Data Mining 2

Topic 03: Review of Model Building

Lecture 03: Classification Models

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Outline

- Confusion matrix
- Precision, Accuracy, Recall and Specificity
- ROC and AUC

Outline

1. Classification Models (Evaluating Categorical Prediction)	
1.1. Imperfect Tests	
1.2. Multiclass Classification	1

2. IRIS Dataset — Classification using Logistic Regression

Consider an imperfect test with two outcomes, there are four possible outcomes:

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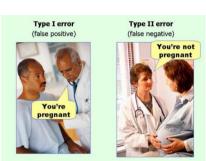
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- How do we combines these quantities into a single metric
- The fraction of correct results seems like a good idea

$$accuracy = \frac{TP + TN}{P + N}$$



- Ideally we want the probability of either error to be zero but that may not be possible.
- Depending on the conditions we often modify the test to reduce probability of the type of error we don't want at the expense of increasing the probability of the other — think court case vs medical condition.

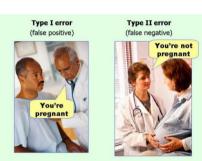
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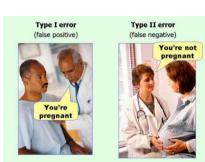
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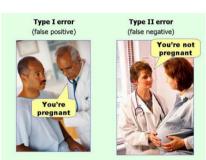
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N

Predicted

Positive

Type I error

False Positive (FP)

True Positive (TP)

Negative

True Negative (TN)

Type II error False Negative (FN)

Ñ

Negative

Confusion matrix (Contingency table) Metrics

 $\textbf{Accuracy} = \frac{TP + TN}{P + N}$

(How often is the classifier correct?)	∢	Positive
False negative rate (FNR) = $\frac{FN}{P}$ = 1 - TPR	-	

- Sensitivity = Recall = True positive rate (TPR) = $\frac{TP}{R}$ = 1 FNR
- Specificity = $\frac{TN}{N}$ = 1 FPR
- False positive rate (FPR) = false acceptance = $\frac{FP}{N}$ = 1 Specificity
- **Precision** = positive predictive value (PPV) = $\frac{TP}{\hat{\mathbf{p}}} = \frac{TP}{TP + FP}$

 $Accuracy = \frac{TP + TN}{P + N}$ (How often is the classifier correct?)

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 $\begin{aligned} \textbf{Accuracy} &= \frac{TP + TN}{P + N} \\ \text{(How often is the classifier correct?)} \end{aligned}$

Negative Positive T True Negative T True Negative T Positive T

Predicted

- False negative rate (FNR) = $\frac{FN}{P}$ = 1 TPR
- Sensitivity = Recall = True positive rate (TPR) = $\frac{TP}{P}$ = 1 FNR (Of positive cases that exist how many did we mark positive?)
- Specificity = $\frac{TN}{N}$ = 1 FPR (When it's actually no, how often does we predict no?) (Of cases that are negative, how many did we mark negative)
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- **Precision** = positive predictive value (PPV) = $\frac{TP}{\hat{P}} = \frac{TP}{TP + FP}$ (Of cases that we marked positive, how many were correct?)

 $\textbf{Accuracy} = \frac{TP + TN}{\mathbf{p} \perp \mathbf{N}}$ (How often is the classifier correct?)

• False negative rate (FNR) = $\frac{FN}{D}$ = 1 - TPR

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Actual		True Negative (TN) Type II error	raise Positive (FF)	
	Positive	False Negative (FN)	True Positive (TP)	
		\hat{N}	\hat{P}	

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 = 1 - TPR

Predicted

Negative Positive

Negative Type I error

True Negative
$$(TN)$$
 False Positive (FP) Positive

Positive \hat{N} True Positive (TP) \hat{N} \hat{P} T

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Accuracy — how well model is trained and perform in general

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Precision — important when the costs of false positives are high

Recall — important when the

costs of false negatives are high

$\overline{\rangle}$ F₁ Score \rangle

The F-measure or balanced F-score (F_1 score, is a special case of the F_β score) is the harmonic mean of precision and recall:

$$F_1 = 2\left[\frac{1}{\frac{1}{\text{precision}} + \frac{1}{\text{recall}}}\right] = 2\left[\frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}}\right]$$

A word of Caution . . .

Consider the three binary classifiers A, B and C

A		3	С	
		0.8	0.12	0.78

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	A		A B		C	
	F	T	F	T	F	T
F	0	0.1 0.9	0.1	0	0.1	0
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Metric	A	В	C	(best)
Accuracy	0.9	0.9	0.88	AB
Precision	0.9	1.0	1.0	BC
Recall	1.0	0.888	0.8667	A
F-score	0.947	0.941	0.9286	A

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Yet look at the performance metrics – B is never the clear winner.

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We use some metrics because they are easy to understand, and not because they always give the "correct" result.

