Computational Physics

Topic 03 : Computational Problems involving Cellular Systems

Lecture 01 : Introduction to Cellular Systems

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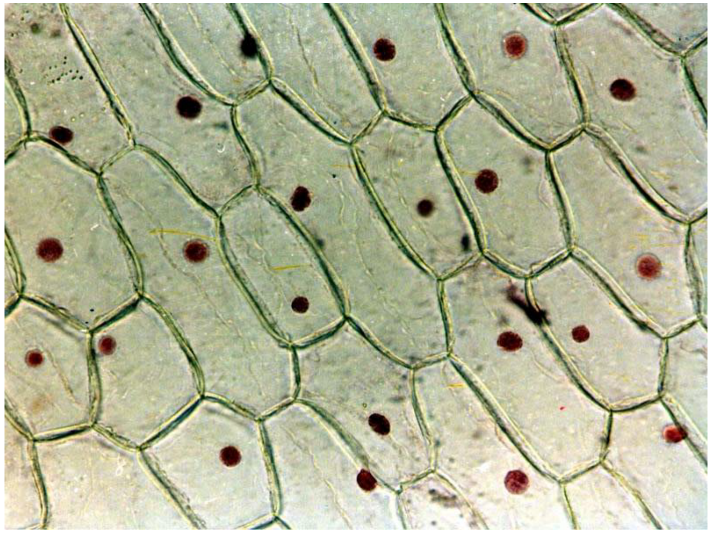
Outline

- Some simple models
- Terminology and definitions

Motivation

Evolution has rediscovered several times multicellularity as a way to build complex living systems*.



- Multicellular systems are composed by many copies of a unique fundamental unit — the cell.
- The local interaction between cells influences the fate and the behaviour of each cell
- The result is an heterogeneous system composed by differentiated cells that act as specialised units, even if they all contain the same genetic material and have essentially the same structure.

Based on *Bio-Inspired Artificial Intelligence: Theories, Methods, and Technologies* by Dario Floreano and Claudio Mattiussi, MIT Press.

*Or maybe god keeps reusing the same idea.

Cellular Automaton

Many complex phenomena are the result of the collective dynamics of a very large number of parts obeying simple rules.
Unexpected global behaviours and patterns can emerge from the interaction of many systems that "communicate" only locally.

A Cellular Automaton is ...

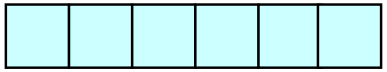
- a **geometrically structured** and
- **discrete** collection of
- **identical** (simple) systems called **cells**

$\}$ $\rightarrow$ **space**

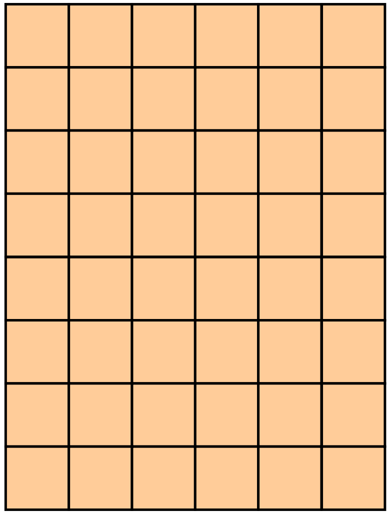
- that **interact** only **locally** $\rightarrow$ **neighbourhood**
- with each cell in one of a predefined finite number of **states** $\rightarrow$ **states**
- and a (simple) **rule** used to **update** the state of all cells $\rightarrow$ **transition rules**
- at **discrete time** steps
- and **synchronously** for all the cells of the automaton (global "signal").

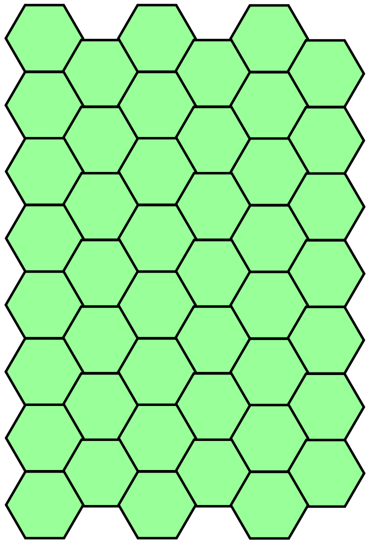
Cellular Space

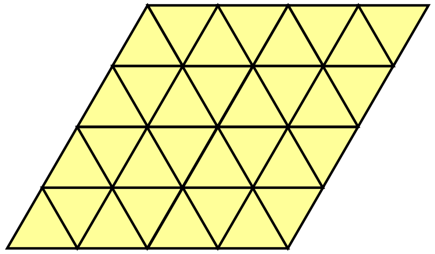
1D



2D

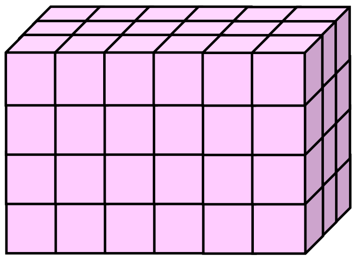






...

