

## Assignment 3

### Problem

Solve the following linear programming problem using the Simplex Method:

$$\begin{aligned} \min \quad & 3x_1 - 3x_2 - x_3 \\ \text{s.t.} \quad & 4x_1 + 2x_2 + x_3 \leq 4 \\ & 2x_1 + 4x_2 + 2x_3 \leq 3 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

**You are required to write the solving process in the simplex table.**

Introducing slack variables  $x_4$  and  $x_5$ , the standard form is:

$$\begin{aligned} \min \quad & 3x_1 - 3x_2 - x_3 \\ \text{s.t.} \quad & 4x_1 + 2x_2 + x_3 + x_4 = 4 \\ & 2x_1 + 4x_2 + 2x_3 + x_5 = 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

Basis	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$B^{-1}b$
$x_4$	4	2	1	1	0	4
$x_5$	2	4	2	0	1	3
$C_N^T - C_B^T B^{-1}N$	3	-3	-1	0	0	

We choose  $x_2$  as the entering variable and  $x_5$  as the leaving variable (3/4 is the minimum ratio). After performing the Gaussian elimination operation, we can get:

Basis	$x_1$	$x_2$	$x_3$	$x_4$	$x_5$	$B^{-1}b$
$x_2$	3	0	0	1	$-\frac{1}{2}$	$\frac{5}{2}$
$x_4$	$\frac{1}{2}$	1	$\frac{1}{2}$	0	$\frac{1}{4}$	$\frac{3}{4}$
$C_N^T - C_B^T B^{-1}N$	$\frac{9}{2}$	0	$\frac{1}{2}$	0	$\frac{1}{4}$	$-\frac{9}{4}$

Since all the coefficients in the last row ( $C_N^T - C_B^T B^{-1}N$ ) are non-negative, the optimal solution is reached.

### Optimal Solution

The final answer is:

$$x_1 = 0, \quad x_2 = \frac{3}{4}, \quad x_3 = 0, \quad x_4 = \frac{5}{2}, \quad x_5 = 0$$

The optimal value of the objective function is  $\frac{9}{4}$ .