LP and MILP-Based Cyclic Scheduling Optimization for Crude Oil Pipeline Networks: From Economic Optimality to System Robustness Analysis

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***Abstract*—Addressing the prevalent physical mutual exclusion constraints and continuous production boundary issues in crude oil pipeline scheduling, this paper constructs a Mixed Integer Linear Programming (MILP) model that is more aligned with engineering practice, building upon traditional linear programming. By introducing the Big-M method to handle the logical mutual exclusion of valve switching and imposing 24-hour cyclic steady-state constraints, the deficiencies of the basic model in physical feasibility and production continuity are overcome. Furthermore, this paper breaks through the single economic perspective by constructing extreme scenarios of "main pipeline congestion" and "hub inventory crisis" to conduct in-depth stress testing of the system. Experimental results demonstrate that the model can not only output economically optimal solutions under normal conditions but also automatically activate the backup berth (S4) and the reverse backflow mechanism of bidirectional pipelines under extreme pressure, proving the important value of bidirectional flow facilities as system "safety redundancy" and validating the robustness of the proposed model in complex dynamic environments.**

***Index Terms*—Crude oil pipeline scheduling, Mixed integer programming, Mutual exclusion constraints, Cyclic scheduling, Robustness analysis**

I. Basic Problem Analysis

A. Problem Description

Crude oil transportation is a crucial component of petrochemical industrial production. The basic structure of the crude oil transportation network studied in this problem is shown in Figure 1, consisting of four parts: oil tanker berths, oil transportation stations (including storage tanks), oil transportation pipelines, and petrochemical enterprises. Multiple types of crude oil are transported from the storage tanks at oil transportation stations to various petrochemical enterprises through pipelines.

The objective of this problem is to develop a preliminary planning scheme for crude oil transportation. The known conditions include: (a) the transportation cost per ton of crude oil per kilometer of pipeline; (b) the pipeline lengths between various nodes (berths, stations, and enterprises); (c) the demand for each type of crude oil from each enterprise; (d) the inventory of each type of crude oil at each oil transportation station. The decisions to be made are: the quantity of each type of crude oil to be transported from each oil transportation station to each enterprise, such that the crude oil demand of each enterprise is satisfied while the total transportation cost is minimized.

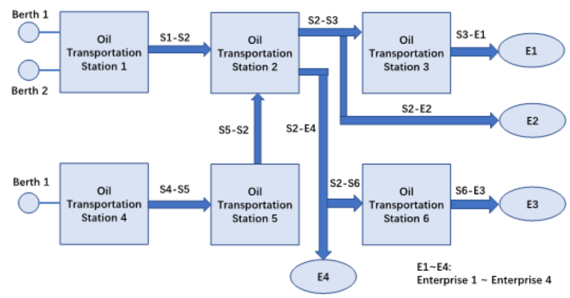

Fig. 1. Schematic diagram of crude oil transportation network structure: including 6 oil transportation stations (S1-S6), 4 petrochemical enterprises (E1-E4), and connecting pipelines

1) Network Parameters: The distance parameters of the pipeline network are shown in Table I. The network contains 9 pipelines connecting 6 oil transportation stations and 4 petrochemical enterprises.

TABLE I
Pipeline Network Distance Parameters (Unit: km)

Pipeline	Distance	Pipeline	Distance
S1-S2	300	S2-S3	250
S3-E1	200	S4-S5	350
S5-S2	150	S2-E4	400
S2-S6	250	S2-E2	450
S6-E3	200		

2) Enterprise Demand: The demand for 6 types of crude oil (O1-O6) from each petrochemical enterprise is shown in Table II. It can be observed that different enterprises have varying demands for crude oil types: E1 mainly requires O1, O2, and O5; E2 requires O3 and O4; E3 requires O1, O4, O5, and O6;

E4 requires O3, O4, and O6.

TABLE II
CRUDE OIL DEMAND OF EACH ENTERPRISE (UNIT: 10^4 TONS)

Enterprise	Oil Type	Demand
E1	O1	5.0
E1	O2	9.0
E1	O5	8.5
E2	O3	9.3
E2	O4	10.0
E3	O1	4.2
E3	O4	9.1
E3	O5	3.4
E3	O6	3.5
E4	O3	3.4
E4	O4	10.0
E4	O6	2.8

3) Station Inventory: The crude oil inventory situation at each oil transportation station is shown in Table III. The types and quantities of crude oil stored at each station vary, which determines the feasibility constraints of the transportation scheme.

