MEMO: Beamforming, phasing, and other practicalities

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Beamforming, in the receiving paradigm, is the process of coherently summing signals from receivers to increase sensitivity towards a particular direction. To achieve that, one needs to delay the signal from the different receivers by amounts to compensate for the fact that receivers are distributed geometrically on the ground. These delays are called the geometric delays. On the other hand, instrumental delays arise due to the differences in path lengths from each element to the central processing system. Here we will focus on geometric delays, as instrumental delays are considered to be fixed (for the duration of a given observation) and easily measured as part of any delay calibration scheme.

Both delay terms, in practice, are relative to some reference point. This reference point can be anywhere, but it is best if it coincides with the location of the array's reference antenna, usually one of the most performant. Therefore, the delay applied on the reference antenna would always be exactly 0, whereas the rest of the receivers will have +ve and -ve values.

1) Determining delays:

Computing the needed delays requires an accurate position of the receiving stations, in some coordinate system, and the pointing position that we are trying to phase the array to. The Earth-Centered-Earth-Fixed (ECEF) coordinate system is widely used, and is what we will be utilizing in our case.

The ECEF coordinate system is a 3D cartesian coordinate system used to describe positions on earth. The Z-axis is always pointed towards the North pole, and the X-axis lies on the equator plane and points towards the prime meridian. This is described in the image below:

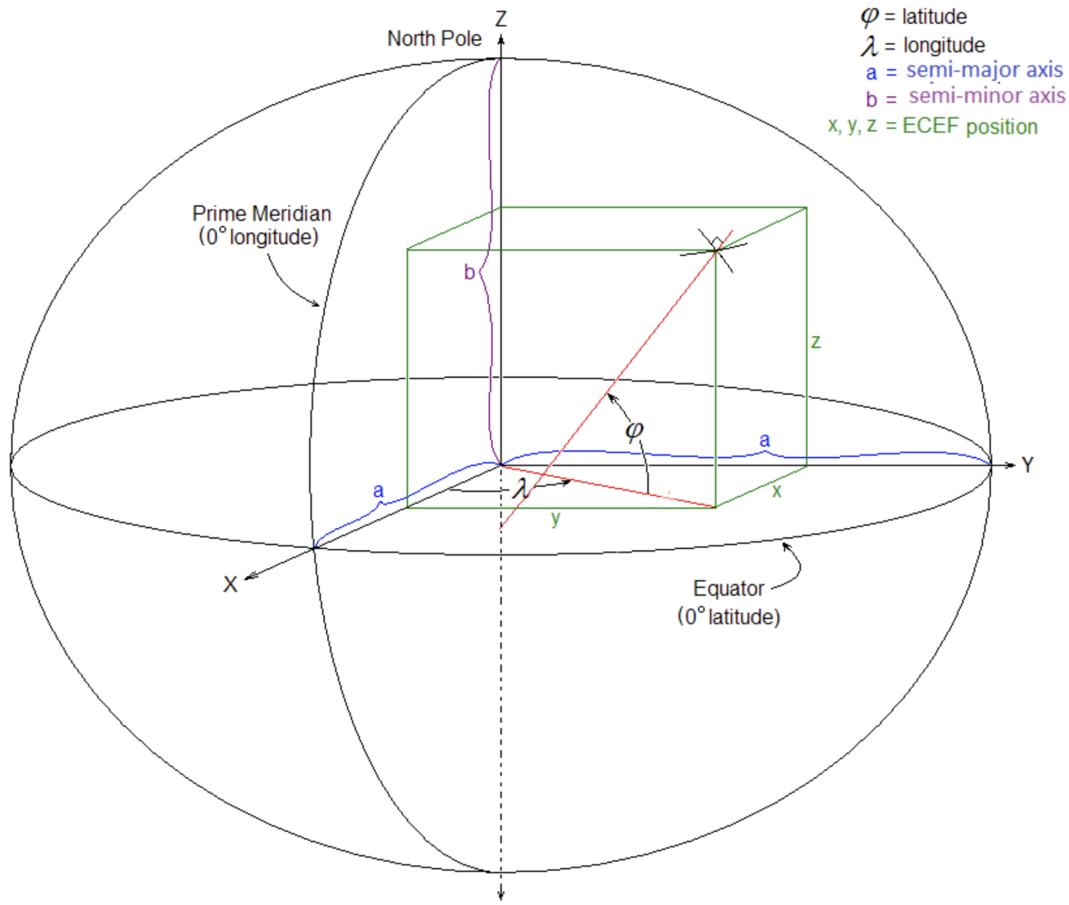


Image taken from wikipedia

Every ATA antenna position can be described by an XYZ coordinate triplet.

