Problem 1.

(a)
$$B=\begin{pmatrix} 2 & 2 & 2 \\ 5 & 8 & 0 \\ 0 & 2 & 3 \\ 0 & 0 & 10 \end{pmatrix}$$

(b)
$$A=\begin{pmatrix} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 0 & 10 \end{pmatrix}$$

(c) $AB\vec{v}_2 = (AB)\cdot\vec{v}_2 = \vec{x}$

$$AB=\begin{pmatrix} 9 & 12 & 4 \\ 7 & 12 & 15 \\ 0 & 0 & 120 \end{pmatrix}$$

$$\det(AB) = 2400$$

$$(AB)^T = \begin{pmatrix} 9 & 7 & 0 \\ 12 & 12 & 0 \\ 4 & 15 & 120 \end{pmatrix}$$

$$\det(AB)\cdot(AB)^{-1} = \begin{pmatrix} 1200 & -1200 & 132 \\ -700 & 900 & -107 \\ 0 & 0 & 24 \end{pmatrix}$$

$$\Rightarrow (AB)^{-1} = \begin{pmatrix} 0.5 & -0.5 & 0.055 \\ -0.291 & 0.375 & -0.044 \\ 0 & 0 & 0.01 \end{pmatrix}$$

$$\vec{v}_2 = (AB)^{-1}\cdot\vec{x} = \begin{bmatrix} 11/2 \\ 53/24 \\ 1 \end{bmatrix} = \begin{bmatrix} 5.5 \\ 2.2083 \\ 1 \end{bmatrix}$$

Problem 2.

(a) $\sigma(-x) = \dfrac{1}{1+e^{x}}$

$$1-\sigma(x) = 1-\frac{1}{1+e^{-x}}$$
$$= 1-\frac{e^{x}}{e^{x}+1}$$
$$= \frac{1}{e^{x}+1}$$

$\therefore \sigma(-x) = 1-\sigma(x)$

(b) $d\sigma(x)/dx$

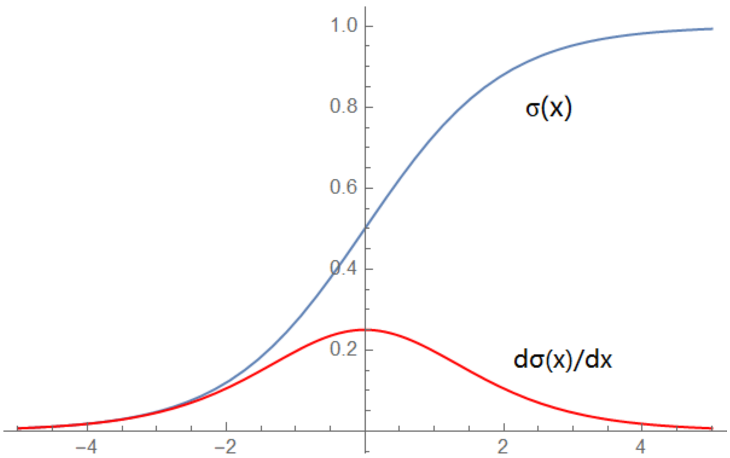
$$= d\frac{1}{1+e^{-x}} \Big/ d(1+e^{-x})\cdot(-e^{-x})$$
$$= \frac{e^{-x}}{(1+e^{-x})^2} = \frac{e^{x}}{(1+e^{x})^2}$$

$\sigma(x)(1-\sigma(x))$

$$= \sigma(x)\cdot\sigma(-x)$$
$$= \frac{e^{x}}{1+e^{x}}\cdot\frac{1}{1+e^{x}}$$
$$= \frac{e^{x}}{(1+e^{x})^2}$$

$\therefore d\sigma(x)/dx = \sigma(x)(1-\sigma(x))$

(c)



Problem 3.

$$f(c) = \frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$$

$$= \frac{1}{n}\sum_{i=1}^{n}(x_i^2 + c^2 - 2x_i \cdot c)$$

$$= \frac{1}{n}\left(\sum_{i=1}^{n} x_i^2 + n \cdot c^2 - 2\sum_{i=1}^{n} x_i \cdot c\right)$$

$$= c^2 - 2E[X] \cdot c + E[X^2]$$

$$= (c - E[X])^2 + E[X^2] - E[X]^2$$

where $E[X] = \sum_{i=1}^{n} x_i / n$

$E[X^2] = \sum_{i=1}^{n} x_i^2 / n$

Then for any c, $f(c) \geq E[X^2] - E[X]^2$

and Iff $c = E[X]$, $f(c)$ is minimized.

So required $c = \sum_{i=1}^{n} x_i / n$.

Problem 4.

Let A = Has cancer

B = Test positive

$$P[A|B] = \frac{P[B|A] \cdot P[A]}{P[B|A] \cdot P[A] + P[B|\neg A] \cdot P[\neg A]}$$

$$= \frac{80\% \cdot 1\%}{80\% \cdot 1\% + 9.6\% \cdot 99\%}$$

$$= 0.078$$

Problem 5

The closest standard deviation should be b, 6.1

We can view the data from the histogram similarly to a normal distribution $\sim N(\mu, \sigma^2)$

The distribution $f(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right)$

P is equal to $\frac{1}{\sigma\sqrt{2\pi}}$ when $x = \mu$.

From the histogram $f(\mu) \approx 0.064$

Then $\sigma \approx \frac{1}{f(\mu)\sqrt{2\pi}} \approx 6.23$

so 6.1 is the closest answer.