HW2 wf2099

1. Finding Memory bugs using valgrind

1. Run in CIMS Linux Server. 4 AMD EPYC Processor. 7.6G Memory.
2. GCC 8.2, OpenMP4.0

Val_test01

First Run:

```
valgrind ./val_test01_solved
==9586== Memcheck, a memory error detector
==9586== Invalid write of size 4
==9586== Invalid read of size 4
==9586== Mismatched free() / delete / delete []
==9586== ERROR SUMMARY: 3 errors from 3 contexts (suppressed: 0 from 0)
```

Two Bug:

```
    x = (int *)malloc((n + 1) * sizeof(int)); // It should allocate n+1
 space not n. Otherwise it would out of index.

    // delete[] x;
    // Here is the bug, it should be free, but not delete.
    free(x);
```

After fix Bug:

```
  ==10502== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
```

Val_test02

First Run

```
  ==12136== Memcheck, a memory error detector
  ==12136== Conditional jump or move depends on uninitialised value(s)
  ==12136== Use of uninitialised value of size 8
  ==12136== ERROR SUMMARY: 24 errors from 4 contexts (suppressed: 0 from 0)
```

One Bug, Initialize the array:

```
    // Here is the bug, it didn't initialize, but default is 0?
    for (int i = 0; i < 10; i++)
    {
        x[i] = 0;
    }
```

After fix Bug:

```
  ==12870== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
  [wf2099@access1 hw2]$
```

2. Optimizing matrix-matrix multiplication.

Comparing different loop arrangements for MMult0.

Performance (Dimension 1984)	n * k * m	m * n * k	k * m * n
time (s)	4.5	40.7	79.7
flops (Gflop/s)	3.4	0.38	0.13

Performance (Dimension 1984)	n * k * m	m * n * k	k * m * n
bandwidth (GB/s)	13.79	1.53	0.5
ERR	0	0	0

- Details are in the appendix.

I think the reason why the arrange has such a great impact on performance is the locality. Both Space and time locality. Cause when change i, you can find its nearby in cache.

```
double A_ip = a[i + p * m];
double B_pj = b[p + j * k];
double C_ij = c[i + j * m];
```

one level blocking scheme

Performance (Dimension 1984)	BS = 16	BS = 4	BS = 32
time (s)	1.55	2.77	1.76
flops (Gflop/s)	10	5.52	8.87
bandwidth (GB/s)	40	22.12	35.54
ERR	0	0	0

- Details are in the appendix. code:

```
void MMult1(long m, long n, long k, double *a, double *b, double *c)
{
    for (long j = 0; j < n; j += BLOCK_SIZE)
    {
        for (long p = 0; p < k; p++)
        {
            for (long i = 0; i < m; i += BLOCK_SIZE)
            {
                for (long jj = j; jj < j + BLOCK_SIZE; ++jj)
                {
                    for (long ii = i; ii < i + BLOCK_SIZE; ++ii)
                    {

                        double A_ip = a[ii + p * m];
                        double B_pj = b[p + jj * k];
                        double C_ij = c[ii + jj * m];
                        C_ij = C_ij + A_ip * B_pj;
                        c[ii + jj * m] = C_ij;
                    }
                }
            }
        }
```

```
        }
    }
```

When use OpenMP

```
  Dimension        Time      Gflop/s        GB/s          Error
       1984   13.307310     1.173721    4.701981 2.176428e+03
```

3. Fast Sin

I do this for sin4_vec(). result

```
  Reference time: 0.3226
  Taylor time:     1.1904      Error: 6.928125e-12
  Intrin time:     0.0021      Error: 2.454130e-03
  Vector time:     0.0022      Error: 6.928125e-12
```

code:

```cpp
  void sin4_vector(double *sinx, const double *x)
  {
    // The Vec class is defined in the file intrin-wrapper.h
    typedef Vec<double, 4> Vec4;
    Vec4 x1, x2, x3, x5, x7, x9, x11;
    x1 = Vec4::LoadAligned(x);
    x2 = x1 * x1;
    x3 = x1 * x2;
    x5 = x3 * x2;
    x7 = x5 * x2;
    x9 = x7 * x2;
    x11 = x9 * x2;

    Vec4 s = x1;
    s += x3 * c3;
    s += x5 * c5;
    s += x7 * c7;
    s += x9 * c9;
    s += x11 * c11;
    s.StoreAligned(sinx);
  }
```

