## HW2 wf2099

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## 1. Finding Memory bugs using valgrind

- 1. Run in CIMS Linux Server. 4 AMD EPYC Processor. 7.6G Memory.
- 2. GCC 8.2, OpenMP4.0

#### Val\_test01

#### First Run:

```
valgrind ./val_test01_solved
==9586== Memcheck, a memory error detector
==9586== Invalid write of size 4
==9586== Invalid read of size 4
==9586== Mismatched free() / delete / delete []
==9586== ERROR SUMMARY: 3 errors from 3 contexts (suppressed: 0 from 0)
```

#### Two Bug:

```
x = (int *)malloc((n + 1) * sizeof(int)); // It should allocate n+1
space not n. Otherwise it would out of index.

// delete[] x;
// Here is the bug, it should be free, but not delete.
free(x);
```

#### After fix Bug:

```
==10502== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0)
```

#### Val\_test02

#### First Run

```
==12136== Memcheck, a memory error detector
==12136== Conditional jump or move depends on uninitialised value(s)
==12136== Use of uninitialised value of size 8
==12136== ERROR SUMMARY: 24 errors from 4 contexts (suppressed: 0 from 0)
```

#### One Bug, Initialize the array:

```
// Here is the bug, it didn't initialize, but default is 0?
for (int i = 0; i < 10; i++)
{
    x[i] = 0;
}</pre>
```

#### After fix Bug:

```
==12870== ERROR SUMMARY: 0 errors from 0 contexts (suppressed: 0 from 0) [wf2099@access1 hw2]$
```

### 2. Optimizing matrix-matrix multiplication.

Comparing different loop arrangements for MMult0.

Performance (Dimension 1984)	n * k * m	m * n * k	k * m * n
time (s)	4.5	40.7	79.7
flops (Gflop/s)	3.4	0.38	0.13

Performance (Dimension 1984)	n * k * m	m * n * k	k * m * n
bandwidth (GB/s)	13.79	1.53	0.5
ERR	5.441071e+02	5.441071e+02	5.441071e+02

• Details are in the appendix.

I think the reason why the arrange has such a great impact on performance is the locality. Both Space and time locality. Cause when change i, you can find its nearby in cache.

```
double A_ip = a[i + p * m];
double B_pj = b[p + j * k];
double C_ij = c[i + j * m];
```

one level blocking scheme

Performance (Dimension 1984)	BS = 16	BS = 4	BS = 32
time (s)	1.55	2.77	1.76
flops (Gflop/s)	10	5.52	8.87
bandwidth (GB/s)	40	22.12	35.54
ERR	5.441071e+02	5.441071e+02	5.441071e+02

• Details are in the appendix. code:

```
void MMult1(long m, long n, long k, double *a, double *b, double *c)
{
    for (long j = 0; j < n; j += BLOCK_SIZE)
    {
        for (long p = 0; p < k; p++)
             for (long i = 0; i < m; i += BLOCK_SIZE)
             {
                 for (long jj = j; jj < j + BLOCK_SIZE; ++jj)</pre>
                 {
                     for (long ii = i; ii < i + BLOCK_SIZE; ++ii)</pre>
                          double A_{ip} = a[ii + p * m];
                          double B_pj = b[p + jj * k];
                          double C_{ij} = c[ii + jj * m];
                          C_{ij} = C_{ij} + A_{ip} * B_{pj};
                          C[ii + jj * m] = C_ij;
                     }
                 }
             }
```

```
}
```

#### When use OpenMP

```
Dimension Time Gflop/s GB/s Error
1984 13.307310 1.173721 4.701981 2.176428e+03
```

### 3. Fast Sin

I do this for sin4\_vec(). result

```
Reference time: 0.3226
Taylor time: 1.1904 Error: 6.928125e-12
Intrin time: 0.0021 Error: 2.454130e-03
Vector time: 0.0022 Error: 6.928125e-12
```

#### code:

```
void sin4_vector(double *sinx, const double *x)
{
 // The Vec class is defined in the file intrin-wrapper.h
 typedef Vec<double, 4> Vec4;
 Vec4 x1, x2, x3, x5, x7, x9, x11;
 x1 = Vec4::LoadAligned(x);
 x2 = x1 * x1;
 x3 = x1 * x2;
 x5 = x3 * x2;
 x7 = x5 * x2;
 x9 = x7 * x2;
 x11 = x9 * x2;
 Vec4 s = x1;
 s += x3 * c3;
 s += x5 * c5;
 s += x7 * c7;
 s += x9 * c9;
 s += x11 * c11;
 s.StoreAligned(sinx);
}
```

