Beamformer Design Options

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Numbers for ATA-42:

Baseline length: B = 300m
Maximum RF Frequency: $\nu = 10$GHz
Maximum geometric delay: $\tau_g = \frac{B}{c} = 1\mu$s
Maximum fixed/instrumental delay: $\tau_i = 1\mu$s
Total maximum delay expected: $\tau = \tau_g + \tau_i = 1 + 1 = 2\mu$s
Maximum delay rate: $\frac{d\tau}{dt} = \Omega_e \frac{B}{c} = 7.27 \times 10^{-11} s/s$, where Ω_e is the angular speed of earth: $2\pi/(24 \times 60 \times 60) = 7.27 \times 10^{-5} rad/s$

RF cycle rate (at maximum RF Frequency) (fringe rate) $= \Omega_e \frac{B}{\lambda} = 0.7$Hz. Update cycle on any beamforming operation must be faster than this rate. If we adopt 25x, then update cycles shall be 17.5 Hz $\sim$ 50 ms^{-1}.

DSP/RFSoC:

Sampling rate: 2.048 GSamples/s
Bandwidth: 1.024 GHz
ADC Sample: $1/(1.024 \times 10^9) = 0.9765625$ns $\sim$ 1ns
Number of PFB channels: 4096
PFB channel width: 0.25 MHz
PFB channel time-sample: $1/(0.25 \times 10^6) = 4\mu$s

Spec:

- Ingest all the usable bandwidth from all antennas/LOs.
- Track sources moving across the sky close to sidereal rates.
- Form multiple beams (number to be determined, but range 16 - 64) within the primary beam.

Narrowband approximation to beamforming: is only valid if the bandwidth of the signal is small compared to the reciprocal of the delay being applied. In other words, the phase difference between the edges of the band is small:

$$\phi_1 - \phi_2 = 2\pi\tau(\nu_1 - \nu_2) \tag{1}$$

Our PFB channel is 0.25 MHz wide. Applying the above equation, we end up with a phase difference across the band (worst case scenario) of ± 90 degrees! That is obviously not good. **So even after coarse channelisation, narrowband approximation does not apply**.

What are our options? I list 3 solutions to the problem, with their description, advantages and disadvantages:

1. Apply sample + fractional delay corrections on RFSoC boards (hybrid time/frequency domain)
   - Description: we phase center all antennas to boresight on the RFSoC boards. Sample delays shall be applied in the time domain, straight after ADC, with programmable timeshift registers. Subsample delays shall be applied as phase shifts in the frequency domain; narrow band approximation in this case DOES APPLY (see advantages).
   - Advantages:
     - Backends downstream do not have to worry about phase centering. This is particularly true for a correlator, for example.
     - The time-domain sample delays can take care of most of the delay required to phase up the array. The fractional delay phasors, post PFB, have to correct for values only within 1 ADC sample, i.e. $\sim$ 1ns. Plugging 1ns into equation 1, we see that the phase difference across a PFB channel is $\pm 45 \times 10^{-3}$ degrees, which is very small.
     - All antenna data are local to the FPGAs. Not necessarily a major advantage, but can be handy to elevate edge effects. Beamforming nodes will only ingest a subset of the total bandwidth
   - Disadvantages:
     - Obviously requires FPGA programming + debugging, which I list as a disadvantage.
     - Phase, delay and delay rate are computed/provided from an external compute server. Additional care must be taken to ensure proper communication with the RFSoC boards.
     - FPGA resources are running low. Previous generation SNAP boards could do the phasing, but at this point I'm not sure what we have to compromise on the RFSoC boards.
2. Phase centre on boresight on compute nodes, then beamform.
   - Description: 2 stage beamforming: apply the phasing to the primary beam centre on the beam-former compute nodes. Then apply offsets to beamform within primary beam. This is a 2 stage operation
   - Advantages:
     - GPU programming is easier than FPGA programming.

- Controlling delay and phase updates is easier in theory because the delay and phase computation can be done locally on the compute nodes.

- Disadvantages:
  - We will have to treat every PFB frequency channel as its own "wideband beamformer"; the narrowband approximation does not apply as mentioned in the intro. Sample delays shall be applied with pointer arithmetics and fractional delays shall be applied as FIR filters (with appropriate coefficients/number of coefficients).
  - Other backends, like a correlator, will have to use the phasing kernel if phase centring is required.

3. Up-channelise every antenna stream BEFORE any beamforming operation.

   - Description: ingest data at the beam-forming nodes, and up-channelise to reach a frequency resolution that makes the narrowband approximation valid. Channelising from 250kHz to 10kHz is enough.
   - Advantages:
     - Narrowband approximation: phasing is simply a phase shift to each frequency channel. No sample-time delay required.
     - For SETI search purposes, we already need to perform high resolution spectroscopy. Thus beam-forming and SETI DSP can be combined. This is even more advantageous if/when we want to produce Nbeams > Nants (i.e. doing Nants×FFTs vs Nbeams×FFTs)
   - Disadvantages:
     - Limited to SETI observations: other backend modes, e.g. pulsar/FRB/etc..., cannot directly (or at all) benefit from this beamforming approach as they require less frequency resolution, but definitely more time resolution.

NOTES:

- Assuming antennas have already been phase aligned on boresight, the maximum additional/lesser delay that we need to apply to synthesise a beam at the edge of the primary beam (let's say 2 degrees) is:
  $\Delta\tau = \tau_g - \tau_{g2} = \frac{B}{c} - \frac{B}{c} \times \cos(2\text{deg}) = \frac{B}{c} \sim 6 \times 10^{-10}\text{s}$. Narrow-band approximation is still valid with this delay, following equation 1. In other words, we can simply use phase-shifts on post-PFB data to synthesise our multiple beams within the primary beam.