4. Pipelining and optimization

x86_64, AMD EPYC Processor (with IBPB), CPU MHz: 2894.562

compute

optimize method	seconds	cycles/eval	Glop/s
O3	1.646260	4.775115	1.214629
O0	4.551409	13.199648	0.439405
02	1.563473	4.534315	1.279134
O1	4.923109	14.277421	0.406235

Compute vec

report

```
[wf2099@access1 lecture4]$ g++ —fopenmp —std=c++11 —O3 —march=native
compute—vec.cpp && ./a.out —n 1000000000
time = 1.631802
flop—rate = 4.902162 Gflop/s

time = 1.507671
flop—rate = 5.306070 Gflop/s

time = 1.510943
flop—rate = 5.294581 Gflop/s
```

The different is:

1. compute_fn0 is standard for loop.
2. compute_fn1 is vectorized using AVX intrinsics.
3. compute_fn2 is vectorized using Vec Class.

Compute vec pipeline

report

Compare different M

Observation

1. As M increase from 1 to 8, both AVS intrinsics and Vec class method have significant improve. I think is the multi process/core result.
2. When M increase from 4 to 9, the standard method also improve significantly. And when M = 16, it is better than vectorized method.

M = 1

```
[wf2099@access1 lecture4]$ g++ —fopenmp —std=c++11 —O3 —march=native
compute—vec—pipe.cpp && ./a.out —n 1000000000
```

```
time = 1.514795
flop-rate = 5.280798 Gflop/s

time = 1.509405
flop-rate = 5.299920 Gflop/s

time = 1.506867
flop-rate = 5.308907 Gflop/s
```

M = 4

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -O3 -march=native
compute-vec-pipe.cpp && ./a.out -n 1000000000
time = 5.312123
flop-rate = 6.023828 Gflop/s

time = 1.506742
flop-rate = 21.237194 Gflop/s

time = 1.508429
flop-rate = 21.213565 Gflop/s
```

M = 8

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -O3 -march=native
compute-vec-pipe.cpp && ./a.out -n 1000000000
time = 1.740536
flop-rate = 36.768448 Gflop/s

time = 1.747677
flop-rate = 36.609474 Gflop/s

time = 1.588393
flop-rate = 40.289816 Gflop/s
```

M = 16

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -O3 -march=native
compute-vec-pipe.cpp && ./a.out -n 1000000000
time = 4.097863
flop-rate = 31.234891 Gflop/s

time = 5.596113
flop-rate = 22.872821 Gflop/s

time = 5.658605
flop-rate = 22.620046 Gflop/s
```

Appendix

Optimizing matrix-matrix multiplication output

MMULT0 (n * k * m)

