

Set Operations

Question 1

Let $A = \{0, 2, 3\}$, $B = \{2, 3\}$, $C = \{1, 5, 9\}$, $D = \{3, 2\}$, and $E = \{2, 3, 2\}$. and let the universal set be $U = \{0, 1, 2, \dots, 9\}$.

(a) Determine:

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|------------------|----------------------|-----------------------|------------------|
| (i) $A \cap B$ | (iv) $A \cup C$ | (vii) \overline{A} | (x) $A \oplus B$ |
| (ii) $A \cup B$ | (v) $A \setminus B$ | (viii) \overline{C} | |
| (iii) $B \cup A$ | (vi) $B \setminus A$ | (ix) $A \cap C$ | |

(b) Determine which of the following are true. Give reasons for your decisions

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|---------------|----------------------------|---------------------------------------|
| (i) $A = B$ | (iv) $E = D$ | (vii) $A \setminus B = B \setminus A$ |
| (ii) $B = C$ | (v) $A \cap B = B \cap A$ | |
| (iii) $B = D$ | (vi) $A \cup B = B \cup A$ | (viii) $A \oplus B = B \oplus A$ |

Question 2

Let $U = \{1, 2, 3, \dots, 9\}$. Give examples of sets A , B , and C for which:

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| (a) $A \cap (B \cap C) = (A \cap B) \cap C$ | (d) $A \cup A^c = U$ |
| (b) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ | (e) $A \subseteq A \cup B$ |
| (c) $(A \cup B)^c = A^c \cap B^c$ | (f) $A \cap B \subseteq A$ |

Note: I used alternative notation here for complement: $A^c = \overline{A}$. This did not come to light until after the first tutorial, so rather than correcting the question, I have included this note.

Question 3

Draw a Venn diagram to represent each of the following:

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|-------------------------|-----------------------------------|
| (a) $A \cup \bar{B}$ | (d) $(A \cap B) \cup C$ |
| (b) $(A \cup B)$ | (e) $\bar{A} \cap B \cap \bar{C}$ |
| (c) $A \cap (B \cup C)$ | (f) $(A \cup B) \setminus C$ |

Question 4

Construct an example of sets A and B such that $A \cap B = \{3, 5\}$ and $A \cup B = \{2, 3, 5, 7, 8\}$.

Question 5

Construct an example of sets A and B such that $A \subseteq B$ and $A \in B$.

Indirect Questions

This questions are based on set relationships and set operations also but may require a little more thought.

Question 6

Let $U = \{1, 2, 3, \dots, 9\}$. Give examples to illustrate the following facts:

- (a) If $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.
- (b) There are sets A and B such that $A \setminus B \neq B \setminus A$
- (c) If $U = A \cup B$ and $A \cap B = \emptyset$, it always follows that $A = U \setminus B$.
- (d) $A \oplus (B \cap C) = (A \oplus B) \cap (A \oplus C)$

Question 7

Suppose that U is an infinite universal set, and A and B are infinite subsets of U . Answer the following questions with a brief explanation.

- (a) Must \overline{A} be finite?
- (b) Must $A \cup B$ infinite?
- (c) Must $A \cap B$ be infinite?