(MSc) Data Mining

Topic 06 : Classification 1

Part 02 : Logistic Regression

Dr Bernard Butler and Dr Kieran Murphy

Department of Computing and Mathematics, SETU Waterford.
(bernard.butler@setu.ie; kmurphy@wit.ie)

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Outline

- Relationship with linear regression
- (Sigmoid) Logistic function

Linear Regression I

Dataset

Take the IRIS dataset (4 features, target discrete with 3 levels $[0, 1, 2]$) and simplify it by* taking only the first feature, and merging class 1 ('`versicolor`') and class 2 ('`virginica`').

```
X, y, target_names = iris.data[:,:1], iris.target, iris.target_names
y[y>1] = 1
target_names = np.array([target_names[0], 'other'])
```

So we have target

```
y
```

```
array([0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
       0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
       1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1])
```

and target names of

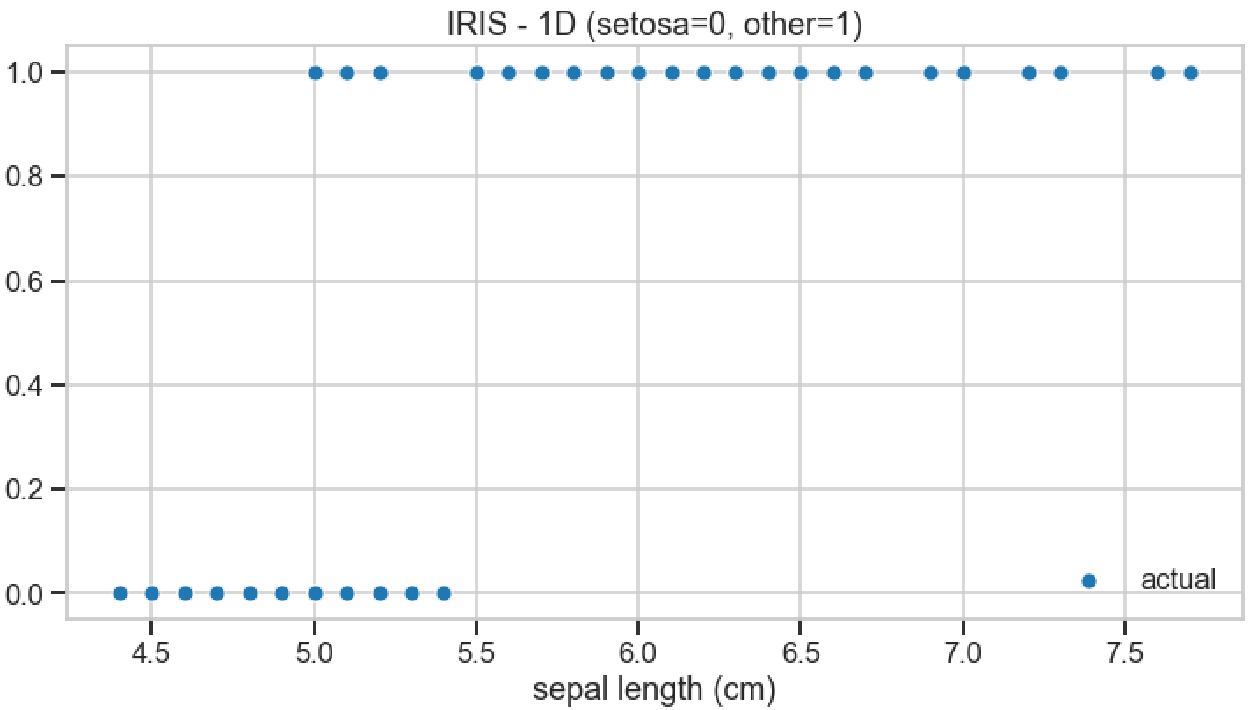
```
target_names
```

```
array(['setosa', 'other'], dtype='<U6')
```

*Python for Data Science — Cheat Sheet Numpy Basics

Linear Regression II

4
```
plt.figure(figsize=(12,6))
sns.scatterplot(x=X_test.T[0],y=y_test, label="actual")
plt.xlabel(iris.feature_names[0])
plt.title("IRIS - 1D (setosa=0, other=1)")
plt.legend(loc="lower right")
plt.show()
```



From the graph, it looks like we could predict class using sepal length, in particular if sepal length is <5.5 then predict `setosa` else predict `other`.
How many observations will be miss-classified then?