```
  Dimension        Time     Gflop/s         GB/s        Error
         16    0.389202    5.138730    24.408965 1.385951e+06
         64    0.210830    9.487061    39.727066 9.440258e+04
        112    0.219419    9.117795    37.448087 2.693089e+04
        160    0.283867    7.070349    28.811673 1.233188e+04
        208    0.247918    8.130742    32.992048 7.315887e+03
        256    0.301170    6.684813    27.052601 4.826596e+03
        304    0.225179    8.983074    36.286891 3.267805e+03
        352    0.228382    8.784631    35.438002 2.471286e+03
        400    0.220176    9.301666    37.485714 1.891448e+03
        448    0.236387    9.128978    36.760440 1.574612e+03
        496    0.233813    9.393980    37.803192 1.312905e+03
        544    0.238176    9.462967    38.060609 1.104834e+03
        592    0.224794    9.229538    37.105238 8.560616e+02
        640    0.235487    8.905594    35.789356 7.445709e+02
        688    0.277906    9.374715    37.662372 7.953869e+02
        736    0.255033    9.379679    37.671647 6.299729e+02
        784    0.318209    9.086290    36.484238 6.724575e+02
        832    0.326867    7.047882    28.293181 4.889743e+02
        880    0.289321    9.421684    37.815213 4.996540e+02
        928    0.343411    9.308723    37.355262 5.309204e+02
        976    0.413475    8.994143    36.087156 5.570778e+02
       1024    0.548490    3.915262    15.706928 2.879263e+02
       1072    0.458317    5.375868    21.563650 3.032963e+02
       1120    0.328639    8.549974    34.291501 3.193277e+02
       1168    0.385964    8.256793    33.112002 3.318287e+02
       1216    0.473931    7.587798    30.426070 3.425009e+02
       1264    0.579649    6.967972    27.938041 3.541980e+02
       1312    0.726666    6.215792    24.920018 3.661419e+02
       1360    0.866787    5.804095    23.267593 3.795629e+02
       1408    1.386899    4.025251    16.135312 3.887651e+02
       1456    1.567307    3.938764    15.787518 4.048556e+02
       1504    1.635187    4.161079    16.677516 4.184804e+02
       1552    1.904722    3.925307    15.731577 4.322260e+02
       1600    2.415287    3.391729    13.592353 4.431469e+02
       1648    2.308219    3.878150    15.540838 4.596910e+02
       1696    2.548807    3.827988    15.339038 4.692641e+02
       1744    2.950545    3.595566    14.407002 4.866494e+02
       1792    3.732496    3.083505    12.354668 4.954003e+02
       1840    3.859976    3.227742    12.932020 5.048999e+02
       1888    4.575723    2.941550    11.784896 5.153721e+02
       1936    4.023079    3.607343    14.451732 5.383513e+02
       1984    4.534441    3.444540    13.798995 5.441071e+02
```

m * n * k

```
  Dimension        Time      Gflop/s          GB/s         Error
         16    0.482501     4.145075     19.689105  1.385951e+06
         64    0.604529     3.308621     13.854850  9.440258e+04
        112    0.803658     2.489388     10.224271  2.693089e+04
        160    0.742537     2.702950     11.014520  1.233188e+04
        208    0.828276     2.433678      9.875117  7.315887e+03
        256    3.220182     0.625202      2.530116  4.826596e+03
        304    0.966262     2.093429      8.456351  3.267805e+03
        352    0.991519     2.023413      8.162633  2.471286e+03
        400    1.010062     2.027598      8.171222  1.891448e+03
        448    1.399642     1.541801      6.208501  1.574612e+03
        496    1.119387     1.962173      7.896165  1.312905e+03
        544    1.068389     2.109576      8.484840  1.104834e+03
        592    1.063374     1.951098      7.843942  8.560616e+02
        640    1.673656     1.253036      5.035640  7.445709e+02
        688    1.387061     1.878278      7.545873  7.953869e+02
        736    1.114985     2.145436      8.616723  6.299729e+02
        784    1.329093     2.175425      8.734997  6.724575e+02
        832    1.697939     1.356775      5.446670  4.889743e+02
        880    1.352970     2.014744      8.086451  4.996540e+02
        928    1.590583     2.009775      8.065090  5.309204e+02
        976    1.778209     2.091349      8.391110  5.570778e+02
       1024    8.628961     0.248869      0.998394  2.879263e+02
       1072    1.295606     1.901697      7.628077  3.032963e+02
       1120    1.512499     1.857757      7.450935  3.193277e+02
       1168    2.501759     1.273835      5.108426  3.318287e+02
       1216    3.348520     1.073934      4.306336  3.425009e+02
       1264    1.999995     2.019493      8.097144  3.541980e+02
       1312    2.727618     1.655953      6.638957  3.661419e+02
       1360    3.207850     1.568313      6.287088  3.795629e+02
       1408    6.826230     0.817819      3.278245  3.887651e+02
       1456   17.276323     0.357325      1.432243  4.048556e+02
       1504    4.312942     1.577611      6.323030  4.184804e+02
       1552    5.241086     1.426540      5.717189  4.322260e+02
       1600    9.288686     0.881933      3.534347  4.431469e+02
       1648    6.823761     1.311831      5.256875  4.596910e+02
       1696   10.991228     0.887690      3.557041  4.692641e+02
       1744   13.449212     0.788810      3.160669  4.866494e+02
       1792   25.899131     0.444384      1.780513  4.954003e+02
       1840   25.461491     0.489328      1.960501  5.048999e+02
       1888   28.001645     0.480676      1.925759  5.153721e+02
       1936   33.351404     0.435143      1.743269  5.383513e+02
       1984   40.794496     0.382872      1.533803  5.441071e+02
```

k * m * n

