)\cdot(\hat{n}-\hat{n}_n)}{2c}(-1)\right](-1)$$
$$\cos\left[2\pi(\nu-\nu_0)\left(t-\frac{\tau_i+\tau_{i+1}+\tau_{in}}{2}\right)+\phi(\nu)\right]d\nu$$

$$v''_{1+i}(t) = 0 \quad \text{for } \hat{n} = \hat{n}_n$$

In the direction of the beam to be formed, $a(\nu)$ is multiplied by the factor

$$\sin 2\pi(\nu-\nu_0)\left[\frac{(\vec{S}_{i+1}-\vec{S}_i)\cdot(\hat{n}_o-\hat{n}_n)}{2c}\right]$$

which reduces the output of each antenna by different amounts at different frequencies and for the different directions of the baseline pairs.

The wide dish spacings (in wavelengths) means multiple nulls. This scheme will work best for nulls within the main beam envelope of the single dishes.

Leo: Sunset Review   doc. in by Nov. 15 -

Site review Jan. or Feb.

Reviewers external to Berkeley - compared with other phys. sciences

little danger of RAL cutoff - but what if - look?

Leo will request ~+50k (inc. visitor's line)

hope to take visitors to Hat Creek

J+J do 5 pages for 1HT for reviews

outreach with SETI

letters of support: Mirabel, G. Papadopolis, G. Moore (?), Harvey Butcher, Ron Ekers

MRI - Yes, go through SETI Inst.

Tom   Proj. Management and Communication

Need a Project Manager -

Mike Davis is a prospect

Frank proposes Robert Hall, GBT most recently

He would be available spring of 2000.

John: Need a manager in the build phase; Mike is more a Project Scientist type -

Jack: Broaden the Search

1. Site Selection

What about UHF. Jill showed that 10 GHz and 300-700 MHz are about equal. John: even L-band picks Hat Creek.

Hat Creek is superior wrt RFI

Primary Pattern of one of the dishes : 5m

For the −9.6dB taper case.

R (m)	E(R)	E(R) − 0.291	$[1-(R/2.5)^2]$	$[\ ]^{1.7}$	$[\ ]^{1.5}$
0	1.00	.709 (1.00)	1.00	1.00	1.00
.25	.986	.695 (.980)	.99	.983	.985
.50	.951	.660 (.931)	.96	.93	.941
.75	.898	.607 (.856)	.91	.852	.868
1.00	.813	.522 (.736)	.840	.743	.770
1.25	.726	.435 (.614)	.750	.613	.650
1.50	.633	.342 (.482)	.640	.468	.512
1.75	.540	.249 (.351)	.510	.318	.364
2.00	.452	.161 (.227)	.360	.176	.216
2.25	.369	.078 (.110)	.19	.059	.083
2.50	.291	0 0	0	0	0

$$E(R) \cong 0.291 + .709\left[1-(R/2.5)^2\right]^{1.5}$$

The Sonine Integral (Erdelyi et al)

$$\int_0^{\pi/2} J_\nu(\alpha \sin\theta)(\sin\theta)^{\nu+1}(\cos\theta)^{2p+1}\,d\theta = 2^p\,\Gamma(p+1)\,\alpha^{-p-1} J_{p+\nu+1}(\alpha)$$

Take $\nu = 0$, $\sin\theta = R$; $\cos\theta\, d\theta\ dR$; $\sin^2\theta = R^2 = 1-\cos^2\theta$
$\cos\theta\, d\theta = dR$; $\sin 1 = 1$; $\cos^2\theta = 1-R^2$

$$\int_0^{\pi/} J_0(\alpha \sin\theta)(\sin\theta)(\cos\theta)^{2p+1}\,d\theta = 2^p\,\Gamma(p+1)\,\alpha^{-p-1} J_{p+1}(\alpha)$$

$$\int_0^1 J_0(\ r\ )\ r\ (1-r)^p \cos\theta\, d\theta\ (dr) = 2^p\,\Gamma(p+1)\,\alpha^{-p-1} J_{p+1}(\alpha)$$

$r = R/a$; $a\,dr = dR$, $ar = R$; when $R = a$, $r = 1$.

$$\int_0^1 J_0(\alpha r)\ (1\ r)^p\, r\,dr = 2^p\,\Gamma(p+1)\,\alpha^{-p-1} J_{p+1}(\alpha)$$

$$\int_0^a J_0(\alpha R/a)\left(1-(R/a)^2\right)^p \frac{R}{a}\frac{dR}{a} = 2^p\,\Gamma(p+1)\,\alpha^{-p-1} J_{p+1}(\alpha)$$

Let $\alpha = ka \sin\theta$

$$\int_0^a \left[1-\left(\tfrac{R}{a}\right)^2\right]^p J_0(kR\sin\Theta)\,\frac{R\,dR}{a^2} = 2^p\,\Gamma(p+1)\,\frac{J_{p+1}(ka\sin\Theta)}{(ka\sin\Theta)^{p+1}}$$

$$\boxed{\int_0^a \left[1-\left(\tfrac{R}{a}\right)^2\right]^p J_0(kR\sin\Theta)\,R\,dR = a^2\,2^p\,\Gamma(p+1)\,\frac{J_{p+1}(ka\sin\Theta)}{(ka\sin\Theta)^{p+1}}}$$

$$E(\Theta) \propto \int_0^a E(R)\,J_0(kR\sin\Theta)\,R\,dR$$

$$= \int_0^{a=2.5\,m} \left\{.291 + .709\left[1-\left(\tfrac{R}{2.5}\right)^2\right]^{3/2}\right\} J_0(kR\sin\Theta)\,R\,dR$$

$$= (2.5)^2\left\{\Gamma(1)(.291)\,\frac{J_1(ka\sin\Theta)}{(ka\sin\Theta)} + (0.709)\,2^{3/2}\,\Gamma\!\left(\tfrac{3}{2}+1\right)\frac{J_{5/2}(ka\sin\Theta)}{(ka\sin\Theta)^{5/2}}\right\}$$

For $\lambda = 0.20$ m, $ka = \left(\frac{2\pi}{.2}\right)2.5 = 75.54$
$\Gamma(1) = 1$; $\Gamma(5/2) = 3\sqrt{\pi}/4 = 1.329$; $(75.54)^{5/2} = 49,595$

$$\frac{E(\Theta)}{(2.5)^2} \propto \left\{0.291\,\frac{J_1(75.54\sin\Theta)}{75.54\sin\Theta} + 2.665\,\frac{J_{5/2}(75.54\sin\Theta)}{(75.54\sin\Theta)^{5/2}}\right.$$

$$= (.0039)\,\frac{J_1(75.54\sin\Theta)}{\sin\Theta} + (.000053)\,\frac{J_{5/2}(75.54\sin\Theta)}{(\sin\Theta)^{5/2}}$$

$$= f_1[x\_] \qquad\qquad + f_2[x\_]$$

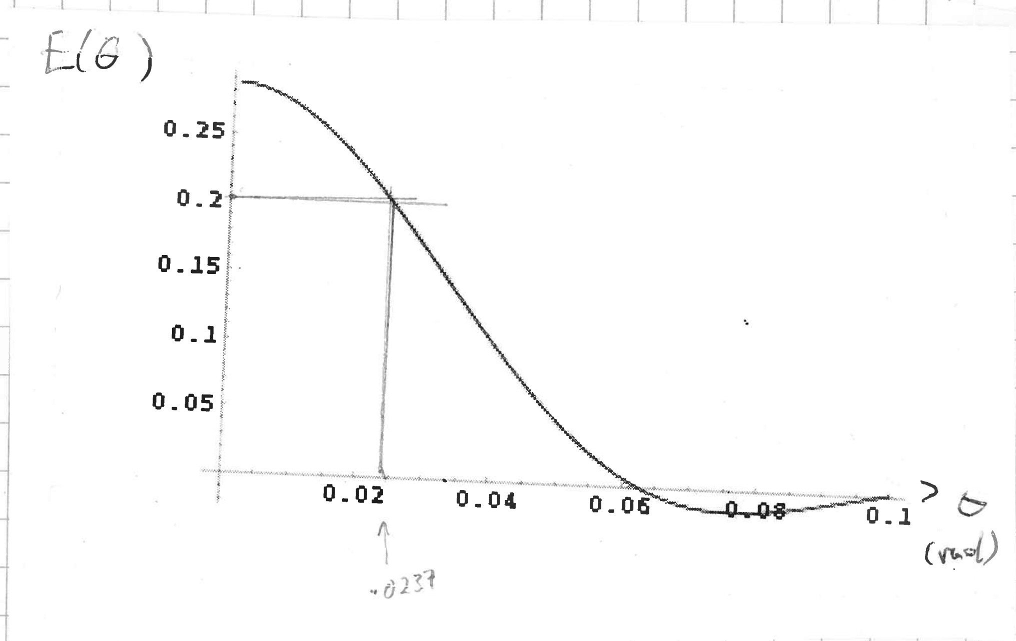
$\Theta_{1/2} \cong 1.8\,\lambda/D = .052$ rad $\approx 3°$ (?)

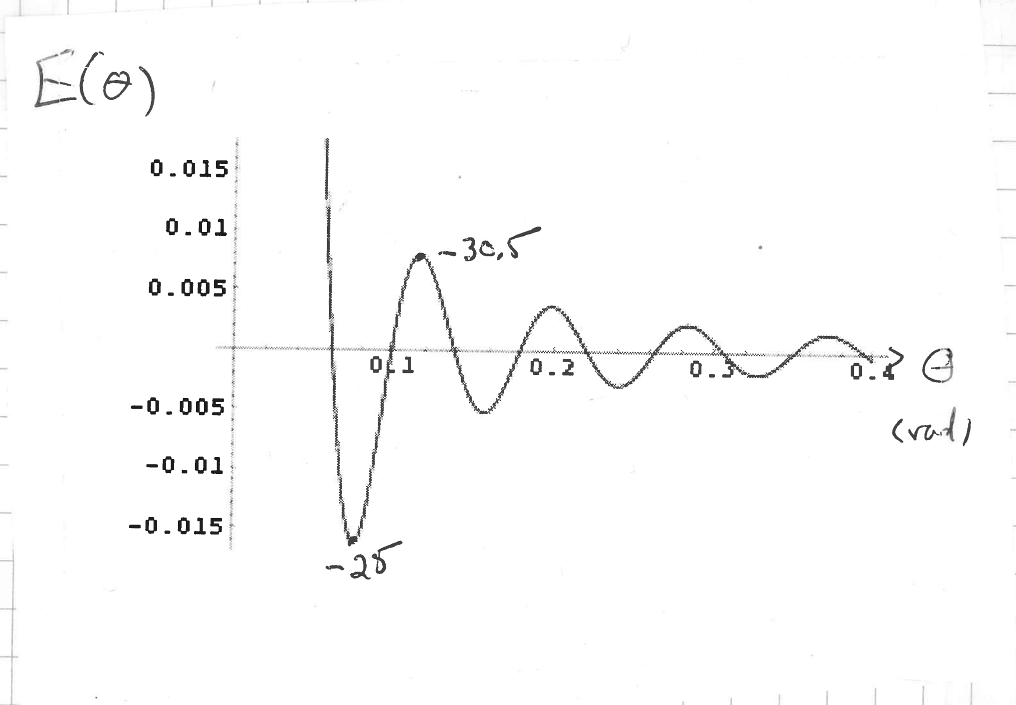
Plot Range $\quad .001 < \Theta < 0.4 \qquad \Theta \Rightarrow x$

At $x = .001$, $E = .2869 = .287$
First Null at $x \geq .061$
1st sidelobe at $x \cong .075$, $E = -0.016$, $\div .287 = .056$, -25 db.

$.287/\sqrt{2} = .2029 \qquad .0237$ rad $= \Theta_{1/2}/2 \qquad \Theta_{1/2} = .0475 = x\,\frac{\lambda}{D} = x\,\frac{(.200)}{5}$

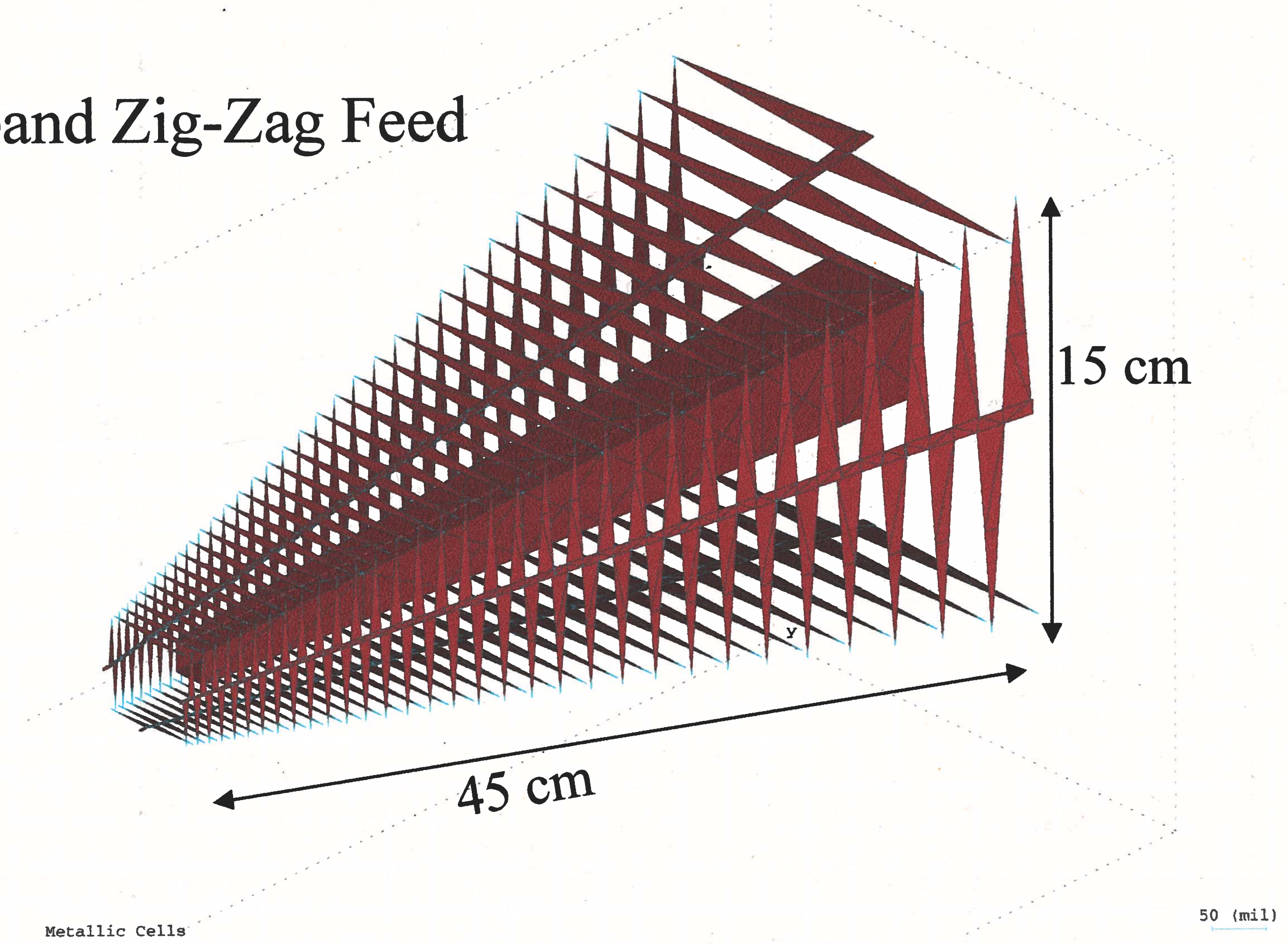
$x = (.0475)\frac{5}{.2} = 1.186 \cong 1.2 \qquad$ So $\Theta_{1/2} = 1.2\,\lambda/D$ for ~ -10 db illumination


E(θ)
0.25
0.2
0.15
0.1
0.05
0.02
0.04
0.06
0.08
0.1
θ
(rad)
.0237


E(θ)
0.015
0.01
0.005
-0.005
-0.01
-0.015
0.1
0.2
0.3
0.4
θ
(rad)
-30,5
-25

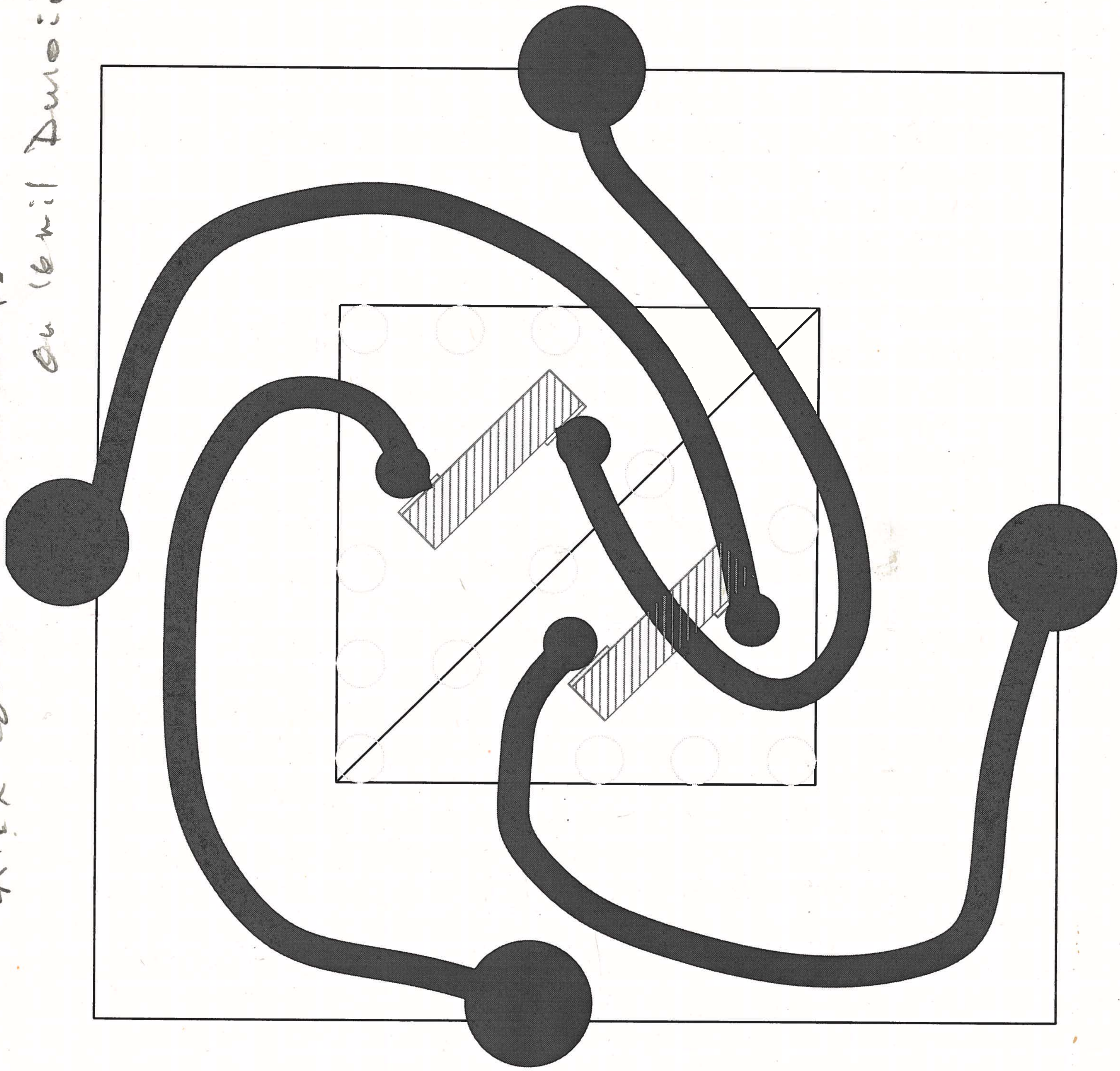
Progress to date

Wideband Zig-Zag Feed

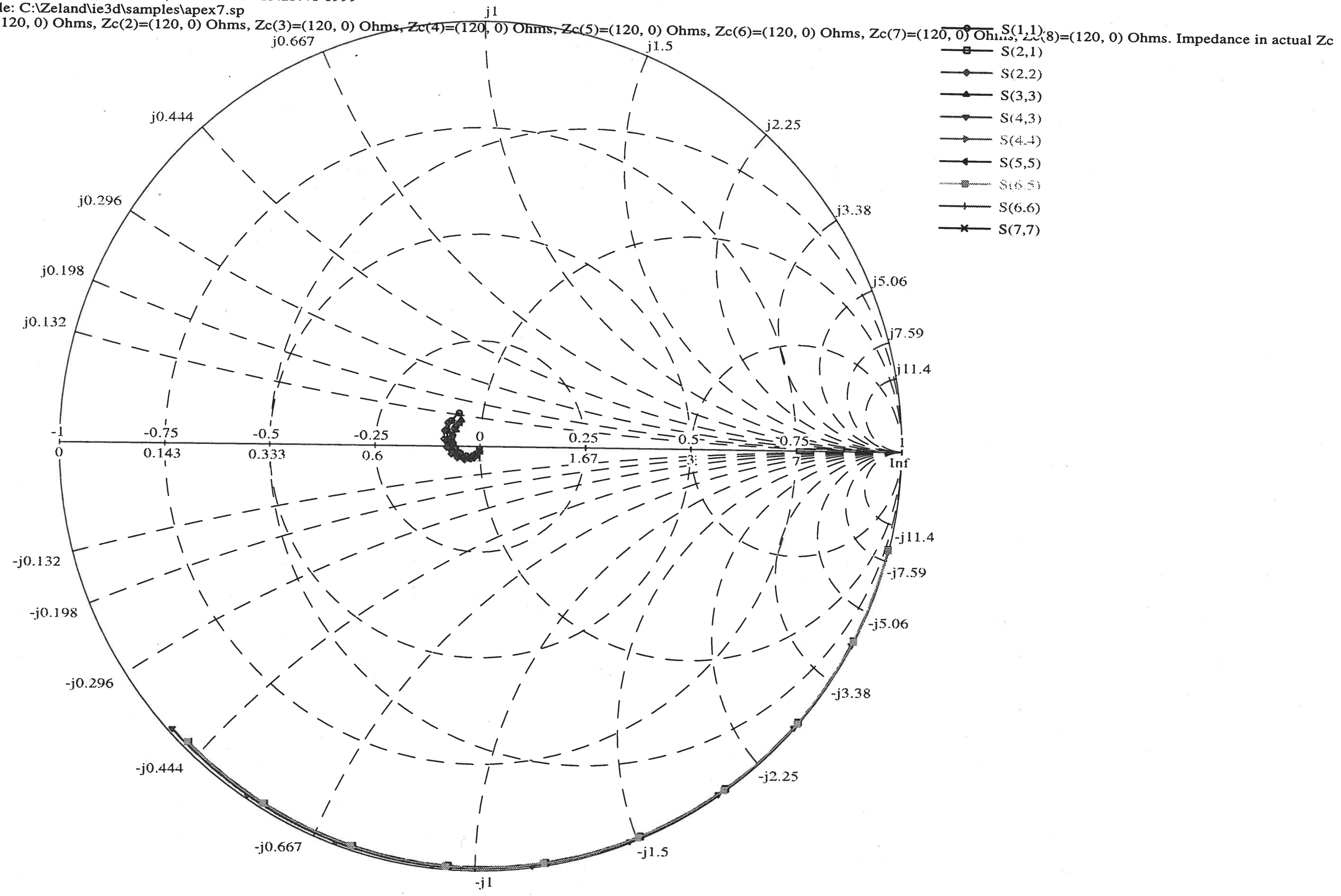


APEX Connection Circuit.

By Kenil Dumoit.



Zeland Software, Inc., IE3D 6.0, Thu Nov 04 15:23:41 1999
Data File: C:\Zeland\ie3d\samples\apex7.sp
Zc(1)=(120, 0) Ohms, Zc(2)=(120, 0) Ohms, Zc(3)=(120, 0) Ohms, Zc(4)=(120, 0) Ohms, Zc(5)=(120, 0) Ohms, Zc(6)=(120, 0) Ohms, Zc(7)=(120, 0) Ohms, Zc(8)=(120, 0) Ohms. Impedance in actual Zc



Apex connection circuit:
transmission + reflection

Another possibility for the array pattern: make the distribution follow one of the ~~circular~~ aperture distributions $(1-(r/a)^2)^p$, with p large enough to give low sidelobes.

Take $p = 2$

$$E(\theta) \propto \int_0^a \left[1-\left(R/a\right)^2\right]^2 J_0(kR\sin\theta)\, R\,dR = a^2 2^2 \Gamma(3) \frac{J_3(ka\sin\theta)}{(ka\sin\theta)^3}$$

$$E_2(\theta) \propto \frac{J_3[ka\sin\theta]}{[ka\sin\theta]^3} \qquad ka = \frac{2\pi}{\lambda}a$$

To get this grading, make the no./area of elements be proportional to $\left[1-\left(R/a\right)^2\right]^2$

(not interesting)

12/8/99 meeting w/ Sandy W and John Lupton

Cooling options   Cost of high pressure N_2 system? at room temperature

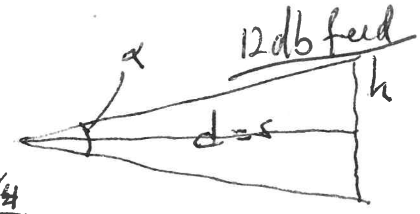
John's pulse tube design looks good.

Sandy's Amps   InP   (~15K @ 6GHz)
20K @ 80K temp.   ≤25 for ≤10GHz

[TRW] single LNA   8K @ 19K temp.   ≤10K for ≤11GHz
~5K min @ 6GHz
~70K at room temperature

A second run from HRL is coming along.

① 12db feed



$\tan(\frac{\alpha}{2}) \doteq \frac{\lambda/4}{s}$

$\frac{s}{\lambda} = \frac{1}{4\tan\alpha/2} \cong \frac{1}{2\alpha}$

$\tan\frac{\alpha}{2} = \frac{h}{d}$; $d = \frac{h}{\tan\frac{\alpha}{2}} \cong \frac{2h}{\alpha}$

For $\alpha = 20° = 0.32$ rad. and $h = \lambda/4$; $d = \frac{\lambda/2}{\tan\alpha/2}$

$d = \frac{2\cdot(\frac{\lambda}{4})}{0.32} \cong 6\cdot\frac{\lambda}{4} = 1.5\lambda = s$

For $\alpha = 30° \cong 0.5$ rad.

$d \cong \frac{\lambda/2}{0.5} = \lambda$

$\boxed{\frac{s}{\lambda} \lesssim \frac{1}{2\alpha}}$ for small α otherwise $\frac{1}{4\tan\alpha/2}$

Gain loss vs. F and s/λ (-13db taper)

~~prime focus~~
Mirror diffraction neglected

$s/\lambda = 0.6$

F	$A_e(0)$	X ✓
0.41	.750	0.60
0.45	.825	0.503
0.50	.880	0.411
0.55	.920	0.34
0.60	.940	0.29
0.65	.960	0.25
0.70	0.970	0.21

$X = \frac{10.2}{(1+16(F/.41)^2)}$

File: focus_gain_loss
lines 2-14
F is first col., the .6, .8, 1.0, 1.2, and 1.4 = s.

$s/\lambda = 0.8$

F	$A_e(0)$	X
0.41	.60	0.80
0.45	.705	.671
0.50	.79	.548
0.55	.845	.453
0.60	.885	.387
0.65	.925	.333
0.70	0.94	.280
0.75	.960	.250
0.80	.97	.220

$X = \frac{13.6}{(1+16(F/.41)^2)}$

2

$s/\lambda = 1.0$

F	$A_e(0)$	$\Sigma = \dfrac{17}{(1+16(F/.41)^2)}$
.41	.44	1.00
.45	.57	.838
.50	.69	.685
.55	.78	.567
.60	.835	.483
.65	.88	.417
.70	0.91	.350
.75	.93	.312
.80	.94	.275
.90	.97	.218

$s/\lambda = 1.2$

F	$A_e(0)$	$\Sigma = 20.4/(1+16(F/.41)^2)$
.41	.29	1.20
.45	.44	1.01
.50	~~[illegible]~~ .58	.822
.55	.69	.68
.60	.77	.58
.65	.82	.50
.70	.87	.42
.75	.895	.374
.80	.925	.330
.90	.95	.261
1.00	.970	.212

$s/\lambda = 1.4$

F	$A_e(0)$	$\Sigma = 23.8/(1+16(F/.41)^2)$
.41	.15	~~1.37~~ 1.40
.45	~~.42~~ .30	1.15
.50	.47	.941
~~.55~~	~~[illegible]~~	~~.800~~
.60	~~.76~~ .68	.690
.70	.835	.481
.80	.885	.384
.90	.94	.305
1.00	.96	.247
1.20	.98	.172

(1)

$|E| = ?$. 12db feed

Mathematica fit:

$$|E| \cong 0.9966 - .0007548\,\theta^2 + 2.2861\times10^{-7}\,\theta^4 - 2.4205\times10^{-11}\,\theta^6 = f_1[x\_]$$

$\theta \lesssim 60°$

Total = 3.3049

	θ'	$\sqrt{E_\theta^2+E_\phi^2}$	$\sin\theta$	$\lvert E\rvert^2$	$\sum^i \lvert E\rvert^2 \sin\theta_i$	Fraction	Total fraction
	0	1.0	0	1.0	0		
	2	.997	.0349	.994	.0347		
	4	.988	.0698	.976	.1028		
	6	.9727	.1045	.946	.2017		
	8	.9519	.1392	.9061	.3278		
	10	.9256	.1736	.857	.4766		
(.2094)	12	.894	.2079	.799	.6427		
	14	.855	.2419	.731	.8195		
	16	.817	.2756	.668	1.0036		
	18	.773	.3090	.598	1.1884		
	20	.726	.3420	.527	1.3686		
	22	.677	.3746	.458	1.5402		
	24	.626	.4067	.392	1.7000		
	26	.576	.4384	.332	1.8455		
	28	.526	.4695	.277	1.9756 – 2.0611		
	30	.477	.5000	.228	2.0896		
	32	.432	.5299	.187	2.1887		
	34	.389	.5592	.151	2.2731		
	36	.350	.5878	.123	2.3452		
	37°				2.3756		
	38	.314	.6157	.0986	2.4059		
	40	.282	.6428	.0785	2.4563	85.9	71.9%
	42	.253	.6691	.0640	2.4991	90.45	75.6%
	44	.227	.6947	.0515	2.5349	92.8	76.7% (92.4)
	46	.203	.7193	.0412	2.5645		77.8%
	48	.182	.7431	.0331	2.5891		
(.8727)	50	.164	.7660	.0269	2.6097		
	52	.150	.7880	.0225	2.6275	95.4%	
	54	.134	.8090	.0180	2.6420		

Mathematica Fit.

$$\eta_{spill} = 0.8323 + 0.11300\,F - 0.35370\,F^2$$

over the range $32° \lesssim \theta \lesssim 44°$

with $.589 < F < .87$

.632

$$F = \frac{1}{4}\sqrt{\frac{1+\cos\theta_e}{1-\cos\theta_e}}$$

.7096

Θ'	$\lvert E \rvert$	$\sin\Theta$	$\lvert E \rvert^2$	$\sum_i \lvert E \rvert^2 \sin\Theta$	% $\Theta \leq 90^\circ$	total fraction
56	.121	.8290	.0146	2.6541		
58	.113	.8480	.0127	2.6649		
60	.102	.8660	.0104	2.6739		
62	.093	.8829	.0086	2.6815		
64	.086	.8988	.0074	2.6882	97%	
66	.084	.9135	.0070	2.6945		
68	.086	.9272	.0074	2.7014		
70	.091	.9397	.0083	2.7092	82%	
72	.097	.9511	.0093	2.7181		
74	.100	.9613	.0099	2.7267		
76	.098	.9703	.0096	2.7360		
78	.091	.9781	.0083	2.7441		
80	.079	.9848	.0063	2.7503		
82	.064	.9903	.0041	2.7544		
84	.050	.9945	.0025	2.7569		
86	.042	.9976	.0018	2.7587		
88	.043	.9994	.0019	2.7606		
90	.050	1.000	.0025	2.7631	100%	83.6%
92	.0540	.9994	.0029	2.7660		
94	.0548	.9976	.0030	2.7690		
96	.0537	.9945	.0029	2.7719		
98	.0562	.9903	.0032	2.7750		
100	.0676	.9848	.0046	2.7796		
102	.0868	.9781	.0075	2.7869		
104	.1075	.9703	.0116	2.7982		
106	.1277 ~~.0788~~	.9613	.0163 ~~.0062~~	2.8138		
108	.1391 ~~.0826~~	.9511	.0193 ~~.0068~~	2.8322		
110	.1465 ~~.0847~~	.9397	.0215 ~~.0072~~	2.8524		
112	.1436 ~~.0842~~	.9272	.0206 ~~.0071~~	2.8715		
114	.1346 ~~.0830~~	.9135	.0181 ~~.0069~~	2.8880		

3

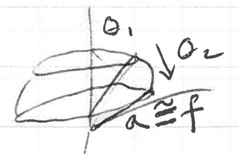
θ	$\lvert E \rvert$	$\sin\theta$	$\lvert E \rvert^2$	$\sum_i \lvert E_i \rvert^2 \sin\theta_i$
116	~~.0830~~ .1240	.8988	~~.0069~~ .0154	2.9018
118	.1192	.8829	.0142	2.9144
120	.1261	.8660	.0159	2.9281
122	.1443	.8480	.0208	2.9457
124	.1682	.8290	.0283	2.9692
126	.1917	.8090	.0367	2.9989
128	.2689	.7880	.0723	3.0559
130	.2216	.7660	.0491	3.0935
132	.2244	.7431	.0503	3.1309
134	.2196	.7193	.0482	3.1656
136	.2073	.6947	.0430	3.1954
138	.1896	.6691	.0360	3.2195
140	.1674	.6428	.0280	3.2375
142	.1444	.6157	.0208	3.2503
144	.1214	.5878	.0147	3.2589
146	.0998	.5592	.0100	3.2645
148	.0803	.5299	.0064	3.2679
150	.0642	.5000	.0041	3.2700
152	.0528	.4695	.0028	3.2713
154	.0477	.4384	.0023	3.2723
156	.0504	.4067	.0025	3.2733
158	.0595	.3746	.0035	3.2746
160	.0730	.3420	.0053	3.2764
162	.0890	.3090	.0079	3.2789
164	.1061	.2756	.0113	3.2820
166	.1236	.2419	.0153	3.2857
168	.1398	.2079	.0196	3.2898
170	.1550	.1736	.0240	3.2939
172	.1679	.1392	.0282	3.2979

θ'	$\lvert E \rvert$	$\sin\theta$	$\lvert E \rvert^2$	$\sum_i \lvert E_i \rvert^2 \sin\theta$
174	.1798	.1045	.0323	3.3012
176	.1860	.0698	.0346	3.3037
178	.1916	.0349	.0367	3.3049
180	.1924	.0000	.0370	$\boxed{3.3049}$

Angle vs. F

$$\cos\theta_e = \frac{(4F)^2 - 1}{(4F)^2 + 1}$$

F	θ_{edge}	% forward spillover	Edge Illum.
0.4	64°	3%	~~−21.3 db~~
0.5	53.1°	4.6%	~~−17.0 db~~
0.6	45.2°	7.5%	~~−13.5 db~~
0.65	42.1°	9.5%	~~−12.0~~
0.70	39.3°	11.7%	~~−10.7~~
0.75	36.9°	14.1%	~~−9.5~~
0.80	34.7°	16.8%	
0.85	32.8°	19.6%	
0.90	31.0°	22.6%	
0.95	29.5°	25.4%	



$$\text{Area} = 2\pi f^2 \int_{\theta_1}^{\theta_2} \sin\theta \, d\theta = 2\pi f^2 (\cos\theta_1 - \cos\theta_2)$$

$\boxed{\text{Back spill} \sim 17\%}$

For F = .75, f = 3.75 m

Screen area 36.9 − 64° (3% spill) $2\pi (3.75)^2 (\cos 36.9 - \cos 64) = 32\ m^2$

36.9 − 53.1 (4.6%)   36.9   53.1) = 18 m^2

36.9 − 45 (7.5%)   8 m^2

~~53 − 64~~   ~~14 m^2~~

Very large generally

The 7.5% case, 8 m^2, might not be too bad.

