

# CS 525: Advanced Database Organization

## 06: Even more index structures

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Slides: adapted from a [course](#) taught by  
[Hector Garcia-Molina](#), Stanford InfoLab

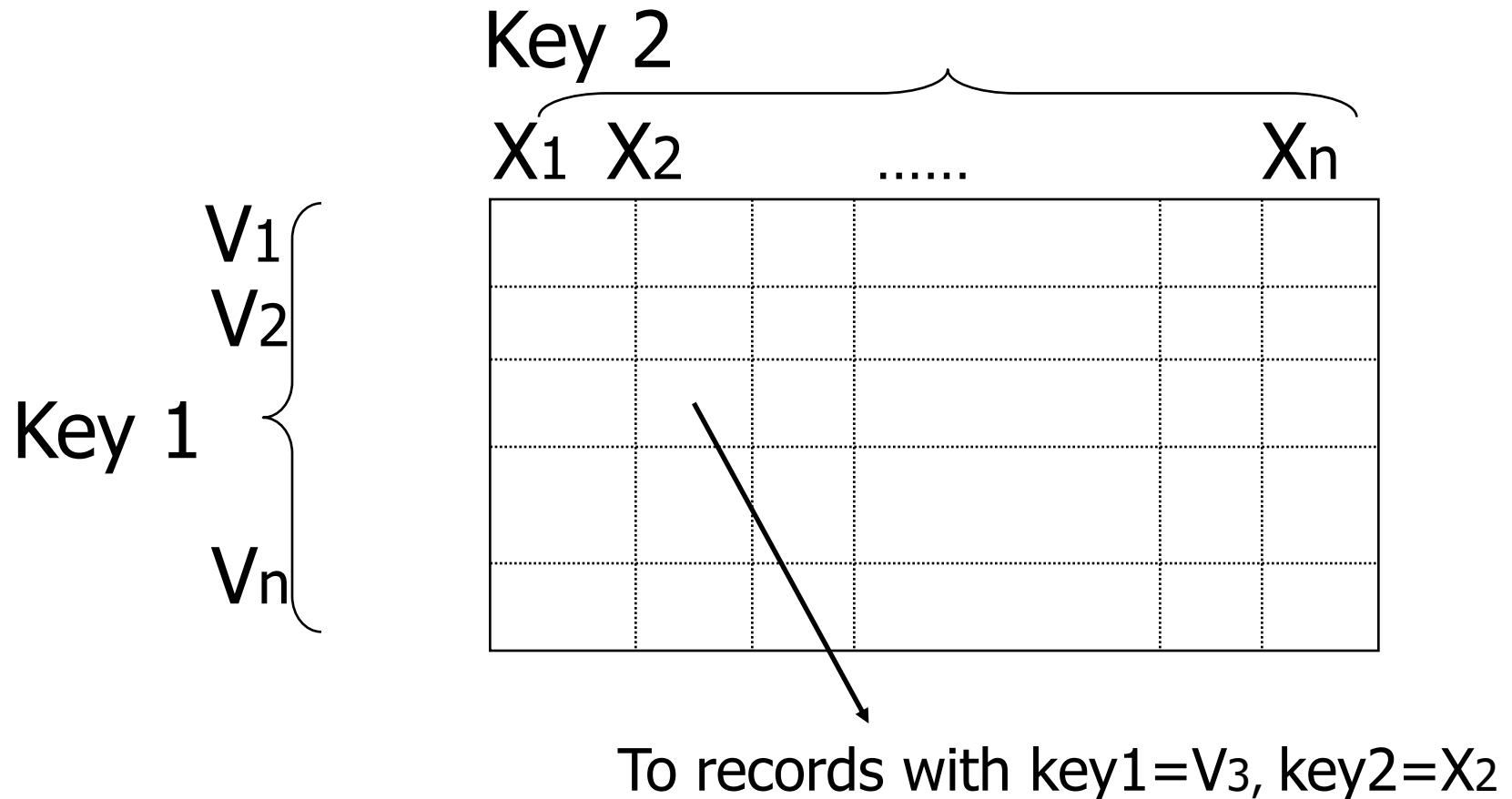
# Recap

- We have discussed
  - Conventional Indices
  - B-trees
  - Hashing
  - Trade-offs
  - Multi-key indices
  - Multi-dimensional indices
    - ... but no example

# Today

- **Multi-dimensional index structures**
  - kd-Trees (very similar to example before)
  - **Grid File (Grid Index)**
  - Quad Trees
  - **R Trees**
  - **Partitioned Hash**
  - ...
- **Bitmap-indices**
- **Tries**

# Grid Index



# CLAIM

- Can quickly find records with
  - key 1 =  $V_i \wedge$  Key 2 =  $X_j$
  - key 1 =  $V_i$
  - key 2 =  $X_j$

# CLAIM

- Can quickly find records with
  - key 1 =  $V_i \wedge$  Key 2 =  $X_j$
  - key 1 =  $V_i$
  - key 2 =  $X_j$
- And also ranges....
  - E.g.,  $\text{key 1} \geq V_i \wedge \text{key 2} < X_j$

- How do we find entry  $i,j$  in linear structure?

$\text{pos}(i, j) =$

max number of  
i values  $N=4$

$i, j$	
0, 0	position $S+0$
0, 1	position $S+1$
0, 2	position $S+2$
0, 3	position $S+3$
1, 0	position $S+4$
1, 1	
1, 2	
1, 3	
2, 0	
2, 1	position $S+9$
2, 2	
2, 3	
3, 0	

- How do we find entry  $i,j$  in linear structure?

max number of  
i values  $N=4$

$$\text{pos}(i, j) = S + iN + j$$

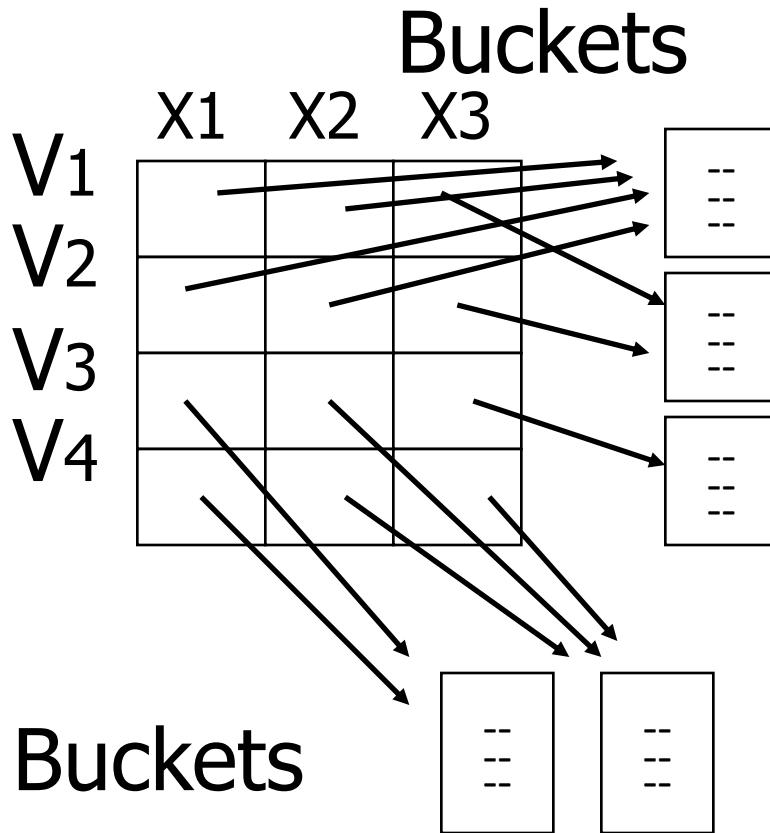
Issue: Cells must be same size,  
and N must be constant!