Linear Regression III

Following standard procedure we split data . . .

5
```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size=0.6, random_state=666)
```

. . . import and create instance of model (`LinearRegression`) . . .

6
```
from sklearn.linear_model import LinearRegression
model = LinearRegression()
```

. . . fit model (using training data — feature(s) and target) . . .

7
```
model.fit(X_train, y_train)
```

. . . predict (using feature(s) in test data) . . .

8
```
y_pred = model.predict(X_test)
```

Linear Regression III

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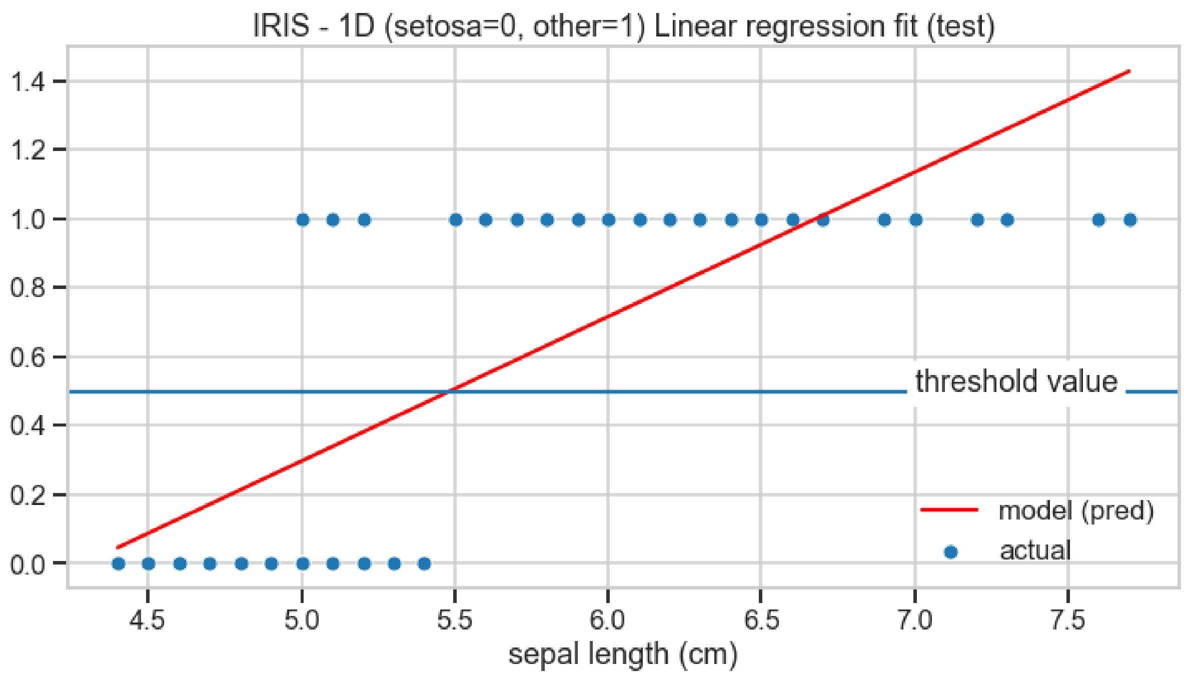
```
plt.figure(figsize=(12,6))
sns.scatterplot(x=X_test.T[0],y=y_test, label="actual")
sns.lineplot(x=X_test.T[0], y=y_pred, color='red', label="model (pred)")
plt.xlabel(iris.feature_names[0])
plt.title("IRIS - 1D (setosa=0, other=1) Linear regression fit (test)")
plt.legend(loc="lower right")
plt.show()
```

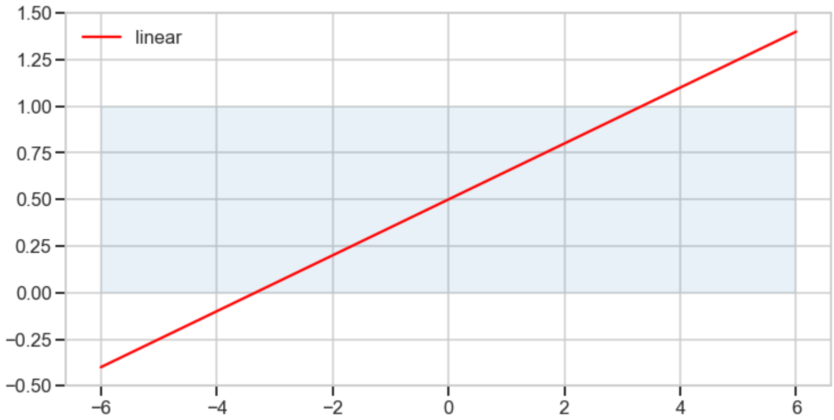


This is called a linear probability model. It has the advantages that much of the regression statistical theory can be applied to it.
But output is in range $(-\infty, \infty)$, we want 0 or 1

