WATERFORD INSTITUTE OF TECHNOLOGY

OUTLINE MODEL ANSWERS & MARKING SCHEME

Course: BSc (H) in AC, in CF, in the IoT	Semester: 1	Page 1 of 6
Subject: Discrete Mathematics	Examiner: Dr D. Flynn, Dr K Murphy	

Question 1

(a)

$$A = \{1, 3, 5, 6\}, \qquad B = \{3, 4, 5, 6\}$$

(2 marks)

(b)

Partial marks for correct parsing of expression, demonstrating ability to compute logical expression, using satisfiability/tautology/contradiction definitions.

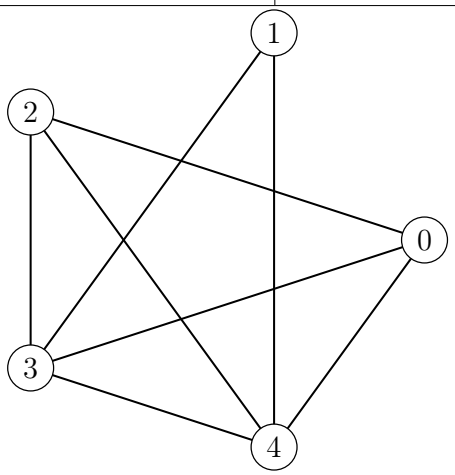
$$\underbrace{\underbrace{\underbrace{(\underbrace{(x \vee z)}_{1}) \wedge \underbrace{(x \vee y)}_{2}}_{3} \rightarrow \underbrace{(\neg x \wedge \neg z)}_{4}}_{E}}$$

					$\overbrace{\quad\quad}^{3}$			
			$\overbrace{\quad}^{1}$	$\overbrace{\quad}^{2}$	$\overbrace{\quad}^{1}$	$\overbrace{\quad}^{2}$	$\overbrace{\quad\quad}^{4}$	
x	y	z	$(x \vee z)$	$(x \vee y)$	$((x \vee z)$	$\wedge (x \vee y))$	$(\neg x \wedge \neg z)$	E
0	0	0	0	0	0		1	0
0	0	1	1	0	0		0	1
0	1	0	0	1	0		1	0
0	1	1	1	1	1		0	0
1	0	0	1	1	1		0	0
1	0	1	1	1	1		0	0
1	1	0	1	1	1		0	0
1	1	1	1	1	1		0	0

Since there is one row in which the output is True, the expression is satisfiable. It is *not* a tautology (does not have all True outputs) and is not a contradiction (has at least one True output). It is also a contingency. **(5 + 2 marks)**

(c)

(i) Student should produce a graph similar to the following:



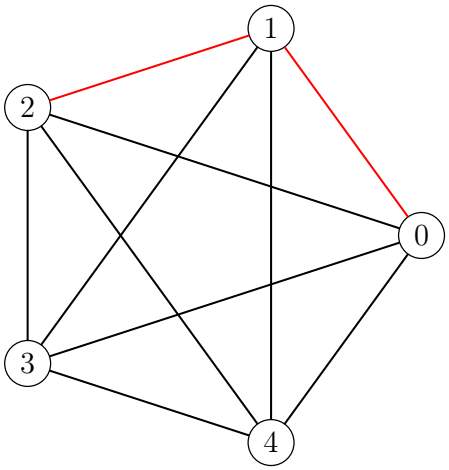
(2 marks)

(ii) Yes + reason **(2 marks)**

(iii) Handshaking Lemma: sum of degrees $= 2+3+3+4+4 = 16 \Rightarrow |E| = 8$. **(1 marks)**

(iv) Changes to make to graph which would make it a complete graph:

- Student should add in edges (0,1) and (1,2) in the plot (e.g. see below) or write down edges using appropriate vertex labels.
- Alternatively, student could remove node 1 to make it a complete graph. **(2 marks)**



(d)

Marks awarded for proving identity by using meembereship tables or using set operation properties.