TABLE III
CRUDE OIL INVENTORY AT EACH STATION (UNIT: 10^4 TONS)

Station	Oil Type	Inventory
S1	O1	7.1
S1	O4	6.5
S2	O1	0.9
S2	O2	8.6
S2	O3	11.6
S2	O4	8.4
S2	O5	11.7
S2	O6	3.2
S3	O3	0.4
S3	O4	29.5
S3	O6	4.9
S4	O3	4.5
S4	O4	10.4
S4	O5	3.4
S4	O6	3.8
S5	O1	5.0
S5	O2	2.2
S5	O4	38.4
S5	O6	6.1
S6	O4	2.5

B. Solution Approach

1) Problem Simplification Strategy: To reduce the complexity of the problem, we adopt the following strategies to simplify the decision variables:

(1) Eliminating necessarily zero decision variables: Only when an oil transportation station has inventory of a certain type of crude oil and an enterprise requires that type of crude oil, the corresponding transportation quantity variable is meaningful. By traversing the inventory table and demand table, we filter out all valid (station, enterprise, oil type) combinations, reducing the original $6 \times 4 \times 6 = 144$ potential variables to the actual number of effective variables.

(2) Floyd algorithm for shortest path calculation: Since transportation cost is proportional to distance, when transporting crude oil from any oil transportation station to any enterprise, the shortest path must be chosen. Therefore, we use the Floyd-Warshall algorithm to pre-calculate the shortest distance matrix from all oil transportation stations to all enterprises, transforming the network topology problem into a simple point-to-point transportation problem.

The core iteration formula of the Floyd-Warshall algorithm is [2], [3]:

$$d_{ij}^{(k)} = \min\{d_{ij}^{(k-1)}, d_{ik}^{(k-1)} + d_{kj}^{(k-1)}\} \tag{1}$$

where $d_{ij}^{(k)}$ represents the shortest distance from node i to node j passing only through intermediate nodes numbered no greater than k. (If two points are unreachable, the distance is infinity)

2) Linear Programming Model: Based on the fundamental theory of operations research [1], [8], let x_{ijk} be the quantity of crude oil type k transported from oil transportation station i to enterprise j (Unit: 10^4 tons), and d_{ij} be the shortest distance from station i to enterprise j (Unit: km).

Objective Function: Minimize total transportation cost

$$\min Z = \sum_{i \in S} \sum_{j \in E} \sum_{k \in O} d_{ij} \cdot x_{ijk} \tag{2}$$

Constraints:

(1) Supply constraint: The outflow of each type of crude oil from each station does not exceed its inventory

$$\sum_{j \in E} x_{ijk} \leq Stock_{ik}, \quad \forall i \in S, k \in O \tag{3}$$

(2) Demand constraint: The supply of each type of crude oil to each enterprise meets its demand

$$\sum_{i \in S} x_{ijk} \geq Demand_{jk}, \quad \forall j \in E, k \in O \tag{4}$$

(3) Non-negativity constraint:

$$x_{ijk} \geq 0, \quad \forall i, j, k \tag{5}$$

Combining the above objective function and constraints, the standard form of the linear programming model for the crude oil transportation optimization problem is:

$$\begin{aligned} \min \quad & Z = \sum_{i \in S} \sum_{j \in E} \sum_{k \in O} d_{ij} \cdot x_{ijk} \\ \text{s.t.} \quad & \sum_{j \in E} x_{ijk} \leq Stock_{ik}, \quad \forall i \in S, k \in O \\ & \sum_{i \in S} x_{ijk} \geq Demand_{jk}, \quad \forall j \in E, k \in O \\ & x_{ijk} \geq 0, \quad \forall i \in S, j \in E, k \in O \end{aligned} \tag{6}$$

where $S = \{S1, S2, S3, S4, S5, S6\}$ is the set of oil transportation stations, $E = \{E1, E2, E3, E4\}$ is the set of enterprises, and $O = \{O1, O2, O3, O4, O5, O6\}$ is the set of crude oil types.

3) Solution Method: The above linear programming problem is solved using MATLAB's `linprog` function [7]. This function employs the Dual-Simplex algorithm, which can efficiently solve large-scale linear programming problems.

C. Solution Results

After optimization, the optimal transportation scheme yields a total cost of **39,400 units of transportation cost** (10^4 tons·km). Figure 2 shows the visualization of the optimal transportation scheme.

