Optimal energy efficient packet scheduling with arbitrary individual deadline guarantee over a fading channel

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Abstract

The wireless communication for Industrial Internet of Things (IIoT) faces many new challenges, for instance, critical industrial machines require low communication delay, complex manufacture environments affect communications greatly, and energy is limited for some wireless devices. We hence define and study the energy efficient packet scheduling problem with arbitrary individual deadline guarantee over a fading channel. Such problem is a longstanding open problem and solving it is very much desired for the IIoT wireless communications. This paper optimally solves this problem by designing the Highest-water-level Interval First (HIF) policy which extends the recently proposed DIF policy by Shan, Luo and Shen. This paper introduces the novel concept of data interval and its water level for HIF policy. By repeatedly locating the highest-water-level interval, the optimal offline transmission power can be determined. An online policy is also proposed, and simulations show its efficiency by comparing it against the offline optimal results.

1. Introduction

Emerging topics on Industrial Internet of Things (IIoT) is attracting numerous attentions from both industrial and academic communities. Wireless communication and connectivity of the industrial things is one of the most fundamental topics for IIoT. This is because, only after the connectivity of every industrial thing is guaranteed, other topics, such as the identification of things, the integration of industrial big data, the decision systems based on the knowledge of data, become meaningful.

The wireless communication for IIoT faces many new challenges. Firstly, control command delivery for critical industrial machines usually has high Quality of Service (QoS) requirement, while different machines may have different

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QoS requirements, hence packets on the communication channel may carry individual deadlines. Secondly, the deployment environment is both complex and dynamic, especially in manufacture industry, thus the wireless communication channel suffers severe dynamic fading which affects the efficiency of the transmission. Thirdly, some industrial sensors are deployed in harsh conditions, where battery replacement is difficult, therefore the wireless communications are required to be conservative on energy consumption.

To address the above three challenges, this paper studies the wireless transmission power scheduling problem for the delivery of a set of packets such that 1) each packet has an individual deadline and they are guaranteed, and 2) the wireless transmission channel changes over time *e.g.,* fading. While the schedule goal is to minimize the energy consumption. Such problem is named the energy efficient packet scheduling problem with arbitrary individual deadline guarantee over a fading channel.

Previous work has already studied some simpler versions of the problem. Prabhakar, Uysal-Biyikoglu, and El Gamal [1, 2] are among the first group of researchers who formulated the delay-constrained energy efficient packet transmission problem more than a decade ago. They consider the case where all packets have a common deadline and the arrival time and size of each packet are known in prior to the scheduling. An optimal scheduling algorithm is presented to guarantee to deliver all packets before the deadline with minimum energy consumption. Zafer and Modiano [7, 8] thus present an optimal algorithm that allows each packet to have an individual deadline. They propose the cumulative curves to track packet arrivals and packet departures. The key observation is that a feasible departure curve always lies between the arrival curve and minimum departure curve. However, they still need to make an undesirable assumption that a packet arriving earlier carries an earlier deadline, which will be referred to as aligned deadlines in this paper. Shan, Luo and Shen [11] solve the most general case in which arbitrary deadlines are allowed. The common deadline model and the aligned deadline model are both special cases of this more general model. They arise the concept of data interval and propose the Densest Interval First (DIF) policy to control the transmission power and schedule the transmission rate that minimize the energy consumption.

These above papers investigate over a static channel. In the real world, especially in industrial environment, a wireless communication channel is almost certainly a time-varying fading channel. El Gamal *et al.* [3, 4] propose the MoveRight and FlowRight algorithms that solve this problem when packets have aligned deadlines. The main idea of the MoveRight/FlowRight algorithm is to iteratively calculate the local optimal solution for every two adjacent time-slots, and this iterative local optimization is proved to lead to the globally optimum solution. However, such an iteration based algorithm has a high computational complexity, where hundreds of seconds may be required in actual computation [3, 4]. Moreover, it can not handle the more general arbitrary deadline model.

Other than the above directly related work, there are also some other related work. Ozel *et al.* [15, 16] investigate the transmission rate scheduling problem for energy harvesting nodes in fading channels. The goal is to maximize the

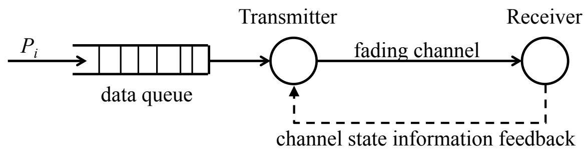


Figure 1: A set of dynamically arrived packets are to be delivered to the receiver over a time-varying fading channel.

throughput in a given period and minimize the completion time to transmit a bundle of given data. The proposed directional water-filling algorithm provides a simple and concise interpretation of the necessary optimality conditions.

From the above discuss, we can conclude that the energy efficient packet scheduling problem with arbitrary individual deadline guarantee over a fading channel is still open, which aim to design a time efficient power control policy that transmits a sequence of dynamically arrived packets, each with an arbitrary arrival time and an arbitrary size, over time-varying fading channels. The contributions of this paper are summarized as follows.

- We study the energy efficient packet scheduling problem with arbitrary individual deadline guarantee over a fading channel. This problem allows the arbitrary individual deadline, which is a longstanding open question and is very much desired in scenarios such as Industrial Internet of Things.
- For the offline setting, we design a novel algorithm and prove its optimality. Novel concepts, namely *data interval* and its *water level*, are introduced for solving the problem. By iteratively locating the data interval with the highest water level, we compute the optimal power control policy.
- Based on the optimal offline results, we also present an online power control policy. Simulations show that this policy is efficient comparing to the offline optimal results.

