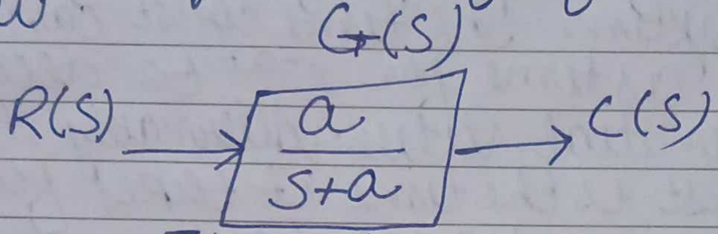
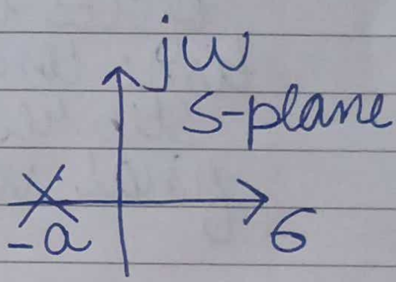
Q1 Short answers

a Name the performance specification for first order systems.

Ans The performance specification for first order system is the time constant.
A first order system without zeros can be described by the transfer function shown below.



First order system



Poleplot

Taking input as unit step and taking inverse laplace transform we get output

$$C(t) = C_f(t) + C_n(t) = 1 - e^{-at}$$

We call $1/a$ the time const of the response.
The time const can be described as the time for e^{-at} to decay to 37% of its initial value.

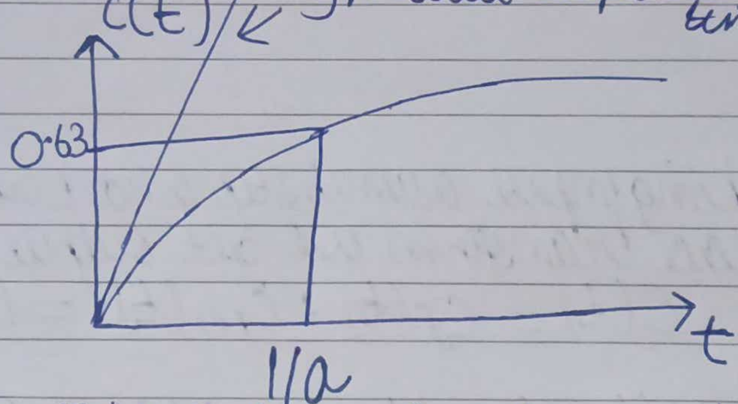
5 What does the performance specification for a first order system tell us?

Ans We know total response of first order system is $c(t) = c_f(t) + c_n(t) = 1 - e^{-at}$

$\frac{1}{a}$ called time constant of the response

is the performance specification of a first order system. The time const can be defined as time for e^{-at} to decay 37% of its initial value. Alternately, the time const is the time it takes for the step response to rise to 63% of its final value

Initial slope = $\frac{1}{\text{time const}} = a$

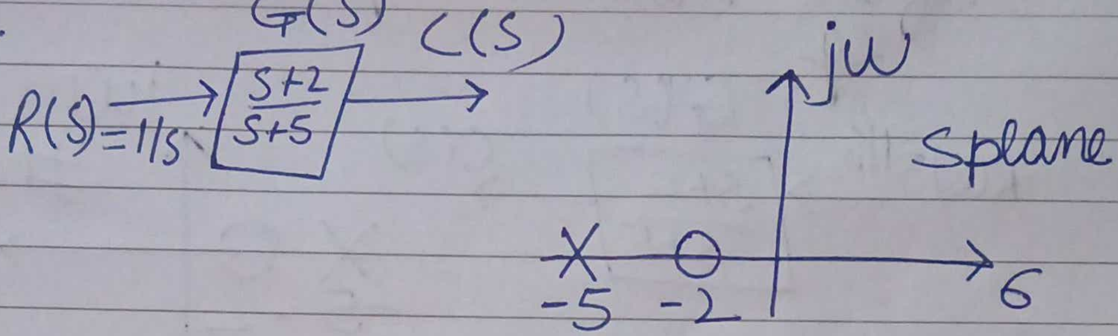


First order system response to a unit step.

C In a system with an input and an output, what poles generate the steady-state response?

