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Efficient Genetic-Based Detection Algorithm for Large-Scale MIMO Systems

Ya Wang, Zheng Wang, Feng Shen, and Qingjiang Shi

***Abstract*—In this paper, a genetic-based signal detection algorithm is proposed for large-scale multi-input multi-output (MIMO) systems. First of all, the random Gaussian noise is utilized to serve for the population initialization in Genetic algorithm (GA), where candidates in the population can be easily generated. Then, the Euclidean distance $\|\mathbf{y}-\mathbf{H}\mathbf{x}\|$ is applied as the fitness function for the candidate selection. After that, two-point recombination as well as random mutation is introduced for the following evolution, thus completing an iteration of the proposed algorithm. Meanwhile, a flexible trade-off is established between detection performance and complexity, which can be adjusted by the population size and the iteration numbers. Furthermore, a pre-detection stage that relies on decoding radius is also proposed for the efficient detection without any performance loss. Finally, simulation results confirm that considerable performance gain can be achieved in a low complexity cost.**

Keywords: MIMO detection, Genetic algorithm, signal optimization.

I. Introduction

Nowadays, multi-input multi-output (MIMO) systems have become a core technique in 5G wireless communications by effectively improving the channel capacity. With respect to MIMO, its detection algorithm for the received signal has emerged as a key problem in both point-to-point systems and multiuser systems, especially when the MIMO dimension goes large. Therefore, how to achieve the reliable detection performance under acceptable complexity cost has become an urgent problem for the application of large-scale MIMO systems in 5G.

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Y. Wang is from State Key Laboratory of Complex Electromagnetic Environment Effects on Electronics and Information System, CEMEE, Luoyang, 451200, China (Email: cemee_wangya@163.com)

Z. Wang is with National Mobile Communications Research Laboratory, Southeast University. Z. Wang is also with Key Laboratory of Dynamic Cognitive System of Electromagnetic Spectrum Space, Ministry of Industry and Information Technology, Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, 211106, China (z.wang@nuaa.edu.cn).

F. Shen is with College of Electronic and Information Engineering, Nanjing University of Aeronautics and Astronautics (NUAA), Nanjing, China (Email:sfjx_nuaa@163.com).

Q. Shi is with School of Software Engineering, Tongji University, Shanghai, 200000, China (Email:shiqj@tongji.edu.cn).

In [1], lattice reduction (LR) from number theory was introduced to MIMO detection to improve the decoding performance. Specifically, according to lattice reduction, the channel matrix is converted into an equivalent but more orthogonal one, where the detection based on it behaves better by significantly removing the effect of noise [2]. However, though full receive diversity can be achieved by lattice-reduction-aided detection, the performance gap between suboptimal detection and ML detection is still substantial especially in high dimensional systems [3]. On the other hand, efficient detection schemes were also developed to adopt the requirement of large-scale MIMO systems. In [4], sampling strategy is introduced for MIMO detection to obtain the optimal solution by probabilistic sampling. In [5], the Gauss-Seidel (GS) method was applied to iteratively realize the MMSE algorithm without the complicated matrix inversion, which reduces the computational complexity from $O(n^3)$ to $O(n^2)$. Meanwhile, in pursuit of efficient detection, the technique of polynomial expansion is also applied to approximate the matrix inversion in MMSE [6]. Other detection schemes for large-scale MIMO systems can be found in [7]–[11].

As a classic method to generate high-quality solutions for optimization and search problems, Genetic algorithm, which is inspired by natural selection, has drawn a lot of attention in various research fields [12]. In particular, the genetic algorithm, which belongs to the larger class of evolutionary algorithms (EA), usually starts with a population of randomly generated individuals, where the population in each iteration is called a generation. During each iteration, the metric of each individual is evaluated by the fitness function, and the more fit individuals are stochastically selected from the current population. Meanwhile, further modifications for individuals are also carried out by crossover and random mutation, thereby completing the current iteration to form a new generation. Consequently, the algorithm terminates when a satisfactory result has been reached for the population.

