

# Assignment 3

## Problem

Solve the following linear programming problem using the Simplex Method:

$$\begin{aligned} \min \quad & 3x_1 - 3x_2 - x_3 \\ \text{s.t.} \quad & 4x_1 + 2x_2 + x_3 \leq 4 \\ & 2x_1 + 4x_2 + 2x_3 \leq 3 \\ & x_1 \geq 0, x_2 \geq 0, x_3 \geq 0 \end{aligned}$$

You are required to write the solving process in the simplex table.

Introducing slack variables  $x_4$  and  $x_5$ , the standard form is:

$$\begin{aligned} \min \quad & 3x_1 - 3x_2 - x_3 \\ \text{s.t.} \quad & 4x_1 + 2x_2 + x_3 + x_4 = 4 \\ & 2x_1 + 4x_2 + 2x_3 + x_5 = 3 \\ & x_1, x_2, x_3, x_4, x_5 \geq 0 \end{aligned}$$

| Basis                    | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $B^{-1}b$ |
|--------------------------|-------|-------|-------|-------|-------|-----------|
| $x_4$                    | 4     | 2     | 1     | 1     | 0     | 4         |
| $x_5$                    | 2     | 4     | 2     | 0     | 1     | 3         |
| $C_N^T - C_B^T B^{-1} N$ | 3     | -3    | -1    | 0     | 0     |           |

We choose  $x_2$  as the entering variable and  $x_5$  as the leaving variable ( $3/4$  is the minimum ratio). After performing the Gaussian elimination operation, we can get:

| Basis                    | $x_1$         | $x_2$ | $x_3$         | $x_4$ | $x_5$          | $B^{-1}b$      |
|--------------------------|---------------|-------|---------------|-------|----------------|----------------|
| $x_2$                    | 3             | 0     | 0             | 1     | $\frac{-1}{2}$ | $\frac{5}{2}$  |
| $x_4$                    | $\frac{1}{2}$ | 1     | $\frac{1}{2}$ | 0     | $\frac{1}{4}$  | $\frac{3}{4}$  |
| $C_N^T - C_B^T B^{-1} N$ | $\frac{9}{2}$ | 0     | $\frac{1}{2}$ | 0     | $\frac{1}{4}$  | $\frac{-9}{4}$ |

Since all the coefficients in the last row ( $C_N^T - C_B^T B^{-1} N$ ) are non-negative, the optimal solution is reached.

## Optimal Solution

The final answer is:

$$x_1 = 0, \quad x_2 = \frac{3}{4}, \quad x_3 = 0, \quad x_4 = \frac{5}{2}, \quad x_5 = 0$$

The optimal value of the objective function is  $\frac{9}{4}$ .