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South East Technological University

BACHELOR OF SCIENCE (HONS) IN
- APPLIED COMPUTING
- COMPUTER FORENSICS & SECURITY

EXAMINATION:

DISCRETE MATHEMATICS (COMMON MODULE)

SEMESTER 1 - YEAR 1

DECEMBER 2023 **DURATION: 2 HOURS**

INTERNAL EXAMINERS:	**DR DENIS FLYNN**	**DATE:**	**15 DEC 2023**
	DR KIERAN MURPHY	**TIME:**	**11.45 AM**
		VENUE:	**MAIN HALL**
EXTERNAL EXAMINER:	**MS MARGARET FINNEGAN**		

INSTRUCTIONS TO CANDIDATES

1. **ANSWER ALL QUESTIONS.**
2. **TOTAL MARKS = 100.**
3. **EXAM PAPER (5 PAGES EXCLUDING THIS COVER PAGE), FORMULA AND PYTHON SHEETS (2 PAGES)**

MATERIALS REQUIRED

1. **NEW MATHEMATICS TABLES.**
2. **GRAPH PAPER**

SOUTH EAST TECHNOLOGICAL UNIVERSITY

WATERFORD INSTITUTE OF TECHNOLOGY

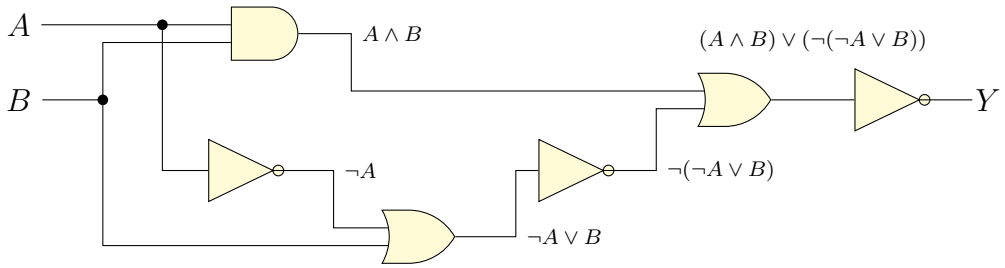
OUTLINE MODEL ANSWERS & MARKING SCHEME

Course: BSc (H) in AC, in CF	Semester: 1	Page 1 of 6
Subject: Discrete Mathematics	Examiner: Dr D. Flynn, Dr K Murphy	

Question 1

(a)

(i) Construct a logical expression to represent the output Y. **(4 marks)**



$$Y = \neg\Big((A \wedge B) \vee (\neg(\neg A \vee B)\Big)$$

(ii) Construct a truth table for this expression and classify the logical expression as a tautology, a contradiction, or satisfiable. **(4 marks)**

A	B	$(A \wedge B)$	$\neg A \vee B$	$\neg(\neg A \vee B)$	$(A \wedge B) \vee (\neg(\neg A \vee B))$	Y
0	0	0	1	0	0	1
0	1	0	1	0	0	1
1	0	0	0	1	1	0
1	1	1	1	0	1	0

Expression is satisfiable, but is not a tautology.

(iii) Compare the inputs to the output and construct a shorter logical expression that is logically equivalent to the original circuit. **(4 marks)**

$$Y = \neg A$$

(b)

Code reports whether set A is a proper subset of set B.
So for the given sets: **True**

4 marks, justification required

(c)

(i) *How many subsets are there of cardinality 4?*

$\binom{6}{4} = 15$ subsets.

(ii) *How many subsets of S are there? That is, find $|\mathcal{P}(A)|$.*

$2^6 = 64$ subsets.

(iii) *How many subsets of S are there where the sum of the elements equals 13?*

The elements of the set S are the powers of 2 so the sum of different subsets of S are unique. Hence answer is 1.

4 marks=1+1+2

Question 2

(a)

(i) $\vee pq$
Not well formed. Logical and, $\vee$ is a binary operator, expected infix notation.

(ii) $p\neg\neg r$
Not well formed. Logical not operator, $\neg$ is a unary operator.

