

# Computational Physics

## Topic 03 : Computational Problems involving Cellular Systems

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### Lecture 01 : Introduction to Cellular Systems

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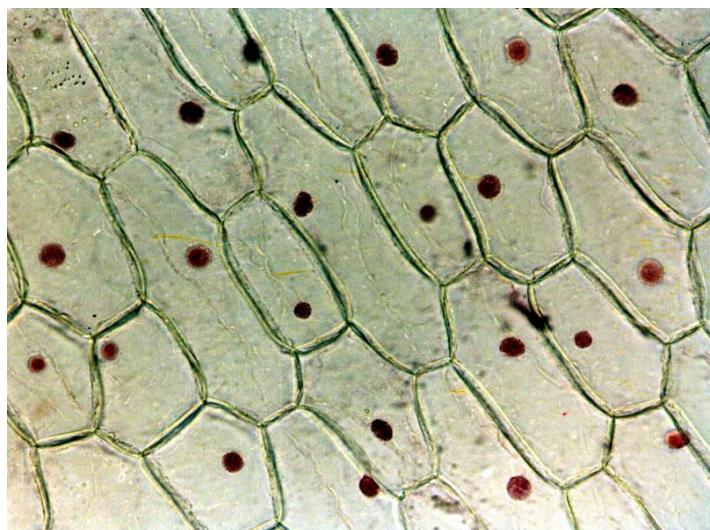
Autumn Semester, 2025/26

#### Outline

- Some simple models
- Terminology and definitions

# Motivation

Evolution has rediscovered several times multicellularity as a way to build complex living systems\*.



- Multicellular systems are composed by many copies of a unique fundamental unit — the cell.
- The local interaction between cells influences the fate and the behaviour of each cell
- The result is an heterogeneous system composed by differentiated cells that act as specialised units, even if they all contain the same genetic material and have essentially the same structure.

Based on *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies* by Dario Floreano and Claudio Mattiussi, MIT Press.

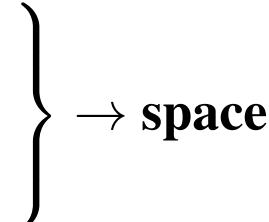
\*Or maybe god keeps reusing her ideas.

# Cellular Automaton

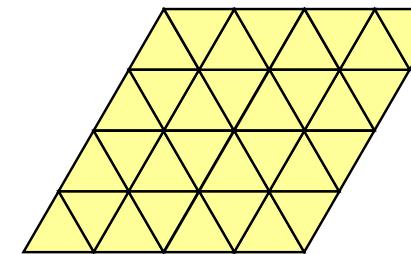
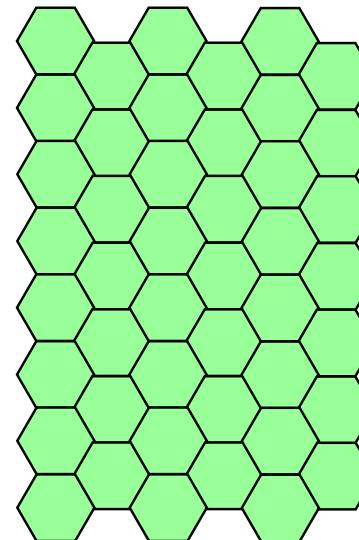
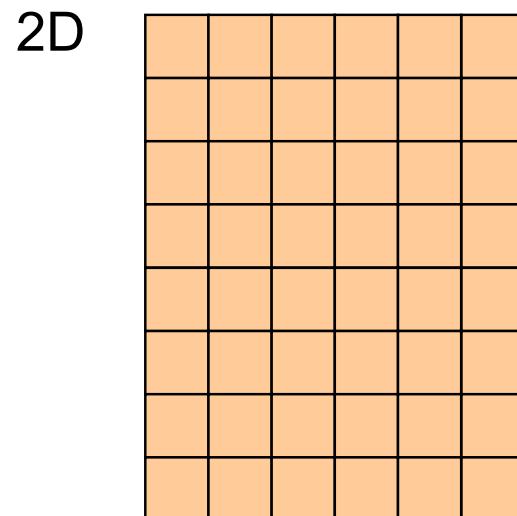
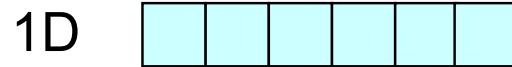
Many complex phenomena are the result of the collective dynamics of a very large number of parts obeying simple rules.

Unexpected global behaviours and patterns can emerge from the interaction of many systems that “communicate” only locally.