In this paper, in order to solve the problem of MIMO detection with low complexity, Genetic algorithm is adopted for the efficient decoding. Different from a few existing GA-based detection algorithms [13], [14], the generation of individuals in population relies on the random Gaussian noise, thus offering an efficient way for the individual generation. Then, the Euclidean distance between the received signal and the candidate individuals is used as the fitness function. Based on it, selection, recombination as well as mutation can be successfully performed to yield the detection solution.

Note that our proposed algorithm is flexible by adjusting the iteration number for the trade-off between performance and complexity. On the other hand, a pre-detection stage based on decoding radius from bounded distance decoding is also given to improve the detection efficiency.

II. System Model

Consider the decoding of an $n \times n$ real-valued system. The extension to the complex-valued system is straightforward [15]. Let $\mathbf{x} \in \mathcal{X}^n$ denote the transmitted signal. The corresponding received signal $\mathbf{y}$ is given by

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n} \tag{1}$$

where $\mathbf{n}$ is the noise vector with zero mean and variance σ_n^2, $\mathbf{H}$ is an $n \times n$ full column-rank matrix of channel coefficients.

Intuitively, the purpose behind MIMO detection is to find an estimation of the transmitted signal and recover the original information bits as accurate as possible. Given the MIMO system shown in (1), the maximum likelihood (ML) detection, which is known as the optimal detection solution, can be described as,

$$\widehat{\mathbf{x}} = \arg\min_{\mathbf{x} \in \mathcal{X}^n} \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2. \tag{2}$$

To solve it, sphere detection (SD) is always used to implement the ML detection. Its core idea is enumerating all the points within a defined radius and then output the closest one in the Euclidean distance. However, the exponentially increased complexity with respect to the dimension n is always a big problem for SD, especially for high dimensional systems, which poses significant challenges for the hardware implementation [16].

Since the complexity of the optimal ML decoding that uses exhaustive search is exponential with the dimension n, some other sub-optimal decoding schemes with low complexity cost are proposed. Although these sub-optimal decoding schemes only solve the problem shown in (2) in an approximate way, they could offer a quite low complexity to implement by sacrificing the system decoding performance. Traditionally, the most common sub-optimal decoding schemes are zero-forcing (ZF) decoding, minimum mean square error (MMSE) decoding and successive interference cancelation (SIC) decoding.

- **ZF Detection** In the linear ZF detection, the received signal $\mathbf{y}$ shown in (1) is multiplied on the left by the pseudoinverse of $\mathbf{H}$, to get the estimate transmit signal

$$\widehat{\mathbf{x}}_{\text{ZF}} = Q\{\mathbf{H}^{\dagger}\mathbf{y}\}, \tag{3}$$

where $Q\{.\}$ denotes the quantization rounding and $\mathbf{H}^{\dagger}$ equals to $(\mathbf{H}^H\mathbf{H})^{-1}\mathbf{H}^H$. A well known drawback of ZF decoding is the effect of noise amplification when the channel $\mathbf{H}$ is ill conditioned.

- **MMSE Detection** Compared to ZF, MMSE achieves a better error performance by taking the effect of noise into account:

$$\widehat{\mathbf{x}}_{MMSE} = Q\{(\mathbf{H}^H\mathbf{H} + \sigma_n^2\mathbf{I}_n)^{-1}\mathbf{H}^H\mathbf{y}\} \tag{4}$$

where σ_n^2 represents the variance of the white Gaussian noise $\mathbf{n}$ while the average transmit power of each antenna is normalized to one. In fact, MMSE is equivalent to ZF with respect to an extended system model as follows [17]:

$$\overline{\mathbf{H}} = \begin{bmatrix} \mathbf{H} \\ \sigma_n^2\mathbf{I}_n \end{bmatrix} \quad \text{and} \quad \overline{\mathbf{y}} = \begin{bmatrix} \mathbf{y} \\ \mathbf{0}_n \end{bmatrix} \tag{5}$$

where $\overline{\mathbf{H}}$ is a $2n \times n$ extended channel matrix and $\overline{\mathbf{y}}$ is the $2n \times 1$ extended receive vector. Such operation is also known as the left preprocessing in the MIMO detection, which is widely used in various detection schemes to boost the detection performance.