```
Dimension       Time     Gflop/s        GB/s       Error
       16   0.629579    3.176731   15.089470 1.385951e+06
       64   0.730679    2.737398   11.462854 9.440258e+04
      112   0.530453    3.771523   15.490184 2.693089e+04
      160   1.224356    1.639262    6.679993 1.233188e+04
      208   1.011874    1.992102    8.083336 7.315887e+03
      256  15.988613    0.125919    0.509577 4.826596e+03
      304   1.257015    1.609210    6.500361 3.267805e+03
      352   1.348202    1.488095    6.003112 2.471286e+03
      400   1.394900    1.468205    5.916868 1.891448e+03
      448   1.689133    1.277560    5.144462 1.574612e+03
      496   1.492252    1.471890    5.923170 1.312905e+03
      544   1.634222    1.379157    5.547049 1.104834e+03
      592   1.336083    1.552857    6.242906 8.560616e+02
      640   2.603997    0.805359    3.236536 7.445709e+02
      688   1.860342    1.400434    5.626161 7.953869e+02
      736   3.004341    0.796224    3.197879 6.299729e+02
      784   2.860353    1.010834    4.058808 6.724575e+02
      832   2.639635    0.872742    3.503557 4.889743e+02
      880   4.678871    0.582595    2.338325 4.996540e+02
      928  11.067304    0.288843    1.159108 5.309204e+02
      976  17.310106    0.214837    0.861991 5.570778e+02
     1024  20.849918    0.102997    0.413196 2.879263e+02
     1072  14.816514    0.166291    0.667025 3.032963e+02
     1120  17.202611    0.163339    0.655106 3.193277e+02
     1168  19.579938    0.162760    0.652711 3.318287e+02
     1216  24.952311    0.144119    0.577897 3.425009e+02
     1264  27.541195    0.146652    0.588001 3.541980e+02
     1312  33.361995    0.135388    0.542789 3.661419e+02
     1360  38.308784    0.131325    0.526460 3.795629e+02
     1408  43.088220    0.129563    0.519354 3.887651e+02
     1456  48.525336    0.127217    0.509917 4.048556e+02
     1504  55.993478    0.121517    0.487036 4.184804e+02
     1552  63.284940    0.118142    0.473482 4.322260e+02
     1600  66.339399    0.123486    0.494871 4.431469e+02
     1648  66.399323    0.134815    0.540241 4.596910e+02
     1696  75.085905    0.129942    0.520687 4.692641e+02
     1744  86.490421    0.122660    0.491482 4.866494e+02
     1792  88.284345    0.130365    0.522332 4.954003e+02
     1840  92.097975    0.135280    0.542002 5.048999e+02
     1888 100.077450    0.134493    0.538827 5.153721e+02
     1936 107.628582    0.134840    0.540195 5.383513e+02
     1984 118.973367    0.131282    0.525922 5.441071e+02
```

one level blocking scheme

BS = 16

```
Dimension       Time     Gflop/s        GB/s       Error
       16   0.151593   13.193213   62.667762 1.385951e+06
```