For F = 0.5 $f_p = 2.5$ $A \approx 2\pi (2.5)^2 (\cos 53 - \cos 64) = 6.4\ m^2$ not bad

F = 0.6 $f_p = 3.0$ $A_s = 2\pi (3.0)^2 (\cos 45 - \cos 64) = 15\ m^2$

Obtain the illumination efficiency by a regular numerical integration using the fit of the E-field from Mathematica.
With θ in radians:

$$f1[r\_, f\_] = E(\theta) = 0.9966 - 2.4779\,\theta^2 + 2.4636\,\theta^4 - 0.8563\,\theta^6 \qquad (12\,db)$$

θ = ArcTan(etc)

$$A_e(0) = \frac{2\pi \left| \int_0^{2.5} E(r)\, r\, dr \right|^2}{\int_0^{2.5} |E(r)|^2\, r\, dr}$$

Scale $E(\theta)$ according to the aperture distance. See p. 38.

$$\theta = \text{Arc Tan}\left\{\frac{r}{f - r^2/4f}\right\} = \theta(r) \qquad f/R = \frac{f \sin\theta}{r} \qquad \pi r_0^2 = 19.63\,m^2$$

$$f/R = f2[r\_, f\_] = (f/r)\, \sin\left[\text{ArcTan}\left[\frac{4fr}{4f^2 - r^2}\right]\right]$$

$$E(r) = f1[r,f] * f2[r,f] \qquad \text{for } f = 2.5 \text{ and } D = 5m.$$

$A_e(0)$ = 6.283*(NIntegrate[r* f1[r,2.5] + f2[r,2.5], {r,.002,2.5}])^2/
(NIntegrate[r*(f1[r,2.5]* f2[r,2.5])^2, {r,.002,2.5}])
(take D = 5m)

F	f = FD	η_I	$A_e(0)$	edge Illum. $\lvert E_0 \rvert$	
0.5	2.5	.683	13.41	0.114	-18.8 db
0.55	2.75	.739	14.51	0.139	-17.1
0.60	3.00	.789	15.49	0.172	-15.3
0.65	3.25	.831	16.32	0.211	-13.5
.70	3.50	.866	16.99	.254	-11.9
.75	3.75	.893	17.527	.299	-10.5
.80	4.00	.914	17.947	.343	-9.3
.85	4.25	.931	18.275	.386	-8.3
.90	4.50	.944	18.53	.427	-7.4
.95	4.75	.954	18.73	.465	-6.7

From Mathematica
$$\eta_{Ill} = -0.54246 + 3.8971\,F - 3.3967\,F^2 + 1.00233\,F^3$$
$$\eta_{Ill} = 0.365632 + 3.73244\,E_0 - 8.82082\,E_0^2 + 7.58361\,E_0^3 \quad \text{within } 1\%$$

Also $E_0 = -.12155 + 0.222264\,F + .52575\,F^2 - 0.108061\,F^3 \sim 1/2\%$

Suppose we use Cassegrain optics?
For $F = .75$ at Cass $= f_c/D_c$. With $d_c = 1.5\,m$,
$f_c = (.75)(1.5m) = 1.12\,m.$

Blockage for 5m primary and 1.5m secondary $\left(\frac{1.5}{5.0}\right)^2 = .09$
The gain loss will be 18%; $\eta_{cass} = 0.82$

<u>How small could the primary be?</u> (F_p)

For $D = 5$ m and $F_p = 0.4$, $f = 2.0$ m

Feed length for $\nu_{min} = 1$ GHz is ~2 λ_{max} = 0.6 m (2 feet)

0.6 m + 1.12 m = 1.72 m < 2 m.

<u>so F = 0.4 is ok</u>

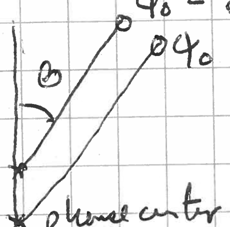
<u>Overall aperture efficiency</u> (12 db feed) (11.5)

Total spillover = Forward spill + 7% (back lobe)

F	Defocus	Illum.	Spillover	Total
0.60	.68	.789	.855	.46
0.65	.767	.831	~~.854~~ .835	.532
0.70	.835	.866	.813	.588
0.75	.860	.893	.789	.606
0.80	.885	.914	.762	.616
0.85	.912	.931	.734	.623
0.90	.940	.944	.704	.625
0.95	.950	.954 (?)	.670	.613

Check John Lugten's estimate of the phase center.

$\psi_0 - \frac{2\pi s}{\lambda}\cos\Theta = \psi$



$\psi = 80°$ at Θ 0, $\psi = 150°$ at $\Theta_x = 0$

$\frac{\pi s}{}($ 30$) =$ 50 - 80 - 70° - 1.2 rad. <u>$s/\lambda = 1.44$</u>

At $\Theta_x = 20$, $\psi = 112$ $s/\lambda = 1.47$

At Θ_x 6 ° $\Delta\psi = 324°$ 5 - rad.) $s/\lambda = 1.79$

This is a little larger, but 60° is well pas the edge of the reflector (at $\Theta_e \sim$). On the reflector, there is a good spherical wave. This agrees with John's e-mail.

How about going to a lower gain (larger α) feed? The feed pattern will be fatter, but the defocus error will be smaller, allowing for a smaller F.

If s is at the point where the width is $\lambda/2$, $s/\lambda \simeq \frac{1}{2\alpha}$

For $\alpha = 20^\circ$, $\frac{1}{2\alpha} = 1.56$; For $\alpha = 34^\circ$, $s/\lambda = 0.83$ {$F = 0.6$ for $A_e = .88$}

$g_0 \Delta\Omega = 4\pi$ $\quad \Delta\Omega \simeq \pi/4 (\Theta_{1/2})^2$ $\quad$ so $\Theta_{1/2} \simeq 4/\sqrt{g_0}$

$\Theta_{1/2} = 42^\circ$ for 12db feed (FW); $\Theta_{1/10} = 75^\circ$ (FW)

From Rumsey $\quad$ Dipole feed with $\tau = 0.95$:

α	gain
20°	10.5db
34°	8.9
50°	7.2

The zig-zag with 0.71 and $\alpha = \psi = 45$ has $g = 7.7$db

$\alpha = 20^\circ$ and $\tau = .975$ gives 12db with pyramid.
Assume $\alpha = 34^\circ$ " " 9.5db "
This is 1db bigger than the effect for the dipole feed, but here we keep $\psi = \alpha$ which gives more gain.

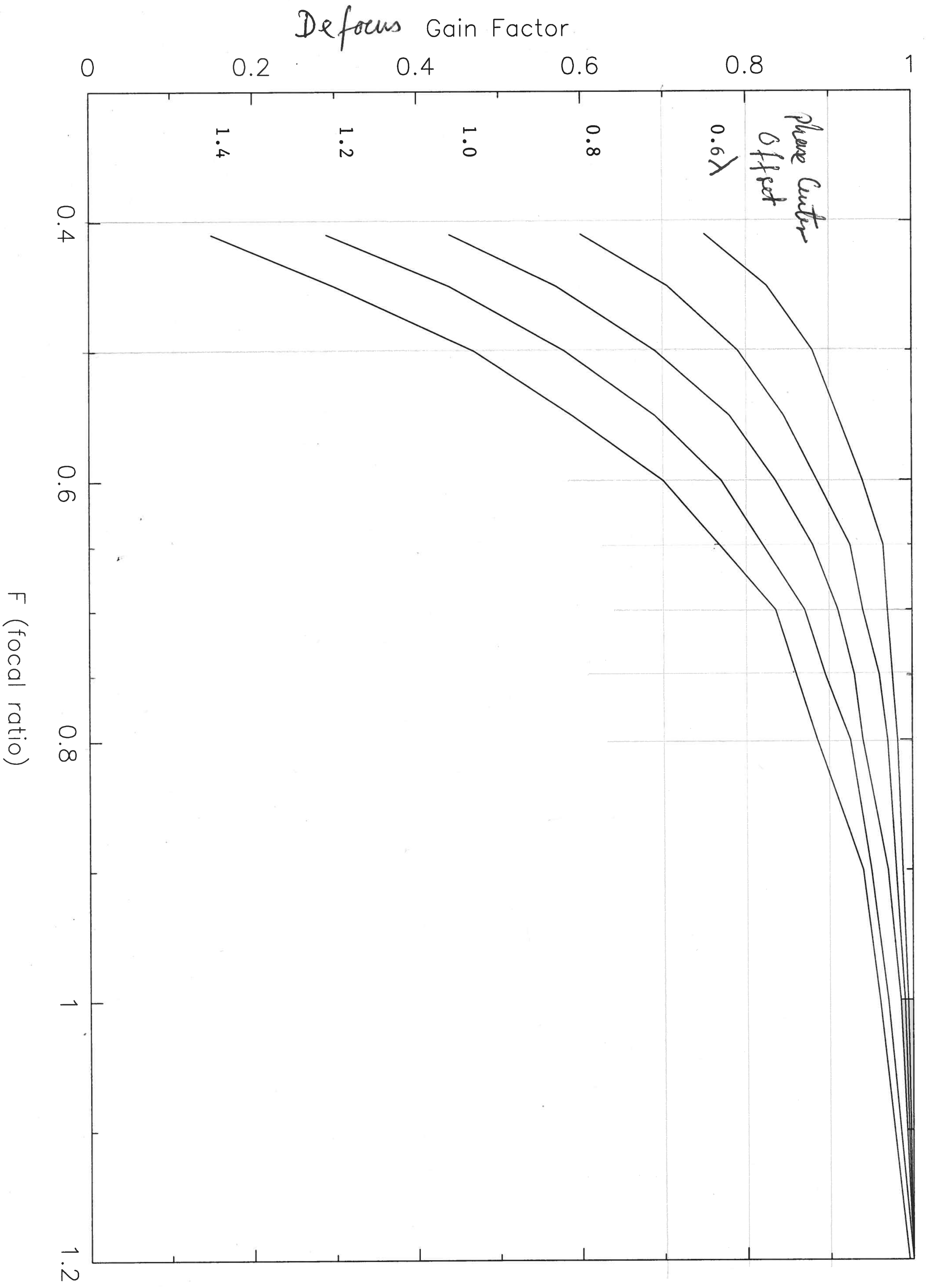
12db $\to$ 9.5db : $15.8 \to 8.9$ $\qquad \Theta_{1/2} \propto \frac{1}{\sqrt{g_0}}$; $\sqrt{\frac{15.8}{8.9}} \times 42 = 56^\circ$ for $\Theta_{1/2}$

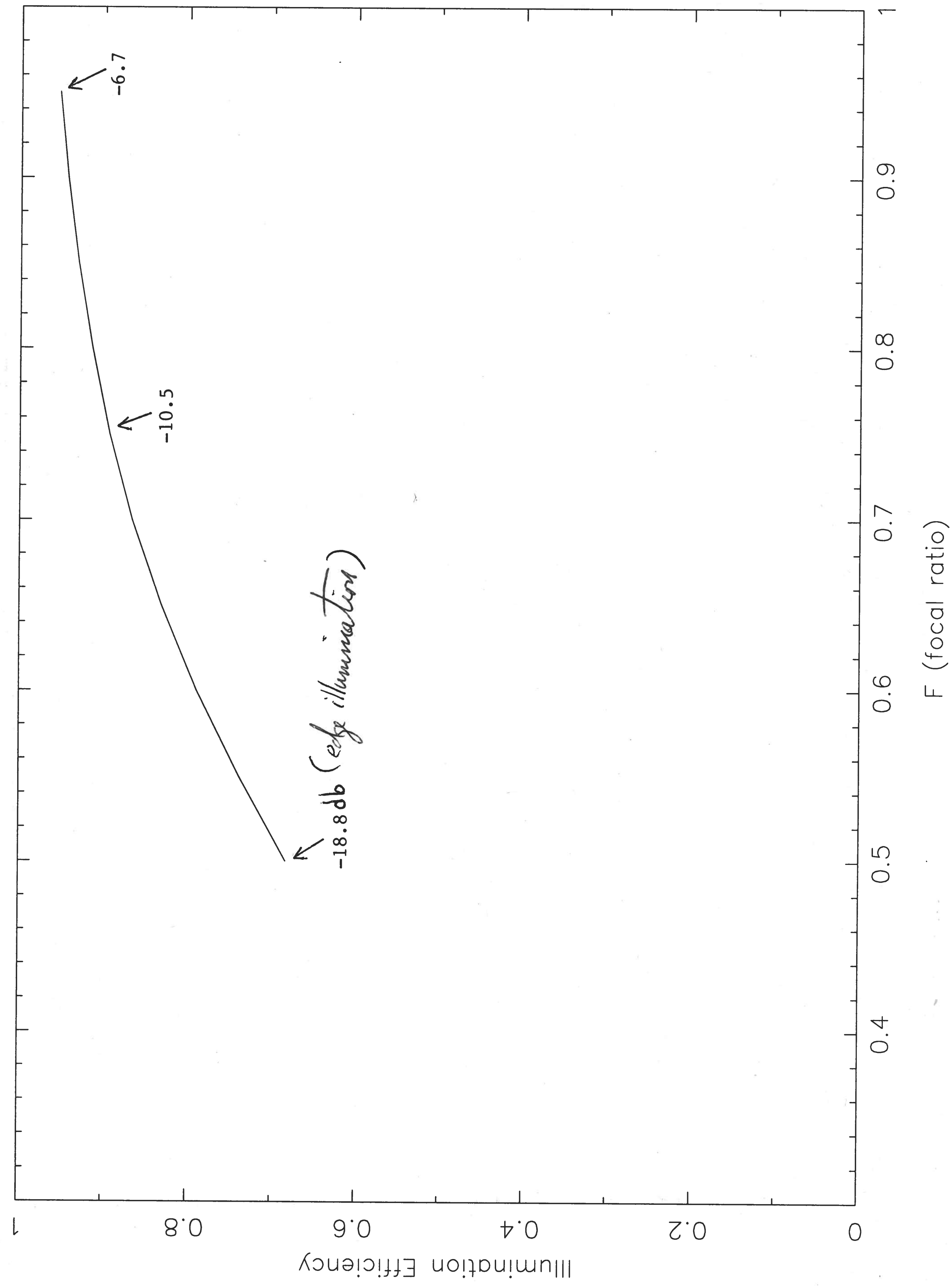
$\sqrt{\frac{15.8}{8.9}} = 1.33$

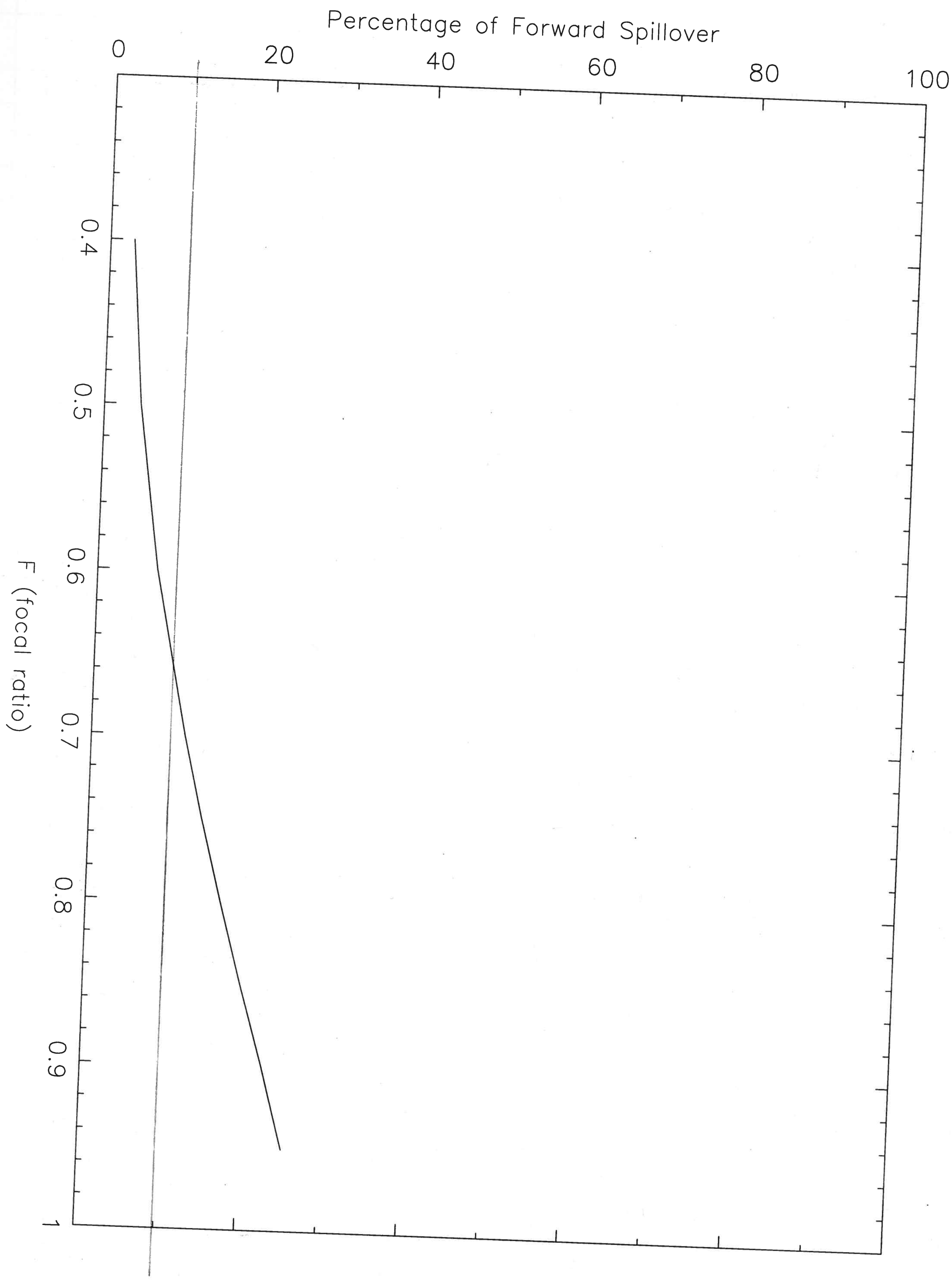
Suppose $\Theta_{1/10}$ also scales by 1.33 : $1.33 \times 75 = 100^\circ$

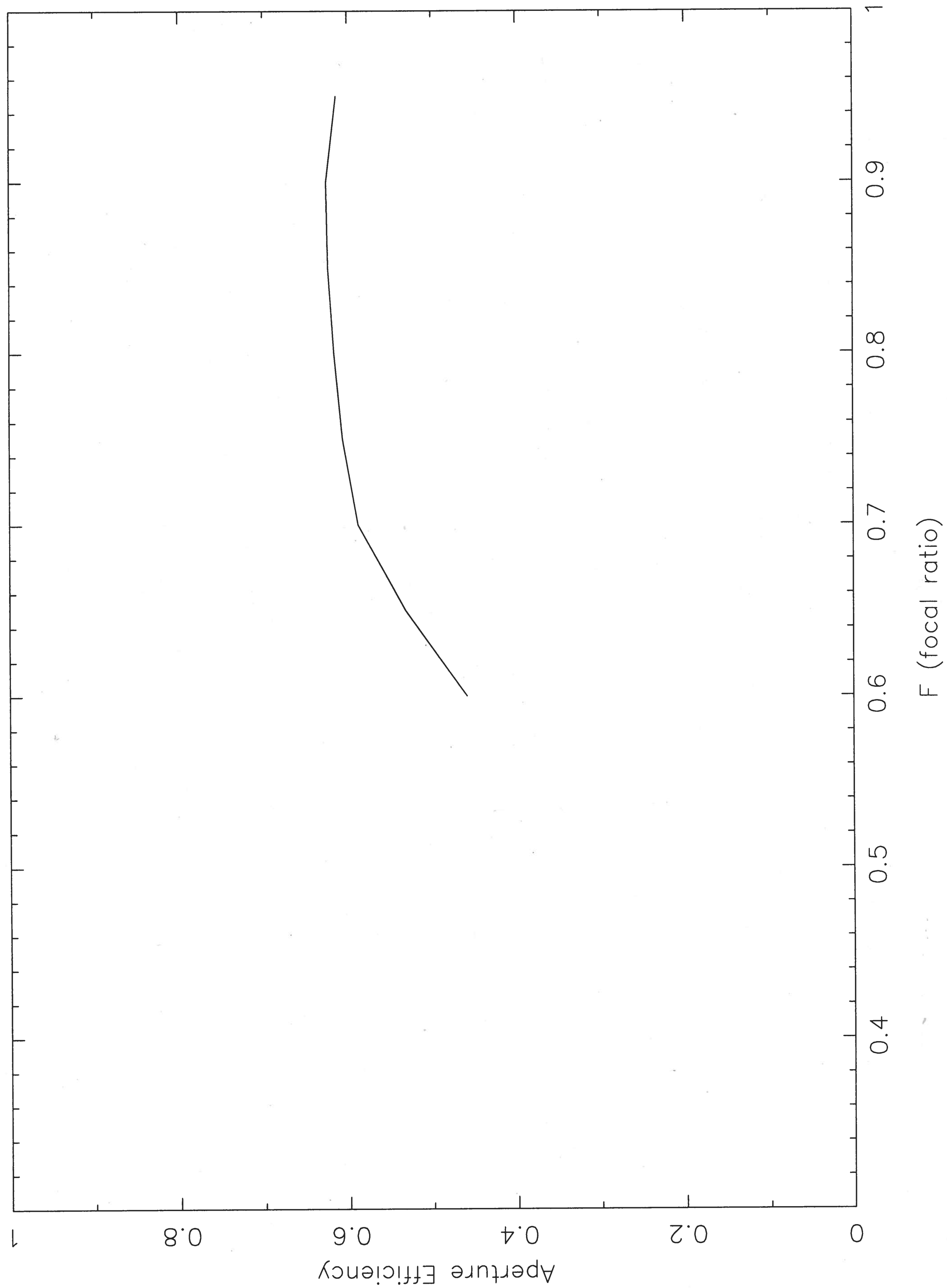
For $F = 0.6$, $2\Theta = 90^\circ$. This is smaller than 100° and suggests that the spillover would be greater at $\alpha = 34^\circ$. The $\alpha = 34^\circ$ (and $\alpha = 28^\circ$) cases should be run to check this out.

Plots of the tables




-6.7
-10.5
-18.8db (edge illumination)
F (focal ratio)
0.4
0.5
0.6
0.7
0.8
0.9
1
Illumination Efficiency
0
0.2
0.4
0.6
0.8
1


Percentage of Forward Spillover
0
20
40
60
80
100
F (focal ratio)
0.4
0.5
0.6
0.7
0.8
0.9
1



F (focal ratio)
0.4
0.5
0.6
0.7
0.8
0.9
1
Aperture Efficiency
0
0.2
0.4
0.6
0.8
1

1/2/2000

Now set the phase center for $\nu = 7.5$ GHz at the mirror focus rather than putting the feed vertex there. Use the curves from p. 7. $\alpha = 20°$ case $S/\lambda = 1.5$ or $S = 1.5\lambda$

Take $F = 0.75$ -10 db taper at the edge.
Then $\frac{17}{1+16(.75/.41)^2} = 0.3117$

After some trial and error, setting the mirror focus at the phase center for <u>$\nu = 7.5$ GHz</u> looks best.

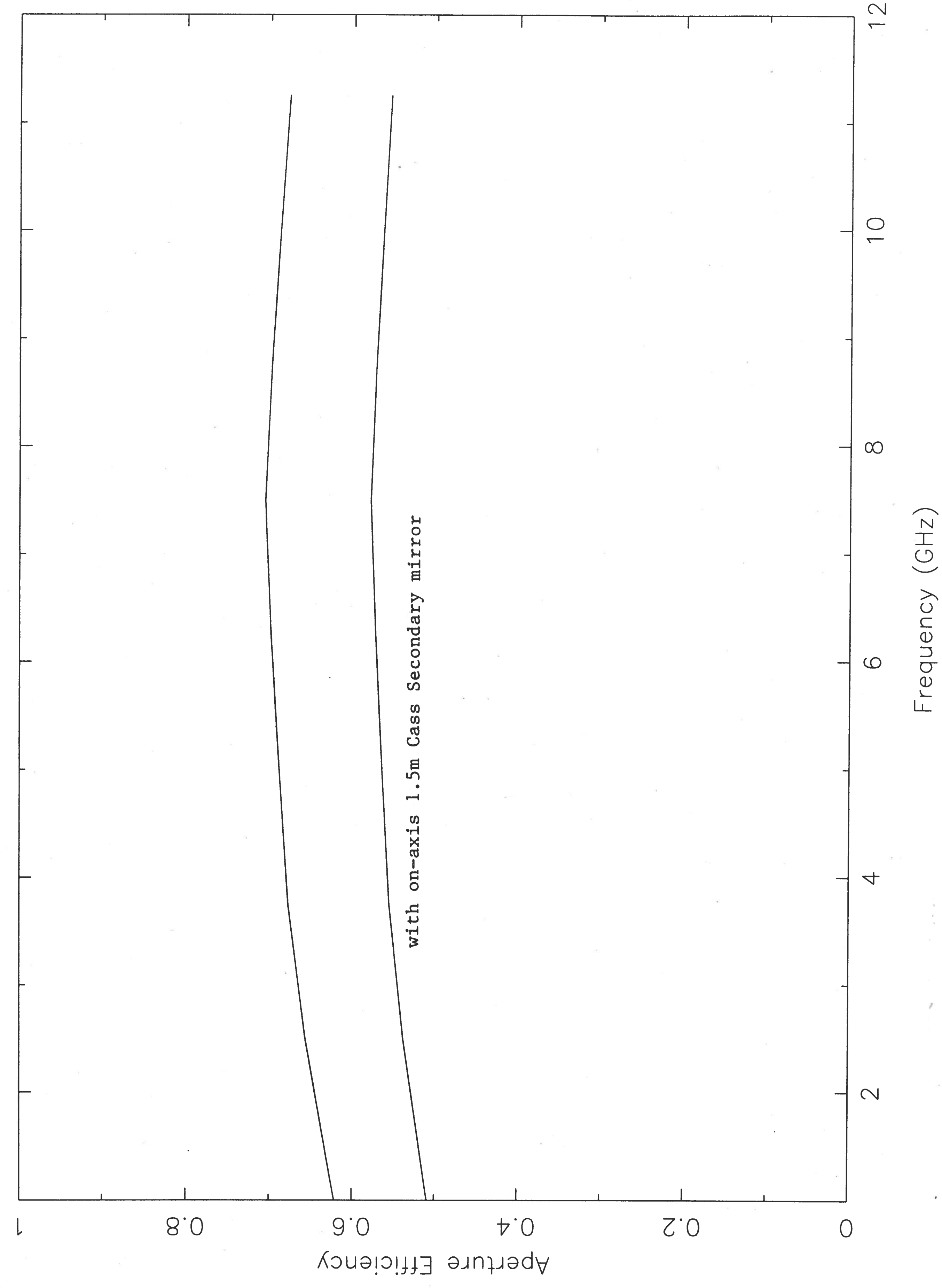
$\Xi = (S/\lambda)(.3117)$

ν	λ(cm)	S	$\lvert\Delta S\rvert$/cm	$\lvert\Delta S\rvert/\lambda$	Ξ	η_{defocus}	η_A no-Cass	η_A with Cass
11.25	2.67	4.0	2.0	.75	.239	.96	.677	.555
10.0	3.00	4.5	1.5	.50	.156	.975	.688	.564
8.75	3.43	5.15	.85	.25	.078	.990	.698	.572
7.50	4.00	$\boxed{6.0}$	0	0	0	1.00	.705	.578
6	4.80	7.2	1.2	.25	.078	.990	.698	.572
5.00	6.00	9.0	3.0	.50	.156	.975	.88	.564
3.75	8.00	12.00	6.0	.75	.239	.96	.677	.555
2.50	12.00	18.00	12.0	1.00	.312	.93	656	.538
1.25	24.0	36 0	30.0	.2	.390	.89	.2	.514
1.00	30.0	45.0	39.0	13	.405	.88	.62	.509
.5	60	90	84	1 0	.4 6	.85	600	

For $\cdot$ .75 $\eta_{Iu} = .893$ $\eta_{\text{spillover}} = .789$; $\eta_A = \eta_{Iu}\,\eta_{\text{spill}} \cdot \eta_{\text{cass}}$.