The paper is organized as follows. Section 2 introduces the system model and define the problem. Section 3 describes basic optimality properties and introduces the HIF policy solving the offline problem. The online algorithm is presented in Section 4 and the simulation results are discussed in Section 5. Section 6 concludes the paper.

2. System model and problem formulation

We consider a single user point-to-point fading channel, where the channel states can be feedback by *channel state information feedback* mechanism. The system model is illustrated in Fig. 1 and Fig. 2. Note that the *channel state information feedback* mechanism is commonly used in many wireless communication scenarios.

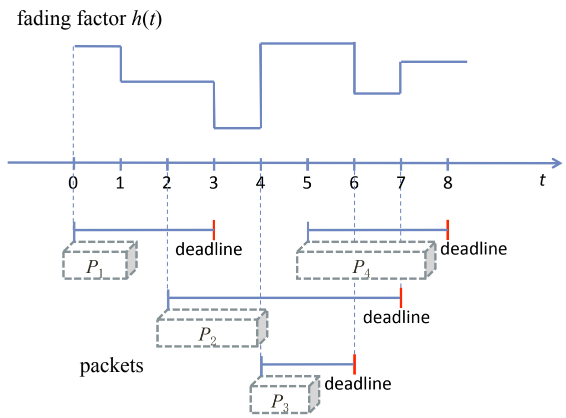


Figure 2: Packets are allowed to have arbitrary arrival times, arbitrary deadlines and arbitrary sizes. The time-varying fading factor $h(t)$ is allowed to be an arbitrary piecewise constant function. The goal is to determine a transmission power control policy that guarantees all packet deadlines and consumes the minimum energy.

Although the fading factor[1] fluctuates as time progresses, we assume it is constant in a small time interval. Let $h(t)$ denote the fading factor at time instance t. Then, it is a piecewise constant function as illustrated in the upper part of Fig. 2.

The transmitter is assumed to be able to adaptively change its transmission power and its corresponding transmission rate according packets in data queue and the channel states. Following previous works [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 15, 16], we assume transmission power $p(t)$ is related with transmission rate $r(t)$ and fading factor $h(t)$ through the following function,

$$r(t) = \frac{1}{2}\log(1 + h(t)p(t)) \tag{1}$$

This equation is called the fading channel constraint.

Assume a set of n packets, $\mathcal{P} = \{P_1, P_2, \cdots, P_n\}$, is to be transmitted to the receiver. Each packet is assumed to have an arbitrary arrival time $a_i \geq 0$, an arbitrary deadline $d_i (> a_i) > 0$, and an arbitrary packet size $B_i > 0$. We use triple $P_i = \{B_i, a_i, d_i\}$ to refer to the i-th packet, $1 \leq i \leq n$. For convenience, we assume packets are sorted such that $0 \leq a_1 \leq a_2 \leq \cdots \leq a_n$. We further assume deadlines can be sorted such that $0 \leq d_{q_1} \leq d_{q_2} \leq \cdots \leq d_{q_n}$ where $q_1, q_2, \cdots, q_n$ is a permutation of $1, 2, \cdots n$. Denote the largest deadline d_{q_n} as T, we have $d_{q_n} = T$. An example of the packets are illustrated in the lower part of Fig. 2.

The transmission of packet P_i can start only after its arrival time a_i and must finish before its deadline d_i. Such constraints are called the causality

[1]In this paper, we use the terms *channel state* and *fading factor* interchangeably for a fading channel.

constraint [3] which is formalized by the following equation.

$$\sum_{d_i \leq t} B_i \leq \int_0^t r(t)\, \mathrm{d}t \leq \sum_{a_i \leq t} B_i, \forall t \in [0, T]. \tag{2}$$

It is assumed that a packet can be transmitted at any transmission rate or by any transmission power, and a packet can be transmitted in segments. For a given $p(t)$ and $h(t)$, then $r(t)$ can be immediately computed by Eq. (1); while for a given $r(t)$, the packet transmission schedule can be uniquely determined by applying the Earlies Deadline First (EDF) rule to select packets from the data queue to transmit. Therefore, the ultimate goal is to develop a power control policy $p(t)$ that consumes a minimum amount of energy. Such ultimate goal is formalized in Eq. (3),

$$E = \int_0^T p(t)\, \mathrm{d}t. \tag{3}$$

With all notations prepared, we now define the E4AD problem.

Definition 1 (Energy Efficient packEt schEduling problem with Arbitrary Deadline guarantee over a fading channel, E4AD)**.** *Given a set of packet $\mathcal{P}$, and the time-varying fading factor $h(t)$, the energy efficient packet scheduling problem with arbitrary deadline guarantee over a fading channel (E4AD problem) is to determine a power control policy $p(t)$ that minimizes the overall energy consumption Eq. (3), while the fading channel constraint Eq. (1) and causality constraint Eq. (2) are satisfied.*

The optimal power control policy is denoted as $p^{opt}(t)$. We now define an important notation that is directly related to $p^{opt}(t)$.

Definition 2 (**Water Level**)**.** *Suppose the fading factor $h(t)$ is given. Then the water level $w(t)$ and the transmission power $p(t)$ are related to each other through the following equation.*

$$w(t) = p(t) + \frac{1}{h(t)}, \text{ if } p(t) \neq 0$$

At time t when $p(t) = 0$, water level $w(t)$ is undefined.

The notation *water level* $w(t)$ is introduced and inspired by the analysis in Appendix, in which the optimal transmission policy is related with water level.