Using membership tables ...

$$\underbrace{\underbrace{(A \setminus B)}_{1} \cap \underbrace{(B \setminus C)}_{2}}_{3} = \underbrace{A \setminus B}_{4}$$

A	B	C	$\overbrace{A \setminus B}^{1}$	$\overbrace{B \setminus C}^{2}$	$\overbrace{\overbrace{(A \setminus B)}^{1} \cap \overbrace{(B \setminus C)}^{2}}^{3}$	$\overbrace{A \cap B \cap \overline{C}}^{4}$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	1	0	0
0	1	1	0	0	0	0
1	0	0	1	0	0	0
1	0	1	1	0	0	0
1	1	0	0	1	1	1
1	1	1	0	0	0	0
					$\uparrow$	$\uparrow$

equal output $\Rightarrow$ expression true

(4 marks)

Question 2

(a)

(i) Construct a logical expression to represent the output Y. **(4 marks)**

$$Y = (\neg A \vee B) \wedge (A \vee \neg B) = \neg(A \oplus B)$$

(ii) Is there an input case for which both outputs, X and Y, are True? (justify ...) **(3 marks)**

Yes, Set $A = True$ and $B = True$.

(iii) Is there an input case for which both outputs, X and Y, are False? (justify ...) **(3 marks)**

Yes, $X = A \wedge B$ (only True when both A and B are True) while $Y = \neg(A \oplus B)$ (True when A and B the same). Hence X and Y can both be True.

(b)

(i) False, $x = 5 \Rightarrow 9 < 9$, which is false. **(1 marks)**

(ii) True, $x = 4, x = 5$. **(1 marks)**

(iii) False, e.g. $x = 5, y = 5$. **(2 marks)**

(iv) True, for all $x, y = 1$ is sufficient for the statement to be true. **(2 marks)**

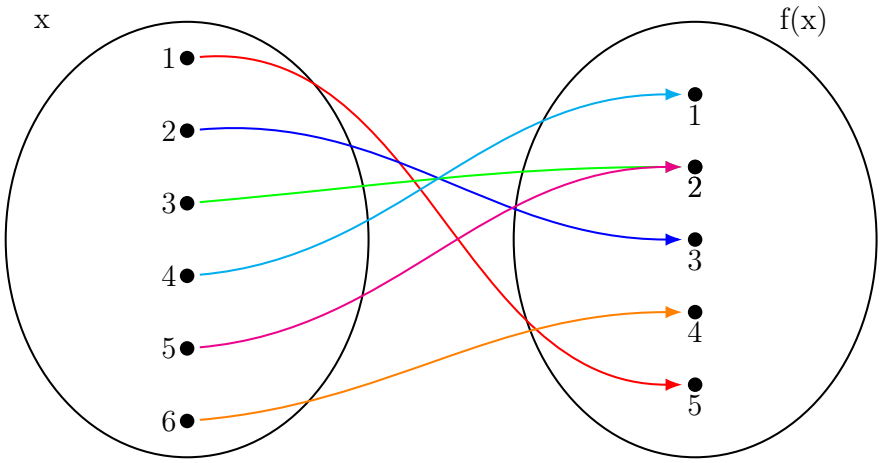
(c)

(i) x=38

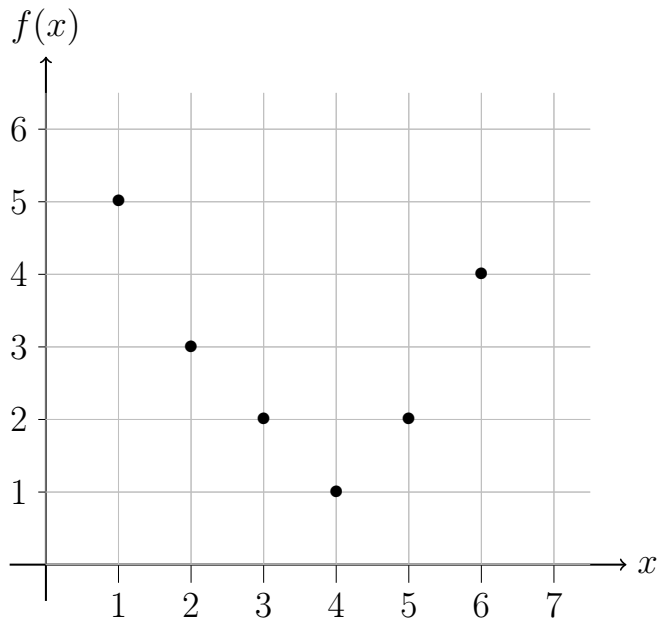
(4 marks)

Question 3

(a)



Venn/Mapping diagram for part (1) of the question



Answer for part (2), graph of the function in the plane.