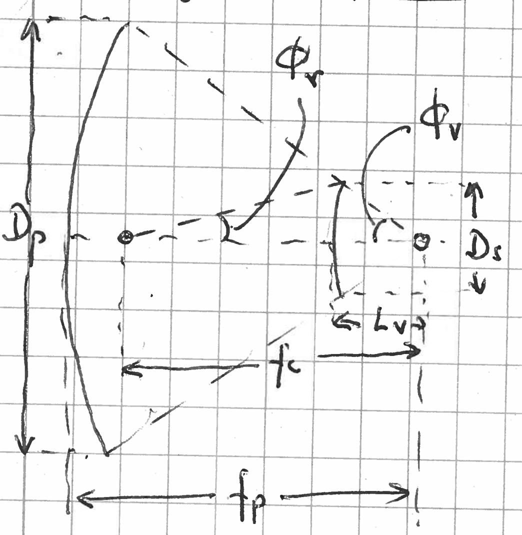
The last factor will be .82 for a Cass of 1.5 m and primary of 5 m
Then $\eta_{Iu} \cdot \text{spill}\ \eta_{\text{cas}} = 0.578$

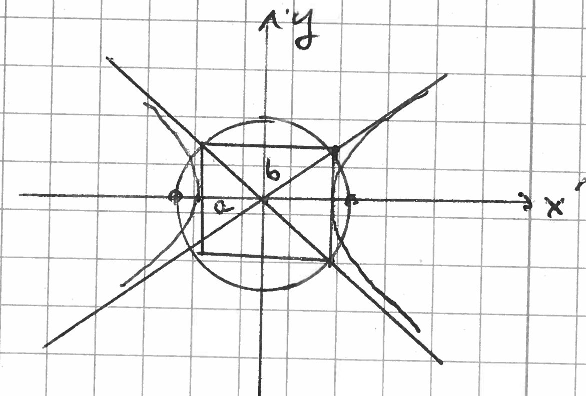
Another possibility would be to use an off-axis Cassegrain. That would avoid the blockage. We could also then make it bigger (say 2–3 m) and the go to lower frequencies.


with on-axis 1.5m Cass Secondary mirror
Frequency (GHz)
Aperture Efficiency
0
0.2
0.4
0.6
0.8
1
2
4
6
8
10
12

CRC p. 178

Cassagrain Options





Hyperbola Equation: $\dfrac{x'^2}{a^2} - \dfrac{y^2}{b^2} = 1$

Eccentricity: $e = \dfrac{\sqrt{a^2+b^2}}{a} > 1$

$e^2a^2 = a^2+b^2$ so

$b = a\sqrt{e^2-1}$

$f_c = 2ae$

$a^2+b^2 = a^2e^2 \qquad (f_c/2)^2 = a^2+b^2$

$f_c = 2ae$

$\tan\left(\dfrac{\Phi_v}{2}\right) = \dfrac{D_p}{4f_p}$

$\dfrac{2f_c}{D_s} = \dfrac{1}{\tan\Phi_r} + \dfrac{1}{\tan\Phi_v}$

$$e = \frac{\tan\left(\frac{\Phi_v}{2}\right) + \tan\left(\frac{\Phi_r}{2}\right)}{\tan\left(\frac{\Phi_v}{2}\right) - \tan\left(\frac{\Phi_r}{2}\right)}$$

Parabola Equation: $y^2 = 4f_p x$ with (0,0) the vertex

$\cos\Phi_v = \dfrac{(4F_p)^2-1}{(4F_p)^2+1} \qquad F_p = f_p/D_p$

$L_v = \dfrac{f_c}{2} - a$

hyperbola x axis relative to vertex of parabola: $f_p - f_c/2$

$x' = x - (f_p - f_c/2)$

Cass focus $= f_p - f_c$

I. A Symmetric Cassagrain System : $\alpha = 20°$ feed

$D_p = 5\,m$ and $D_s = 1.5\,m$ $F_p = 0.4$, so that $f_p = 2.0\,m$, and $\phi_V = 64°$

$y^2 = 4 f_p x \rightarrow y^2 = 8x$, $y = \sqrt{8x}$ vertex at $(0,0)$

For $F = f/D = 0.75$ for the feed, $\phi_r = 36.9°$

$$f_c = \frac{D_s}{2}\left\{\frac{1}{\tan\phi_r} + \frac{1}{\tan\phi_V}\right\} = 1.3647\,m$$

$$e = \frac{\tan\left(\frac{\phi_V}{2}\right) + \tan\left(\frac{\phi_r}{2}\right)}{\tan\left(\frac{\phi_V}{2}\right) - \tan\left(\frac{\phi_r}{2}\right)} = \frac{.3346 + .6249}{.6249 - .3346} = 3.2904 = e$$

$a = f_c/2e = 1.3647/2(3.2904) = .2074$

$b = a\sqrt{e^2 - 1} = .2074\sqrt{(3.2904)^2 - 1} = 0.6501$

center point $f_p - f_c/2 = 2.0 - 1.3647/2 = 1.3176$

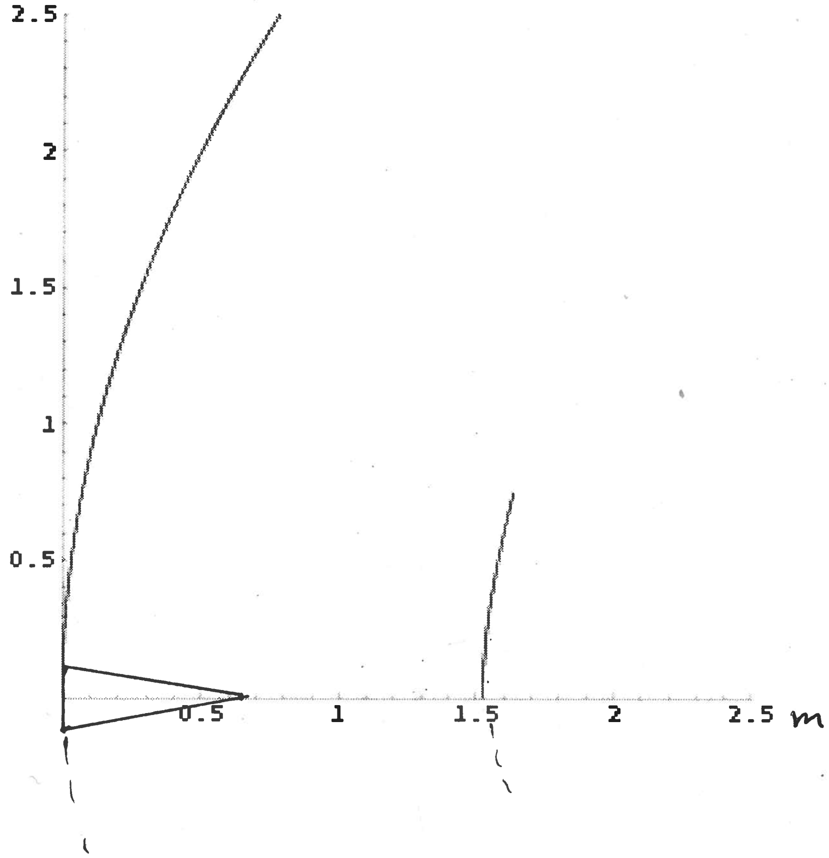
Equations primary $y_p = \sqrt{8x}$ when $y = 2.5\,m$, $x_{edge} = 0.7813\,m$

hyperboloid: $\dfrac{x'^2}{(.2074)^2} - \dfrac{y^2}{(.6501)^2} = 1$; $x' = x - 1.3176$

$$y_s = 0.6501\left\{\left(\frac{x - 1.3176}{.2074}\right)^2 - 1\right\}^{1/2}$$

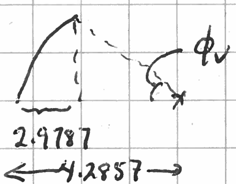
At the edge, $y_s = .75\,m$, $x_{edge} = 1.6342$

At the center, $y_s = 0$, $x_{center} = 1.5250$


2.5
2
1.5
1
0.5
0.5
1
1.5
2
2.5
m

II. Offset Cassagrain   Take $F_p = 0.3$ $D_p(\text{total}) = 2 \times 7.1428 = \underline{14.2856}$

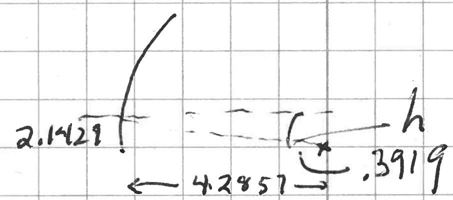
$$f_p = 4.2857$$
$$y_p^2 = 17.1427x$$
$$\text{At } y_p = 7.1428,\ x_{edg} = \frac{(7.1428)^2}{17.1427} = 2.9787$$



$$\phi_v = \tan^{-1}(7.1458/1.3620) = 79.6°$$

Take $\phi_v = \underline{65°}$ $\underline{D_s/2 = 2.1429}$

$$f_c = 2.1429\{.1829 + .4663\} = \underline{1.3912}$$
$$e = \frac{\tan(39.8) + \tan(32.5)}{\tan(39.8) - \tan(32.5)} = \underline{7.4976}$$
$$a = f_c/2e = 1.3912/2(7.4976) = \underline{.0928}$$
$$b = a\sqrt{e^2-1} = .0928\sqrt{(7.4976)^2-1} = \underline{.6896}$$



$$h = \frac{.3919}{4.2857}(2.1429) = 0.1960$$

Cass focus: $f_p - f_c = \underline{2.8945}$

Base of secondary: $f_p - f_c/2 = \underline{3.3732}$

equation of secondary $\left(\frac{x - 3.3732}{.0982}\right)^2 - \left(\frac{y}{.6896}\right)^2 = 1$

Curve limits $y_p = (17.1427x)^{1/2}$

when $y_p = 7.1428$, $x = (7.1428)^2/17.1427 = \underline{2.9762}$

when $y_p = 2.1429$, $x = \underline{.2679}$

Secondary $y_s = 0.6896\left\{\left(\frac{x - 3.3732}{.0982}\right)^2 - 1\right\}^{1/2}$

when $y_s = 2.1429$, $x_e = \underline{3.6938}$

when $y_s = h = 0.1960$, $x_h = \underline{3.4753}$

Aperture diameter is 5m. Secondary diameter is 1.95m

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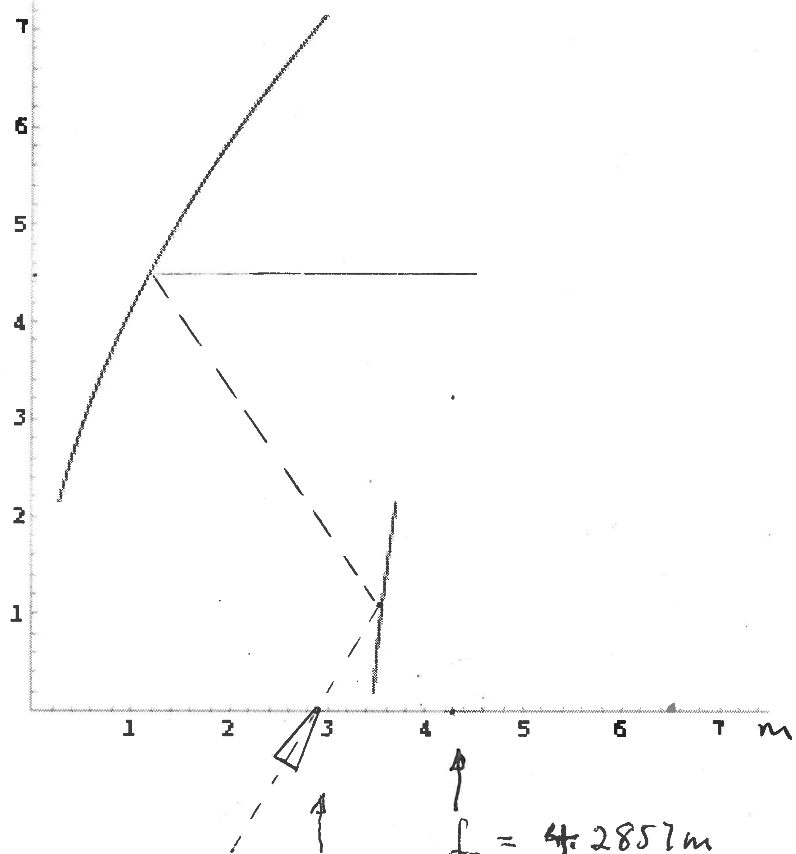
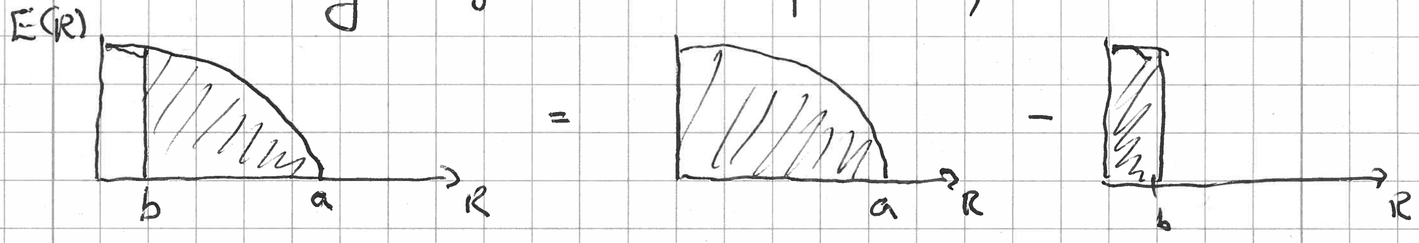


$f_p = 4.2857m$

Cass focus 2.8945 m

Patterns of the symmetric Cassagrain; Edge diffraction at the secondary is neglected. $D_p = 5\,m$, $D_s = 1.5\,m$



The middle distribution field is given on p. ~~6~~61-62.

The distribution for the secondary:

R	$E_b(R)$	$E_b(R) - .898$	$.102\left[1-\left(R/.75\right)^2\right]^{1.5}$
0	1.00	.102	.102
.25	.986	.088	.086
.50	.951	.053	.042
.70	.898	.00	0

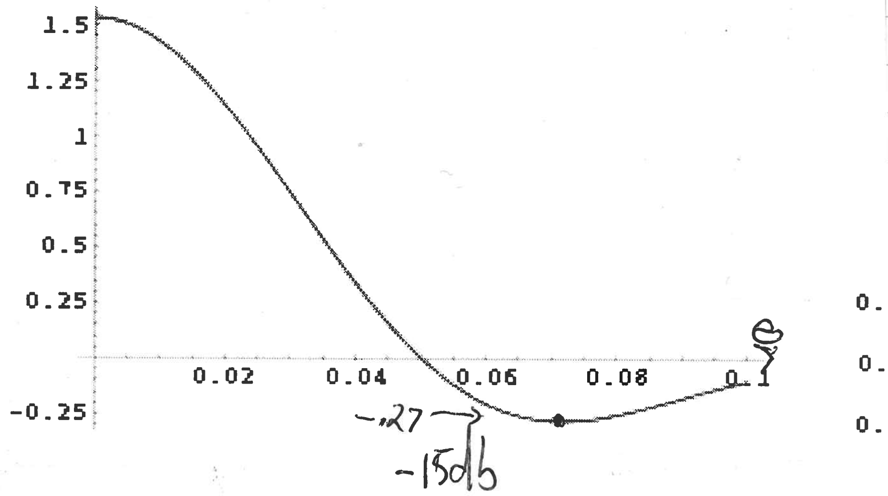
$$E_b(R) \doteq 0.898 - .102\left[1-\left(R/.75\right)^2\right]^{3/2}$$

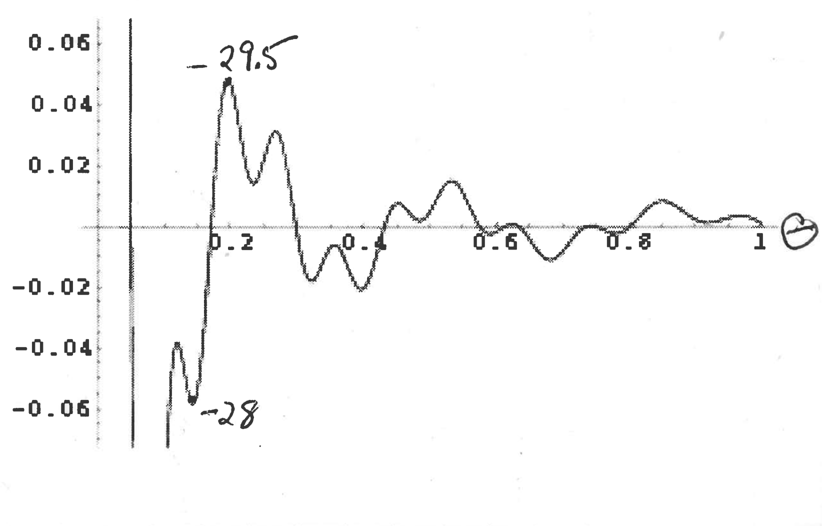
Use the integral on p. 62. The difference patterns are:

$$E(\theta) \propto (2.5)^2\left\{0.291\,\frac{J_1\left(\frac{ka\sin\theta}{\lambda}\right)}{(ka\sin\theta/\lambda)} + 2.6651\,\frac{J_{5/2}\left(\frac{ka\sin\theta}{\lambda}\right)}{\left(ka\frac{\sin\theta}{\lambda}\right)^{5/2}}\right\}$$

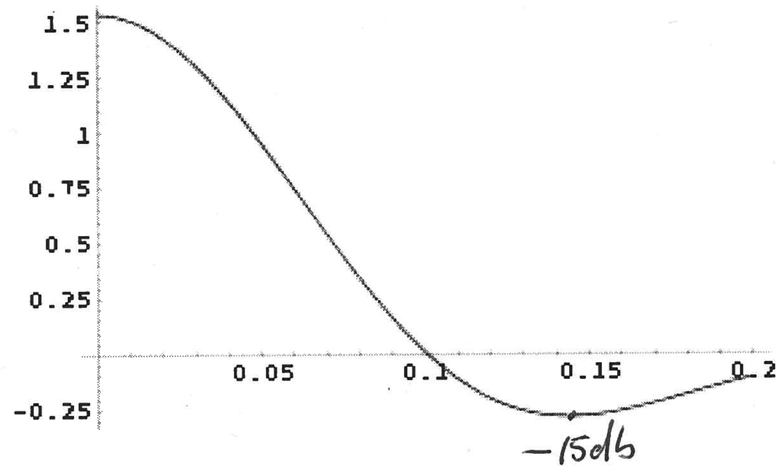
$$-(.75)^2\left\{(.898)\,\frac{J_1\left(\frac{kb\sin\theta}{\lambda}\right)}{\left(\frac{kb\sin\theta}{\lambda}\right)} + .3834\,\frac{J_{5/2}\left(\frac{kb\sin\theta}{\lambda}\right)}{\left(\frac{kb\sin\theta}{\lambda}\right)^{5/2}}\right\}$$

$\lambda = 20$ cm

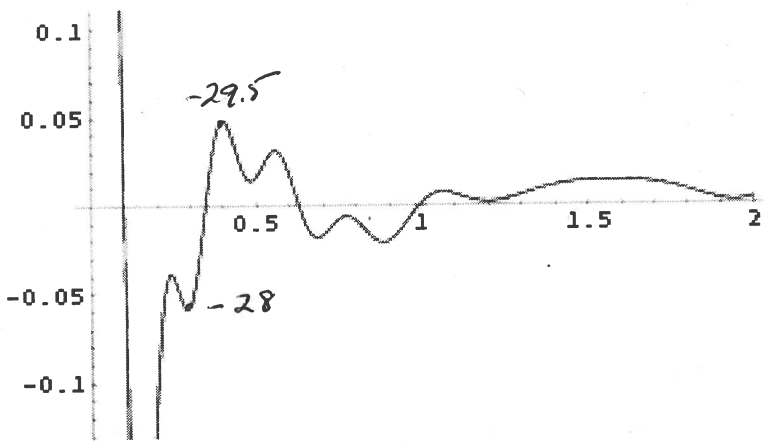


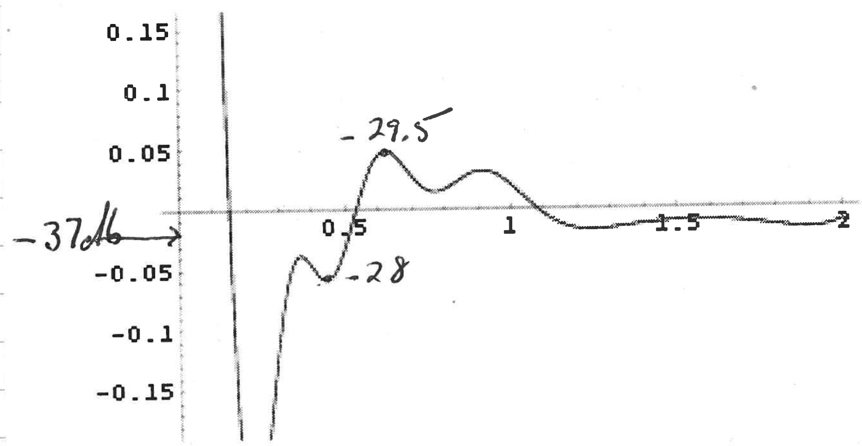


The secondary raises the first side lobe from −25db to −15db.



$\lambda = 40$ cm





$\lambda = 60$ cm.

Circular Aperture Diffraction Papers

Bekefi, G. J. App. Phys., vol 24, p. 1123, 1953
Millar, R.F. Proc. Inst. Elec. & Eng (Brit.) Monograph No. 152R, Oct. 1955
Meixner + Andrejewski, Ann. der Physic, vol. 7, 1950.

→ The most useful paper appears to be Per-Simon Kildal PGAP, AP-31, #6 Nov. 1983

Cassagrain aperture efficiency including both blockage and diffraction effects is expressed as:

$$\eta_a = \eta_{Iu}\,\eta_i = \eta_{Iu}\left|1 - C_b\left(\frac{d}{D}\right)^2 - C_d\sqrt{\frac{\lambda}{d}}\sqrt{1-\left(\frac{d}{D}\right)}\,A_0\right|^2$$

D and d are primary and secondary diameters, respectively.

$C_b\left(\frac{d}{D}\right)^2$ is the aperture geometrical blockage. $C_b = 1$ for uniform illumination.

$C_b = 1.5$ for -10db edge taper. His figure 4 gives a curve for other values.

The last term is the edge diffraction contribution. A_0 is the amplitude edge taper.

$$C_d = \frac{1}{\pi}\frac{\cos^2(\phi_r/2)}{\sqrt{\sin\phi_v}}C_b = \frac{0.95}{\pi}C_b = .454 \quad \text{for } C_b = 1.5,\ \phi_r = 36.9^\circ, \text{ and } \phi_v = 64^\circ$$

For -10db taper, $A_0 = .312$

For $D = 5$

d	$(d/D)^2 1.5$
1.5	.135
1.2	.086
1.0	.06

The factor is x2 because of the $|\ |^2$ (27 by itself)

The diffractive loss

$$l_d = C_d\sqrt{\frac{\lambda}{d}}\sqrt{1-\frac{d}{D}}\,A_0 = (.454)(.312)\sqrt{\frac{\ }{d}}\sqrt{1-\frac{d}{D}} = 0.142\sqrt{\frac{\ }{d}}\sqrt{1-\frac{d}{D}}$$

(.299 written above)

For $d = 1.5$m, $D = 5$m, and $\lambda = 0.3$m (1GHz), this is .05 (10%)

$D = 5m$, $\lambda = 0.3m$

d	l_d
1.5	.054
1.2	~~.064~~ .062
1.0	.070

$D = 5m$, $\lambda = 0.6m$

d	l_d
1.5	.076
1.2	.088
1.0	.098

Symmetrical Cassegrain with $D = 5m$ and $d = 1m$ $F_{cass} = 0.75$
from p. 83 for everything but the last column. focused at 7.5 GHz

ν	$\lambda^{(cm)}$	η_0 (no blockage)	η_i	$(1-.06-.127\sqrt{\lambda})^2$	η (Total)	diffractive spillover	diff spill for d = 2m
11.25	2.67	.677	.919	.845	.572	2.1%	1.3%
10.00	3.0	.688	.918	.843	.580	2.2	1.36
8.75	3.43	.698	.917	.841	.587	2.4	1.44
7.50	4.0	.705	.915	.837	.590	2.5	1.56
6.25	4.8	.698	.912	.832	.581	2.8	1.70
5.00	6.0	.688	.909	.824	.570	3.1%	1.9
3.75	8.0	.677	.904	.817	.553	3.6	2.2
2.50	12.0	.656	.896	.803	.527	4.4	2.7
1.25	24.0	.627		.771	.483	6.2%	3.8
1.00	30.0	.621		.757	.470	7.0%	4.3%
0.50	60.0	.600		.708	.425	10%	6.0%

This argues for $d = 2m$

Note that about half the edge diffraction loss will spillover past the primary onto the ground; e.g. 7% at 1 GHz
A primary skirt would help some.
Increasing d to 1.2 m will tilt the curve a little more in favor of the lower frequencies

Curves for $D_s = 1.0m$

$f_c = .9098\,m$
$a = .1383$
$b = .4334$

center point : $f_p - f_c/2 = 1.5451$

Primary $y_p = \sqrt{8x}$ $y = 2.5m$, $x_{edge} = 0.7813$

hyperboloid: $$y_s = .4334 \left\{ \left(\frac{x - 1.5451}{.38} \right)^2 - 1 \right\}^{1/2}$$

At the edge $y_s = 0.50m$
$x_{edge} = 1.7562\,m$

At the center $y_s = 0$, $x_{center} = 1.6834$

Antenna gain implied by the pattern of pp 72-75, assuming axial symmetry.

$\int g \frac{d\Omega}{4\pi} = 1$ $\quad g(\theta,\phi) = g_0 f(\theta)$, where $f(\theta) = |E(\theta)|^2$ and $f(0) = |E(0)|^2 = 1$.

$$\frac{g_0}{4\pi}\int_0^{2\pi}\int_0^{\pi} f(\theta)\sin\theta\, d\theta\, d\phi = \frac{g_0}{2}\int_0^{\pi} f(\theta)\sin\theta\, d\theta$$

$$\cong \frac{g_0}{2}\sum_i f(\theta_i)\sin\theta_i\,\Delta\theta \qquad ; \ \Delta\theta = 2^\circ = .0349 \text{ radians.}$$

$$\frac{g_0}{2}\sum f(\theta_i)\sin\theta_i\,\Delta\theta = \frac{g_0}{2}(3.3049)(.0349) = 1.$$

Hence $g_0 = \frac{2}{(3.3049)(.0349)} = 17.3388$, $\underline{12.390 \text{ db}}$

This is close to the ~12db for the feed simulation.

This suggests that the assumption of a completely symmetric pattern is correct (?!).

Look at a solution for $F_p = 0.65$ instead of 0.75 with the large spillover implied by the above calculation.

$\theta_e = 42^\circ$, total spillover is now 24.4%. Forward spillover is 9.5%.

Now A_0 is $\underline{.253}$ instead of .332. $\quad .253 \leftrightarrow -11.94$ db

$\eta_{spill} = 0.756 \qquad \eta_{I\cdot n} = 0.831$

Assume an offset Gregorian so there's no blockage.

$\eta_{sp\,ll}\ \eta_I = 0.628$

For F_p $\quad$ 5, $\Sigma = .05/\lambda \quad 17/\left[1+16(.65/\alpha)^2\right] = \cancel{\Delta/\lambda}\ .2$

A_{gain} at 7 GHz ( )

$s/\lambda = 1.4$

ν	λ(cm)	S	ΔS	$\Delta S/\lambda$	X	$\eta_{diffrac.}$	η_{total}	forward diff spillover
11.25	2.67	3.74	1.86	.697	.2875	.940	.590	1.1[illegible]%
10.0	3.00	4.20	1.40	.467	.193	.980	.615	1.16
8.75	3.43	4.80	0.80	.233	.096	.99	.622	1.24
7.50	4.00	$\boxed{5.60}$	0	0	0	1.00	.628	1.34
6.25	4.80	6.72	1.12	.250	.103	.99	.622	1.47
5.00	6.00	8.40	2.80	.467	.193	.98	.615	1.65
3.75	8.00	11.20	5.60	.700	.289	.940	.590	1.90
2.50	12.00	16.80	11.20	.933	.385	.885	.556	2.33
1.25	24.00	33.6	28.0	1.17	.4826	.835	.524	3.29
1.00	30.00	42.0	36.4	1.213	.5004	.825	.518	3.68%
0.50	60.00	84.0	78.4	1.307	.539	.800	.502	5.2%

The diffraction spillover will change as A_0 goes from .33 → .25

C_b : 1.5 → 1.6 ✓ $C_d \to (.95/\pi)(1.6) = .484$

$(.484)(.253) = .122$ (instead of .142) no, (.484)(.2 = .102

take $d = 2m$, $D = 5m$

$$l_d = 0.122 \sqrt{\frac{\lambda}{d}} \sqrt{1-\frac{2}{5}} = \cancel{.095}\ .079 \sqrt{\frac{\lambda(m)}{2}}$$

no blockage

$$\eta_i = \left(1 - \cancel{.095}\ .072 \sqrt{\frac{\lambda}{2}}\right)^2$$

Careful here

Estimate Tsys at 5GHz

ant.	.2	
spillover	.165	
	65	11K at 300K
dewar Xmission	1.5	
2.7° bb	2.7	
atmos	3.0	
	18.20K	

For 10K RCVR → 28.2
K → 33.2
20K → 38.2

IHT Board Meeting 2/14/00

next meeting May 12, 11AM @ SETI

JB summary
MD: P Napier warns that the skirt may cause base line problems
HRL run starts later this month
MITECH estimates ~$500 / amplifier
Refer developments : J. Layton Looks good

F/O link Ortel maybe Thompson/CFF

Sun computers will be on loan

FY 2000 Budget 1.44M$
160K equipment loan from Sun – computers

Leo's funding estimate ~1.5M/yr. from NSF + UC for ops.

John L may have something pattentable in the Compressor for the pulse Tube – Also what about Orbitron upgraded mount –
Leo talked with Harold about looking into the patent issues – ① mount ② compresser ③ feed ④ possibly Gregorian antenna

[call Ammny. again]

Advisory Panel { Rick Fisher, P. Napier, R. Ekers, Lou Sheffer, Dan. Werthimer, Horowitz, Arnold van Ardenne, Alan Rogers, Greg Papadopoulos, Brian Glendenning, Tim Cornwell, Barry Clark, Wm Brown, Bruce Veidt

Jack send the list around for priorities

PDR December

2/16/00 meeting
Jack discussed patterns

shadowing Dave
Mike 15m
what single dish?
Woodbury?

Put shadowed ants to the South edge of the array
Smith Chart like.

Douglas/Matt on the RPA
a) cables pulled
b) Niles has a second card
c) internet connection - beginning of April to be done - AT&T
d) Matt will take one dish out there tomorrow
e) feeds: nearly done - all assembled -
final amp needed -
f) Tak room - sheet rock on most walls - new window

next meeting March 8 3:30

Shop → storage / vehicle storage
Camera Shelter → Visitor Center

Antenna diameter optimisation wrt to cost.

$$\text{Cost} = n A \left[\left(D/D_0 \right)^{\alpha} + 1 \right]$$

First term is antenna cost
Second is electronics.
At $D = D_0$, they are the same.

Single pointing sensitivity $S = k n D^2$, $n = \frac{S}{k \cdot D^2}$

$$\text{Cost} = A \frac{S}{k D^2} \left[\left(D/D_0 \right)^{\alpha} + 1 \right] \; ; \; \frac{\partial \text{cost}}{\partial D} = 0 \text{ gives } \left(D/D_0 \right)^{\alpha} = \frac{2}{\alpha - 2}$$

Then antennas relative to electronics should be $\frac{2}{\alpha-2}$

For mosaicing sensitivity $S = k' D n$

$$\text{Cost} = \frac{A S}{D} \left[\left(D/D_0 \right)^{\alpha} + 1 \right] \; ; \; \frac{\partial \text{cost}}{D} = 0 \text{ gives } \left(D/D_0 \right)^{\alpha} = \frac{1}{\alpha - 1}$$

				D/D_0	
α	$\frac{2}{\alpha-2}$	$\frac{1}{\alpha-1}$	average	$\left(\frac{2}{\alpha-2}\right)^{1/\alpha}$	$\left(\frac{1}{\alpha-1}\right)^{1/\alpha}$
2.5	4	0.7	2.3	1.7	.87
2.7	2.9	0.6	1.8	1.48	.83
3.0	2	0.5	1.3	1.26	.79
3.3	~~0.3~~ 1.5	0.4		1.14	.78

10/3

$\alpha = 2.7$ is a frequent choice. This gives $\frac{\text{ant}}{\text{ele}}$ 2.9 or 0.6.
Taking into account the correlator p stes to a smaller number of antennas of larger diameter.

The average is: antenna cost ~ 1.8 electronics

1. Autogrid to larger number
2. 1/2 sized pyramid (1/4 sized) what difference?
3. Calculate the pattern integral.
4. Wina zig-zag with the pyramid and without → patterns?
   do it with 4 arms

Front end meeting 4/13/00

Sandy on schedules (a) active balun chips by end of year (HRL)
(b) next InP HEMT from TRW

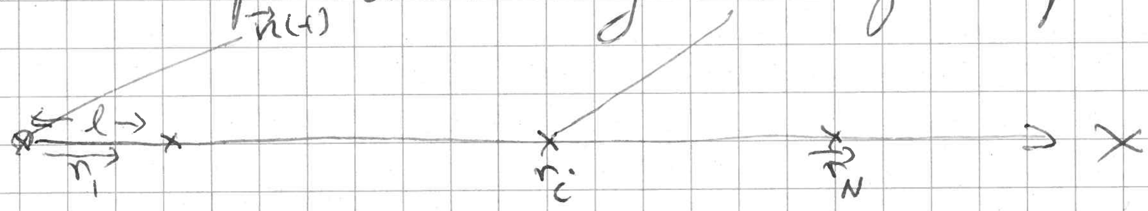
Paying for run ~100k. Start within a few weeks, then 4-6 weeks to get the chips -
2 iterations/yr. if you pay; 1/yr. from piggy back -

JPL plans. 4000 ants to 32 GHz or 8000 ants to GHz

Mike Davis - bldg keys inc. 635
check with Leo   get a computer connection

The effect of array size on filtering out narrow band interference

Use a simple linear array with equal spacing.



delays are (relative to $x=0$) $\tau_i = -\frac{\hat{n}\cdot\vec{r}_i}{c}$

$v(t)$ arrives at $x=0$
$v(t-\tau_i)$ arrives at $x=\vec{r}_i$

$$v(t) = \int_0^\infty a(\nu)\cos(2\pi t + \phi(\nu))\,d\nu$$

$$v(t-\tau_i) = \int_0^\infty a(\nu)\cos\left[2\pi(t-\tau_i) + \phi(\nu)\right]d\nu$$

Add the outputs with equal wgts.

$$O(t) = \sum_{i=1}^{N} v(t-\tau_i) = \sum_{i=1}^{N}\int_0^\infty a(\nu)\cos\left[2\pi\left(t - \frac{\hat{n}\cdot\vec{r}_i}{c}\right) + \phi(\nu)\right]d\nu$$

Heterodyne to baseband and put in delays.