(iii) $p\neg \rightarrow (q \wedge q)$
Not well formed. $\neg$ applied to conditional operator

(iv) $(p \wedge \neg q) \vee (q\neg \rightarrow q)$
Not well formed. Conditional operator, $\rightarrow$ is a binary operator.

4 marks $= 4 \times 1$**, justification required**

(b)

The completed table is **(4 marks)**

char	A	B	C	D
H	False	True	True	False
i	True	False	True	False
,	False	False	False	False
	False	False	False	False
I	False	True	True	False
	False	False	False	False
a	True	False	True	False
m	True	False	True	False
	False	False	False	False
2	False	False	False	True
.	False	False	False	False

(i) `all([c.isupper() for c in message])` **(3 marks)**

False (with any valid example)

(ii) $\exists c \left[B(c) \wedge C(c)\right]$ **(3 marks)**

True (with any valid example)

(iii) $\forall c \left[C(c) \rightarrow (A(c) \vee B(c))\right]$ **(3 marks)**

True (with any valid reason)

(iv) `any([c. isdigit () and c.isalpha() for c in message])` **(3 marks)**

False (with any valid reason)

Question 3

(a)

Partial marks for correct parsing of expression, demonstrating ability to compute logical expression, implication of satisfiability definition.

$$\underbrace{\Big(\underbrace{\underbrace{(P \wedge Q)}_{1} \vee \underbrace{(\neg P \wedge R)}_{2}}_{3}\Big) \rightarrow \underbrace{(\neg Q \vee \neg R)}_{4}}_{E}$$

P	Q	R	$\neg P$	$\neg Q$	$\neg R$	$\overbrace{(P \wedge Q)}^{1}$	$\overbrace{(\neg P \wedge R)}^{2}$	$\overbrace{(P \wedge Q) \vee (\neg P \wedge R)}^{3}$	$\overbrace{(\neg Q \vee \neg R)}^{4}$	E
F	F	F	**T**	**T**	**T**	F	F	F	**T**	**T**
F	F	**T**	**T**	**T**	F	F	**T**	**T**	**T**	**T**
F	**T**	F	**T**	F	**T**	F	F	F	**T**	**T**
F	**T**	**T**	**T**	F	F	F	**T**	**T**	F	F
T	F	F	F	**T**	**T**	F	F	F	**T**	**T**
T	F	**T**	F	**T**	F	F	F	F	**T**	**T**
T	**T**	F	F	F	**T**	**T**	F	**T**	**T**	**T**
T	**T**	**T**	F	F	F	**T**	F	**T**	F	F

The expression is satisfiable but is not a tautology.

6 **marks**

(b)

The term in the loop is odd and since the range is semi-open, the total is -5.

4 **marks**

(c)

(i) *Construct a recursive definition for the sequence.*

$$a_n = a_{n-1} + 5 \text{ with } a_1 = 7$$

(ii) *Construct a closed formula for the nth term of the sequence.*

$$a_n = 7 + 5(n-1)$$

(iii) *Is* 2023 *a term in the sequence? Explain.*

No, since $(2023 - a)/d = 2016/5 = 403.2$ is not an integer.

(iv) *How many terms does the sequence* $7 + 12 + 17 + 22 + 27 + \cdots + 437$ *have?*

87

(v) *Determine the sum:* $7 + 12 + 17 + 22 + 27 + \cdots + 437$

$\frac{(437+7)\times 87}{2} = 19314$

$10 = 5 \times 2$ **marks**

Question 4

(a)

The function returns **True** iff relation is an **onto** relation from A to B.