Mutual Information is a Better Metric

The mutual information between predicted and actual label (case) is defined

$$I(\hat{y}, y) = \sum_{\hat{y} = \{0,1\}} \sum_{y = \{0,1\}} p(\hat{y}, y) \log \frac{p(\hat{y}, y)}{p(\hat{y})p(y)}$$

where $p(\hat{y}, y)$ is the joint probability distribution function.

This gives the intuitively correct rankings B > C > A

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F-score	0.947	0.941	0.9286
Mutual information	0	0.1865	0.1735

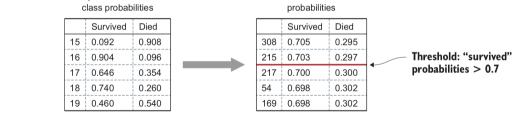
Sorted

Receiver Operating Characteristic (ROC)

Output from classifier:

Many classification systems have thresholds and parameters that can vary their performance*. In such cases, it can be useful to look at the TPR and FPR as the threshold/parameters are changed. One can seek to determine the best possible values for the threshold/parameters by finding a particular TPR vs FPR ratio. This is done by plotting an ROC curve.

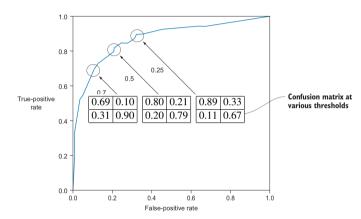
For example, consider the Titanic survivor dataset, a part of the output from a classifier is given bellow:



After sorting the full table by decreasing survival probability, we can set a threshold and consider all rows above this threshold as survived.

^{*}Think logistic regression – at what probability value do we switch from predicting 0 to predicting 1?

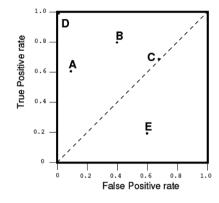
Receiver Operating Characteristic (ROC)



The ROC curve defined by calculating the confusion matrix and ROC metrics at various threshold points from 0 to 1. By convention, we plot the false-positive rate on the x-axis and the true-positive rate on the v-axis.

ROC Space

- Each point in ROC represents a classifier.
- Lower left point (0,0) represents the strategy of never issuing a positive classification: such a classier commits no false positive errors but also gains no true positives
- Upper right corner (1, 1) represents the opposite strategy, of unconditionally issuing positive classifications.
- Point (0, 1) represents perfect classification D's performance is perfect as shown.
- Informally, one point in ROC space is better than another if it is to the northwest of the first: TPR is higher, FPR is lower, or both.
- The diagonal line y = x represents the strategy of randomly guessing a class.

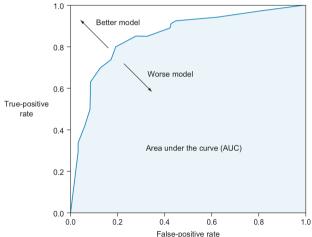


An ROC graph with five classifiers: A,..., E.

Area Under an ROC Curve (AUC)

Rather than looking at individual points (based on particular threshold values) we can compare classifiers

by using the area under the ROC.



The AUC of a classifier is equivalent to the probability that the classier will rank a randomly chosen positive instance higher than a randomly chosen negative instance.

The bigger AUC the better

Micro Average vs Macro Average Performance

In a multi-class classifier we have more than two classes (iris dataset):

- The ROC is computed for each class using the one-vs-all for each class, we denote the particular class as the positive class, and everything else as the negative class, and we draw the ROC curves as usual.
- To combine the metrics for individual classes to get an overall system metrics, we can apply either Micro-Average Method

Sum up the individual true positives, false positives, and false negatives of the system for different classes and then apply totals to get the statistics.

Macro-average Method

Average the precision and recall of the system on different classes.

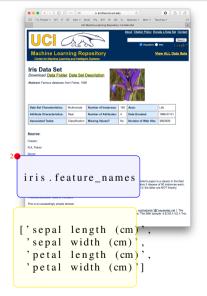
See classification_report from sklearn.metrics.

Outline

1. Classification Models (Evaluating Categorical Prediction)
1.1. Imperfect Tests
1.2 Multiclass Classification

2. IRIS Dataset — Classification using Logistic Regression

Example: IRIS Dataset — Load



```
from sklearn import datasets
iris = datasets.load_iris()

df = pd.DataFrame(iris.data)
df.columns = iris.feature_names
df['target'] = iris.target_names[iris.target]
df.sample(4)
```

sepal lengt	h (cm) sepal width	(cm) petal leng	th (cm) petal width	(cm) target
17 5.1	3.5	1.4	0.3	setosa
80 5.5	2.4	3.8	1.1	versicolor
97 6.2	2.9	4.3	1.3	versicolor
99 5.7	2.8	4.1	1.3	versicolor

The data set contains, four numeric features, 3 classes of 50 instances each, where each class refers to a type of iris plant. One class is linearly separable from the other 2; the latter are NOT linearly separable from each other.