3D

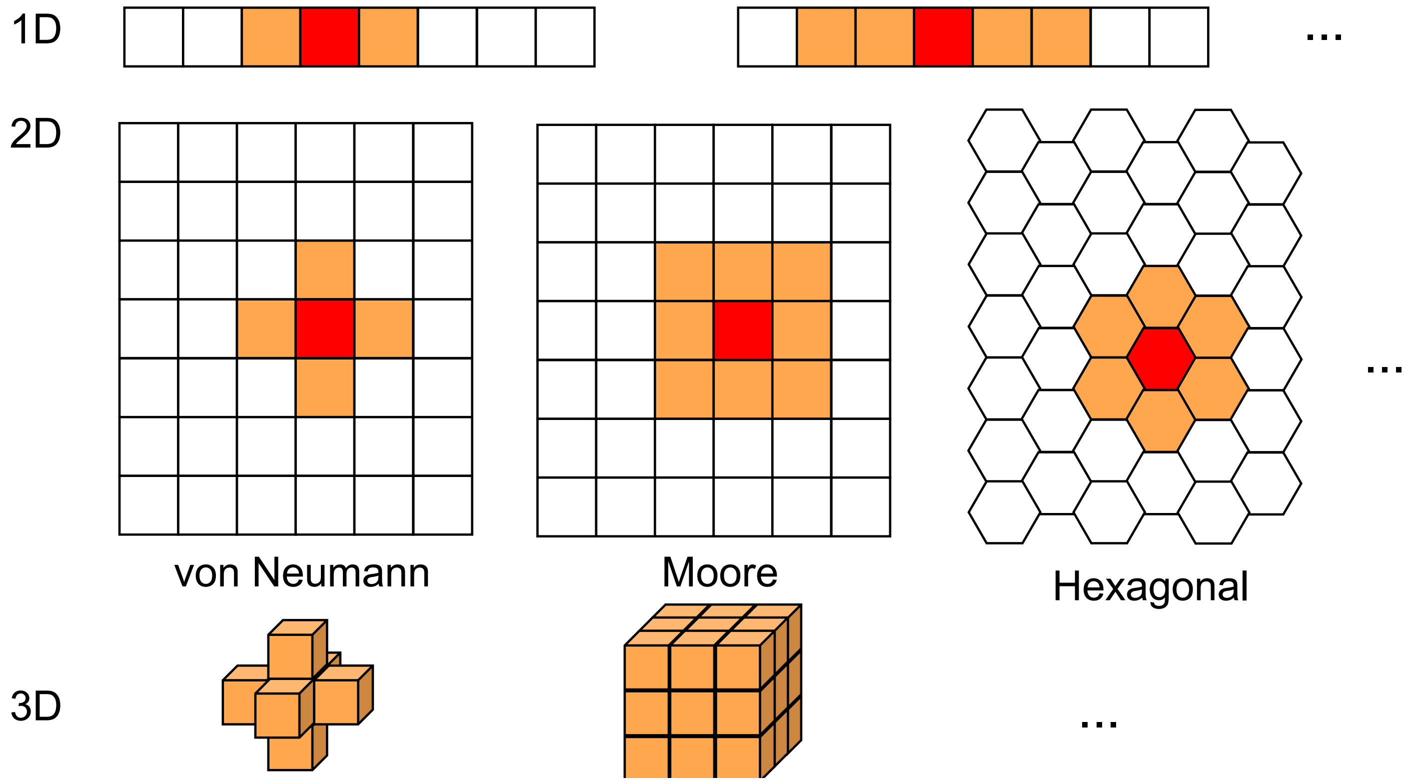


...

and beyond...

