Sources of Variations in T_{sys} Measurements

Ronald Maddalena
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I believe I have a reasonable, potential causes of the variations in the high-frequency values for T_{SYS}, T_{Rcvr}, and SEFD from one observing run to the next. The same cause will also explain why we are seeing variations between the X and Y polarization for a single antenna between different sessions. Essentially, the variations come from Tau A and Cas A having weak intensities as one goes higher in frequency and the algorithm that is required for processing on-off observations.

The clearest way to show this is to consider a perfect antenna with perfect electronics, and with all ancillary quantities known to infinite precision. I will then add small, expected perturbations to the performance of this perfect system to see if these could induce the variations we are seeing.

In two back-to-back on-off observations with a single antenna and single polarization, we measure 4 spectra, $P_{on,1}$, $P_{off,1}$, $P_{on,2}$, $P_{off,2,}$ which are in units of backend counts. Subscripts designate the observing session (1 and 2) and whether the spectra were with the antenna pointing at the source or at the off-source position. In a perfect system, the gain (G), in units of K-per-backend count, is the same for all P. The system is also linear, and the following identities between Tsys and the measured P are absolutely true:

$$\begin{aligned} Tsys_{on,1} &= G \cdot P_{on,1} \\ Tsys_{off,1} &= G \cdot P_{off,1} \\ Tsys_{on,2} &= G \cdot P_{on,2} \\ Tsys_{off,2} &= G \cdot P_{off,2} \end{aligned} \quad (1)$$

I will break up Tsys into the sum of three components:

Designation	Definition
T_{Rcvr}	Contribution that is different between polarization (and antennae). This is the ‘classic’ value for T_{rcvr} plus anything else that is not common between polarization (and antennae).
T_{Common}	Contributions from CMB, atmosphere, spillover, Milky Way, etc., all of which are common between polarization (and antennae).
T_A	Contribution from the astronomical source in the ‘on’ position that is common between polarization (and antennae). It is zero in the ‘off’ position.

All three of these vary with frequency. In this ‘perfect’ system we also assume we know T_A and T_{Common} to infinite precision and that they do not vary with observing session. For observing session 1,

$$\begin{aligned} G \cdot P_{on,1} = Tsys_{on,1} &= T_{Rcvr,1} + T_{Common} + T_A \\ G \cdot P_{off,1} = Tsys_{off,1} &= T_{Rcvr,1} + T_{Common} \end{aligned} \quad (2)$$

T_{rcvr} is the quantity we have been lately comparing between observing sessions. Since we have two equations in two unknowns, G and T_{Rcvr}, there is only one way to derive T_{rcvr} from the measured P:

$$T_{Rcvr,1} = T_A \cdot \frac{P_{off,1}}{P_{on,1} - P_{off,1}} - T_{Common} \qquad (3)$$

Note that equation 3 involves a large number in the numerator and a difference between two large numbers in the denominator. Unfortunately, the value in the denominator becomes smaller with increasing frequency for Cas A and Tau A due to their negative, synchrotron spectral indices.

Perturbation 1 to the Perfect System: Since we actually don't know with infinite precision T_A and T_{Common}, could our lack of knowledge produce the variations we are seeing between sessions, from antenna to antenna, and from polarization to polarization on the same antenna? Since we are using the same T_{Common} for all systems in a single observing session, any error in the value we use in the pipeline for T_{Common} would produce similar (smooth) frequency offsets for all polarization and antenna. Since we are using the same T_A for all systems in a single observing session, any error in the value for T_A would produce similar (smooth) frequency scaling for all polarization and antenna. These are not the kind of seemingly random variations we are noticing within a session at high frequencies. **In the model presented here, the observed variations is probably not due to uncertainties in the values we use for T_{Common} and T_A.**

Perturbation 2 to the Perfect System: From the definition of the various quantities we have been using in our perfect system, a second on-off observation will produce an identical T_{Rcvr} to that derived in the first on-off. Mathematically, the perfect system requires $P_{on,2} = P_{on,1}$ and $P_{off,2} = P_{off,1}$. Here we will break perfection by having $P_{off,2}$ be slightly different from $P_{off,1}$ (i,e., $P_{off,2} = P_{off,1} + \Delta P_{off}$) and then calculate whether reasonable values for ΔP_{off} could produce the observed variations in T_{Rcvrr}.

