

## Chapter Summary — Relations and Functions

*This document is a very rough summary of the concepts and tasks that we covered in this chapter. The plan is to write a similar document at the end of each chapter, but time will tell.*  
*I hope this document will be useful to you when revising the chapter. However, please do not think that this comes with any guarantee of completeness — the probability of me overlooking something is large. Please ask if you think I have omitted anything.*  
— kmurphy, 17 Nov, 2024

### Section A: Concepts

#### A.1: Relations .....

- **Definition of relation based on subsets of a Cartesian product**
  - A relation is a set so properties/concepts of sets carry over to relations.
  - Terminology: source and target, and domain and image
- **Properties of relations from set A to set B (i.e., relating to the output values)**
  - one to one (injective)
  - into (not surjective) vs onto (surjective)
  - bijective = injective + surjective
- **Properties of relations on a set (source=target)**
  - Main three properties: reflexive, symmetric and transitive
  - anti-symmetric
  - irreflective and asymmetric
- **Equivalence relation = reflexive, symmetric and transitive**
  - Decomposition of a set into equivalence classes.
- **Representation of relations**
  - Set of ordered pairs
  - Venn diagrams — good for discrete (any usually finite) sets
  - Digraph — for relations on a set (source=target)

#### A.2: Functions .....

- **Definition of function as a restricted relation — exactly one outgoing arrow for each element in the source**
  - A function is a relation so properties/concepts of relations carry over to functions.
  - Formal vs informal definition of functions
- **Representation of functions (in addition to those for relations (above))**
  - Lookup table
  - Formula — good for continuous or infinite sets
  - 2D Cartesian Plots — good for continuous or infinite sets
- **Algebra of functions**
  - Notation: addition/subtraction/multiplication/division of functions
  - Function composition
    - Repeated iteration of functions
- **Function inverse**
  - bijective = necessary and sufficient condition for existence of inverse function pair.

## Section B: Tasks

### B.1: Relations .....

- Verify that a set is a relation from set  $A$  to set  $B$ .
- Represent a relation using suitable format (3 options)
- Verify that a relation has/does not have various properties

### B.2: Functions .....

- Represent a function using suitable format (6 options)
- Verify that a function has/does not have various properties in particular injective, surjective and bijective
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