Issue: Some cells may overflow,  
some may be sparse...

$i, j$	position $S+0$
0, 0	position $S+1$
0, 1	position $S+2$
0, 2	position $S+3$
0, 3	position $S+4$
1, 0	
1, 1	
1, 2	
1, 3	
2, 0	
2, 1	position $S+9$
2, 2	
2, 3	
3, 0	

# Solution: Use Indirection



\*Grid only  
contains  
pointers to  
buckets

# With indirection:

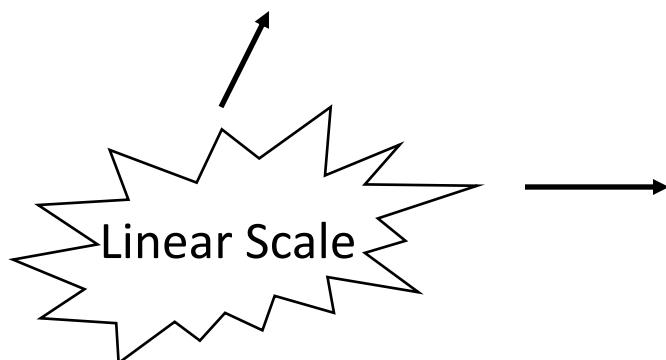
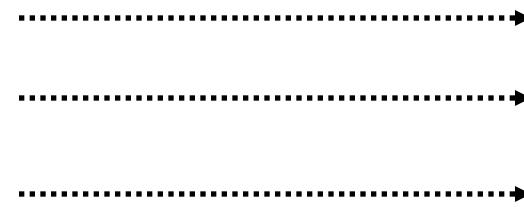
- Grid can be regular without wasting space
- We do have price of indirection

# Can also index grid on value ranges

Salary

0-20K	1
20K-50K	2
50K- $\infty$	3

Grid

1	2	3
Toy	Sales	Personnel

# Grid files

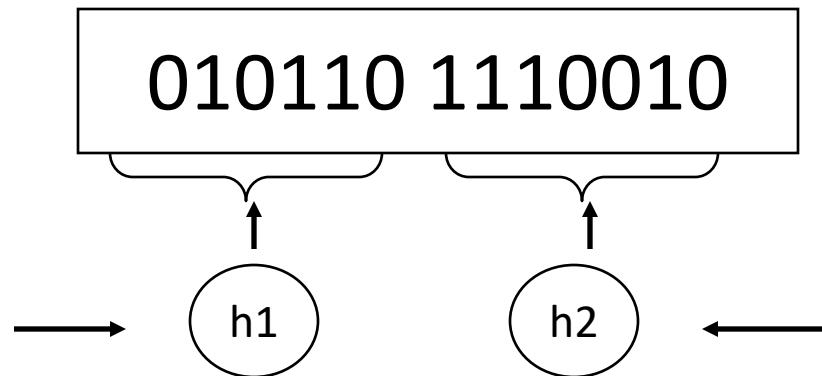
- Good for multiple-key search
- Space, management overhead  
(nothing is free)
- Need partitioning ranges that evenly split keys

# Partitioned hash function

Idea:

Key1

Key2



# EX:

$h_1(\text{toy}) = 0$

$h_1(\text{sales}) = 1$

$h_1(\text{art}) = 1$

.

$h_2(10k) = 01$

$h_2(20k) = 11$

$h_2(30k) = 01$

$h_2(40k) = 00$

.

Insert

$\langle \text{Fred}, \text{toy}, 10k \rangle, \langle \text{Joe}, \text{sales}, 10k \rangle$   
 $\langle \text{Sally}, \text{art}, 30k \rangle$

000
001
010
011
100
101
110
111

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 $\langle \text{Sally}, \text{art}, 30k \rangle$

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001	<Fred>
010	
011	
100	
101	<Joe><Sally>
110	
111	

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.

Find Emp. with Dept. = Sales  $\wedge$  Sal=40k

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110	<Tom><Bill>
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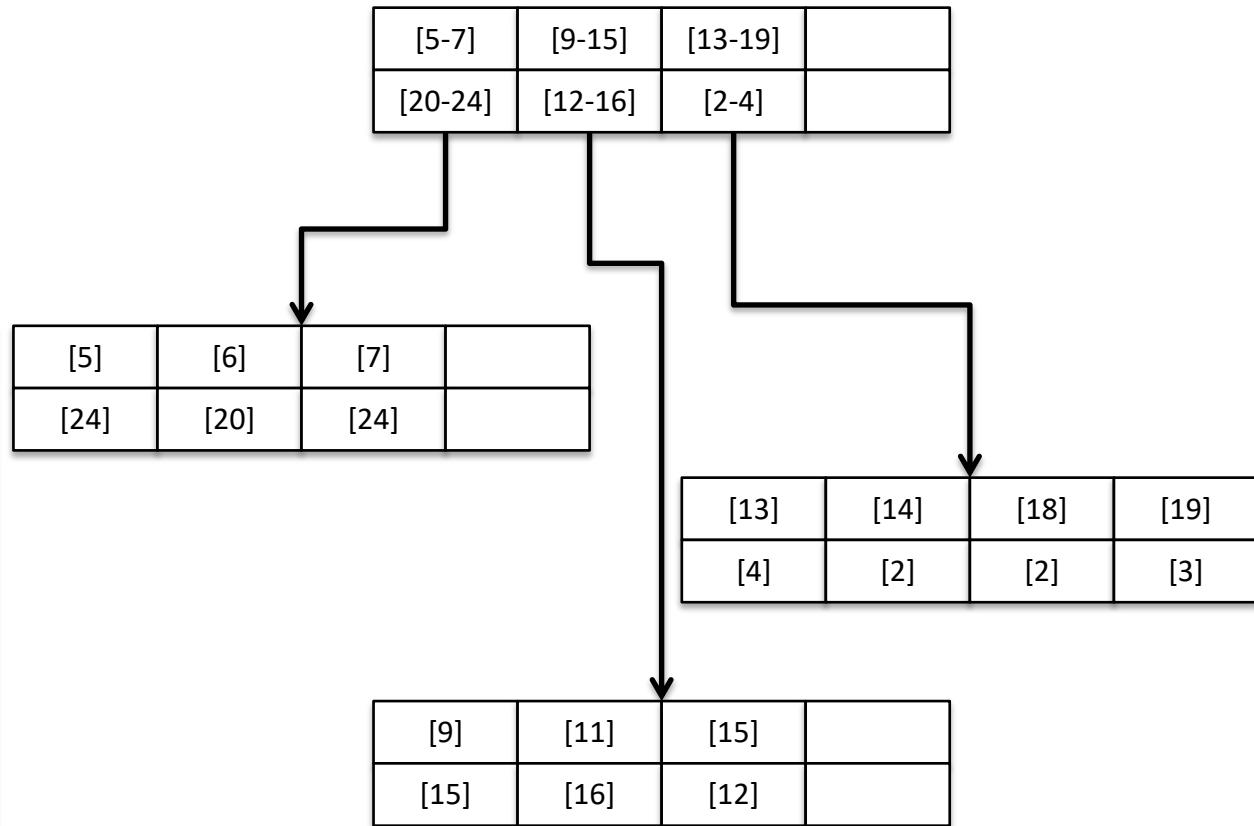
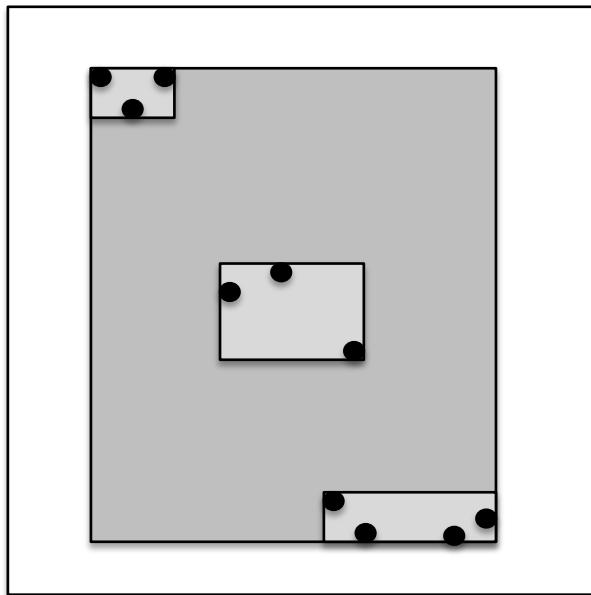
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100	<Sally>
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110	
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# R-tree