Observe a source in the direction $\hat{n}_0(t)$. Heterodyne to baseband taking out the fringe and put in delays to cover a finite bandwidth. Add up all the signals with equal weights. The delays and fringe compensation are for the direction $\hat{n}_0$. Then imagine a signal (interference) coming from the direction $\hat{n}$. For ground based interference $\hat{n}$ is a fixed direction, not varying in time. For an interference signal with spectrum $a(\nu)\angle\phi(\nu)$ arriving from the direction $\hat{n}$ the output is given by the expression on page 52.

$$O(t) = \sum_{j=1}^{N}\int_0^\infty a(\nu)\cos\left\{2\pi\nu_e\left(t - \frac{\tau_m}{2}\right) + \frac{2\pi\nu}{c}\vec{r}_j\cdot(\hat{n}-\hat{n}_0) + \phi(\nu)\right\}d\nu$$

Since $\cos x = \mathrm{Re}\{e^{ix}\}$, we can write this as

$$O(t) = \sum_{j=1}^{N}\int_0^\infty a(\nu)\,\mathrm{Re}\left[e^{i\left\{2\pi\nu_e\left(t-\frac{\tau_m}{2}\right) + \frac{2\pi\nu}{c}\vec{r}_j\cdot(\hat{n}-\hat{n}_0) + \phi(\nu)\right\}}\right]d\nu$$

Now consider a single frequency of interference at $\nu = \nu_0$ so that $\nu_\ell = 0$. This is probably the worse case. Look at the effect of the natural fringe rates. $\hat{n}_0(t)$ varies in time.

let $\hat{n}_0(t) = \hat{n}_{00} + \dot{\hat{n}}_0 t$ $\qquad a(\nu) = \delta(\nu - \nu_0) \quad (\phi = 0)$

Also, $\vec{r}_j = \hat{a}_x x_j = \hat{a}_x j\ell = \hat{a}_2 j \ell$

ℓ is the array spacing. Make it a uniform E-W array.

$$\frac{2\pi\nu}{c}\vec{r}_j \cdot (\hat{n} - \hat{n}_0) = \frac{2\pi}{\lambda}\hat{a}_2 j \ell \cdot (\hat{n} - \hat{n}_{00} + \dot{\hat{n}}_0 t)$$

$$= \frac{2\pi\ell}{\lambda} j \hat{a}_2 \cdot (\hat{n} - \hat{n}_{00}) + \frac{2\pi}{\lambda} j \ell \hat{a}_2 \cdot \dot{\hat{n}}_0 t \qquad \dot{\hat{n}}_0(t) = \frac{\partial}{\partial t}\hat{n}_0(t)$$

The first term is a constant in time with $|\hat{a}_2 \cdot (\hat{n} - \hat{n}_{00})| \cong 1$.

$$e^{i\left\{\frac{2\pi}{\lambda}\ell j \hat{a}_2 \cdot (\hat{n} - \hat{n}_{00}) + \frac{2\pi\ell}{\lambda} j \hat{a}_2 \cdot \dot{\hat{n}}_0 t\right\}} = \left[e^{i\{\quad\}}\right]^j = \left[e^{i(a+bt)}\right]^j$$

j is a factor.

Summing up the voltages:

$$\sum_{j=1}^{N}\left[e^{i(a+b(t))}\right]^j = \frac{1 - e^{iN(a+bt)}}{1 - e^{i(a+bt)}} = \frac{e^{i\frac{N}{2}(a+bt)}}{e^{i\frac{(a+bt)}{2}}}\left[\frac{e^{-i\frac{N}{2}(a+bt)} - e^{i\frac{N}{2}(\;)}}{e^{-i\frac{(a+bt)}{2}} - e^{i\frac{(a+bt)}{2}}}\right]$$

$$= e^{i\frac{(N-1)}{2}(a+bt)} \frac{\sin\frac{N}{2}(a+bt)}{\sin\frac{(a+bt)}{2}}$$

$$a(t) = \mathrm{Re}\left\{\quad\right\} = \cos\left(\frac{N-1}{2}\right)(a+bt)\frac{\sin\frac{N}{2}(a+bt)}{\sin\left(\frac{a+bt}{2}\right)} \cong \frac{\cos\frac{N}{2}(a+bt)\sin\frac{N}{2}(a+bt)}{\sin\left(\frac{a+bt}{2}\right)}$$

$$a(t) = \frac{1}{2}\frac{\sin N(a+bt)}{\sin(a+bt)/2} \qquad \sin 2x = 2\sin x\cos x$$

Assume an East-West linear array with spacings ℓ.

$$\hat{n}_0(t)\cdot\hat{a}_2 = \hat{a}_2 \cdot \left[\hat{a}_3 \sin\delta + \cos\delta\left(\hat{a}_1\cos h - \hat{a}_2 \sin h\right)\right] = -\cos\delta_0 \sin h_0$$

$$t\frac{\partial}{\partial t}\hat{n}_0(t) = -\cos\delta\cos h\,\dot{h}\,t$$

$$bt = \frac{2\pi\nu l}{c} \cos\delta \cosh \dot{h} t = \frac{2\pi l}{\lambda} \cos\delta \cosh \dot{h} t$$

For h in radians $\dot{h} = 2\pi/24\,3600 = 7.3\times10^{-5}$

Take $\lambda = 0.2$m then (with $l = 7.5$m)

$$b = \frac{2\pi(7.5)}{0.2} \cos\delta \cosh \; 7.3\times10^{-5} = .0172 \cos\delta \cosh$$

$$a = \frac{2\pi\, 7.5}{0.2} \cos_0 \cosh_0 \Rightarrow 236(\cos\delta_0 \cosh_0 - \cos\delta \cosh)$$

$$a + bt = 236(\cos\delta_0 \cosh_0 - \cos\delta \cosh) + .0172 \cos\delta_0 \cosh_0 t$$

A .01 Hz filter is equivalent to $\frac{1}{T}\int_0^T f(t)\, t$

where $T =$ 100 secs.

$$\frac{1}{T}\int_0^T \frac{1}{2}\frac{\sin N(a+bt)}{\sin(a+bt)/2}\, dt \cong \frac{1}{T \sin a}\,\frac{1}{2}\int_0^T \sin N(a+bt)\,dt$$

$$= \frac{1/2}{T \sin a\, Nb} \cos N(a+b\;)\Big|_0^T = \frac{\cos Na - \cos N(a+bT)}{2\, TNb \sin}$$

$$\le \frac{1}{\sin a\; TNb}$$

$$TNb = N\frac{2\pi l}{\lambda} = 2\pi\left(\frac{Nl}{\lambda}\right) T \cos\delta \cosh_0$$

For a 2D array of 500 antennas, it's E-W extent is ~23 ants,

Take $l = 7.5$m, $\lambda = 0.2$ m, $T = 100$

Then $2\pi\left(\frac{Nl}{\lambda}\right)T = 2\pi(23)\frac{(7.5)}{0.2}100 = 540$; $(540)^{-1} = \sim 55$db

If $Nl \rightarrow 10Nl$, the signal is reduced another 20db

Another option is to leave $\nu_e \neq 0$.
In this case the output is:

$$O(t) = \int_0^\infty a(\nu) \cos\left\{ 2\pi \nu_e \left(t - \frac{\tau_m}{2}\right) + \left(\frac{N-1}{2}\right)(a+bt) + \phi(\nu) \right\} \frac{\sin \frac{N}{2}(a+bt)}{\sin\left(\frac{a+bt}{2}\right)} d\nu$$

Again look at a single interfering signal, but now let $2\pi\nu_e t$ cancel $\left(\frac{N-1}{2}\right) bt$, a diff. channel. Again $\phi(\nu) = 0$.

Then $$O(t) = \cos\left\{ 2\pi\nu_e \left(-\frac{\tau_m}{2}\right) + \left(\frac{N-1}{2}\right) a \right\} \frac{\sin \frac{N}{2}(a+bt)}{\sin(a+bt)}$$

$$O(t) \propto \frac{\sin \frac{N}{2}(a+bt)}{\sin(a+bt)}$$

In this channel, $N \to N/2$, half the frequency for the envelope

Once again average over $(0, T)$

$$\frac{1}{T}\int_0^T \frac{\sin \frac{N}{2}(a+bt)}{\sin(a+bt)/2} dt \cong \frac{1}{T \sin a} \int_0^T \sin \frac{N}{2}(a+bt)\, dt$$

$$= \frac{1}{T\sin a} \left[- \frac{\cos \frac{N}{2}(a+bt)}{\frac{N}{2} b} \right]_0^T = \frac{2}{TNb \sin a}\left[\cos \frac{N}{2} a - \cos \frac{N}{2}(a+bT) \right]$$

$$\leq \frac{2 \cdot 3}{TNb} = \underline{\underline{\frac{4}{TNb}}}$$

This assumes $\langle \sin Na \rangle \sim \langle \cos NA \rangle$ and $\langle O \rangle \sim \frac{1}{2}$ peak.

For the numbers chosen above, $\frac{4}{TNb} = \frac{4}{540} = .0074$
-42 db

$N\ell = 23 \times 7.5$ m $= 172$ m for the East-West extent

At $t = 1$ sec. $.74 \to -2.6$ db not much

For $N\ell = 1720$ m, get a factor of 20db more:

-62 db or -22.6 db.

1HT Board 5/12/00 at SETI Inst.

JD : personel – [ad. for computer programmer – to be located at SETI
[15 good candidates (top 1/4)
inst. post-doc (RAL) 2 cands. decision iminent
[note also Bolatto]
Dave de Boer – JD wants to high him.
Douglas Bock – 1HT system scientist

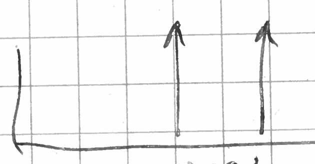
Funding status – Susan Pearson Brown etc.

JW RPA ~~meeting~~ report

F/O contract (JW) with place in Boston
MIMIC design with NFRA (JW)

JW get together advisory lists – a scientific advisory committee
Mike Davis – get younger pple –

Delay decorrelation of wide band signals e.g.
~~the~~ spread spectrum, like GPS:



$\Delta\nu \sim 10 MHz - 10^7 Hz$

10 MHz, with switched keying

Correlation time $\Delta t \sim \frac{1}{\Delta\nu} = 10^{-7}$ sec. = 100 ps.
In terms of distance 100 ps → 100 feet.

a) cross correlation (interferometry no correlation at distances ...

b) beam forming. The outputs at each port remain but are uncorrelated. The sum voltage goes from
$N\nu\ t \rightarrow \sqrt{N}\ \nu_i(t)$ same help.

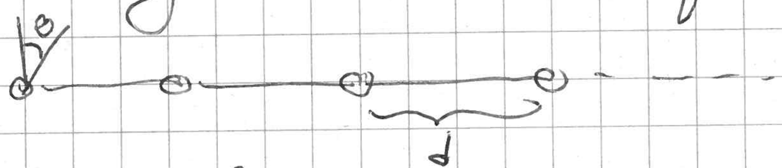
iAT Wed. meeting 6/14/00

Jack get the PDR date set. JD has Sept. 7 now -
phone in this room?

Possibility of forming nulls on satellites

1. Are their positions well enough known?
2. Can we measure the antenna positions well enough?

Model the array as linear with N equi-spaced elements.



$Nd \cong 2\text{km}$, the length of the array.

The voltage pattern is $V = \frac{1}{N} \frac{\sin\left[\frac{2\pi d}{\lambda} N \sin\theta\right]}{\sin\left[\frac{2\pi d}{\lambda}\sin\theta\right]}$ $\to 1$ at $\theta = 0$

The interesting part of the beam is near $\theta \cong 0$, so $\sin\theta \cong \theta$

The first null occurs when $2\pi N d/\lambda \; \theta_n = \pi$, i.e. $\frac{2\pi d}{\lambda}\theta_n = \frac{\pi}{N}$

Find the 30 db width of the null. Let ϵ = -30db half width.

$$\frac{1}{N} \frac{\sin\left[\frac{2\pi d N}{\lambda}(\theta_n + \epsilon)\right]}{\sin\left[\frac{2\pi d}{\lambda}(\theta_n + \epsilon)\right]} = .03 \quad (-30\,db)$$

The denominator varies slowly for large N; put $\theta_n + \epsilon = \theta_n$ there.

Then $N \sin\left[\frac{2\pi d}{\lambda}\theta_n\right] = N\sin(\pi/N) \cong N \cdot \pi/N = \pi$ for large N.

$\frac{1}{\pi}\sin\left[\frac{2\pi d N}{\lambda}(\theta_n + \epsilon)\right] = .03$, $\sin\left[\pi + \frac{2\pi N d}{\lambda}\epsilon\right) = .03\pi = .0942$

$$\sin\pi \cos\left(\frac{2\pi N d}{\lambda}\epsilon\right) + \cos\pi \sin\left(\frac{2\pi N d}{\lambda}\epsilon\right) = -\sin\left(\frac{2\pi N d}{\lambda}\epsilon\right)$$

$$-\sin\left(\frac{2\pi N d}{\lambda}\epsilon\right) = .0942$$

$$-\frac{2\pi N d \epsilon}{\lambda} \cong .0942 \quad \text{and} \quad \epsilon = -\frac{(.0942)\lambda}{2\pi N d},$$

At $\lambda \cong 20$ cm (GPS) and $Nd \cong 2000$ m

$$\epsilon = -\frac{(.0942)(.20)}{2\pi\,(2000)} = -1.5 \times 10^{-6} \text{ rad}; \text{ or } -0.''30$$

That's the half null width. The full width is $0''.6$

GPS: The orbit is $\sim 20,000$ km, about half the geo-synch orbit (42,700 km).

The earth radius is 6378 km, so the GPS satellites are $\sim 14,000$ km up.

With SA turned off, the RSS satellite position error is 5.8 m

The angular error is $5.8\text{m}/14,000,000 = 4\times10^{-7}$ rad $\rightarrow .09''$ ($\sim 0''.1$)

This is substantially less than the 30db null width of $0''.6$

The beamwidth is $\sim \frac{\lambda}{2 V_b} = \frac{.20}{2\,(2000)} = .00005 \Rightarrow \underline{10''}$

With the illumination tapered, both the beam width and the null width will increase, by a factor of 2-3. Hence the null width will be on the order of $\underline{1''}$. This leaves considerable margin for the GPS satellites.

For LEO satellites, more linear accuracy may be required. For the same 5.8 m error 1'' corresponds to a height of about 1200 m above the ground. That's an orbital radius of 1200 + 6400 = 7600 km.

orbital: $a \propto p^{2/3}$ or $p \propto a^{3/2}$ so $p_{LEO} = 24\text{hours}\left(\frac{7600}{42,000}\right)^{3/2} = \underline{1.8\text{hours}}$

That may be the limit. Or maybe better linear accuracy may be obtained at the lower orbit.

The $0''.6$ error is about 1/20 of a beam. ~~He~~ Measuring to this accuracy should not be difficult.

Geosynch. Orbit. $a^3 = \frac{p^2 MG}{(2\pi)^2}$ $a = \left[\frac{p^2 MG}{(2\pi)^2}\right]^{1/3}$

$p = 1$ day $= 86,400$ seconds, $M = M_e = 5.976\times10^{27}$ gms $G = 6.67\times10^{-8}$

$$a = \left[\frac{(86,400)^2(5.976\times10^{27})(6.67\times10^{-8})}{(2\pi)^2}\right]^{1/3} = 4.2\times10^{9}\text{ cm} = 4.2\times10^{4}\text{ km}.$$

$r_e = 6378$ km

What precision in the array construction is required to achieve nulls that are as deep as -30 db?

Beckmann and Spizzichino ~~give a form~~ ("The Scattering of Electromagnetic Waves from Rough Surfaces", Macmillan 1963) give a formula for the reflection from a rough mirror which we can adapt to this question.

$$|\psi(\theta)|^2 = |\psi_0(\theta)|^2 e^{-g} + g\left(\frac{\pi T^2}{\lambda^2}\right) e^{-\left(\frac{\pi T\theta}{\lambda}\right)^2}$$

$g = \left(\frac{4\pi\sigma}{\lambda}\right)^2$ where σ is the surface RMS, "T" is the correlation length of the roughness in the surface. We should replace 2σ by σ since there is no reflection here. σ will be the uncertainties in our antenna heights (positions). Also, $T \sim d$ the typical horizontal antenna separation. The first term is the unperturbed beam pattern. The second is the scattering. At the nulls, only the second term remains, and we can compare its strengths to the gain peak.

Let $T = 7.5$ m $= d$, and $\lambda = 0.2$ m

For $\sigma/\lambda = 1/100$ $(2\pi\sigma/\lambda)^2$ $.0039$, $e^{-.0039} \cong 1$

At the first null $\theta_n = \frac{\lambda}{2Nd} = 5\times10^{-5}$ rad.

$\frac{\pi T}{\lambda} = \frac{\pi\, 7.5}{0.2} = 118$ $\quad e^{-\left(\frac{\pi T\theta_n}{\lambda}\right)^2} = e^{-(118\cdot 5\times10^{-5})^2} \cong 1$

The scattered width is large compared with the 1st null location.

$$g\left(\frac{\pi T^2}{\lambda^2}\right) = (.0039)\frac{1}{\pi}\left(\frac{T}{\lambda}\right)^2 = \frac{(.0039)}{\pi}(118)^2 = 17$$

For the relative strength of the null consider 2 cases (a) the array only. In this case $(\psi_0)^2 = G_a$ N

Then with $N = 500$, the depth of the null is

$\frac{17}{500} = .0340$ $\quad \underline{-14.6 \text{ db}}$ not great!

(b) The array, including the dishes.

In this case, the peak gain is $G_o N$, where G_o is the individual antenna gain.

$$G_o = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi(0.\cancel{65})\,\pi/4\,(5)^2}{(.2)^2} = 3700 \quad , +35\,db.$$

$$G_o N = (500)(3700) = 1.85 \times 10^6 \qquad \underline{62.6\,db}$$

Now with $\sigma/\lambda = 1/100$ and $T = 7.5\,m$, the null depth is $17/1.85\times10^6 = 9.2\times10^{-6}$, very low. ($\sim 50\,db$)

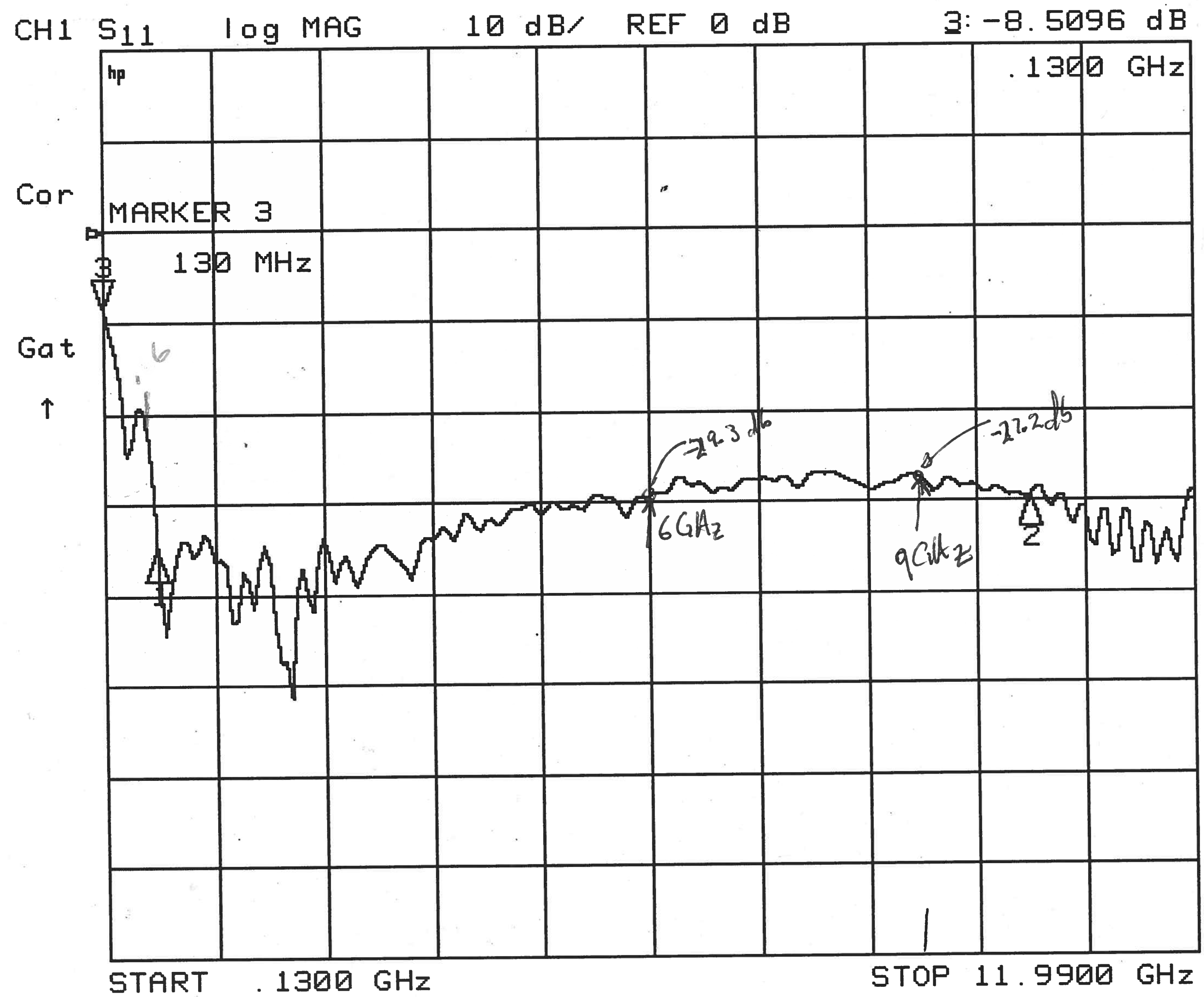
For $\sigma/\lambda = 1/10$, it's $9.2\times10^{-4} \simeq 10^{-3}$ $-\cancel{30\,db}$ $-25\,db$ i.e. e^{-5}

This seems reasonable. The phase error corresponding to $\sigma/\lambda = 10$ is $36°$.

For $\sigma/\lambda \simeq 10$ @ $\lambda 3\,cm$, $\sigma \sim 3\,mm$.

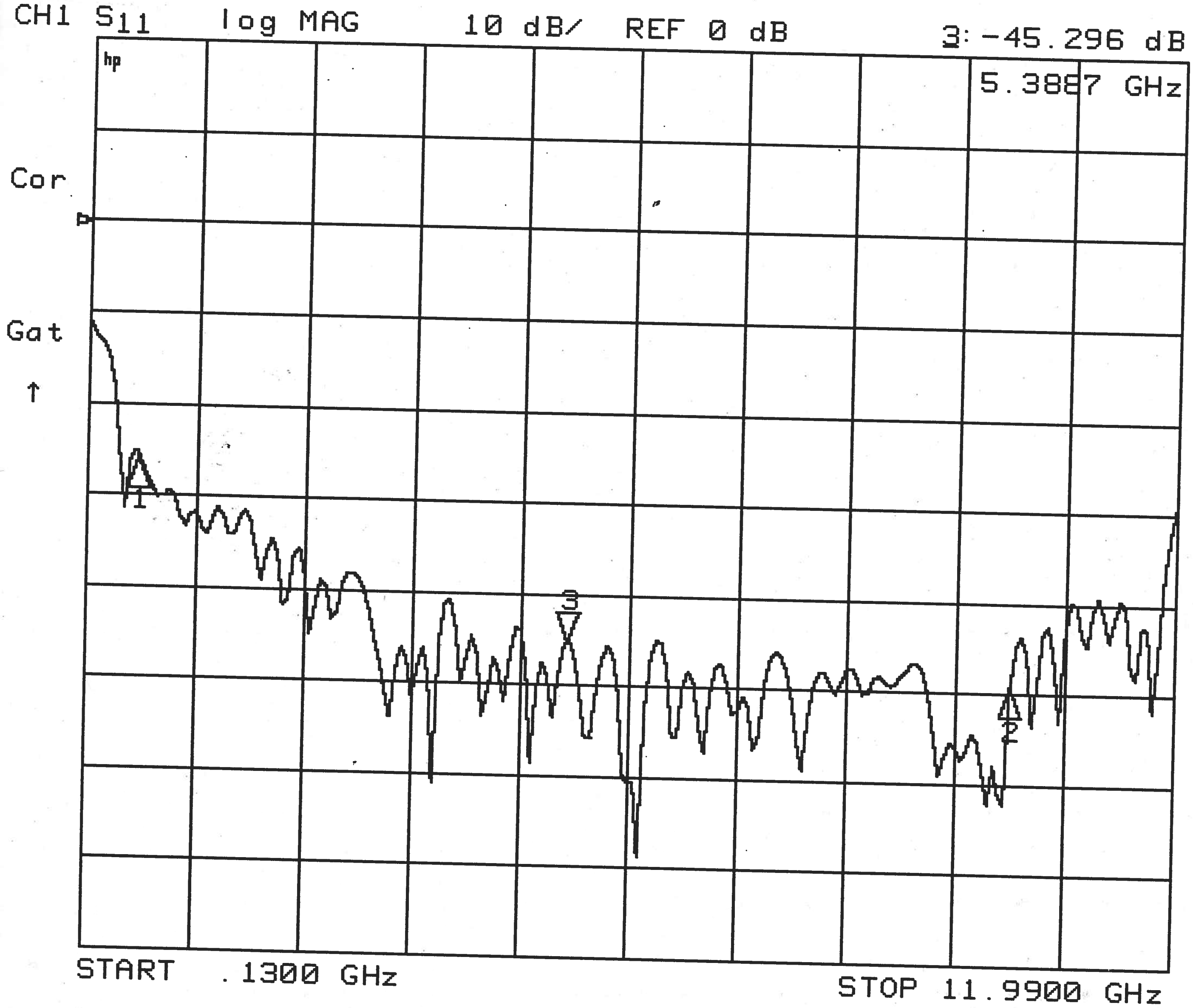
For $\sigma/\lambda = 1/30$ g →

Tests of the passive wide-band balun for the feed.



Plot 2B
Output connector gated out.
6/20/00

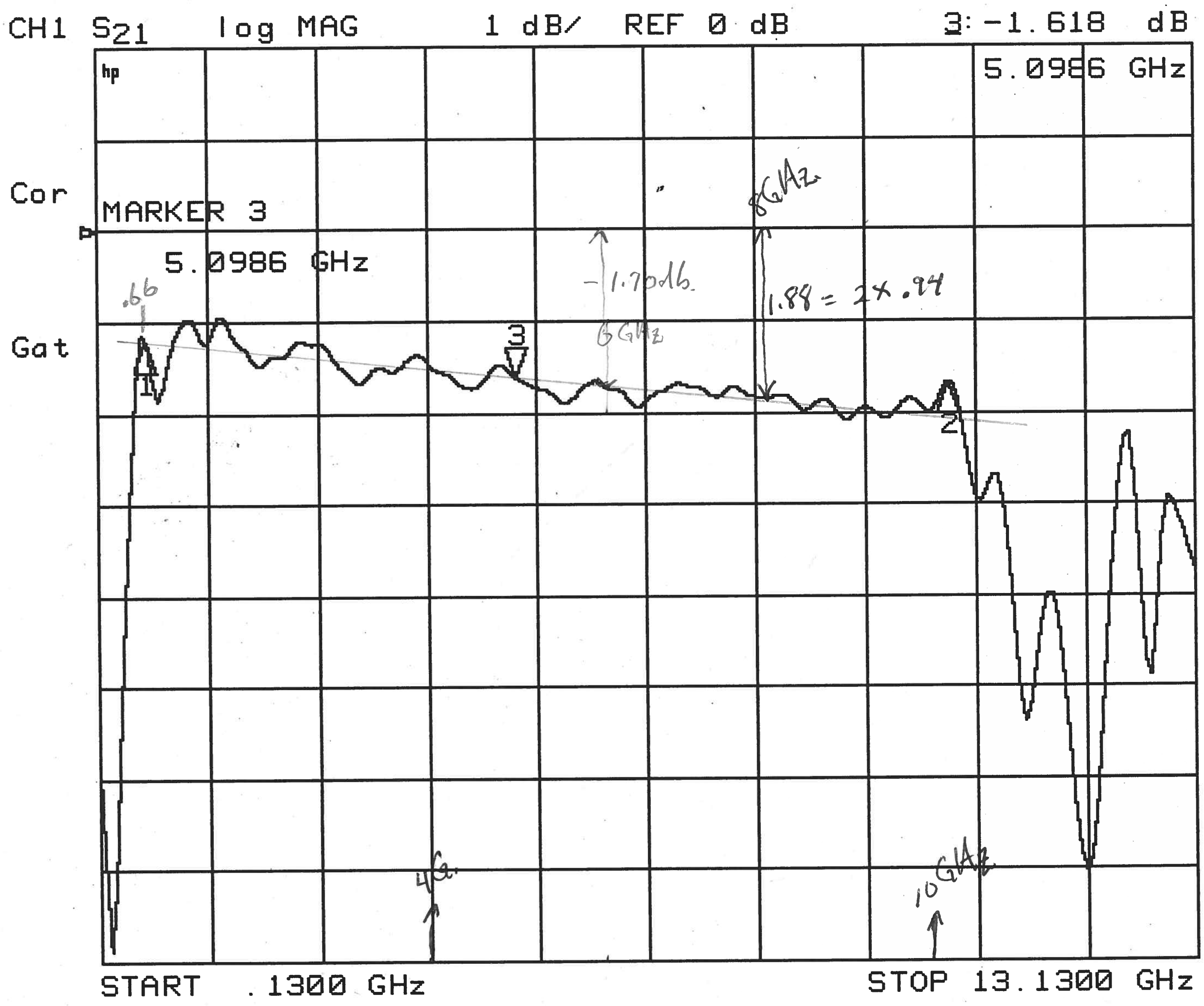
Input Match on the back-to-back balun.
The output connector is gated out.
Good internal match of the balun structure



Plot 1B

Both connectors gated out

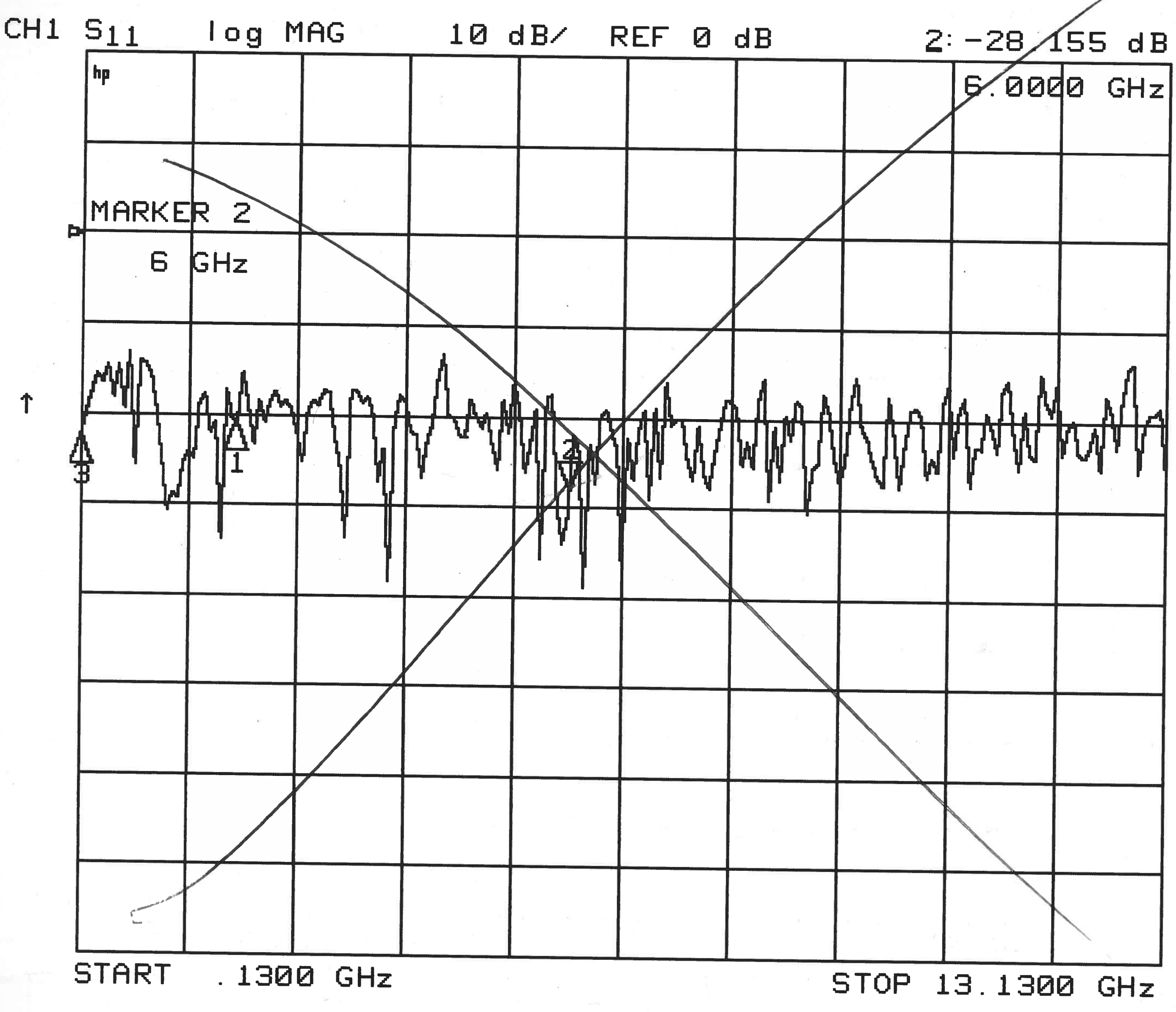
6/20/00



PLOT 2A
Output connector gated out
6/20/00

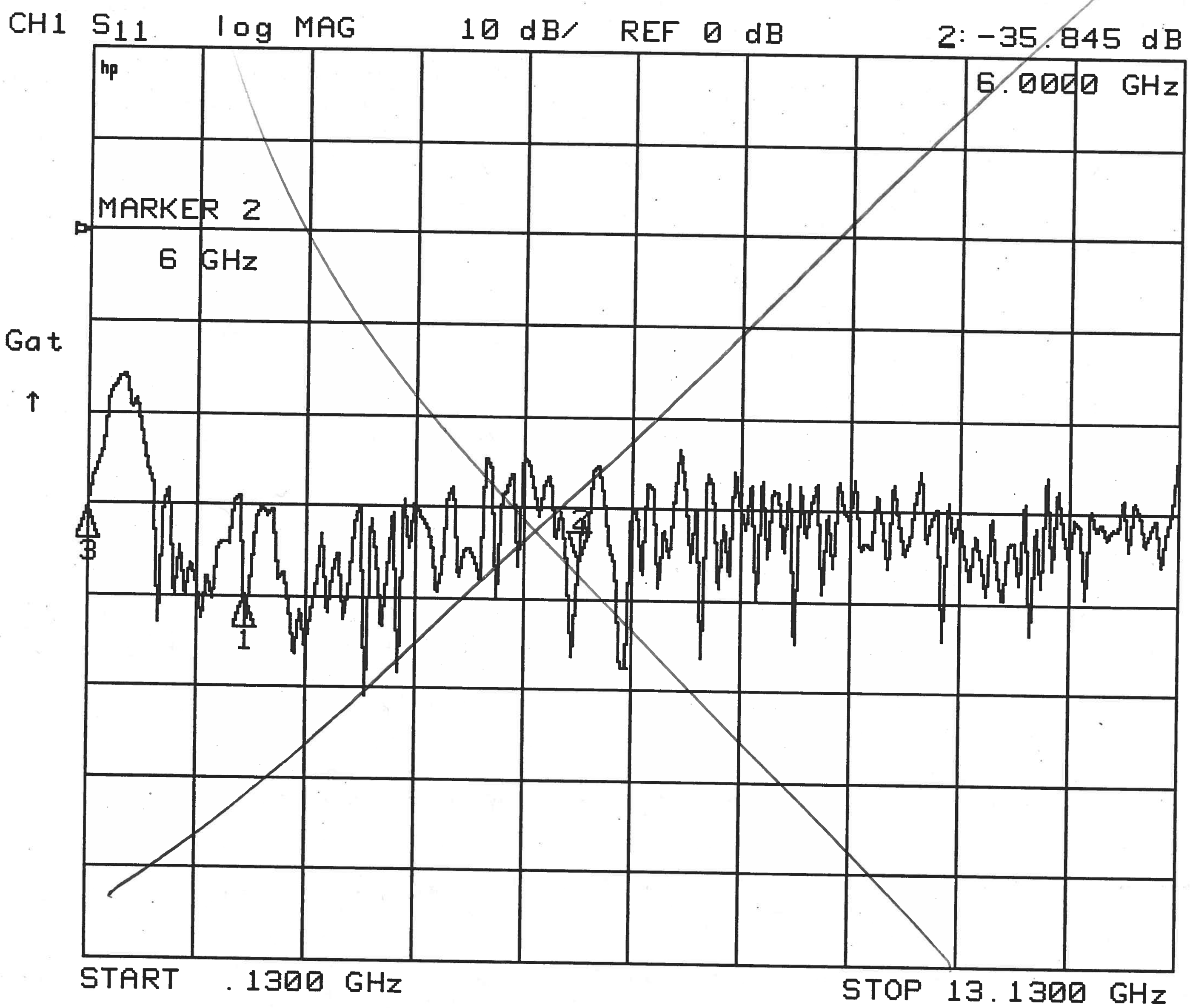
Double balun transmission output connector gated out. One way loss at 3GHz is 0.8 db

Zig-zag feed Tests



Input match to the balun connected to the feed.