When trying to measure the geometric delays towards a particular source S that moves across the sky following a sidereal rate, it is advantageous to transform the ECEF coordinate system into another that is more useful. In this case, we can use the widely used in radio astronomy, the UVW, described in the image below:

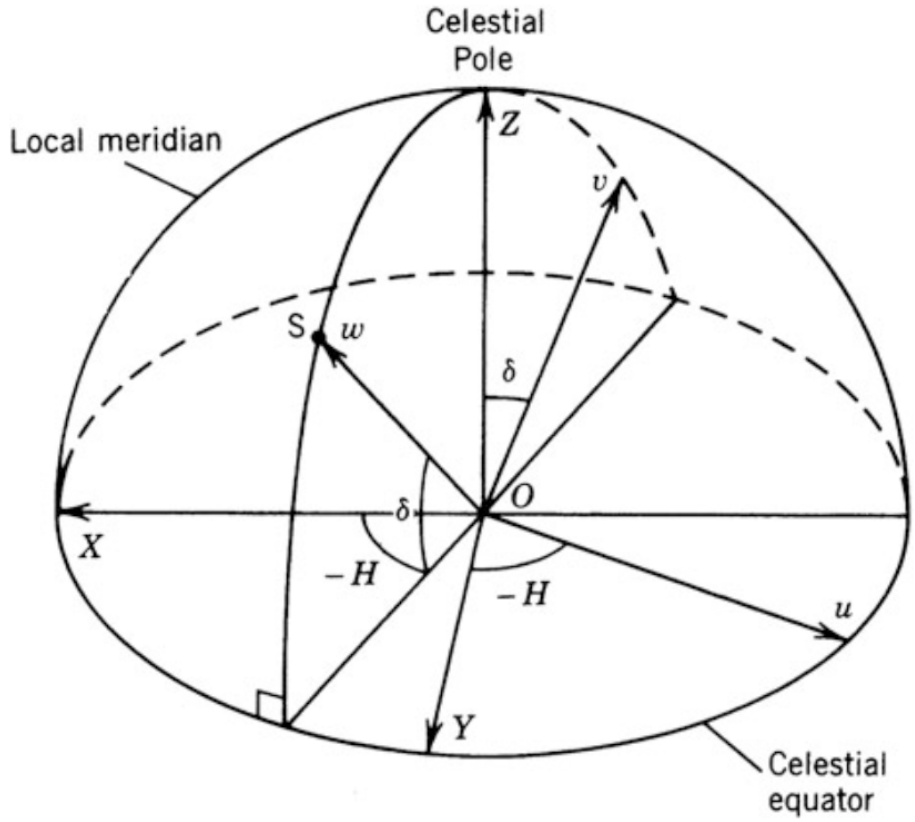


Figure 4.2 from Interferometry and Radio Synthesis, Thompson, Moran and Swenson, third edition.

The W-axis in this coordinate system always points towards the source S. Therefore, the W-coordinates represent the radial position of all the antennas with respect to the line of sight. As radio waves propagate at the speed of light, the geometric delay across the array will then be the W-coordinates divided by c.

Transforming between the 2 coordinate systems, and obtaining geometric delays, is straightforward, requiring <u>1 translation and 2 rotations:</u>

- <u>Translation:</u> changes the reference position of the ECEF system from earth's center of mass to the local reference (which could be the reference antenna that is being used, the center of the array, or anything else…). This translation can be done before or after the following rotations.
- <u>Rotation 1:</u> around the Z-axis, anticlockwise, by the value `longitude-HA`, producing (X', U, Z) frame

