**BACHELOR OF SCIENCE (HONS) IN**
**- APPLIED COMPUTING**
**- COMPUTER FORENSICS & SECURITY**
**- ENTERTAINMENT SYSTEMS**
**- THE INTERNET OF THINGS**

EXAMINATION:

# DISCRETE MATHEMATICS (COMMON MODULE)

## SEMESTER 1 - YEAR 1- REPEAT

**AUGUST 2023** **DURATION: 2 HOURS**

| | | |
|---|---|---|
| INTERNAL EXAMINERS: | DR KIERAN MURPHY<br>DR DENIS FLYNN | DATE: 24/08/2023<br>TIME: 11:45 AM<br>VENUE: MAIN HALL, CORK ROAD CAMPUS |
| EXTERNAL EXAMINER: | MS MARGARET FINNEGAN | |

INSTRUCTIONS TO CANDIDATES

1. ANSWER ALL QUESTIONS.
2. TOTAL MARKS = 100.
3. EXAM PAPER (5 PAGES) AND FORMULA SHEET (1 PAGE)

MATERIALS REQUIRED

1. NEW MATHEMATICS TABLES.
2. GRAPH PAPER

**SOUTH EAST TECHNOLOGICAL UNIVERSITY**

**Question 1**

**(a)** Construct sets $A$ and $B$ satisfying the following three properties:

$$A \setminus B = \{1, 3\}, \qquad B \setminus A = \{2, 4\}, \qquad A \cap B = \{5, 6\}.$$

**(2 marks)**

**(b)** Construct a truth table for the logical expression

$$(x \vee z) \wedge ((x \vee y) \rightarrow (\neg x \wedge \neg z))$$

Hence or otherwise, state whether the proposition is satisfiable, is a tautology or a contradiction. (Justify your answer.) **(7 marks)**

**(c)** **(i)** Draw a graph with degree sequence $(2, 3, 3, 4, 4)$.

**(ii)** Does there exist a *simple* graph with this degree sequence? Justify your answer.

**(iii)** How many edges does this graph have?

**(iv)** What changes to this graph would make it a complete graph?

**(7 marks)**

**(d)** Use a membership table, or otherwise, to determine whether the following expression involving sets is true

$$(A \setminus B) \cap (B \setminus C) = A \cap B \cap \overline{C}$$

**(4 marks)**

**(Total 20 marks)**

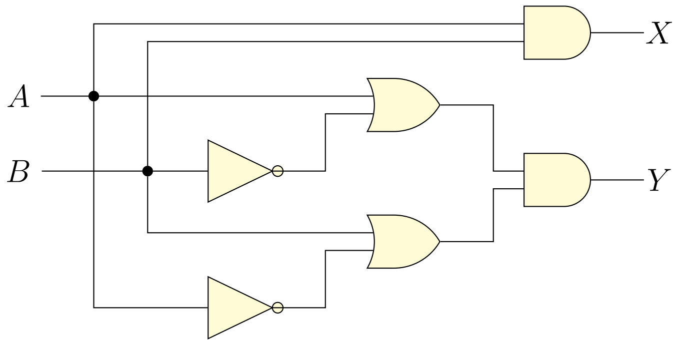
## Question 2

**(a)** Consider the following logical circuit with two inputs, $A$ and $B$, and two outputs, $X$ and $Y$.

**(i)** Construct a logical expression to represent the output $Y$.

**(ii)** Is there an input case for which both outputs, $X$ and $Y$, are True? (Justify your answer)

**(iii)** Is there an input case for which both outputs, $X$ and $Y$, are False? (Justify your answer)




**(10 marks)**

**(b)** Let $A = \{1, 2, 3, 4, 5\}$. Determine the truth value of each of the following statements, and justify your answer.

**(i)** $\forall x\big[x + 4 < 9\big]$

**(ii)** $\exists x\big[x^2 > 15\big]$

**(iii)** $\forall x \forall y\big[x + y < 9\big]$

**(iv)** $\forall x \exists y\big[x^2 + y^2 < 27\big]$

**(6 marks)**

**(c)** In a Python program, the variable x stores an unknown integer. Running the following Python code (where % denotes mod and // denotes integer division)

```
print(x%7, ',', x//7)
```

produces the output:

```
3 , 5
```

Determine a value for x that would produce the above output.

**(4 marks)**

**(Total 20 marks)**

**Question 3**

**(a)** Consider the function $f : \{1, 2, 3, 4, 5\} \to \{1, 2, 3, 4, 5\}$ given by the table below:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f(x)$ | 5 | 3 | 2 | 1 | 2 | 4 |

**(i)** Use a Venn diagram to represent the function $f$.

**(ii)** Graph the function $f$ as points on the plane.

**(iii)** Is $f$ injective? Explain.

**(iv)** Is $f$ surjective? Explain.

**(6 marks)**

**(b)** Let $R$ be the relation on the set $A$ as given in the Python code below:

```
A = {1,2,3,4,5,6}
R = {(a,b) for a in A for b in A if a%3 == b%3}
print(R)
```