With the new notation *water level* $w(t)$, Eq. (1) can be re-written as

$$r(t) = \frac{1}{2} \log h(t) w(t). \tag{4}$$

Obviously, the optimal power control policy $p^{opt}(t)$ and the optimal *water level* $w^{opt}(t)$ uniquely determine each other. In this paper, we will therefore focus on the optimal water level $w^{opt}(t)$. An illustration of the optimal water level is given in Fig. 3.

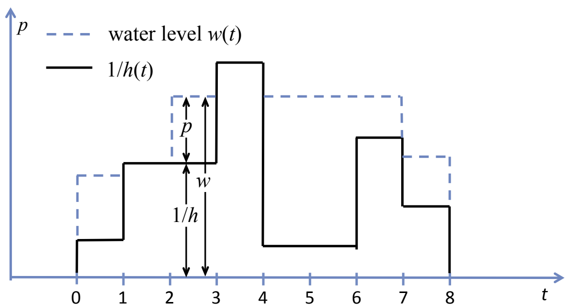


Figure 3: An illustration of the optimal water level that this paper aims to determine. Since $h(t)$ is known in the offline problem, an optimal water level and an optimal transmission power uniquely determine each other. Note that the *water level* is undefined in time durations $[1, 2)$ and $[3, 4)$, where transmission power is zero.

In the offline E4AD problem, all information about packets are assumed to be known before the scheduling, including the packet arrival time a_i, deadline d_i and packet size B_i, $1 \leq i \leq n$, as well as the channel fading factor $h(t)$. An arrival time instance $a_i, i = 1, 2, \ldots, n$ is called *arrival (event) point* and an arrival event occurs at t if $t = a_i$. Similarly, a deadline time instance $d_i, i = 1, 2, \ldots, n$ is called *deadline (event) point* and a deadline event occurs at t if $t = d_i$. Since $h(t)$ is piecewise constant function, if it changes at time t, we say that a *changing event* occurs at t and t is called a *changing (event) point*. It is assumed that there are m changing event points in duration $[0, T]$. Correspondingly, there are altogether $2n + m$ event points, including n arrival points and n deadline points.

Although, events may occur simultaneously and event points may overlap each other, we focus on a simple case where $2n + m$ events are distinct. Our results can be easily extended to the more general case, by treating simultaneously occurred events as occurred in sequence with extremely small interval. As a result, event points can be sorted in the order they occur. Let $e_k, 1 \leq k \leq 2n + m$ be the sorted event points, so that $0 \leq e_1 < e_2 < e_3 < \cdots < e_{2n+m}$. It is known that $e_1 = a_1$ and $e_{2n+m} = d_{q_n} = T$. The time interval between any two adjacent event points is defined as an *epoch* and the i-th epoch is denoted as E_i. Any *epoch* $E_i, i = 1, 2, \ldots, 2n + m$, has a non-zero length l_i.

3. An optimal offline power control policy

Given information of packets and the channel states in advantage, the offline E4AD problem is still difficult to solve because the optimal power control policy must adapt to the time-varying channel states, and in the meantime guarantee the packet causality constraint to minimize total energy consumption. In this section, basic optimality properties are firstly presented, and then the HIF policy is introduced to solve the offline E4AD problem.

3.1. Basic properties of an optimal power policy

We first present some basic properties that any optimal water level $w^{opt}(t)$ must have.

It is easy to see that, in any epoch $[e_k, e_{k+1}), 1 \leq k < m$, only one transmission power should be used. This is because, inside any epoch, the fading factor $h(t)$ stays constant; and according to Eq. (1), $r(t)$ is a convex function of $p(t)$. By its convexity, if two transmission powers $p_1 < p_2$ were used in transmission, we can always find a single power $p_1 < p < p_2$ to transmit more data with the same amount of energy. Hence, the water level $w(t) = p(t) + \frac{1}{h(t)}$ must be constant for each epoch too. The technique to use p to replace p_1 and p_2 is called *equalization* [11, 18].

The technique is called *equalization* [11, 18]. Such technique can not only equal water level inside a epoch but also between epochs where the fading factor may vary. That is, regardless of the different value of the fading factors, using a single water level is superior in order to transmit more data or consume less energy.

Lemma 1 (Water level equalization)**.** *The water levels of any two epochs can be equalized to transmit more data or consume less energy unless the causality constraints do not allow.*

Proof. Suppose two epochs E_1 and E_2 have the lengths l_1 and l_2, fading channel factors h_1 and h_2, the transmission power p_1 and p_2 respectively. We assume water level is defined in both epoch E_1 and epoch E_2, let them be w_1 and w_2 respectively. Then according to Definition 2, we have $w_1 = p_1 + \frac{1}{h_1}$ and $w_2 = p_2 + \frac{1}{h_2}$ as well as $p_1 > 0$ and $p_2 > 0$. So $p_1 = w_1 - \frac{1}{h_1}$ and $p_2 = w_2 - \frac{1}{h_2}$. Therefore, the energy consumption E is

$$\begin{aligned} E &= p_1 l_1 + p_2 l_2 \\ &= w_1 1_1 + w_2 l_2 - l_1/h_1 - l_2/h_2. \end{aligned} \tag{5}$$

The data transmission B is

$$\begin{aligned} B &= r_1 l_1 + r_2 l_2 \\ &= \frac{l_1}{2} \log h_1 w_1 + \frac{l_2}{2} \log h_2 w_2. \end{aligned} \tag{6}$$

Suppose $w_1 \neq w_2$ and we equalize the water levels of the two epochs. By *equalizing the water levels* we mean to control transmission power to reduce higher water level and increase lower water level. Assume the resulting new water level is w.