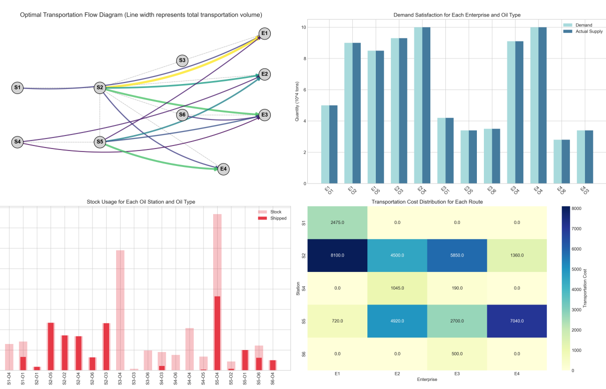

Fig. 2. Visualization of optimal transportation scheme

D. Result Analysis

From the visualization results in Figure 2, the following analysis can be drawn:

(1) Transportation path selection: The optimization results show that the transportation scheme fully utilizes the shortest path principle. For example, station S2, due to its central geographical location, becomes an important oil supply hub, supplying crude oil to multiple enterprises. Station S3 mainly serves enterprise E1 because the direct distance S3-E1 is the shortest (200km).

(2) Inventory utilization efficiency: The inventory at each oil transportation station is reasonably utilized. Some types of crude oil at certain stations are completely shipped out (100% utilization), while others have surplus, depending on the comprehensive trade-off between enterprise demand and transportation cost.

(3) Demand satisfaction: All enterprise crude oil demands are fully satisfied, verifying the correctness of the model constraints. The supply of each type of crude oil received by each enterprise exactly equals its demand.

(4) Cost composition analysis: Of the total cost of 39,400 units, long-distance transportation (such as the S1-S2-E4 route with a total distance of 700km) contributes a larger proportion of the cost, while short-distance transportation (such as S3-E1, 200km) has relatively lower cost. This indicates that in actual operations, optimizing network topology and shortening critical path distances has significant economic value.

(5) Model limitations: The basic model assumes that the transportation process is static and unidirectional, without considering time factors, pipeline capacity limitations, valve switching constraints, and other practical engineering issues. These limitations will be addressed in the extended model.

II. Extended Section: Modeling and Solution

A. Introduction: From Theory to Engineering Practice

As the "arteries" of modern energy systems, the quality of crude oil pipeline scheduling schemes directly determines the continuity and economy of chemical production. However, the basic model oversimplifies physical constraints and cannot explain the necessity of redundant facilities (such as backup berths and bidirectional pipelines) in the network. To bridge the gap between theoretical models and engineering practice, this paper conducts in-depth extensions based on the basic transportation problem, introducing logical mutual exclusion constraints and cyclic steady-state constraints, with a focus on exploring the system's robustness under extreme conditions [4], [5].

B. Mathematical Expression of Mutual Exclusion Constraints

In actual oil transportation hubs (such as node S2), the physical manifold structure determines that the same pump station often cannot simultaneously pump oil in two opposite directions. To precisely describe this physical limitation in the mathematical model, we introduce binary decision variables $y_t \in \{0, 1\}$.

Using the classic Big-M Method [6], we transform logical mutual exclusion into algebraic linear inequalities. For example, for two groups of oil transportation paths A and B that need to be mutually exclusive, we construct constraints:

$$\sum_{e \in A} x_{e,t} \leq M \cdot y_t \tag{7}$$

$$\sum_{e \in B} x_{e,t} \leq M \cdot (1 - y_t) \tag{8}$$

where M is a sufficiently large positive number. This modeling approach ensures that the scheduling scheme strictly complies with physical valve operation specifications.

C. Cyclic Steady-State Constraints

To prevent the optimization algorithm from performing "short-sighted" optimization using a limited time window, cyclic boundary constraints must be introduced. We force the inventory level I_{24} at time $t = 24$ to equal the initial inventory I_0 at time $t = 0$. Although this constraint significantly compresses the feasible solution space and increases solving difficulty, it ensures that the generated schedule can be executed in an infinite loop on a daily basis.

III. Robustness Analysis Under Extreme Conditions

Under basic parameter settings, the optimization solver tends to use only the lowest-cost main pipeline, causing the higher-cost S4 berth and bidirectional pipelines to remain idle for extended periods. To explore the potential value of these redundant designs, we designed a set of "stress test" experiments.

A. Stress Scenario Construction

We constructed an extreme condition combination called the "Perfect Storm": 1. The main oil supply line (S1 to S2) experiences a sudden failure, with capacity dropping from 60,000 tons/hour to 20,000 tons/hour. 2. The storage tank area of the core hub S2 is restricted, with the inventory upper limit compressed from 400,000 tons to 15,000 tons.

