How Do Changes In The Weather Between Trials Produces Changes In The Calculated T_{sys} And T_{rcvr}?

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Below are recommendations for the weather conditions that should provide a repeatability of no worse than 5 K when using astronomical sources.

Following the same exposition as last week, consider two trials of on-off observation with a perfect system.

$$\begin{gathered} T_{sys,on,1} = G_{on,1} \cdot P_{on,1} = T_{Rcvr,on,1} + T_{Common} + T_A \\ T_{sys,off,1} = G_{off,1} \cdot P_{off,1} = T_{Rcvr,off,1} + T_{Common} \\ \\ T_{sys,on,2} = G_{on,2} \cdot P_{on,2} = T_{Rcvr,on,2} + T_{Common} + T_A \\ T_{sys,off,2} = G_{off,2} \cdot P_{off,2} = T_{Rcvr,off,2} + T_{Common} \end{gathered} \tag{1}$$

From last week:

$$\begin{gathered} T_{sys,off,1}^{Calculated} = T_{Rcvr,1}^{Calculated} + T_{Common} = \frac{P_{off,1}}{P_{on,1} - P_{off,1}} \cdot T_A \\ \\ T_{sys,off,2}^{Calculated} = T_{Rcvr,2}^{Calculated} + T_{Common} = \frac{P_{off,2}}{P_{on,2} - P_{off,2}} \cdot T_A \end{gathered} \tag{2}$$

Changes in atmospheric opacity alter the numerator and denominator in opposite ways. For example, an increase in opacity increases the system temperature (numerator) while decreasing the amount of power from the observed source due to increased attenuation, which decreases the difference in the denominator.

$$\begin{gathered} P_{2,off} = P_{1,off} + \left[T_{Atm} \cdot \left(1 - \mathrm{e}^{-\tau_2 \cdot A_2}\right) - T_{Atm} \cdot \left(1 - \mathrm{e}^{-\tau_1 \cdot A_1}\right) \right] / G \\ P_{2,on} = P_{2,off} + \left[T_A' \cdot e^{-\tau_2 \cdot A_2} \right] / G \\ P_{1,on} = P_{1,off} + \left[T_A' \cdot e^{-\tau_1 \cdot A_1} \right] / G \end{gathered} \tag{3}$$

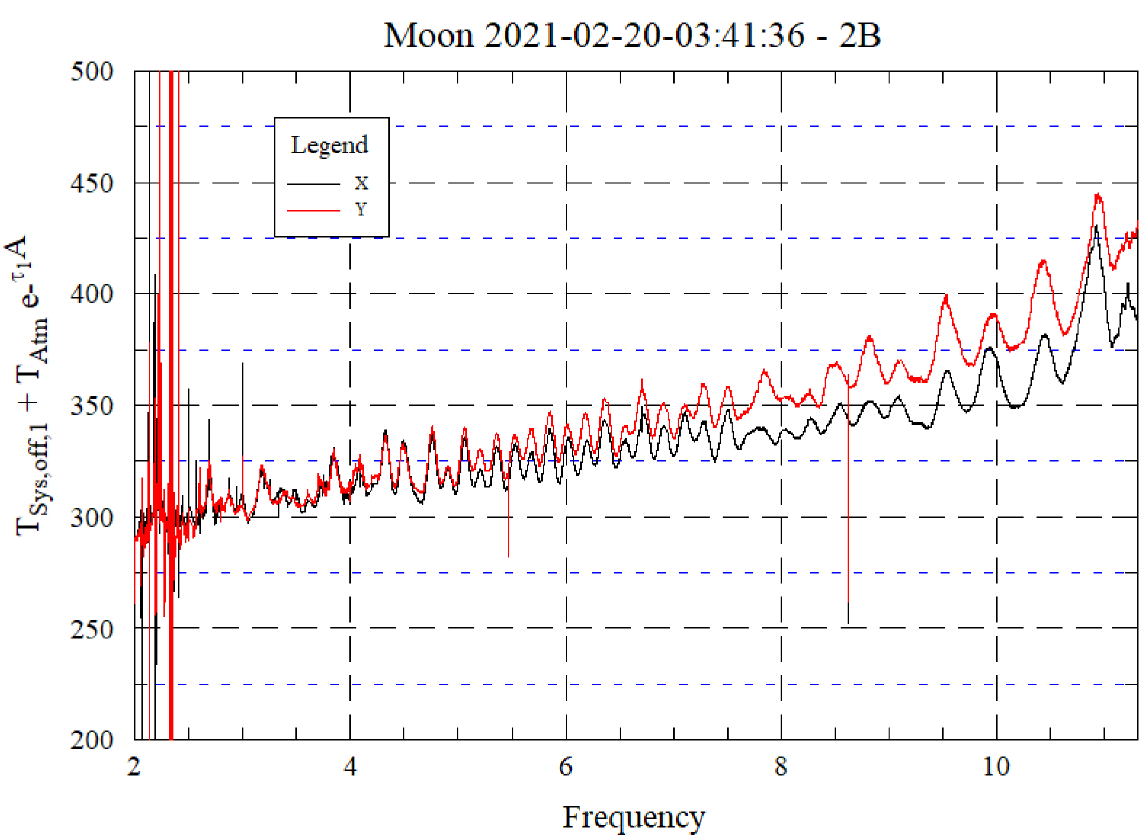
For each trial, τ is zenith opacity, A is the air mass (we will use the plane-parallel approximation of 1/sin(Elev), and T_{Atm} is the equivalent black-body temperature of the atmosphere derived from NWS data. T_A' is the antenna temperature of the source as if it were observed above the Earth's atmosphere (i.e., $T_A = T_A' \cdot e^{-\tau_1 \cdot A_1}$). Substitute (3) and the definition of T_A into (2), assume for simplicity that T_{Atm} and A do not change between trials, and after some algebra you get:

$$\Delta T_{Sys}^{Calculated} = \Delta T_{Rcvr}^{Calculated} = \left(e^{\Delta\tau \cdot A} - 1\right) \cdot \left(T_{Sys,off,1} + T_{Atm} \cdot e^{-\tau_1 \cdot A}\right) \qquad (4)$$

$\Delta\tau$ = the change in zenith opacity between the two trials.

The pipeline uses vertical weather profiles supplied by the NWS to estimate τ_1 and $T_{Atm.}$ Since A, τ_1 and T_{atm} will be similar between antenna and polarization, but each system has different $T_{sys.}$, **we can expect ΔT_{Rcvr} and ΔT_{Sys} will be somewhat but not fully correlated between systems.** Even though eq. 4 does not depend upon the strength of the observed object, we saw last week that a strong source like the Moon increases the accuracy of the calculated T_{sys} used in eq 4.

As an example, the value of the last term in eq. 4 for a recent Moon observation taken under moderate weather are:

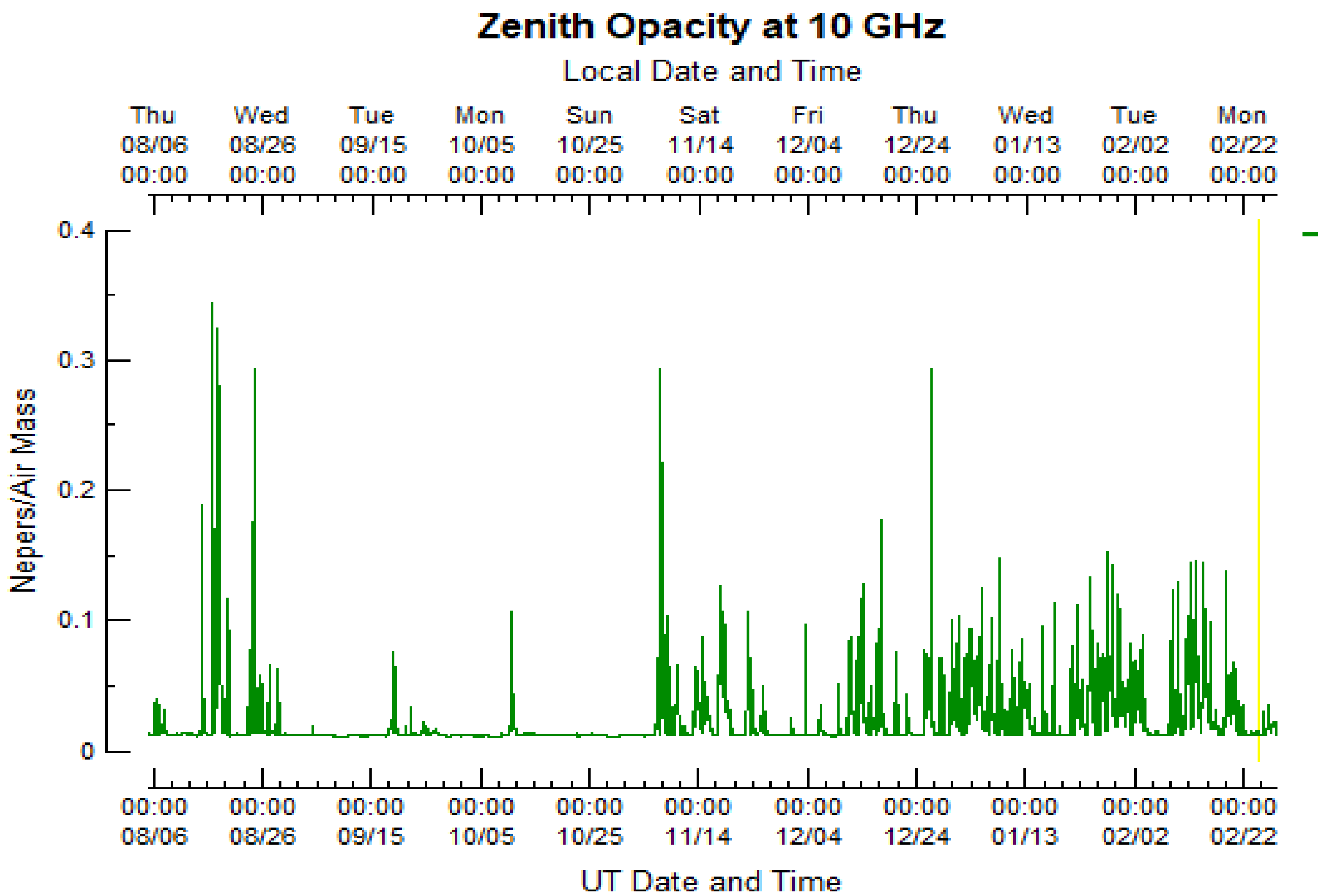


Since 2B is a well-performing system, with other systems having similar or higher T_{sys} we can use the above graph and eq. 4 to ballpark the most stringent fluctuations we can tolerate for accurate, repeatable measurements of T_{rcvr}. For example, if we desire a repeatability of 5 K at 10 GHz (~10% repeatability on T_{rcvr}), we should not observe under conditions when we expect $\tau \cdot A$ to change by $\ln(1 + 5/375) = 0.013$. The allowable change in τ is somewhat smaller since A can range from 1 (at zenith) to ~3 (at elev=18º).

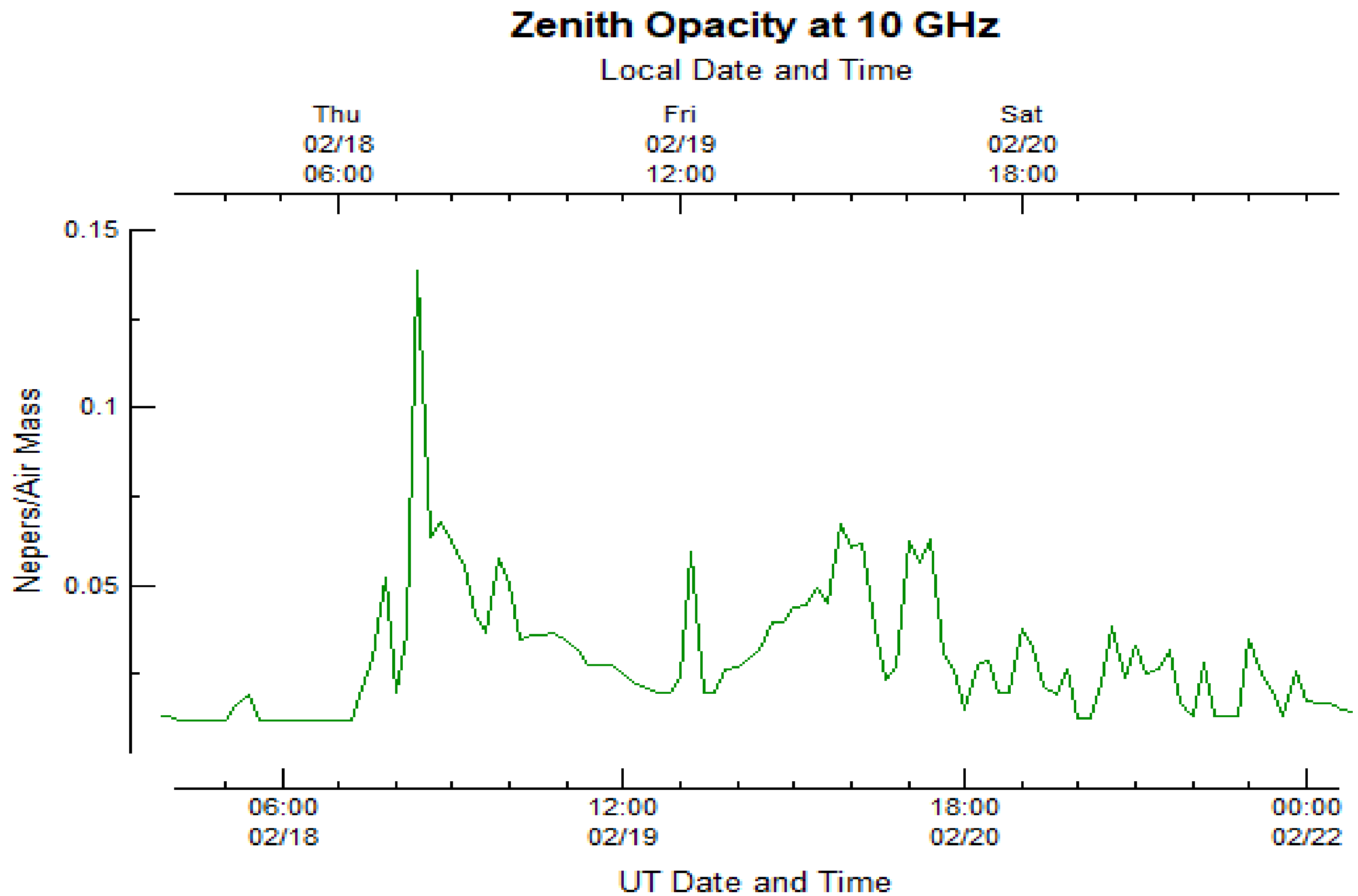
Thus, I suggest observations should only be made when the zenith opacity is expected not to change by ~0.01 at 10 GHz. The Moon remains the best source.