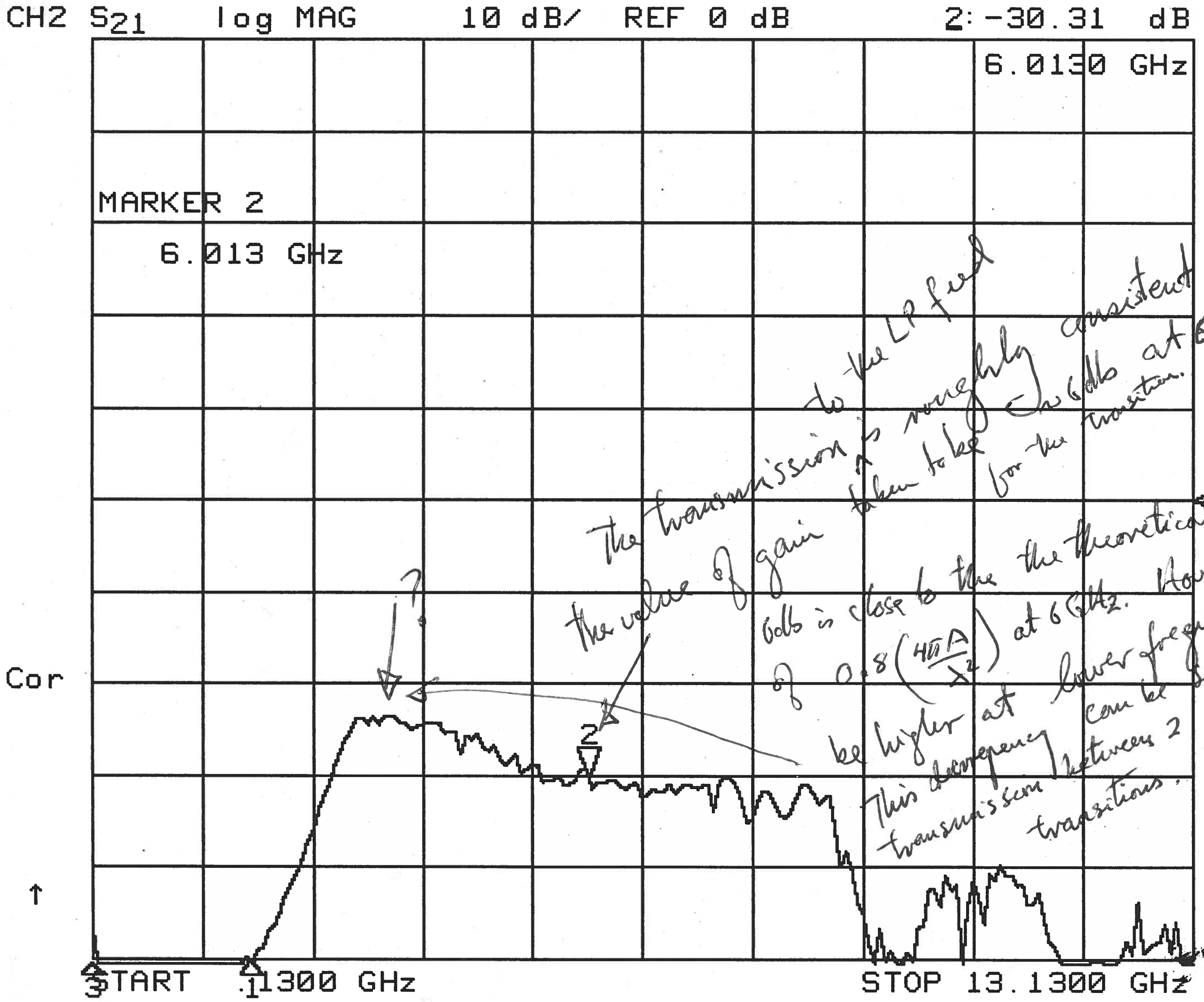
no, cal ~~off~~ here



Input match. The vertex connection and other reflections in front of it are gated out. This is the pure antenna match.

no

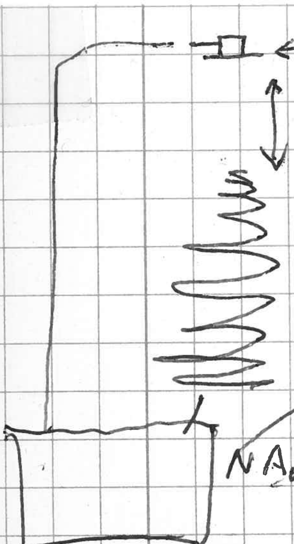
call off here.



?

The transmission is roughly consistent with to the LP feed
the value of gain taken to be ~6db at 6GHz (5cm)
for the transition.

6db is close to the theoretical gain of $0.8\left(\frac{4\pi A}{\lambda^2}\right)$ at 6GHz. How can it be higher at lower frequencies?
This divergency can be seen in the transmission between 2 4-8 GHz transitions.



← 4-8 GHz waveguide to coax transition

With 12db for LP feed the predicted transmission at 6GHz is about -30db as observed.

Repeat meas. showed -29.5db

N Anel.

First Tests of the first prototype Zig-Zag Feed

1 The test antenna is one of the waveguide/coaxial transitio The coaxial connection is Type N The waveguide R 87 [1 X /i]. 1 T, cutoff is 3.15 GHz and nominal opera ing range is GHz 5 85 GHz

The theoretical gain of this antenna is $\left(\frac{4\pi}{\,^{2}}\right) 0.81$

[from Balanis' Textbook]

At 6 GHz (λ 5 cm . $\frac{4\pi\,(4.76\,cm)(2.22\,cm)\,(.81)}{5^2}$ 4.30 .3 db)

(a) The loss is found from connecting two transitions back to back and measuring the loss. md −.44 db at 6 GHz. That's .22 db for each. The mismatch of the better transition is −17.2 db. This mismatch loss is −.08 db, for an overall of $\underline{-.30\,db}$.

(b) Measured gain from transmission between the two transitions at different separations at 6 GHz, λ 5 cm.

$$t = g_1 g_2 \left(\frac{\lambda}{4\pi R}\right)^2 \qquad \text{For identical antennas, in dbs.}$$

$$t_{db} = 2G + 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) - \text{losses}\,; \quad G = \frac{1}{2}\left\{t_{db} - 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) + \text{losses}\right\}$$

From above, the losses for the two transition should be $\underline{0.6\,db}$

Separation	$20\log_{10}\left(\frac{\lambda}{4\pi R}\right)$	t	G (no losses)
22.5 (57.15 cm)	−43.14	−30.7	6.2
22.0	−42.95	−31.3	5.8
23.0	−43.3	−30.3	6.5
23.5	−43.5	−30.6	6.5
21.5	−42.7	−30.8	6.0
21.0	−42.5	−30.4	6.1
24.0	−43.7	−31.5	6.1
24.5	−43.9	−31.3	6.3
20.5	−42.3	−30.7	5.8
20.0	−42.1	−30.7	5.7
19.5	−41.9	−30.6	5.7
19.0	−41.7	−30.5	5.6
18.5	−41.4	−29.1	6.2

average = 6.0 , with loss correction $\boxed{6.3}$

remarkable agreement

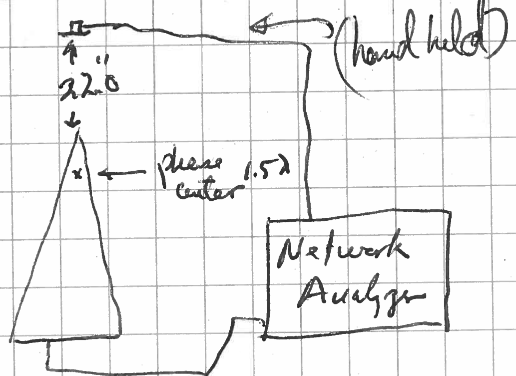
2. Loss in the balun at 6 GHz (p 112): $\frac{1.70}{2}$ = .85 db

3. Semi-rigid coax loss at 6 GHz. Two extra pieces were measured ⓐ .75 db and ⓑ .95 db. Average: .85 db

4. Overall input match is −25 db at 6 GHz roughly the lowest reflection over the band: −.01 db correction, negligible

5. Calculated gain: 11.5 db

6. Transmission measurement



a) near zone for test antenna:

$2D^2/\lambda = 2\frac{(4.76\text{ cm})^2}{5} = 9\text{ cm}$ (3.5")

near zone for feed: at ~12 db gain, $D_{eff} \approx 10$ cm.

$2\frac{(10)^2}{5} = 40\text{ cm} \rightarrow 16"$, close to 22"

Measured transmission: −28.7 db

Estimated: distance	$22.0 \times 2.54 + 1.5 \times 5 = 63.4\text{ cm}$; $20\log_{10}\left(\frac{\lambda}{4\pi R}\right) =$	−44.0 db
coax		−0.85
balun		−0.85
loss in test ant.		−0.30
gain of test an		6.3
gain of feed		11.5
		−28.2

Comments: The small microstrip losses are probably ~.1 db. $\boxed{.05}$
mm of the dielectric may lower them. $\boxed{\text{NO}}$
2' is just a little more than $2D^2/\lambda$ for the feed.

Input Transmission Losses Revisted

Copper resistivity at low temperature

T(K)	ρ ($\mu\Omega$ cm)
14.6	.00016
23.3	.00157
43.1	.0274
49.0	.04516
66.4	.1268
73.7	.1694
84.9	.2420
98.17	.333
110.6	.421
152.7	.719
188.2	.969
250.19	1.387
297.9	1.705
(273.2)	1.54
(80)	.18

An approximate interpolation formula

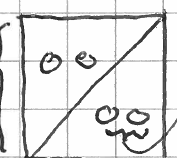
$$\rho = .67\left(\frac{T-20}{132}\right)^{1.35} 10^{-6}\ \Omega\,\text{cm}$$

Brass at 273K is 3.9 $\mu\Omega$ cm
Gold is 2.4 $\mu\Omega$ cm
Copper is 1.54

@ 49.0K ρ (formula) = .087, not .045

1. A section of Cu line at a constant 80K in the dewar.
Assume $Z_0 = 200\,\Omega$ — o s o d
from p. 12 s/d = ~~2.73~~ 2.55 for $Z_0 = 200\,\Omega$.

$\boxed{\text{For } Z_0 = 220,\ s/d = 3.2}$

4mm  1.4mm = s (34 mils) $\boxed{1.81\text{mm}}$ $\boxed{32}$, then d = 0.55mm = ~~12.5 mils~~ 14 mils(?)

$\boxed{\text{For } 220\,\Omega\text{, use } s = 32,\ d = 10,\ s/D = 3.2}$ .0254 cm.

from p. 31 $R = \frac{2R_s}{\pi d}\frac{s/}{\sqrt{(s/d)^2-1}}$ and $\alpha_p = 2\alpha_D = \frac{R}{Z_0}$

$\boxed{\text{Twin Line impedance : } Z_0 = 120\cosh^{-1}(s/ \text{, or } s/d = \cos\left(\frac{Z_0}{120}\right)}$

At 273 $\rho(\text{cu}) = 1.54\times10^{-6}\,\Omega\,\text{cm} = 1.54\times10^{-8}$ m m (at D.C.)
$= 1.72\times10^{-8}$ ohm m + 100 MHz
s The above table by this factor of 1.12

$$R_s = \sqrt{\frac{\omega\mu}{2\sigma}} = \sqrt{\frac{\omega\mu\rho}{2}} = \sqrt{f\mu\pi\rho} = \sqrt{\pi(4\pi\times10^{-7})f\rho} = 2.0\sqrt{f\rho}$$

henry/m

Must use ρ in ohm-m

$R_s(273) = .0020\sqrt{1.72\times10^{-8}}\sqrt{f} = 2.62\times10^{-7}\sqrt{f} = .0262\sqrt{f/10}$

$R_s(80) = .0020\sqrt{f}\sqrt{(.18)(1.12)\times10^{-8}} = 8.48\times10^{-8}\sqrt{f}$ @ 80K

$$\alpha_p(80) = \frac{2R_s(80)}{\pi d Z_0}\frac{(S/d)}{\sqrt{(S/d)^2-1}} = \frac{2}{\pi(.055cm)200}\frac{2.55}{\sqrt{(2.55)^2-1}}R_s = .063\,R_s(80)$$

$\boxed{=0.1199\,R_s \text{ for } 220\Omega}$

$= (.063)(8.98\times10^{-8})\sqrt{f} = 5.65\times10^{-9}\sqrt{f} \rightarrow .0004\ cm^{-1}$ @ 5GHz

$\boxed{1.077\times10^{-8}\sqrt{f}}$

$\alpha_p(80) = .0004\ cm^{-1}$ at 80K $\boxed{.0011\sqrt{f/10} \text{ for } 220\Omega}$ at 5GHz

Transmission line emission $\Delta T_B = T_0\,\alpha_p(80)\ cm^{-1}$

$\Delta T_B = 0.032\ °K/cm.$ $\boxed{=.086\sqrt{f/10}\ °K/cm}$

~~L = 30~~ $\rightarrow$ 1K $L = 12 \rightarrow 1K$

.36°K/cm at 300K

2. Emission from the transmission line with a temperature gradient. Assume it's linear from 300 to 80 over L.

$T(z) = 300 - \frac{z}{L}(220)$ (Modify this?)

The (scaled by 1.12) interpolation for $\rho(T)$ is

$\rho(T)\quad 75\times10^{-8}\left(\frac{T-20}{32}\right)^{1.35}$ ohm m ~~[illegible]~~?

$R_s = .0020\sqrt{f\rho} = 1.73\times10^{-}\left(\frac{T\ 20}{1}\right)^{.67}\sqrt{f}$

$\alpha_p = .063$ $\boxed{0.12}$ $= 1.73\times 0^{-7}\left(\frac{T}{2}\right)^{?}\sqrt{5GHz}\,(\frac{.06}{.12}) = .07\left(\frac{-20}{12}\right)^{67}$ at 5 z z

$\boxed{.\ \left(\frac{T\ 20}{132}\right)\sqrt{f/10}}$

Emission from the line of length L (cm)

$T_B(L) = \int_0 T(z)\,\alpha_p(z)\,dz$

$\int_0\left[300-\frac{z}{L}(220)\right](.\ \boxed{.\sqrt{f/10}}\ 77)\left\{\frac{[00-\frac{z}{L}(220)]-20}{132}\right\}^{0.\ 7}dz$

use Mathematica

Linear Fit $T_B(L) = .184L$ $\boxed{.50\sqrt{f/10}\ L\ °K}$

L(cm)	$T_B(L)$
	8
2	.369
3	553
4	.738
5	922
6	1.107
7	290
8	4
9	1.660
10	84

(3) The top circuit board has ~1% absorption for 2 traces at 10GHz.

(see next page)

③ Losses in the tip end circuit board.

The material is Cu Flon $K_e = 2.1$ loss tan = .00045 @ 10 GHz

Using the Rogers software for microstrip:
board thickness = 15 mils = h
trace width = 10 mils = w
with trace thickness = 1.4 mils $Z_0 = 109\,\Omega$ $\epsilon_{eff} = 1.653$

Losses $\alpha_{diel.}$ = .397 db/m @ 10 GHz
α_{copper} = 8.068 db/m @ 10 GHz $\alpha_d + \alpha_c = 8.465$ db/m
= .0085 db/mm

The copper losses should scale as $\sqrt{f}$ and they dominate.

The trace lengths are 4.77 millimeters
4.77 x .0085 = .0404 db ½ The signal divides, then

The total loss per polarization is ~~twice this:~~ ~~.0808 db~~ .04 db

~~.0808 db ⇒ .9816 = 1/1.019~~ ~~@ 10 GHz~~.

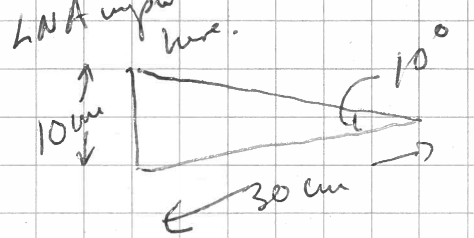
i.e. ~ ~~2~~ $\frac{8}{10}$ %

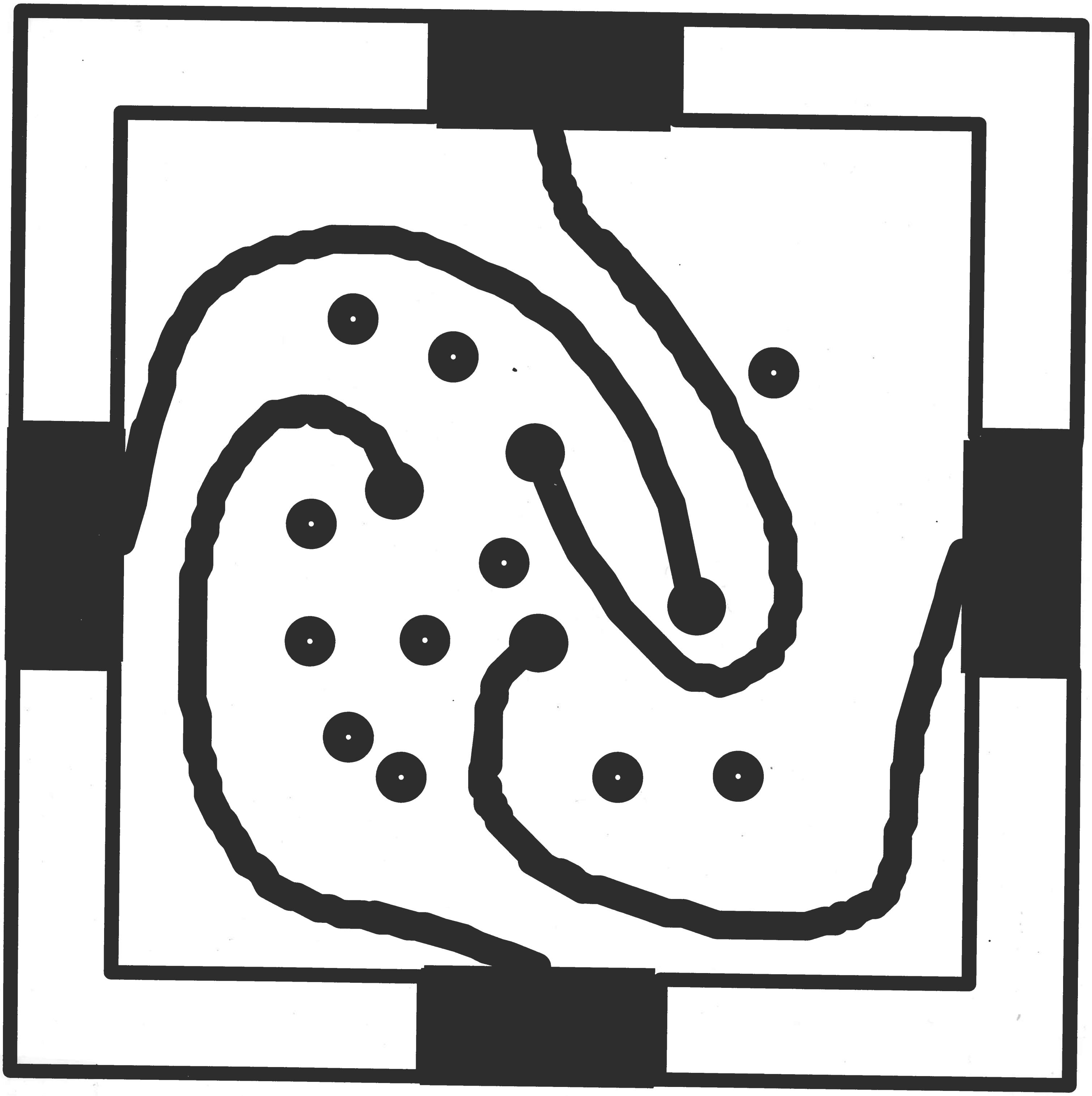
Summary of the losses in terms of noise contribution

at 10 GHz

Feed alone	3% at 300 K	9°K
Circuit Board	1% at 300 K	3°K
Transition Line (5cm)	0.5 X 5cm	°K
Cold Line (25cm)	~~25 X .03~~ 25 X .8	2.15 ~~.75K~~

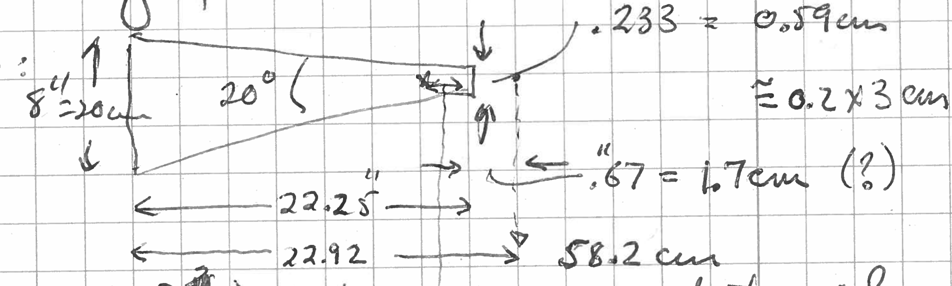
15/28 K
16.
2.15





Dimensions and Scaling for the Feed (rough)

The prototype dimensions:



$20° = .35$ rad

20 cm = $\frac{1}{2}$ 40cm λ. (750 MHz) 3cm phase center: 2.8cm from input terminals

The input match appears flat out to (at least) 13 GHz and down to 750 MHz. The input reflection rises to a peak at 400 MHz. To scale from 750 → 500 MHz requires scaling the length to
(700/800) 58.2 cm = 81.5 cm overall! (750/800) 58.2 87.3 cm

Altogether 100 cm = 1 m should offer sufficient margin (20%). The input impedance may not be as sensitive as, say, the gain.

Leave the small dimension as it is 0.5 cm (?) not .76cm!

The radiating phase center at 10 GHz is 1.5 × 3.0 cm − 4.5 cm
i.e. 4.5 − 1.7 = 2.8 cm from the input terminals, about 1 wavelength

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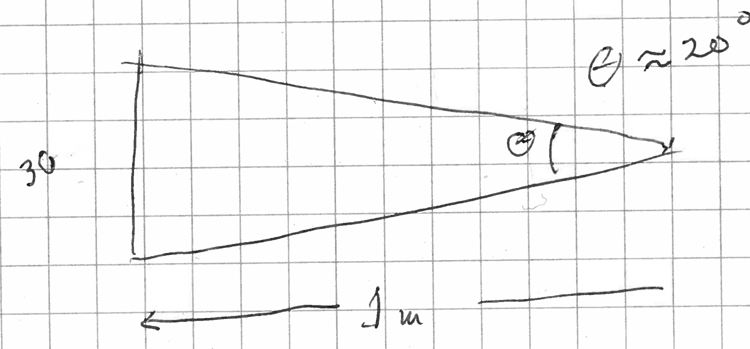
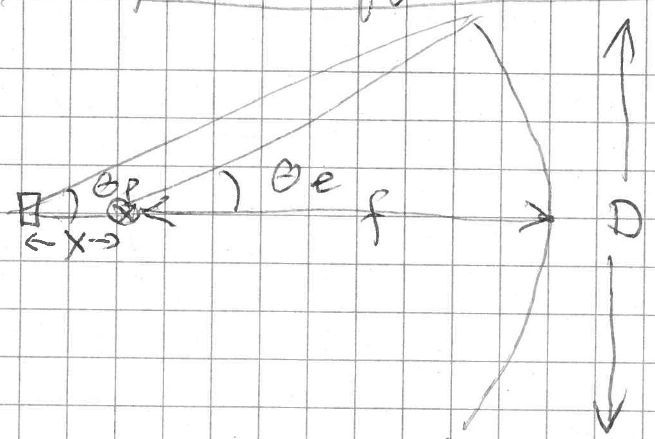


One further effect - the scaling of the phase center on spillover.



$F = f/D$

$$\cos\Theta_e = \frac{(4F)^2-1}{(4F)^2+1}$$

$D = 5m$
$f = 3.75m$

$$F' = \frac{X+f}{D}$$

$$\cos\Theta_p = \frac{\left[4\left(\frac{X}{D}+F\right)\right]^2-1}{\left[4\left(\frac{X}{D}+F\right)\right]^2+1}$$

$F = .75$

X(cm)	Θ_p	Total Fraction	spillover rel. to X=0	remaining fraction
−6	37.4°			
−9	37.7°	72.57%	−.67%	1.0067
0	36.9°	71.9%	0	1.00
9	36.06°	70.96%	1.0	.990
27	34.54°	69.51	2.4	.976
36	33.83°	68.5	3.4	.966
60	32.06	66.23	5.7	.943
81	30.66°	64.22%	7.7%	.923

For $F = 0.65$ and $\Theta_e = 42.1°$ ← at $\lambda = 4.8$ cm (6.25 GHz)

X(cm)	Θ_p	Spillover fraction
−9	43.16	
−6	42.80	.762
0	42.1	.756
9	41.03	.750
27	39.1	.736
36	38.20	.730
60	35.97	.7096
81	34.22	.690

An updated overall summary for the Optical Systems

$F = 0.65$ gives an edge illumination of -13.5 db for the higher frequencies. The ~~dr~~ illumination efficiency drops, but the edge diffraction is less. This is important at the lower frequencies.

Anderson claims to be making dishes at $D = 5m$ with $\sigma = .025''$ (.0635 cm) and $\sigma = .008''$ for 2m diameter. This requires no correction for the secondary and a small correction for the primary $e^{-\left(\frac{4\pi\sigma}{\lambda}\right)^2} = e^{-\left[\frac{.798}{\lambda(cm)}\right]^2}$ ✓

Apply this correction.

For the shorter wavelengths, $F = 0.65$ provides -13.5 db edge illumination, an illumination efficiency of .831 and low edge diffraction. There is no geometrical blocking due to the offset gregorian.

At the longer wavelengths the focus moves, increasing the edge illumination, the geometric spillover, and the edge diffraction.

A reasonable compromise for the focus puts the feed focus ($\delta = 1.4\lambda$) at the geometric focus for $\nu = 6.25$ GHz (4.80 cm λ)

For $\nu \leq 2.5$ GHz the motion of the focus requires following the edge illumination and the illumination efficiency as the ~~focal pon~~ phase center of the feed moves.

A_0 is the electric field at the reflector edge.

The diffractive loss is: $l_D = C_d \sqrt{\frac{\lambda}{d}} \sqrt{1 - \frac{d}{D}}\, A_0$

from Kildal $d = 2m$ for the secondary and $5m$ for the primary $C_d \cong 1.6$ for roughly our illumination. At $F = .65$, (see p. 76) $A_0 = .211$

$$l_D = 1.6 \sqrt{\frac{\lambda(m)}{2}} \sqrt{1 - \frac{2}{5}}\, A_0 = .056\sqrt{\lambda m} \quad \cancel{.185\sqrt{\lambda(m)}}$$

Way off!

$$\eta_c = (1 - l_D)^2 \cong 1 - 2l_D = 1 - .112\sqrt{\lambda cm} \quad \cancel{1 - .370\sqrt{\lambda(m)}}$$

For the de focus loss $X = (\Delta s/\lambda) \dfrac{17}{\left[1+16\left(\frac{.65}{.4}\right)^2\right]} = \dfrac{\Delta s}{\lambda}(.4125)$

use the curves on. p. 7 (−13db)

Select the correct focus for $\nu = 6.25$ ($\lambda = 4.8$ cm)

Follow the change in edge illumination from $\cos\Theta_p = \dfrac{\left[4\left(\frac{x}{D}+F\right)\right]^2-1}{\left[4\left(\frac{x}{D}+F\right)\right]^2+1}$ using the pattern from pp 72 − 75.

η_i	ν	λ(cm)	s (1.4λ)	$\lvert\Delta s\rvert$	$\frac{\lvert\Delta s\rvert}{\lambda}$	X	η_{defoc}	η_i	η_{spill}	η_{Zu}	Ruze $e^{-\left(\frac{4\pi\sigma}{\lambda}\right)^2}$	η_T	d=2.0 η_T	d=2.0 η_i	2.4 η_i	(2.4) η_T
.986	11.25	2.67	3.74	2.98	1.11	.458	.85	.982	.758	.831	.915	.491	.46	.94	.95	.47
.985	10.0	3.00	4.20	2.52	.84	.365	.91	.981	.757	.831	.932	.523	.502	.94	.946	.505
.984	8.75	3.43	4.80	1.92	.56	.231	.985	.979	.756	.831	.947	.574	.55	.933	.943	.55
.982	7.50	4.00	5.60	1.12	.28	.116	.99	.978	.756	.832	.961	.585	.555	.927	.938	.56
.981	6.25	4.80	6.72	0	0	0	1.00	.976	.756	.832	.973	.596	.564	.921	.932	.57
.978	5.00	6.00	8.40	1.68	.28	.116	.99	.973	.756	.833	.983	.595	.560	.911	.925	.57
.975	3.75	8.00	11.20	4.48	.56	.231	.985	.968	.753	.833	.990	.591	.550	.898	.913	.56
.970	2.50	12.00	16.80	10.08	.83	.365	.91	.961	.748	.837	.996	.546	.50	.876	.894	.51
.960	1.25	24.00	33.60	22.88	.95	.392	.89	.937	.736	.845	1.00	.518	.46	.827	.852	.47
.952	1.00	30.00	42.00	35.28	1.176	.485	.84	.925	.730	.866	1.00	.491	.43	.808	.835	.44
.944	.75	40.00	56.00	49.28	1.23	.507	.82	.906	.710	.893	1.00	.471	.41	.780	.811	.42
.932	.50	60.00	84.00	77.28	1.29	.531	.805	.870	.693	.914	1.00	.444	.36	.71	.772	.393

For a d=2.4 secondary $l_D = \frac{1.6}{\sqrt{24}}\sqrt{\lambda(\text{cm})}\sqrt{1-\frac{2.4}{5}}\,.211 = .157\sqrt{\lambda(\text{m})}$

$(1-l_D)^2$ at $\lambda = 60$ cm. 0.77

For Kildal's formulae $\phi_r = \phi_e = 42°$ $\phi_v \cong 75°$

$C_b \cong 1.6$ for −13db taper.

