

# Computational Physics

## Topic 03 — Computational Problems involving Marko Chains

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### Lecture 02 — The Collector Problem

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#### RESOURCE OUTLINE LABEL

- Problem statement
- Sample run

# Markov chain approach

## Model

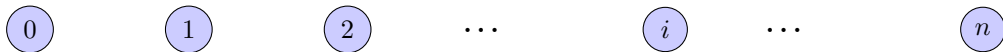
- **State:**  $i$ ,  $i = 0, \dots, n$ , where  $i$  is number of collected coupons.

*Have  $i$  of the  $n$  available coupons, so  $(n - i)$  coupons are unseen.*

- Initial state is 0. State  $n$  is terminal.

- **Stage:** Number of coupons purchased.

*How long does it take to travel from 0 to  $n$ ?*



# Markov chain approach

## Model

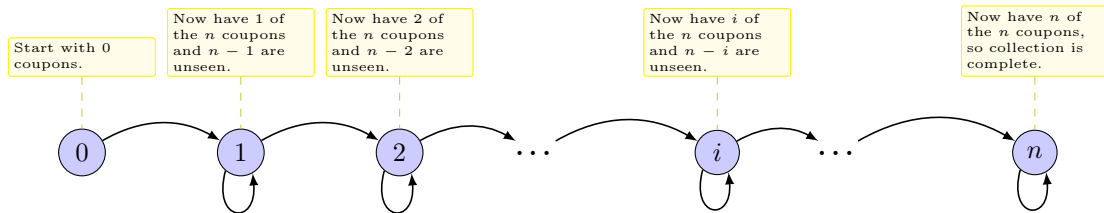
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*Have  $i$  of the  $n$  available coupons, so  $(n - i)$  coupons are unseen.*
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- **Stage:** Number of coupons purchased.  
*How long does it take to travel from  $0$  to  $n$ ?*



# Markov chain approach

## Model

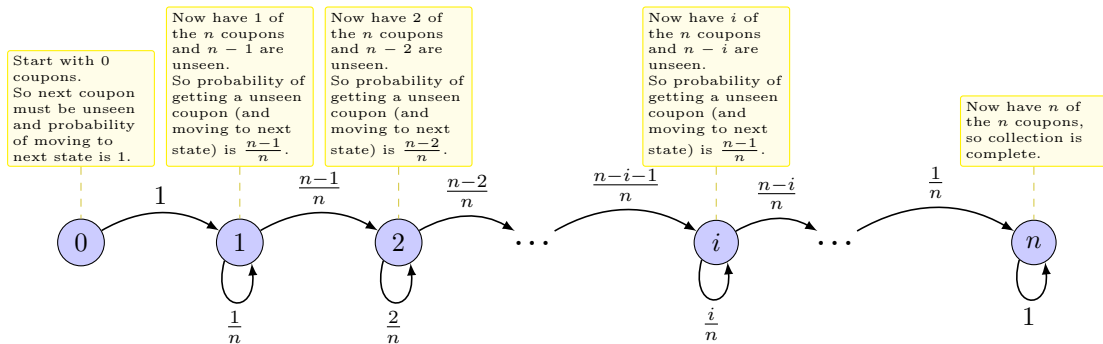
- **State:**  $i, i = 0, \dots, n$ , where  $i$  is number of collected coupons.  
*Have  $i$  of the  $n$  available coupons, so  $(n - i)$  coupons are unseen.*
- Initial state is  $0$ . State  $n$  is terminal.
- **Stage:** Number of coupons purchased.  
*How long does it take to travel from  $0$  to  $n$ ?*



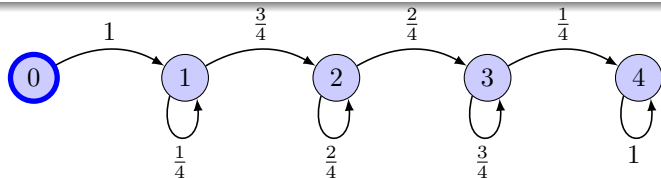
# Markov chain approach

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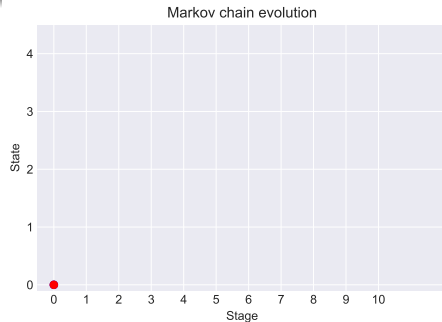
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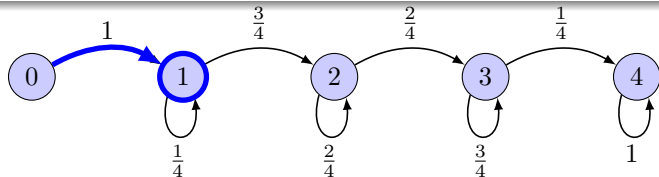
## Viewing our first simulation run as a Markov chain ...