The first plot below shows the zenith opacity, derived from NWS vertical weather profiles, for every hour from August 1,2020 to today. The second plot is the same as the first but zoomed-in around the time of recent Moon observations. The variability we encounter is larger than that plotted as the values are smoothed in time (1 hour) and averaged across the whole sky.

The variations in opacity in the second plot suggests that the most recent Moon observations should show changes of a few tens of K in T_{rcvr} and T_{sys} from trial to trial. The top plot suggests that the times we can make accurate T_{rcvr} and T_{sys} must be picked judiciously during the winter months, as there are few times when the weather is stable. There appears to be many more opportunities of stable weather during the summer and early fall months.



Zenith Opacity at 10 GHz
Local Date and Time
Thu 08/06 00:00
Wed 08/26 00:00
Tue 09/15 00:00
Mon 10/05 00:00
Sun 10/25 00:00
Sat 11/14 00:00
Fri 12/04 00:00
Thu 12/24 00:00
Wed 01/13 00:00
Tue 02/02 00:00
Mon 02/22 00:00
Nepers/Air Mass
0.4
0.3
0.2
0.1
0
00:00 08/06
00:00 08/26
00:00 09/15
00:00 10/05
00:00 10/25
00:00 11/14
00:00 12/04
00:00 12/24
00:00 01/13
00:00 02/02
00:00 02/22
UT Date and Time