$C_d = \frac{1}{\pi}\dfrac{\cos^2\left(\frac{\phi_r}{2}\right)}{\sqrt{\sin\phi_v}} \cdot C_b = 0.45$; $l_d = .45\sqrt{\dfrac{\lambda}{24}}\sqrt{1-\dfrac{2.4}{5}}\,.211 = .044\sqrt{\lambda}$ −13.5db

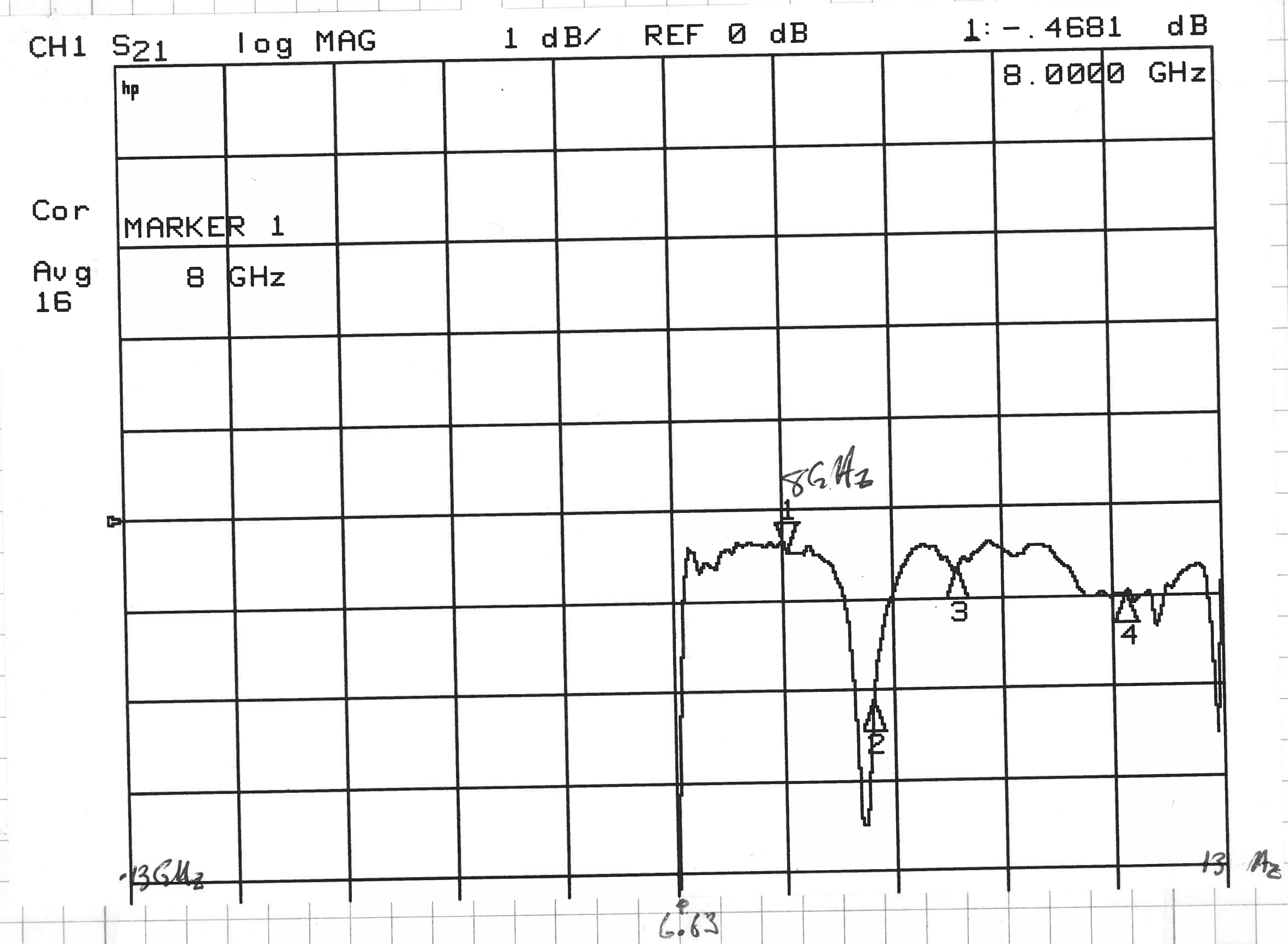
$l_d = [1-.044\sqrt{\lambda}]^2 \cong 1 - .088\sqrt{\lambda(\text{m})}$ ← meters

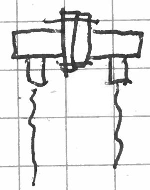
Note that A_o increases at low frequencies $l_d = .2095\sqrt{\lambda(\text{m})}\,A_o$

C_b goes down

Evaluation of the 2 x band horns and transitions

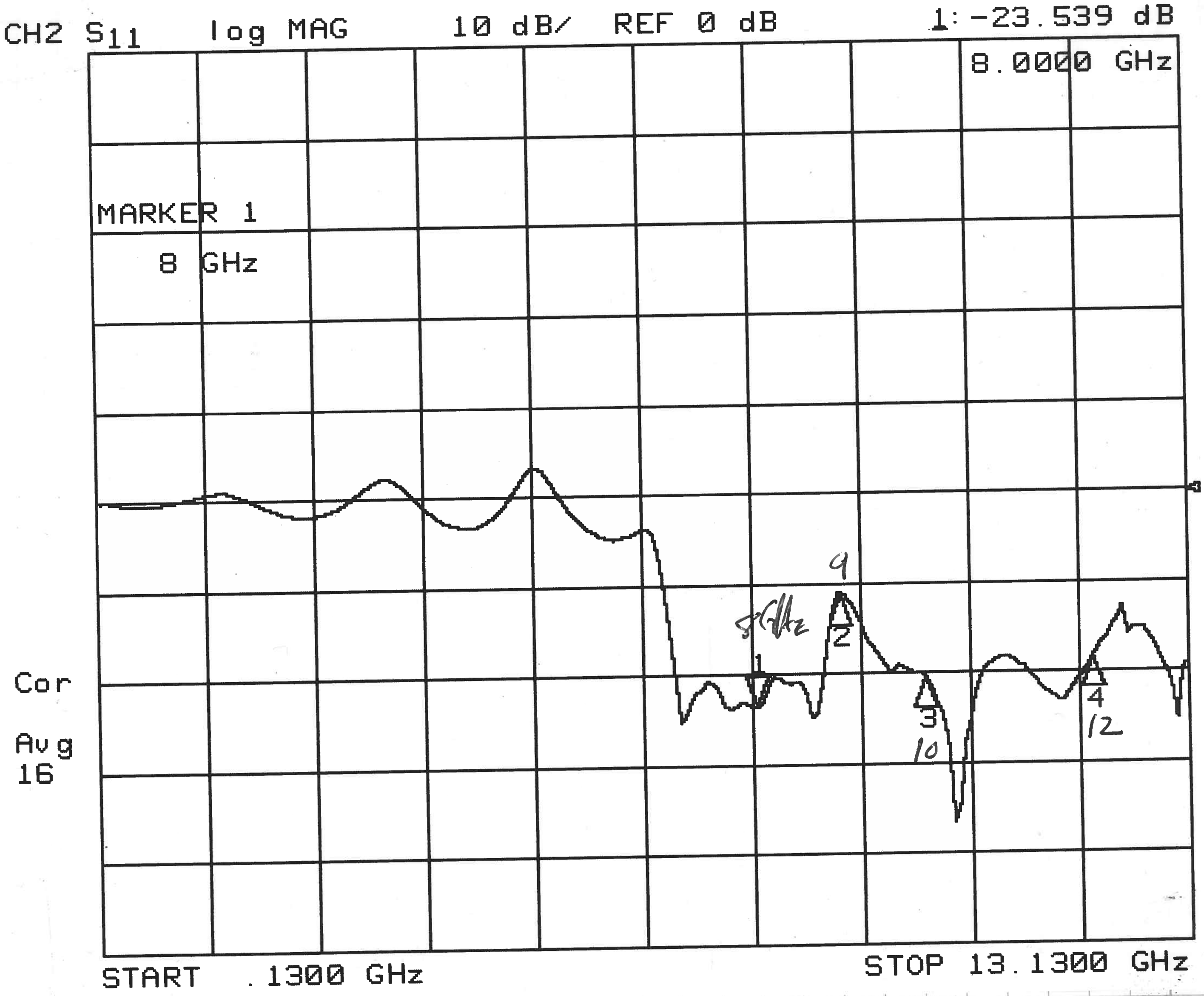
Losses in the waveguide/coax transitions. Back to back transmission. Markers are at 8, 9, 10, and 12 GHz. Below



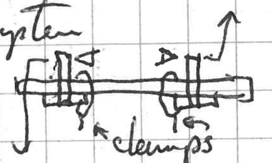


The network analyzer was used with the small heavy duty coax cables. The transmission was calibrated with a brass dble female SMA in place - then removed. The NA stability is excellent. After 24hours the transmission was sitting at .004db at GHz at reconnection.

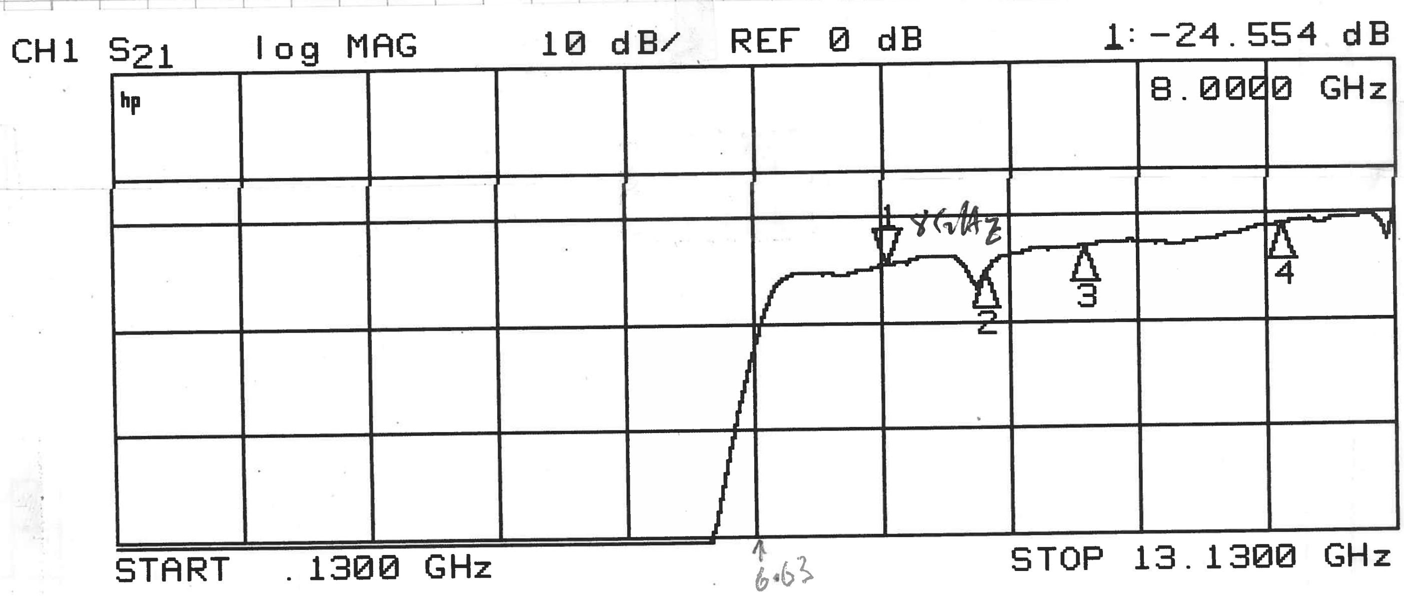
The next graph shows the " into one transition with one feed horn attached. 8GHz seems good choice for the calibration frequency.



Transmission between two horns is measured with the NA. with a wood sliding system



Everything measured carefully and cut square. One clamp can be loosened to allow one horn to be slide along the bar. Distance between horn apertures measured with an tape.



Transmission $\lambda = 3.75$ cm $\nu = 8$ GHz

R(cm)		t(db)	$-20\log_{10}\left(\frac{\lambda}{4\pi R}\right)$	G
28.25	1	−21.47	+39.524	9.027
29.02	17	−21.67	39.758	9.044
29.40	2	−21.82	39.871	9.026
29.50	4	−21.83	39.900	9.035
30.00	16	−21.94	40.046	9.053
30.51	15	−22.03	40.193	9.080
30.92	14	−22.17	40.309	9.069
31.05	3	−22.26	40.345	9.043
31.33	5	−22.36	40.423	9.032
31.50	13	−22.43	40.470	9.020
32.18	12	−22.59	40.656	9.033
32.72	11	−22.76	40.800	9.020
33.03	10	−22.87	40.882	9.006
33.32	9	−22.97	40.958	8.994
33.76	8	−23.12	41.072	8.976
34.32	7	−23.25	41.215	8.983
35.32	6	−23.43	41.464	9.017
40.40	18	−24.58	42.632	9.026

The order of measurement is given by the circled numbers.

Since $t = g_1 g_2 \left(\frac{\lambda}{4\pi R}\right)^2$, and $G = g_1 = g_2$; $G = \frac{1}{2}\left\{t_{db} - 20\log_{10}\left[\frac{\lambda}{4\pi R}\right]\right\}$

The readings were taken from the Marker # reading, Ave = 16.
There appears to be a slight systematic effect, probably a standing wave. The averages is $\boxed{9.023 \text{ db}}$ Max = ~~9.069~~ 9.080, + .057 ~~.046~~
Min = 8.976, −.047 db ± 1% max. $\sigma \sim \frac{1}{2}\%$; $\frac{\sigma}{\sqrt{18}} \sim 0.1\%$

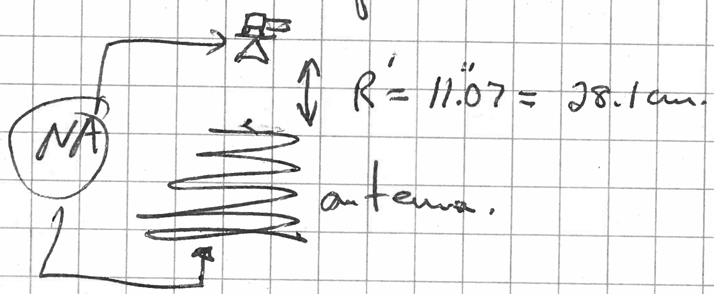
The loss of $\frac{.468}{2} = .234$ db is included in the above gain measurement.

The input mismatch loss of −23.539 db = Γ will cause a loss of $1 - |\Gamma|^2 = 1 - (.0644)^2 =$ ~~[illegible]~~ 0.9956
A correction of 2 percent

Antenna Tests with the circuit board of p. 65 at the tip

I. At 8 GHz with ~~a~~ one of the calibrated horns.

Setup.



True R is between the phase centers. At 8 GHz (3.75 cm λ) the phase center is $1.4 \times 3.75 = 5.25$ cm from the vertex. The distance of the input plane to the vertex is 1.70 cm (p. 122). So,
$R = R' + (5.25 - 1.70) = 28.1 + 3.55 = \underline{31.7 \text{ cm}}$.

The measured transmission is -26.2 db.

The estimated (expected) transmission:
$t \quad G_1 + G_2 + 20\log_{10}\left(\frac{\lambda}{4\pi R}\right) - \text{losses}.$

The losses in the cables and balun at 8 GHz: $.94 + 1.3 = 2.4$

$t_{exp} = 9. \quad 15 \quad 2.4 + 20\log_{10}\left(\frac{3.75}{4\pi \times 31.7}\right) = 9.03 + 11.5 - 2.24 - 40.2$ (feed)
$= -22.23$ db

$$\boxed{\Delta = -4.0 \text{ db}}$$

II $\quad$ 6 GHz measurement with one of the coax/wguide transitions
$\lambda = 5.0$ cm $\quad R' = 22.$ $\quad 1.4\lambda = 4$; 7.0 cm, $-1.7 = 5$ cm
$R = (22.5)(\times 2.54) + 5.3 = 62.5$ cm

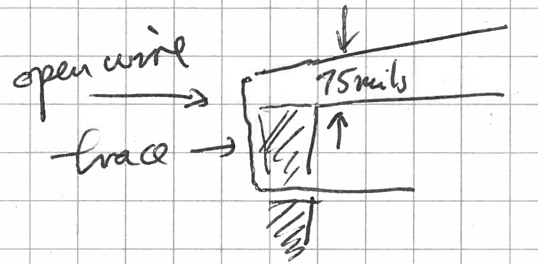
$$20\log_{10}\left(\frac{5.0}{4\pi \times 62.5}\right) = -43.92 \text{ db}$$

$$t_{exp} = G_1 + G_2 - \text{losses} + 20\log_{10}\left(\frac{\lambda}{4\pi R}\right)$$
$$= .0 + 11.5 - 2.0 - 43.92 = \underline{-28.4 \text{ db}}$$

$t_{meas} = -31.4$ db $\qquad \boxed{\Delta = -3.0 \text{ db}}$

Modifications in the Circuit Board

The large amount of groundplane covering 0.3" appears to cause standing waves in the simulations). Cut it back to just the 0.150" covering the end of the pyramid. Then running the wire over the 75 mils between the board and the end of the feed will leave some inductance.



From Montgomery, Dicke, and Purcell p. 168 the inductance/m of a short piece of wire of length l and radius r is

$$L = \frac{\mu}{2\pi} \ln\left(\frac{2l}{er}\right) = \frac{\overset{2}{\cancel{4}}\pi \times 10^{-7}}{\cancel{2\pi}} \ln\left(\frac{2 \cdot 75\text{mils}}{2.71 \cdot 10\text{mils}}\right)$$

$$= 2 \times 10^{-7} \ln 5.531 = 3.4 \times 10^{-7} \text{ henrys/m}$$

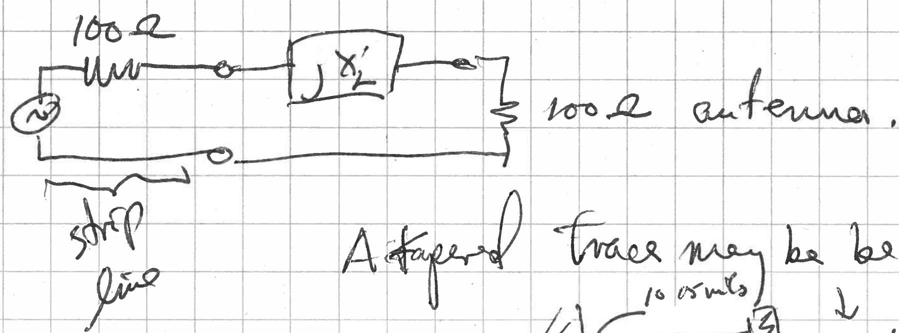
For the 75 mil length: $(.075") \times (.0254 \frac{m}{in}) = .0019 m$

$$\mathcal{L} = Ll = 3.4 \times 10^{-7} \cdot (.0019) = 6.5 \times 10^{-10} \text{ henrys.}$$

Reactance at 10 GHz: $X_L = 2\pi\, \cancel{10^{10}} \times 6.5 \times \cancel{10^{-10}} = 40.8\ \Omega$.

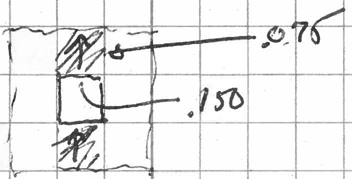
Thicker wire will lower the inductance and reactance.

r	$\mathcal{L}$ (nH)	X_L (10 GHz)
10 mils	.65	40.8 Ohms.
20	.39	24.5
50	.039	2.45



A tapered trace may be best

How much radiation in the forward direction from the tip will result?



A crude overestimate comes from the Gain/Area formula.

$$G = \frac{4\pi \, \text{Area}}{\lambda^2} = \frac{4\pi \; 2(.150 \times .075)}{(1.25'')^2} = 0.18 \;@\; \lambda\, 3\text{cm}.$$

The expected gain of the antenna in that direction is 11.5 db or 14.12. This is 100 times larger. The formula is an overestimate because the radiation crosssection for a small source really goes down as λ^{-4}, not λ^{-2}.

Gain summary: table of p. 125 cleaned up. $d = 2.4m$

ν	λ	ΔS	$\eta_{defoc.}$	η_i	η_{spill}	η_{Z11}	$F_{eff.}$	A_0	l_d	$\bar{e}^{(4\pi\sigma/\lambda)^2}$	η_T	2/3 l_d Gnd spill	new η_{spill}	η_T
11.25	2.67	-2.98	.85	.986	.758	.832	.644	.211	.0072	.915	.484	.6%	.820	.524
10.0	3.00	-2.52	.91	.985	.757		.645		.0077	.932	.526	.6	.820	.570
8.75	3.43	-1.92	.985	.984	.756		.646		.0082	.947	.577	.7	.820	.626
7.50	4.00	-1.12	.99	.982	.756		.649		.0088	.961	.588	.7	.817	.635
6.25	4.80	0	1.00	.981	.756	.832	.650	.211	.0097	.973	.600	.7	.817	.648
5.00	6.00	1.68	.99	.978	.756	.838	.653	.217	.0111	.983	.603	.7	.815	.650
3.75	8.00	4.48	.985	.974	.753	.842	.659	.222	.0132	.990	.602	.9	.812	.649
2.50	12.00	10.08	.91	.953	.748	.851	.670	.231	.0237	.996	.550	1.5	.806	.593
1.25	24.00	22.88	.89	.945	.736	.863	.696	.248	.0255	1.00	.534	1.7%	.791	.574
1.00	30.00	35.28	.84	.942	.730	.867	.721	.251	.0288		.501	1.9	.777	.533
.75	40.00	49.28	.82	.922	.710	.893	.749	.294	.039		.479	2.6	.760	.513
.50	60.0	77.28	.805	.888	.693	.914	.865	.342	.056		.453	3.7%	.725	.474

spillover from diffraction

$D = 5m \quad d = 2.4m$

$l_d = .2095 A_0 \sqrt{\lambda(cm)}$

$\eta_i = 1 - 2l_d$

For the spillover from diffraction it's $\frac{1}{2}(2l_d) = l_d$. However, the feed shield will eliminate 2/3 of the secondary diffraction(?). Thus, $\frac{2}{3} l_d$ is the expected diffraction to ground.

A small correction: (1/17/09)
a) The total diffractive spillover is $2l_d$
b) Without the shroud 1/2 of that, l_d, would go to the ground.
c) The shroud should take out most of this ground spillover
d) Assume that only $\frac{1}{2}$ of that spill over reaches the ground.

altogether $\frac{1}{2} \cdot \frac{1}{2} \cdot 2l_d = \frac{1}{2} l_d$

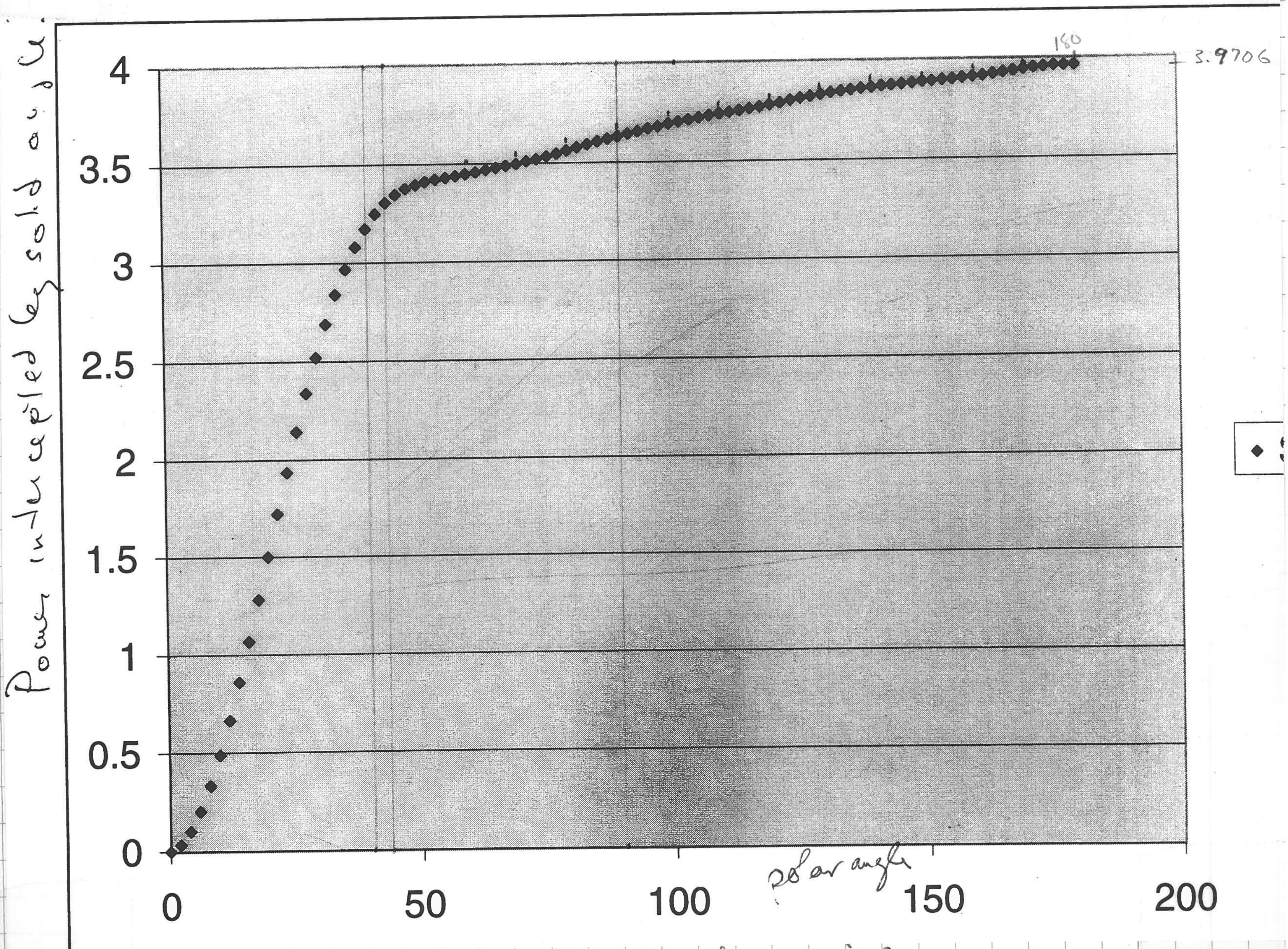
For $l_d = .2095 A_0 \sqrt{\lambda(cm)}$ with $A_0 = .211$

at 1 GHz: $\frac{1}{2} l_d = \frac{1}{2}(.2095)(.211)\sqrt{.3m} = .0121$, $\times 300K = 3.6K$

We are now using $d = 2.4$ and $D = 6m$, the same ratio

Greg's New Design with Interior Fins 8/30/00

Thin Boom + shield + 2 polarizations + Fins



Θ_e	$\int_0^{\Theta_e} P(\theta)\, d\Omega$	η_s frac.	$1-\eta_s$	F[Ψ/d]	Forward frac.
90	3.6324	.9148	.0852		
44	3.2941	.8296	.1704	.6188	.0852
→ 42	3.2560	.8185	.1815	.6513	.0963
40	3.1647	.797	.2030	.6869	.1178
38	3.0824	.776	.2240	.7261	.1388
36	2.9706	.748	.2519	.7694	.1667
34	2.8382	.7148	.2852	.8177	.2000
32	2.6912	.6778	.3222	.8719	
30	2.5294	.637	.3630	.9330	

$= 1 - \eta_s + \eta_s(90)$

Fit: for $30 \leq \Theta_e < 44^\circ$

$\eta_s = .987783 - .00065526\,F - .404364\,F$

$$F = \frac{1}{4}\sqrt{\frac{1+\cos\Theta_e}{1-\cos\Theta_e}}$$

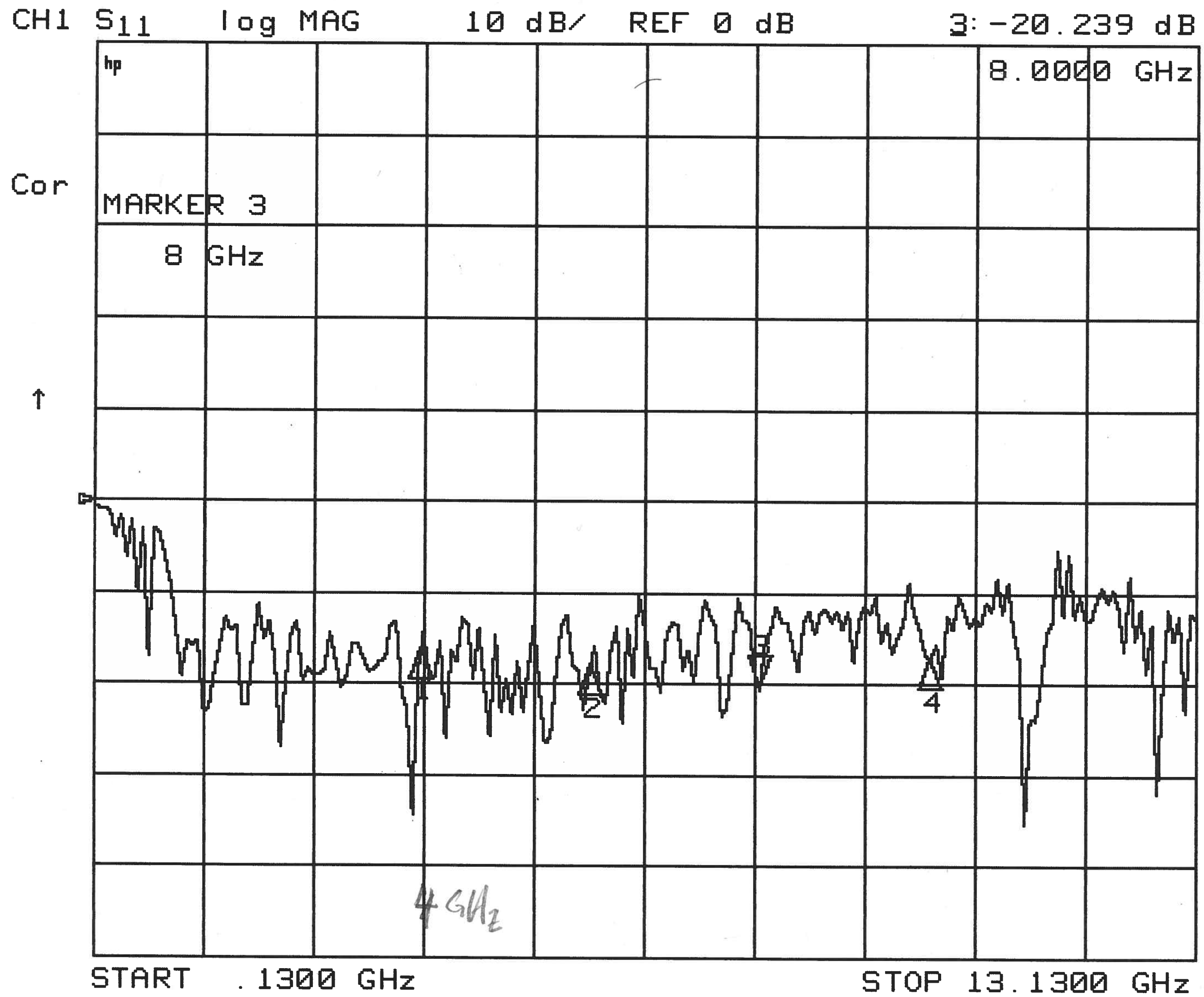
Further angles

θ_e	$\int_0^{\theta_e} P(\theta)d\theta$	η_s (frac)	$1-\eta_s$	F	η_{fd} forward frac. $=1-\eta_s-\eta_s(90)$
46	3.3470	.843	.157	.589	.0718
48	3.4000	.856	.144	.562	.0588
50	3.403	.857	.143	.536	.0578
60	3.444	.867	.133	.433	.0478
70	3.500	.881	.119	.357	.0338
80	3.559	.896	.104	.298	.0188
90	3.632	.915	.085		0
100	3.700	.932	.068		
110	3.738	.941	.059		
120	3.779	.952	.048		
130	3.838	.967	.039		
140	3.879	.977	.023		
150	3.897	.981	.019		
160	3.911	.985	.015		
170	3.950	.995	.005		
180	3.971	1.000	0		

Total Back lobe is 8.5%

9/4/00 Final measurements on the feed after stiffeners had been added to the top ends of the elements. →