- **SIC Detection** To further improve the decoding performance, the nonlinear SIC detection was introduced with the decision feedback in the detection stages. Specifically, after QR decomposition of the channel matrix $\mathbf{H}$, the system model becomes:

$$\mathbf{y}' = \mathbf{Q}^{\dagger}\mathbf{y} = \mathbf{R}\mathbf{x} + \mathbf{n}'. \tag{6}$$

The general idea behind of SIC is to process the received vector $\mathbf{y}$ to estimate each component of transmitted signal $\mathbf{x}$ in a recursive way, thus canceling the effect of those symbols already decoded and nulling those yet unknown. If a symbol $\widehat{x}_i$ is estimated, the decoder will exploit this decision to further estimate the remaining symbols $\widehat{x}_{i-1}, \ldots, \widehat{x}_1$, forming a nonlinear decoding structure.

$$\widehat{x}_i = \left\lceil \frac{y_i' - \sum_{j=i+1}^{n} r_{i,j}\widehat{x}_j}{r_{i,i}} \right\rfloor. \tag{7}$$

Unfortunately, the performance of SIC detection is affected by the error propagation especially when incorrectly detected symbols are applied to perform the further detection. To this end, necessary ordering about the channel matrix $\mathbf{H}$ is normally carried and at each decoding stage the column with the maximum SNR is always detected for the first. However, even if these sub-optimal decoders offer rather low computational complexities, their performance are very poor and they cannot completely take advantages of the diversity offered by MIMO systems.

III. Genetic-based Random Noise Detection

In this section, the proposed Genetic-based random noise detection is firstly presented. Furthermore, a pre-detection stage based on the judgement by decoding radius is added in order to improve the detection efficiency without any performance loss.

A. Algorithm Description

Specifically, the proposed Genetic-based detection algorithm can be described by the following 6 steps.

- **Population initialization**: The generation of initial individuals fully exploit the potential of random Gaussian noises. As shown in (1), the received signal $\mathbf{y}$ suffers from the Gaussian noise $\mathbf{n}$ while the aim of ML detection given in (2) is trying to remove the effect of noises.
  Because of the randomness of Gaussian noise, here we propose to use the Gaussian noise to generate the individuals by

$$\mathbf{y}_j = \mathbf{y} + \mathbf{n}_j, \tag{8}$$

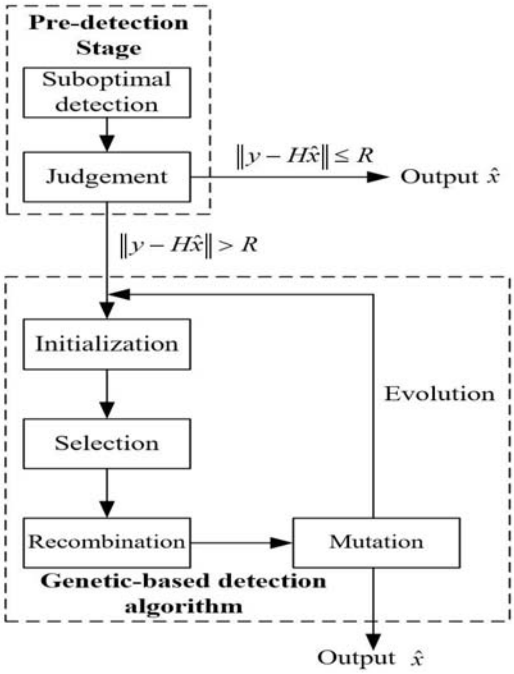

Fig. 1. Illustration of the proposed Genetic-based MIMO detection.

where $\mathbf{n}_j$ follows zero mean with variance σ_n^2, $1 \leq j \leq K$ is the index of individuals and K indicates the size of the population.

An open question about GA is the size of the initial population. Clearly, a small size has the risk that the optimal solution is omitted while the a large size may result in overloaded complexity consumption. To this end, a moderate size K is encouraged to guarantee a number of individuals can survive in the individual selection, thereby ensuring that the new generation can be obtained by the following recombination and random mutation.