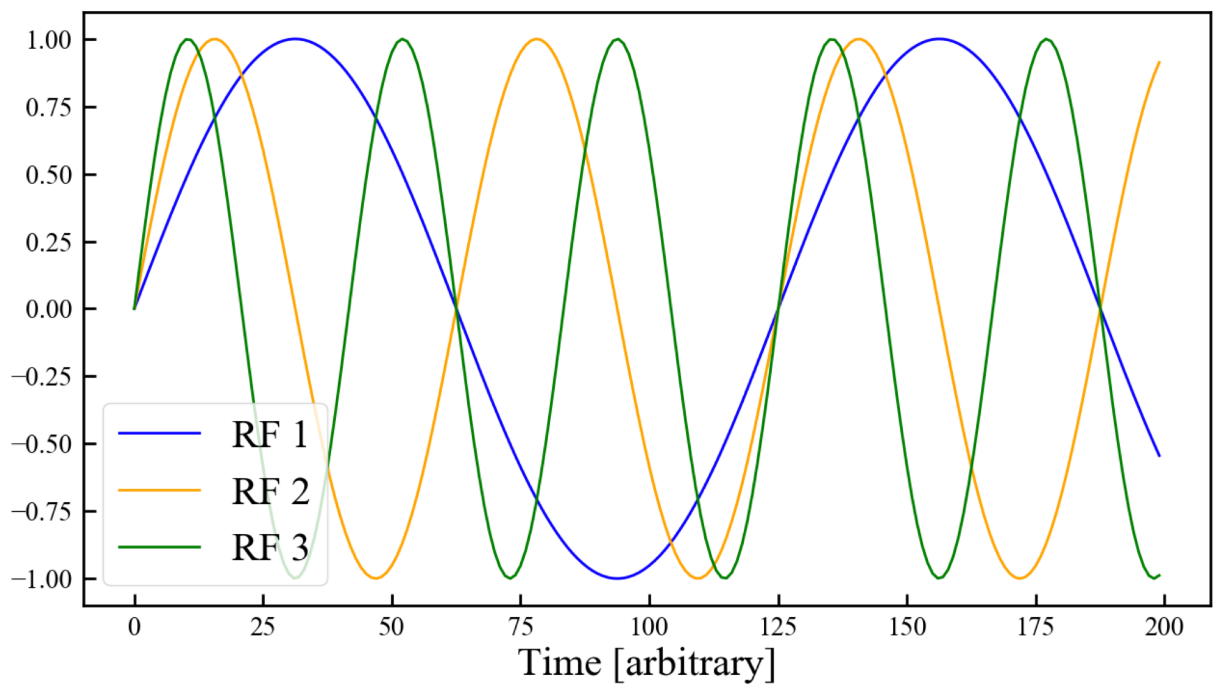
- Rotation 2: around the U-axis, by the value `-declination`, producing the (W, U, V) frame. Shuffle axii to produce (U, V, W).

The translation in the above matrix transformation is commutative, meaning that the translation can either be performed before or after the set of rotations.

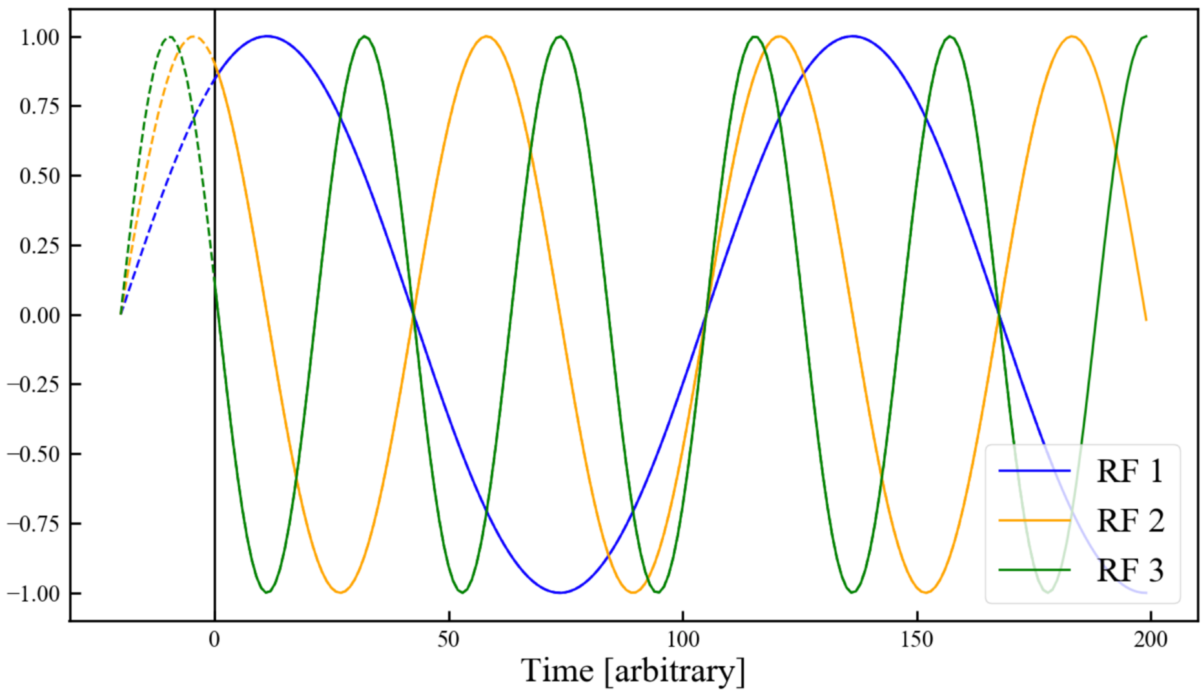
In summary, determining geometric delays, towards a particular source on sky, requires the following:
1) An accurate position of receiving elements, along with a desired pointing direction
2) A translation to recenter the coordinate system at the center of the array (or a reference antenna).
3) Two rotations, one counter-clockwise along the Z-axis, and another counter-clockwise along the u-axis.
4) The W-coordinates are divided by the speed of light to obtain delays in units of time.