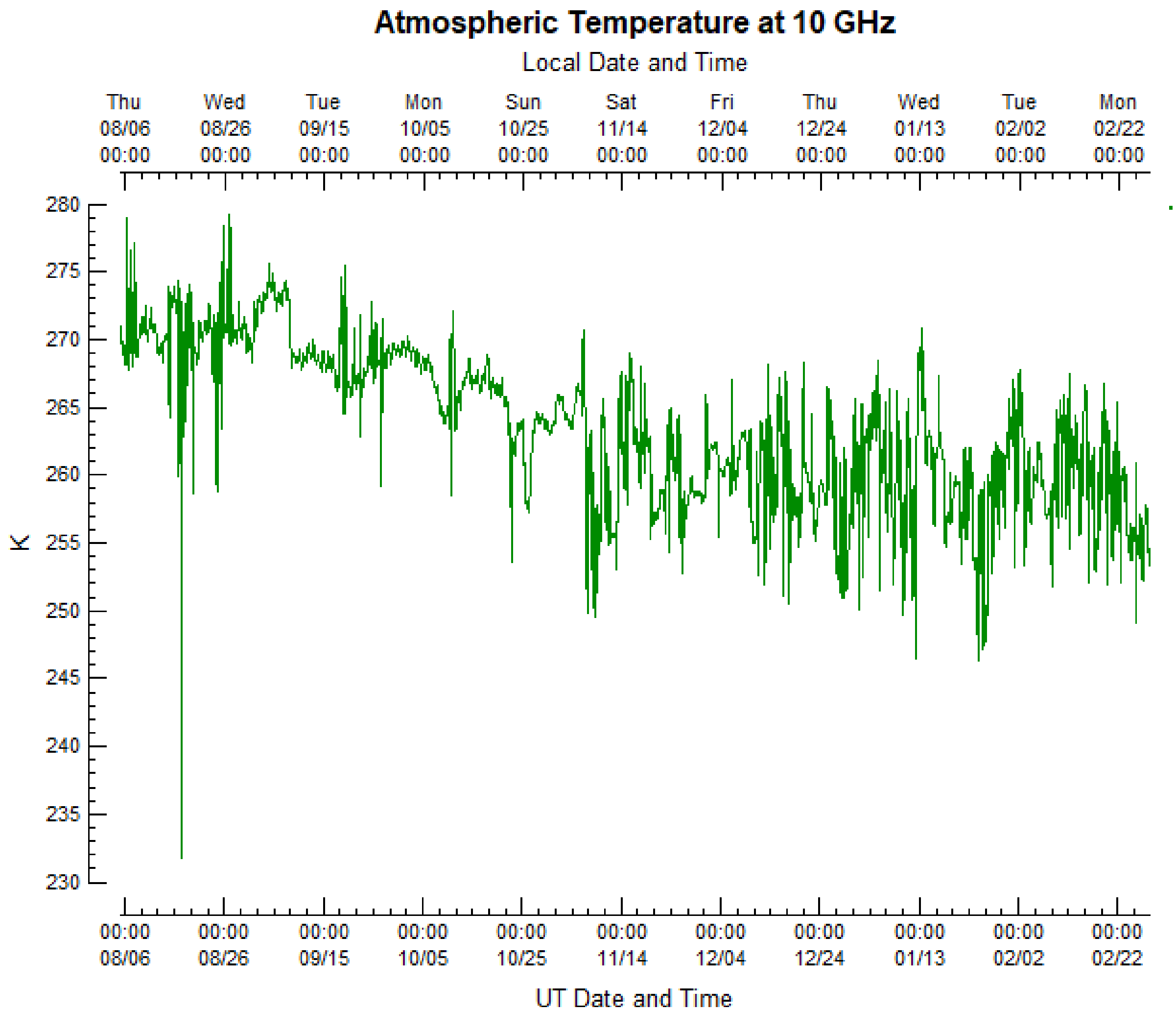
Just to be complete: do changes in T_{Atm} from trial to trial alter significantly the calculated values for T_{rcvr} and T_{sys}? Assuming the opacity does not change between trials, only the numerators change between trials:

$$P_{2,off} = P_{1,off} + [\Delta T_{Atm} \cdot (1 - e^{-\tau_1 \cdot A_1})]/G \qquad (4)$$

Substitute (4) and the definition of T_A into (2), gives:

$$\Delta T_{Sys}^{Calculated} = \Delta T_{Rcvr}^{Calculated} = \Delta T_{Atm} \cdot (1 - e^{-\tau \cdot A}) \qquad (5)$$

The following plot, which shows $T_{Atm\ for}$ every hour since Aug 1, suggests that ΔT_{Atm} between trials is probably no more than 10 K. Since the worse value for τ from the above graph is 0.3, eq. 5 suggests that changes in T_{Atm} between trials will produce changes in the calculated T_{sys} and T_{rcvr} that are typically always less than 3 K. That is, changes in τ between trials reduce repeatability far more than changes in T_{Atm}.



Atmospheric Temperature at 10 GHz
Local Date and Time
Thu 08/06 00:00
Wed 08/26 00:00
Tue 09/15 00:00
Mon 10/05 00:00
Sun 10/25 00:00
Sat 11/14 00:00
Fri 12/04 00:00
Thu 12/24 00:00
Wed 01/13 00:00
Tue 02/02 00:00
Mon 02/22 00:00
K
280
275
270
265
260
255
250
245
240
235
230
00:00 08/06
00:00 08/26
00:00 09/15
00:00 10/05
00:00 10/25
00:00 11/14
00:00 12/04
00:00 12/24
00:00 01/13
00:00 02/02
00:00 02/22
UT Date and Time