There are two cases, either w utilizes the same energy E or w transmits the same amount of date B. We consider the formal case and show the data transmission is increased, which is illustrated in Fig. 4. The other case is similar and therefore ignored. The w can be computed as follows.

$$w = \frac{w_1 l_1 + w_2 l_2}{l_1 + l_2}. \tag{7}$$

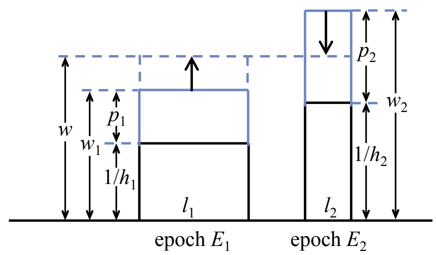


Figure 4: The water levels of any two epochs can be equalized to transmit more data or consume less energy unless the causality constraints do not allow.

It is easy to check that the consumed energy does not change in this new policy. But the transmitted packets B' in new policy has changed.

$$\begin{aligned} B' &= \frac{l_1}{2}\log h_1 w + \frac{l_2}{2}\log h_2 w. \\ \Delta B &= B - B' \\ &= \frac{1}{2}(l_1 \log w_1 + l_2 \log w_2 - (l_1 + l_2)\log w) \\ &\leq 0. \end{aligned}$$

The last inequality follows from the fact that the function log is a concave function. As a conclusion, the water levels of any two epochs can be equalized to transmit more with the same amount of energy consumption as long as the casuality constraints allow. In fact, it is similar to show that water levels of any two epochs can be equalized to save energy with the same amount of data transmission as long as the casuality constraints allow, we omit the detailed proof. □

Lemma 1 is illustrated in Fig. 4.

As an immediate result, if there is only one packet $P = \{B, a, d\}$, then the optimal water level is constant over epochs regardless of the fading level. This result is consistent with the famous *water-filling* method, however, our Lemma 1 is much more powerful. If multiple packets are assumed in this problem, the optimal water level no longer stay constant. It changes, only at event points, according to the previous discuss. The following lemma about these event points must be satisfied.

Lemma 2. *If the optimal water level increases/decreases from one epoch to the next epoch, there must be a(n) arrival/deadline point in between the two epochs.*

Proof. We formulize the lemma as follows.

Suppose w_1 and w_2 are two adjacently defined water levels in epoch $E_1 = [e_i, e_{i+1})$ and $E_2 = [e_j, e_{j+1})$. Obviously, e_{i+1} may or may not equals e_j, since it is possible interval $[e_{i+1}, e_j)$ has a non-zero length and the *water level* is undefined inside.

- If the two water levels increase, e.g. $w_1 < w_2$, and if $e_{i+1} = e_j$, then it must an arrival point; if $e_{i+1} \neq e_j$, then there must be an arrival point in interval $[e_{i+1}, e_j)$.
- If the two water levels decreases, e.g. $w_1 > w_2$, and if $e_{i+1} = e_j$, then it must a deadline point; if $e_{i+1} \neq e_j$, then there must be a deadline point in interval $[e_{i+1}, e_j)$.

We now prove the first part and the proof of the second part is similar.

In the seek of contradiction, we assume e_{i+1} is not an arrival point if $e_{i+1} = e_j$. No packet can arrive during time interval $[e_i, e_{j+1})$. Obviously, the water levels of the two epochs can be equalized by moving a certain amount of data transmitted in Epoch E_2 to Epoch E_1. According to Lemma 1, such equalization transmits more data with the same amount of energy or consume less energy to transmit the same amount of data. We now show such equalization satisfies the *casuality constraints*. First, every packet is finished before its deadline, because more data is transmitted in an earlier epoch. Second, no packet will be transmitted before its arrival time, because no packet arrival in interval $[e_{i+1}, e_j)$. This conflicts the optimality of the policy.

If $e_{i+1} \neq e_j$, the *water level* is undefined in interval $[e_{i+1}, e_j)$. According to Definition 2, the optimal transmission power in $[e_i, e_{j+1})$ must be zero. By contradiction, we assume that there does not exist an arrival point in (e_{i+1}, e_j). Thus, we can equalize the water level by moving some amount of data from E_2 to E_1 without violating causality constraint. According to Lemma 1, such equalization transmits more data with the same amount of energy or consume less energy to transmit the same amount of data. This contradicts the optimality. □

Such two epochs may or may not be adjacent, since in some epochs, the water level may be undefined. For example, in Fig. 3, the water level increases from epoch $[0, 1)$ to epoch $[2, 3)$, and water level is undefined in epoch $[1, 2)$. Then, according to Lemma 2, there must be an arrival point in $[1, 2]$. The water level decreases from epoch $[6, 7)$ to $[7, 8)$, then there must be a deadline point at time 7.

Lemma 3. *Let $P_k = \{B_k, a_k, d_k\}$ be any packet transmitted according to the optimal water level $w^{opt}(t)$. Let H be the set of all epochs contained in time interval $[a_k, d_k)$, and $H' \subseteq H$ be the subset of H which is not used to transmit P_k. The following two statements are true:*

1. *The overall water level $w^{opt}(t)$ used for any epoch of $H - H'$ must be the same water level w.*
2. *The overall water level $w^{opt}(t)$ used for any epoch of H' must be higher or equal to the water level w.*

Proof. We prove (1) by contradiction. Assume that two water levels $w_1 < w_2$ are used in Epoch E_1 and E_2 which are contained inside the set $H - H'$ in $w^{opt}(t)$. The water level of two epochs can be equalized by moving some amount of packets from E_2 to E_1 according to Lemma 1. By doing this, we can transmit the same amount of data with less energy which contradicts to the optimality of $w^{opt}(t)$. Then, we prove (2). By contradiction, we assume $w^{opt}(t)$ is lower than w in the Epoch x, where Epoch x is contained inside in H'. We can always equalize the water level between some epoch in $H - H'$ and Epoch x by moving some amount of data from the former epoch to the latter epoch. But no constraint is violated. So (2) is true as well. □

3.2. Highest water level Interval First (HIF) policy

With the optimal properties prepared, we now introduce the Highest water level Interval First (HIF) policy in this subsection. But before that, we first introduce some related definitions.