2) Delays in the frequency domain:

Imagine the below is an RF signal output of a wideband receiver composed of 3 simple sin waves (RF1, RF2, RF3) of increasing frequencies (w1, w2, w3).



Say that we would like to delay this signal by some amount $\tau = 20$ [some arbitrary time unit]. As seen in the figure below, we *push* the sin waves 20 time samples towards the positive direction. This time-lag introduces a *phase shift* for each sin wave that is proportional to each of their frequencies, as seen in the below plot:



The value of the phase shift is determined by: $\phi = 2\pi\omega\tau$, where ω is the frequency in Hz and τ is the delay in seconds.

Given the above equation, we notice that <u>a delay in the time-domain</u> would manifest as <u>a linear phase ramp in the frequency domain</u>.

3) Beamformers and heterodyne receivers:

It is fairly common in radio astronomy to downconvert high RF frequencies to lower baseband intermediate frequencies (IF) by mixing the signal with local oscillators. Although this complicates the DSP design of the system at hand, it is reasonably cheaper to digitize signals at lower frequencies than it is to do so at high frequencies.

Delaying RF signals in the IF domain requires an additional frequency-independent phase offset. The math is briefly described here.
Say x_1 and x_2 are the output of 2 receivers mounted on 2 antennas, pointed at the source S:

$x_1(t) = s(t)$
$x_2(t) = s(t - \tau)$

x_2 is a delayed version of x_1 by a value of τ. To down-convert the RF signals from antennas 1 and 2, the signals are multiplied by an oscillator at frequency $-f_0$, to produce IF baseband signals, y_1 and y_2.

$$y_1(t) = x_1(t)e^{-j2\pi f_0 t} = s(t)e^{-j2\pi f_0 t}$$
$$y_2(t) = x_2(t)e^{-j2\pi f_0 t} = s(t - \tau)e^{-j2\pi f_0 t}$$

The beamfomer has to compensate for the delay of τ between antennas 1 and 2 by introducing a lag of value + τ, to produce $y'_2(t)$

