Assignment 5

For a constrained nonlinear programming problem

$$\begin{aligned} \max \quad & f(x) \\ \text{s.t.} \quad & g_i(x) \geq 0, i = 1, ...m \\ & x \in \mathbb{R}^n \end{aligned}$$

what is condition that a feasible ascent direction at a point $x_0 \in \mathbb{R}^n$ must satisfy when (1) x_0 is in the interior of the feasible domain and (2) x_0 is on the boundary of the feasible domain, respectively.

Firstly, We can find the condition for feasible ascent direction of the original condition:

$$\boldsymbol{\exists p \in R^n, \alpha_0 \geq 0, \forall \alpha \in [0, \alpha_0], g_i(x_0 + \alpha p) \geq 0} \text{ and } \boldsymbol{f(x_0 + \alpha p) > f(x_0)}$$

Using Taylor expansion, we can get:

$$f(x_0 + \alpha p) = f(x_0) + \alpha \nabla^T f(x_0) p + o(\alpha), f(x_0 + \alpha p) - f(x_0) = \alpha \nabla^T f(x_0) p + o(\alpha)$$
$$g_i(x_0 + \alpha p) = g_i(x_0) + \alpha \nabla^T g_i(x_0) p + o(\alpha), g_i(x_0 + \alpha p) - g_i(x_0) = \alpha \nabla^T g_i(x_0) p + o(\alpha)$$

For 2 cases, we can get sufficient conditions separately.

1 $\mathbf{x_0}$ is in the interior of the feasible domain

Because x_0 is in the interior, so we can get:

$$g_i(x_0) > 0$$

If α_0 is small enough, any direction p will satisfy the feasible condition

$$g_i(x_0 + \alpha p) - g_i(x_0) = \alpha \nabla^T g_i(x_0) p + o(\alpha)$$

In order to satisfy the ascent condition

$$f(x_0 + \alpha p) > f(x_0)$$

We should let

$$\boldsymbol{\nabla^T f(x_0) p > 0}$$

This is the sufficient condition of this case.

2 $\mathbf{x_0}$ is on the boundary of the feasible domain

Because x_0 is on the boundary, so we can get:

$$g_i(x_0) = 0, i = 1, 2, ..., m$$

In order to satisfy the feasiable condition

$$g_i(x_0 + \alpha p) \geq 0, i = 1, 2, ..., m$$

We should let

$$\nabla^T g_i(x_0)p \geq 0$$

In order to satisfy the ascent condition

$$f(x_0 + \alpha p) > f(x_0)$$

We should let

$$\nabla^T f(x_0)p > 0$$

So we can get the sufficient condition of this case:

$$\begin{aligned} &\mathbf{\nabla^T f(x_0) p > 0} \\ &\mathbf{\nabla^T g_i(x_0) p \geq 0, i = 1, 2, ..., m} \end{aligned}$$