## 4. Pipelining and optimization

x86\_64, AMD EPYC Processor (with IBPB), CPU MHz: 2894.562

#### compute

optimize method	seconds	cycles/eval	Glop/s
O3	1.646260	4.775115	1.214629
00	4.551409	13.199648	0.439405
02	1.563473	4.534315	1.279134
01	4.923109	14.277421	0.406235

#### Compute vec

#### report

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -03 -march=native
compute-vec.cpp && ./a.out -n 10000000000
time = 1.631802
flop-rate = 4.902162 Gflop/s

time = 1.507671
flop-rate = 5.306070 Gflop/s

time = 1.510943
flop-rate = 5.294581 Gflop/s
```

#### The different is:

- 1. compute\_fn0 is standard for loop.
- 2. compute\_fn1 is vectorized using AVX intrinsics.
- 3. compute\_fn2 is vectorized using Vec Class.

#### Compute vec pipeline

report

#### Compare different M

#### Observation

- 1. As M increase from 1 to 8, both AVS intrinsics and Vec class method have significant improve. I think is the multi process/core result.
- 2. When M increase from 4 to 9, the standard method also improve significantly. And when M = 16, it is better than vectorized method.

M = 1

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -03 -march=native compute-vec-pipe.cpp && ./a.out -n 1000000000
```

```
time = 1.514795
flop-rate = 5.280798 Gflop/s

time = 1.509405
flop-rate = 5.299920 Gflop/s

time = 1.506867
flop-rate = 5.308907 Gflop/s
```

#### M = 4

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -03 -march=native
compute-vec-pipe.cpp && ./a.out -n 10000000000
time = 5.312123
flop-rate = 6.023828 Gflop/s

time = 1.506742
flop-rate = 21.237194 Gflop/s

time = 1.508429
flop-rate = 21.213565 Gflop/s
```

#### M = 8

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -03 -march=native
compute-vec-pipe.cpp && ./a.out -n 1000000000
time = 1.740536
flop-rate = 36.768448 Gflop/s

time = 1.747677
flop-rate = 36.609474 Gflop/s

time = 1.588393
flop-rate = 40.289816 Gflop/s
```

#### M = 16

```
[wf2099@access1 lecture4]$ g++ -fopenmp -std=c++11 -03 -march=native
compute-vec-pipe.cpp && ./a.out -n 1000000000
time = 4.097863
flop-rate = 31.234891 Gflop/s

time = 5.596113
flop-rate = 22.872821 Gflop/s

time = 5.658605
flop-rate = 22.620046 Gflop/s
```