Neighbourhood (local interaction)

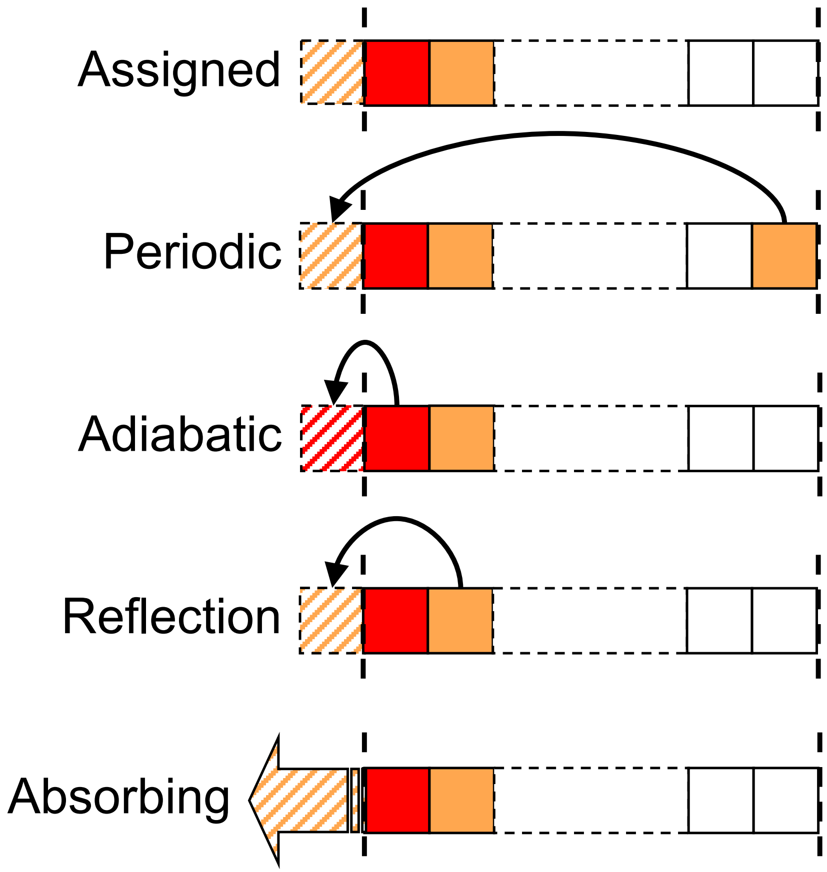
The neighbourhood of a cell, is the set of cells that can influence directly a given cell.
In homogeneous cellular models the neighbourhood has the same shape for all cells.



Boundary Conditions

- If the cellular space has a boundary, cells on the boundary may lack the cells required to form the prescribed neighbourhood.
- Boundary conditions specify how to build a "virtual" neighbourhood for boundary cells.
- Common boundary conditions:

Assigned — set fixed value.
Periodic — copied from opposite edge
Adiabatic — cells "outside" are set to be equal to boundary cells
Reflection — cells "outside" mirror cells inside boundary cells.
Absorbing — acts like infinite heat sink boundary



Cell State and Transition Rule

State Space

Each cell can be in one of a predetermined finite set of states.

- Without loss of generality the states can be represented by integers $\{0, 1, 2, ..k-1\}$ where k is the number of states.
- Alternatively, colours are used to indicate state in visualisations.

$$S = \{\text{state}_0, \text{state}_1, \ldots, \text{state}_{k-1}\} \rightarrow \{0, 1, 2, \ldots, k-1\} \rightarrow \{\color{red}{\bullet}, \color{green}{\bullet}, \color{blue}{\bullet}, \ldots, \color{orange}{\bullet}\}$$

Transition rule

For each possible configuration of states of the cells in the neighbourhood (this may include the current cell), we need a transition rule to determine the new state.