5 marks, -2 if not stating into/onto

(b)

(i) $5 \in C =$ **True** **(ii)** $\{9\} \in C =$ **False**

(iii) $\{3\} \subseteq A =$ **True** **(iv)** A.intersection(B).issubset(B.union(C)) = **True**

(v) $B \subseteq A =$ **True** **(vi)** B.union(C).intersection(A). isdisjoint (C) = **True**

6 marks$= 6 \times 1$**, justification required**

(c)

(i) $$\sum_{k=0}^{8} 2^k (k \bmod 2) = \underbrace{[0]}_{k=0} + \underbrace{[2^1]}_{k=1} + \underbrace{[0]}_{k=2} + \underbrace{[2^3]}_{k=3} + \underbrace{[0]}_{k=4} + \underbrace{[2^5]}_{k=5} + \underbrace{[0]}_{k=6} + \underbrace{[2^7]}_{k=7} + \underbrace{[0]}_{k=8} = 170$$

(ii) $$\sum_{k=1}^{5} (2k+1) = \underbrace{[3]}_{k=1} + \underbrace{[5]}_{k=2} + \underbrace{[7]}_{k=3} + \underbrace{[9]}_{k=4} + \underbrace{[11]}_{k=5} = 35$$

(iii) $$\prod_{k=0}^{6} (k+1) = \underbrace{[1]}_{k=0} + \underbrace{[2]}_{k=1} + \underbrace{[3]}_{k=2} + \underbrace{[4]}_{k=3} + \underbrace{[5]}_{k=4} + \underbrace{[6]}_{k=5} = 21$$

9 marks$= 3 \times 3$

Question 5

(a)

(i) *Start with the sub-string 10.*

No constraints on remaining 4 bits, so $2^4 = 16$.

(ii) *Have weight 3 (i.e., contain exactly three 1's) and start with the sub-string 01.*

In the remaining 4 bits, two of which must be 1, so $\binom{4}{2} = 6$

(iii) *Either start with* 01 *or end with* 11 *(or both).*

Start with 01: No constraints on remaining 4 bits, so $2^4 = 16$.

Ends with 11: No constraints on preceding 4 bits, so $2^4 = 16$.

Start with 01 and ends with 11: No constraints on middle 2 bits, so $2^2 = 4$.

Ans (remove double counting): $2^4 + 2^4 - 2^2 = 16 + 16 - 4 = 28$

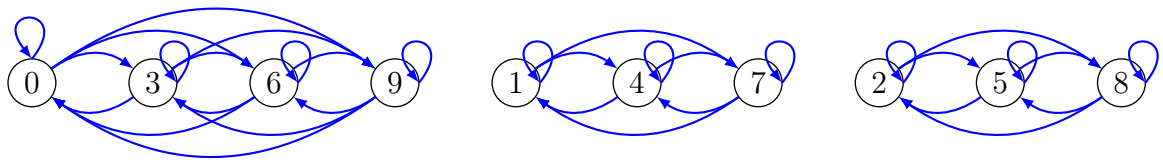
(iv) *Have weight 3, starts with* 01*, and ends with* 11*.*

The middle 2 digits must have weight 0 so that the entire string has weight 3. Hence have 1 possibility.

6 marks $= 1 + 1 + 2 + 2$**, justification required**

(b)

(i) *Represent R using a digraph.*



(ii) *Is R reflexive? symmetric? transitive?*

R is reflexive, symmetric and transitive + reason

(iii) *Is R an equivalence relation? and if yes, what the resulting equivalence classes?*

R an equivalence relation with classes

$$\{0, 3, 6, 9\} \qquad \{1, 4, 7\} \qquad \{2, 5, 8\}$$

8 marks $= 3 + 3 + 2$

(c)

(i) $f(4) = 2 \cdot 4 + 3 = 8 + 3 = 11$

(ii) $g(2) = 2^3 - 1 = 8 - 1 = 7$

(iii) $f(5) + g(2) - h(4) = 2 \cdot 5 + 3 + 7 - \frac{5}{4} = 10 + 3 + 7 - \frac{5}{4} = 20 - \frac{5}{4} = \frac{75}{4} = 18.75$

(iv) $f(g(1)) = f(1^3 - 1) = f(0) = 2 \cdot 0 + 3 = 3$

(v) $g(h(2)) = g(\frac{5}{2}) = (\frac{5}{2})^3 - 1 = \frac{125}{8} - 1 = \frac{117}{8} = 14.63$

(vi) $h(f(3)) = h(2 \cdot 3 + 3) = h(9) = \frac{5}{9} = 0.556$

6 marks$= 6 \times 1$