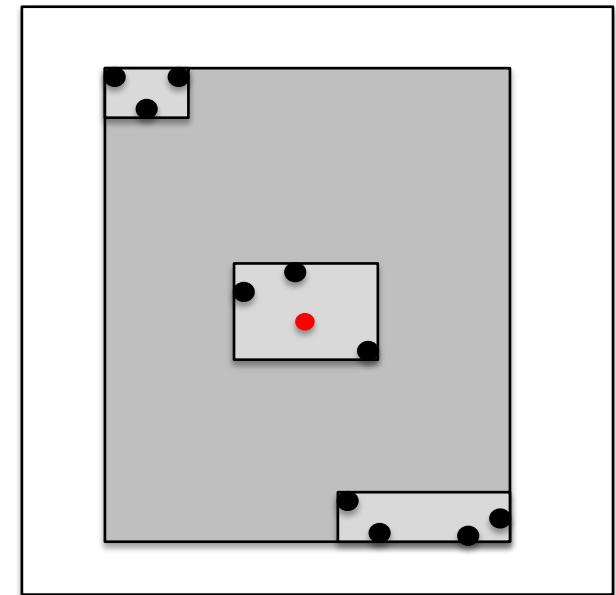
- Nodes can store up to **M** entries
  - Minimum fill requirement (depends on variant)
- Each node rectangle in **n**-dimensional space
  - Minimum Bounding Rectangle (MBR) of its children
- MBRs of siblings are allowed to overlap
  - Different from B-trees
- balanced

Data Space



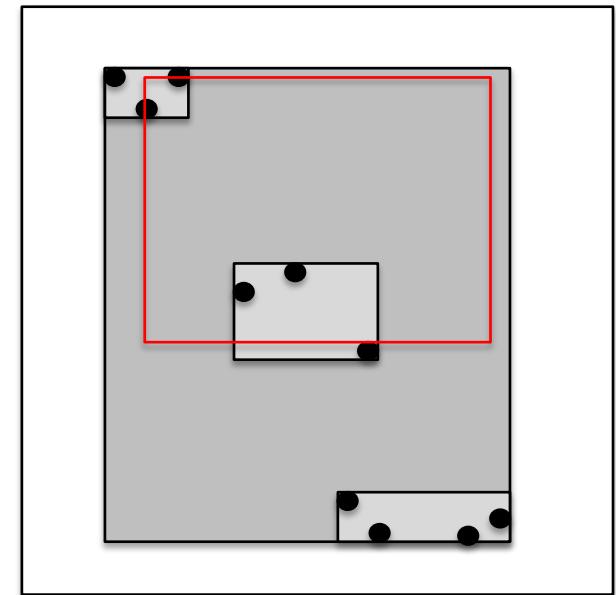
# R-tree - Search

- Point Search
  - Search for  $p = \langle x_i, y_i \rangle$
  - Keep list of potential nodes
    - Needed because of overlap
  - Traverse to child if MBR of child contains  $p$



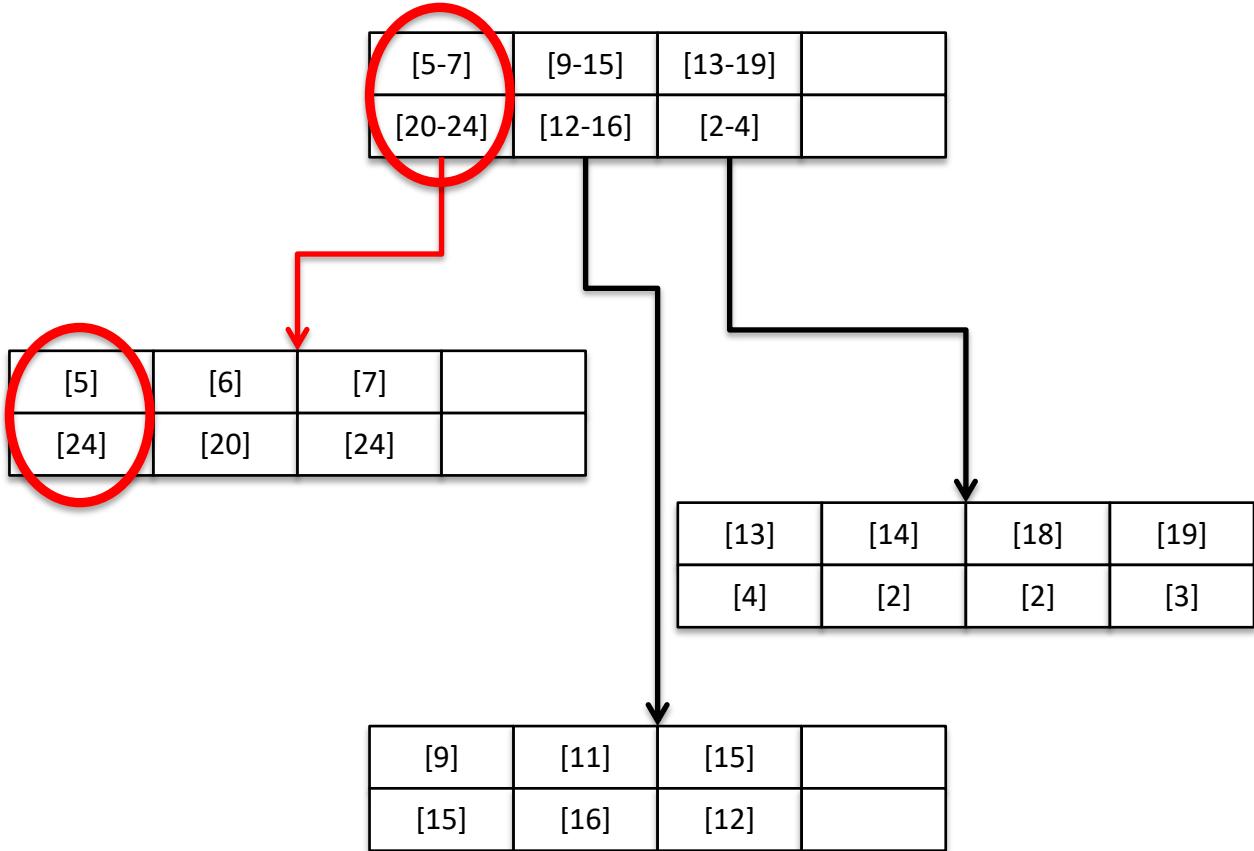
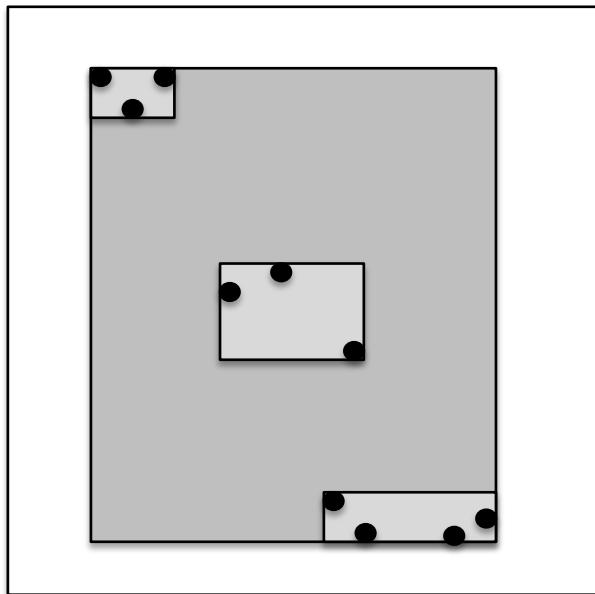
# R-tree - Search