This scenario aims to simulate the catastrophic moment when "external supply interruption" and "internal buffer failure" occur simultaneously.

B. Backup Activation Mechanism of S4 Berth

Experimental results show that under the pressure of main pipeline obstruction, the S4 berth, which was previously neglected due to poor economics, is automatically activated by the system. The cumulative daily oil supply reaches 204,000 tons, effectively compensating for the supply gap left by the S1 line. This result proves that the S4 berth is not a useless "sunk cost" but an indispensable "hot backup" for the system to cope with main pipeline failures.

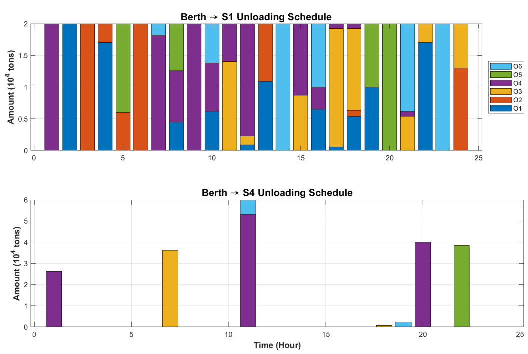

Fig. 3. Activation of S4 berth under stress test: S4 (bottom figure) is no longer idle and undertakes critical oil supply tasks.

C. Reverse Backflow Effect of Bidirectional Pipelines

An even more remarkable phenomenon occurs in the scheduling of bidirectional pipelines. Due to the extreme compression of S2 hub's storage capacity, facing sudden nighttime oil consumption peaks, the model demonstrates remarkable intelligence: it uses off-peak periods to reversely transport oil to neighboring S3 and S6 stations for temporary storage (as shown in the reverse flow during hours 1-6 in Figure 4), and withdraws it during peak periods.

Our data monitoring captured the flow from S3→S2 and S6→S2. This "reverse backflow" mechanism successfully resolves the inventory crisis at node S2 by utilizing the remaining storage capacity of neighboring nodes to construct a virtual "external reservoir."

Meanwhile, Figure 5 shows the dramatic fluctuations in inventory at each node. The inventory curve of S2 oscillates at extremely low levels for extended periods, proving that the system indeed maintains balance through precise scheduling at the edge of its limits.

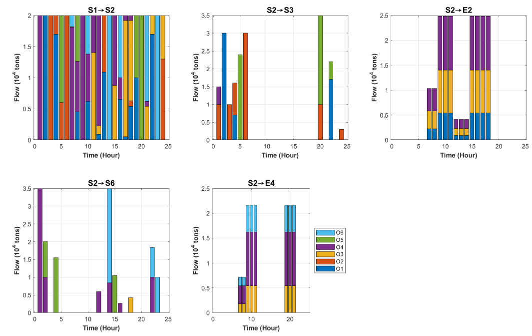

Fig. 4. Key pipeline flow analysis: Under extreme conditions, due to inventory and capacity limitations, the pipeline network exhibits complex dynamic flow patterns (including reverse backflow).

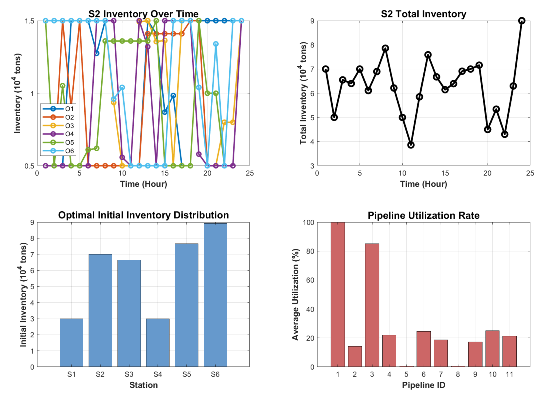

Fig. 5. Operating status of hub S2 under extreme inventory constraints: Inventory consistently hovers at low levels, maintaining supply through rapid turnover.

IV. Conclusion

Through extended modeling and stress testing of the crude oil pipeline scheduling problem, this paper not only achieves a physically feasible engineering scheduling scheme but also deeply explores the intrinsic value of redundant designs in pipeline network systems. Research shows that the seemingly redundant S4 berth and complex mutually exclusive bidirectional pipelines are actually "fuses" for the system to cope with uncertainty and sudden failures.

Appendix A
Thinking Process: From a Textbook Exercise to Engineering Reality

Looking back at the research journey of this project, we went through three stages: from "textbook exercise" to "engineering modeling," and then to "system-level reflection."