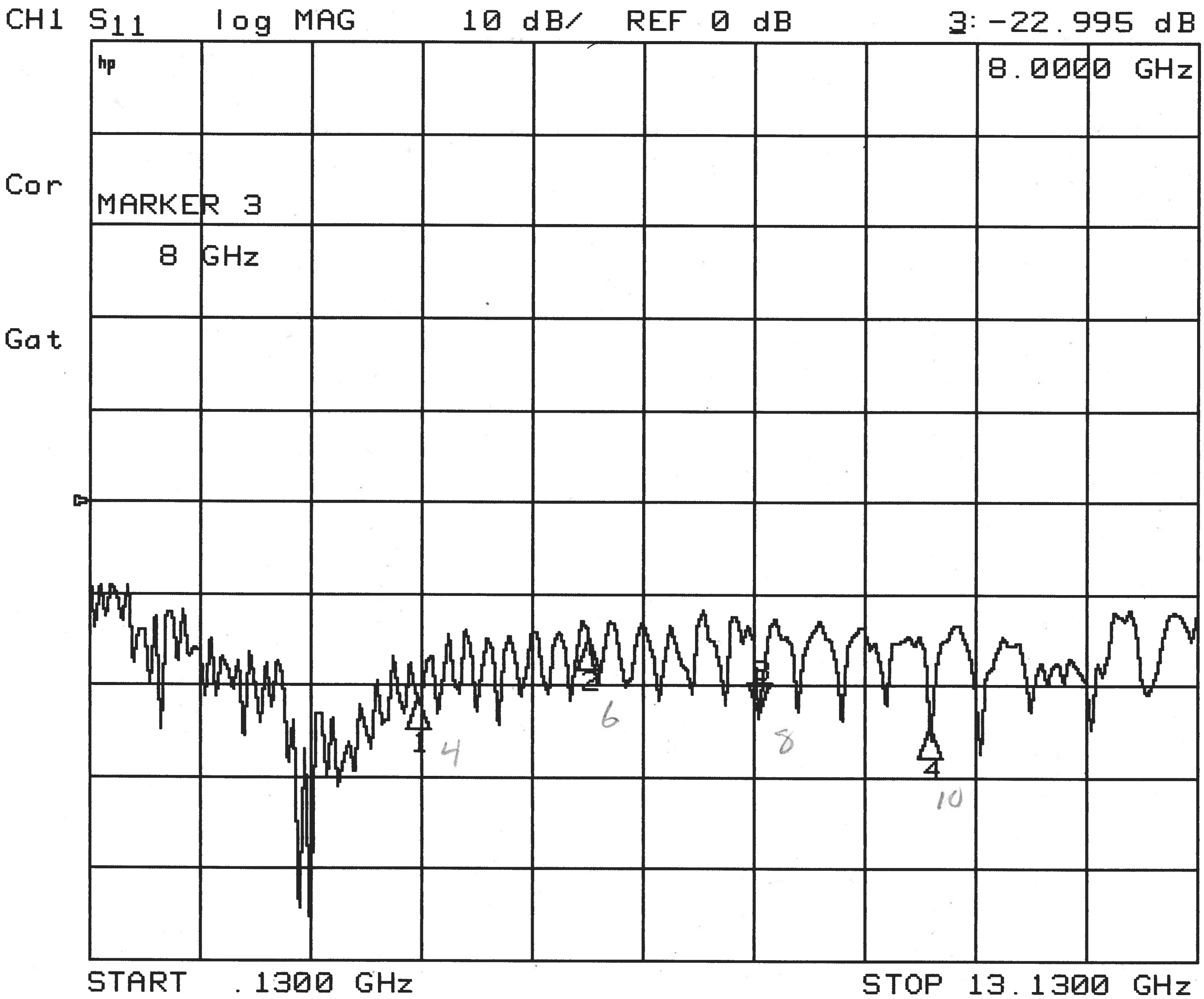
9/4/00



Antenna input Z; no gates

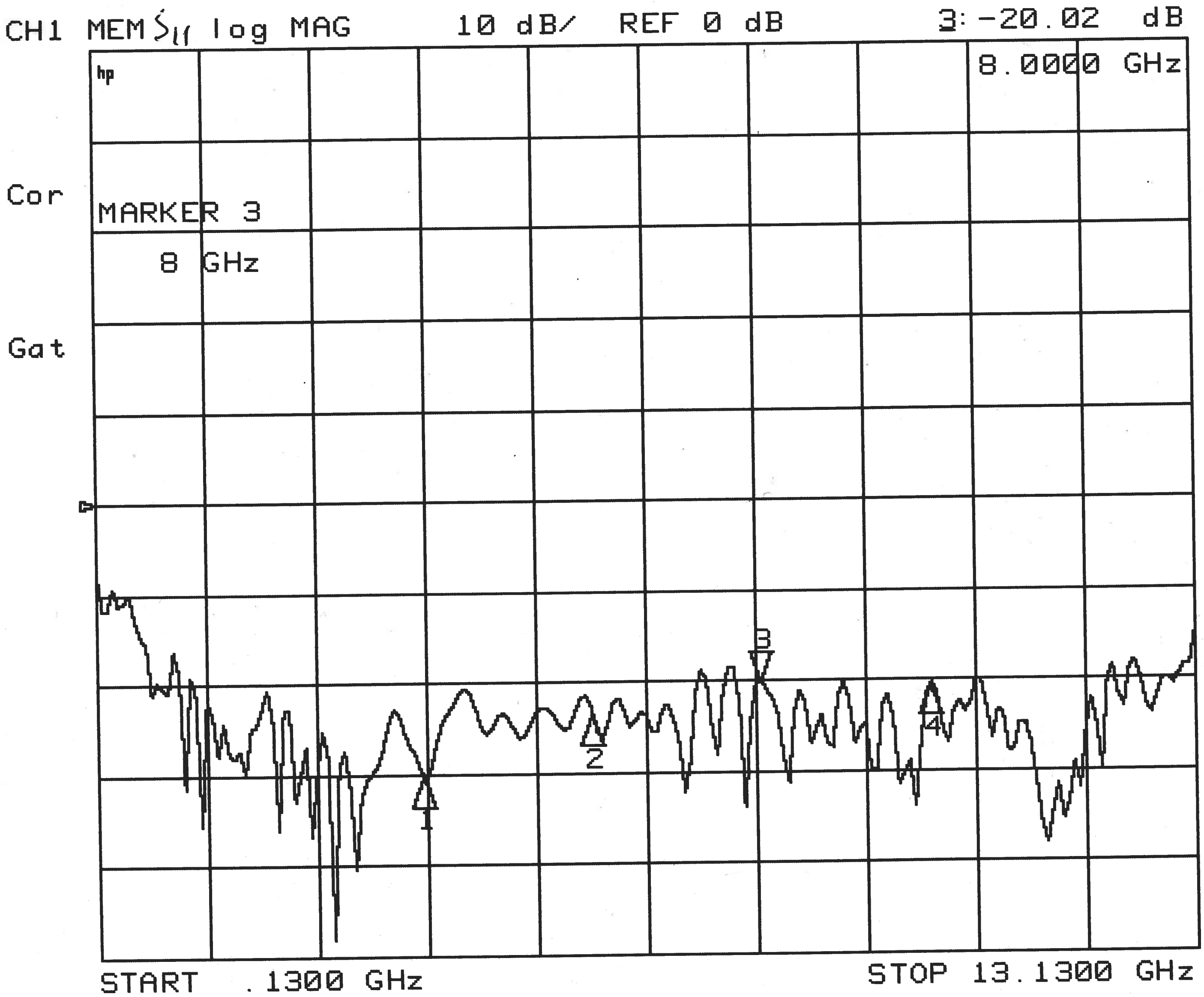
N.B. Input cable losses are ~2db (4-5 db round trip). These should be added to the losses above

9/4/00



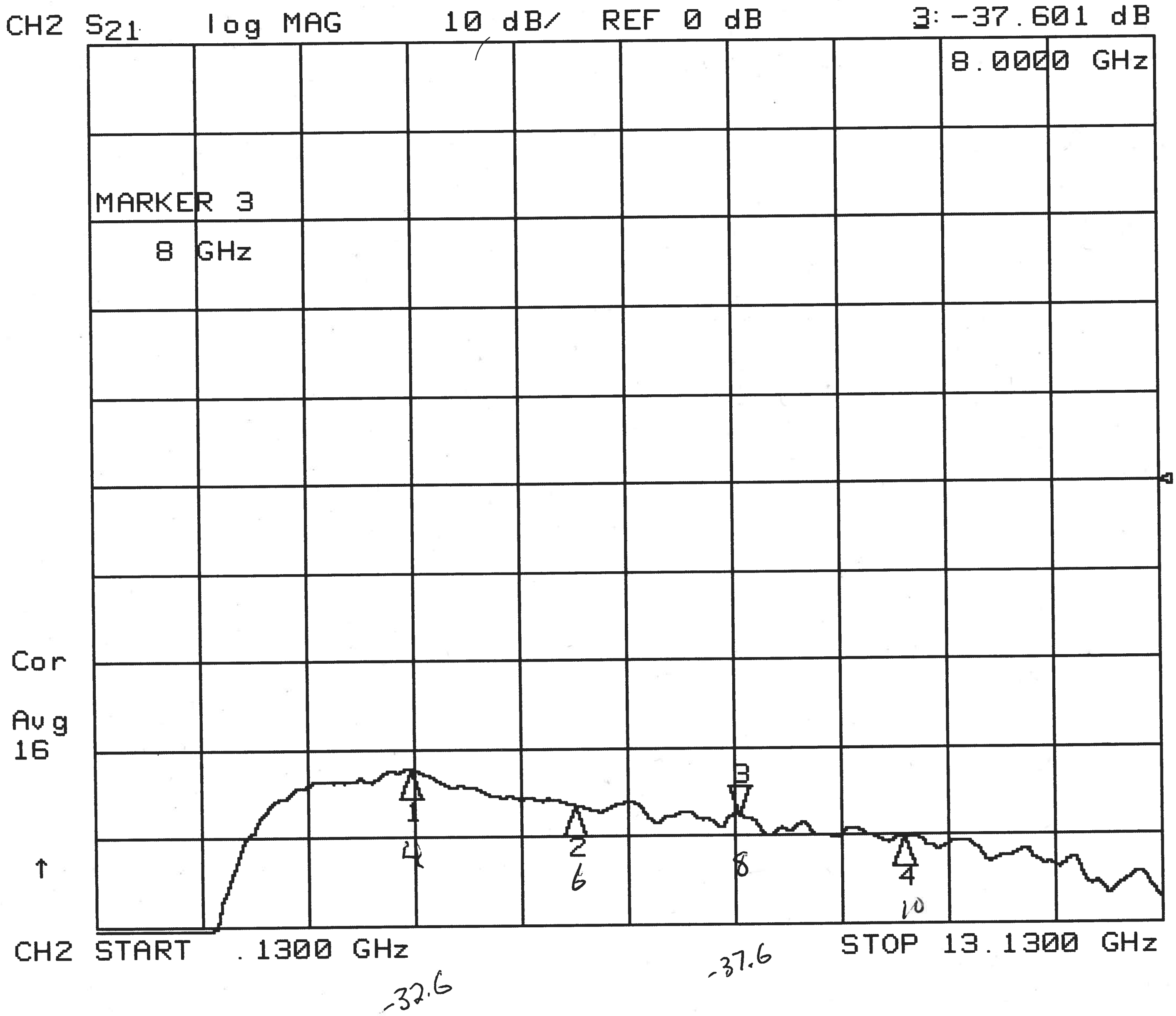
tip circuit board included. Grate in front of it.

9/4/02



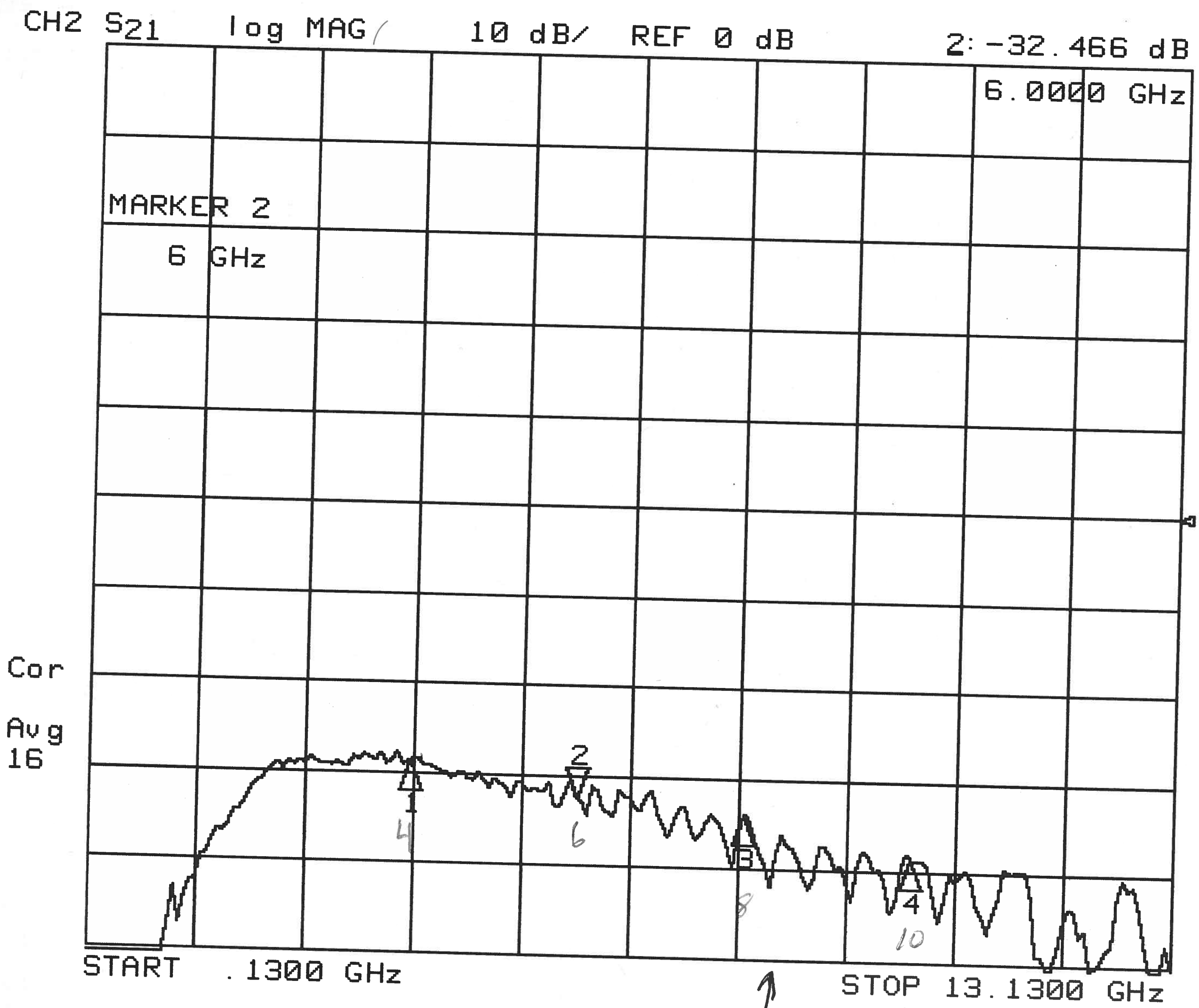
tip circuit board gated out

9/4/00



Transmission between 2 spiral antennas

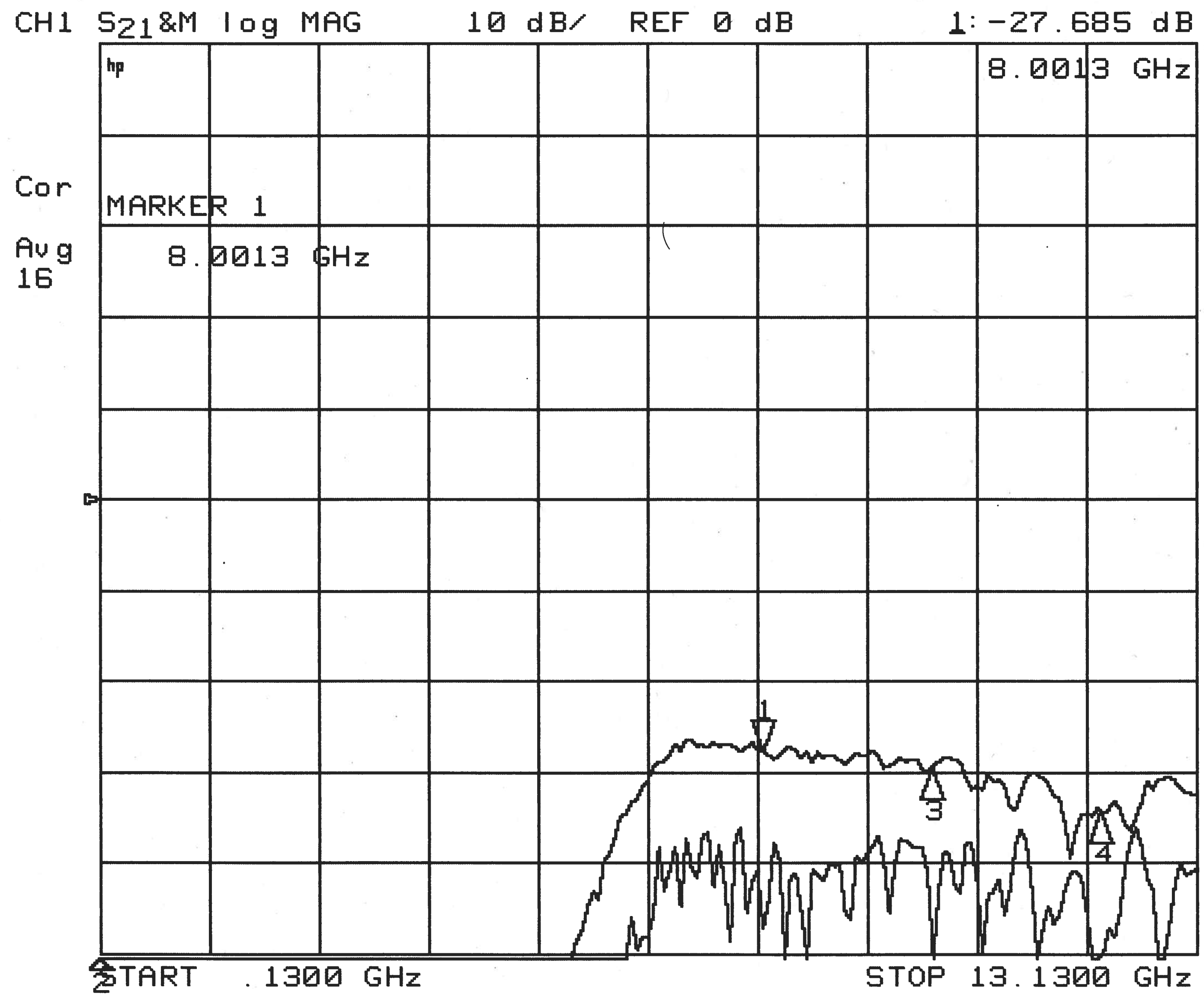
9/4/00



see next page

transmission between spiral antenna and LP Feed

9/4/00



Cross polarization
x band standard horn

see p 126 for horn dropout at 9 GHz.

1AT board 9/8/00

Comparisons between the Symmetric Cassagrain and the Gregorian feed patterns.

I. Size of Cassagrain Primary with 2.4m secondary that has the same area as the 5m primary of the offset Gregorian.

Assume a taper to 1/3 in voltage at the edge: $E_0 = .333$ (from 1)

$E(r) = \frac{1}{3} + \frac{2}{3}(1-r^2)$

Evdel $\propto \int_0^1 E(r) r dr = \int_0^1 \left[\frac{1}{3} + \frac{2}{3}(1-r^2)\right] r dr = \frac{1}{3}$

$\int_a^b \left[1 - \frac{2}{3}r^2\right] r dr = \frac{b^2-a^2}{2} - \frac{1}{6b^2}(b^4-a^4) \longrightarrow \frac{b^2}{3}$ for $a=0$.

Make $\frac{b_1^2-a^2}{2} - \frac{1}{6b_1^2}(b_1^4-a^4) = \frac{b^2}{3}$ $\quad b_1 > b$

With $a = 1.25$m and $b = 2.5$m find $b_1 = 2.92$m. For the taper to $E_0 = .25$ b_1 will be even larger i.e. 3m or more.

Relative mirror cost: The secondarys have the same diameter

If the primary scales as $D^{2.7}$, $(6/5)^{2.7} = 1.63$

II. Diffraction patterns

i) Unblocked (Gregorian) Case First

For $\theta \lesssim 55°$ (1rad)

$f_1[r\_] \cdot |E_f(\theta)| = .997 \quad 2.4779\theta^2 + 2.4636\theta^4 - 0.853\theta^6$ $\quad \theta$ in radians.

$F \quad .6 \quad , E_0 = .211$ Use transformation of pp 38 and 76.

$\theta = A\ T \left\{\frac{r}{f - \frac{\ }{f}}\right\}$ $\qquad \frac{f}{R} = \frac{f \sin\theta}{r}$

$f/R = f_2[r\_, f\_] = f/r \sin\left[\mathrm{ArcTan}\left[\frac{4fr}{4f^2-r^2}\right]\right]$

$E(r) = f_1[r, f\_] * f\ [r, f]$

$D = 5$m $\quad f = F D = .6 * 5 = 3.25$m aperture distribution

r(m)	E(r)	E(r) − .211		$[1-(r/2.5)^2]^{1.8}$
0	.997	.786	(1.00)	1.00
.25	.981	.770	(.980)	.982
.50	.934	.723	(.920)	.929
.75	.861	.650	(.827)	.844
1.00	.768	.557	(.709)	.731
1.25	.663	.452	(.575)	.600
1.50	.554	.343	(.436)	.450
1.75	.449	.238	(.303)	.300
2.00	.354	.143	(.182)	.159
2.25	.274	.063	(.080)	.050
2.50	.211 (−13.5 dB)	0	(0)	0

$E(r) \cong .211 + .786\,[1-(r/2.5)^2]^{1.8}$ Use equation from p. 62

$E(\theta) \propto \int_0^{2.5} [.211 + .786\,[1-(r/2.5)^2]^{1.8}]\, J_0[kr \sin\theta]\, r\,dr$

$\lambda = .20$, $a = 2.5$, $ka = \frac{2.5}{.2} 2\pi = 78.54$, $\Gamma(2.8) = 1.676$, $2^{1.8} = 3.482$, $\Gamma(1) = 1$

$E(\theta) \propto (2.5)^2 \left\{ .211\, \Gamma(1) \dfrac{J_1[78.54 \sin\theta]}{78.54 \sin\theta} + (.786)(3.482)(1.676) \dfrac{J_{2.8}[78.54 \sin\theta]}{[78.54 \sin(\theta)]^{2.8}} \right.$

$= (2.5)^2 \left\{ .0027 \dfrac{J_1[78.54 \sin\theta]}{\sin\theta} + 2.266\times10^{-5} \dfrac{J_{2.8}[78.54 \sin\theta]}{(\sin\theta)^{2.8}} \right\}$

$= (2.5)^2 \{ f_3[x\_] + f_4[x\_] \}$ $x = \theta$

From p. 76 $A_e(0) = 16.32$; $G_0 = 5127$.

$G(\theta) \propto \{ f_3 + f_4 (.001) \}^2 = .0605$; scale by 84,721 up to $G_0 = 5127$.

g_1 = Plot[10* Log[10, .001+ 84721*(f3[x]+f4[x])^2], {x, .001, 1.5},
PlotRange→{-30, 40}, Frame -> True, GridLines -> Automatic]

2) The Symmetric Cassagrain $D_p = 3$ m, $D_s = 2.4$ m
$F = .65$, $f = .65 \times 6 = 3.90$ m

Proceeding as above, find $E(r) \cong .211 + .786\,[1-(r/3)^2]^{1.9}$; $0 \le r \le 3$ m.

The blocked distribution is found in the same way.
$E_b(r) \cong .751 + .246\,[1-(r/1.25)^2]^{1.1}$; $0 \le r \le 1.25$ m.

The total field is $E(r) - E_b(r)$

The difference patterns are:

$$E(\theta) \propto (3.0)^2 \left\{ 0.211 \frac{J_1[ka\sin\theta]}{(ka\sin\theta)} + (.786)\, 2^{1.9}\,\Gamma(2.9)\, \frac{J_{2.9}[ka\sin\theta]}{(ka\sin\theta)^{2.9}} \right\}$$
$$- (1.25)^2 \left\{ (.751) \frac{J_1[kb\sin\theta]}{(kb\sin\theta)} + (.246)\, 2^{1.1}\,\Gamma(2.1)\, \frac{J_{2.1}[kb\sin\theta]}{[kb\sin\theta]^{2.1}} \right\}$$

$\lambda = 0.2$, $ka = \frac{2\pi}{.2} 3.0 = 94.25$; $\Gamma(2.9) = 1.827$, $2^{1.9} = 3.732$
$kb = 39.27$; $\Gamma(2.1) = 1.046$, $2^{1.1} = 2.144$

$$E(\theta) \propto (1.899) \frac{J_1[94.25\sin\theta]}{[94.25\sin\theta]} + (48.23) \frac{J_{2.9}[94.25\sin\theta]}{[94.25\sin\theta]^{2.9}} \qquad f_6[x\_]$$
$$- (1.1734) \frac{J_1[39.27\sin\theta]}{[39.27\sin\theta]} - (.8620) \frac{J_{2.1}[39.27\sin\theta]}{[39.27\sin\theta]^{2.1}} \qquad f_7[x\_]$$

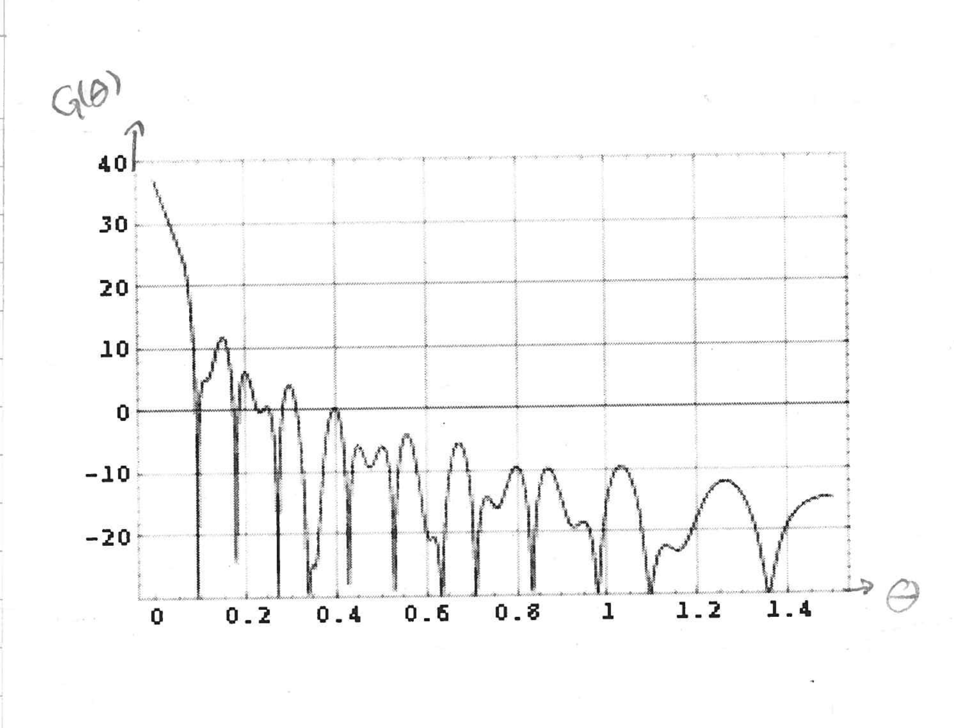
For $G_{cas}(.001) = 5127$, scale by 2312.
i.e: $G_{cas}[x] = 2312\, [f_6[x] + f_7[x]]^2$

3) At wide angles the feed looks past the secondary in both cases and ~~gains the~~ dominates the gain.

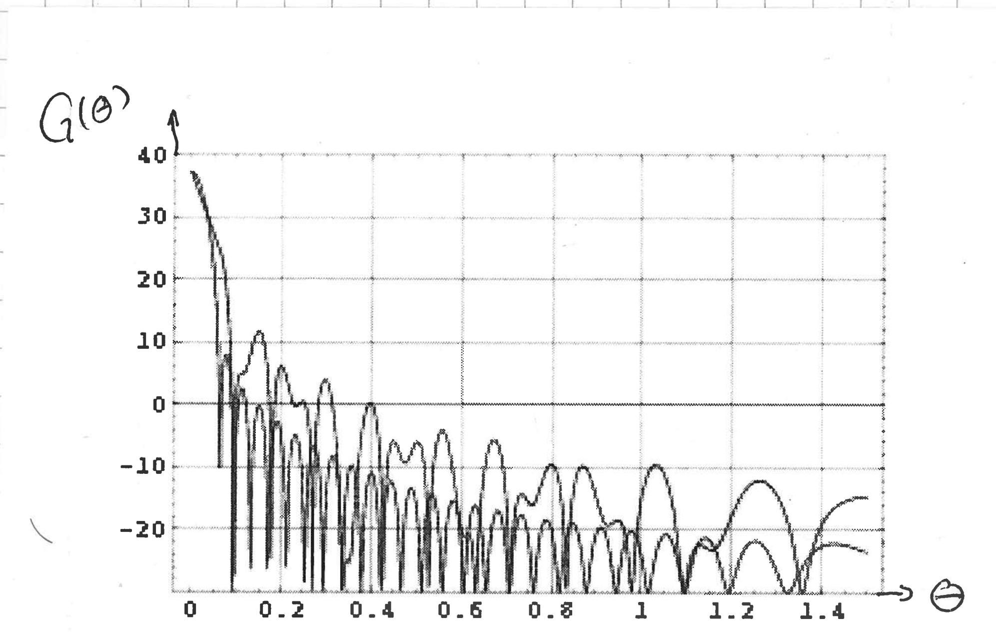
For a gain of 11.5 db for the feed, $G_{feed}(\theta) = 14.1\, |E_f(\theta)|^2$
$E_f(\theta)$ is only correct up to $\theta \sim 55°$ ~ 1 radian.
For $55° < \theta < 90°$ a fit to the pattern of p.73 had to be found.

For the Symmetrical Cassegrain the feed pattern appears on both sides of the peak between 42° and 90° away.

For the Gregorian there is no direct feed contribution to the gain on one side of the maximum of the aperture pattern, the lower elevation side. It is there on the upper side. In addition there is a contribution from the feed pattern that is reflected from the ground.

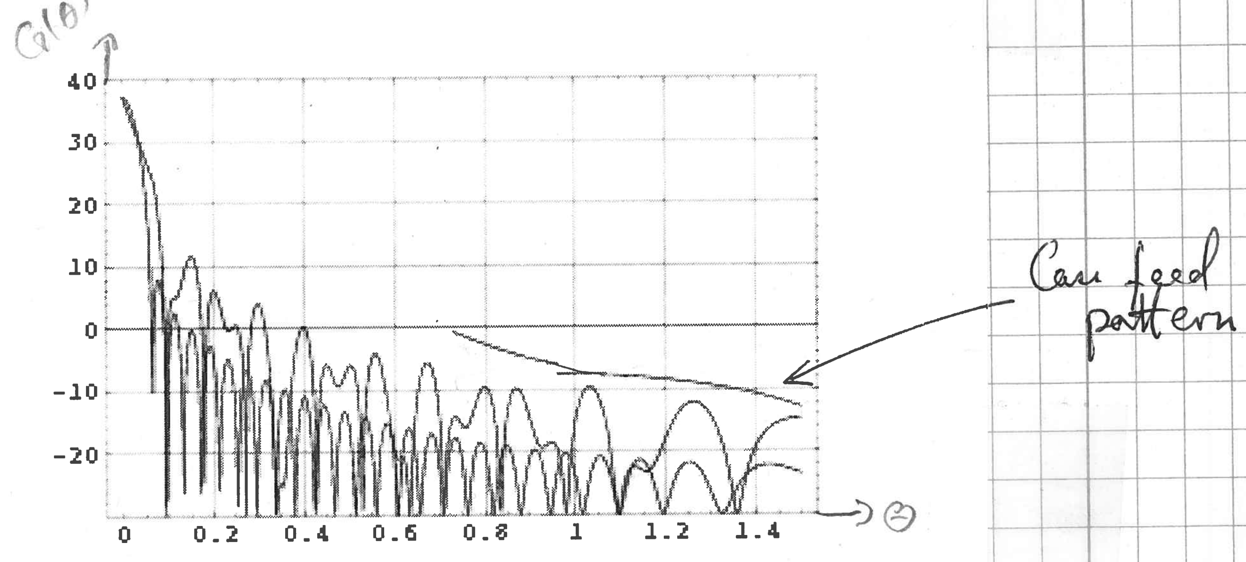


Gain pattern of the symmetric Cassagrain: $D_p = 6m$, $D_s = 2.4m$

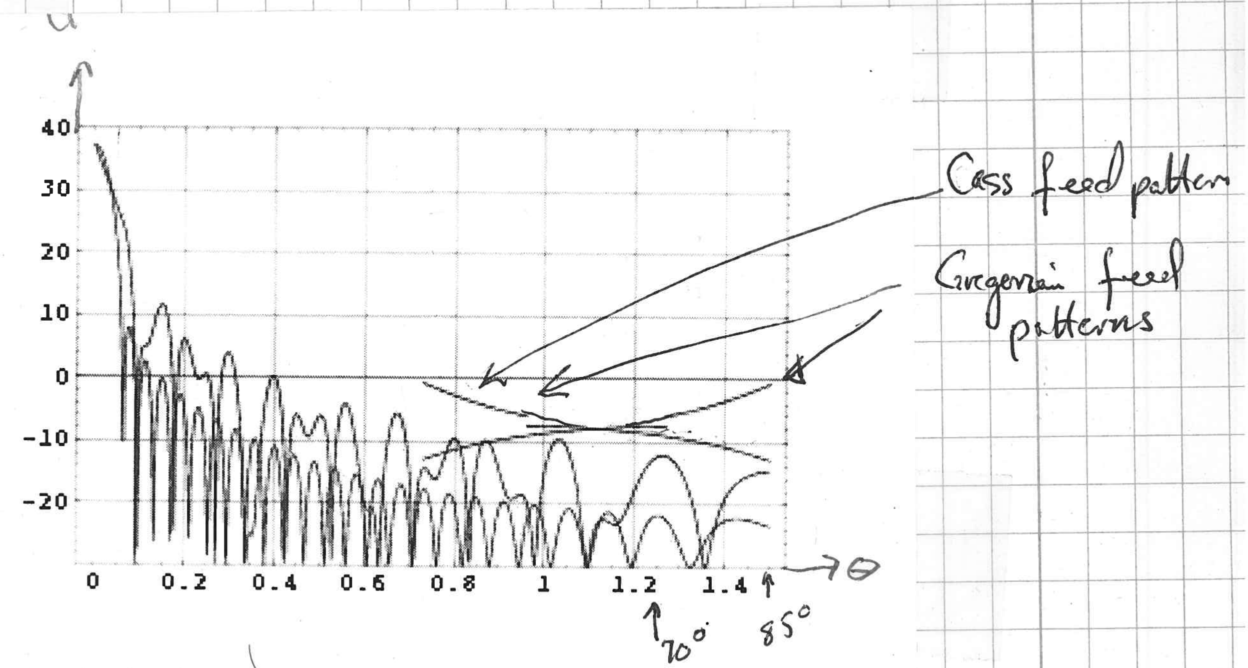


Gain patterns of the symmetrical Cassagrain and the unblocked (Gregorian) apertures superposed. The envelope of sidelobes is about 10 db lower for the Gregorian over $0 < \theta < 90°$

$D_p = 5m$. for the Gregorian



For the lower elevation side of the beam the Cass feed pattern is added for $42° < \theta < 90°$. The feed is shielded by the shroud for the Gregorian. In this angular range ($42° < \theta < 90°$), the Gregorian has about 15db lower response and it is $\simeq -20$ db below the isotropic level. This is largely an advantage for the Gregorian against ground based interference.