```
      64    0.151174   13.230858   55.404216 9.440258e+04
     112    0.153431   13.039161   53.553698 2.693089e+04
     160    0.154294   13.007859   53.007025 1.233188e+04
     208    0.156780   12.857269   52.170842 7.315887e+03
     256    0.201870    9.973102   40.359896 4.826596e+03
     304    0.171511   11.794025   47.641653 3.267805e+03
     352    0.174994   11.464695   46.249623 2.471286e+03
     400    0.179123   11.433492   46.076972 1.891448e+03
     448    0.189355   11.396404   45.890878 1.574612e+03
     496    0.198799   11.048507   44.461329 1.312905e+03
     544    0.198435   11.358130   45.683069 1.104834e+03
     592    0.183022   11.336021   45.573868 8.560616e+02
     640    0.188480   11.126627   44.715132 7.445709e+02
     688    0.231793   11.239695   45.154821 7.953869e+02
     736    0.212373   11.263806   45.238873 6.299729e+02
     784    0.317768    9.098902   36.534876 6.724575e+02
     832    0.204866   11.245009   45.142223 4.889743e+02
     880    0.242609   11.235724   45.096111 4.996540e+02
     928    0.352915    9.058036   36.349274 5.309204e+02
     976    0.330528   11.251258   45.143368 5.570778e+02
    1024    0.392235    5.474986   21.964106 2.879263e+02
    1072    0.218754   11.263122   45.178570 3.032963e+02
    1120    0.247892   11.335022   45.461534 3.193277e+02
    1168    0.282861   11.266398   45.181344 3.318287e+02
    1216    0.370530    9.705260   38.916815 3.425009e+02
    1264    0.362179   11.151862   44.713320 3.541980e+02
    1312    0.402927   11.209977   44.942437 3.661419e+02
    1360    0.454995   11.057071   44.325848 3.795629e+02
    1408    0.607806    9.184872   36.817768 3.887651e+02
    1456    0.570394   10.822787   43.380348 4.048556e+02
    1504    0.694809    9.792833   39.249466 4.184804e+02
    1552    0.708622   10.550927   42.285286 4.322260e+02
    1600    0.746216   10.978061   43.994580 4.431469e+02
    1648    0.983984    9.097321   36.455527 4.596910e+02
    1696    0.941365   10.364525   41.531434 4.692641e+02
    1744    1.022224   10.378232   41.584338 4.866494e+02
    1792    1.402369    8.206946   32.882743 4.954003e+02
    1840    1.201316   10.371133   41.552169 5.048999e+02
    1888    1.286984   10.458338   41.899824 5.153721e+02
    1936    1.388576   10.451447   41.870569 5.383513e+02
    1984    1.550995   10.070350   40.342309 5.441071e+02
```

BS = 4

```
  Dimension       Time     Gflop/s        GB/s       Error
          4    0.376417    5.313256   37.192792 1.930508e+07
         52    0.364523    5.486649   23.212746 1.294194e+05
        100    0.317827    6.299015   25.951944 3.279681e+04
        148    0.319967    6.261363   25.553131 1.488839e+04
        196    0.304124    6.585667   26.745871 8.132360e+03
        244    0.305241    6.567593   26.593369 5.150747e+03
```

```
 292    0.311638    6.551064    26.473476 3.653748e+03
 340    0.311827    6.554306    26.448550 2.678501e+03
 388    0.324041    6.489292    26.157869 2.105263e+03
 436    0.335658    6.420007    25.856727 1.675396e+03
 484    0.314064    6.498163    26.153765 1.309623e+03
 532    0.324576    6.494517    26.124562 1.105332e+03
 580    0.359575    6.511427    26.180428 1.011378e+03
 628    0.380578    6.507813    26.155604 9.125553e+02
 676    0.379880    6.505552    26.137693 7.759236e+02
 724    0.353200    6.446822    25.894143 6.326553e+02
 772    0.426639    6.470576    25.982883 6.613743e+02
 820    0.341261    6.462710    25.945416 4.724340e+02
 868    0.407783    6.414901    25.748288 4.941193e+02
 916    0.483479    6.358711    25.518146 5.210746e+02
 964    0.679066    5.276902    21.173295 5.516814e+02
1012    0.329557    6.289861    25.234028 2.870719e+02
1060    0.377798    6.305040    25.291537 2.958791e+02
1108    0.432964    6.283445    25.201830 3.127530e+02
1156    0.494686    6.245599    25.047227 3.248432e+02
1204    0.559511    6.238796    25.017365 3.360267e+02
1252    0.630086    6.229361    24.977150 3.500426e+02
1300    0.713514    6.158254    24.689859 3.640703e+02
1348    0.802933    6.101271    24.459399 3.794245e+02
1396    0.927079    5.869074    23.526748 3.892219e+02
1444    1.085201    5.549087    22.242464 3.994090e+02
1492    1.115541    5.954575    23.866191 4.176378e+02
1540    1.234421    5.917372    23.715599 4.284149e+02
1588    1.364255    5.870645    23.526944 4.393153e+02
1636    1.517443    5.771219    23.127210 4.523815e+02
1684    1.661568    5.748283    23.034093 4.711033e+02
1732    1.828779    5.682146    22.767953 4.806263e+02
1780    2.184768    5.162792    20.685973 4.884137e+02
1828    2.192141    5.573013    22.328637 5.033187e+02
1876    2.374111    5.561954    22.283395 5.170853e+02
1924    2.568666    5.545460    22.216429 5.273173e+02
1972    2.776654    5.523686    22.128355 5.442381e+02
```

BS = 32