Pick a threshold value, default is 0.5.
If $y <$ `threshold`
    predict 0
else
    predict 1

...it would be nice ...

Can we ~~replace~~ map the infinite interval $(-\infty, \infty)$ to a finite interval, say to $(0, 1)$?



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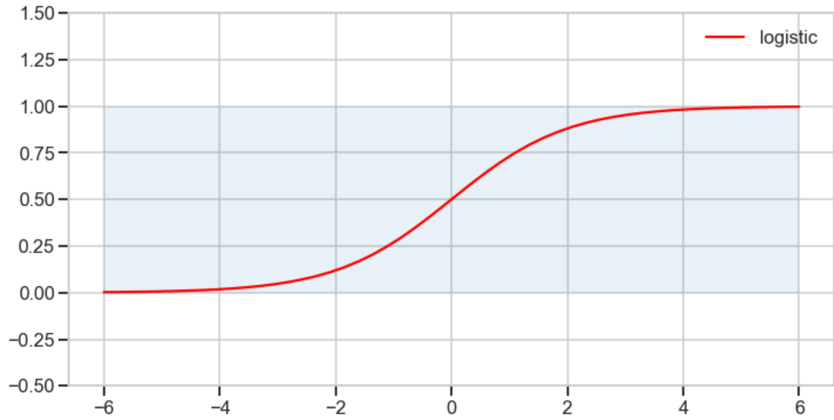


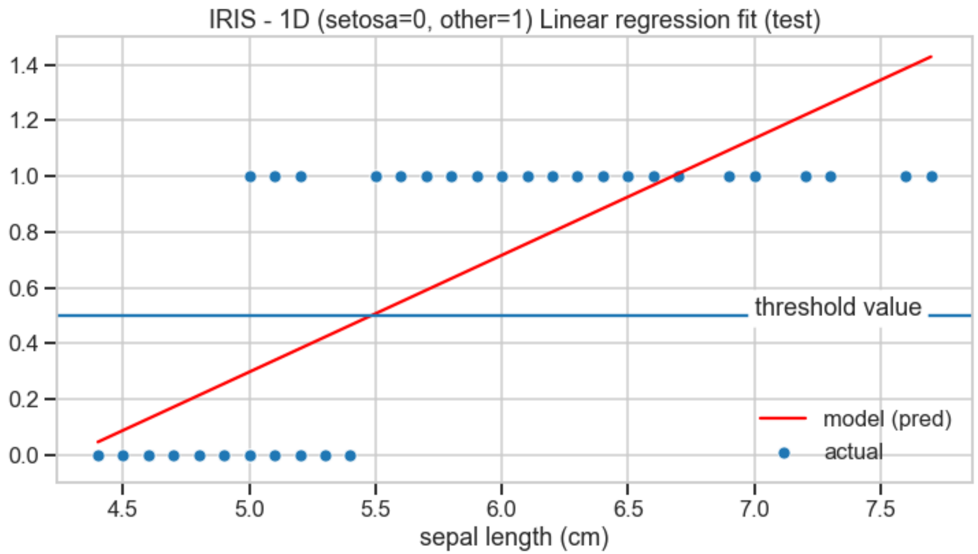
A Sigmoid function† is any function that has a stretched S-shaped curve. One example of a sigmoid function is the