This is the upper elevation side of the beam. Both Cassegrain and Gregorian sidelobes are dominated by the feed gain for $42 < \theta < 90°$. In addition, the Gregorian shows the feed pattern reflected by the shroud strongest near 85°. In the range $70° < \theta < 85°$, the Gregorian gain is higher by about 5-10db. (than just the antenna)

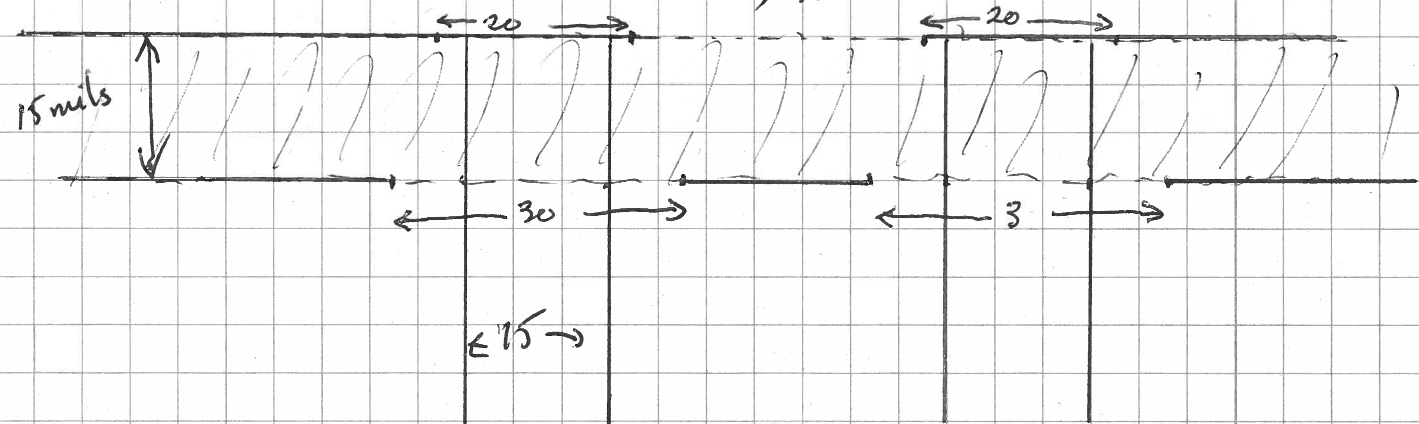
More details about the input circuit at the Tip of feed

As built dimension: Cuflon board: $h = 15$ mils trace width is 10 mils
$Z_0 = 109\,\Omega \approx 220/2$.
clear diameter on ground side is 30 mils.
pad spacing is 52 mils
wire used is #34 6 mils

Line impedance for $s = 52$ and $D = 6$
$Z_0 = 120 \cosh^{-1}(s/D) = 120 \cosh^{-1}(8.667) = \underline{345\,\Omega}$

For the present, assume that the antenna impedance is $\underline{240\,\Omega}$

Do an optimization for match on pad diameter and hole diameter in the ground plane. The line trace width must be selected for 120 Ω & not 109 Ω. (8 mils) For 240 Ω, $s/D = 3.7$ ∴ $s = 52$ mils has $D = 14.5$ mils



Note: Cuflon from Polyflon has $K_e - 2.10$ and dissipation factor $= .0005$ at 10 GHz

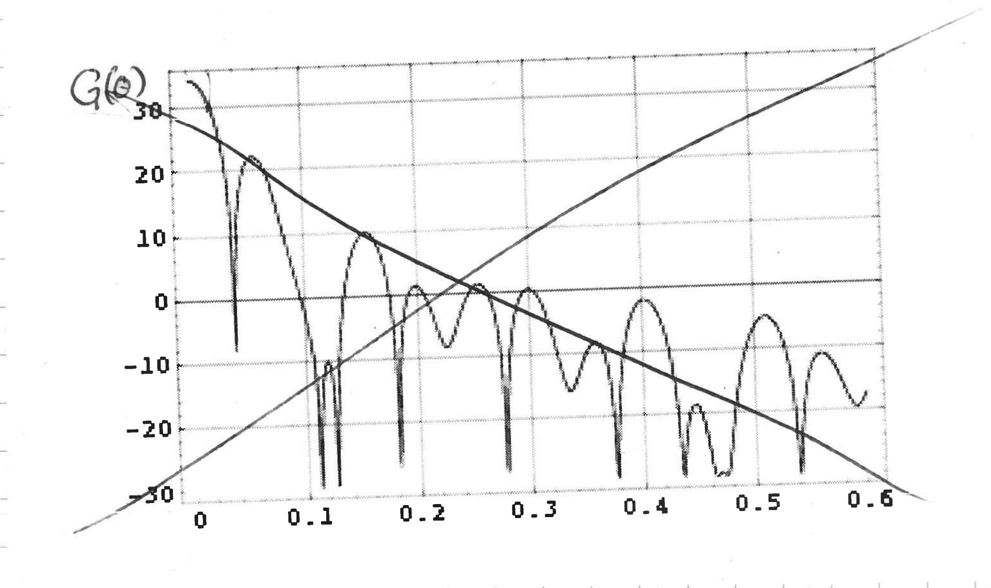
The Cassagrain with smoothed out illumination

Assume $D = 6\,m$; $D_s = 2.4\,m$. We might be able to get a smoothed Gaussian: $E = f(r) = e^{-[1.41(r-2.1)]^2}$; r in meters. This has the value 0.2 at both r = 1.2 m and 3.0 m, and it is equal to 1 at r = 2.1 m.

The field pattern: $E(\theta) \propto \int_{1.2}^{3.0} e^{-[1.41(r-2.1)]^2} J_0(kr\sin\theta)\, r\,dr$

For $\lambda = 20\,cm$ $K = \frac{2\pi}{\lambda} = \frac{2\pi}{.2} = 31.4$ (Scales by $\frac{\sin\theta}{\lambda}$)

Plot this, $|E_0(\theta)|^2$ for $.001 < \theta < 0.4$ rads. Normalize it so that its gain at $\theta = 0$ is 5127, the same as a clear aperture of 5 m with edge illumination of .2, and $\eta_{Ill} = 0.83$.



something wrong here.

The sidelobes are a little less than those of the sharply illuminated Cassagrain but still 10db worse than for the clear aperture case.

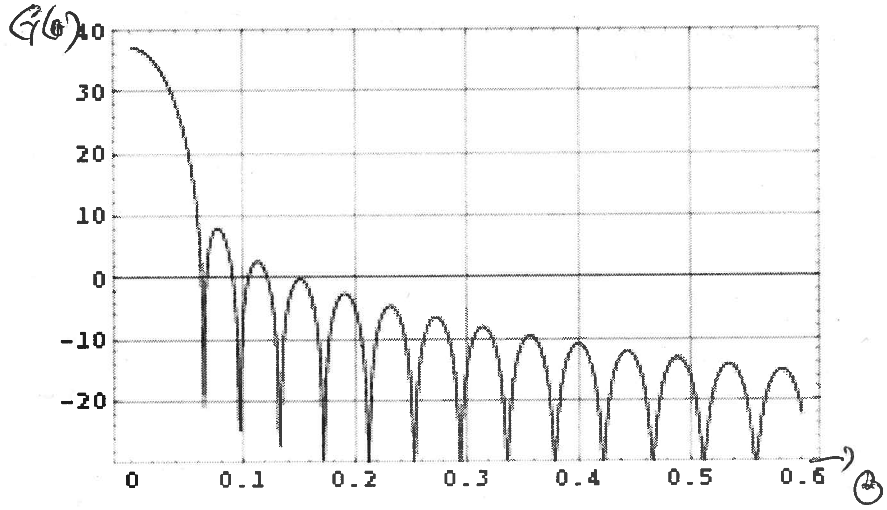
The aperture illumination efficiency is $\eta_{Il} = \frac{2\pi\left|\int_{app} E(r)\, r\,dr\right|^2}{\int_{app} |E(r)|^2 r\,dr} = 20.29\,m^2$

$\pi/4\,(6)^2 = 28.27$ so $\eta_{ILL} = 0.72$

unblocked aperture

$E_0(\text{edge}) = 0.211$

$D = 5\,m$



Case with Gaussian donut distribution

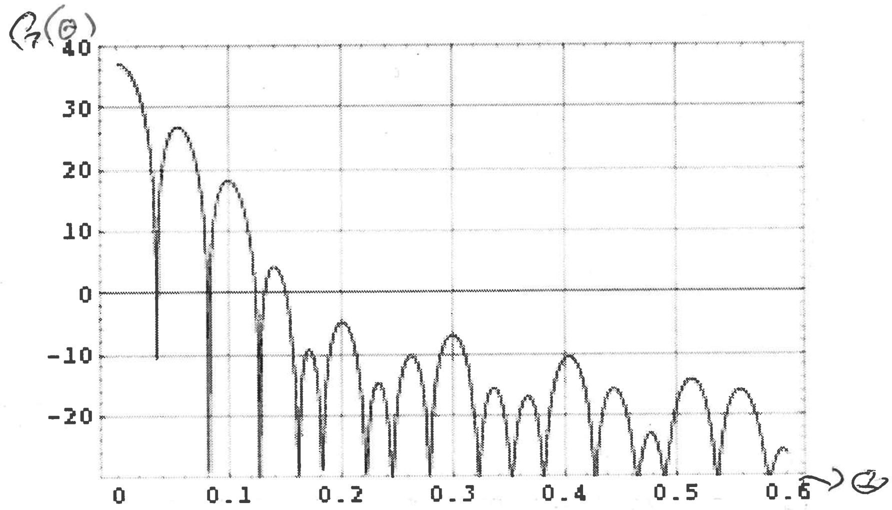
$D_p = 6\,m$

$D_s = 2.4\,m$

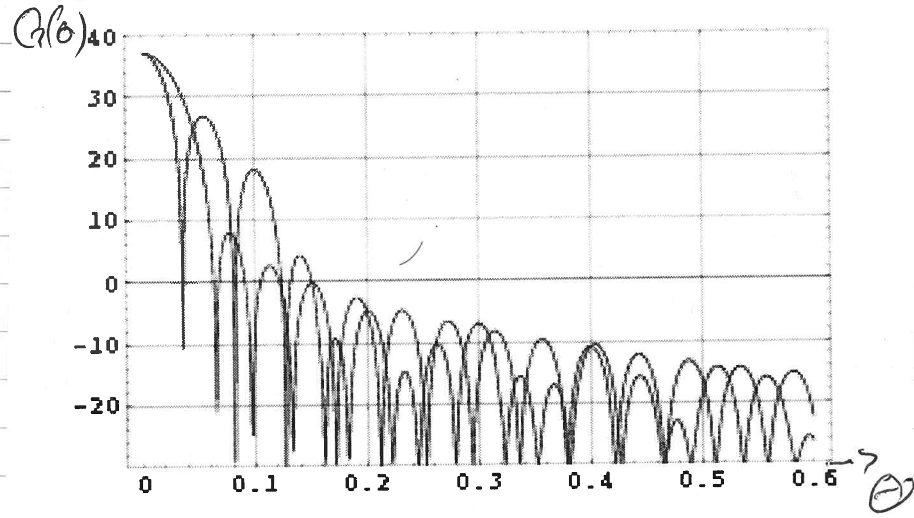
$E(r) \propto e^{-[1.41(r-2.1)]^2}$

$= 0.2$ at $r = 1.2$

$= 0.2$ at $r = 3.0$



Both of the above curves combined



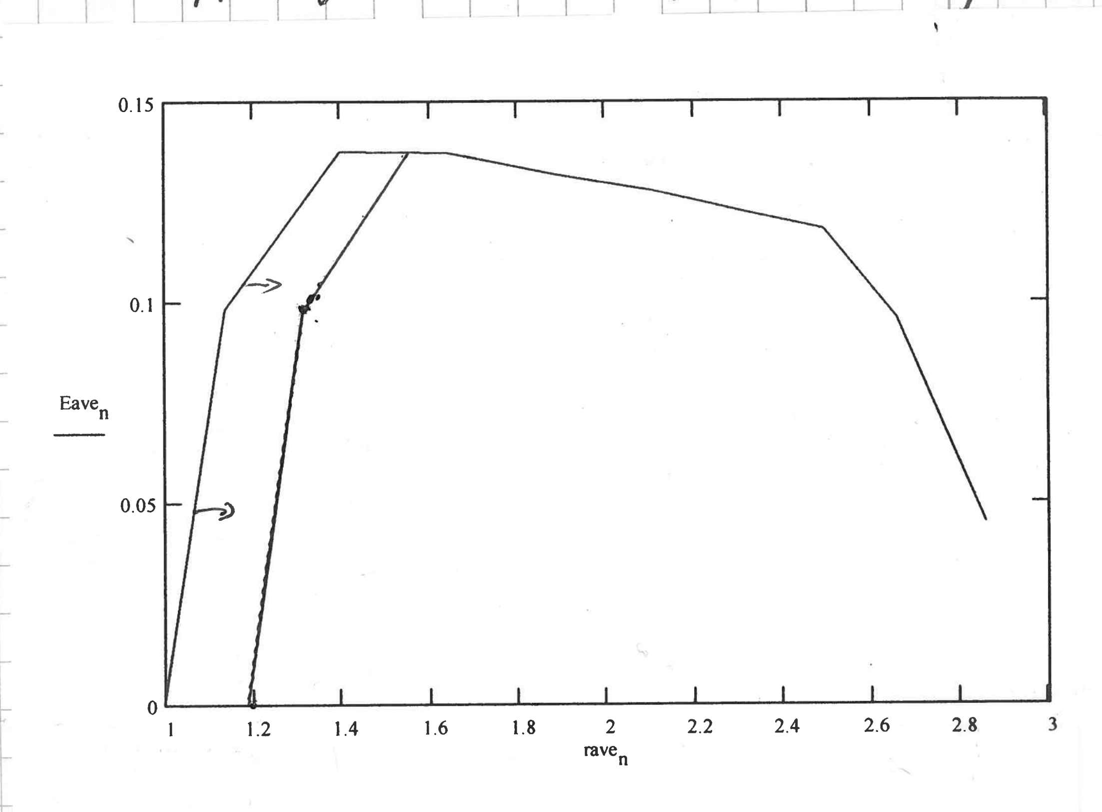
This would describe the diffraction above ~2 GHz

1. The diffraction at the central edge for the Cassagrain is at a peak of the incident signal. Kildal gives the diffraction loss as $l_d = C_d \sqrt{\frac{\lambda}{d}} \sqrt{1 - \frac{d}{D}}\, A_0$

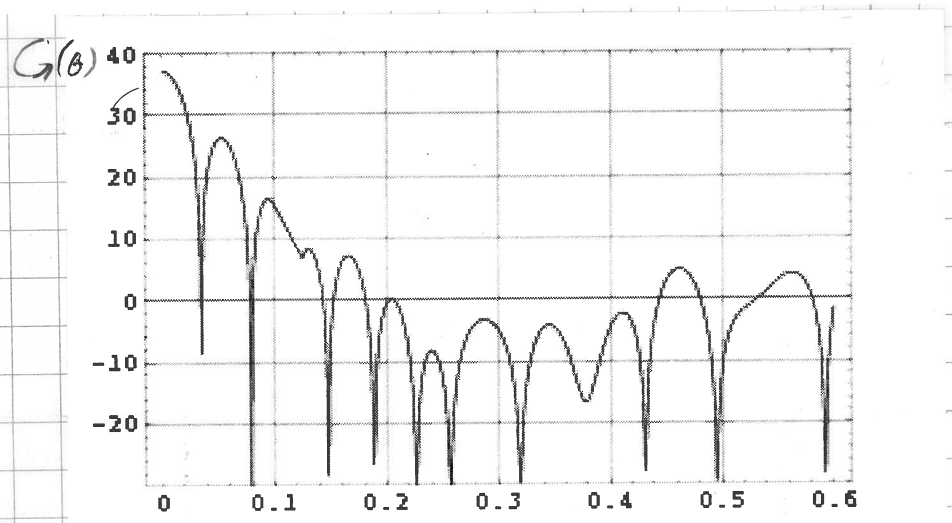
where A_0 is the strength of the incident signal. $A_0 \sim 0.3$ at the outer edge. It's ~ 1. ~~and~~ at the central edge. This would give a 30% loss due to diffraction at 1 GHz. We might do about as well with a conventional Cassagrain.

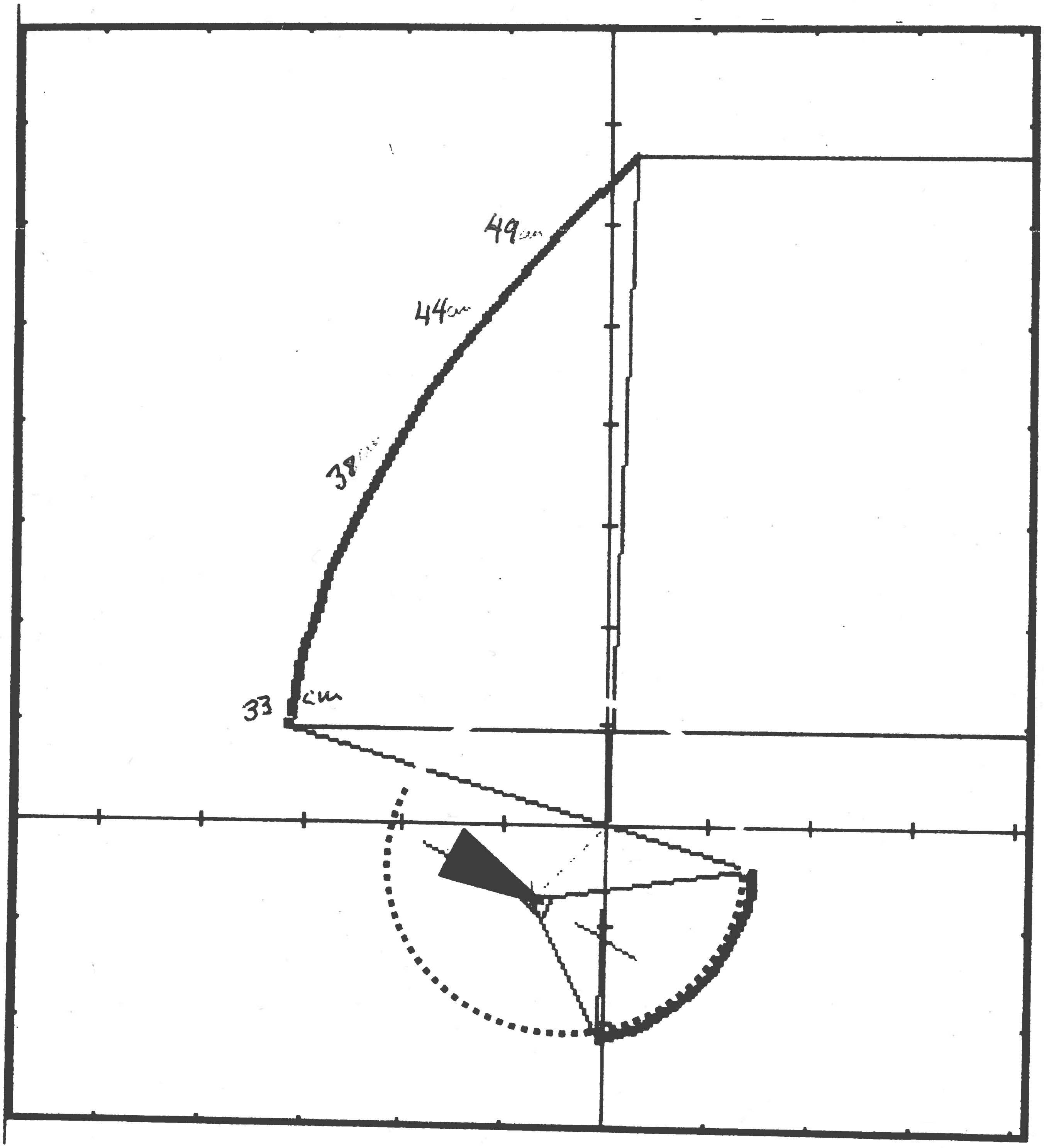
2. Other features
   i) spillover to ground at all frequencies
   ii) difficulty in manufacturing the secondary

3. John Lugten's Geometric Optics Cassagrain distribution →



The gain pattern





The numbers represent the distance of the geometric edge ray (reflected from the secondary) from the edge of the primary! This is the result of the motion of the feed phase center which is correct for 6.25 MHz.

$\lambda = 30$ cm here

Costs from John Anderson (from Matt) 11/27/00

6.1m	Symmetric Primary (tool)	\$215,000	\$5560 each
2.4m	Secondary Tool	\$45,000	\$426 each
5.0m	Offset Primary tool (16.5')	\$350,000	\$4323 each
2.4m	Offset Secondary tool (8')	\$75,000	\$426 each
7.3m	Offset tool (24') tool	\$700,000	\$11,945 each

unit cost

5.0m tool/500 = \$700, +4323 = 5,023
[secondary: 75,000/500 = 150, + 426 = 576] } 5596

6.1m tool/500 = 430, + 5560 = \$5990
[secondary: 45,000/500 = 90, + 426 = 516] } 6506

7.3m tool/500 = 1400, + \$11,945 = 13,345 !

} D^4

Kildal's Diffraction Correction with no geometrical blockage

$\eta_i = |1 - l_d|^2$

$l_d = .2095 A_0 \sqrt{\lambda(m)}$

A_0 is the edge electric field.

For the summary Gregorian Design

			total η_{eff}
At $\lambda = 30$ cm	$\eta_i = .92$	with $A_0 = .251$	.82
$\lambda = 60$ cm	$\eta_i = .88$	with $A_0 = .342$	.81

Now increase A_0 to 1. $l_d = .2095 \sqrt{\lambda(m)}$

		η_{III}	total
At $\lambda = 30$ cm	η_i 0.78	1	.78
$\lambda = 60$ cm	η = 0.70	1	.70

This gives a feeling for the effect of the inner shadow boundary of the Cass.

The beam pattern is approximately symmetric on the secondary. It is on the primary. The extreme ray angle is 53°. The energy lost is between 42.1° and 32.1° w.r.t. the feed. From p 2, that's a difference in total fraction of 75.6 − 71.9 = 3.7% where it 33cm and $\left(\frac{44}{33}\right)$5° at 44cm ≈ 4.9%

The average of this is 4.3%. This much gets to the ground at zenith. Anyway, it is lost. Double at ≈ 60cm. 8.3%

Diffraction at the shadow boundary

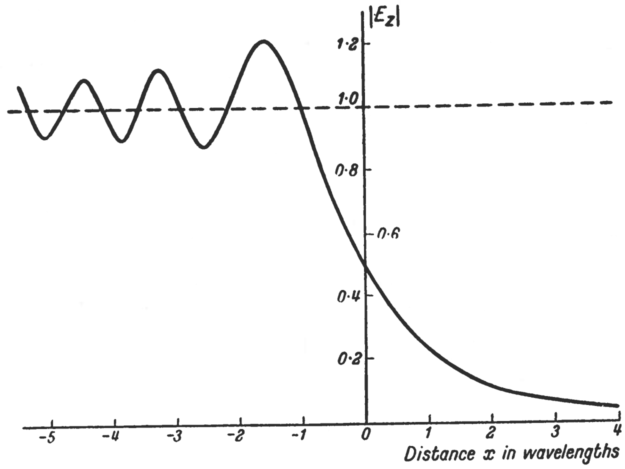


Fig. 11.11. Diffraction of a normally incident E-polarized plane wave of amplitude unity by a perfectly conducting half-plane. The variation of $|E_z|$ with x at a distance of three wavelengths behind the screen.

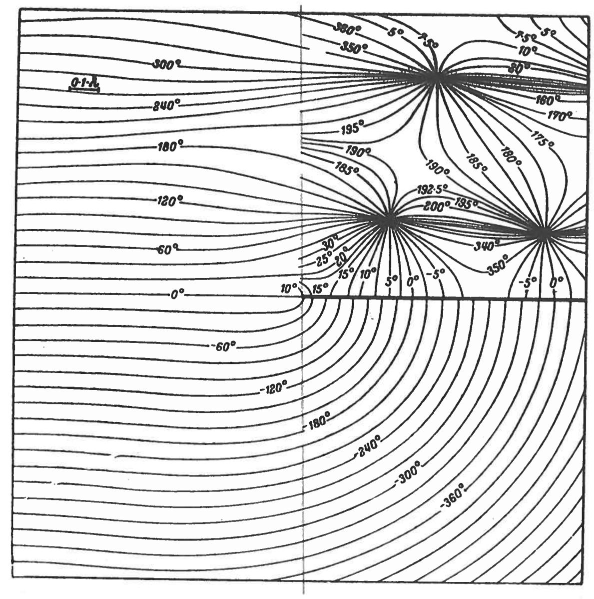


Fig. 11.13. Phase contours of H_z.

Diffraction of a normally incident H-polarized plane wave by a perfectly conducting half-plane.
[After W. Braunbek and G. Laukien, *Optik*, 9 (1952), 174.]

$$E_d \sim \sqrt{\frac{2}{}}\; e^{\frac{\pi}{4}} \frac{\sin\frac{}{2}\;\sin\theta/2\; e^{-ikr}}{(\cos\theta + \cos\alpha_0)\sqrt{kr}}$$

note symmetry α_0 = incident angle

out of phase with the neighboring geometric field by 45

The energy in the strip between < 10 wavelengths, the strip that was sent past the gregorian edge 30 cm has a power that is about that of the geometric field. This is the part that gets to the ground when the antenna is pointed toward the zenith

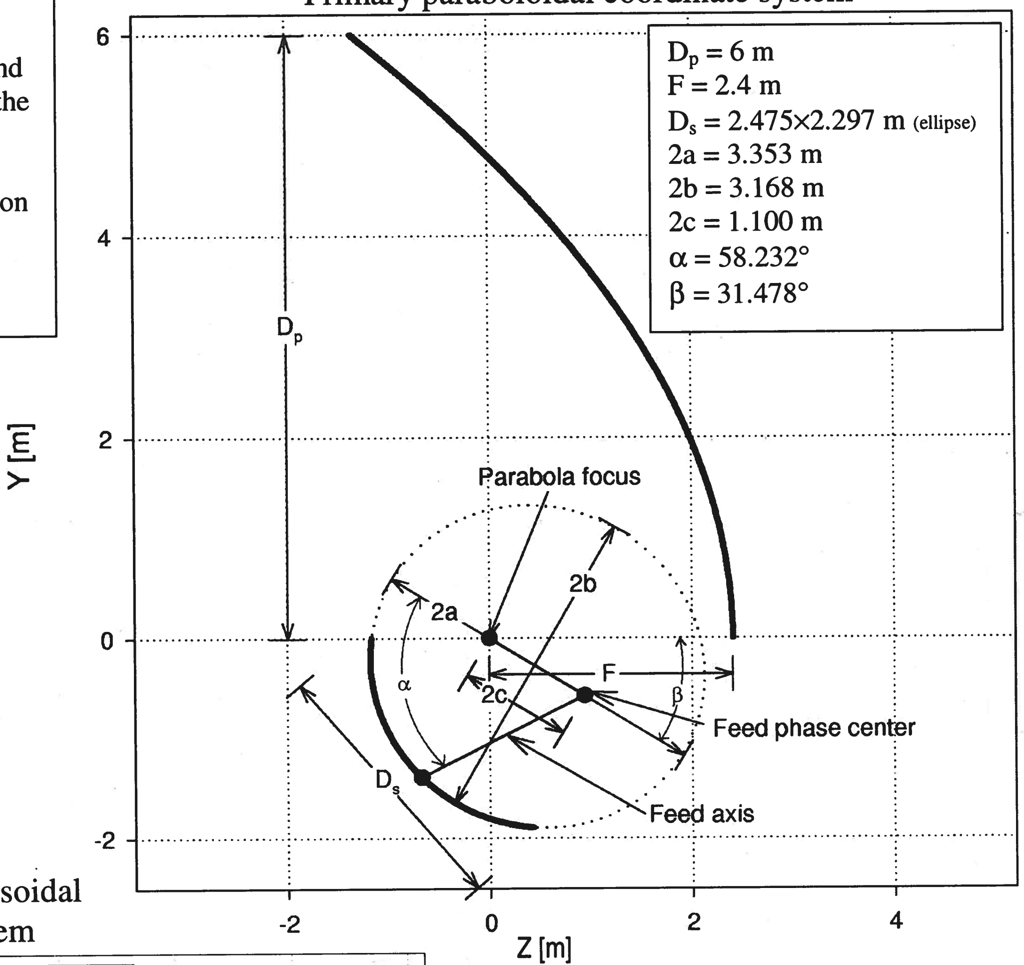
$\simeq 0.55 \times 4.3\% = \boxed{2.4\%}$ 4.8% at 60 cm.

Note also that when the edge is ... side the shadow boundary the ... the out of phase pulled the primary. This goes against making the ... ndary larger the Gregorian for the ... release

Offset Gregorian Strawman Design

The offset Gregorian utilizes a portion of a parabola as the primary and a portion of an ellipse as the secondary. The designs used always satisfy the conical projection condition to mitigate cross-polarization.

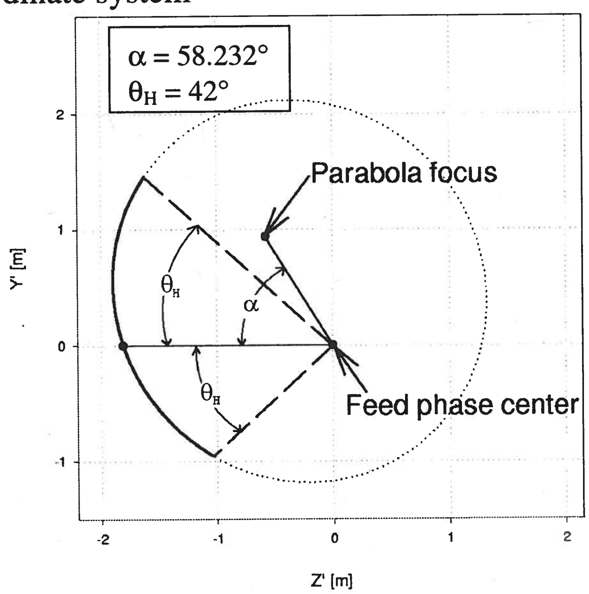
Primary paraboloidal coordinate system



$D_p = 6$ m
$F = 2.4$ m
$D_s = 2.475 \times 2.297$ m (ellipse)
$2a = 3.353$ m
$2b = 3.168$ m
$2c = 1.100$ m
$\alpha = 58.232°$
$\beta = 31.478°$

Secondary ellipsoidal coordinate system

$\alpha = 58.232°$
$\theta_H = 42°$



The unprimed coordinate system (above) places the origin at the focal point of the parabola with the z-axis perpendicular to the vertex. The "unoccupied" focal point of the ellipse is placed at that point. The feed phase center is placed at the other focal point of the ellipse. The projection of the parabola is a circle of diameter D_p (6m).

The primed coordinate system (left) places the origin at the focal point of the ellipse with the feed phase center and the z-axis extends along the feed axis. The projection of the sub-reflector is an ellipse with major and minor axes of 2.475×2.297 m (note D_s above is 2.475 m). The boundaries of the sub-reflector are determined by the intersection of θ_H with the ellipse.

X' → 2.298
Y' → 2.407