**(i)** What does the print statement output?

**(ii)** Represent $R$ using a *digraph*.

**(iii)** Is $R$ reflexive? symmetric? transitive?

**(iv)** Is $R$ an equivalence relation? and if yes, what are the resulting equivalence classes?

**(8 marks)**

**(c)** The **girth** of a graph is the length of its shortest cycle. State the girth of each of the following graphs.

**(i)** $C_5$

**(ii)** Petersen graph

**(iii)** $K_{3,3}$



**(6 marks)**

**(Total 20 marks)**

**Question 4**

**(a)** What does the following function compute? (Justify your answer.)

```python
def findWhat(A, B):
    result = set()
    for a in A:
        if a not in B:
            result.append(a)
    return result
```

**(4 marks)**

**(b)** Let $S = \{1, 2, 3, \ldots, 8\}$

**(i)** How many subsets of $A$ are there?

**(ii)** How many subsets of $A$ contain exactly 4 elements, *i.e.*, have cardinality 4?

**(iii)** How many subsets with cardinality 5 contain *at least* one odd number?

**(iv)** How many subsets of cardinality 5 contain *exactly* one odd number?

**(v)** How many subsets of cardinality 6 contain *exactly* one even number?

**(10 marks)**

**(c)** Let $A$ and $B$ be defined as

$$A = \{n \in \mathbb{N} | n \text{ is a multiple of } 14\}$$

and

$$B = \{n \in \mathbb{N} | n \text{ is a multiple of } 2 \textbf{ or } n \text{ is a multiple of } 7\}$$

Which of the following is true? (Justify your answer.)

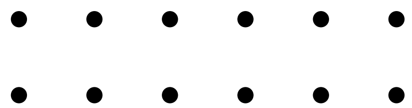
| | | | |
|---|---|---|---|
| **(i)** | $A \subset B$ | **(ii)** | $A \subseteq B$ |
| **(iii)** | $B \subset A$ | **(iv)** | $A \cap B = A$ |
| **(v)** | $A \cap B = \{n, k \in \mathbb{N} | n = 7k\}$ | **(vi)** | $A \cap B = B$ |

**(6 marks)**

**(Total 20 marks)**

**Question 5**

**(a)** Consider the following diagram, consisting of two rows of six dots.



How many

**(i)** Squares **(ii)** Right-angled triangles **(iii)** Triangles

can be drawn using the dots as vertices (corners).

**(8 marks)**

**(b)** How many shortest lattice paths start at $(0,0)$ and

**(i)** end at $(8,9)$?

**(ii)** end at $(8,9)$ and pass through $(3,6)$?

**(iii)** end at $(8,9)$ and avoid $(3,6)$?

**(7 marks)**

**(c)** **(i)** What does the following function do? (Justify your answer.)

```
def calculateWhat(n):
    if n == 0 or n == 1:
        return 1
    return n*calculateWhat(n-1)
```

**(ii)** What value will calculateWhat(5) return?

**(5 marks)**

**(Total 20 marks)**

# Laws of Logic

| **Logical Connective** | **Symbol** | **Python Operator** | **Precedence** | **Logic Gate** |
|---|---|---|---|---|
| Negation (NOT) | $\neg$ | **not** | Highest |  |
| Conjunctive (AND) | $\wedge$ | **and** | Medium |  |
| Disjunctive (OR) | $\vee$ | **or** | Lowest |  |

## Basic Rules of Logic

Commutative Laws

$p \vee q \Leftrightarrow q \vee p \qquad p \wedge q \Leftrightarrow q \wedge p$

Associative Laws

$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r) \qquad (p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

Distributive Laws

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r) \qquad p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Identity Laws

$p \vee \mathbf{F} \Leftrightarrow p \qquad p \wedge \mathbf{T} \Leftrightarrow p$

Negation Laws

$p \wedge (\neg p) \Leftrightarrow \mathbf{F} \qquad p \vee (\neg p) \Leftrightarrow \mathbf{T}$

Idempotent Laws

$p \vee p \Leftrightarrow p \qquad p \wedge p \Leftrightarrow p$

Null Laws

$p \wedge \mathbf{F} \Leftrightarrow \mathbf{F} \qquad p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$

Absorption Laws

$p \wedge (p \vee q) \Leftrightarrow p \qquad p \vee (p \wedge q) \Leftrightarrow p$

DeMorgan's Laws

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q \qquad \neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

Involution Law

$\neg(\neg p) \Leftrightarrow p$

## Implications and Equivalences

Detachment (Modus Ponens)

$(p \rightarrow q) \wedge p \Rightarrow q$

Indirect Reasoning (Modus Tollens)

$(p \rightarrow q) \wedge \neg q \Rightarrow \neg p$

Disjunctive Addition

$p \Rightarrow (p \vee q)$

Conjunctive Simplification

$(p \wedge q) \Rightarrow p \qquad (p \wedge q) \Rightarrow q$

Disjunctive Simplification

$(p \vee q) \wedge \neg p \Rightarrow q \qquad (p \vee q) \wedge \neg q \Rightarrow p$

Chain Rule

$(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$

Resolution

$(\neg p \vee r) \wedge (p \vee q) \Rightarrow (q \vee r)$

Conditional Equivalence

$p \rightarrow q \Leftrightarrow \neg p \vee q$

Biconditional Equivalences

$$\begin{aligned}(p \leftrightarrow q) &\Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)\\ &\Leftrightarrow (p \wedge q) \vee (\neg q \wedge \neg q)\end{aligned}$$

Contrapositive

$p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p$