

Data Mining 2

Topic 03 : Review of Model Building

Lecture 03 : Classification Models

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Outline

- Confusion matrix
- Precision, Accuracy, Recall and Specificity
- ROC and AUC



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A Non-perfect Test — Type I and Type II Errors

Consider an imperfect test with two outcomes, there are four possible outcomes:

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		Negative	Positive	
Actual	Negative	 True Negative (TN)		N
	Positive		 True Positive (TP)	P
		\hat{N}	\hat{P}	T

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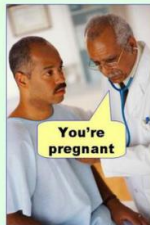
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- If the test is applied to $T = P + N = \hat{P} + \hat{N}$ observations / subjects / instances then we have four independent quantities TP , TN , FP , and FN .
- How do we combine these quantities into a single metric?
- The fraction of correct results seems like a good idea

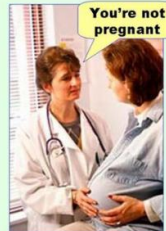
$$\text{accuracy} = \frac{TP + TN}{P + N}$$

But what happens, if we are testing for an rare event? Maximising accuracy will result in the test always returning negative.

Type I error
(false positive)



Type II error
(false negative)



- Ideally we want the probability of either error to be zero but that may not be possible.
- Depending on the conditions we often modify the test to reduce probability of the type of error we don't want at the expense of increasing the probability of the other — think court case vs medical condition.

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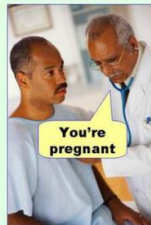
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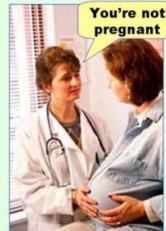
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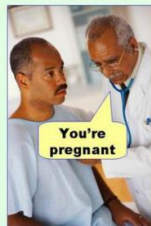
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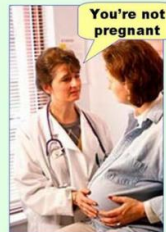
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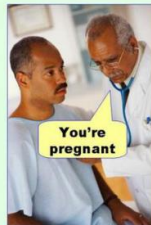
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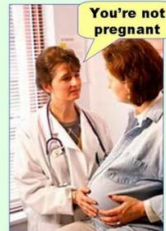
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Confusion matrix (Contingency table) Metrics

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- Sensitivity** = **Recall** = True positive rate (TPR) = $\frac{TP}{P} = 1 - FNR$
(Of positive cases that exist how many did we mark positive?)
- Specificity** = $\frac{TN}{N} = 1 - FPR$
(When it's actually no, how often does we predict no?)
(Of cases that are negative, how many did we mark negative?)
- False positive rate (FPR) = false acceptance = $\frac{FP}{N} = 1 - \text{Specificity}$
- Precision** = positive predictive value (PPV) = $\frac{TP}{\hat{P}} = \frac{TP}{TP + FP}$
(Of cases that we marked positive, how many were correct?)

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Accuracy — how well model is trained and perform in general

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Recall — important when the costs of false negatives are high

Precision — important when the costs of false positives are high

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F₁ Score

The F-measure or balanced F-score (F₁ score) is the harmonic mean of precision and recall:

$$F_1 = 2 \left[\frac{1}{\frac{1}{\text{precision}} + \frac{1}{\text{recall}}} \right] = 2 \left[\frac{\text{precision} \times \text{recall}}{\text{precision} + \text{recall}} \right]$$

A word of Caution ...

Consider the three binary classifiers A, B and C

	A		B		C	
	T	F	T	F	T	F
T	0.9	0.1	0.8	0	0.78	0
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Metric	A	B	C	(best)
Accuracy	0.9	0.9	0.88	AB
Precision	0.9	1.0	1.0	BC
Recall	1.0	0.888	0.8667	A
F-score	0.947	0.941	0.9286	A

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We use some metrics because they are easy to understand, and not because they always give the “correct” result.

Mutual Information is a Better Metric

The **mutual information** between predicted and actual label (case) is defined

$$I(\hat{y}, y) = \sum_{\hat{y}=\{0,1\}} \sum_{y=\{0,1\}} p(\hat{y}, y) \log \frac{p(\hat{y}, y)}{p(\hat{y})p(y)}$$

where $p(\hat{y}, y)$ is the **joint probability distribution** function.

This gives the intuitively correct rankings $B > C > A$

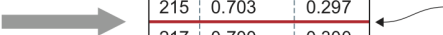
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Mutual information	0	0.1865	0.1735

Receiver Operating Characteristic (ROC)

Many classification systems have thresholds and parameters that can vary their performance*. In such cases, it can be useful to look at the **TPR** and **FPR** as the threshold/parameters are changed. One can seek to determine the best possible values for the threshold/parameters by finding a particular **TPR** vs **FPR** ratio. This is done by plotting an ROC curve.