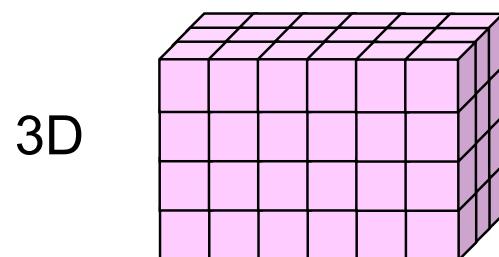
A Cellular Automaton is ...

- a **geometrically structured** and
  - **discrete** collection of
  - **identical** (simple) systems called **cells**
  - that **interact only locally**
  - with each cell in one of a predefined finite number of **states**
  - and a (simple) **rule** used to **update** the state of all cells
  - at **discrete time steps**
  - and **synchronously** for all the cells of the automaton (global “signal”).
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# Cellular Space



...



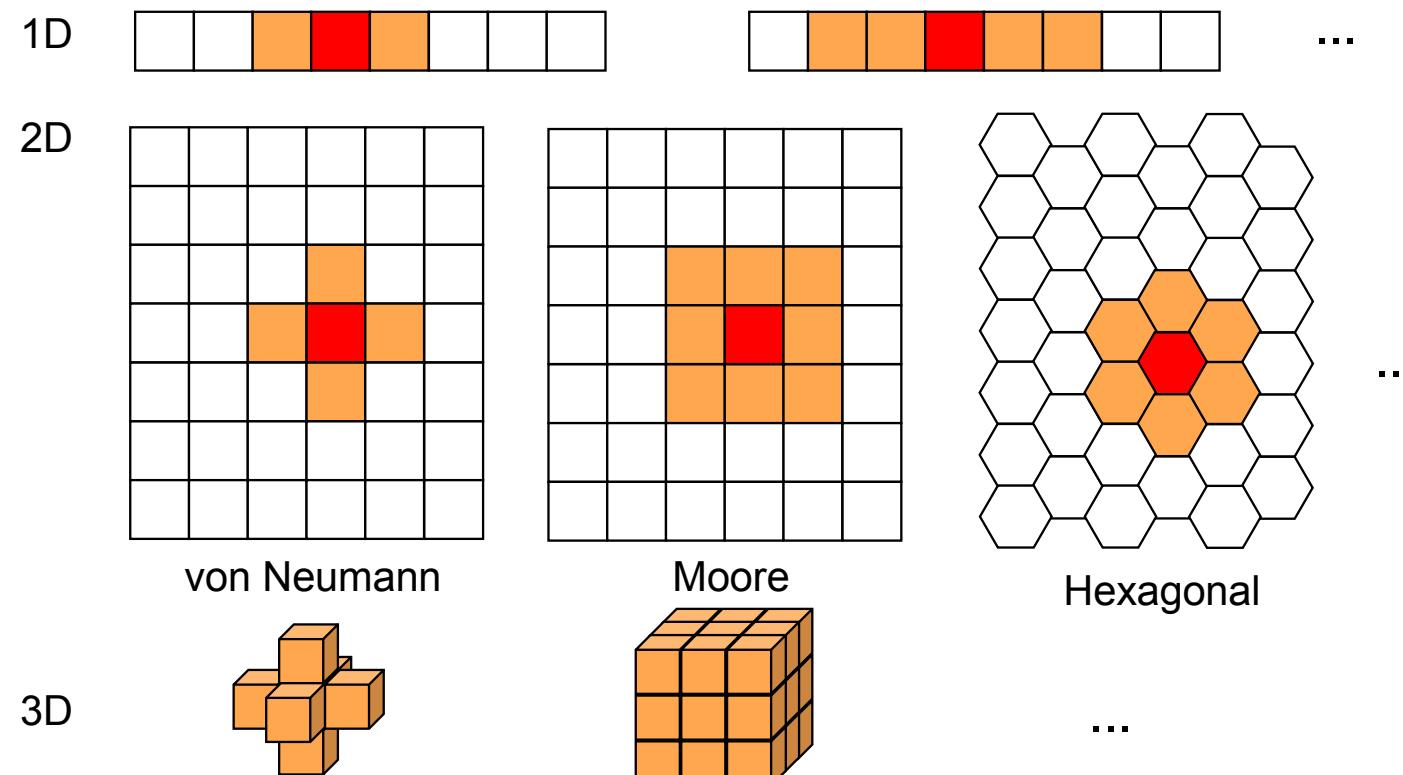
...

and beyond...

# Neighbourhood (local interaction)

The **neighbourhood** of a cell, is the set of cells that can influence directly a given cell.

In **homogeneous** cellular models the neighbourhood has the same shape for all cells.