- Point Search
  - Search for points in region =  $\langle [x_{\min} - x_{\max}], [y_{\min} - y_{\max}] \rangle$
  - Keep list of potential nodes
  - Traverse to child if MBR of child overlaps with query region



Search <5,24>

Data Space



# R-tree - Insert

- Similar to B-tree, but more complex
  - Overlap -> multiple choices where to add entry
  - Split harder because more choice how to split node (compare B-tree = 1 choice)
- 1) Find potential subtrees for current node
  - Choose one for insert (heuristic, e.g., the one that would grow the least)
  - Continue until leaf is found

# R-tree - Insert

- 2) Insert into leaf
- 3) Leaf is full? -> split
  - Find best split (minimum overlap between new nodes) is hard ( $O(2^M)$ )
  - Use linear or quadratic heuristics (original paper)
- 4) Adapt parents if necessary

# R-tree - Delete

- 1) Find leaf node that contains entry
- 2) Delete entry
- 3) Leaf node underflow?
  - Remove leaf node and cache entries
  - Adapt parents
  - Reinsert deleted entries

# Bitmap Index

- Domain of values  $D = \{d_1, \dots, d_n\}$ 
  - Gender {male, female}
  - Age {1, ..., 120?}
- Use one vector of bits for each value
  - One bit for each record
    - 0: record has different value in this attribute
    - 1: record has this value

# Bitmap Index Example

Age			Todlers			Gender	
1	2	3	Name	Age	Gender	male	female
1	0	0	Peter	1	male	1	0
0	1	0	Gertrud	2	female	0	1
1	0	0	Joe	1	male	1	0
0	0	1	Marry	3	female	0	1

# Bitmap Index Example

Age			Todlers			Gender	
1	2	3	Name	Age	Gender	male	female
1	0	0	Peter	1	male	1	0
0	1	0	Gertrud	2	female	0	1
1	0	0	Joe	1	male	1	0
0	0	1	Marry	3	female	0	1

Find all todlers with age **2** and sex **female**:

Bitwise-and between vectors



# Bitmap Index Example

Age			Todlers			Gender	
1	2	3	Name	Age	Gender	male	female
1	0	0	Peter	1	male	1	0
0	1	0	Gertrud	2	female	0	1
1	0	0	Joe	1	male	1	0
0	0	1	Marry	3	female	0	1

Find all todlers with age **2 or sex female**:  
Bitwise-or between vectors

0
1
0
1

# Compression

- Observation:
  - Each record has one value in indexed attribute
  - For  $N$  records and domain of size  $|D|$ 
    - Only  $1/|D|$  bits are 1
  - -> waste of space
- Solution
  - Compress data
  - Need to make sure that **and** and **or** is still fast

# Run length encoding (RLE)

- Instead of actual 0-1 sequence encode length of 0 or 1 runs
- One bit to indicate whether 0/1 run + several bits to encode run length
- But how many bits to use to encode a run length?
  - Gamma codes or similar to have variable number of bits

# RLE Example

- 0001 0000 1110 1111                   **(2 bytes)**
- 3, 1,4,       3, 1,4                   **(6 bytes)**
- -> if we use one byte to encode a run we have  
7 bits for length = max run length is 128(127)

# Elias Gamma Codes

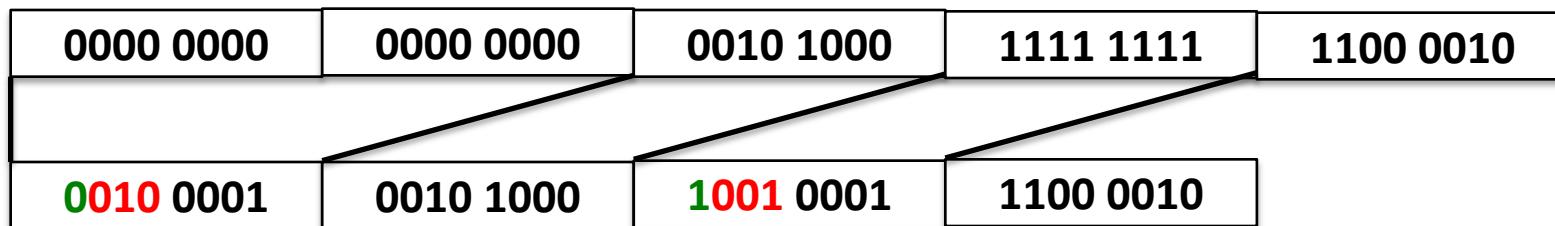
- $X = 2^N + (x \bmod 2^N)$ 
    - Write N as N zeros followed by one 1
    - Write  $(x \bmod 2^N)$  as N bit number
  - $18 = 2^4 + 2 = 000010010$
  - 0001 0000 1110 1111 (2 bytes)
  - 3, 1,4, 3, 1,4 (6 bytes)
  - 0111 0010 0011 1001 00 (3 bytes)

# Hybrid Encoding

- Run length encoding
  - Can waste space
  - And/or run length not aligned to byte/word boundaries
- Encode some bytes of sequence as is and only store long runs as run length
  - EWAH
  - BBC (that's what Oracle uses)

# Extended Word aligned Hybrid (EWAH)

- Segment sequence in machine words (64bit)
- Use two types of words to encode
  - Literal words, taken directly from input sequence
  - Run words
    - $\frac{1}{2}$  word is used to encode a run
    - $\frac{1}{2}$  word is used to encode how many literals follow



# Bitmap Indices

- Fast for read intensive workloads
  - Used a lot in datawarehousing
- Often build on the fly during query processing
  - As we will see later in class

# Trie

- From Retrieval
- Tree index structure
- Keys are sequences of values from a domain D
  - $D = \{0,1\}$
  - $D = \{a,b,c,\dots,z\}$
- Key size may or may not be fixed
  - Store 4-byte integers using  $D = \{0,1\}$  (32 elements)
  - Strings using  $D=\{a,\dots,z\}$  (arbitrary length)