Definition 3. *Given a data set $P_k = \{B_k, a_k, d_k\}, 1 \le k \le n$, the data interval $I[i,j]$ is defined as the time interval from the arrival time a_i to the deadline d_{q_j}, $1 \le i, j \le n$, e.g. $I[i,j] = [a_i, d_{q_j})$ when $a_i \le d_{q_j}$. $I[i,j]$ is undefined, when $a_i > d_{q_j}$.*

The HIF policy works in iteration. We define the following four parameters for each data interval $I[i,j], 1 \le i, j \le n$, which will be modified in each iteration.

- The data set $S[i,j]$ is the set of packets whose arrival time and deadline are contained inside $I[i,j]$ and have not been assigned epochs yet. Initially $S[i,j] = \{P_k | [a_k, b_k) \subseteq I[i,j]\}$;
- The data load $B[i,j]$ is the sum amount of data contained in $S[i,j]$, i.e., $B[i,j] = \sum_{P_k \in S[i,j]} B_k$.
- The available epochs $T[i,j]$ is the set of all available epochs contained in interval $I[i,j]$, and its total length is $|T[i,j]|$. Initially, $|T[i,j]| = d_{q_j} - a_i$.
- The water level $W[i,j]$ is a constant which transmits $B[i,j]$ data during $T[i,j]$, e.g. $B[i,j] = \int_{t \in T[i,j]} \frac{1}{2} \log(h(t)W[i,j])\, dt$.

For the example illustrated in Fig. 2, $I[2,3] = [a_2, d_{q_3}) = [2,7)$, the data set $S[2,3] = \{P_2, P_3\}$, the data load $B[2,3] = B_2 + B_3$, the available epochs $T[2,3]$ includes all epochs inside interval $[2,7)$, the water level $W[2,3]$ is a constant and is illustrated in Fig. 3 as dash lines inside interval $[2,7)$. The exact value of water level $W[2,3]$ can be computed by the classic water-filling technique. That is, water can be gradually filled into interval $[2,7)$, and stops filling as soon as the corresponding transmission power can support transmitting $B[2,3] = B_2 + B_3$ data. The final water level is $W[2,3]$.

Since computing $W[i,j]$ is one of the most important steps of HIF policy, we specifically design an efficient Algorithm IntervalWaterLevel which directly computes $W[i,j]$. We assume $T[i,j]$ contains x epochs and $h[z]$ is the channel fading factor in the z-th epoch. We sort the x epochs such that the channel fading factors are in non-increasing order, e.g. $H[1] > H[2] > \cdots > H[x]$, where $H[z]$ is the channel fading factor of new sorted epoch sequence. Hence, $\frac{1}{H[1]} <$

Algorithm 1 IntervalWaterLevel($I[i,j]$)

$H[x] = sort(h[x])$; //sort epochs such that the channel fading factors are in non-increasing order
for $q \leftarrow 1$ to x **do**
  $Q = 0$
  for $z \leftarrow 1$ to x **do**
    if $h[z] > H[q]$ **then**
      $Q = Q + \frac{l_z}{2}\log(\frac{h[z]}{H[q]})$
    end if
  end for
  if $Q \geq B[i,j]$ **then**
    solve $B[i,j] = \sum_{z:h[z]>H[q]} \frac{l_z}{2}\log(h[z]W[i,j])$
    return $(W[i,j])$
  end if
end for
if $Q < B[i,j]$ **then**
  Solve $B[i,j] = \sum_{z=1}^{x} \frac{l_z}{2}\log(h[z]W[i,j])$
  return $(W[i,j])$
end if

$\frac{1}{H[2]} < \cdots < \frac{1}{H[x]}$. As the water is gradually filled in, the water level reach these values one by one. When the water level $W[i,j]$ equals $\frac{1}{H[q]}, q = 1, 2, \cdots, x$, the total data transmitted with current water level is $Q = \sum_{z:h[z]>H[q]} \frac{l_z}{2}\log(\frac{h[z]}{H[q]})$. If $Q < B[i,j]$, then we must have the water level $W[i,j] > 1/H[q]$, and otherwise, $W[i,j] \geq 1/H[q]$. By computing Q for each $\frac{1}{H[q]}, q = 1, 2, \cdots, x$, we can determine a q such that $1/H[q+1] > W[i,j] > 1/H[q]$. So, by solving $B[i,j] = \sum_{z:h[z]>H[q]} \frac{l_z}{2}\log(h[z]W[i,j])$, we can compute $W[i,j]$ directly.

To compute the offline optimal water level $w^{opt}(t)$, the HIF policy works in iteration. In each iteration, the water level is computed for every data interval. Amongst all the data intervals, locates the one with the highest water level. Let it be $I[i,j]$ and its water level be $W[i,j]$. Then, the HIF policy transmits all packets from $S[i,j]$ in the epochs $T[i,j]$. It will be proved that any optimal water level should use $W[i,j]$ in data interval $I[i,j]$, and exactly $B[i,j]$ data from $S[i,j]$ can be delivered in $I[i,j]$. We then update the available packet set by subtracting $S[i,j]$ and update the available epoch set by subtracting $T[i,j]$. After such updates, the same problem appears and we again locate the highest water level interval by the same procedure. The details are presented in Algorithm HighestIntervalFisrt.