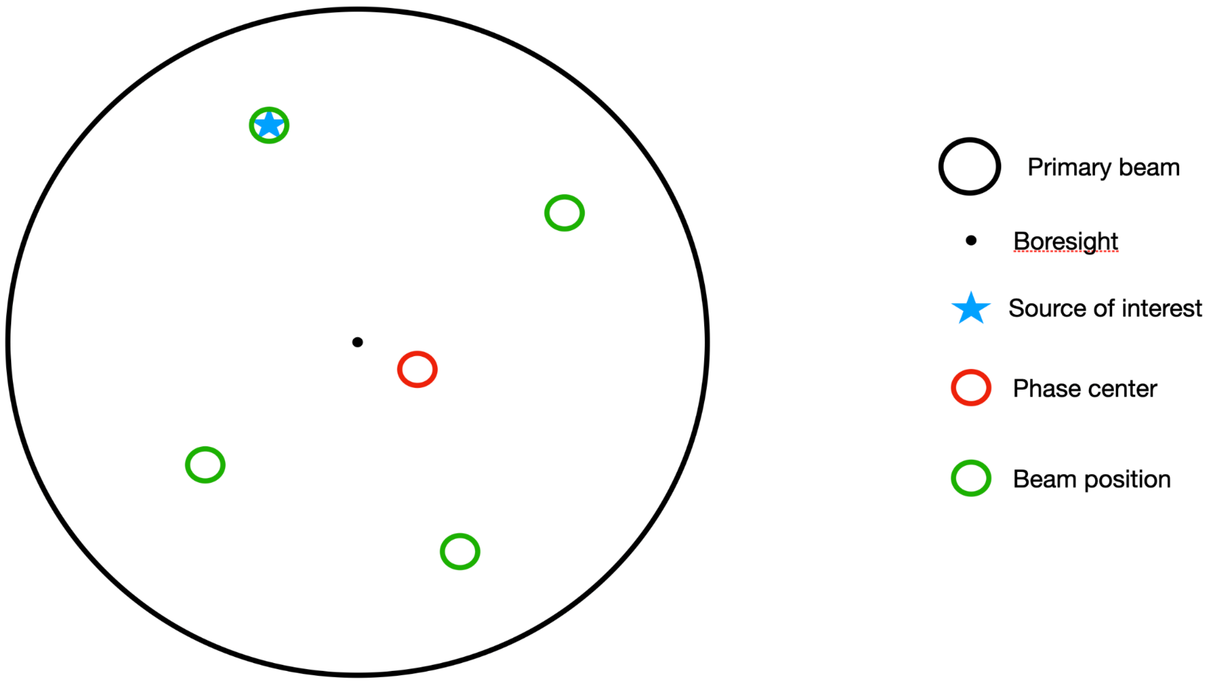
$$y'_2(t) = y_2(t + \tau) = x_2(t + \tau)e^{-j2\pi f_0 (t+\tau)} = s(t)e^{-j2\pi f_0 t}e^{-j2\pi f_0 \tau}$$

We notice that $y'_2(t)$ differs by the frequency-independent phase of $e^{-j2\pi f_0 \tau}$, which have to be compensated for in the beamformer. This phase shift is seen because the delay is exhibited in RF, but removed in IF.

4) Multi-beam synthesize; steering from phase center:

Terminology:

Before describing multi-beam synthesise, let us first define the nomenclature. This is all depicted in the following figure:

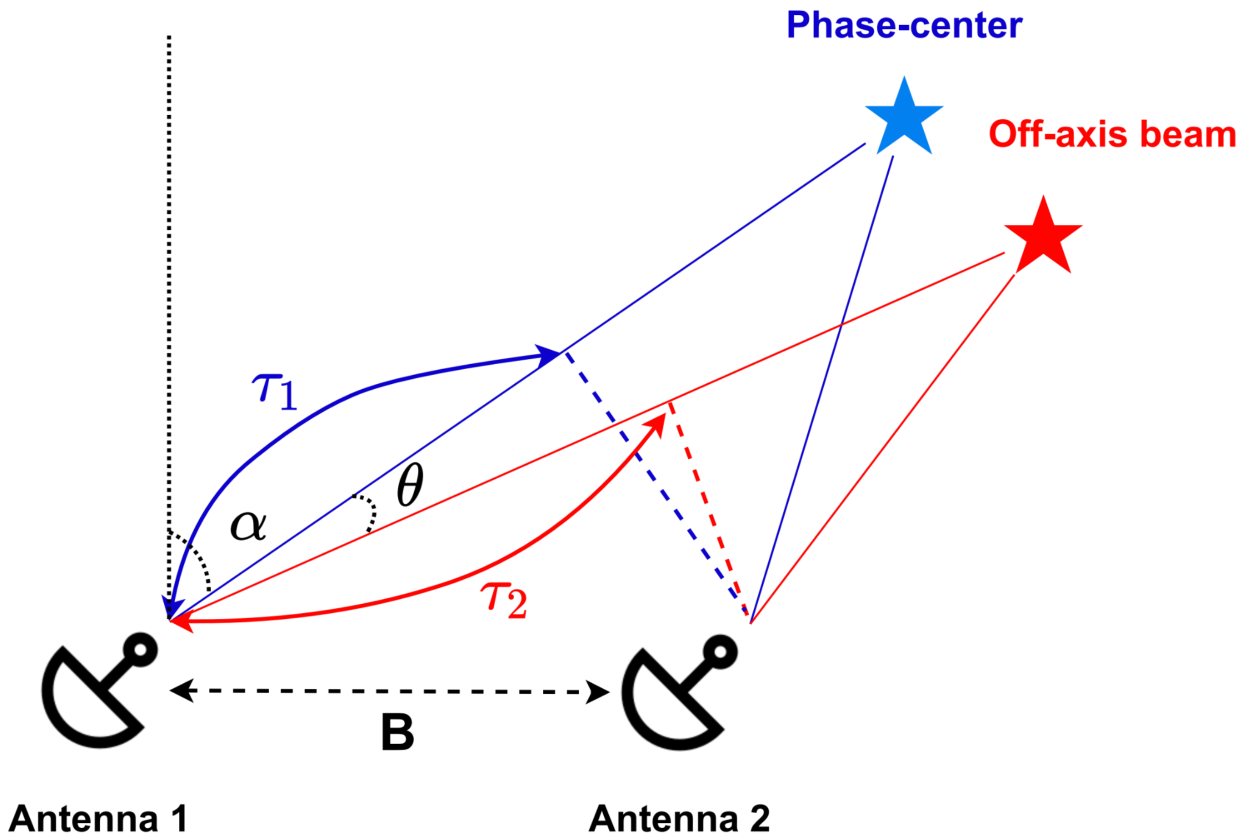


Primary beam: the first order spatial response of individual antennas on sky.
Boresight: antenna's mechanical pointing - center of the primary beam
Phase center: the delay engine's electronic tracking position
Source(s) position: position of the source(s) of interest
Beam position: (ideally) anywhere within the primary beam. One "special" beam is the one directly on the phase center where no delay differentials have to the applied. For any other beam, one has to steer away from phase center.

… confusing, but all of the above can be spatially non-coincident, as seen in the figure.

Multi-beam synthesise:

The ATA's digitizer boards supports a delay engine that can is utilized to phase center the antenna voltages to a particular point in sky. We call this mechanism the "delay engine". By summing the complex voltages straight out of the delay engine without alteration, we synthesize an "on-phase center" beam.



Synthesizing multiple beams on the sky at multiple positions within the primary beam requires delaying antennas appropriately to *steer* the electronic beam from phase-center towards the desired locations. This is illustrated in the above figure. Antenna 1 lags behind antenna 2 by a value of τ_1 when electronically pointing at the phase center. As the delay engine already compensates for τ_1, in order to form a beam on an off-phase center position, the delay that needs to applied on antenna 1 is: $\Delta\tau = \tau_2 - \tau_1$. The geometric delays τ_1 and τ_2 are calculated using the transformations defined above.

If $\Delta\tau$ is small enough such that the narrowband approximation to beamforming still holds (i.e. the phase difference between the edges of the polyphase filterbank channel is small), one can simply alter the phase of each frequency channel of each antenna in order to steer them away from the phase center to another position of preference.

Given the above, and in order to steer the beam from the phase center, the equation that describes the phase offset, for each frequency channel of each polyphase filterbank, is the below:

$$\Phi_{total}(\nu_{IF}) = \phi_{delay}(\nu_{IF}) + \phi_{fr}$$

Where Φ_{delay} is the delay term and Φ_{fr} is the term compensating for delay RF signals in the IF as described above.

$$\Phi_{delay}(\nu_{IF}) = 2\pi \, \Delta\tau \, \nu_{IF}$$
$$\Phi_{fr} = 2\pi \, \Delta\tau \, f_0$$

ν_{IF} is the IF frequency of the polyphase filterbank channel in the range of 0 to BW Hz, f_0 is the observing RF "sky" frequency, both in Hz.

For the case of the ATA, f_0 is replaced by $f_0 - BW/2$ where BW is the total bandwidth delivered by the digitizers, currently $BW = 1.024$ GHz. The reason behind this is that the ATA system is a lower side-band. Moreover, here we have also omitted the instrumental phase, $\phi_{inst}(\nu_{IF})$, and we assume that this tern has been removed by the delay engine prior to the beamformer module. Otherwise, this term will need be added to the above equation.