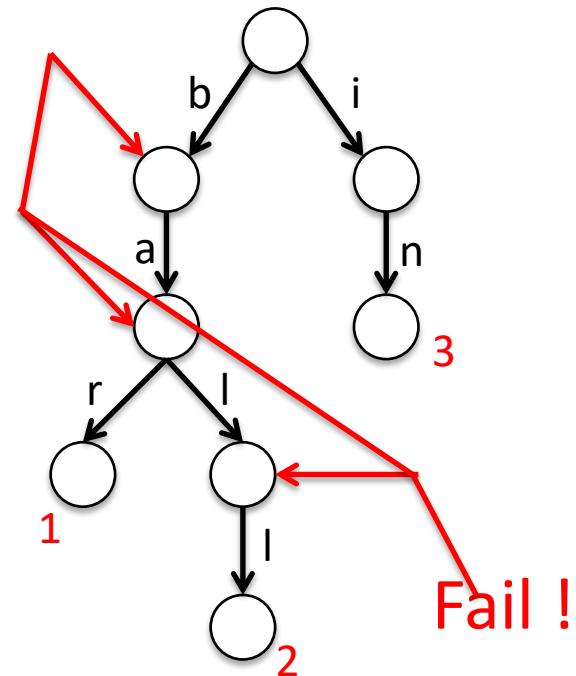
# Trie

- Each node has pointers to  $|D|$  child nodes
  - One for each value of D
- Searching for a key  $k = [d_1, \dots, d_n]$ 
  - Start at the root
  - Follow child for value  $d_i$

# Trie Example

Words: bar, ball, in

Search for bald



# Tries Implementation

- 1) Each node has an array of child pointers
- 2) Each node has a list or hash table of child pointers
- 3) array compression schemes derived from compressed DFA representations

# Index structures in the Main Memory DBMS era

- Larger and large portions of the data fit into main memory
  - Disk I/O no longer the (only) bottleneck
  - Highly optimized and specialized operator code
    - Difference of the constant factor for full scan versus index increase
  - Increasing amounts of parallelism
    - Traditional methods for parallel access to indexes no longer effective enough
- => Do not use indexes anymore?

# Index structures in the Main Memory DBMS era

- Solutions
  - More Light-weight and coarse-grained data structures
  - Use data-structures that have less parallelization bottle-necks

# Index structures in the Main Memory DBMS era

- **Solutions**
  - More Light-weight and coarse-grained data structures, e.g.:
    - Data skipping (small materialized aggregates)
    - Database cracking
  - Use data-structures that have less parallelization bottle-necks, e.g.,
    - Skip lists
    - B<sup>w</sup>-trees

# Data skipping

- Consider a relation stored in an unsorted page file
  - Regular DBMS
  - HDFS parquet file
  - ...
- Main idea of data skipping
  - For each page store min/max values of each attribute
- To evaluate a selection predicate on attribute A say  $c1 \leq A \leq c2$ 
  - if for page P:  $A_{\max} < c1$  or  $A_{\min} > c2$  then none of the tuples on that page will qualify and we can skip reading this page

R		
A	B	C
a	1	10
	5	20
	2	10
d	2	35
e	3	45
f	4	40

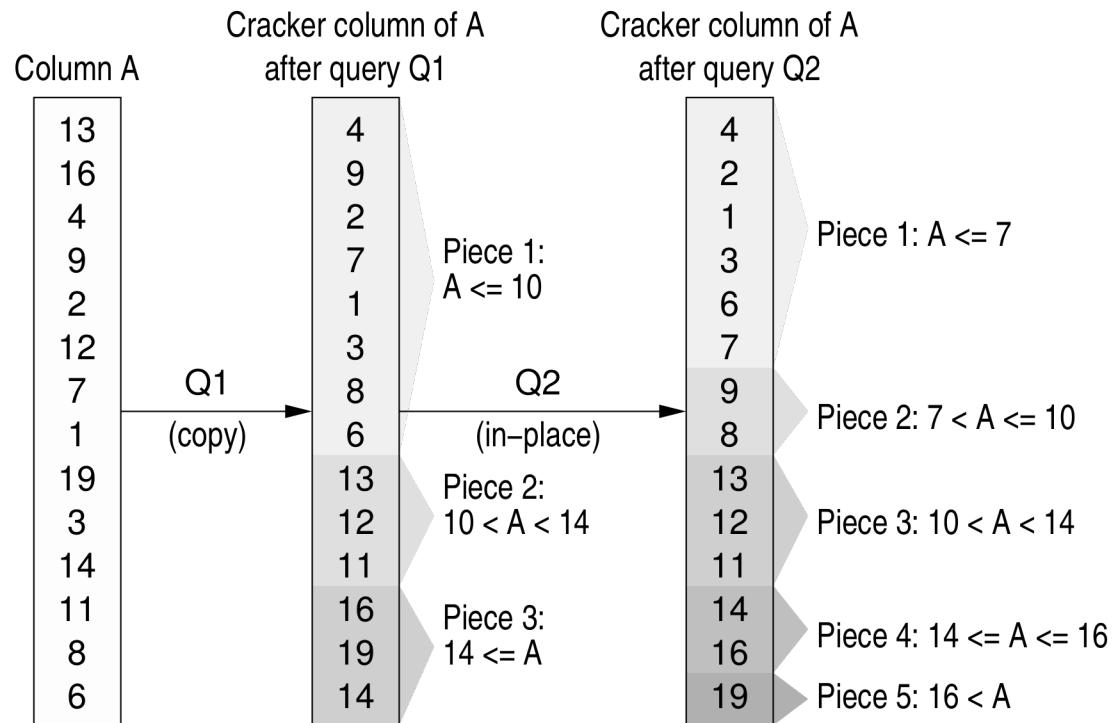
# Database cracking

- Main rationale
  - Originally designed for columnar databases
  - The amount of indexing effort we spend for a part of the key space should be based on how frequently this part of the keyspace is accessed
- Basic idea
  - Start with an unsorted file
  - Whenever a query applies a selection condition on a column A, say  $A < 50$ , then split the current partition containing 50 into two fragments one with data  $< 50$  and one with the remaining data (partial sort)
  - Keep a small in-memory tree index for these fragments