$$\Delta T_{Rcvr} = T_{Rcvr,1} - T_{Rcvr,2} = \left[T_A \cdot \frac{P_{off,1}}{P_{on,1} - P_{off,1}} - T_{Common} \right] - \left[T_A \cdot \frac{P_{off,2}}{P_{on,2} - P_{off,2}} - T_{Common} \right]$$

$$\Delta T_{Rcvr} = T_A \cdot \left[\frac{P_{off,1}}{P_{on,1} - P_{off,1}} - \frac{P_{off,1} + \Delta P_{off}}{P_{on,1} - (P_{off,1} + \Delta P_{off})} \right]$$

Assuming ΔP_{on} is small in comparison with P, a first order expansion of the above equation produces:

$$\Delta T_{Rcvr} = \left[\frac{1}{P_{on,1} - P_{off,1}} - \frac{P_{off,1}}{(P_{on,1} - P_{off,1})^2} \right] \cdot T_A \cdot \Delta P_{off}$$

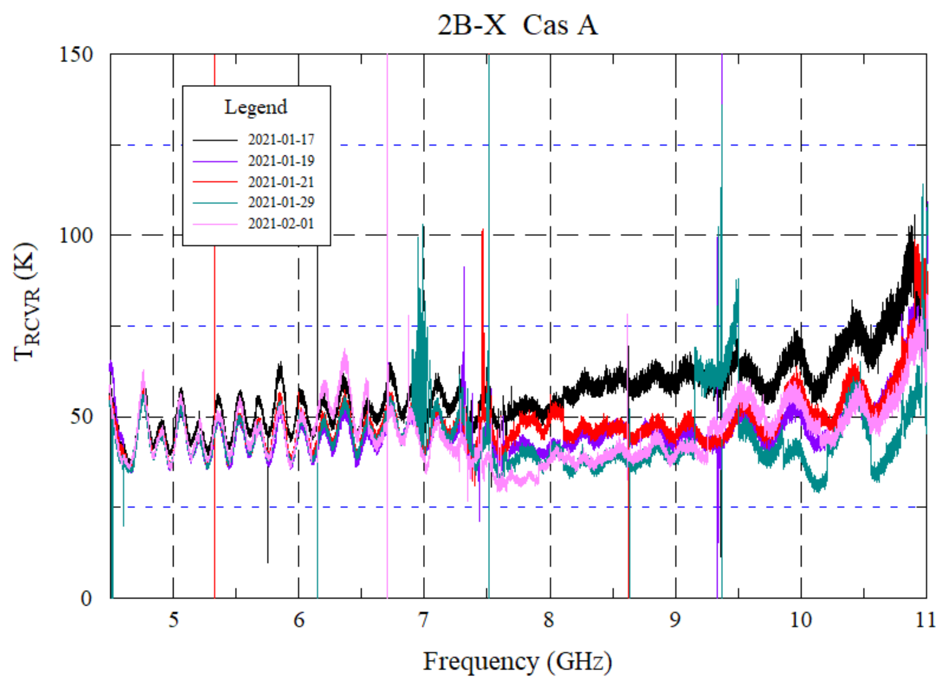
Substituting the definitions asserted in equation 2:

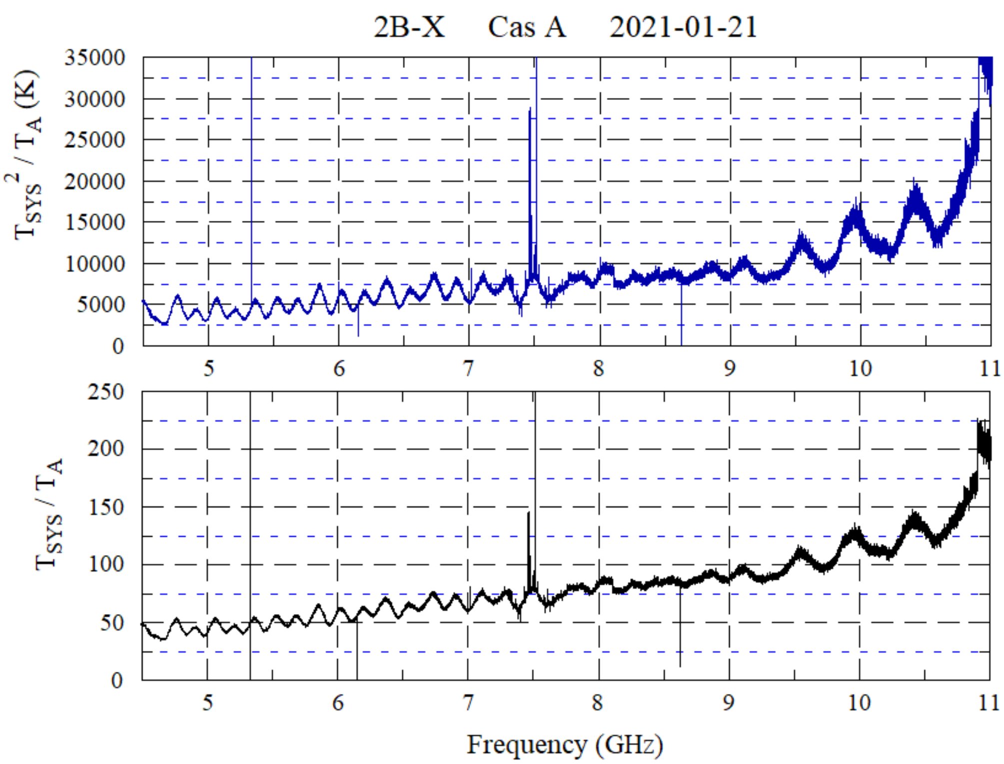
$$\Delta T_{Rcvr} = \left[1 - \frac{Tsys_{off}}{T_A} \right] \cdot G \cdot \Delta P_{off}$$

Finally, substituting an identity implied from equation 1:

$$\Delta T_{Rcvr} = \left[1 - \frac{Tsys_{off}}{T_A} \right] \cdot \Delta Tsys_{off} \qquad (4)$$

Using a 'good' receiver and antenna (2B-X) as an example, at 11 GHz the measured $Tsys/T_A$ is about 200 and ΔT_{rcvr} is about ±20 K. **Thus, a ±0.1 K change in Tsys, no matter the cause, is sufficient to produce the ±20 K variations we are seeing between observing runs in 2B.** Since the performance of the different polarization of an antenna is mostly independent of each other, we can expect variations in X to not be correlated to those in Y, which is what we observe. It is not clear how one can measure the time variability of such small differences in T_{rcvr}.