Example: IRIS Dataset — Preprocess Data

We will cover some classifiers in a moment, but for now just treat the classifiers (LogisticRegression) as a black box and focus on the general process:

Extract the data (features and target)

Split dataset into train and test

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```
Extract the data (features and target)
```

The IRIS dataset has 4 features, but to simplify visualisation we are only going to use the first two † ('sepal length' and 'sepal width'):

```
dataset_name = "IRIS"

X, y, target_names = iris.data[:,:2], iris.target, iris.target_names
```

```
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```
Split dataset into train and test
```

We will keep 40% of the data for testing. Setting the parameter random_state to a value means that we will get a random — but still reproducible — split.

```
from sklearn.model_selection import train_test_split
X_train, X_test, y_train, y_test = train_test_split(X, y, train_size=0.6, random_state=666)
```

Select classifier

Train model



Select classifier

Scikit-learn supports a large set of classifiers, and aims to have a consistent interface to all. First import classifier and create instance . . .

from sklearn.linear_model import LogisticRegression model = LogisticRegression(max_iter=500)

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Train model

Then we train (fit) the classifier/model using only the features (X_train) and targets (y_train) from the train dataset . . .

```
model.fit(X_train, y_train)
```

```
Predict
```

Logistic Regression (max_iter=500)

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```
Predict
```

1.4...4

Logistic Regression (max_iter=500)

Now that model is trained, we can use it to generate predictions, using the features (X_test) from the test

Scoring and confusion matrix

Scoring and confusion matrix

We could just compute the score using whatever metric we have picked ...

But this needs context, and even if "good" it can hide critical flaws. Lets look at the confusion matrix ...

or, to get a nicer output, convert to a DataFrame ...

```
df_cm = pd.crosstab(target_names[y_test], target_names[y_pred])
df_cm.index.name = 'Actual'
df_cm.columns.name = 'Predicted'

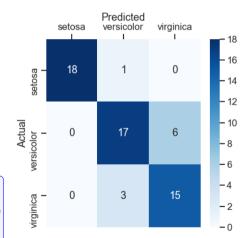
df_cm.
```

The confusion matrix is fundamental in evaluating a classifier, so find a presentation/visualisation that you like and use it. Here I have a heat map representation that I tend to use.

Predicted	setosa	versicolor	virginica

Actual			
setosa	18	1	0
versicolor	0	17	6
virginica	0	3	15

plt.figure(figsize=(6,6))
g = sns.heatmap(df_cm, annot=True, cmap="Blues")
g.xaxis.set_ticks_position("top")
g.xaxis.set_label_position('top')



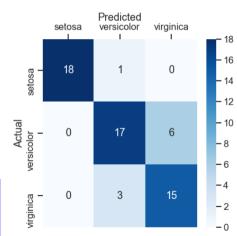
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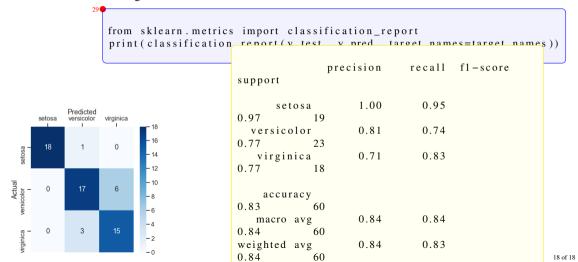
Actual			
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The first class setosa was only misclassified once, while the classifier had more difficulty between the second two classes

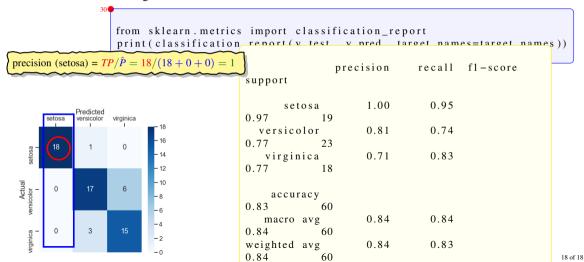
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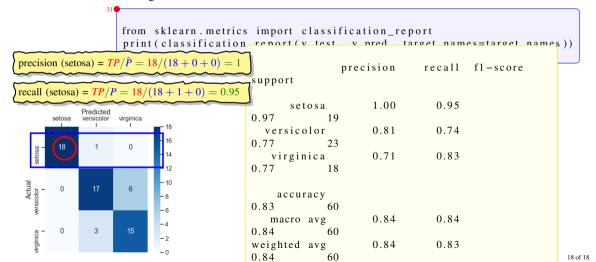
The classification report, constructed from the confusion matrix, summaries the most common metrics per class and for overall averages ...



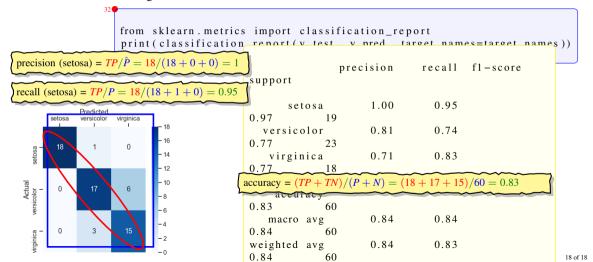
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