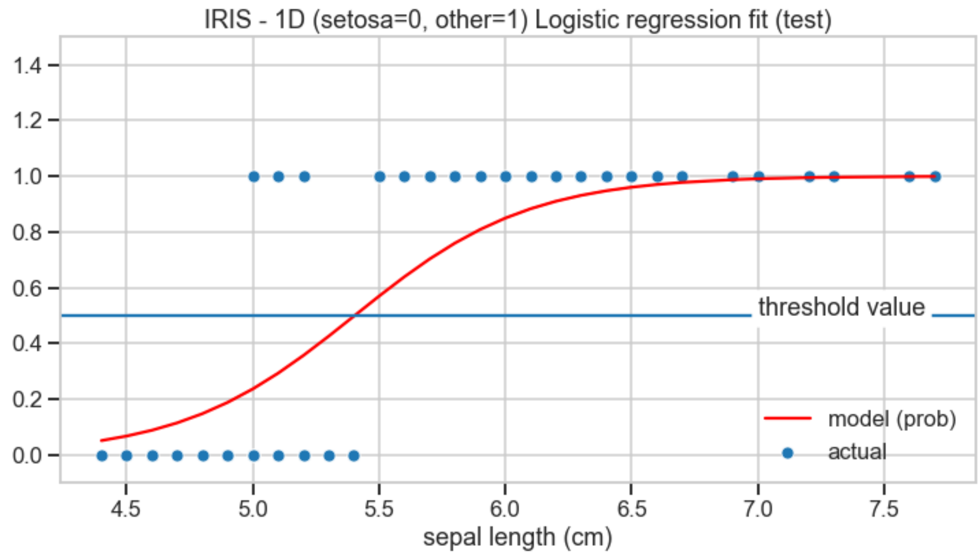
Logistic function, σ

$$\sigma(x) = \frac{1}{1 + \exp(-x)} = \frac{1}{1 + e^{-x}} = \frac{e^x}{1 + e^x}$$

† Sigmoid function (Wikipedia), and Logistic function (Wikipedia),

Linear Regression vs Logistic Regression





Linear Regression Model

$$y = mx + c$$

$$x \curvearrowright y$$

Logistic Regression Model

$$y = \sigma(z) \quad \text{where} \quad z = mx + c$$

$$x \curvearrowright z \curvearrowright y$$

Aside: The function linking, z, the output of linear step, to y, the model output is called a link function.

Aside: Of all the Sigmoid functions, why pick Logistic? I

First a little bit of mathematical manipulation . . .

$$
\begin{aligned}
y = \sigma(z) &= \frac{1}{1+\mathrm{e}^{-z}} && \text{definition of } \sigma(z) \\
y + y\mathrm{e}^{-z} &= 1 && \text{multiply both sides by } (1+\mathrm{e}^{-z}) \\
\mathrm{e}^{-z} &= \frac{1-y}{y} && \text{solve for } \mathrm{e}^{-z} \\
\mathrm{e}^{z} &= \frac{y}{1-y} && \text{invert both sides} \\
z &= \ln\left(\frac{y}{1-y}\right) && \text{apply logs}
\end{aligned}
$$

So we have

$$
\boxed{y = \frac{1}{1+\mathrm{e}^{-z}} \quad \Longleftrightarrow \quad z = \ln\left(\frac{y}{1-y}\right)}
$$

Aside: Of all the Sigmoid functions, why pick Logistic? II

- If y represents a probability, then

$$\frac{y}{1-y}$$

represent the odds — an alternative measure of the likelihood of a particular outcome‡.

- Then

$$z = \ln\left(\frac{y}{1-y}\right)$$

is the log-odds, or the logit.

- This implies that increasing one of the independent variables multiplicatively scales the odds of the given outcome at a constant rate, with each independent variable having its own parameter. So the feature coefficients in a logistic regression have a similar interpretation (but not the same!) as in linear regression.

‡Odds are defined as the ratio of the number of events that produce that outcome relative to the number that do not.

Logistic Regression — Probability of being in Predicted Class

Unlike some classifiers, the `LogisticRegression` classifier can also report on the probability of an observation being in the predicted class[§].

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```
y_prob = model.predict_proba(X_test)
y_prob
```