(i) Student should show a Venn diagram similar to the above. **(1 marks)**

(ii) Student should produce a graph similar to the one shown above, indicating all $(x, f(x))$ values. **(1 marks)**

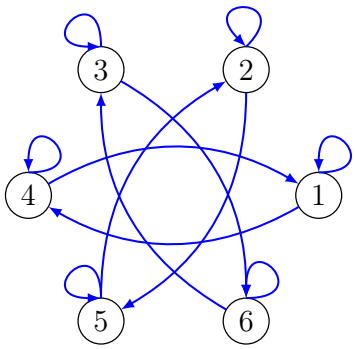
(iii) Is f injective? No. Explain: at most one arrow to each element of target set is not satisfied, $(3, 2)$ and $(5, 2)$. **(2 marks)**

(iv) Is f surjective? Yes. Explain: at least one arrow to each elements of the target set is satisfied. **(2 marks)**

(b)

(i) $R = \{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(2,5),(5,2),(3,6),(6,3),(1,4),(4,1)\}$

(ii) *Represent R using a digraph.*



(iii) *R is reflexive, symmetric and transitive*

(iv) *R is an equivalence relation (since is is ref., symm. and trans.) resulting equivalence classes partition $A = A_0 \cup A_1 \cup A_2$, where $A_0 = \{1,4\}, A_1 = \{3,6\}, A_2 = \{2,5\}$.*

(8 marks)

(c)

Partial marks for identifying graph. 4×2 marks

(i) C_5
Cycle graph has girth is 5. **(2 marks)**

(ii) Petersen Graph
Petersen Graph has girth of 5 **(2 marks)**

(iii) $K_{3,3}$
Complete bipartite graph, so girth is 4. **(2 marks)**

Question 4

(a)

Function finds the difference of the two sets A and B — will return the results. **(4 marks)**

(b)

(i) Number of subsets $= |\mathcal{P}(A)| = 2^8 = 256$. Need to select y/n for each of 8 elements.

(ii) Number of subsets $= \binom{8}{4} = 70$

(iii) $\binom{8}{5} = 56$. All subsets of cardinality 5 contain at least one odd number.

(iv) Number of subsets $= \binom{4}{1} = 4$. i.e. four even elements plus one odd element chosen from $\{1, 3, 5, 7\}$.

(v) Number of subsets $= 0$. There are no such subsets $(5 \times 2 = 10$ **marks)**

(c)

$A = \{0, 14, 28, 42, \ldots\}$, $B = \{0, 2, 4, 6, 7, 8, 10, 12, 14, \ldots\}$

(i) $A \subset B$ is true,
(ii) $A \subseteq B$ is false, for example, $7 \in B$ but $7 \notin A$.
(iii) $B \subset A$ false, e.g. $2 \in B$ but $2 \notin A$.
(iv) True,
(v) False
(vi) False

$(6 \times 1 = 6$ **marks)**

Question 5

(a)

(i) $\binom{5}{1} = 5$ squares. Top left corner, remaining 3 dots to be determined. **(2 marks)**

(ii) $2 \times \binom{6}{1}\binom{5}{1} : \binom{6}{1}$ for the vertical line (formed from the top dot down to the bottom dot), then out of the 6 remaining $\binom{5}{1}$ for the horizontal line. This is repeated for the horizontal line at the top, hence 60 in total. **(3 marks)**

(iii) $2 \times \binom{6}{1}\binom{6}{1}\binom{5}{1} : \binom{6}{1}\binom{6}{1}\binom{5}{1}$ for top vertex and bottom two non-overlapping bottom vertices. Likewise $\binom{6}{1}\binom{5}{1}\binom{6}{1}$ for two top non-overlapping vertices and one bottom vertex. Giving 360 triangles in total. **(3 marks)**

(2+3+3=8 marks)

(b)

(i) $\binom{17}{9} = 24310$. The paths all have length 17 (9 steps up and 8 steps right), we just select which 9 of those 17 should be up. **(2 marks)**

(ii) $\binom{9}{3}\binom{8}{3} = 84 \times 56 = 4704$. First travel to $(3, 6)$, and then continue on to $(8, 9)$. **(3 marks)**

(iii) $\binom{17}{9} - \binom{9}{3}\binom{8}{3} = 24310 - 4704 = 19606$
Remove all the paths found in preceding question. **(2 marks)**

(2+3+2=7 marks)

(c)

Function calculateWhat(n) computes the *factorial* using a *recursive* function call. Student should recognise the terminal conditions and show understanding of recursive nature e.g. function calls itself. **(4 marks)**

Function calculateWhat(5) return 120 i.e. 5! = 120 **(1 marks)**

(4+1=5 marks)