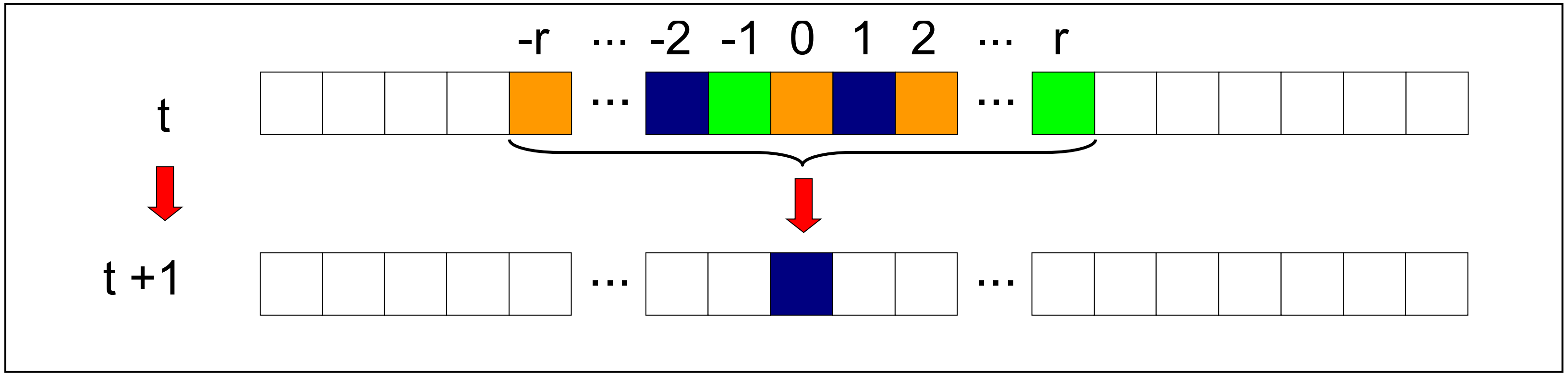
- Rules can be deterministic (usually) or stochastic

Special Rules

The transition table of a generic CA can have an enormous number of entries. Special rules can have more compact definitions:

- A rule is totalistic if the new value of the state depends only on the sum of the values of the states of the cells in the neighbourhood.
  - Example: "Conway's Game of Life" with Rule B3/S23
    - A dead cell becomes alive (Birth) if it has exactly 3 live neighbours. A live cell survives (S) if it has 2 or 3 live neighbours. Otherwise, the cell dies or stays dead.
- A rule is outer totalistic if the new value of the state depends on the current state of the cell in question and on the sum of the values of the states of the other cells in the neighbourhood.
  - Example: "1D Conway's Game of Life" with Rule B3/S23
    - A dead cell becomes alive (Birth) if it has exactly 3 live neighbours. A live cell survives (S) if it has 2 or 3 live neighbours. Otherwise, the cell dies or stays dead.
  - Example: "Brian's Brain"
    - 3 states (on, off, dying)
    - A cell turns "on" if exactly two neighbours are "on"
    - An "on" cell becomes "dying", and "dying" becomes"of"

Rules for 1D Cellular Automata



k states (colours ●,●, ●,...), range (or radius) r

Possible Rules

$$k^{k^{2r+1}}$$

e.g.

- $k = 2, r = 1 \rightarrow 256$
- $k = 3, r = 1 \rightarrow 7,625,597,484,987$

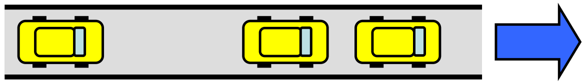
Totalistic Rules

$$k^{(2r+1)(k-1)+1}$$

e.g.

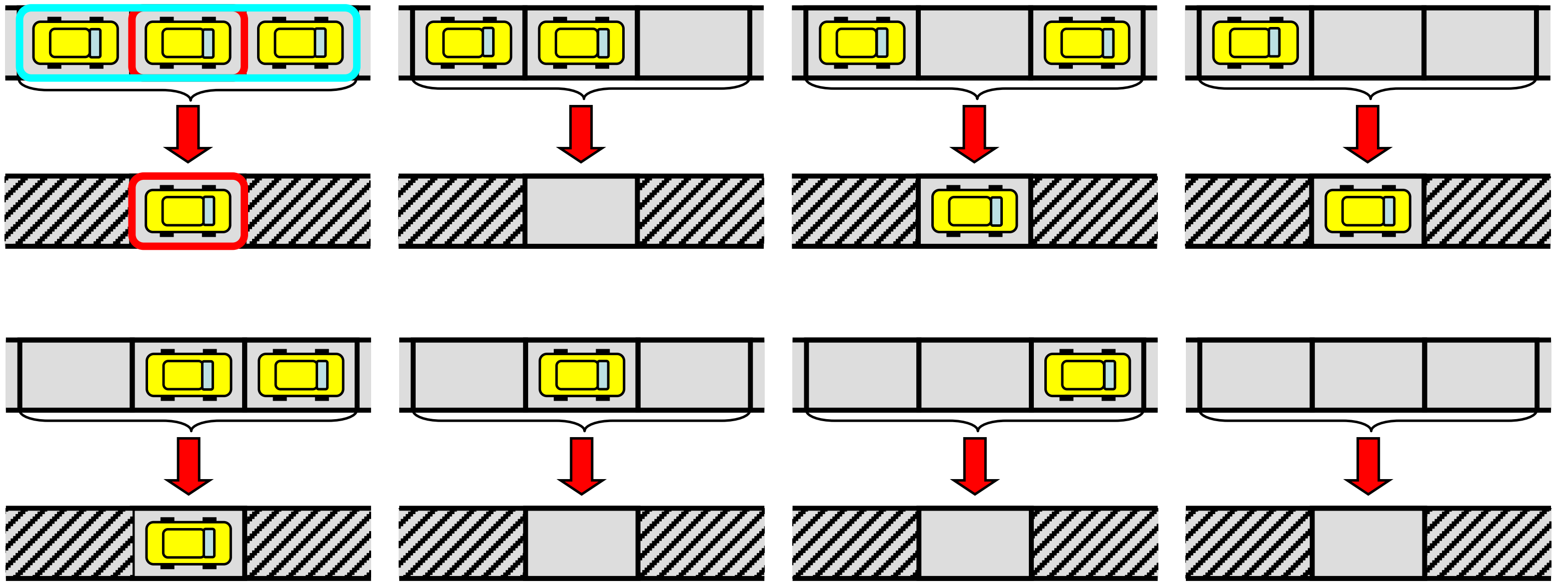
- $k = 2, r = 1 \rightarrow 16$
- $k = 3, r = 1 \rightarrow 2187$

