Discrete Mathematics — Tutorial Sheet 07 — Enumeration

BSc (H) in App Comp, BSc (H) in Comp Foren

Basic Counting Principles

Question 1

Your wardrobe consists of 5 shirts, 3 pairs of pants, and 17 bow ties. How many different outfits can you make?

Question 2

For your college interview, you must wear a tie. You own 3 regular (boring) ties and 5 (cool) bow ties.

(a) How many choices do you have for your neck-wear?

(b) You realise that the interview is for clown college, so you should probably wear both a regular tie and a bow tie. How many choices do you have now?

(c) For the rest of your outfit, you have 5 shirts, 4 skirts, 3 pants, and 7 dresses. You want to select either a shirt to wear with a skirt or pants, or just a dress. How many outfits do you have to choose from?

Question 3

If $|A| = 8$ and $|B| = 5$, what is $|A \cup B| + |A \cap B|$?

Question 4

Consider all 5 letter "words" made from the letters a through h. (Recall, words are just strings of letters, not necessarily actual English words.)

(a) How many of these words are there total?

(b) How many of these words contain no repeated letters?

(c) How many of these words start with the sub-word "aha"?

(d) How many of these words either start with "aha" or end with "bah" or both?

(e) How many of the words containing no repeats also do not contain the sub-word "bad"?

Binomial Coefficients

Question 5

Let $A = \{1, 2, 3, \ldots, 9\}$.

(a) How many subsets of A are there? That is, find $|\mathcal{P}(A)|$. Explain.

(b) How many subsets of A contain exactly 5 elements? Explain.

(c) How many subsets of A contain only even numbers? Explain.

(d) How many subsets of A contain an even number of elements? Explain.

Question 6

How many 9-bit strings (that is, bit strings of length 9) are there which satisfy each of the following criteria? Explain your answers.

(a) Start with the sub-string 101.

(b) Have weight 5 (i.e., contain exactly five 1's) and start with the sub-string 101.

(c) Either start with 101 or end with 11 (or both).

(d) Have weight 5, and starts with 101 and ends with 11.

Question 7

How many shortest lattice paths start at (3,3) and

(a) end at (10,10)?

(b) end at (10,10) and pass through (5,7)?

(c) end at (10,10) and avoid (5,7)?

Question 8

What is the coefficient of x^{12} in $(x+2)^{15}$?

Combinations and Permutations

Question 9

A pizza parlour offers 10 toppings.

(a) How many 3-topping pizzas could they put on their menu? Assume double toppings are not allowed.

(b) How many total pizzas are possible, with between zero and ten toppings (but not double toppings) allowed?

(c) The pizza parluor will list the 10 toppings in two equal-sized columns on their menu. How many ways can they arrange the toppings in the left column?

Question 10

Using the digits 2 through 8, find the number of different 5-digit numbers such that:

(a) Digits can be used more than once.

(b) Digits cannot be repeated, but can come in any order.

(c) Digits cannot be repeated and must be written in increasing order.

(d) Which of the above counting questions is a combination and which is a permutation? Explain why this makes sense.

Question 11

How many triangles are there with vertices from the points shown below? Note, we are not allowing degenerate triangles — ones with all three vertices on the same line, but we do allow non-right triangles. Explain why your answer is correct.

Hint. You need exactly two points on either the x- or y-axis, but don't over-count the right triangles.

Advanced Counting Techniques

Question 12

After gym class you are tasked with putting the 14 identical dodgeballs away into 5 bins.

(a) How many ways can you do this if there are no restrictions?

(b) How many ways can you do this if each bin must contain at least one dodgeball?

Question 13

How many integer solutions are there to the equation $x + y + z = 8$ for which

(a) x, y, and z are all positive?

(b) x, y, and z are all non-negative?

(c) x, y, and z are all greater than -3.

Question 14

Using the digits 2 through 8, find the number of different 5-digit numbers such that:

(a) Digits cannot be repeated and must be written in increasing order. For example, 23678 is okay, but 32678 is not.

(b) Digits *can* be repeated and must be written in *non-decreasing* order. For example, 24448 is okay, but 24484 is not.