# Boundary Conditions

- If the cellular space has a boundary, cells on the boundary may lack the cells required to form the prescribed neighbourhood.
- Boundary conditions specify how to build a “virtual” neighbourhood for boundary cells.
- Common boundary conditions:

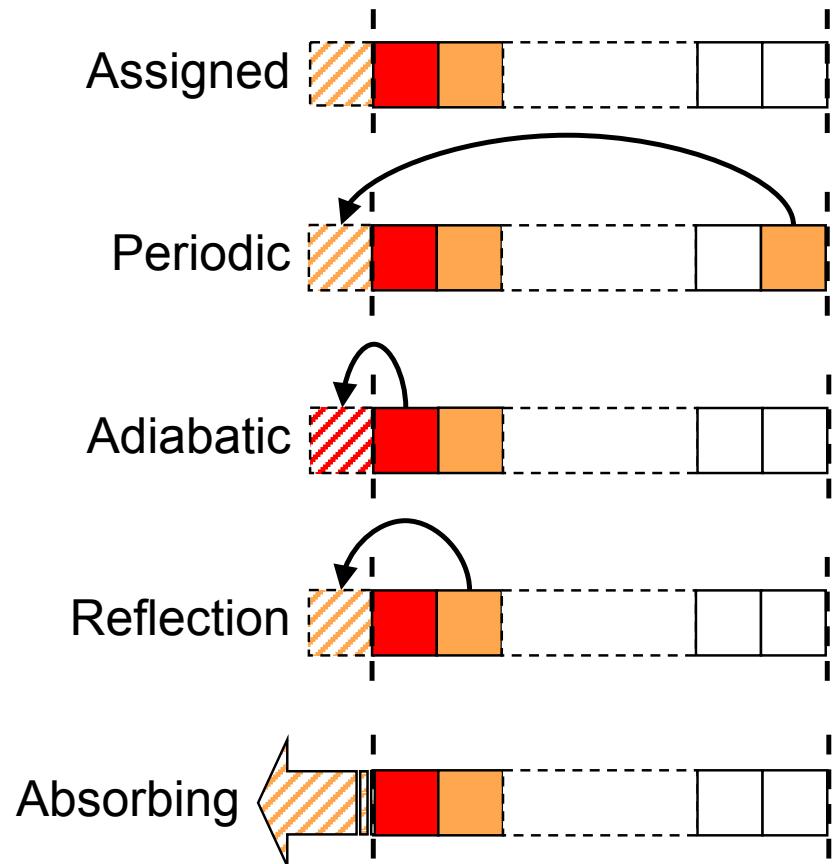
**Assigned** — set fixed value.

**Periodic** — copied from opposite edge

**Adiabatic** — cells “outside” are set to be equal to boundary cells

**Reflection** — cells “outside” mirror cells inside boundary cells.

**Absorbing** — acts like infinite heat sink boundary



# Cell State and Transition Rule

## State Space

Each cell can be in one of a predetermined finite set of **states**.

- Without loss of generality the states can be represented by integers  $\{0, 1, 2, \dots, k - 1\}$  where  $k$  is the number of states.
- Alternatively, colours are used to indicate state in visualisations.

$$S = \{\text{state}_0, \text{state}_1, \dots, \text{state}_{k-1}\} \rightarrow \{0, 1, 2, \dots, k - 1\} \rightarrow \{\bullet\text{red}, \bullet\text{green}, \bullet\text{blue}, \dots, \bullet\text{orange}\}$$

## Transition rule

For each possible configuration of states of the cells in the neighbourhood (this may include the current cell), we need a **transition rule** to determine the new state.

