(MSc) Data Mining

Topic 06 : Classification 1

Part 04 : K Nearest Neighbours

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Outline

- K Nearest Neighbours (KNN) algorithm

k-Nearest Neighbour Methods

General Idea

- Given m labeled observations (▲, ■, and ★), how should we classify a new unlabelled observation (?)?
- We could use the labels of the k-nearest neighbouring points.
    - "Nearest" means distance — how should we calculate this?
    - How do we pick the value for k?
- What is our decision rule?
    - Assign new observation to most frequent occurring class in k-nearest neighbours.
    - What to do if there is a tie?

Distance Functions

We frequently want to measure how close/near/similar two points (think observations/instances/cases) are. For this we need a distance function.

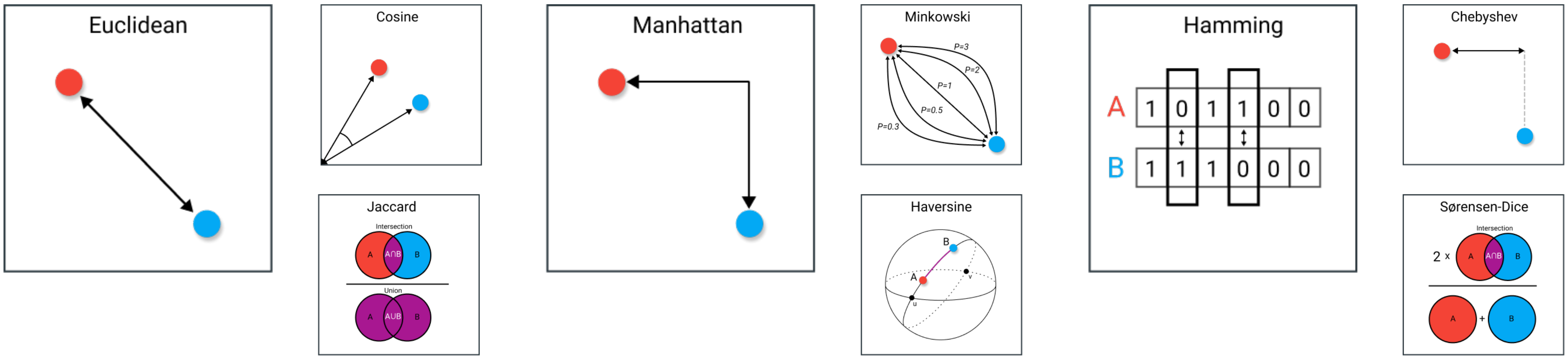
Distance Function

A distance function, $D(a, b)$, is any function that satisfies the properties:

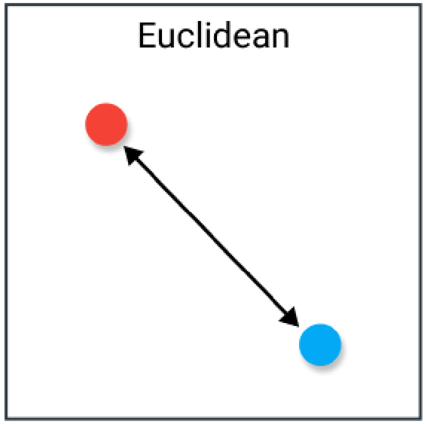
non-negativity: $D(a, b) \geq 0$, distance between any two points is non-negative and is only zero if $a = b$.

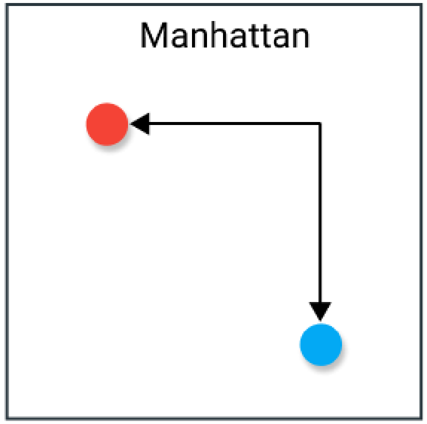
symmetric: $D(a, b) = D(b, a)$

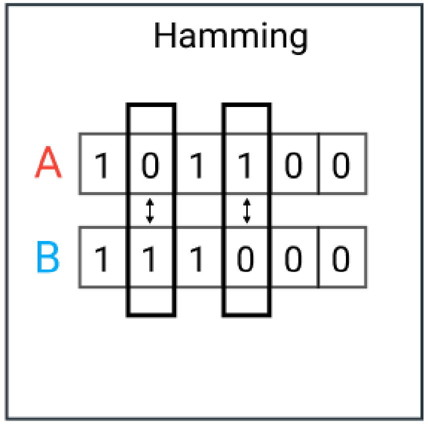
triangular inequality: $D(a, c) \leq D(a, b) + D(b, c)$