```
array([[0.36106274, 0.63893726],
       [0.24038055, 0.75961945],
       [0.76292166, 0.23707834],
       [0.24038055, 0.75961945],
       [0.91120554, 0.08879446],
       [0.43024916, 0.56975084],
       [0.29719846, 0.70280154],
       [0.03992405, 0.96007595],
       [0.06912596, 0.93087404],
       [0.43024916, 0.56975084],
       [0.02275676, 0.97724324],
       [0.93203415, 0.06796585],
```

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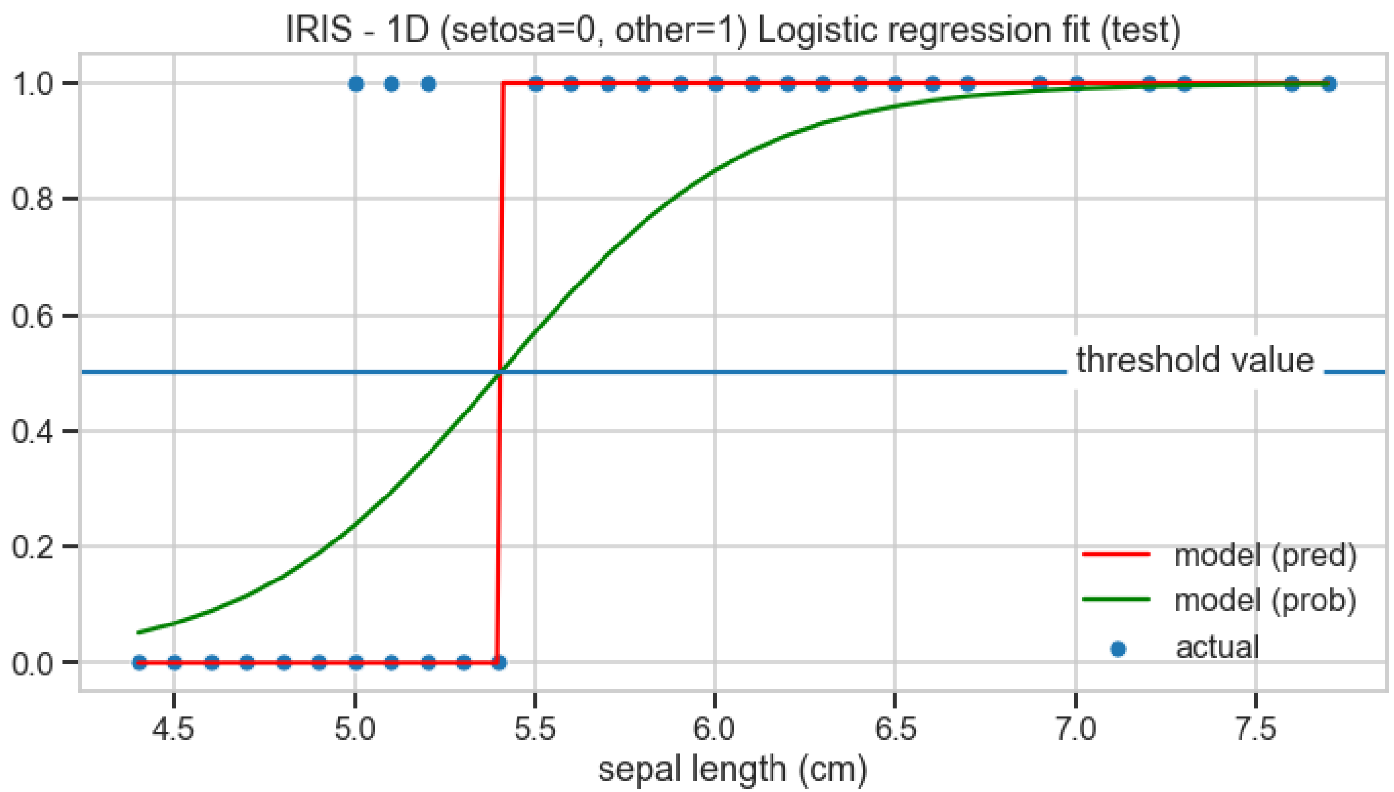
```
y_log_prob = model.predict_log_proba(X_test)
y_log_prob
```

```
array([[-1.01870355e+00, -4.47949007e-01],
       [-1.42553199e+00, -2.74937692e-01],
       [-2.70599925e-01, -1.43936465e+00],
       [-1.42553199e+00, -2.74937692e-01],
       [-9.29867864e-02, -2.42143102e+00],
       [-8.43390801e-01, -5.62556133e-01],
       [-1.21335515e+00, -3.52680729e-01],
       [-3.22077634e+00, -4.07428849e-02],
       [-2.67182500e+00, -7.16313014e-02],
       [-8.43390801e-01, -5.62556133e-01],
       [-3.78289290e+00, -2.30196948e-02],
       [-7.03858210e-02, -2.68874994e+00],
```

[§] This is a big deal as it allows us to punish learners when they miss-classify based on how confident the classifier was — cross entropy loss functions (week 6 & 7).

Logistic Regression



Predicted	other	setosa
Actual		
other	38	3
setosa	0	19