# **Appendix**

### Optimizing matrix-matrix multiplication output

MMULT0 ( n \* k \* m)

```
Dimension
                Time
                         Gflop/s
                                        GB/s
        16
             0.389202
                         5.138730
                                    24.408965 1.385951e+06
        64
             0.210830
                         9.487061
                                    39.727066 9.440258e+04
       112
             0.219419
                         9.117795
                                    37.448087 2.693089e+04
       160
             0.283867
                         7.070349
                                    28.811673 1.233188e+04
       208
             0.247918
                         8.130742
                                    32.992048 7.315887e+03
       256
             0.301170
                         6.684813
                                    27.052601 4.826596e+03
       304
                                    36.286891 3.267805e+03
             0.225179
                         8.983074
       352
             0.228382
                         8.784631
                                    35.438002 2.471286e+03
                                    37.485714 1.891448e+03
       400
             0.220176
                         9.301666
       448
             0.236387
                         9.128978
                                    36.760440 1.574612e+03
       496
                         9.393980
                                    37.803192 1.312905e+03
             0.233813
       544
             0.238176
                         9.462967
                                    38.060609 1.104834e+03
       592
             0.224794
                         9.229538
                                    37.105238 8.560616e+02
       640
             0.235487
                         8.905594
                                    35.789356 7.445709e+02
       688
             0.277906
                         9.374715
                                    37.662372 7.953869e+02
       736
             0.255033
                         9.379679
                                    37.671647 6.299729e+02
       784
             0.318209
                         9.086290
                                    36.484238 6.724575e+02
       832
             0.326867
                         7.047882
                                    28.293181 4.889743e+02
       880
             0.289321
                                    37.815213 4.996540e+02
                         9.421684
       928
             0.343411
                         9.308723
                                    37.355262 5.309204e+02
       976
             0.413475
                         8.994143
                                    36.087156 5.570778e+02
      1024
             0.548490
                                    15.706928 2.879263e+02
                         3.915262
      1072
             0.458317
                         5.375868
                                    21.563650 3.032963e+02
      1120
             0.328639
                         8.549974
                                    34.291501 3.193277e+02
      1168
             0.385964
                         8.256793
                                    33.112002 3.318287e+02
      1216
             0.473931
                         7.587798
                                    30.426070 3.425009e+02
      1264
             0.579649
                         6.967972
                                    27.938041 3.541980e+02
      1312
             0.726666
                         6.215792
                                    24.920018 3.661419e+02
      1360
             0.866787
                         5.804095
                                    23.267593 3.795629e+02
             1.386899
      1408
                         4.025251
                                    16.135312 3.887651e+02
      1456
                                    15.787518 4.048556e+02
             1.567307
                         3.938764
      1504
             1.635187
                         4.161079
                                    16.677516 4.184804e+02
      1552
                                    15.731577 4.322260e+02
             1.904722
                         3.925307
      1600
             2.415287
                         3.391729
                                    13.592353 4.431469e+02
      1648
             2.308219
                         3.878150
                                    15.540838 4.596910e+02
                         3.827988
      1696
             2.548807
                                    15.339038 4.692641e+02
      1744
             2.950545
                         3.595566
                                    14.407002 4.866494e+02
      1792
             3.732496
                         3.083505
                                    12.354668 4.954003e+02
      1840
             3.859976
                         3.227742
                                    12.932020 5.048999e+02
             4.575723
                         2.941550
                                    11.784896 5.153721e+02
      1888
      1936
             4.023079
                         3.607343
                                    14.451732 5.383513e+02
      1984
             4.534441
                         3.444540
                                    13.798995 5.441071e+02
```

### m \* n \* k

Dimension	Time	Gflop/s	GB/s	Error	
16	0.482501	4.145075		1.385951e+06	
64	0.604529	3.308621		9.440258e+04	
112	0.803658	2.489388		2.693089e+04	
160	0.742537	2.702950		1.233188e+04	
208	0.828276	2.433678		7.315887e+03	
256	3.220182	0.625202		4.826596e+03	
304	0.966262	2.093429		3.267805e+03	
352	0.991519	2.023413		2.471286e+03	
400	1.010062	2.027598		1.891448e+03	
448	1.399642	1.541801		1.574612e+03	
496	1.119387	1.962173		1.312905e+03	
544	1.068389	2.109576		1.104834e+03	
592	1.063374	1.951098	7.843942	8.560616e+02	
640	1.673656	1.253036		7.445709e+02	
688	1.387061	1.878278	7.545873	7.953869e+02	
736	1.114985	2.145436	8.616723	6.299729e+02	
784	1.329093	2.175425	8.734997	6.724575e+02	
832	1.697939	1.356775	5.446670	4.889743e+02	
880	1.352970	2.014744	8.086451	4.996540e+02	
928	1.590583	2.009775	8.065090	5.309204e+02	
976	1.778209	2.091349	8.391110	5.570778e+02	
1024	8.628961	0.248869	0.998394	2.879263e+02	
1072	1.295606	1.901697	7.628077	3.032963e+02	
1120	1.512499	1.857757	7.450935	3.193277e+02	
1168	2.501759	1.273835	5.108426	3.318287e+02	
1216	3.348520	1.073934	4.306336	3.425009e+02	
1264	1.999995	2.019493	8.097144	3.541980e+02	
1312	2.727618	1.655953	6.638957	3.661419e+02	
1360	3.207850	1.568313	6.287088	3.795629e+02	
1408	6.826230	0.817819	3.278245	3.887651e+02	
1456	17.276323	0.357325	1.432243	4.048556e+02	
1504	4.312942	1.577611		4.184804e+02	
1552	5.241086	1.426540		4.322260e+02	
1600	9.288686	0.881933	3.534347	4.431469e+02	
1648	6.823761	1.311831		4.596910e+02	
1696	10.991228	0.887690		4.692641e+02	
1744	13.449212	0.788810		4.866494e+02	
1792	25.899131	0.444384		4.954003e+02	
1840	25.461491	0.489328		5.048999e+02	
1888	28.001645	0.480676		5.153721e+02	
1936	33.351404	0.435143		5.383513e+02	
1984	40.794496	0.382872	1.533803	5.441071e+02	