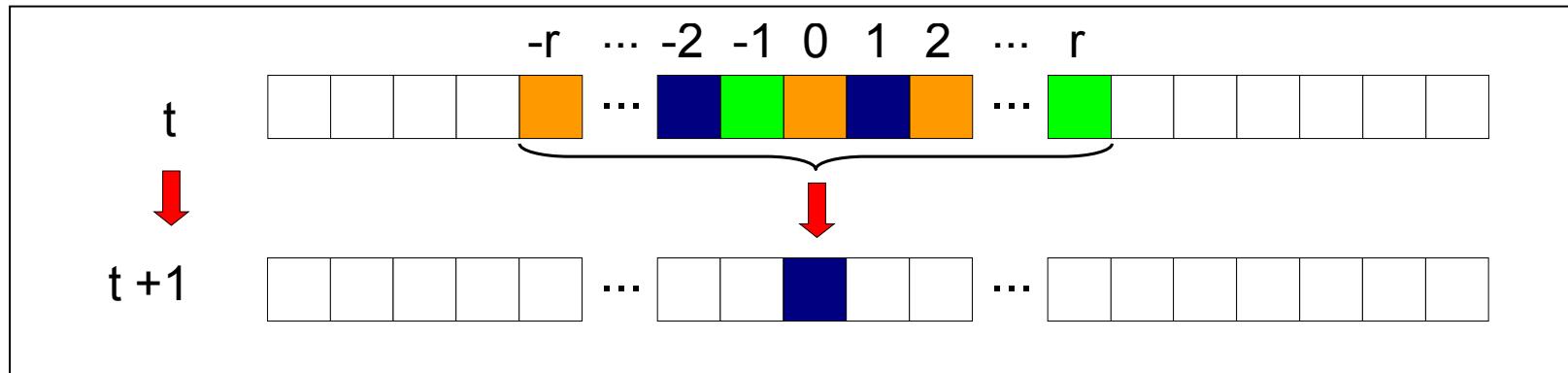
- Rules can be deterministic (usually) or stochastic

# Special Rules

The transition table of a generic CA can have an enormous number of entries. Special rules can have more compact definitions:

- A rule is **totalistic** if the new value of the state depends only on the sum of the values of the states of the cells in the neighbourhood.
  - Example: “Conway’s Game of Life” with Rule B3/S23
    - A dead cell becomes alive (Birth) if it has exactly 3 live neighbours. A live cell survives (S) if it has 2 or 3 live neighbours. Otherwise, the cell dies or stays dead.
- A rule is **outer totalistic** if the new value of the state depends on the current state of the cell in question and on the sum of the values of the states of the other cells in the neighbourhood.
  - Example: “1D Conway’s Game of Life” with Rule B3/S23
    - A dead cell becomes alive (Birth) if it has exactly 3 live neighbours. A live cell survives (S) if it has 2 or 3 live neighbours. Otherwise, the cell dies or stays dead.
  - Example: “Brian’s Brain”
    - 3 states (on, off, dying)
    - A cell turns “on” if exactly two neighbours are “on”
    - An “on” cell becomes “dying”, and “dying” becomes “off”

# Rules for 1D Cellular Automata



$k$  states (colours  $\bullet, \bullet, \bullet, \dots$ ), range (or radius)  $r$

## Possible Rules

$$k^{k^{2r+1}}$$

## Totalistic Rules

$$k^{(2r+1)(k-1)+1}$$

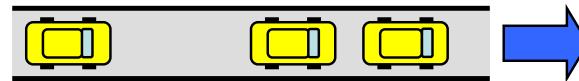
e.g.

- $k = 2, r = 1 \rightarrow 256$
- $k = 3, r = 1 \rightarrow 7, 625, 597, 484, 987$

e.g.

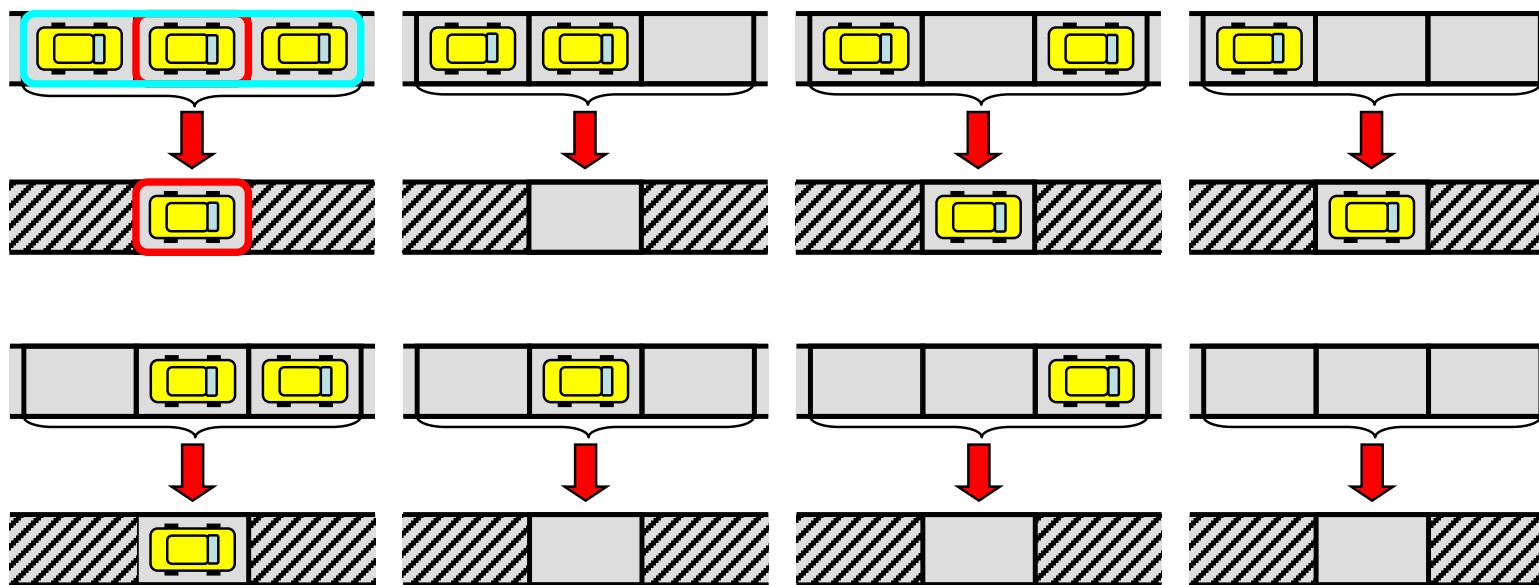
- $k = 2, r = 1 \rightarrow 16$
- $k = 3, r = 1 \rightarrow 2187$