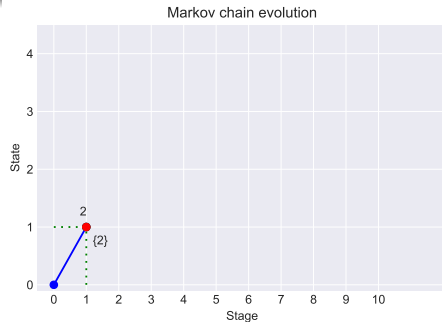
count: 0 found: collected: set()



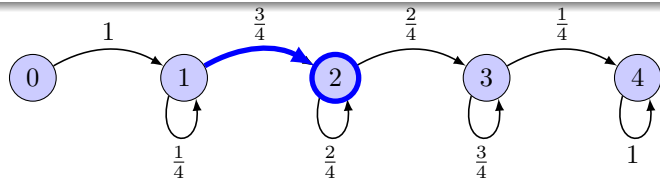
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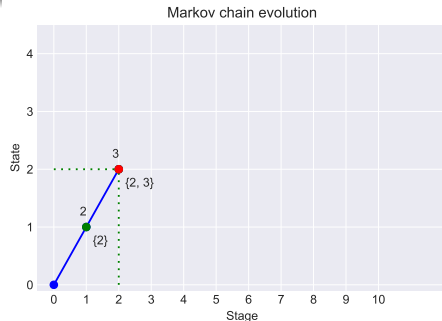
```
count:    0      found:      collected: set()
count:    1      found: {2}   collected: {2}
```



## Viewing our first simulation run as a Markov chain ...

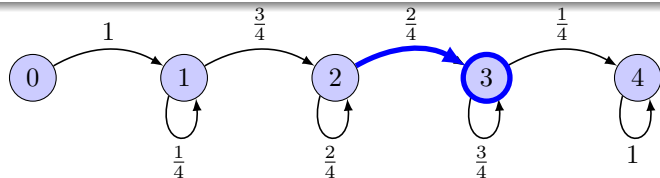


count:	0	found:		collected:	set()
count:	1	found:	{2}	collected:	{2}
count:	2	found:	{3}	collected:	{2, 3}

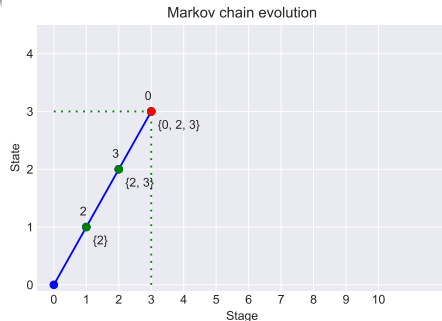




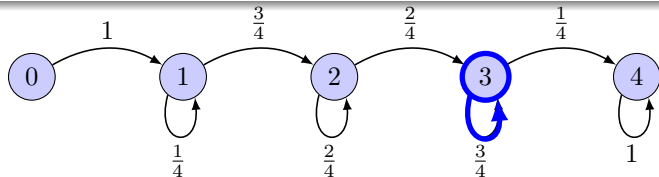
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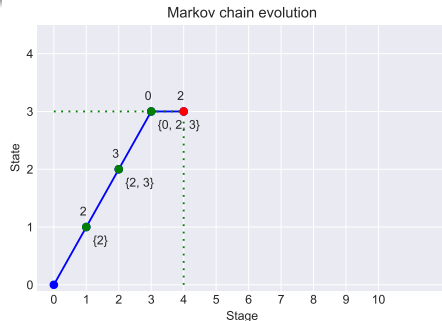
count:	0	found:		collected:	set()
count:	1	found:	{2}	collected:	{2}
count:	2	found:	{3}	collected:	{2, 3}
count:	3	found:	{0}	collected:	{0, 2, 3}



