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South East Technological University

BACHELOR OF SCIENCE (HONS) IN
- APPLIED COMPUTING
- COMPUTER FORENSICS & SECURITY
- ENTERTAINMENT SYSTEMS
- THE INTERNET OF THINGS

EXAMINATION:

DISCRETE MATHEMATICS (COMMON MODULE)

SEMESTER 1 - YEAR 1

DECEMBER 2022 **DURATION: 2 HOURS**

INTERNAL EXAMINERS: DR DENIS FLYNN, DR KIERAN MURPHY

DATE: 15 DEC 2022
TIME: 11.45 AM
VENUE: MAIN HALL

EXTERNAL EXAMINER: MS MARGARET FINNEGAN

INSTRUCTIONS TO CANDIDATES

1. ANSWER ALL QUESTIONS.
2. TOTAL MARKS = 100.
3. EXAM PAPER (5 PAGES EXCLUDING THIS COVER PAGE) AND FORMULA SHEET (1 PAGE)

MATERIALS REQUIRED

1. NEW MATHEMATICS TABLES.
2. GRAPH PAPER

SOUTH EAST TECHNOLOGICAL UNIVERSITY

WATERFORD INSTITUTE OF TECHNOLOGY

OUTLINE MODEL ANSWERS & MARKING SCHEME

Course: BSc (H) in AC, in CF, in the IoT	Semester: 1	
Subject: Discrete Mathematics	Examiner: Dr D. Flynn, Dr K Murphy	

Question 1

(a)

$$A = \{1, 4, 9\}, \qquad B = \{1, 8\}$$

(b)

Partial marks for correct parsing of expression, demonstrating ability to compute logical expression, using satisfiability/tautology/contradiction definitions.

$$\underbrace{\underbrace{(\underbrace{(\neg(a \wedge b))}_{1} \wedge \underbrace{(c \vee b)}_{2})}_{3} \rightarrow \underbrace{(a \rightarrow c)}_{4}}_{E}$$

a	b	c	$\overbrace{(\neg(a \wedge b))}^{1}$	$\overbrace{(c \vee b)}^{2}$	$\overbrace{(\overbrace{(\neg(a \wedge b))}^{1} \wedge \overbrace{(c \vee b)}^{2})}^{3}$	$\overbrace{(a \rightarrow c)}^{4}$	E
0	0	0	1	0	0	1	1
0	0	1	1	1	1	1	1
0	1	0	1	1	1	1	1
0	1	1	1	1	1	1	1
1	0	0	1	0	0	0	1
1	0	1	1	1	1	1	1
1	1	0	0	1	0	0	1
1	1	1	0	1	0	1	1

Since there is one row in which the output is **T**, the expression is satisfiable. It is tautology (has at all **T** output) and not a contradiction (has at least one **T** output). **($5 + 2$ marks)**

(c)

(i) Since the matrix contains entries other than 1s and 0s, G is not simple. For example, there are 2 edges from vertex 1 to vertex 4.

(ii) The sum of the entries in any row is the degree of the vertex corresponding to that row. The degree sequence is therefore $(2, 2, 3, 3, 4)$.

(iii) The sum of the degrees is 14, and so G has 7 edges.

(d)

Function computes whether $A \subseteq B$ — will return **True** iff all elements of A are elements of B.

Question 2

(a)

(i) Construct a logical expression to represent the output Y. **(4 marks)**

$$Y = (\neg A \wedge B) \vee (A \wedge \neg B) = A \oplus B$$

(ii) Is there an input case for which both outputs, X and Y, are **F**? (justify . . .) **(3 marks)**

Yes, Set $A = \mathbf{F}$ and $B = \mathbf{F}$.

(iii) Is there an input case for which both outputs, X and Y, are **T**? (justify . . .) **(3 marks)**

No, $X = A \wedge B$ (only **T**when both A and B are **T**) while $Y = A \oplus B$ (only **T**when A and B different). Hence X and Y cannot both be **T**.

(b)

(i) "Everyone sent an email to everyone." $\rightarrow$ $\forall x \forall y M(x, y)$

(ii) "There is a student who sent an email to everyone." $\rightarrow$ $\exists x \forall y M(x, y)$

(iii) "There is a student who sent an email to someone." $\rightarrow$ $\exists x \exists y M(x, y)$

(3×2 **marks)**

(c)

The number of different possible outcomes is 2^8, all are equally likely. So the required probability is $1/2^8 = 0.0039$.

Question 3

(a)

Determine the cardinality of the following sets.

