



Discrete Mathematics

Number Theory

Topic 01 — Logic

Mathematical Proofs

Lecture 01 — Introduction to Propositional Logic

Dr Kieran Murphy 

Recurrence Relations

Department of Computing and Mathematics,
SETU (Waterford).
(kieran.murphy@setu.ie)

Set Theory

Autumn Semester, 2022

Outline

- Propositions and fundamental logical operators (AND, OR and NOT).
- Evaluating logical expression using truth tables.
- Satisfiability, Tautologies and Contradictions.

Thought for the day ...

While walking through a fictional forest, you encounter three identical trolls guarding a bridge. Each troll is either a knight, who always tells the truth, or a knave, who always lies. The trolls will not let you pass until you correctly identify each as either a knight or a knave. Each troll makes a single statement:

If I am a knave, then there are exactly two knights here.

Troll 1 is lying.

Either we are all knaves or at least one of us is a knight..



Which troll are knights? and which are knaves?

- | | |
|---|----|
| 1. Introduction | 3 |
| <ul style="list-style-type: none">• Propositional logic is concerned with analysing propositions (true or false statements).• A proposition may be atomic or compound (build up using logical connectives).• Constructing compound propositions using <i>And</i>, <i>Or</i> and <i>Not</i>. | |
| 2. Truth tables | 13 |
| <ul style="list-style-type: none">• Evaluating an expression for all possible input combinations. | |
| 3. Tautologies and Contradictions | 21 |
| <ul style="list-style-type: none">• Statements that are always true or always false. | |

Logic

Logic is “science of reasoning”

- Allows us to represent knowledge in precise, unambiguous way.
- Allows us to make valid inferences using a set of consistent rules.
- Roots of logic date back to the ancient Greeks, e.g., Aristotle.
- Greeks were interested in valid logical inference rules, such as syllogisms:

“All men are mortal.

Socrates is a man.

Therefore, Socrates is mortal.”



The Partially Examined Life podcast: www.partiallyexaminedlife.com

The Fallacy-a-Day Podcast: <http://fallacyaday.com>

Propositional Logic

I

- The building blocks of propositional logic are propositions

Definition 1 (Proposition)

A **proposition** (**statement**) is a sentence that is either **True** or **False**.

- Examples:

“Java is a programming language.”

True

“Cork is the capital of Ireland.”

False

“ $1 + 2 = 3$ ”

True

“Today is Tuesday.”

depends

“The universe is fine-tuned.”

unknown (at present)

- Examples of sentences that are not propositions/statements:

- *“How are you?”* — A question cannot be assign a **True/False** value.
- *“Stop sleeping in class!”* — An order cannot be assign a **True/False** value.
- *“Correct horse battery staple.”* — Not a sentence.
- *“This sentence is false.”* — Pathological example.

Propositional Variables, Truth Value

Given a proposition we are interested in knowing its **truth value**.

Definition 2 (Truth Value)

The **truth value** of a proposition identifies whether a proposition is true (written **True** or **T** or 1) or false (written **False** or **F** or 0)

Question

What is truth value of “*Tuesday in the day after Sunday*” ?

F

Notation

- Variables that represent propositions are called propositional variables.
- Denote propositional variables using lower-case letters, such as p , p_1 , p_2 , q , r , s , ...
- Truth value of a propositional variable is either **T** or **F**.

Compound vs Atomic Propositions

- Propositional logic allows constructing more complex propositions from atomic ones.
- More complex propositions formed using **logical connectives** (also called **boolean connectives** or **logical operators**).
- The three basic logical connectives:

Connective	Symbol	Python
conjunction (AND)	\wedge	and
disjunction (OR)	\vee	or
negation (NOT)	\neg	not

- Propositions formed using these logical connectives called **compound propositions**; otherwise called **atomic propositions**.

Today is wet and I am hungry

Compound vs Atomic Propositions

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Today is wet and I am hungry
 { atomic } { atomic }
 { compound }

Exercise

Classify each of the sentences below as an atomic statement, a compound statement, or not a statement at all.