Distance Functions — Euclidean, Manhattan, and Hamming







- Pythagorean theorem

$$D(\boldsymbol{a}, \boldsymbol{b}) = \sqrt{\sum_{i=1}^{n} \left[a^{(i)} - b^{(i)}\right]^2}$$

- “As the crow flies”
- Features should be normalised before use

✔ Most commonly used metric.

✘ Becomes less useful for large dimensions

- Taxi-cab distance

$$D(\boldsymbol{a}, \boldsymbol{b}) = \sum_{i=1}^{n} \left|a^{(i)} - b^{(i)}\right|$$

✔ Seems to work better than Euclidean for high-dimensional data

✔ Suitable for datasets with discrete and/or binary features.

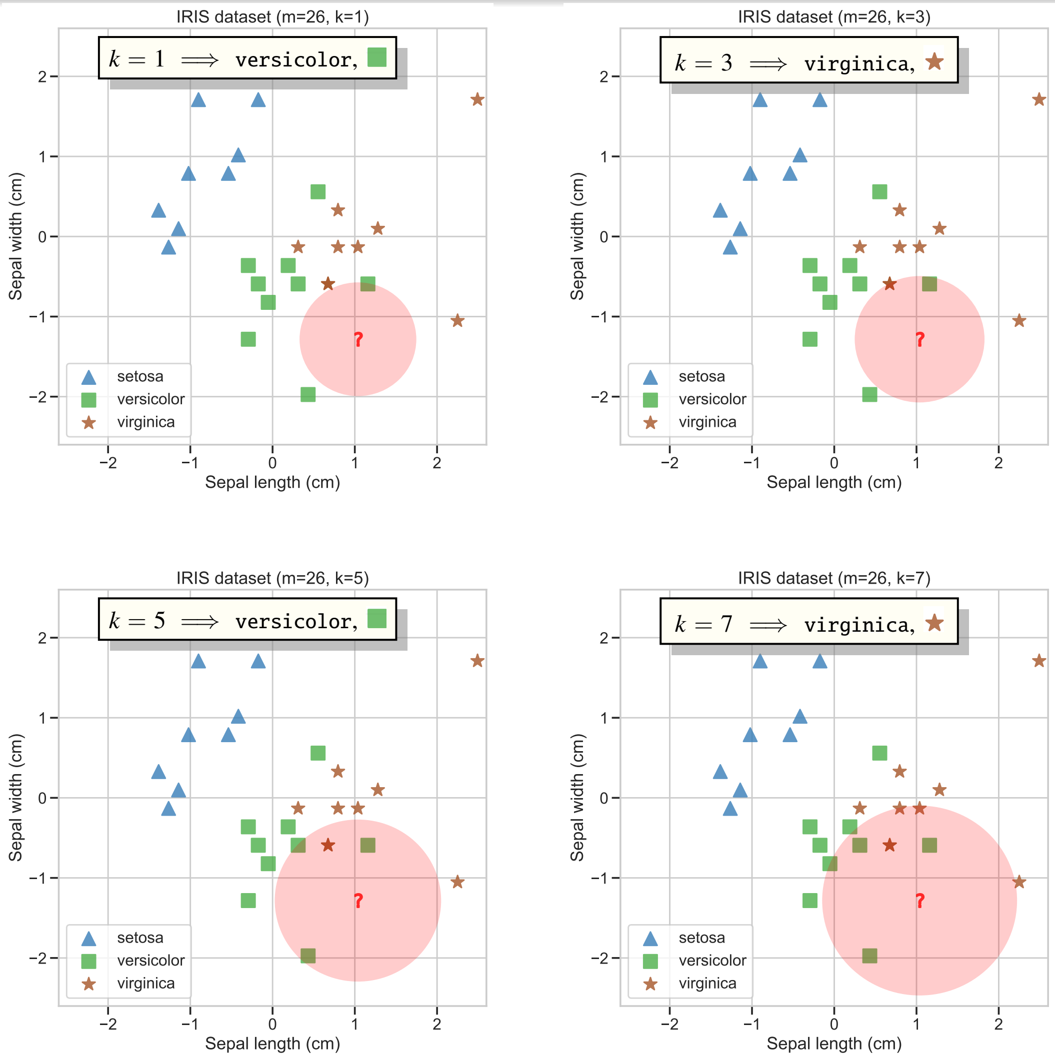
- Count of the number of differences (bits/letters/levels etc) between two points.