## k \* m \* n

Dimension	Time	Gflop/s	GB/s	Error	
16	0.629579	3.176731	15.089470	1.385951e+06	
64	0.730679	2.737398	11.462854	9.440258e+04	
112	0.530453	3.771523	15.490184	2.693089e+04	
160	1.224356	1.639262	6.679993	1.233188e+04	
208	1.011874	1.992102	8.083336	7.315887e+03	
256	15.988613	0.125919	0.509577	4.826596e+03	
304	1.257015	1.609210	6.500361	3.267805e+03	
352	1.348202	1.488095	6.003112	2.471286e+03	
400	1.394900	1.468205	5.916868	1.891448e+03	
448	1.689133	1.277560	5.144462	1.574612e+03	
496	1.492252	1.471890	5.923170	1.312905e+03	
544	1.634222	1.379157	5.547049	1.104834e+03	
592	1.336083	1.552857	6.242906	8.560616e+02	
640	2.603997	0.805359	3.236536	7.445709e+02	
688	1.860342	1.400434	5.626161	7.953869e+02	
736	3.004341	0.796224	3.197879	6.299729e+02	
784	2.860353	1.010834	4.058808	6.724575e+02	
832	2.639635	0.872742	3.503557	4.889743e+02	
880	4.678871	0.582595	2.338325	4.996540e+02	
928	11.067304	0.288843	1.159108	5.309204e+02	
976	17.310106	0.214837	0.861991	5.570778e+02	
1024	20.849918	0.102997	0.413196	2.879263e+02	
1072	14.816514	0.166291	0.667025	3.032963e+02	
1120	17.202611	0.163339	0.655106	3.193277e+02	
1168	19.579938	0.162760	0.652711	3.318287e+02	
1216	24.952311	0.144119	0.577897	3.425009e+02	
1264	27.541195	0.146652	0.588001	3.541980e+02	
1312	33.361995	0.135388	0.542789	3.661419e+02	
1360	38.308784	0.131325	0.526460	3.795629e+02	
1408	43.088220	0.129563	0.519354	3.887651e+02	
1456	48.525336	0.127217		4.048556e+02	
1504	55.993478	0.121517		4.184804e+02	
1552	63.284940	0.118142		4.322260e+02	
1600	66.339399	0.123486	0.494871	4.431469e+02	
1648	66.399323	0.134815	0.540241	4.596910e+02	
1696	75.085905	0.129942		4.692641e+02	
1744	86.490421	0.122660		4.866494e+02	
1792	88.284345	0.130365		4.954003e+02	
1840	92.097975	0.135280		5.048999e+02	
	100.077450	0.134493		5.153721e+02	
	107.628582	0.134840		5.383513e+02	
	118.973367	0.131282		5.441071e+02	
255.					
1304		0.131202	0.020022		

