Discrete Mathematics — Tutorial Sheet 03 — Sets

BSc (H) in App Comp, BSc (H) in Comp Foren

Set Operations

Question 1

Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, $C = \{1, 5, 9\}$, $D = \{3, 2\}$, and $E = \{2, 3, 2\}$. and let the universal set be $U = \{0, 1, 2, ..., 9\}$.

(a) Determine:

- **(i)** $A \cap B$
- **(ii)** $A \cup B$
- **(iii)** $B \cup A$
- **(iv)** $A \cup C$
- **(v)** $A \setminus B$
- **(vi)** $B \setminus A$
- **(vii)** $\overline{A}$
- **(viii)** $\overline{C}$
- **(ix)** $A \cap C$
- **(x)** $A \oplus B$

(b) Determine which of the following are true. Give reasons for your decisions

- **(i)** $A = B$
- **(ii)** $B = C$
- **(iii)** $B = D$
- **(iv)** $E = D$
- **(v)** $A \cap B = B \cap A$
- **(vi)** $A \cup B = B \cup A$
- **(vii)** $A \setminus B = B \setminus A$
- **(viii)** $A \oplus B = B \oplus A$

Question 2

Let $U = \{1, 2, 3, ..., 9\}$. Give examples of sets A, B, and C for which:

- **(a)** $A \cap (B \cap C) = (A \cap B) \cap C$
- **(b)** $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
- **(c)** $(A \cup B)^c = A^c \cap B^c$
- **(d)** $A \cup A^c = U$
- **(e)** $A \subseteq A \cup B$
- **(f)** $A \cap B \subseteq A$

Note: I used alternative notation here for complement: $A^c = \overline{A}$. This did not come to light until after the first tutorial, so rather than correcting the question, I have included this note.

Question 3

Draw a Venn diagram to represent each of the following:

- **(a)** $A \cup \bar{B}$
- **(b)** $(A \bar{\cup} B)$
- **(c)** $A \cap (B \cup C)$
- **(d)** $(A \cap B) \cup C$
- **(e)** $\bar{A} \cap B \cap \bar{C}$
- **(f)** $(A \cup B) \setminus C$

Question 4

Construct an example of sets A and B such that $A \cap B = \{3, 5\}$ and $A \cup B = \{2, 3, 5, 7, 8\}$.

Question 5

Construct an example of sets A and B such that $A \subseteq B$ and $A \in B$.

Indirect Questions

This questions are based on set relationships and set operations also but may require a little more thought.

Question 6

Let $U = \{1, 2, 3, ..., 9\}$. Give examples to illustrate the following facts:

(a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

(b) There are sets A and B such that $A \setminus B \neq B \setminus A$

(c) If $U = A \cup B$ and $\mathrm{A} \cap \mathrm{B} = \emptyset$, it always follows that $A = U \setminus B$.

(d) $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$

Question 7

Suppose that U is an infinite universal set, and A and B are infinite subsets of U. Answer the following questions with a brief explanation.

(a) Must $\overline{A}$ be finite?

(b) Must $A \cup B$ infinite?

(c) Must $A \cap B$ be infinite?