For example, consider the Titanic survivor dataset, a part of the output from a classifier is given bellow:

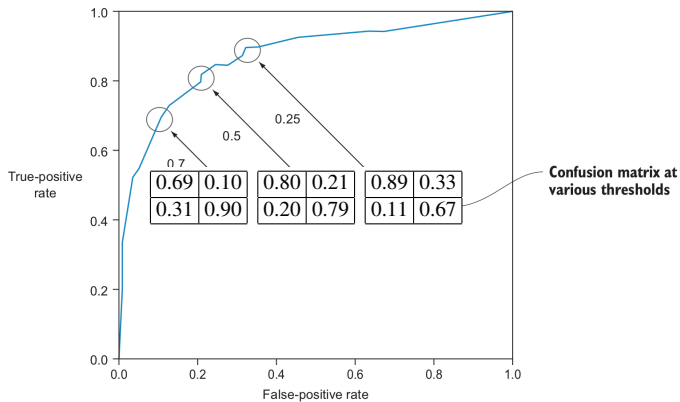
Output from classifier: class probabilities			Sorted probabilities		
	Survived	Died		Survived	Died
15	0.092	0.908	308	0.705	0.295
16	0.904	0.096	215	0.703	0.297
17	0.646	0.354	217	0.700	0.300
18	0.740	0.260	54	0.698	0.302
19	0.460	0.540	169	0.698	0.302


Threshold: "survived" probabilities > 0.7

After sorting the full table by decreasing survival probability, we can set a threshold and consider all rows above this threshold as survived.

*Think logistic regression – at what probability value do we switch from predicting 0 to predicting 1?

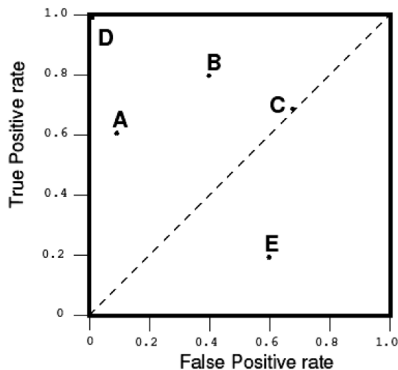
Receiver Operating Characteristic (ROC)



The ROC curve defined by calculating the confusion matrix and ROC metrics at various threshold points from 0 to 1. By convention, we plot the false-positive rate on the x-axis and the true-positive rate on the y-axis.

ROC Space

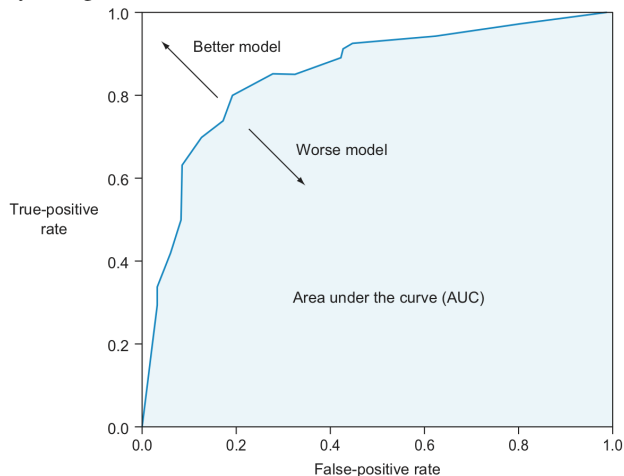
- Each point in ROC represents a classifier.
- Lower left point $(0, 0)$ represents the strategy of never issuing a positive classification: such a classifier commits no false positive errors but also gains no true positives
- Upper right corner $(1, 1)$ represents the opposite strategy, of unconditionally issuing positive classifications.
- Point $(0, 1)$ represents perfect classification D's performance is perfect as shown.
- Informally, one point in ROC space is better than another if it is to the northwest of the first:
TPR is higher, FPR is lower, or both.
- The diagonal line $y = x$ represents the strategy of randomly guessing a class.



An ROC graph with five classifiers: A, ..., E.

Area Under an ROC Curve (AUC)

Rather than looking at individual points (based on particular threshold values) we can compare classifiers by using the area under the ROC.



The AUC of a classifier is equivalent to the probability that the classifier will rank a randomly chosen positive instance higher than a randomly chosen negative instance.

The bigger AUC the better

Micro Average vs Macro Average Performance

In a multi-class classifier we have more than two classes (iris dataset):

- The ROC is computed for each class using the one-vs-all — for each class, we denote the particular class as the positive class, and everything else as the negative class, and we draw the ROC curves as usual.
- To combine the metrics for individual classes to get an overall system metrics, we can apply either

Micro-Average Method

Sum up the individual true positives, false positives, and false negatives of the system for different classes and then apply totals to get the statistics.

Macro-average Method

Average the precision and recall of the system on different classes.

See `classification_report` from `sklearn.metrics`.