- ❶ The sum of the first 100 odd positive integers.
- ❷ Everybody needs somebody sometime.
- ❸ Waterford will win the All-Ireland or I'll eat my hat.
- ❹ Go to your room!
- ❺ Every natural number greater than 1 is either prime or composite.
- ❻ This sentence is false.

Exercise

Classify each of the sentences below as an atomic statement, a compound statement, or not a statement at all.

- ❶ The sum of the first 100 odd positive integers.

—*This is not even a sentence (no verb).*

- ❷ Everybody needs somebody sometime.

—*This is an atomic statement.*

- ❸ Waterford will win the All-Ireland or I'll eat my hat.

—*This is a compound statement.*

- ❹ Go to your room!

—*This is an order, not a statement*

- ❺ Every natural number greater than 1 is either prime or composite.

—*This is a compound statement.*

- ❻ This sentence is false.

—*This is sentence but is not a statement.*

Conjunction (AND)

- **Conjunction** of two propositions, p and q , written as $p \wedge q$, is the proposition:

“ p and q ”

- What is the relationship between the truth value of p and of q and the truth value of $p \wedge q$?

$$p \wedge q = \begin{cases} \mathbf{T} & \text{if both } p \text{ is } \mathbf{T} \text{ and } q \text{ is } \mathbf{T} \\ \mathbf{F} & \text{otherwise} \end{cases}$$

Example

What is the conjunction and the truth value of $p \wedge q$ for ...

- $p = \text{“It is a autumn semester”}$, $q = \text{“Today is Thursday”}$
- $p = \text{“It is Tuesday”}$, $q = \text{“It is morning”}$

Disjunction (OR)

- **Disjunction** of two propositions, p and q , written as $p \vee q$, is the proposition

“ p or q ”

- What is the relationship between the truth value of p and of q and the truth value of $p \vee q$?

$$p \vee q = \begin{cases} \mathbf{T} & \text{if either } p \text{ is } \mathbf{T} \text{ or } q \text{ is } \mathbf{T}, \text{ or both are } \mathbf{T} \\ \mathbf{F} & \text{otherwise} \end{cases}$$

Example

What is the disjunction and the truth value of $p \vee q$ for ...

- $p = \text{“It is a autumn semester”}$, $q = \text{“Today is Thursday”}$
- $p = \text{“It is Friday”}$, $q = \text{“It is morning”}$

Negation (NOT)

- **Negation** of a proposition, p , written, $\neg p$, represents the proposition:
“It is not the case that p .”
- What is the relationship between the truth value of p and $\neg p$?

If p is **T**, then $\neg p$ is **F** and vice versa.

- In simple English, what is $\neg p$ if p stands for ...

p	$\neg p$
<i>“Today is Tuesday.”</i>	<i>“Today is not Tuesday.”</i>
$1 + 1 = 2$	$1 + 1 \neq 2$

- Properties of NOT
 - $\neg \neg p = p$

Python Implementation

I

Python supports the fundamental logical connectives (programmers call them “logical operators”)

Logical Connective	Math	Python Operator
conjunction (AND)	\wedge	and
disjunction (OR)	\vee	or
negation (NOT)	\neg	not

- | | |
|---|----|
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Propositional Formulas and Truth Tables

- A **propositional formula** is logical expression constructed from atomic and compound propositions and logical connectives.
- A **truth table** for a propositional formula, A , shows the truth value of A for every possible value of its constituent atomic propositions.

