

Assignment 2

Write the standard formulation and the dual problem of the following linear programming problem:

$$\begin{array}{ll}\min & 2x_1 + x_2 + 3x_3 + x_4 \\ \text{s.t.} & x_1 + 2x_2 + x_3 + x_4 \leq 7 \\ & 2x_1 - 3x_2 + 5x_3 = -8 \\ & x_1 - 2x_3 + 2x_4 \geq 1 \\ & x_1 \geq 0, \quad x_2 \leq 0, \quad x_3 \geq 0\end{array}$$

Let $x_5 = -x_2, x_4 = x_8 - x_9$ and introduce slack variables ($x_6, x_7 \geq 0$), the standard formulation:

$$\begin{array}{ll}\min & 2x_1 - x_5 + 3x_3 + x_8 - x_9 + 0(x_6 + x_7) \\ \text{s.t.} & x_1 - 2x_5 + x_3 + x_8 - x_9 + x_6 = 7 \\ & 2x_1 + 3x_5 + 5x_3 = -8 \\ & x_1 - 2x_3 + 2x_8 - 2x_9 - x_7 = 1 \\ & x_1, x_3, x_5, x_6, x_7, x_8, x_9 \geq 0\end{array}$$

The dual problem of the programming problem is:

$$\begin{array}{ll}\max & 7y_1 - 8y_2 + y_3 \\ \text{s.t.} & y_1 + 2y_2 + y_3 \leq 2 \\ & 2y_1 - 3y_2 \geq 1 \\ & y_1 + 5y_2 - 2y_3 \leq 3 \\ & y_1 + 2y_3 = 1 \\ & y_1 \leq 0, y_2 \text{unlimited}, y_3 \geq 0\end{array}$$