Example — Traffic Flow Model (a.k.a. 1D Rule 184)



Discretise space (road) and store occupancy (not track individual cars)

We construct an elementary model of car motion in a single lane, based only on the **local** traffic conditions. The cars advance at discrete time steps and at discrete space intervals. A car can advance (and must advance) only if the destination interval is free.



$2^3 = 8$ different input possibilities

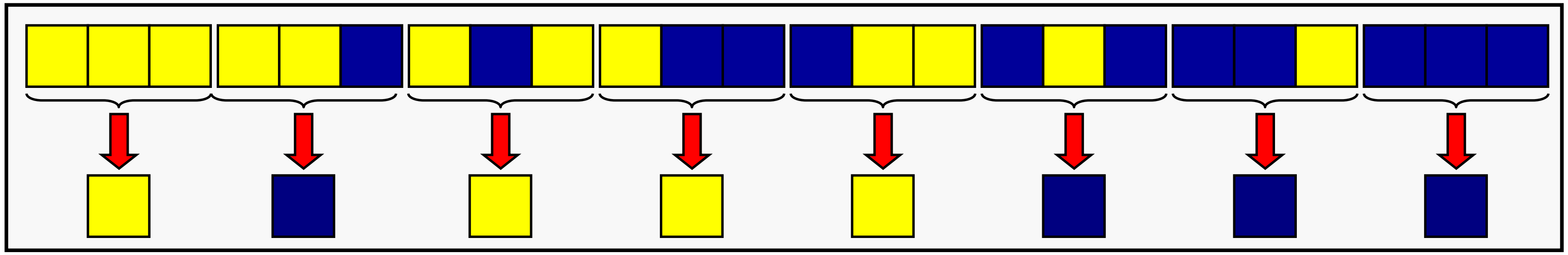
Encoding Traffic Flow Model → Rule 184

Elementary CA

256 1D binary CA (k=2) with minimal range (r=1)

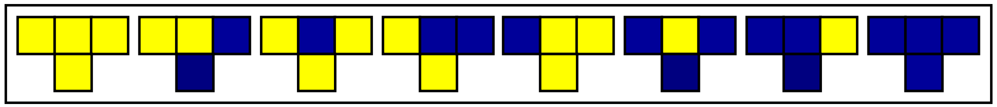
Wolfram's Rule Code (here, = 0 , = 1)