# one level blocking scheme

BS = 16

```
Dimension Time Gflop/s GB/s Error
16 0.151593 13.193213 62.667762 1.385951e+06
```

```
64
       0.151174
                 13.230858
                             55.404216 9.440258e+04
 112
       0.153431
                             53.553698 2.693089e+04
                 13.039161
160
       0.154294
                 13.007859
                             53.007025 1.233188e+04
208
       0.156780
                 12.857269
                             52.170842 7.315887e+03
256
                  9.973102
                             40.359896 4.826596e+03
       0.201870
304
       0.171511
                 11.794025
                             47.641653 3.267805e+03
352
       0.174994
                 11.464695
                             46.249623 2.471286e+03
                             46.076972 1.891448e+03
400
       0.179123
                 11.433492
448
       0.189355
                 11.396404
                             45.890878 1.574612e+03
496
       0.198799
                 11.048507
                             44.461329 1.312905e+03
544
       0.198435
                 11.358130
                             45.683069 1.104834e+03
592
                             45.573868 8.560616e+02
       0.183022
                 11.336021
640
       0.188480
                 11.126627
                             44.715132 7.445709e+02
688
       0.231793
                 11.239695
                             45.154821 7.953869e+02
736
       0.212373
                 11.263806
                            45.238873 6.299729e+02
784
       0.317768
                  9.098902
                             36.534876 6.724575e+02
832
       0.204866
                 11.245009
                             45.142223 4.889743e+02
880
       0.242609
                 11.235724
                             45.096111 4.996540e+02
                            36.349274 5.309204e+02
928
       0.352915
                  9.058036
976
       0.330528
                 11.251258
                             45.143368 5.570778e+02
1024
       0.392235
                  5.474986
                            21.964106 2.879263e+02
1072
       0.218754
                 11.263122
                             45.178570 3.032963e+02
1120
       0.247892
                 11.335022
                             45,461534 3,193277e+02
1168
       0.282861
                 11.266398
                             45.181344 3.318287e+02
1216
       0.370530
                  9.705260
                            38.916815 3.425009e+02
1264
       0.362179
                 11.151862
                            44.713320 3.541980e+02
1312
       0.402927
                 11.209977
                             44.942437 3.661419e+02
1360
       0.454995
                 11.057071
                             44.325848 3.795629e+02
1408
       0.607806
                  9.184872
                             36.817768 3.887651e+02
1456
       0.570394
                 10.822787
                             43.380348 4.048556e+02
1504
       0.694809
                  9.792833
                             39.249466 4.184804e+02
1552
       0.708622
                 10.550927
                             42.285286 4.322260e+02
1600
       0.746216
                 10.978061
                             43.994580 4.431469e+02
1648
       0.983984
                  9.097321
                             36.455527 4.596910e+02
1696
       0.941365
                 10.364525
                             41.531434 4.692641e+02
       1.022224
                             41.584338 4.866494e+02
1744
                 10.378232
1792
       1.402369
                  8.206946
                             32.882743 4.954003e+02
1840
       1.201316
                 10.371133
                            41.552169 5.048999e+02
1888
       1.286984
                 10.458338
                             41.899824 5.153721e+02
1936
       1.388576
                 10.451447
                             41.870569 5.383513e+02
1984
       1.550995
                 10.070350
                            40.342309 5.441071e+02
```

#### BS = 4

Dimension	Time	Gflop/s	GB/s	Error
4	0.376417	5.313256	37.192792	1.930508e+07
52	0.364523	5.486649	23.212746	1.294194e+05
100	0.317827	6.299015	25.951944	3.279681e+04
148	0.319967	6.261363	25.553131	1.488839e+04
196	0.304124	6.585667	26.745871	8.132360e+03
244	0.305241	6.567593	26.593369	5.150747e+03

292	0.311638	6.551064	26.473476 3.653748e+03
340	0.311827	6.554306	26.448550 2.678501e+03
388	0.324041	6.489292	26.157869 2.105263e+03
436	0.335658	6.420007	25.856727 1.675396e+03
484	0.314064	6.498163	26.153765 1.309623e+03
532	0.324576	6.494517	26.124562 1.105332e+03
580	0.359575	6.511427	26.180428 1.011378e+03
628	0.380578	6.507813	26.155604 9.125553e+02
676	0.379880	6.505552	26.137693 7.759236e+02
724	0.353200	6.446822	25.894143 6.326553e+02
772	0.426639	6.470576	25.982883 6.613743e+02
820	0.341261	6.462710	25.945416 4.724340e+02
868	0.407783	6.414901	25.748288 4.941193e+02
916	0.483479	6.358711	25.518146 5.210746e+02
964	0.679066	5.276902	21.173295 5.516814e+02
1012	0.329557	6.289861	25.234028 2.870719e+02
1060	0.377798	6.305040	25.291537 2.958791e+02
1108	0.432964	6.283445	25.201830 3.127530e+02
1156	0.494686	6.245599	25.047227 3.248432e+02
1204	0.559511	6.238796	25.017365 3.360267e+02
1252	0.630086	6.229361	24.977150 3.500426e+02
1300	0.713514	6.158254	24.689859 3.640703e+02
1348	0.802933	6.101271	24.459399 3.794245e+02
1396	0.927079	5.869074	23.526748 3.892219e+02
1444	1.085201	5.549087	22.242464 3.994090e+02
1492	1.115541	5.954575	23.866191 4.176378e+02
1540	1.234421	5.917372	23.715599 4.284149e+02
1588	1.364255	5.870645	23.526944 4.393153e+02
1636	1.517443	5.771219	23.127210 4.523815e+02
1684	1.661568	5.748283	23.034093 4.711033e+02
1732	1.828779	5.682146	22.767953 4.806263e+02
1780	2.184768	5.162792	20.685973 4.884137e+02
1828	2.192141	5.573013	22.328637 5.033187e+02
1876	2.374111	5.561954	22.283395 5.170853e+02
1924	2.568666	5.545460	22.216429 5.273173e+02
1972	2.776654	5.523686	22.128355 5.442381e+02