# Database cracking

Q1:  
select \*  
from R  
where R.A > 10  
and R.A < 14

Q2:  
select \*  
from R  
where R.A > 7  
and R.A >= 16

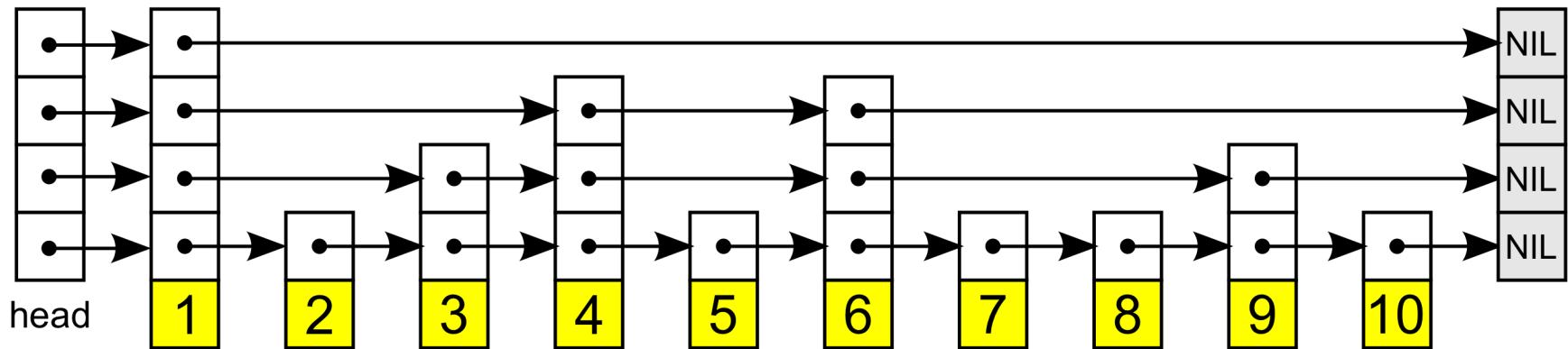


From **Database Cracking – CIDR 2007**

# Skip lists

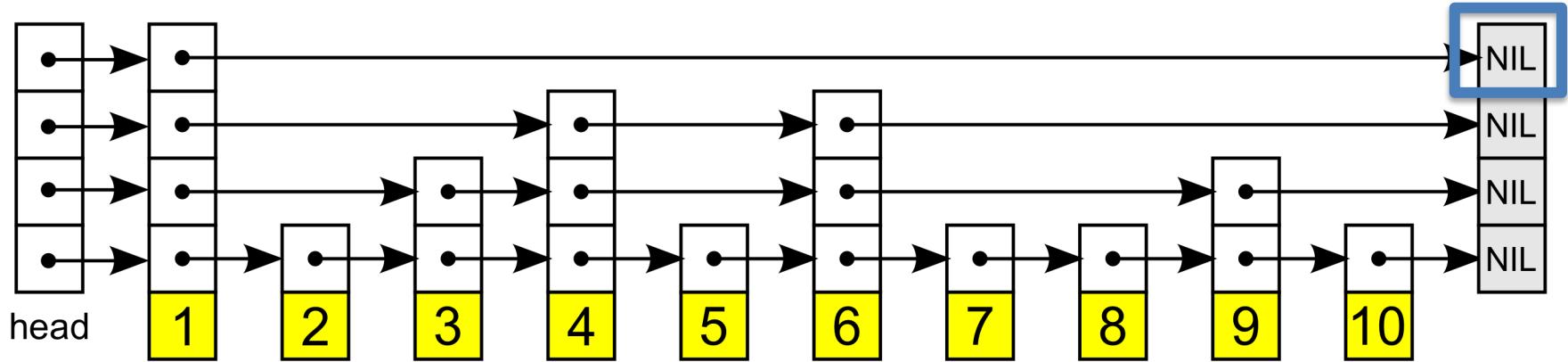
- Probabilistic datastructure
  - Behavior depends on randomization
  - Gives only probabilistic guarantees
    - => with high probability will guarantee good performance
  - Approximates a search tree using the much simpler (and easier to parallelize linked list datastructure)

# Skip lists



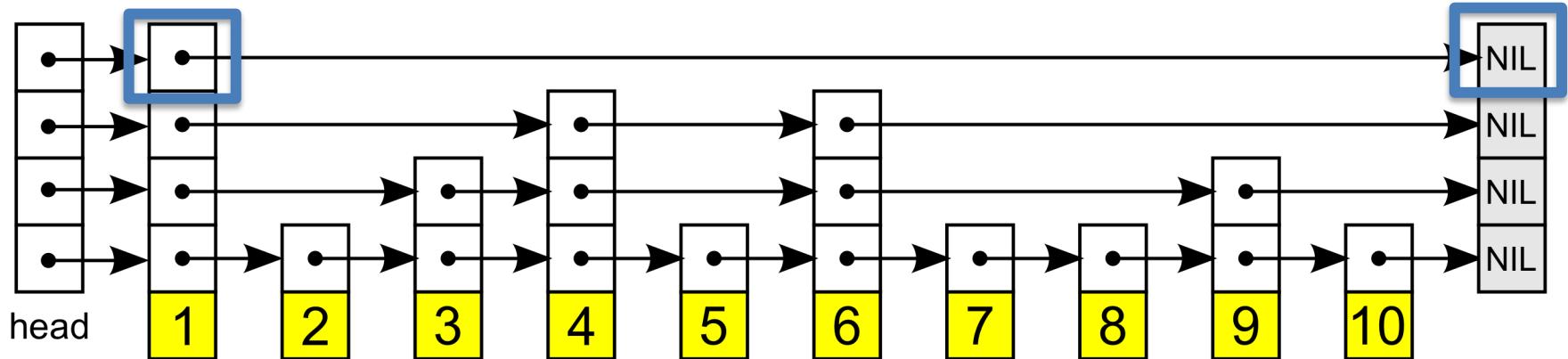
- **Search:**
  - Start from the top list
  - 1) Move through list until element is found or we are at a larger element/end of the list
  - 2) move to previous element (smaller than search key) and follow a down pointer to the next deeper level
  - 3) Goto 1)

