Discrete Mathematics — Tutorial Sheet 08 — Graphs and Networks

BSc (H) in App Comp, Ent Sys, Comp Foren, and the IoT

Fundamental Concepts

Question 1

Draw a graph with degree sequence $(3, 3, 5, 5, 5, 5)$. Does there exist a *simple* graph with this degree sequence? Justify your answer.

Question 2

State, with an explanation, which of the following sequences are the degree sequences of a simple graph. For those sequence that are degree sequences, draw a simple graph with that degree sequence.

(a) $(1, 1, 2, 2, 3, 4, 4, 5, 5,)$ **(b)** $(2, 2, 2, 3, 5, 6, 6, 6)$ **(c)** $(1, 2, 3, 4, 5, 6, 7)$

Question 3

If G is a simple graph with at least two vertices, prove that G must contain two or more vertices of the same degree.

Question 4

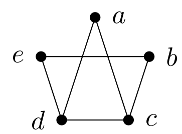
(Hard) Is it possible for two *different* (non-isomorphic) graphs to have the same number of vertices and the same number of edges? What if the degrees of the vertices in the two graphs are the same (so both graphs have vertices with degrees 1, 2, 2, 3, and 4, for example)? Draw two such graphs or explain why not.

Question 5

Are the two graphs below equal? Are they isomorphic? If they are isomorphic, give the isomorphism. If not, explain.

Graph 1:
$V = \{a, b, c, d, e\}$
$E = \{\{a, b\}, \{a, c\}, \{a, e\}, \{b, d\}, \{b, e\}, \{c, d\}\}$.

Graph 2:



Some Common Graphs

Question 6

Give an example of, or explain why it doesn't exist, each of the following.

(a) A bipartite graph that is regular of degree 5.

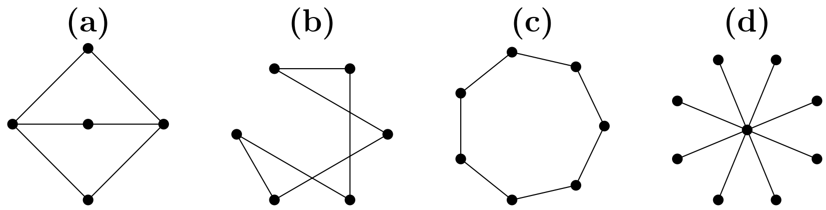
(b) A complete graph that is a wheel.

(c) A cubic graph with 11 vertices.

(d) A graph (other than K_5, $K_{4,4}$, or Q_4) that is regular of degree 4.

Question 7

Which of the graphs below are bipartite? Justify your answers.



Question 8

For which $n \geq 3$ is the graph C_n bipartite?

Representing Graphs using Matrices

Paths and Walks

Question 9

In a Peterson graph, find

(a) a trail of length 5;
(b) a path of length 9;
(c) cycles of length 5, 6, 8, and 9;
(d) cutsets with 3, 4, and 5 edges.

Question 10

The **girth** of a graph is the length of its shortest cycle. Write down the girth of each of the following graphs.

(a) K_9 **(b)** $K_{5,7}$ **(c)** C_8 **(d)** W_8

How Connected is a Graph?

Question 11

Write down $\kappa(G)$ and $\lambda(G)$ for each of the following graphs.

(a) The cycle graph, C_6.

(b) The wheel graph, W_6.

(c) The complete bipartite graph, $K_{4,7}$.

Question 12

Show that, if G is a connected graph with minimum degree k, then $\lambda(G) \leq k$.

Question 13

Draw a graph G with minimum degree k for which $\kappa(G) < \lambda(G) < k$.

Eulerian Graphs

Question 14

Which of the following are Eulerian? semi-Eulerian?

(a) The complete graph, K_5.

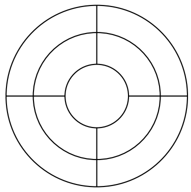
(b) The complete bipartite graph, $K_{2,3}$.

(c) The Peterson graph.

Question 15

Let G be a connected graph with k (> 0) vertices of odd degree.

(a) How many continuous pen-strokes are needed to draw the diagram without repeating any line?



Question 16

An Eulerian graph is **randomly traceable** from a vertex, v, if whenever we start from v and traverse the graph in an arbitrary way never using any edge twice, we eventually obtain an Eulerian trail.

(a) Give an example of an Eulerian graph that is not randomly traceable.

(b) Why might a randomly traceable graph be suitable for the layout of an exhibition?

Hamiltonian Graphs

Question 17

(a) For which values of n is K_n Hamiltonian?

(b) Which complete bipartite graphs are Hamiltonian?

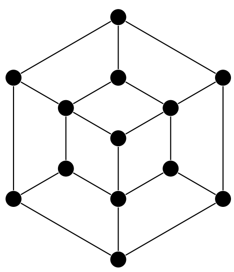
(c) For which values of n is the wheel W_n Hamiltonian?

(d) For which values of n is the k-cube Q_k Hamiltonian?

Question 18

(a) Prove that, if G is a bipartite graph with an odd number of vertices then G is non-Hamiltonian.

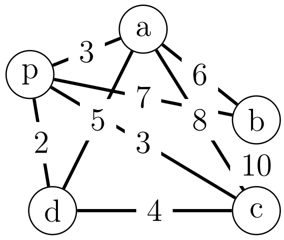
(b) Deduce that the graph bellow is non-Hamiltonian.



(c) Show that if n is odd, it is not possible for a knight to visit all the squares of an $n \times n$ chessboard exactly once by knight's moves and return to the starting point.

Question 19

A company is considering building a gas pipeline to connect 4 wells (a, b, c and d) to a process plant p. The possible pipelines that they can construct and their costs (in millions of euro) are shown in the accompanying graph.



What pipelines do you suggest be built and what is the total cost of your suggested pipeline network?

Question 20

Apply Kruskal's algorithm to determine a minimum cost spanning tree for the graph with the following cost matrix. How many such trees are there?

	A	B	C	D	E	F	G	H
A	0	12	0	14	11	0	17	8
B	12	0	9	0	12	15	10	9
C	0	9	0	18	14	31	0	9
D	14	0	18	0	0	6	23	14
E	11	12	14	0	0	15	16	0
F	0	15	31	6	15	0	8	16
G	17	10	0	23	16	8	0	22
H	8	9	9	14	0	16	22	0