Discrete Mathematics

Topic 02 — Methods of Mathematical Proof

Lecture 03 — Other Proof Techniques

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Outline

- Proof by Contradiction, Construction, Induction …

Outline

1. Proof by Contradiction 2

- We prove a statement using the process:
  - assume reverse of statement …
  - derive conclusions from assumption …
  - show conclusions are contradictory …
  - hence assumption must be **False**, so original statement is **True**.

Proof by Contradiction I

Proof by Contradiction

In a proof by contradiction argument you:

- Assume the negative of the claim
  - So a universal claim will become an existence claim, and an existence claim will become a universal claim.
- Then show that the assumption leads to a contradiction.

Proof by Contradiction (Formal Structure)

Given claim

$$P \implies Q$$

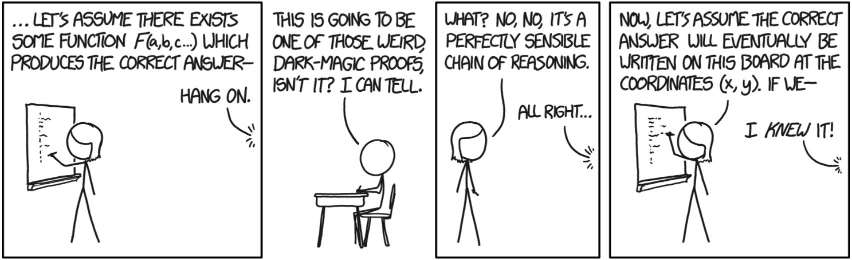
Show that the negative, i.e. $P \Rightarrow \neg Q$, leads to a contradiction, by

1. Assume P.
2. Assume $\neg Q$.
3. Use P and $\neg Q$ to demonstrate a contradiction.

Proof by Contradiction II

Proofs by contradiction can be tricky, you

- Need to be very clear as to what statement you are assuming in order to generate a contradiction.
- In particular, take case when the statement involves a qualifier.



*https://xkcd.com/1724/

Examples

a) Prove that a triangle cannot have more than one right angle.
b) Prove that the $\sqrt{2}$ is irrational.[†]
c) Prove that $\log_2(3)$ is irrational.
d) Let n be an integer. If $3n + 2$ is odd, then n is odd.
e) Prove that there are an infinite number of primes.[‡]
f) There are no integers x and y such that $x^2 = 4y + 2$.
g) The Pigeonhole Principle: If more than n pigeons fly into n pigeon holes, then at least one pigeon hole will contain at least two pigeons. Prove this.

[†] "irrational"= "not rational". A rational number is a number that can be expressed as quotient of two integers p and p which don't have a common factor.

[‡] A prime is an integer greater than one with exactly two divisors.