# Skip lists



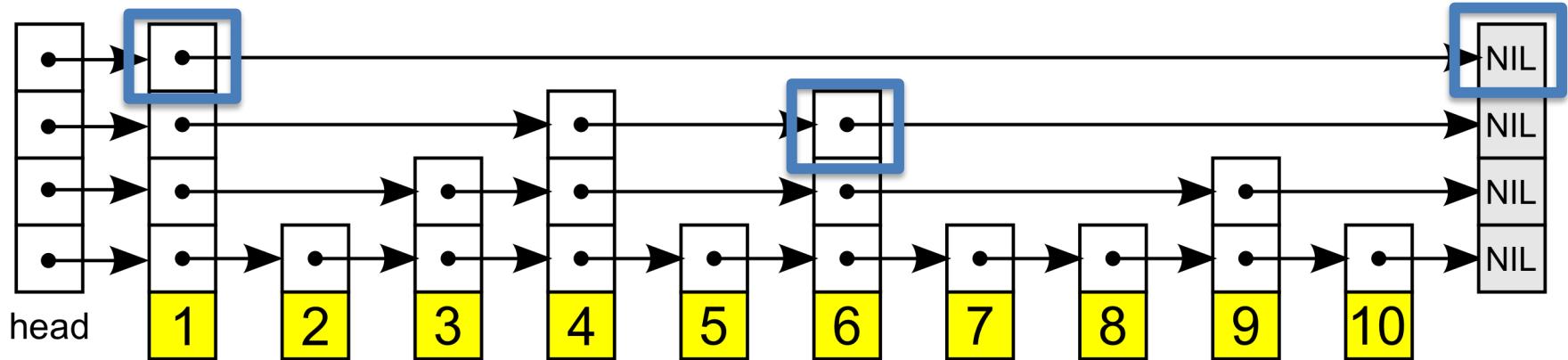
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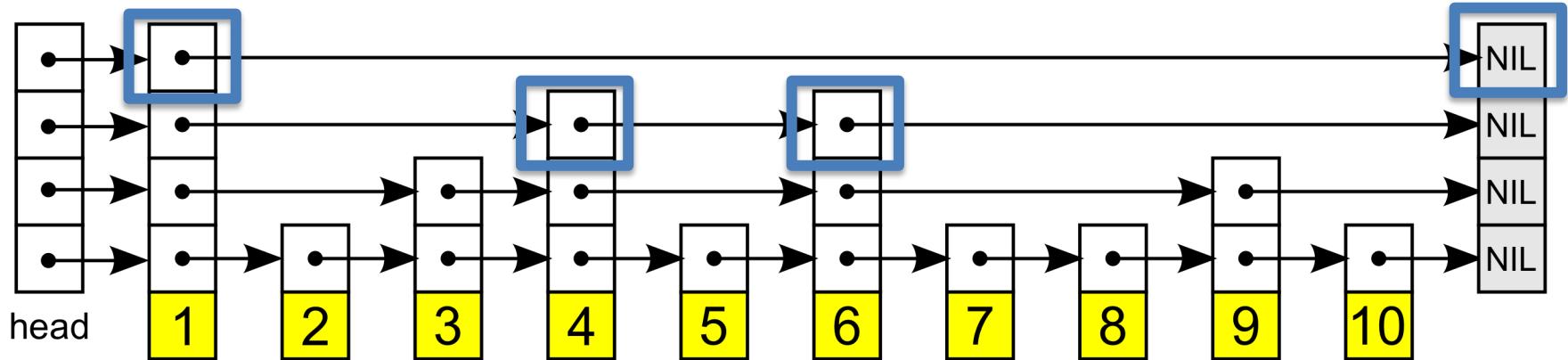
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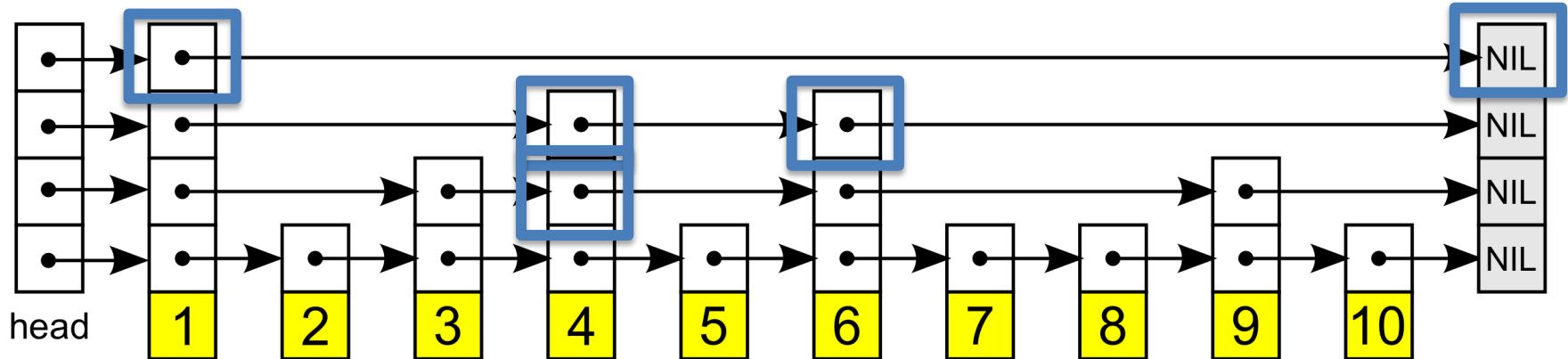
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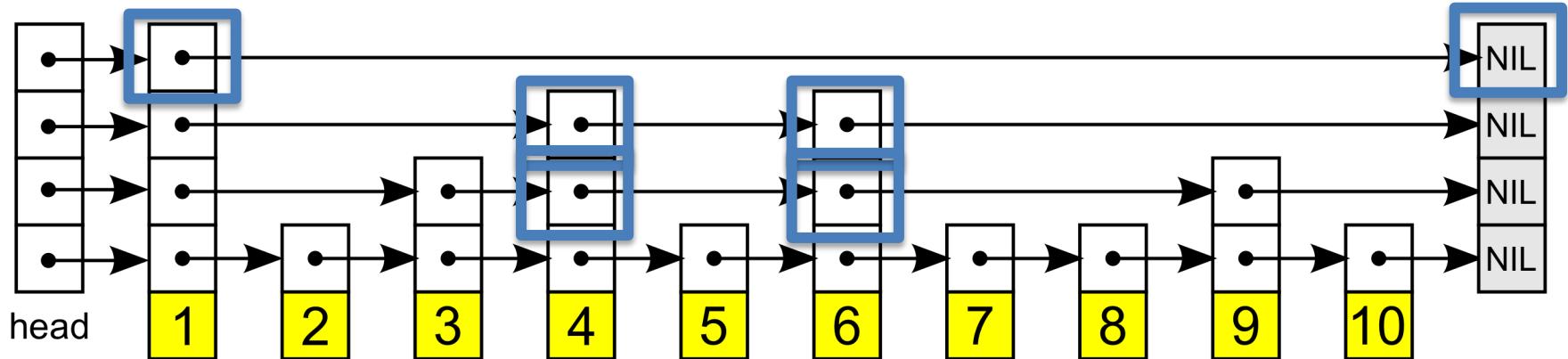
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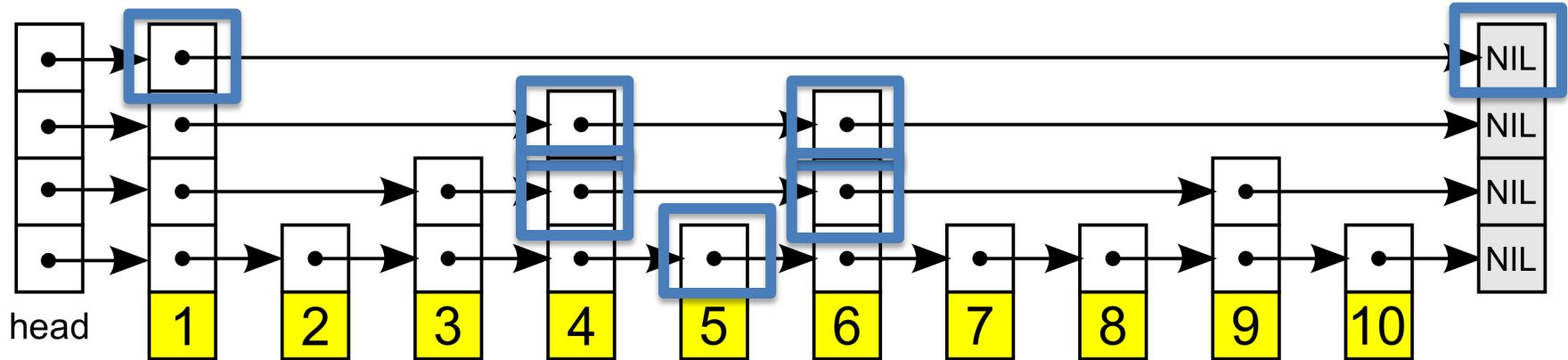
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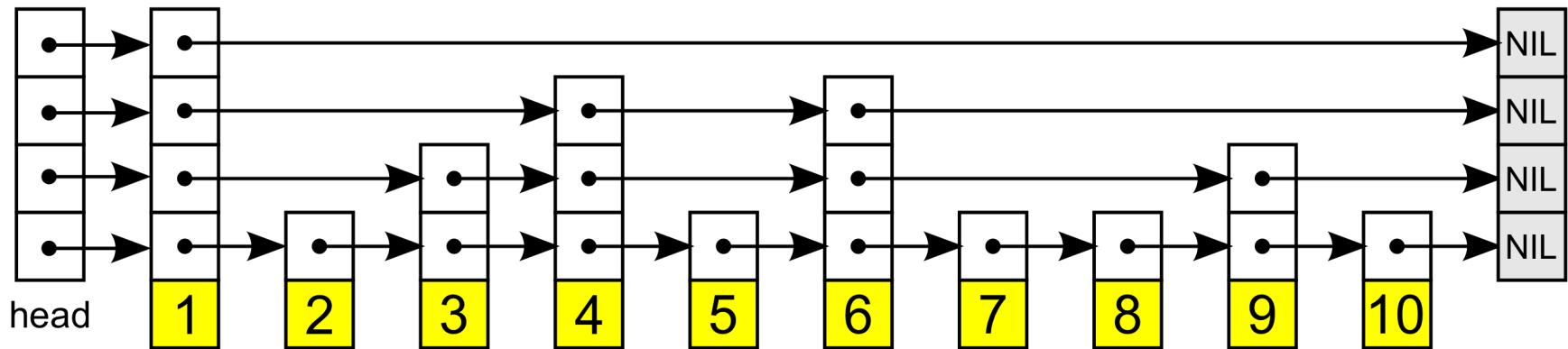
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# Skip lists



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# Skip lists



- **Insert:**
  - Use search to find insertion position at the lowest level (keep pointers at the higher levels)
  - Insert element in the lowest list
  - Then for every level throw a dice and insert key with probability  $p$  (typically  $\frac{1}{2}$ )

Observation: in expectation each level has  $p$  as many nodes as the next lower level