3.3. Correctness analysis of HIF policy

The correctness of the HIF policy depends on whether using $W[i,j]$ as the water level in data interval $I[i,j]$ is optimal. If this is true for the first iteration, then, by the recursive native of HIF policy, we can conclude that it is optimal

Algorithm 2 HIGHESTINTERVALFISRT

```
while not all packets are assigned epoch do
    Identify the data set S[i, j] for every data interval I[i, j], 1 ≤ i, j ≤ n.
    Compute the data load B[i, j] and the available epochs T[i, j] for each I[i, j].
    Compute the water level W[i, j] for each I[i, j] by Algorithm INTERVALWATERLEVEL.
    Locate the highest water level interval I[i, j].
    Assign the water level w = W[i, j] to epochs in T[i, j].
    Mark all the epochs in T[i, j] as unavailable.
    Mark all the packets of S[i, j] as assigned epoch.
end while
```

in every iteration. The following theorem states that this is true for the first iteration.

Theorem 1. *Given a set of packets $P = \{P_i|1 \le i \le n\}$, $P_i = \{B_i, a_i, d_i\}$, among all the data intervals, if the highest water level interval is $I[i,j]$, then the following statements must be true:*

1. *Any optimal transmission policy must assign water level $W[i,j]$ to every epoch of $I[i,j]$,*
2. *Any optimal transmission policy must transmit exactly the packets $S[i,j]$ in $I[i,j]$.*

Proof. We prove (1) by contradiction. Assume $w^{opt}(t)$ is the optimal water level used in $I[i,j]$ and $w^{opt}(t) \ne W[i,j]$ in some epochs inside $[a_i, d_{q_j})$. There must be an epoch $[e_k, e_{k+1}) \subseteq [a_i, d_{q_j})$ where the optimal water level $w^{opt}(t) > W[i,j]$. Because if $w^{opt}(t) \le W[i,j]$ for entire $I[i,j]$, then $\int_{t\in[a_i,d_{q_j})} \log(w^{opt}(t)h(t))\,\mathrm{d}t < \int_{t\in[a_i,d_{q_j})} \log(W[i,j]h(t))\,\mathrm{d}t$, which implies some packets must miss their deadlines. Therefore, $w^{opt}(t) > W[i,j]$ holds in epoch $[e_k, e_{k+1}) \subseteq [a_i, d_{q_j})$. We then extend this epoch $[e_k, e_{k+1})$ to the longest time interval $[e_u, e_v)$ where every epoch has their $w^{opt}(t) > W[i,j]$. Note, $[a_i, d_{q_j})$ may not contain the time interval $[e_u, e_v)$ or vice versa. Thus, the water level increases/decreases at e_u/e_v. Otherwise, we can extent the $[e_u, e_v)$ to be a larger time interval. Note, it is possible that water level is undefined in $[e_{u'}, e_u)$ or in $[e_v, e_{v'})$, for some $u' < u$ and $v' > v$.

By Lemma 2, e_u is an arrival point, or an arrival point is inside $[e_{u'}, e_u)$, which is assumed to be a_p. Similarly, a deadline point d_{q_q} is at e_v or inside $[e_v, e_{v'})$. Thus, there must exist a data interval $[a_p, d_{q_q})$, and its water level is no higher than $W[i,j]$, because the $I[i,j]$ is the highest water level interval. However, we have $w^{opt}(t) \ge W[i,j]$ for every epoch in $[e_u, e_v)$ and $w^{opt}(t) > W[i,j]$ for Epoch $[e_k, e_{k+1}) \subseteq [e_u, e_v)$,

$\int_{t\in[a_p,d_{q_q})} \log((w^{opt}(t)h(t))\,\mathrm{d}t$

$= \int_{t\in[e_u,e_v)} \log w^{opt}(t)h(t))\,\mathrm{d}t$

$> \int_{t\in[e_u,e_v)} \log(W[i,j]h(t))\,\mathrm{d}$
$= \int_{t\in[a_p,d_{q_q})} \log(W[i,j]h(t))\,\mathrm{d}\ t > B[p,q]$.
Thus, the optimal water level $w^{opt}(t)$ transmits more data than the $B[p,q]$ in the data interval $I[p,q]$. We therefore conclude that there must be a packet P_x not belonging to $S[i,j]$ is transmitted in the data interval $I[p,q]$. Packet P_x either arrives before a_p or has a deadline after d_{q_q}. This contradicts Lemma 3. Therefore, $W[i,j]$ is the optimal water level for every epoch in $I[i,j]$. The statement (1) is proved.

According to Algorithm IntervalWaterLevel, the water level $W[i,j]$ transmits exactly $B[i,j]$ data in $I[i,j]$, and all packets in $S[i,j]$ have arrival times and deadlines inside $I[i,j]$. Hence, packets in $S[i,j]$ must be transmitted in $I[i,j]$. $\square$

All the water levels computed by the HIF policy is optimal by the recursive native of HIF policy.

After the optimal water levels are calculated, the transmission power is determined, hence the packet transmission schedule can be uniquely determined by applying Earlies Deadline First (EDF) rule to select packets from the data queue to transmit.