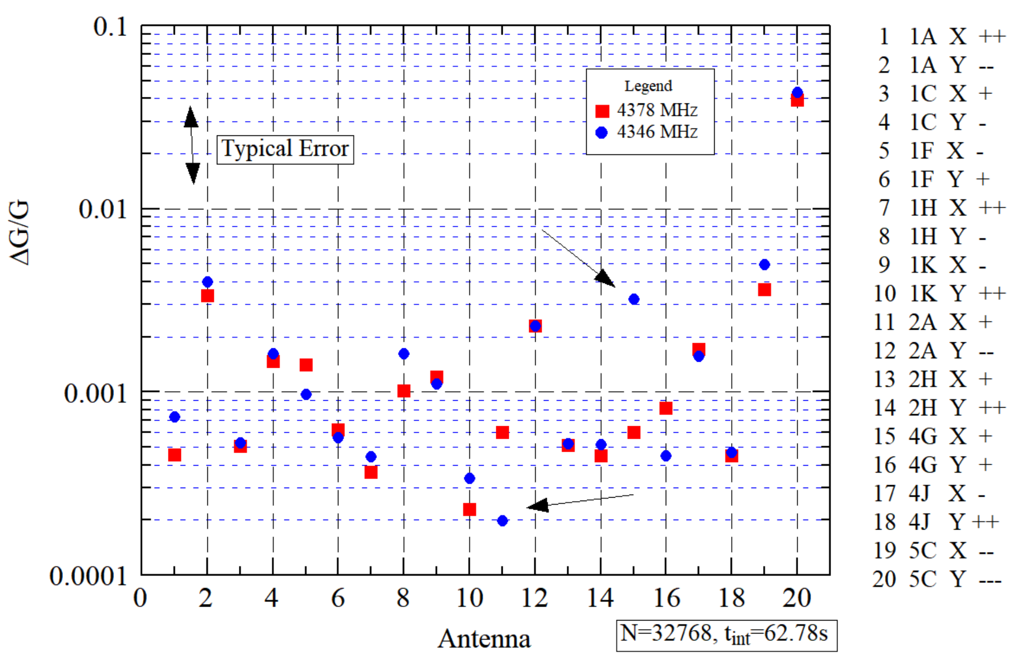
Perturbation 3 to the Perfect System: Another expected deviation of a real system from the perfect one we are modeling is if the gain, G, changes between on and off observations of different observing runs. Here, we revert back to having $P_{on,2} = P_{on,1}$ and $P_{off,2} = P_{off,1}$, but now allow the gain to to be different for **only** the 'on' source observation of the second trial. That is, $G = G_{on,1} = G_{off,1} = G_{off,2}$, but $G_{off,2} = G\cdot(1 + \Delta G/G)$. Following the same steps and substitutions as before:

$$\Delta T_{Rcvr} = T_A \cdot \left[\frac{P_{off,1}}{P_{on,1} - P_{off,1}} - \frac{(1+\frac{\Delta G}{G}) \cdot P_{off,2}}{P_{on,2} - (1+\frac{\Delta G}{G}) \cdot P_{off,2}} \right]$$

$$\Delta T_{Rcvr} = T_A \cdot \frac{\Delta G}{G} \cdot \left[\frac{P_{off_1}}{P_{on_1} - P_{off_1}} - \frac{P_{off_1}^2}{(P_{on_1} - P_{off_1})^2} \right] \quad (5)$$

$$\Delta T_{Rcvr} = \frac{\Delta G}{G} \cdot \left[T_{sys,off} - \frac{T_{sys,off}{}^2}{T_A} \right]$$

For antenna 2B-X $T_{sys,off}^2 / T_A$ is 30,000 at 11 GHz. While we have yet to measure the gain stability of 2B-X, we have measurements for ten other antennae. The median measured value for ΔG/G is 0.0006, but the range in values for different antennae and polarization is rather large (0.0002 to 0.005). **If 2B-X has the median measured value of gain stability, then we would expect to see ±18 K variations in T_{rcvr}, which is very similar to the ±20 K range we actually observe across various observing sessions for Cas A and Tau A for 4B-X.** Since the gain is mostly independent for each polarization, we can expect time variations in X not to be correlated to those in Y.



Conclusion and Recommendations: Small changes iwe expect or have measured in either the system temperatures or gains between the on and off observations for different observing sessions can produce the types of variations we have been seeing at high frequencies. This includes the seemingly random variations between different observing sessions for the same antennae and polarization, and between polarizations for the same antennae. These variations are a direct consequence of the low value of T_A for Cas A and Tau A at high frequencies and the algorithm (eq. 3) we must employ.

As a test of these conclusions: (1) Antennae and polarizations with the worse gain stability should produce the largest variations in T_{Rcvr} between different observing sets. (2) Antennae and polarizations with the worse T_{rcvr} will have a tendency to exhibit large variations. (3) Antennae and polarizations whose amplifiers have the least stable Tsys should also exhibit large variations. (4) On-off observations using the

Moon should show low variations at high frequencies but large variations at low frequencies, which is suggested by a few Moon observing sessions from October.

Cas A and Tau A have, respectively, $T_A \sim 2$ and 4 K at 11 GHz. If we used a source that was significantly brighter, like the Moon, we should expect variations in T_{Rcvr} from one observing session to another to be small. However, at 1 GHz, the Moon is very weak, ~ 2 K, when observed with the ATA. Thus, Cas A is the preferred source for determining performance below about 3 GHz.