✔ Can be used between categorical variables.

✘ Difficult to use when two vectors are not of equal length.

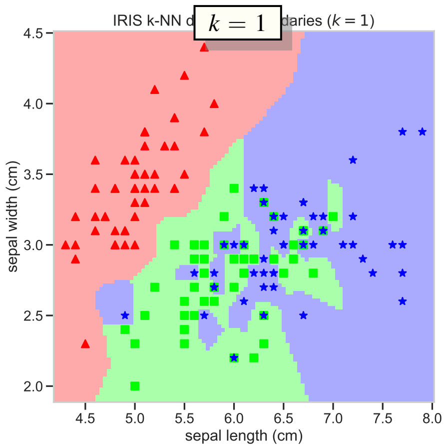
✘ Should not be used when magnitude is important.

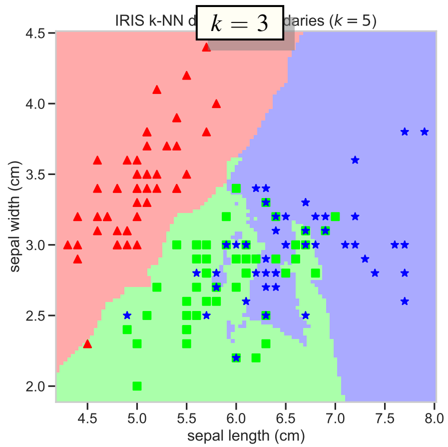
Effect of k

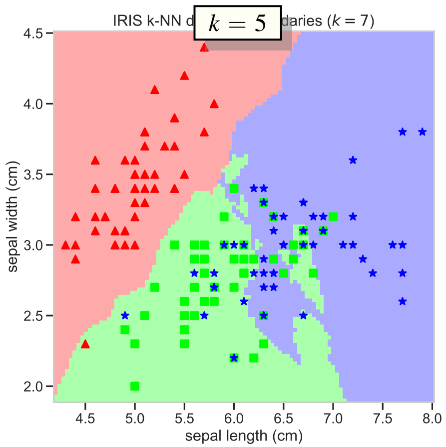


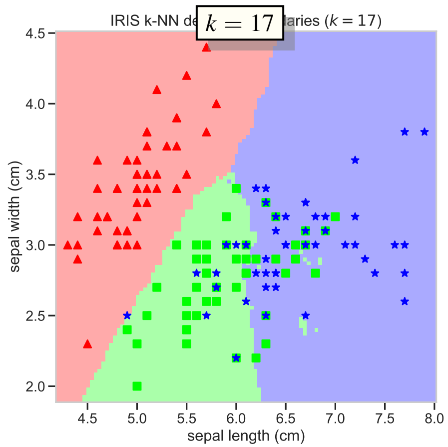
IRIS dataset (m=26, k=1)
$k = 1 \implies$ versicolor,
IRIS dataset (m=26, k=3)
$k = 3 \implies$ virginica,
IRIS dataset (m=26, k=5)
$k = 5 \implies$ versicolor,
IRIS dataset (m=26, k=7)
$k = 7 \implies$ virginica,
?
setosa
versicolor
virginica
Sepal width (cm)
Sepal length (cm)

Effect of k on Decision Boundary









k-Nearest Neighbour Methods — Review

When to Consider

- Observations/instances map to points in $\mathbb{R}^n$ (quantitative/numerical features)
- Less than 20 features/attributes per instance (low dimensionality)
- Lots of training data (more points means closer neighbours)

Advantages

- Training is very fast (instantaneous, since lazy learner)
- Learn complex target functions
- Do not lose information (lazy learner)

Disadvantages

- Slow at query time (uses training data not model to predict)
- Memory-based technique (must pass over (nearly) all points for each classification)
- Easily fooled by irrelevant features/attributes

Hyper-Parameters

- Distance metric (Euclidean — "as the crow flies")
- Number of neighbours, k (Increasing k reduces variance, increases bias)

Resources

- 9 Distance Measures in Data Science
  `towardsdatascience.com/9-distance-measures-in-data-science-918109d069fa`
  Non-technical comparison of common distances functions (source of images used here).