111_2 110_2 ... ... 010_2 001_2 000_2



1 0 1 1 1 0 0 0

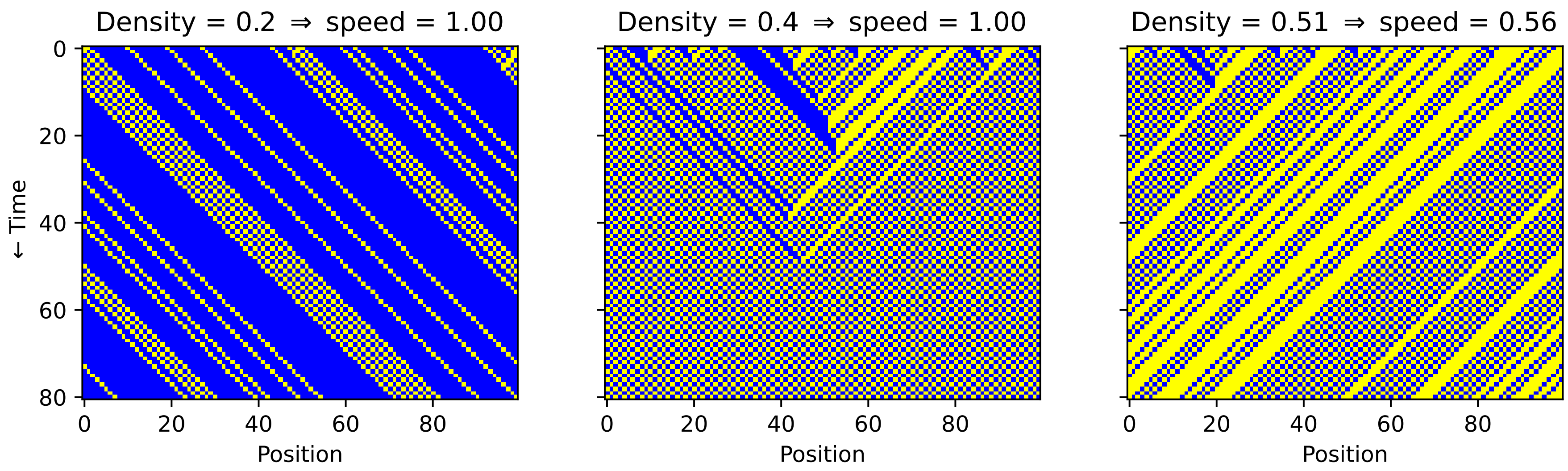
$10111000_2 = 1 \cdot 2^7 + 0 \cdot 2^6 + \ldots + 0 \cdot 2^0 = 184_{10}$ ⇨ Rule 184

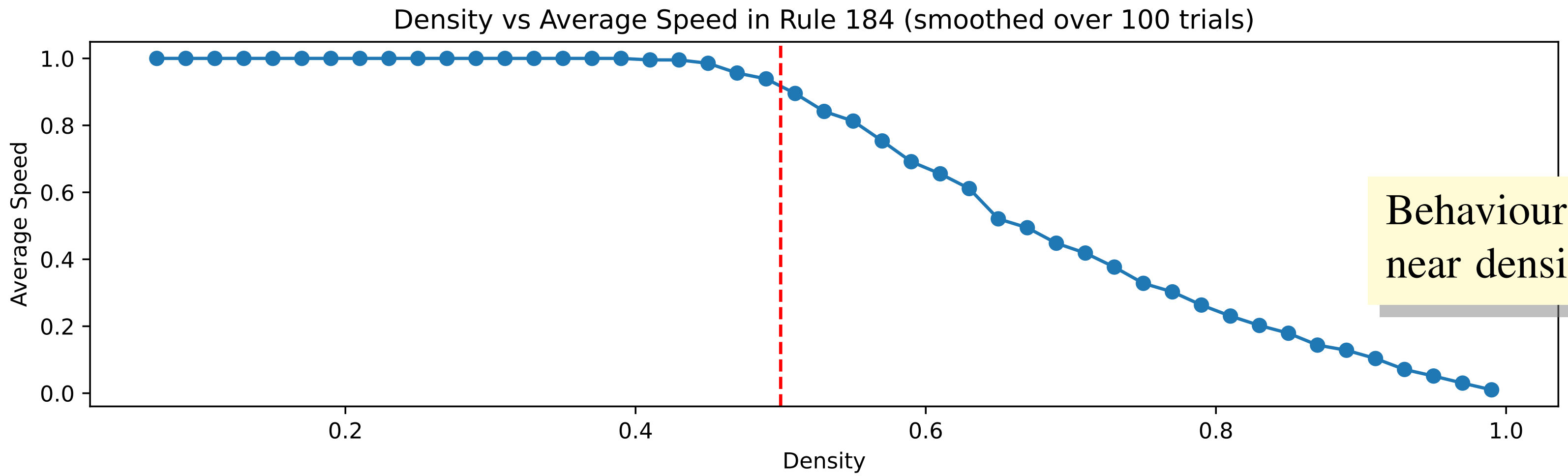


R184

(the "car traffic" rule!)

Global Behaviour of Traffic Flow Model → Rule 184





Behaviour changes near density of 0.5