- **Fitness function**: The key that GA could yield the solution of global optimum rather than local optimum chiefly depends on the selection of fitness function. As shown in (2), the detection problem can be written as an optimization problem, where $\widehat{\mathbf{x}}$ that minimizes the Euclidean distance $\|\mathbf{y}-\mathbf{Hx}\|$ is just the global optimum. To this end, we use $\|\mathbf{y}-\mathbf{Hx}\|$ as the fitness function to evaluate the metric of each individual

$$f(\widehat{\mathbf{x}}) = \|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\|, \tag{9}$$

and this is also accordance with the fact that the fitness is usually the value of the objective function in the optimization problem being solved.

- **Individual selection**: Given the fitness function in (9), the metric for each individual can be estimated, where individual selection can be performed thereafter. Specifically, with respect to the generation $\mathbf{Y} = [\mathbf{y}, \mathbf{y}_1, \ldots, \mathbf{y}_K]$, traditional detection schemes like ZF or MMSE is applied to output the corresponds results, which can be expressed as

$$\mathbf{X} = [\widehat{\mathbf{x}}, \widehat{\mathbf{x}}_1, \ldots, \widehat{\mathbf{x}}_K]. \tag{10}$$

Intuitively, the one with smallest distance $\|\mathbf{y} - \mathbf{Hx}\|$ in set $\mathbf{X}$ can be directly outputted as the detection solution. However, in order to further exploit the potential of Genetic algorithm, individual selection is performed to form the list

$$\overline{\mathbf{X}} = [\widehat{\mathbf{x}}, \ldots, \widehat{\mathbf{x}}_j] \tag{11}$$

according to the threshold

$$f(\widehat{\mathbf{x}}_j) \geq f(\widehat{\mathbf{x}}) = \|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\|. \tag{12}$$

Clearly, the metric of the original individual $\mathbf{y}$ serves as the threshold for the individual selection, where the individuals with the same or better metric can be saved. Accordingly, the saved individuals after the selection can be written as

$$\overline{\mathbf{Y}} = [\widehat{\mathbf{y}}, \ldots, \widehat{\mathbf{y}}_j]. \tag{13}$$

Then, based on it, the following operations from GA is performed for a better detection performance.

- **Recombination**:Similar to crossover in GA, recombination here means to generate a new individual by random combining two individuals in a certain way. For simplicity, here we generate the new individual by applying the mean of two random selected individuals as

$$\mathbf{y}_{j+1} = (\mathbf{y}_a + \mathbf{y}_b)/2, \tag{14}$$

where index a and b are randomly selected. Note that the total amount of generation is K, which means the number of generated individuals by recombination is $K - j$.

- **Random mutation**: In GA, genetic diversity can be achieved by random mutation, which makes a sudden change to the existing individuals for a broader space. In essence, the proposed population initialization that utilizes Gaussian noises can be viewed as a variation of Gaussian mutation. Here, we apply the idea of uniform mutation by randomly selecting a fraction (10%) of individuals for mutation. Furthermore, regarding to the chosen individuals, Gaussian noises with zero means and variance $\gamma \cdot \sigma_n^2$ is added:

$$\widehat{\mathbf{y}}_j = \widehat{\mathbf{y}}_j + \mathbf{n}', \tag{15}$$

where $\gamma \leq 1$ represents the scale parameter of the random Gaussian noise.

- **Stopping criteria**:As shown in Fig. 1, the proposed detection algorithm operates iteration by iteration, which implies the detection can terminate at the end of each iteration. Clearly, after the first iteration, a detection result with less or equal Euclidean distance $\|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\|$ can be issued, therefore offering a better decoding performance than those of suboptimal detection schemes. Since the detection performance improves with the number of iterations, a reasonable iteration number N can be determined by the required detection performance.

B. *Efficient Detection by Decoding Radius*

The proposed Genetic-based detection algorithm offers an effect way to improve the detection performance. However, one fact has to be confronted in MIMO detection: not all the detection by suboptimal detector should be improved. In other words, the optimal detection result also can be found by suboptimal detection schemes if the effect of noises is modest. For this reason, a judgement based on decoding radius can be applied to determine the detection result that needs to be further improved by the proposed algorithm, which leads to an efficient detection without performance loss.