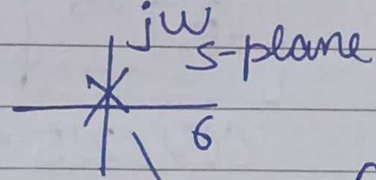
Ans Steady state response also called as forced response. A pole of the input function generates the form of the forced response.

eg



Input pole

$\frac{1}{s}$



Output transform $C(s) = \frac{2/5}{s} + \frac{3/5}{s+4}$

Output time response $c(t) = \frac{2}{5} + \frac{3}{5} e^{-5t}$

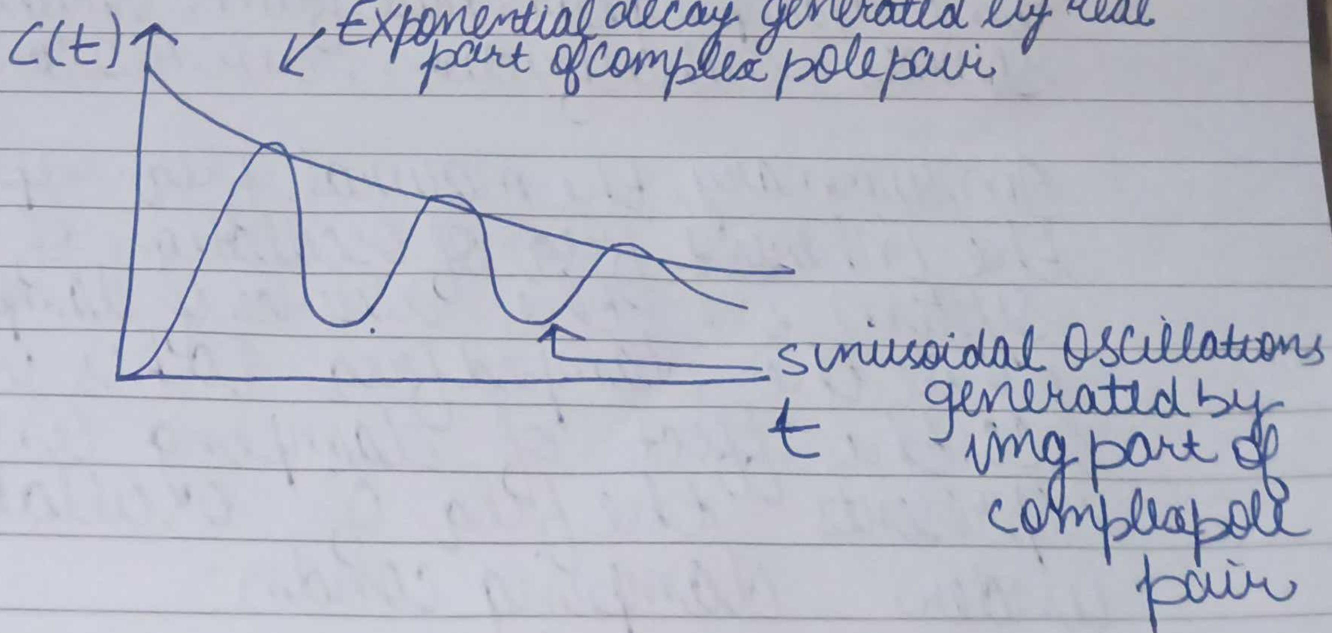
Forced response   Natural response

e The imaginary part of a pole generates what part of a response?

Ans The imaginary part of the pole matches the freq of the sinusoidal oscillation. While as the input pole at the origin generates the constant forced response, the system poles on the imaginary axis generate a sinusoidal natural response whose freq is equal to the location of the imaginary poles.

f The real part of a pole generates what part of a response?

Ans The real part of a pole generates a decay envelope. The real part of the pole matches the exponential decay frequency of the sinusoid's amplitude.



Q What is the difference b/w the natural freq and the damped freq of oscillation?

Ans The natural freq refers to the inherent freq at which a system oscillates in the absence of any external forces or damping. It is determined solely by the system's properties, such as its mass, stiffness. When a system is set into motion and left to oscillate freely, it vibrates at its natural freq. i
The natural freq is the characteristic prop of the system and remains const.

The damped freq, also known as the damped oscillation freq, takes into account the effect of damping on the system's oscillation. Damping refers to the dissipation of energy within the system which causes the amplitude of oscillation to decrease over time. Damping can arise from various sources, such as friction, air resistance. ii

In summary, the natural freq represents the inherent freq of oscillation of a system in the absence of damping, while the damped freq takes into acc the effect of damping and represents the freq of oscillation under damping condn.

$c(t) = 1 - \frac{1}{\sqrt{1-\xi^2}} e^{-\xi\omega_n t}\cos(\omega_n$

h If a pole is moved with const imaginary part what will responses have in common?

Ans Their damped freq of oscillation will be the same. As the img part signifies that the response contains oscillations. Hence const imag part tells their freq of oscillation is the same.

I If a pole is moved with a const real part, what will the responses have in common?

Ans They will all exist under the same exponential decay.

j If a pole is moved along a radial line extending from the origin, what will the responses have in common?

Ans They will all ~~the~~ have the same percent overshoot and the same shape although differently scaled in time.

