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South East Technological University

BACHELOR OF SCIENCE (HONS) IN
- APPLIED COMPUTING
- COMPUTER FORENSICS & SECURITY
- ENTERTAINMENT SYSTEMS
- THE INTERNET OF THINGS

EXAMINATION:

DISCRETE MATHEMATICS (COMMON MODULE)

SEMESTER 1 - YEAR 1

DECEMBER 2022 **DURATION: 2 HOURS**

INTERNAL EXAMINERS:	**DR DENIS FLYNN** **DR KIERAN MURPHY**	**DATE:** **TIME:** **VENUE:**	**15 DEC 2022** **11.45 AM** **MAIN HALL**
EXTERNAL EXAMINER:	**MS MARGARET FINNEGAN**		

INSTRUCTIONS TO CANDIDATES

1. ANSWER ALL QUESTIONS.
2. TOTAL MARKS = 100.
3. EXAM PAPER (5 PAGES EXCLUDING THIS COVER PAGE) AND FORMULA SHEET (1 PAGE)

MATERIALS REQUIRED

1. NEW MATHEMATICS TABLES.
2. GRAPH PAPER

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Question 1

(a) Construct sets A and B satisfying the following three properties:

$$A \setminus B = \{4, 9\}, \qquad B \setminus A = \{8\}, \qquad A \cap B = \{1\}.$$

(2 marks)

(b) Construct a truth table for the logical expression

$$((\neg(a \wedge b)) \wedge (c \vee b)) \rightarrow (a \rightarrow c)$$

Hence or otherwise, state whether the proposition is satisfiable, and is a tautology or is a contradiction. (Justify your answer.) **(7 marks)**

(c) A graph, G, has adjacency matrix $A = \begin{pmatrix} 0 & 1 & 1 & 2 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 2 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 \end{pmatrix}$

(i) Is G a simple graph?

(ii) State the degree sequence of G.

(iii) How many edges does G have?

(6 marks)

(d) What does the following function compute? (Justify your answer.)

```
def isWhat(A, B):
    for a in A:
        if a not in B: return False
    return True
```

(5 marks)

(Total 20 marks)

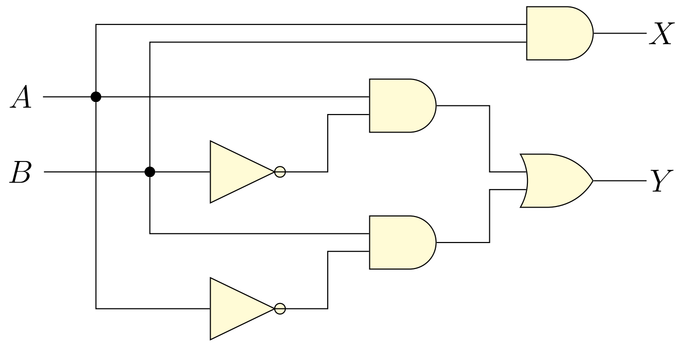
Question 2

(a) Consider the following logical circuit with two inputs, A and B, and two outputs, X and Y.

(i) Construct a logical expression to represent the output Y.

(ii) Is there an input case for which both outputs, X and Y, are **False**? (justify your answer)

(iii) Is there an input case for which both outputs, X and Y, are **True**? (justify your answer)



(10 marks)

(b) The email accounts of four hackers, Alice, Bob, Claire, and Dave, were monitored as part of an investigation. The following table details which hacker sent an email and to whom. (So, for example, Claire sent an email to Dave, but did not send an email to Alice).

from \ to	Alice	Bob	Claire	Dave
Alice	**False**	**False**	**True**	**False**
Bob	**False**	**False**	**True**	**False**
Claire	**False**	**False**	**False**	**True**
Dave	**False**	**True**	**False**	**False**

Let $M(x, y)$ represent the logical proposition that "x sent an email to y.". State whether the following statements are true (justify your answers).

(i) "Everyone sent an email to someone else."

(ii) $\forall x \exists y \big[(x \neq y) \wedge M(x, y)\big]$

(iii) $\exists x \forall y \big[(x \neq y) \wedge M(x, y)\big]$

(6 marks)

(c) Snow white is going to a party with the seven dwarves. Each of the eight of them owns a red dress and a blue dress. If each of them is likely to choose either coloured dress randomly and independently of the other's choices, what is the chance that all of them go to the party wearing the same coloured dress? **(4 marks)**

(Total 20 marks)

Question 3

(a) Determine the cardinality of the following sets.

(i) $\{a\}$

(ii) $\{a, \{a\}\}$

(iii) $\{\{a\}\}$

(iv) $\{a, \{a\}, \{a, \{a\}\}\}$

(4 marks)

(b) Let R be the relation on the set $A = \{1, 2, 3, 4, 5, 6\}$ where $(a, b) \in R$ iff a and b are the same length when written in English.

(i) Represent R using a digraph.

(ii) Is R reflexive? symmetric? transitive?

(iii) Is R an equivalence relation? and if yes, what the resulting equivalence classes?