Algorithm 1 Genetic-based algorithm for MIMO Detection

Require: $\mathbf{H}, \mathbf{y}, N, K$

Ensure: $\mathbf{x}$

1: perform sub-optimal detection to obtain $\widehat{\mathbf{x}}$
2: **if** $\|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\| \le R_{\text{detection scheme}}$ **then**
3:     output $\widehat{\mathbf{x}}$ as the solution directly
4: **else**
5:     generate K individuals by (8)
6:     **for** $t = 1, \ldots, N$ **do**
7:         perform individual selection by the threshold (12)
8:         perform recombination by (14)
9:         perform mutation by (15)
10:     **end for**
11:     output $\widehat{\mathbf{x}}_i$ with the smallest $\|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}_i\|$ in the population as the solution
12: **end if**

In theory, the notion of decoding radius comes from bounded distance decoding (BDD). Specifically, the optimal decoding solution is guaranteed if the Euclidean distance $\|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\|$ is less than the decoding radius R, where each detector actually has its own decoding radius. For example, the decoding radius of ZF is

$$R_{\text{ZF}} = \frac{1}{2}\min_i \|\mathbf{b}_i\| \sin\theta_i, \quad i = 1, ..., n \tag{16}$$

and the decoding radius of SIC is given by

$$R_{\text{SIC}} = \frac{1}{2}\min_i \|\widehat{\mathbf{b}}_i\|. \tag{17}$$

Therefore, given the corresponding decoding radius, the detection result $\widehat{\mathbf{x}}$ from the suboptimal detection schemes can be used to make the judgement by

$$\|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\| \le R_{\text{detection scheme}}. \tag{18}$$

Clearly, if (18) is satisfied, then there is no need to recall the proposed detection algorithm as the result of suboptimal detection is actually optimal, which means considerable computational complexity can be saved. Otherwise, the proposed detection algorithm is activated for a better decoding performance. To make it more flexible, a relaxation factor $\alpha \ge 1$ can be added to achieve the trade-off between performance and complexity, that is

$$\|\mathbf{y} - \mathbf{H}\widehat{\mathbf{x}}\| \le \alpha \cdot R_{\text{detection scheme}}. \tag{19}$$

IV. Simulations

In this section, the performance of the proposed Genetic-based detection algorithm is evaluated in the large-scale MIMO detection. Specifically, the ith entry of the transmitted signal $\mathbf{x}$, denoted as x_i, is a modulation symbol taken independently from an M-QAM constellation $\mathcal{X}$ with Gray mapping. Meanwhile, we assume a flat fading environment, where the square channel matrix $\mathbf{H}$ contains uncorrelated complex Gaussian fading gains with unit variance and remains constant over each frame duration. Let E_b represents the average power per bit at the receiver, then the signal-to-noise ratio (SNR) $E_b/N_0 = n/(\log_2(M)\sigma_w^2)$ where M is the modulation level and σ_w^2 is the noise variance.

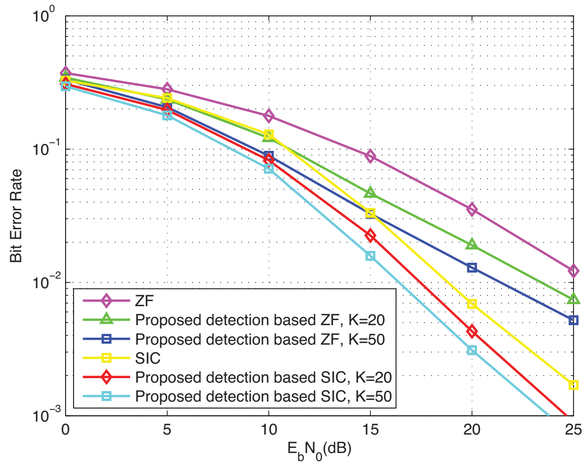

Fig. 2. Bit error rate versus the number of iterations for the uncoded 8×8 MIMO system using 16-QAM.