Not / Negation

p	$\neg p$
F	T
T	F



NOT

And / Conjunction

p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T



AND

Or / Disjunction

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T



OR

Truth tables and Logic Gates

AND

NOT

OR

p	$\neg p$

p	q	$p \wedge q$

p	q	$p \vee q$

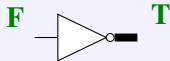
Truth tables and Logic Gates

AND

NOT

OR

p	$\neg p$
F	T



p	q	$p \wedge q$

p	q	$p \vee q$

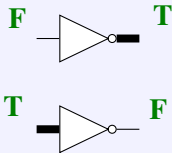
Truth tables and Logic Gates

AND

NOT

OR

p	$\neg p$
F	T
T	F

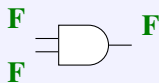


p	q	$p \wedge q$

p	q	$p \vee q$

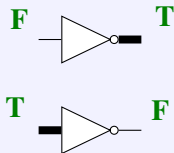
Truth tables and Logic Gates

AND

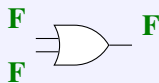


NOT

p	$\neg p$
F	T
T	F



OR

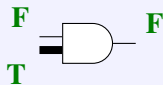
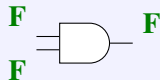


p	q	$p \wedge q$
F	F	F

p	q	$p \vee q$
F	F	F

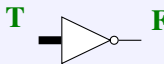
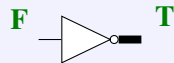
Truth tables and Logic Gates

AND

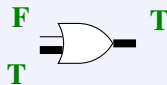
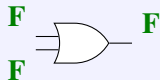


NOT

p	$\neg p$
F	T
T	F



OR

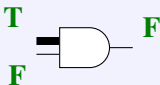
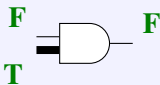
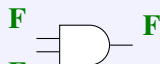


p	q	$p \wedge q$
F	F	F
F	T	F

p	q	$p \vee q$
F	F	F
F	T	T

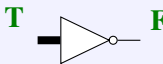
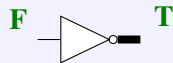
Truth tables and Logic Gates

AND



p	$\neg p$
F	T
T	F

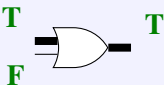
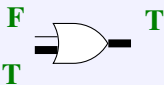
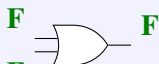
NOT



p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F

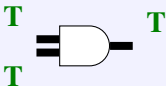
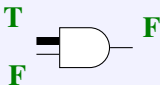
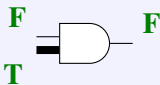
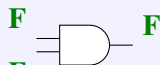
p	q	$p \vee q$
F	F	F
F	T	T
T	F	T

OR



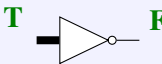
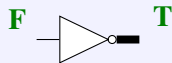
Truth tables and Logic Gates

AND



p	$\neg p$
F	T
T	F

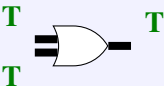
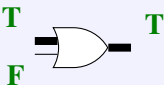
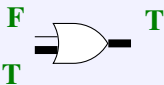
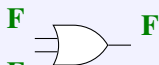
NOT



p	q	$p \wedge q$
F	F	F
F	T	F
T	F	F
T	T	T

p	q	$p \vee q$
F	F	F
F	T	T
T	F	T
T	T	T

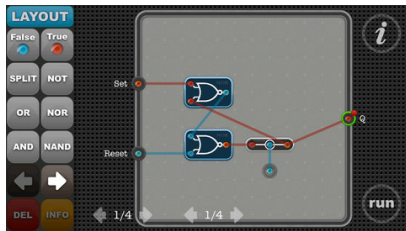
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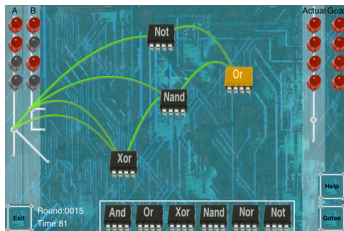
Other Resources

iPad/iPhone Apps (assume similar on Android)

Circuit Coder



Boolean Master



Videos

- <https://class.coursera.org/cs101/lecture/17>
Part of the Computer Science 101 by Nick Parlante on coursera.

Constructing Truth Tables

Useful strategy for constructing truth tables for a formula:

- STEP 1 Identify the constituent atomic propositions of the formula.
- STEP 2 Identify compound propositions in within the formula in increasing order of complexity, including the formula itself.
- STEP 3 Construct a table enumerating all combinations of truth values for atomic propositions.
- STEP 4 Fill in values of compound propositions for each row.