## BS = 32

Dimension	Time	Gflop/s	GB/s	Error
32	0.151868	13.169543	57.616752	3.454625e+05
64	0.274196	7.294640	30.546306	8.645484e+04
96	0.184151	10.867574	44.828743	3.603143e+04
128	0.173076	11.559530	47.321828	2.028065e+04
160	0.181404	11.063931	45.085520	1.285523e+04
192	0.184923	10.870015	44.159437	8.626524e+03
224	0.185809	10.767056	43.645030	6.241077e+03
256	0.219629	9.166680	37.096409	4.740149e+03
288	0.184567	10.871829	43.940308	3.676900e+03
320	0.290033	7.004779	28.281797	3.014212e+03
352	0.197450	10.160840	40.989752	2.393770e+03

```
384
       0.190698
                 10.689302
                             43.091248 2.120384e+03
 416
                             43.836758 1.722277e+03
       0.185259
                 10.880723
 448
       0.201057
                 10.733126
                             43.219997 1.587122e+03
 480
       0.207519
                 10.658489
                            42.900418 1.408996e+03
512
       0.248860
                             34.719381 1.211501e+03
                  8,629283
544
       0.219592
                 10.263806
                             41.281632 1.120362e+03
576
       0.223570
                 10.257336
                             41.243040 1.006499e+03
608
       0.217028
                 10.356085
                             41.628736 8.804057e+02
640
       0.208055
                 10.079785
                             40.508135 7.327325e+02
672
       0.234784
                 10.340199
                             41.545443 7.719212e+02
704
       0.205823
                 10.171267
                             40.858442 6.036365e+02
736
                             41.506043 6.293510e+02
       0.231473
                 10.334387
768
       0.305577
                  8.894355
                             35.716394 6.599235e+02
800
       0.286649
                  7.144630
                            28.685688 4.588220e+02
832
       0.229936
                            40.220298 4.751202e+02
                 10.018948
864
       0.255755
                 10.087362
                             40.489549 4.877100e+02
896
       0.295615
                  9.733256
                            39.063379 5.102025e+02
928
                 10.122719
                            40.621772 5.313950e+02
       0.315796
960
       0.356766
                  9.919503
                             39.802006 5.444863e+02
                             36.429825 5.618113e+02
992
       0.430040
                  9.079997
1024
       0.287092
                  7.480126
                             30.008163 2.906769e+02
                             40.069890 2.992610e+02
1056
       0.235774
                  9.989094
1088
       0.251540
                 10.240243
                             41.073916 3.058866e+02
1120
       0.276178
                 10.174091
                             40.805372 3.146530e+02
1152
       0.315325
                  9.696818
                            38.888280 3.241138e+02
1184
       0.330412
                 10.046835
                            40.289166 3.320246e+02
1216
       0.357537
                 10.057952
                             40.331065 3.475650e+02
1248
       0.386637
                 10.054719
                             40.315555 3.481782e+02
1280
       0.491562
                  8.532604
                             34.210409 3.566596e+02
1312
       0.453977
                  9.949415
                             39.888660 3.676665e+02
1344
       0.508054
                  9.556915
                             38.312989 3.780126e+02
1376
       0.525832
                  9.909195
                             39.723196 3.912954e+02
1408
       0.601612
                  9.279435
                             37.196827 3.904002e+02
1440
       0.625103
                  9.553581
                            38.293937 4.022505e+02
1472
       0.640948
                  9.952460
                             39.890976 4.082739e+02
       0.709962
                  9.583811
                            38.411712 4.174498e+02
1504
1536
       0.940382
                  7.707244
                             30.889189 4.298162e+02
1568
       0.777819
                  9.912652
                            39.726469 4.390920e+02
1600
       0.864818
                  9.472518
                             37.961117 4.482625e+02
                             37.130438 4.580419e+02
1632
       0.938249
                  9.265577
1664
       0.944725
                  9.754040
                             39.086500 4.590257e+02
1696
       1.018764
                  9.577094
                             38.376139 4.684472e+02
1728
       1.177380
                  8.764850
                             35.120265 4.795047e+02
1760
       1.136957
                  9.590117
                             38.425856 4.839984e+02
1792
                             31.241394 4.950327e+02
       1.476047
                  7.797295
1824
       1.350245
                  8.988594
                             36.013513 5.030270e+02
1856
       1.361203
                  9.393795
                             37.635917 5.111048e+02
1888
       1.588570
                  8.472849
                             33.945250 5.251387e+02
1920
       1.857477
                  7.620968
                             30.531504 5.275152e+02
1952
       1.927056
                  7.719251
                             30.924457 5.386715e+02
1984
       1.760171
                  8.873606
                             35.548094 5.445766e+02
```