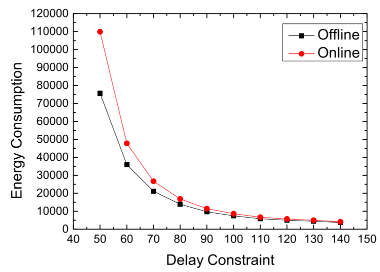
4. Online policy

The optimal HIF policy requires all information before schedule. In this section, we design an online policy that handles the case that no future information is available in advance. The online policy is motivated by the optimality conditions discussed in the previous section. It locates the data interval with the highest water level with no knowledge of any information of arriving packets, including arrival time, deadline, packet size, and fading channel factor.

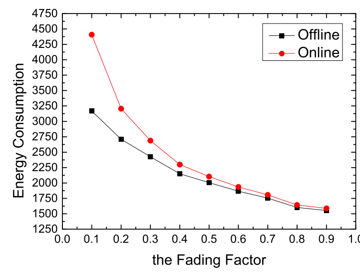
Suppose the current time is t_0 and the current fading channel factor is $h(t_0)$. There are n packets, $P_i = (B_i, a_i, d_i), i = 1, 2, \cdots, n$, remaining unfinished. For convenience, we assume that $d_1 < d_2 \cdots < d_n$. Then, there are altogether n data intervals, $I[t_0, d_i], i = 1, 2, \ldots, n$. Their water level is defined as $W[t_0, d_j], i = 1, 2, \cdots, n,$. The online policy works as follows to compute the highest water level w_{max}.

1. Assume the fading channel factor stay constant at $h(t_0)$ after time t_0, and invoke Algorithm IntervalWaterLevel to compute $W[t_0, d_i], i = 1, 2, \cdots, n,$.

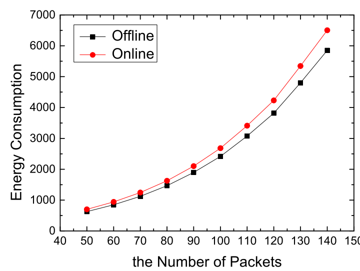
2. Let $w_{max} = \max_{j\le n} W[t_0, d_j]$.
   Let $j_{wmax} = \arg\max_{j\le n} W[t_0, d_j]$.
   Let $t_{wmax} = d_{j_{wmax}}$.
3. Use water level w_{max} to transmit packets until time t_{wmax} or the next event happens.
4. Repeat.



(a)



(b)



(c)

Figure 5: The energy consumption by the online algorithm and by the offline optimal algorithm.

5. Simulations

In our simulation, we investigate the performance of the proposed online algorithm. Since no other existing work studies the same problem as we do, we compare the online algorithm against the offline optimal results. In the comparison, we exam the impact of delay constraint, the number of packets and the diversity of fading factor on the performance.

5.1. Simulation Settings

We assume an additive white Gaussian noise channel and its fading factor follows uniform distribution $u(q,1)$, where q is a variable parameter. The packet size is assumed to follow uniform distribution $u(1,6)$. We assume that the distribution of packet arrival time is a Poisson process and the average inter-arrival time is 4. Packets delay constraint is assumed to follow an uniform distribution $u(d-10, d+10)$, where d is the average delay constraint. The simulated packet number is n.

In our simulation, the default setting is the fading factor $q = 0.3$, the average delay constraint $d = 140$ and the packet number $n = 100$. In Fig. 5, we change these three parameters, one at a time, as shown in (a), (b) and (c), to observe the trend of energy consumption and study the impact on algorithm performances.

Each value shown in figures of this section is the mean value of simulation results from 100 random instances. In each instance, n packets are generated according to the above model.

5.2. Simulation Results

In Fig. 5, the energy consumption of our online algorithm is compared to the offline optimal solution that minimizes the energy consumption. We observe that our online algorithm consumes slightly more energy than offline algorithm does in all three figures.

From (a), it can be observed that energy consumption decreases as the average delay constraint increases. This is because the longer a delay constraint is, the less urgent the packet is, which implies that lower transmission rate can be used to deliver it and therefore consumes less energy. In (b), the two curves descend as the fading factor enlarges. This is because, generally, the higher fading factor, the lower transmission power, and thus the less energy is consumed. It can also be observed that the two curves approaching each other with the increase of the abscissa. Because the online algorithm assumes the fading factor stay constant as the present value and computes the schedule, when q grows, it is more likely that fading factor is close to the present value. Therefore, the gap between the online algorithm and offline algorithm becomes smaller as the scope of fading factor decreases. In (c), the curves show rising trends with the increase of packet number. Since more packets means more data to transfer, more energy needs to be consumed.

6. Conclusions

This paper has optimally solved the problem of energy efficient packet scheduling with arbitrary individual deadline guarantee over a fading channel. First, some optimality properties was presented and proved. Then, novel notations such as date interval and water level was introduced. Based on the optimality properties and the notations, we have proposed the Highest water level Interval First (HIF) policy which has later been proved to be optimal. Finally, we have introduced an online algorithm. Simulations have shown this online algorithm is efficient to determine the transmission power.

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[1] B. Prabhakar, E. Uysal Biyikoglu, and A. El Gamal, “Energy-Efficient Transmission Over a Wireless Link via Lazy packet Scheduling,” in Proc. IEEE INFOCOM, Apr 2001, vol. 1, pp.386-394.

[2] E. Uysal-Biyikoglu, B. Prabhakar, and A. El Gamal, “Energy-Efficient Packet Transmission Over a Wireless Link,” IEEE Trans. on Netw., vol. 10, no. 4, pp. 478-499, Aug. 2002.

[3] A. El Gamal, C. Nair, B. Prabhakar, E. Uysal-Biyikoglu, and S. Zahedi, “Energy-efficient Scheduling of Packet Transmissions over Wireless Networks,” in Proc. IEEE INFOCOM, May 2002, pp.1773-1782.