K List 5 specifications for a second order underdamped system.

Ans Response of a second order underdamped system is $c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}} e^{-\zeta \omega_n t} \cos(\omega_n \sqrt{1-\zeta^2}\, t - \phi)$

A plot of this response for various values of ζ, plotted along a time axis normalized to the natural freq shows lower the value of ζ, the more oscillatory the response.

The natural freq is a time axis scale factor and doesnot affect the nature of the response other than to scale it in time.

Some other specifications other ζ and ω_n are:-

1. Rise Time:- T_r, Time reqd for the waveform to go from 0.1 of the final value to 0.9 of the final value.

2. Peak Time:- T_P, The time reqd to reach the first, or max, peak.

3. %OS:- The amount that the waveform overshoots the steady-state, or final, value at the peak time, expressed as a %age of the steady state value.

For above question, how many specifications completely determines the response?

The nature of the response obtained was related to ζ. Variations of damping ratio alone yield the complete range of overdamped, critically damped, underdamped and undamped responses.

m What pole locations characterize ① the underdamped system, ② the overdamped system ③ the critically damped system?

Ans Underdamped - For this response, there are 2 complex poles that comes from the system.
Overdamped - There are 2 real poles that come from the system.
Critically damped - There are 2 multiple real poles that come from the system.

m Name two conditions under which a response generated by a pole can be neglected?

Ans 1. If a pole is near to zero, then it can be assumed as a pole zero cancellation and response can be neglected.

2. If the pole location is far from the dominant poles then its response can be neglected as the exponential term dies away fast.

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Q How can you justify pole zero cancellation?

Ans Pole zero cancellation is a phenomenon that occurs in control systems and signal processing when a pole and a zero of transfer function coincide, resulting in their cancellation.

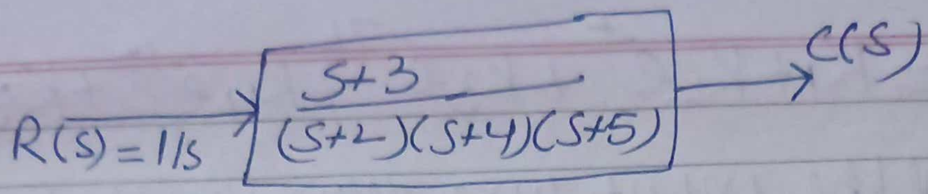
In control systems the location of poles determine the stability of the system. If a system has unstable poles, it will exhibit undesirable behaviour such as oscillations or divergance. This cancellation can lead to improved stability margins and robustness against disturbances or uncertainities.

Q2 The transfer function of the system is given by the eqn $\dfrac{(S+3)}{(S+2)(S+4)(S+5)}$

Find the unit step response of the system (in time domain) specifying forced and natural responses?

Ans Laplace transform of unit step response $= \dfrac{1}{S}$

i.e $R(S) = 1/S$



We know $T(S) = \dfrac{C(S)}{R(S)}$

$$\Rightarrow C(S) = T(S)R(S) = \frac{1}{S}\,\frac{(S+3)}{(S+2)(S+4)(S+5)}$$

Each system pole generates an exponential as part of the natural response. The input's pole generates the forced response.

$$C(S) \equiv \underbrace{\frac{K_1}{S}}_{\text{Force response}} + \underbrace{\frac{K_2}{S+2} + \frac{K_3}{S+4} + \frac{K_4}{S+5}}_{\text{Natural response}}$$

Taking inverse Laplace transform we get

$$C(t) \equiv \underbrace{K_1}_{\text{Forced Response}} + \underbrace{K_2 e^{-2t} + K_3 e^{-4t} + K_4 e^{-5t}}_{\text{Natural response}}$$

Q3 The transfer func of the system is given by $\dfrac{10(S+4)(S+6)}{(S+1)(S+7)(S+8)(S+10)}$. Write by inspection the output for a unit step input.

Ans $C(t) = K_1 + K_2 e^{-t} + K_3 e^{-7t} + K_4 e^{-8t} + K_5 e^{-10t}$

Poles of the input func determine the form of the forced response and poles of the transfer function determines the form of the natural response.

Q4 A system has a transfer func = $\frac{50}{s+50}$

Find the time const T_c, settling time T_s and rise time T_r?