(i) $\{a\}$ 1

(ii) $\{a, \{a\}\}$ 2

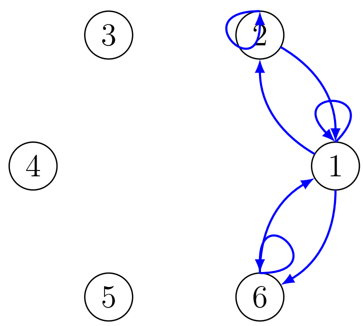
(iii) $\{\{a\}\}$ 1

(iv) $\{a, \{a\}, \{a, \{a\}\}\}$ 3

(4×1 marks)

(b)

(i) *Represent R using a digraph.*



(ii) *Is R reflexive? symmetric? transitive?*

(iii) *Is R an equivalence relation? and if yes, what the resulting equivalence classes?*

(c)

Partial marks for identifying graph. 4×2 **marks**

(i) K_9
Complete graph so girth is 3.

(ii) $K_{5,7}$
Complete bipartite graph, so girth is 4.

(iii) C_8
Cycle graph has girth is 8.

(iv) W_8
Wheel graph has girth of 3.

Question 4

(a)

No + reason

(b)

Function computes whether the two sets A and B are disjoint — will return **True** iff no element in A is an element of B.

(c)

(i) *How many subsets are there of cardinality 4?*

$\binom{6}{4} = 15$ subsets.

(ii) *How many subsets of cardinality 4 have $\{a, b, c\}$ as a subset?*

$\binom{3}{1} = 3$ subsets. We need to select 1 of the 3 remaining elements to be in the subset.

(iii) *How many subsets of cardinality 4 contain at least one vowel?*

$\binom{6}{4} = 15$ subsets. All subsets of cardinality 4 must contain at least one vowel.

(iv) *How many subsets of cardinality 4 contain exactly one vowel?*

$\binom{3}{1} = 3$ subsets. Select 1 of the 3 vowels. The three consonants of S must all be in the set.

(d)

$A \subset B$ since $A \subseteq B$ but, for example, $6 \in B$ but $6 \notin A$.

Question 5

(a)

(i) *Start with the sub-string 101.*

No constraints on remaining 6 bits, so 2^6.

(ii) *Have weight 5 (i.e., contain exactly five 1's) and start with the sub-string 101.*

In the remaining 6 bit, three of which must be 1, so $\binom{6}{3} = 20$

(iii) *Either start with* 101 *or end with* 11 *(or both).*

Start with 101: No constraints on remaining 6 bits, so 2^6.

Ends with 11: No constraints on preceeding 7 bits, so 2^7.

Start with 101 and ends with 11: No constraints on middle 4 bits, so 2^4.

Ans (remove double counting): $2^6 + 2^7 - 2^4 = 64 + 128 - 16 = 176$

(iv) *Have weight 5, and starts with* 101 *and ends with* 11.

The middle 4 digits must have weight 1 so that the entire string has weight 5. Hence have $\binom{4}{1} = 4$ possibilities.

(b)

$n\%3$	n	n^2	n^2+5	$n(n^2+5)$	
0	$3k$	$9k^2$	$9k^2+5$	$(3k)(9n^2+5)$	$= \underbrace{3\,\underbrace{k(9n^2+5)}_{\text{int}}}_{\text{mult. of 3}}$
1	$3k+1$	$9k^2+6k+1$	$9k^2+6k+6$	$(3k+1)(9n^2+6n+6)$	$= \underbrace{3\,\underbrace{(3k+1)(3n^2+2n+2)}_{\text{int}}}_{\text{mult. of 3}}$
2	$3k+2$	$9k^2+12k+4$	$9k^2+12k+9$	$(3k+2)(9n^2+12n+9)$	$= \underbrace{3\,\underbrace{(3k+2)(3n^2+4n+3)}_{\text{int}}}_{\text{mult. of 3}}$

(c)

Marks awarded for proving identity by using meembereship tables or using set operation properties.

Using membership tables . . .

$$\underbrace{\underbrace{(A \setminus B)}_{1} \setminus \underbrace{(B \setminus C)}_{2}}_{3} = \underbrace{A \setminus B}_{4}$$

A	B	C	$\overbrace{A \setminus B}^{1}$	$\overbrace{B \setminus C}^{2}$	$\overbrace{\overbrace{(A \setminus B)}^{1} \setminus \overbrace{(B \setminus C)}^{2}}^{3}$	$\overbrace{A \setminus B}^{4}$
0	0	0	0	0	0	0
0	0	1	0	0	0	0
0	1	0	0	1	0	0
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	0	1	1	0	1	1
1	1	0	0	1	0	0
1	1	1	0	0	0	0
					$\uparrow$	$\uparrow$

equal output $\Rightarrow$ expression true