# Skip lists

- **Characteristics**
  - $O(\log(n))$  expected performance (insert, delete, search)
  - Easy to parallelize (linked lists)
  - Simpler to implement (also less CPU ops) than B-trees
- **Example implementations**
  - MemSQL (main memory database system)
  - Lucene
  - leveldb

# Improving insert/update performance

- B-tree
  - $O(\log(n))$  I/O
- Hash-index
  - $O(1)$  I/O, but potential reorg cost
- Consider Key-value store (e.g., Cassandra) application
  - Need fast write-throughput
  - Need fast point-lookup

# One Solution: LSM-trees

- **Log-structured merge (LSM) trees**
  - Have small index that is memory resident (**memtable**)
  - When memtable exceeds a size threshold write it as one sorted run to disk (will explain algorithm when talking about query execution)
    - Sequential I/O!
    - Runs are immutable after being written (exception compaction)
    - Runs may contain outdated values for keys that exist in newer runs of the memtable
    - Over time we have multiple sorted runs
  - **Inserts/Updates**
    - Always applied to memtable
  - **Lookup**
    - If we find a key in the memtable then return it
    - Otherwise lookup keys in the sorted runs in reverse chronological order

# LSM-trees

- **Performance**
  - Inserts/Updates
    - $O(1)!$
  - Lookup
    - $O(\#runs)$
    - $\Rightarrow$  want to make sure the number of runs does not grow indefinitely
- **Compaction**
  - Merge sorted runs on disks to reduce  $\#runs \Rightarrow$  improve lookup performance

# Basic Compaction

- Have levels
  - Once there are more than  $x$  runs on a level these are merged into one larger run
  - Run sizes increase exponentially per level
- E.g., threshold is 4 runs
  - first level: runs are of same size as memtable
  - 2<sup>nd</sup> level:  $4 * \text{size of memtable}$
  - 3<sup>rd</sup> level:  $4 * 4 * \text{size of memtable}$
  - ...

# LSM-trees

- **Other lookup improvements**
  - Block index in memory (similar to sparse index)
  - Bloomfilters
    - A bloom filter is a small over-approximation of set
      - Can be used to test if a key K is contained in a set S
        - » Returns yes, then the key **may** be in the set
        - » Returns no, then the key is guaranteed to not be in the set
      - => fast way to avoid looking at runs that are guaranteed to not contain a key

# Bw-trees

- **Motivation**
  - Improve concurrency properties of B-trees
  - Improve cache effectiveness of B-trees
  - Designed for flash-storage
    - Fast random/sequential reads
    - Fast sequential writes
    - Comparably slower random writes (albeit smaller factor)

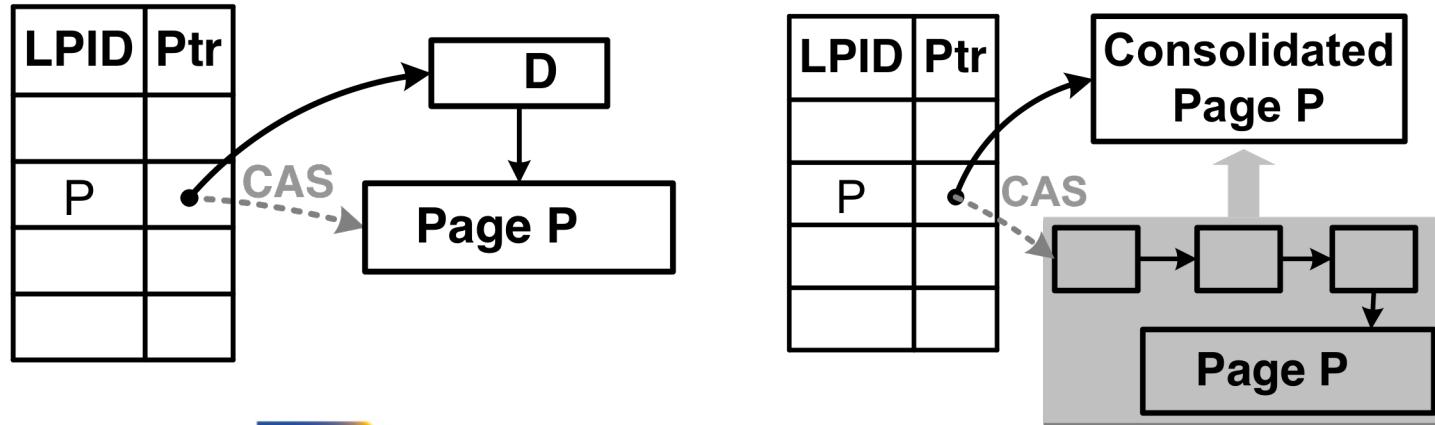
# Bw-trees

- **Overview**
  - Updateable B-tree without latches
    - Threads almost never block
      - => Improved instruction cache performance
  - Backed up by log-structured storage
  - Updates never modify pages but append deltas to a page
    - Deltas are “installed” using CAS (atomic compare and swap)

# Bw-trees

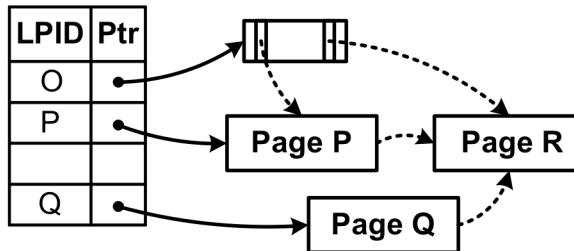
- **Mapping table**

- Pages are logically identified by a LPID which is stable
- Locations and size of pages can change over time
- Updates create a delta record that points to the previous address of the page
- The delta record's address is swapped for the current address in the mapping table using CAS

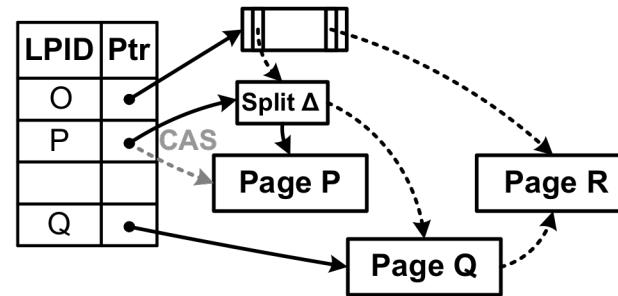


# Bw-trees

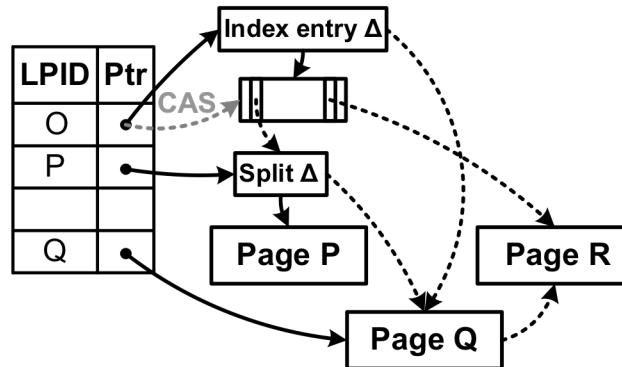
- Making page splits atomic



(a) Creating sibling page  $Q$



(b) Installing split delta



(c) Installing index entry delta

# Summary

## Discussion:

- Conventional Indices
- B-trees
- Hashing (extensible, linear)
- SQL Index Definition
- Index vs. Hash
- Multiple Key Access
- Multi Dimensional Indices
  - Variations: Grid, R-tree,
- Partitioned Hash
- Bitmap indices and compression
- Tries
- Database cracking
- Data skipping (small materialized aggregates/zone maps)
- Skip-lists
- Log-structured merge trees (LSM)