[4] E. Uysal-Biyikoglu, A. El Gamal, “On adaptive transmission for energy efficiency in wireless data networks,” IEEE Trans. Inf. Theory, vol. 50, no. 12 pp. 3081-3094, Dec. 2004.

[5] W. Chen and U. Mitra, “Energy efficient scheduling with individual packet delay constraints,” in Proc. IEEE INFOCOM, Apr. 2006.(individual packet)

[6] https://en.wikipedia.org/wiki/Fading

[7] M. Zafer and E. Modiano, “A Calculus approach to minimum energy transmission policies with quality of service guarantees,” in Proc. IEEE INFOCOM, Mar. 2005, vol. 1, pp. 548-559.

[8] M. Zafer and E. Modiano, “A Calculus Approach to Energy- Efficient Data Transmission With Quality-of-Service Constraints,” IEEE Trans. on Netw., vol. 17, no. 3, pp. 898-911, June 2009.

[9] M. Zafer and E. Modiano, “Delay-Constrained Energy Efficient Data Transmission over a Wireless Fading Channel,” in Proc. Information Theory and Applications Workshop, 2007, pp. 289-298.

[10] M. Zafer and E. Modiano, “Minimum Energy Transmission Over a Wireless Channel With Deadline and Power Constraints,” IEEE Trans. on Automatic Control, vol. 54, no. 12, pp. 2841-2852, Dec. 2009.

[11] F. Shan, J. Luo, and X. Shen, “Optimal energy efficient packet scheduling with arbitrary individual deadline guarantee,” Comput. Netw., vol. 75, no. 2014, pp. 351-366. Dec. 2014

[12] A. Fu, E. Modiano, and J. N. Tsitsiklis, “Optimal transmission scheduling over a fading channel with energy and deadline constraints,” IEEE Trans. Wireless Commun., vol. 5, pp. 630-641, March 2006.

[13] A. Goldsmith and P. Varaiya, “Capacity of fading channels with channel side information,” IEEE Trans. Inf. Theory, vol. 43, pp. 1986-1992, Nov.1997.

[14] W. Chen, U. Mitra, M. Neely, "Energy-efficient scheduling with individual delay constraints over a fading channel," in: Proceedings of WiOpt, April 2007.

[15] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener. "Resource management for fading wireless channels with energy harvesting nodes." Proc. INFOCOM, 2011 IEEE, pp. 456-460.

[16] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener. "Transmission with energy harvesting nodes in fading wireless channels: Optimal policies." IEEE J. on Sel. Areas in Comm., 29, no. 8 (2011): 1732-1743.

[17] O. Ozel, K. Tutuncuoglu, J. Yang, S. Ulukus, and A. Yener, "Adaptive transmission policies for energy harvesting nodes in fading channels," in Proc. 2011 Conf. Information Sciences Systems.

[18] J. Yang and S. Ulukus, "Optimal Packet Scheduling in an Energy Harvesting Communication System," IEEE Trans. On Comm., vol. 60, no 1, pp. 220-230, Jan. 2012.

[19] J. Yang, S. Ulukus, "Optimal packet scheduling in a multiple access channel with rechargeable nodes," in: Proceedings of IEEE ICC, June 2011

[20] J. Yang, O. Ozel, S. Ulukus, "Optimal packet scheduling in a broadcast channel with an energy harvesting transmitter," in: Proceedings of IEEE ICC, June 2011.

[21] M. Akif Antepli, E. Uysal-Biyikoglu, H. Erkal, "Optimal packet scheduling on an energy harvesting broadcast link," IEEE J. Sel. Area Comm. 29 (8) (2011) 1721-1731

Appendix .1. Introduction of water level

Given just one packet $P = \{B, 0, 2\}$ to be transmitted where arrival time $a = 0$, deadline $d = 2$, packet size B and the fading channel factor h changes at time $t = 1$. There are three event points and two epochs that $e_1 = a, e_3 = d$ and e_2 is channel states change point at time $t = 1$. We assume that h_1 is the channel fading factor in epoch $E_1 = [e_1, e_2) = [0, 1)$, and h_2 is the channel fading factor in epoch $E_2 = [e_2, e_3) = [1, 2)$, respectively. The total consumed energy is E. From previous reference ([11]), the optimal transmission power policy must satisfy the basic property that the transmission power/rate remains constant for each epoch. By applying the basic property, the optimal transmission power must be constant in E_1 and E_2 which can be defined by p_1 and p_2. Thus the optimal power transmission must satisfy the following statements:

$$\min.\ E = p_1 + p_2 \tag{.1}$$

$$\text{s.t. } \frac{1}{2}\log(1 + h_1 p_1) + \frac{1}{2}\log(1 + h_2 p_2) = B \tag{.2}$$

$$p_i \geq 0, i = 1, 2. \tag{.3}$$

Through the formulas, we can compute

$$\begin{aligned} p_1 &= 2^B/\sqrt{h_1 h_2} - 1/h_1 \\ p_2 &= 2^B/\sqrt{h_1 h_2} - 1/h_2. \end{aligned} \tag{.4}$$

The two equations have the common item $2^B/\sqrt{h_1 h_2}$ if the $2^B/\sqrt{h_1 h_2}$ is larger than $1/h_1$ or $1/h_2$. This implies that $p_1 + 1/h_1 = p_2 + 1/h_2$, whatever the value of the channel fading factor, packet data size is. Thus we make sure that the optimal transmission policy is related with the formula $p + 1/h$ called water level.