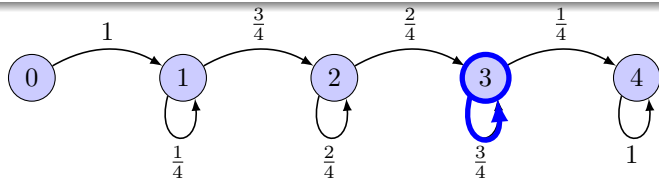
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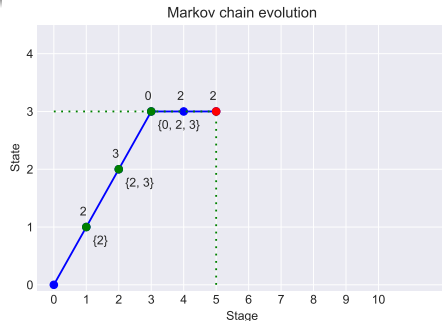
count:	0	found:		collected:	set()
count:	1	found:	{2}	collected:	{2}
count:	2	found:	{3}	collected:	{2, 3}
count:	3	found:	{0}	collected:	{0, 2, 3}
count:	4	found:	{2}	collected:	{0, 2, 3}



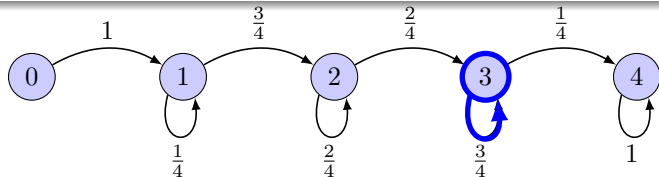
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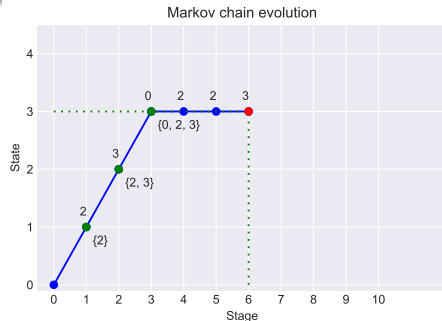
count:	0	found:		collected:	set()
count:	1	found:	{2}	collected:	{2}
count:	2	found:	{3}	collected:	{2, 3}
count:	3	found:	{0}	collected:	{0, 2, 3}
count:	4	found:	{2}	collected:	{0, 2, 3}
count:	5	found:	{2}	collected:	{0, 2, 3}



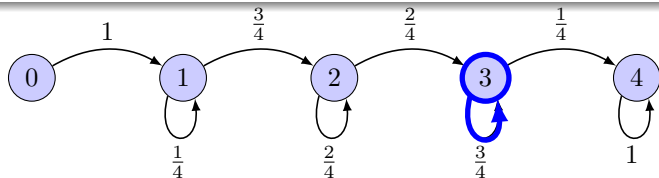
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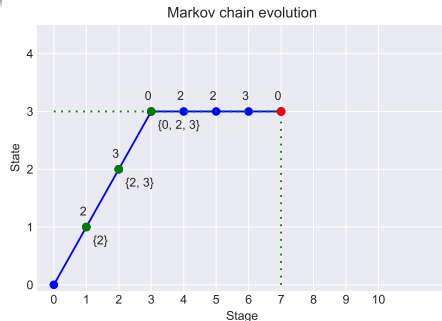
count:	0	found:		collected:	set()
count:	1	found:	{2}	collected:	{2}
count:	2	found:	{3}	collected:	{2, 3}
count:	3	found:	{0}	collected:	{0, 2, 3}
count:	4	found:	{2}	collected:	{0, 2, 3}
count:	5	found:	{2}	collected:	{0, 2, 3}
count:	6	found:	{3}	collected:	{0, 2, 3}