```
Dimension        Time      Gflop/s         GB/s        Error
       32    0.151868   13.169543    57.616752 3.454625e+05
       64    0.274196    7.294640    30.546306 8.645484e+04
       96    0.184151   10.867574    44.828743 3.603143e+04
      128    0.173076   11.559530    47.321828 2.028065e+04
      160    0.181404   11.063931    45.085520 1.285523e+04
      192    0.184923   10.870015    44.159437 8.626524e+03
      224    0.185809   10.767056    43.645030 6.241077e+03
      256    0.219629    9.166680    37.096409 4.740149e+03
      288    0.184567   10.871829    43.940308 3.676900e+03
      320    0.290033    7.004779    28.281797 3.014212e+03
      352    0.197450   10.160840    40.989752 2.393770e+03
```

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 384    0.190698   10.689302   43.091248  2.120384e+03
 416    0.185259   10.880723   43.836758  1.722277e+03
 448    0.201057   10.733126   43.219997  1.587122e+03
 480    0.207519   10.658489   42.900418  1.408996e+03
 512    0.248860    8.629283   34.719381  1.211501e+03
 544    0.219592   10.263806   41.281632  1.120362e+03
 576    0.223570   10.257336   41.243040  1.006499e+03
 608    0.217028   10.356085   41.628736  8.804057e+02
 640    0.208055   10.079785   40.508135  7.327325e+02
 672    0.234784   10.340199   41.545443  7.719212e+02
 704    0.205823   10.171267   40.858442  6.036365e+02
 736    0.231473   10.334387   41.506043  6.293510e+02
 768    0.305577    8.894355   35.716394  6.599235e+02
 800    0.286649    7.144630   28.685688  4.588220e+02
 832    0.229936   10.018948   40.220298  4.751202e+02
 864    0.255755   10.087362   40.489549  4.877100e+02
 896    0.295615    9.733256   39.063379  5.102025e+02
 928    0.315796   10.122719   40.621772  5.313950e+02
 960    0.356766    9.919503   39.802006  5.444863e+02
 992    0.430040    9.079997   36.429825  5.618113e+02
1024    0.287092    7.480126   30.008163  2.906769e+02
1056    0.235774    9.989094   40.069890  2.992610e+02
1088    0.251540   10.240243   41.073916  3.058866e+02
1120    0.276178   10.174091   40.805372  3.146530e+02
1152    0.315325    9.696818   38.888280  3.241138e+02
1184    0.330412   10.046835   40.289166  3.320246e+02
1216    0.357537   10.057952   40.331065  3.475650e+02
1248    0.386637   10.054719   40.315555  3.481782e+02
1280    0.491562    8.532604   34.210409  3.566596e+02
1312    0.453977    9.949415   39.888660  3.676665e+02
1344    0.508054    9.556915   38.312989  3.780126e+02
1376    0.525832    9.909195   39.723196  3.912954e+02
1408    0.601612    9.279435   37.196827  3.904002e+02
1440    0.625103    9.553581   38.293937  4.022505e+02
1472    0.640948    9.952460   39.890976  4.082739e+02
1504    0.709962    9.583811   38.411712  4.174498e+02
1536    0.940382    7.707244   30.889189  4.298162e+02
1568    0.777819    9.912652   39.726469  4.390920e+02
1600    0.864818    9.472518   37.961117  4.482625e+02
1632    0.938249    9.265577   37.130438  4.580419e+02
1664    0.944725    9.754040   39.086500  4.590257e+02
1696    1.018764    9.577094   38.376139  4.684472e+02
