

Logic

# Discrete Mathematics

Number Theory

Mathematical  
Proofs

## Topic 02 — Methods of Mathematical Proof

### Lecture 03 — Other Proof Techniques

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Recurrence  
Relations

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Set Theory

Autumn Semester, 2022

#### Outline

- Proof by Contradiction

Enumeration

## 1. Proof by Contradiction

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- We prove a statement using the process:
  - assume reverse of statement ...
  - derive conclusions from assumption ...
  - show conclusions are contradictory ...
  - hence assumption must be **False**, so original statement is **True**.

# Proof by Contradiction

## Proof by Contradiction

In a **proof by contradiction** argument you:

- Assume the negative of the claim
  - So a universal claim will become an existence claim, and an existence claim will become a universal claim.
- Then show that the assumption leads to a contradiction.

## Proof by Contradiction (Formal Structure)

Given claim

$$P \implies Q$$

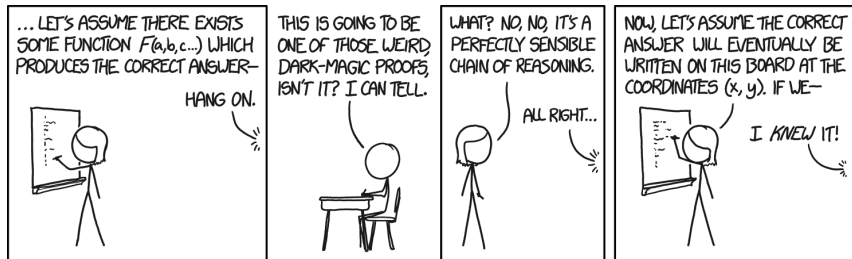
Show that the negative, i.e.  $P \implies \neg Q$ , leads to a contradiction, by

- 1 Assume  $P$ .
- 2 Assume  $\neg Q$ .
- 3 Use  $P$  and  $\neg Q$  to demonstrate a contradiction.

# Proof by Contradiction

Proofs by contradiction can be tricky, you

- Need to be very clear as to what statement you are assuming in order to generate a contradiction.
- In particular, take case when the statement involves a qualifier.



\*<https://xkcd.com/1724/>

# Examples

- a) Prove that a triangle cannot have more than one right angle.
- b) Prove that the  $\sqrt{2}$  is irrational.<sup>†</sup>
- c) Prove that  $\log_2(3)$  is irrational.
- d) Let  $n$  be an integer. If  $3n + 2$  is odd, then  $n$  is odd.
- e) Prove that there are an infinite number of primes.<sup>‡</sup>
- f) There are no integers  $x$  and  $y$  such that  $x^2 = 4y + 2$ .
- g) The Pigeonhole Principle: If more than  $n$  pigeons fly into  $n$  pigeon holes, then at least one pigeon hole will contain at least two pigeons. Prove this.

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<sup>†</sup>“irrational”= “not rational”. A **rational** number is a number that can be expressed as quotient of two integers  $p$  and  $q$  which don't have a common factor.

<sup>‡</sup>A **prime** is an integer greater than one with exactly two divisors.