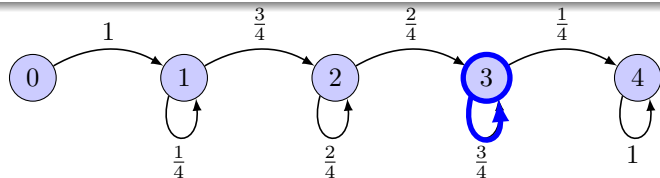
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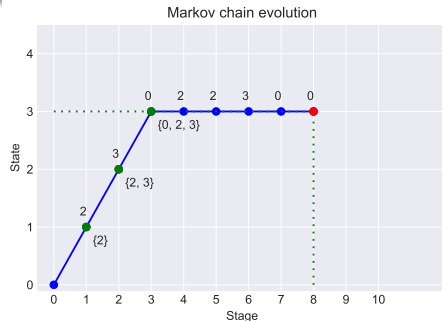
count:	0	found:		collected:	set()
count:	1	found:	{2}	collected:	{2}
count:	2	found:	{3}	collected:	{2, 3}
count:	3	found:	{0}	collected:	{0, 2, 3}
count:	4	found:	{2}	collected:	{0, 2, 3}
count:	5	found:	{2}	collected:	{0, 2, 3}
count:	6	found:	{3}	collected:	{0, 2, 3}
count:	7	found:	{0}	collected:	{0, 2, 3}



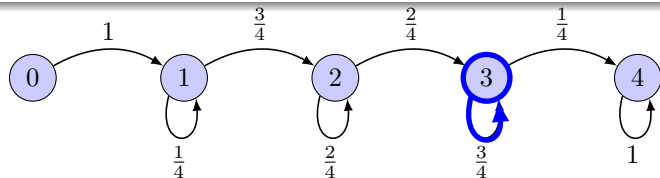
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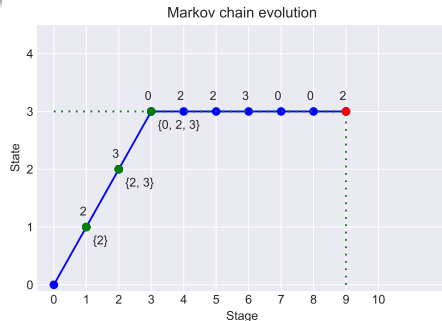
count:	0	found:	collected: set()
count:	1	found: {2}	collected: {2}
count:	2	found: {3}	collected: {2, 3}
count:	3	found: {0}	collected: {0, 2, 3}
count:	4	found: {2}	collected: {0, 2, 3}
count:	5	found: {2}	collected: {0, 2, 3}
count:	6	found: {3}	collected: {0, 2, 3}
count:	7	found: {0}	collected: {0, 2, 3}
count:	8	found: {0}	collected: {0, 2, 3}



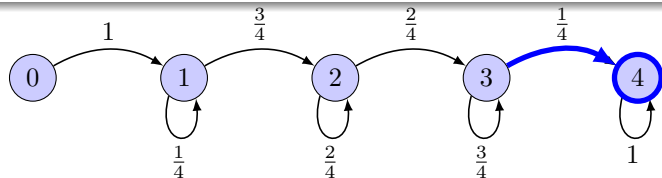
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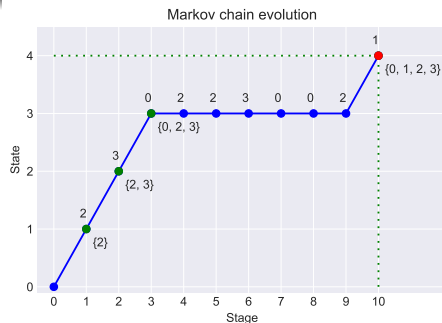
count:	0	found:	collected:	set()
count:	1	found:	collected:	{2}
count:	2	found:	collected:	{2, 3}
count:	3	found:	collected:	{0, 2, 3}
count:	4	found:	collected:	{0, 2, 3}
count:	5	found:	collected:	{0, 2, 3}
count:	6	found:	collected:	{0, 2, 3}
count:	7	found:	collected:	{0, 2, 3}
count:	8	found:	collected:	{0, 2, 3}
count:	9	found:	collected:	{0, 2, 3}



## Viewing our first simulation run as a Markov chain ...



count:	0	found:		collected:	set()
count:	1	found:	{2}	collected:	{2}
count:	2	found:	{3}	collected:	{2, 3}
count:	3	found:	{0}	collected:	{0, 2, 3}
count:	4	found:	{2}	collected:	{0, 2, 3}
count:	5	found:	{2}	collected:	{0, 2, 3}
count:	6	found:	{3}	collected:	{0, 2, 3}
count:	7	found:	{0}	collected:	{0, 2, 3}
count:	8	found:	{0}	collected:	{0, 2, 3}
count:	9	found:	{2}	collected:	{0, 2, 3}
count:	10	found:	{1}	collected:	{0, 1, 2, 3}



Number of stages needed = 10