A. Phase 1: Establishment of Basic Model

The original problem we first encountered (see Appendix B.1 for details) was a standard minimum cost flow problem with simple network structure and single constraints. We solved for the optimal scheduling that meets demand using

MATLAB. However, this "idealized" solution appeared too thin when facing the complexity of the real world.

B. Phase 2: Introduction of Engineering Constraints (Course Presentation Version)

To make the model more "realistic," we introduced numerous practical constraints during the course presentation phase:

- Considering that pump stations cannot simultaneously pump oil in both directions, **mutual exclusion constraints** were added.
- Considering the continuity of chemical production, **24-hour cyclic boundaries** were added.
- To increase scheduling flexibility, we designed **bidirectional pipelines** between S2 and S3/S6.

This version of the model is more rigorous mathematically and has more engineering flavor.

C. Phase 3: Deep Reflection and Further Upgrade (Post-Course Refinement)

In the review after completing the course presentation, we discovered an "abnormal" phenomenon in the solution:

1) **Why has the S4 berth always been idle?**
2) **Why has reverse backflow never occurred in bidirectional pipelines?**

This made us realize that under normal parameters, since the S1 path has the lowest cost and S2 has sufficient storage capacity, the optimization algorithm naturally chose the simplest "single main line + local stockpiling" strategy. In this case, the bidirectional pipelines and S4 berth we painstakingly designed became mere decorations.

To verify that these designs are not redundant, we conducted a new round of **stress test upgrades** after the presentation. We adjusted model parameters (reduced core node capacity, created bottlenecks) to simulate extreme scenarios of main pipeline failure and inventory restrictions. It was in this version that the S4 berth and reverse backflow were finally "activated," proving their key value as system safety redundancy. This process gave us a deep understanding that: **Robustness often comes at the cost of sacrificing economy, and optimization models are precisely the tools for balancing the two.**

APPENDIX B
PROBLEM DESCRIPTION DETAILS

A. Original Problem Description

Crude oil transportation is an important component of petrochemical industrial production. The preliminary planning problem aims to minimize transportation costs. Network parameters are shown in Table IV.

The crude oil demand of each enterprise (regardless of time period) and the initial inventory of each storage and transportation station are as shown in the original assignment data, and the detailed numerical matrices are not repeated here.

B. Extended Problem Description

To align with engineering practice, we conducted comprehensive refinement based on the original problem, constructing a more complex dynamic scheduling environment.

TABLE IV
ORIGINAL PROBLEM PIPELINE NETWORK PARAMETERS

Pipeline	Dist.(km)	Pipeline	Dist.(km)
S1-S2	300	S2-S3	250
S3-E1	200	S4-S5	350
S5-S2	150	S2-E4	400
S2-S6	250	S2-E2	450
S6-E3	200		

1) Time-Varying Demand Patterns: Each enterprise no longer has constant demand but has specific production rhythms, as shown in Table V.

TABLE V
TIME DISTRIBUTION CHARACTERISTICS OF ENTERPRISE DEMAND

Enterprise	Production Period Characteristics
E1	**Stable type**: Uniform production 24 hours a day (4.17% per hour)
E2	**Daytime type**: Concentrated at 9-11h (10%) and 15-18h (8%), shutdown at
E3	**Nighttime type**: Concentrated at 1-6h and 22-24h (about 11%), shutdown d
E4	**Bimodal type**: Double peaks at 9-11h and 19-21h (12%)

2) Extended Version Total Demand Table: We updated the crude oil demand of enterprises to adapt to the new scenario settings (Unit: 10^4 tons):

TABLE VI
TOTAL DEMAND BY OIL TYPE FOR EACH ENTERPRISE

Enterprise	O1	O2	O3	O4	O5	O6	Total
E1	6	8	0	0	6	0	20
E2	5	0	8	10	0	0	23
E3	0	0	0	8	4	5	17
E4	0	0	4	8	0	4	16

3) Key Constraint Set:

1) **Mutual Exclusion Constraints**: Physical limitations for the S2 hub.
   - **Group A**: S2→S3 and S2→E2 cannot operate simultaneously.
   - **Group B**: S2→S6 and S2→E4 cannot operate simultaneously.
2) **Cyclic Steady State**: $I_{i,k,24} = I_{i,k,0}$, ensuring sustainable production.
3) **Asymmetric Oil Supply**: S4 berth cannot handle O1/O2 crude oil.
4) **Stress Test Parameters** (applied only in Phase 3):
   - S1→S2 pipeline capacity reduced from 6.0 to 2.0 (simulating failure).
   - S2 inventory upper limit reduced from 40.0 to 1.5 (simulating storage capacity restriction).

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