Discrete Mathematics

Topic 02 — Methods of Mathematical Proof

Lecture 03 — Other Proof Techniques

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Outline

- Proof by Contradiction, Construction, Induction . . .

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- We prove a statement using the process:
    - assume reverse of statement . . .
    - derive conclusions from assumption . . .
    - show conclusions are contradictory . . .
    - hence assumption must be **False**, so original statement is **True**.

Proof by Contradiction I

Proof by Contradiction

In a proof by contradiction argument you:

- Assume the negative of the claim
    - So a universal claim will become an existence claim, and an existence claim will become a universal claim.
- Then show that the assumption leads to a contradiction.

Proof by Contradiction (Formal Structure)

Given claim

$$P \implies Q$$

Show that the negative, i.e. $P \Rightarrow \neg Q$, leads to a contradiction, by
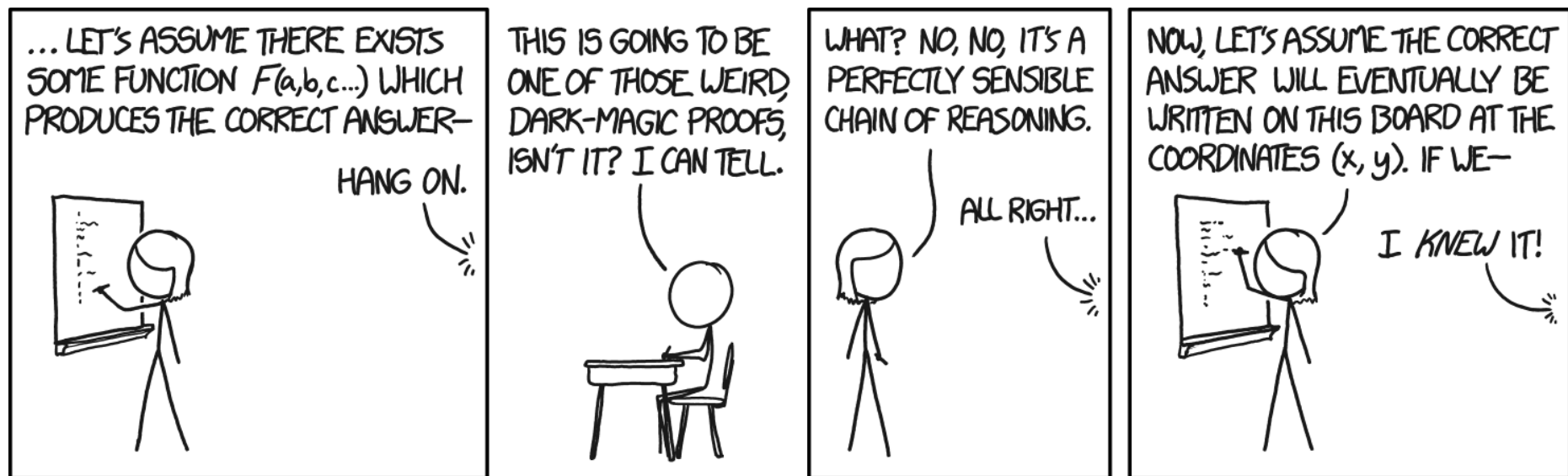
1. Assume P.
2. Assume $\neg Q$.
3. Use P and $\neg Q$ to demonstrate a contradiction.

Proof by Contradiction   II

Proofs by contradiction can be tricky, you

- Need to be very clear as to what statement you are assuming in order to generate a contradiction.

- In particular, take case when the statement involves a qualifier.

Examples

a) Prove that a triangle cannot have more than one right angle.

b) Prove that the $\sqrt{2}$ is irrational.[†]

c) Prove that $\log_2(3)$ is irrational.

d) Let n be an integer. If $3n + 2$ is odd, then n is odd.

e) Prove that there are an infinite number of primes.[‡]

f) There are no integers x and y such that $x^2 = 4y + 2$.

g) The Pigeonhole Principle: If more than n pigeons fly into n pigeon holes, then at least one pigeon hole will contain at least two pigeons. Prove this.

[†]"irrational"= "not rational". A rational number is a number that can be expressed as quotient of two integers p and p which don't have a common factor.

[‡]A prime is an integer greater than one with exactly two divisors.