Examples

Construct truth tables for the following formulas:

- 1 $(p \vee q) \wedge \neg p$
- 2 $(p \wedge q) \vee (\neg p \wedge \neg q)$
- 3 $(p \vee q \vee \neg r) \wedge r$

Example 1: $(p \vee q) \wedge \neg p$

- STEP 1 Identify the constituent atomic propositions ... p and q
- STEP 2 Identify compound propositions ...
- STEP 3 Enumerate all combinations of truth values for atomic propositions ...
- STEP 4 Fill in values of compound propositions for each row ...

Example 1: $(p \vee q) \wedge \neg p$

STEP 1 Identify the constituent atomic propositions ... p and q

STEP 2 Identify compound propositions ...

STEP 3 Enumerate all combinations of truth values for atomic propositions ...

STEP 4 Fill in values of compound propositions for each row ...

p	q
<hr/>	

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STEP 1 Identify the constituent atomic propositions ... p and q

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p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$

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- STEP 1** Identify the constituent atomic propositions ... p and q
- STEP 2** Identify compound propositions ...
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- STEP 4** Fill in values of compound propositions for each row ...

p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$
F	F			
F	T			
T	F			
T	T			

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p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$
F	F	F	T	F
F	T			
T	F			
T	T			

Example 1: $(p \vee q) \wedge \neg p$

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p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$
F	F	F	T	F
F	T	T	T	T
T	F			
T	T			

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p	q	$p \vee q$	$\neg p$	$(p \vee q) \wedge \neg p$
F	F	F	T	F
F	T	T	T	T
T	F	T	F	F
T	T			

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F	F	F	T	F
F	T	T	T	T
T	F	T	F	F
T	T	T	F	F

Example 1: $(p \vee q) \wedge \neg p$

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F	F	F	T	F
F	T	T	T	T
T	F	T	F	F
T	T	T	F	F

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

- STEP 1 Identify the constituent atomic propositions ... p and q
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Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

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p q

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

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p	q	$(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

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F	F					
F	T					
T	F					
T	T					

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

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STEP 4 Fill in values of compound propositions for each row ...

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F	F	F	T	T	T	T
F	T					
T	F					
T	T					

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

STEP 1 Identify the constituent atomic propositions ... p and q

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p	q	$(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
F	F	F	T	T	T	T
F	T	F	T	F	F	F
T	F					
T	T					

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

STEP 1 Identify the constituent atomic propositions ... p and q

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STEP 4 Fill in values of compound propositions for each row ...

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F	F	F	T	T	T	T
F	T	F	T	F	F	F
T	F	F	F	T	F	F
T	T					

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

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STEP 3 Enumerate all combinations of truth values for atomic propositions ...

STEP 4 Fill in values of compound propositions for each row ...

p	q	$(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
F	F	F	T	T	T	T
F	T	F	T	F	F	F
T	F	F	F	T	F	F
T	T	T	F	F	F	T

Example 2: $(p \wedge q) \vee (\neg p \wedge \neg q)$

STEP 1 Identify the constituent atomic propositions ... p and q

STEP 2 Identify compound propositions ...

STEP 3 Enumerate all combinations of truth values for atomic propositions ...

STEP 4 Fill in values of compound propositions for each row ...

p	q	$(p \wedge q)$	$\neg p$	$\neg q$	$(\neg p \wedge \neg q)$	$(p \wedge q) \vee (\neg p \wedge \neg q)$
F	F	F	T	T	T	T
F	T	F	T	F	F	F
T	F	F	F	T	F	F
T	T	T	F	F	F	T

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
- STEP 2 Identify compound propositions ...
- STEP 3 Enumerate all combinations of truth values for atomic propositions ...
- STEP 4 Fill in values of compound propositions for each row ...

Example 3: $(p \vee q \vee \neg r) \wedge r$

STEP 1 Identify the constituent atomic propositions ... p , q , and r

STEP 2 Identify compound propositions ...

STEP 3 Enumerate all combinations of truth values for atomic propositions ...