(8 marks)

(c) The **girth** of a graph is the length of its shortest cycle. Write down the girth of each of the following graphs.

(i) K_9

(ii) $K_{5,7}$

(iii) C_8

(iv) W_8

(8 marks)

(Total 20 marks)

Question 4

(a) Draw a graph with degree sequence $(3, 3, 5, 5, 5, 5)$. Does there exist a *simple* graph with this degree sequence? Justify your answer. **(3 marks)**

(b) What does the following function compute? (Justify your answer.)

```
def isWhat(A, B):
    for a in A:
        if a in B: return False
    return True
```

(5 marks)

(c) Let $S = \{a, b, c, d, e, u\}$

(i) How many subsets are there of cardinality 4?

(ii) How many subsets of cardinality 4 have $\{a, b, c\}$ as a subset?

(iii) How many subsets of cardinality 4 contain at least one vowel?

(iv) How many subsets of cardinality 4 contain exactly one vowel?

(8 marks)

(d) Let A and B be defined as

$$A = \{n \in \mathbb{N} | n \text{ is a multiple of } 12\}$$

and

$$B = \{n \in \mathbb{N} | n \text{ is a multiple of } 2 \text{ and } n \text{ is a multiple of } 6\}$$

Which of the following is true? (Justify your answer.)

(i) $A \subset B$ **(ii)** $B \subset A$ **(iii)** $A = B$

(4 marks)

(Total 20 marks)

Question 5

(a) How many 9-bit strings (that is, bit strings of length 9) are there which satisfy each of the following criteria? Explain your answers.

(i) Start with the sub-string 101.

(ii) Have weight 5 (i.e., contain exactly five 1's) and start with the sub-string 101.

(iii) Either start with 101 or end with 11 (or both).

(iv) Have weight 5, and starts with 101 and ends with 11.

(8 marks)

(b) Let n be a positive integer. Prove that $n(n^2 + 5)$ is divisible by 3.
(**Hint:** Use proof by cases.)

(7 marks)

(c) Use a membership table, or otherwise, to determine whether the following expression involving sets is true

$$(A \setminus B) \setminus (B \setminus C) = A \setminus B$$

(5 marks)

(Total 20 marks)

Laws of Logic

Logical Connective	Symbol	Python Operator	Precedence	Logic Gate
Negation (NOT)	$\neg$	**not**	Highest	
Conjunctive (AND)	$\wedge$	**and**	Medium	
Disjunctive (OR)	$\vee$	**or**	Lowest	

Basic Rules of Logic

Commutative Laws

$p \vee q \Leftrightarrow q \vee p \qquad p \wedge q \Leftrightarrow q \wedge p$

Associative Laws

$(p \vee q) \vee r \Leftrightarrow p \vee (q \vee r) \qquad (p \wedge q) \wedge r \Leftrightarrow p \wedge (q \wedge r)$

Distributive Laws

$p \wedge (q \vee r) \Leftrightarrow (p \wedge q) \vee (p \wedge r) \qquad p \vee (q \wedge r) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Identity Laws

$p \vee \mathbf{F} \Leftrightarrow p \qquad p \wedge \mathbf{T} \Leftrightarrow p$

Negation Laws

$p \wedge (\neg p) \Leftrightarrow \mathbf{F} \qquad p \vee (\neg p) \Leftrightarrow \mathbf{T}$

Idempotent Laws

$p \vee p \Leftrightarrow p \qquad p \wedge p \Leftrightarrow p$

Null Laws

$p \wedge \mathbf{F} \Leftrightarrow \mathbf{F} \qquad p \vee \mathbf{T} \Leftrightarrow \mathbf{T}$

Absorption Laws

$p \wedge (p \vee q) \Leftrightarrow p \qquad p \vee (p \wedge q) \Leftrightarrow p$

DeMorgan's Laws

$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q \qquad \neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$

Involution Law

$\neg(\neg p) \Leftrightarrow p$

Implications and Equivalences

Detachment (Modus Ponens)

$(p \to q) \wedge p \Rightarrow q$

Indirect Reasoning (Modus Tollens)

$(p \to q) \wedge \neg q \Rightarrow \neg p$

Disjunctive Addition

$p \Rightarrow (p \vee q)$

Conjunctive Simplification

$(p \wedge q) \Rightarrow p \qquad (p \wedge q) \Rightarrow q$

Disjunctive Simplification

$(p \vee q) \wedge \neg p \Rightarrow q \qquad (p \vee q) \wedge \neg q \Rightarrow p$

Chain Rule

$(p \to q) \wedge (q \to r) \Rightarrow (p \to r)$

Resolution

$(\neg p \vee r) \wedge (p \vee q) \Rightarrow (q \vee r)$

Conditional Equivalence

$p \to q \Leftrightarrow \neg p \vee q$

Biconditional Equivalences

$$\begin{aligned}(p \leftrightarrow q) &\Leftrightarrow (p \to q) \wedge (q \to p)\\ &\Leftrightarrow (p \wedge q) \vee (\neg q \wedge \neg q)\end{aligned}$$

Contrapositive

$p \to q \Leftrightarrow \neg q \to \neg p$