Assignment 4

Problem

Solving the following integer linear programming problem by using the branch and bound algorithm:

$$\begin{aligned} \max \quad & z = 2x_1 + 3x_2 \\ \text{s.t.} \quad & 5x_1 + 7x_2 \leq 35 \\ & 4x_1 + 9x_2 \leq 36 \\ & x_1 \geq 0,\ x_2 \geq 0 \\ & x_1, x_2 \in Z^+ \end{aligned}$$

First, write the slack problem of this problem:

$$\begin{aligned} \max \quad & z = 2x_1 + 3x_2 \\ \text{s.t.} \quad & 5x_1 + 7x_2 \leq 35 \\ & 4x_1 + 9x_2 \leq 36 \\ & x_1 \geq 0,\ x_2 \geq 0 \end{aligned}$$

Using the simplex method to solve the slack problem, the final simplex table is:

Basis	x_1	x_2	x_3	x_4	$B^{-1}b$
x_1	1	0	$\frac{9}{17}$	-$\frac{7}{17}$	$\frac{63}{17}$
x_2	0	1	-$\frac{4}{17}$	$\frac{45}{153}$	$\frac{40}{17}$
$C_N^T - C_B^T B^{-1} N$	0	0	-$\frac{6}{17}$	-$\frac{1}{17}$	$\frac{246}{17}$

the optimal solution of the slack problem is:

$$x_1 = \frac{63}{17}, \quad x_2 = \frac{40}{17}, \quad z = \frac{246}{17}$$

The answer is not integer. So we need to branch the problem.

Branch and Bound Process

We can set $\underline{z} = 0$, firstly we can branch on x_1:

Branch 1: $x_1 \leq 3, x_1 \geq 4$

Let $x_1 = 3$, we can get the answer:

$$x_1 = 3, \quad x_2 = \frac{20}{7}, \quad z = 14.571$$

Let $x_1 = 4$, we can get the answer:

$$x_1 = 4, \quad x_2 = \frac{20}{9}, \quad z = 14.667$$

Both answers are not integer. So we set $\overline{z} = 14.667$ and branch again:

Branch 2: $x_2 \le 2, x_2 \ge 3$

Let $x_2 = 3$, we can get the answer:

$$x_1 = 2.25, \quad x_2 = 3, \quad z = 13.5$$

Let $x_2 = 2$, we can get the answer:

$$x_1 = 4, \quad x_2 = 2, \quad z = 14$$

We got an integer solution here, so we update $\underline{z} = 14$.

Now the range of z^* is:[14,14.667] and the branch $x_2 \ge 3$ will be cut off. We return to branch 1 and branch on x_2.

Branch 3: $x_2 \le 2, x_2 \ge 3$

Let $x_2 = 3$, we can get the answer:

$$x_1 = 2.25, \quad x_2 = 3, \quad z = 12.75 < 14$$

this branch will be cut off. Let $x_2 = 2$, we can get the answer:

$$x_1 = 3, \quad x_2 = 2, \quad z = 12 < 14$$

this branch will be cut off.

Now the range of z^* is:[14,14] and we have found the optimal integer solution.

Optimal Integer Solution

The final answer is:

$$x_1 = 4, \quad x_2 = 2$$

The optimal value of the objective function is 14.