STEP 4 Fill in values of compound propositions for each row ...

p q r

Example 3: $(p \vee q \vee \neg r) \wedge r$

STEP 2 Identify compound propositions ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$

Example 3: $(p \vee q \vee \neg r) \wedge r$

STEP 1 Identify the constituent atomic propositions ... p , q , and r

STEP 2 Identify compound propositions ...

STEP 3 Enumerate all combinations of truth values for atomic propositions ...

STEP 4 Fill in values of compound propositions for each row ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F			
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
- STEP 2 Identify compound propositions ...
- STEP 3 Enumerate all combinations of truth values for atomic propositions ...
- STEP 4 Fill in values of compound propositions for each row ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T			
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
- STEP 2 Identify compound propositions ...
- STEP 3 Enumerate all combinations of truth values for atomic propositions ...
- STEP 4 Fill in values for compound propositions for each row ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F			
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
- STEP 2 Identify compound propositions ...
- STEP 3 Enumerate all combinations of truth values for atomic propositions ...
- STEP 4 Fill in values of compound propositions for each row ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F	T	T	F
F	T	T			
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
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p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F	T	T	F
F	T	T	F	T	T
T	F	F			
T	F	T			
T	T	F			
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
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p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F	T	T	F
F	T	T	F	T	T
T	F	F	T	T	F
T	F	T			
T	T	F			
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
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- STEP 4 Fill in values of compound propositions for each row ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F	T	T	F
F	T	T	F	T	T
T	F	F	T	T	F
T	F	T	F	T	T
T	T	F			
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
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p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F	T	T	F
F	T	T	F	T	T
T	F	F	T	T	F
T	F	T	F	T	T
T	T	F	T	T	F
T	T	T			

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1 Identify the constituent atomic propositions ... p , q , and r
- STEP 2 Identify compound propositions ...
- STEP 3 Enumerate all combinations of truth values for atomic propositions ...
- STEP 4 Fill in values of compound propositions for each row ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F	T	T	F
F	T	T	F	T	T
T	F	F	T	T	F
T	F	T	F	T	T
T	T	F	T	T	F
T	T	T	F	T	T

Example 3: $(p \vee q \vee \neg r) \wedge r$

- STEP 1** Identify the constituent atomic propositions ... p , q , and r
- STEP 2** Identify compound propositions ...
- STEP 3** Enumerate all combinations of truth values for atomic propositions ...
- STEP 4** Fill in values of compound propositions for each row ...

p	q	r	$\neg r$	$(p \vee q \vee \neg r)$	$(p \vee q \vee \neg r) \wedge r$
F	F	F	T	T	F
F	F	T	F	F	F
F	T	F	T	T	F
F	T	T	F	T	T
T	F	F	T	T	F
T	F	T	F	T	T
T	T	F	T	T	F
T	T	T	F	T	T

Outline

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 - A proposition may be atomic or compound (build up using logical connectives).
 - Constructing compound propositions using *And*, *Or* and *Not*.
2. Truth tables 13
 - Evaluating an expression for all possible input combinations.
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 - Statements that are always true or always false.

Introduction to Propositional Logic — Summary

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| 1. Introduction | 3 |
| <ul style="list-style-type: none">• Propositional logic is concerned with analysing propositions (true or false statements).• A proposition may be atomic or compound (build up using logical connectives).• Constructing compound propositions using <i>And</i>, <i>Or</i> and <i>Not</i>. | |
| 2. Truth tables | 13 |
| <ul style="list-style-type: none">• Evaluating an expression for all possible input combinations. | |
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Satisfiable, Tautologies and Contradictions

Satisfiable

A proposition is **satisfiable** if it is **True** for at least one set of inputs (case).

Tautology

A **tautology** is an expression involving logical variables that is **True** in all cases.

- Examples

- $p \vee \neg p$

“Tomorrow, I will be dead or I will be alive”

- $(p \wedge q) \vee (p \wedge \neg q) \vee \neg p$

Contradiction

A **contradiction** is an expression involving logical variables that is **False** in all cases.

- Examples

- $p \wedge \neg p$

“On Friday, I will win the lottery and not win the lottery.”