WATERFORD INSTITUTE OF TECHNOLOGY

OUTLINE MODEL ANSWERS & MARKING SCHEME

Course: BSc (H) in AC, in CF	Semester: 1	Page 1 of 6
Subject: MODULE NAME	Examiner: Dr D. Flynn, Dr K Murphy	

Question 1

(a)

(i) $a_0 = 5$ and $a_n = a_n + 3$ for $n \geq 1$.

(ii) $a_n = 3n + 5$.

(iii) $2024 = 3n + 5$ so $n = 673 \in \mathbb{N}$, therefore 2024 is a term in the sequence.

(iv) $3n + 5 < 1000$ so $n < 331.667$, therefore there are 331 terms less than 1000.

(v) $S_{99} = (n + 1) \times 5 + \frac{n+1}{2}(nd)(n + 1) = 100 \times 5 + \frac{100}{2}(99 \times 3) = 15350$.

(b)

Returns True if input is prime, otherwise returns False.

(c)

(i) $\binom{7-2+12-3}{5} = \binom{14}{5} = 2002$. The paths all have length 14 (9 steps up and 5 steps right), we just select which 5 of those 14 should be up.

(ii) $\binom{4-2+12-3}{2}\binom{7-4+12-8}{3} = \binom{11}{2} \times \binom{7}{3} = 55 \times 35 = 1925$ First travel to (4,8), and then continue on to (7,12)

(iii) $\binom{14}{5} - \binom{11}{2} \times \binom{7}{3} = 2002 - 1925 = 77$
Remove all the paths found in preceding question.

Question 2

(a)

(i) $g(h(4)) = 4$

(ii) $h(g(5)) = 5$

(iii) $f(g(h(3))) = -8$

(iv) $k(j(7)) = 6$

(v) $j(j(g(2))) = j(j(7)) = j(3) = 1$

(b)

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p	q	r	$\neg r \wedge \neg p$	$q \rightarrow (\neg r \wedge \neg p)$	$p \vee r$	$(q \rightarrow (\neg r \wedge \neg p)) \wedge (p \vee r)$
F	F	F	T	T	F	F
F	F	T	F	T	T	T
F	T	F	T	T	F	F
F	T	T	F	F	T	F
T	F	F	F	T	T	T
T	F	T	F	T	T	T
T	T	F	F	F	T	F
T	T	T	F	F	T	F

Alternatively, if students starts with all True inputs:

p	q	r	$\neg r \wedge \neg p$	$q \rightarrow (\neg r \wedge \neg p)$	$p \vee r$	$(q \rightarrow (\neg r \wedge \neg p)) \wedge (p \vee r)$
T	T	T	F	F	T	F
T	T	F	F	F	T	F
T	F	T	F	T	T	T
T	F	F	F	T	T	T
F	T	T	F	F	T	F
F	T	F	T	T	F	F
F	F	T	F	T	T	T
F	F	F	T	T	F	F

Final column is a mixture of true and false statements $\implies$ satisfiable.

(c)

(i) sum = 3 + 4 + 5 + 6 + 7 = 25

(ii) sum = 1 + 2 + 4 + 8 + 16 = 31

(iii) product = 0 * 1 * 2 * 3 = 0

(d)

(i) Not well formed ('not' is a unary operator).

(ii) Well formed

(iii) Not well formed. 'implication' operator is a binary operator.

(iv) Not well formed. 'Not' operator after propositional variable.

Question 3

(a)

(i)
- $U = \{1, 2, \cdots, 9\}$
- $A = \{1, 3, 5, 7\}$
- $B = \{2, 4, 6, 8\}$
- $C = \{3, 4, 5\}$.

(ii) Venn Diagram as done in class.

(iii) Equivalent mathematical expressions:

- D $= (A \cap C) \cup \overline{B}$.
- E $= \overline{(A \cup B) \cap C}$.
- F $= A \cup \overline{B}$.

(iv)
- D $=\{1, 3, 5, 7, 9\}$.
- E $= \{1, 2, 6, 7, 8, 9\}$.
- F $= \{1, 3, 5, 7, 9\}$.

(b)

(i) $\binom{6}{4} = 15$ subsets.

(ii) $\binom{3}{1} = 3$ subsets.

(iii) $\binom{6}{4} = 15$ subsets. All subsets of cardinality 4 must contain at least one odd number.

(iv) $\binom{3}{1} = 3$ subsets. Select one of the three even numbers. The three odd numbers of S must all be in the set.

(c)

P	Q	R	$(P \vee Q) \to R$	$(P \to R) \vee (Q \to R)$
F	F	F	T	T
F	F	T	T	T
F	T	F	F	T
F	T	T	T	T
T	F	F	F	T
T	F	T	T	T
T	T	F	F	F
T	T	T	T	T

or starting with all p=q=r=TRUE:

P	Q	R	$(P \vee Q) \rightarrow R$	$(P \rightarrow R) \vee (Q \rightarrow R)$
T	T	T	T	T
T	T	F	F	F
T	F	T	T	T
T	F	F	F	T
F	T	T	T	T
F	T	F	F	T
F	F	T	T	T
F	F	F	T	T

Looking at either row with inputs: T F F or F T F we don't have logical equivalence (highlighted above).

Question 4

(a)

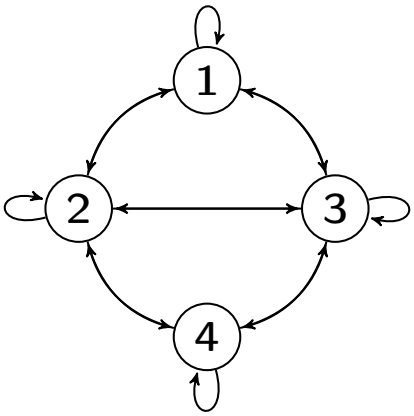
(i) The Python code gives the set $A = \{1, 2, 3, 4\}$, and the relation $R = \{(a, b) | a, b \in A, |a - b| \leq 2\}$
$= \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (2, 4), (3, 1), (3, 2), (3, 3), (3, 4), (4, 2), (4, 3), (4, 4)\}$.

(ii) The digraph is shown below.

(iii) The relation R is reflexive, symmetric, but not transitive (e.g. $(1, 2), (2, 4) \in R$ but $(1, 4) \notin R$.

(iv) R is not an equivalence relation as it's not transitive.

(v) R is: not irreflexive ($\because (1, 1) \in R$ etc.); not antisymmetric (e.g. $(1, 2), (2, 1) \in R$ etc.); not asymmetric (e.g. R is reflexive. R is not antisymmetric, etc.).



(b)

(i) Ending with four bits $\implies (14 - 4) = 10$ yes/no choices, $|B^{10}| = 2^{10} = 1024$.

(ii) Weight 7 and ending with sub-bitstring 0011 i.e. $B_2^4 \implies |B_{7-2}^{14-4}| = \binom{10}{5} = 252$.

(iii) Divisible by 16 $\implies$ ending with 0000, so weight of 4 is unaffected, $|B_7^{14-4}| = \binom{10}{7} = 120$.

Question 5

(a) _____

(i) x=38

(b) _____

(i) (a) TRUE (there is at least a digit in the password
(b) FALSE (special characters $ and # don't satisfy any of the three predicates)
(c) TRUE (no character can be both upper and lower).
(d) TRUE (Special characters satisfies this.)