Fig. 2 shows the bit error rate (BER) of the proposed Genetic-based detection algorithm in a 8×8 uncoded MIMO system with 16-QAM. As a fair comparison, traditional suboptimal detection schemes like ZF and SIC are illustrated. Clearly, the detection performance of the proposed Genetic-based detection algorithms is better than the those of conventional ones. More specifically, with the increase of population size K, the proposed Genetic-based detection algorithm shows an increasing detection performance. This actually poses a flexible trade-off between detection performance and complexity. Note that in this case the iteration number N is set as 1 by default, where the impact of N is shown in the following simulation.

In Fig. 3, the BERs of the proposed Genetic-based detection algorithm with different iteration number N are evaluated in a 6×6 uncoded MIMO system with 4-QAM, where the size of the population is set as $K = 30$. Specifically, as shown in Fig. 3, the detection performance improves gradually with the increase of N, thus establishing another detection trade-off between performance and complexity. Therefore, considering the requirement of detection performance, a reasonable iteration number N can be set while the algorithm terminates to output the final detection solution.

On the other hand, the BERs of the proposed Genetic-based detection algorithm with different relaxation factors $\alpha \ge 1$ by means of decoding radius R_{zf} are also given. Intuitively, with a reasonable α, the detection shows negligible performance loss, but saving considerable computational complexity from it. To be more precisely, the percentage of the direct detection

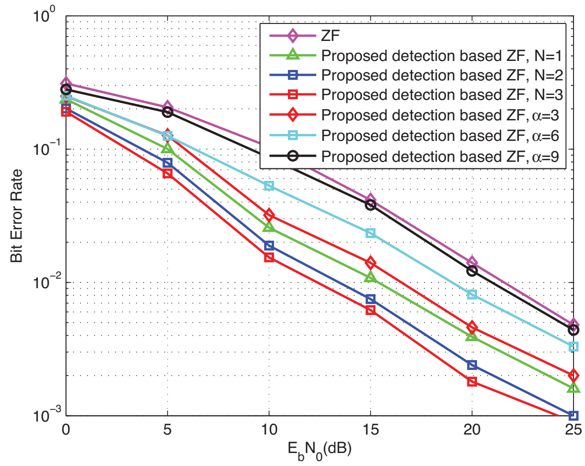

Fig. 3. Bit error rate versus the number of iterations for the uncoded 6×6 MIMO system using 4-QAM.

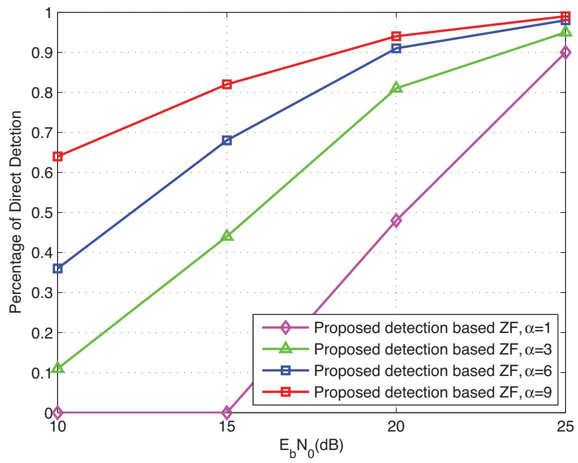

Fig. 4. Bit error rate versus the percentage of direct detection due to the startup mechanism for the uncoded 6×6 MIMO system using 4-QAM.

finished by the suboptimal detection scheme, i.e., $\mathbf{x}_{\text{output}} = \mathbf{x}_{\text{zf}}$, is depicted in Fig. 4. Typically, with the increment of SNR, the percentage of direct detection improves gradually. This is mainly because the noise is suppressed while the detection capability of ZF therefore increases. Meanwhile, as α gets loose gradually, the percentage of direct detection improves gradually, where considerable computational complexity is reduced without any performance loss.

V. Conclusion

In this paper, we proposed a genetic-based detection algorithm for MIMO systems. The proposed algorithm consists of 6 steps, which are population initialization, fitness function, individual selection, recombination, random mutation and stopping criteria respectively. Meanwhile, a pre-detection stage is applied for the efficient detection, which is able to reduce the computational complexity without any performance loss. In the end, the simulations of MIMO detection are presented to illustrate the gain in both detection performance and complexity.

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