1728    1.177380    8.764850   35.120265  4.795047e+02
1760    1.136957    9.590117   38.425856  4.839984e+02
1792    1.476047    7.797295   31.241394  4.950327e+02
1824    1.350245    8.988594   36.013513  5.030270e+02
1856    1.361203    9.393795   37.635917  5.111048e+02
1888    1.588570    8.472849   33.945250  5.251387e+02
1920    1.857477    7.620968   30.531504  5.275152e+02
1952    1.927056    7.719251   30.924457  5.386715e+02
1984    1.760171    8.873606   35.548094  5.445766e+02
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OpenMP

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Dimension       Time    Gflop/s        GB/s       Error
       16  21.989616   0.090952    0.432023 3.953354e+06
       64  10.820056   0.184857    0.774087 3.407899e+05
      112   6.299466   0.317585    1.304368 1.017575e+05
      160   4.549159   0.441189    1.797846 4.831103e+04
      208   2.931880   0.687530    2.789786 2.857763e+04
      256   2.729928   0.737479    2.984487 1.885309e+04
      304   2.219474   0.911388    3.681528 1.294787e+04
      352   2.255125   0.889642    3.588895 9.752712e+03
      400   2.212199   0.925776    3.730875 7.563525e+03
      448   2.248084   0.959915    3.865372 6.295509e+03
      496   2.170400   1.011993    4.072458 5.234917e+03
      544   1.922359   1.172439    4.715620 4.415464e+03
      592   1.876354   1.105733    4.445347 3.395948e+03
      640   1.454913   1.441428    5.792738 2.976941e+03
      688   1.801948   1.445816    5.808481 3.154993e+03
      736   2.127876   1.124186    4.515075 2.518677e+03
      784   2.547320   1.135052    4.557583 2.674568e+03
      832   2.191626   1.051147    4.219749 1.954231e+03
      880   2.397333   1.137050    4.563706 1.998178e+03
      928   2.966563   1.077582    4.324262 2.118935e+03
      976   3.756269   0.990040    3.972333 2.226589e+03
     1024   5.866915   0.366033    1.468421 1.138714e+03
     1072   2.142776   1.149840    4.612233 1.212227e+03
     1120   2.504422   1.121958    4.499853 1.277311e+03
     1168   2.872558   1.109404    4.449014 1.326886e+03
     1216   3.399094   1.057956    4.242264 1.369478e+03
     1264   1.808404   2.233447    8.954993 1.415789e+03
     1312   3.937808   1.147036    4.598634 1.464268e+03
     1360   4.444765   1.131874    4.537482 1.518108e+03
     1408   3.766467   1.482190    5.941391 1.555060e+03
     1456   5.734748   1.076465    4.314731 1.619364e+03
     1504   5.921955   1.148969    4.605044 1.671654e+03
     1552   6.466117   1.156276    4.634045 1.727057e+03
     1600   3.535840   2.316847    9.284763 1.772262e+03
     1648   6.699633   1.336136    5.354272 1.836995e+03
     1696   8.012183   1.217746    4.879600 1.876319e+03
     1744   4.605605   2.303471    9.229734 1.946598e+03
     1792   9.842674   1.169313    4.685084 1.980012e+03
     1840  10.425893   1.195006    4.787819 2.019545e+03
     1888  11.363430   1.184477    4.745435 2.061347e+03
     1936  12.348966   1.175210    4.708124 2.153405e+03
     1984  13.307310   1.173721    4.701981 2.176428e+03
```