Ans $G(s) = \frac{a}{s+a}$ · Comparing we have $a=50$

We know time const $= 1/a = 1/50 = 0.02$

Rise Time, $T_r = \frac{2.31}{a} - \frac{0.11}{a} = \frac{2.2}{a} = \frac{2.2}{50}$

$= 0.044$

Settling Time $T_s = 4/a = \frac{4}{50} = 0.08$

Q6 What is the step response of the system with transfer function $\frac{200}{s^2+10s+200}$

Ans Laplace transform of input $= 1/s$

$$R(S) = 1/S \longrightarrow \boxed{\frac{200}{S^2+10S+200}} \longrightarrow C(S)$$

The poles of the transfer func can be found as $s^2+10s+200=0$

$$S = \frac{-10 \pm \sqrt{100-800}}{2} = \frac{-10 \pm \sqrt{700}\,i}{2}$$

$$= -5 \pm 13.23i$$

The real part -5 is the exponential freq for the damping. It is also the reciprocal of the time constant of the decay of oscillations. The imag part 13.23 is the radian freq ~~t~~ for the sinusoidal oscillations.

$$C(t) = K_1 + e^{-5t}(K_2 \cos 13.23t + K_3 \sin 13.23t)$$

$C(t)$ is a const plus an exponentially damped sinusoid.

Q7 Write by inspection the general form of step response:

a $F(S) = \dfrac{400}{S^2+12S+400}$

Sol poles of the transfer function

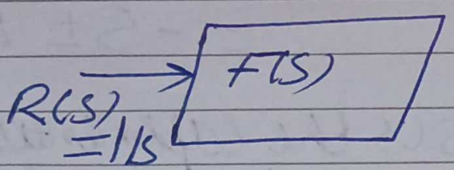
$$S^2+12S+400=0$$

$$S = \frac{-12 \pm \sqrt{144-1600}}{2} = \frac{-12 \pm j\sqrt{1456}}{2}$$

$$= \frac{-12 \pm j\,38.2}{2}$$

$$= -6 \pm 19j$$

underdamped response



$$\Rightarrow C(t) = \underset{\text{for } 1/S}{A} + Be^{-6t}\cos(19.01t+\phi)$$

b $F(S) = \dfrac{900}{S^2+90S+900}$

Sol poles of the transfer func

$$S^2+90S+900=0$$

$$S = \frac{-90 \pm \sqrt{8100-3600}}{2} = \frac{-90 \pm 67.08}{2}$$

$= \frac{-90+67.08}{2}, \frac{-90-67.08}{2}$

$= -11.46, -78.54$

Overdamped response

$c(t) = A + Be^{-78.54t} + Ce^{-11.46t}$

c $F(s) = \frac{225}{s^2+30s+225}$

Sol Poles of the transfer func

$s^2+30s+225=0$

$s = \frac{-30 \pm \sqrt{900-900}}{2} = \frac{-30}{2} = -15$

Critically damped system

$c(t) = A + Be^{-15t} + Cte^{-15t}$

d $F(s) = \frac{625}{s^2+625}$

Sol Poles $s^2+625=0$

$s^2 = -625$

$\Rightarrow s = 25j$

Undamped responses

$\Rightarrow c(t) = A + B\cos(25t+\phi)$

Q8 For a given transfer function find ξ, and w_n = $\frac{36}{S^2+4.2S+36}$?

Sol Comparing with the standard 2nd order system

$$G(S) = \frac{w_n^2}{S^2+2\xi w_n S+w_n^2}$$

we have $w_n^2 = 36$

$\Rightarrow w_n = 6$

Also $2\xi w_n = 4.2$

$\Rightarrow \boxed{\xi = 0.35}$

Q9 Characterize the type of response for various values of ξ, for a unit step input.

a $12/S^2+8S+12$

Ans Comparing with the standard eqn we have $a = 2\xi w_n$ & $w_n = \sqrt{b}$

$\xi = \frac{a}{2\sqrt{b}}$ $\qquad$ $b=12$, $a=8$

Substituting value we get $\xi = 1.155$

Over damped since $\xi > 1$

b $\frac{16}{S^2+8S+16}$

$\xi = \frac{a}{2\sqrt{b}}$ Here $b=16$, $a=8$ $\Rightarrow \xi = 1$

Hence Critically damped system

$S = -90 \pm \sqrt{8100-3600}$

Q10 Find T_p, T_s, T_r & %OS for a TF $\frac{100}{S^2+15S+100}$?

sol We know $T_p = \frac{\pi}{\omega_n\sqrt{1-\xi^2}}$

Here $\omega_n^2 = 100 \Rightarrow \omega_n = 10$

$2\xi\omega_n = 15 \Rightarrow \xi = \frac{15}{2\times10} = 0.75$

$\Rightarrow T_p = \frac{\pi}{10\sqrt{1-(0.75)^2}} = \frac{\pi}{10 \times 0.661} = 0.475$

$T_s = \frac{4}{\xi\omega_n} = \frac{4}{0.75\times10} = .533$

$\cancel{T_\xi} = \%OS = e^{-(\xi\pi\sqrt{1-\xi^2})} \times 100$

$= 2.838$