## Example — Traffic Flow Model (a.k.a. 1D Rule 184)



Discretise space (road) and store occupancy (all cars are identical)

We construct an elementary model of car motion in a single lane, based only on the **local** traffic conditions. The cars advance at discrete time steps and at discrete space intervals. A car can advance (and must advance) only if the destination interval is free.



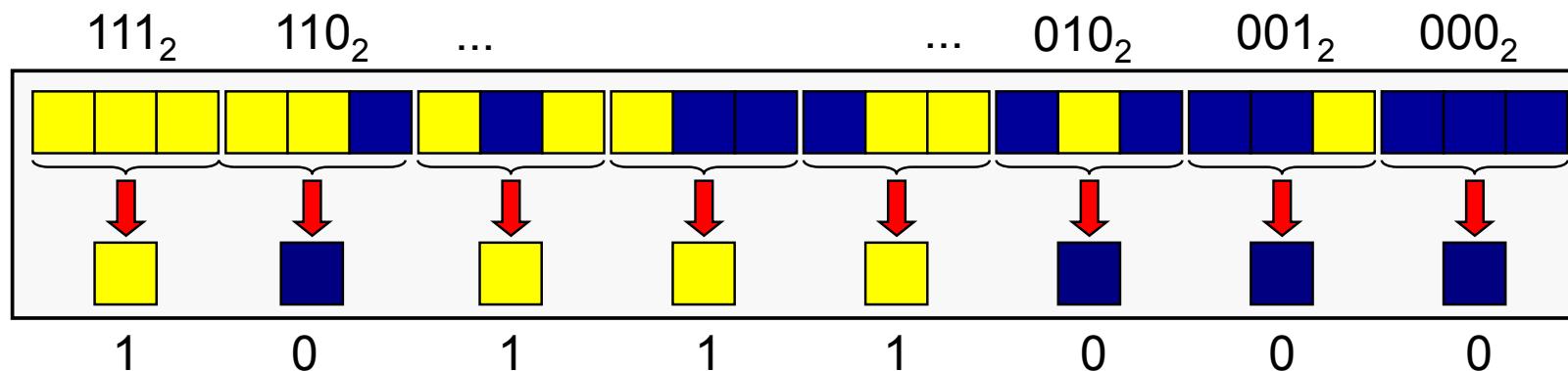
$2^3 = 8$  different input possibilities

# Encoding Traffic Flow Model → Rule 184

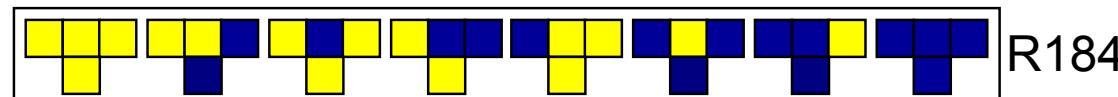
# Elementary CA

256 1D binary CA ( $k=2$ ) with minimal range ( $r=1$ )

## Wolfram's Rule Code (here, $\textcolor{blue}{\square} = 0$ , $\textcolor{yellow}{\square} = 1$ )



$$10111000_2 = 1 \cdot 2^7 + 0 \cdot 2^6 + \dots + 0 \cdot 2^0 = 184_{10} \quad \Rightarrow \text{Rule 184}$$



(the “car traffic” rule!)

# Global Behaviour of Traffic Flow Model → Rule 184