# OpenMP

Dimension	Time	Gflop/s	GB/s Error
16	21.989616	0.090952	0.432023 3.953354e+06
64	10.820056	0.184857	0.774087 3.407899e+05
112	6.299466	0.317585	1.304368 1.017575e+05
160	4.549159	0.441189	1.797846 4.831103e+04
208	2.931880	0.687530	2.789786 2.857763e+04
256	2.729928	0.737479	2.984487 1.885309e+04
304	2.219474	0.911388	3.681528 1.294787e+04
352	2.255125	0.889642	3.588895 9.752712e+03
400	2.212199	0.925776	3.730875 7.563525e+03
448	2.248084	0.959915	3.865372 6.295509e+03
496	2.170400	1.011993	4.072458 5.234917e+03
544	1.922359	1.172439	4.715620 4.415464e+03
592	1.876354	1.105733	4.445347 3.395948e+03
640	1.454913	1.441428	5.792738 2.976941e+03
688	1.801948	1.445816	5.808481 3.154993e+03
736	2.127876	1.124186	4.515075 2.518677e+03
784	2.547320	1.135052	4.557583 2.674568e+03
832	2.191626	1.051147	4.219749 1.954231e+03
880	2.397333	1.137050	4.563706 1.998178e+03
928	2.966563	1.077582	4.324262 2.118935e+03
976	3.756269	0.990040	3.972333 2.226589e+03
1024	5.866915	0.366033	1.468421 1.138714e+03
1072	2.142776	1.149840	4.612233 1.212227e+03
1120	2.504422	1.121958	4.499853 1.277311e+03
1168	2.872558	1.109404	4.449014 1.326886e+03
1216	3.399094	1.057956	4.242264 1.369478e+03
1264	1.808404	2.233447	8.954993 1.415789e+03
1312	3.937808	1.147036	4.598634 1.464268e+03
1360	4.444765	1.131874	4.537482 1.518108e+03
1408	3.766467	1.482190	5.941391 1.555060e+03
1456	5.734748	1.076465	4.314731 1.619364e+03
1504	5.921955	1.148969	4.605044 1.671654e+03
1552	6.466117	1.156276	4.634045 1.727057e+03
1600	3.535840	2.316847	9.284763 1.772262e+03
1648	6.699633	1.336136	5.354272 1.836995e+03
1696	8.012183	1.217746	4.879600 1.876319e+03
1744	4.605605	2.303471	9.229734 1.946598e+03
1792	9.842674	1.169313	4.685084 1.980012e+03
1840	10.425893	1.195006	4.787819 2.019545e+03
1888	11.363430	1.184477	4.745435 2.061347e+03
1936	12.348966	1.175210	4.708124 2.153405e+03
		1.173721	4.701981 2.176428e+03