

# Determining $g$ on an Incline

## BACKGROUND

During the early part of the seventeenth century, Galileo experimentally examined the concept of acceleration. One of his goals was to learn more about freely falling objects. Unfortunately, his timing devices were not precise enough to allow him to study free fall directly. Therefore, he decided to limit the acceleration by using fluids, inclined planes, and pendulums. In this lab exercise, you will see how the acceleration of a rolling ball or cart depends on the ramp angle. Then, you will use your data to extrapolate to the acceleration on a vertical “ramp,” that is, the acceleration of a ball in free fall.

If the angle of an incline with the horizontal is small, a ball rolling down the incline moves slowly and can be easily timed. Using time and distance data, it is possible to calculate the acceleration of the ball. When the angle of the incline is increased, the acceleration also increases. The acceleration is directly proportional to the sine of the incline angle,  $(\theta)$ . A graph of acceleration versus  $\sin(\theta)$  can be extrapolated to a point where the value of  $\sin(\theta)$  is 1. When  $\sin\theta$  is 1, the angle of the incline is  $90^\circ$ . This is equivalent to free fall. The acceleration during free fall can then be determined from the graph.

Galileo was able to measure acceleration only for small angles. You will collect similar data. Can these data be used in extrapolation to determine a useful value of  $g$ , the acceleration of free fall? We will see how valid this extrapolation can be. Rather than measuring time, as Galileo did, you will NOT use a Motion Sensor to determine the acceleration. **You will make quantitative measurements of the motion of a ball rolling down inclines of various small angles. From these measurements, you should be able to decide for yourself whether an extrapolation to large angles is valid.**

## OBJECTIVES

- Use a Motion Sensor to measure the speed and acceleration of a ball and a cart rolling down an incline.
- Determine the mathematical relationship between the angle of an incline and the acceleration of ball rolling down the ramp.
- Determine the value of free fall acceleration,  $g$ , by extrapolating the acceleration vs. sine of track angle graph.
- Compare the results for a ball with the results for a low-friction dynamics cart.
- Determine if an extrapolation of the acceleration vs. sine of track angle is valid.

## MATERIALS



Power Macintosh or Windows P C  
LabPro interface  
Logger *Pro*  
Motion Sensor  
graph paper  
Ramp

hard ball, approximately 8 cm diameter  
rubber ball, similar size  
dynamics cart  
meter stick  
books

## PRELIMINARY QUESTIONS

1. One of the timing devices Galileo used was his pulse. Drop a rubber ball from a height of about 2 m and try to determine how many pulse beats elapsed before it hits the ground. What was the timing problem that Galileo encountered?
2. Now measure the time it takes for the rubber ball to fall 2 m, using a wrist watch or wall clock. Did the results improve substantially?
3. Roll the hard ball down a ramp that makes an angle of about  $10^\circ$  with the horizontal. First, use your pulse and then your wrist watch to measure the time of descent.
4. Do you think that during Galileo's day it was possible to get useful data for any of these experiments? Why?

## PROCEDURE

1. Connect the Motion Sensor to DIG/SONIC 2 of the LabPro.
2. Place a single book under one end of a 1 – 3 m long board or track so that it forms a small angle with the horizontal. Adjust the points of contact of the two ends of the incline, so that the distance,  $x$ , in Figure 1 is between 1 and 3 m.
3. Place the Motion Sensor at the top of an incline. Place it so the ball will never be closer than 0.4 m.
4. Open the file “P2-4 Determining  $g$ ” of the *Physics for HKCEE* folder. Two graphs will be displayed: distance vs. time and velocity vs. time. The vertical axis on the distance graph is scaled from 0 to 3 m. The vertical axis on the velocity graph is scaled from 0 to 2 m/s. The horizontal axis of both graphs is time scaled from 0 to 5 s.
5. Hold the hard ball on the incline about 0.5 m from the Motion Sensor.
6. Click  to begin collecting data; release the ball after the Motion Sensor starts to click. Get your hand out of the Motion Sensor path quickly. You may have to adjust the position and aim of the Motion Sensor several times before you get it right. Adjust and repeat this step until you get a good run showing approximately constant slope on the velocity vs. time graph during the rolling of the ball.
7. Logger Pro can fit a straight line to a portion of your data. First, indicate which portion is to be used by dragging across the graph to indicate the starting and ending times. Then click on the Regression Line tool, , to perform a linear regression of the selected data. Use this tool to determine the slope of the velocity vs. time graph, using only the portion of the data for times when the ball was freely rolling. From the fitted line, find the acceleration of the ball. Record the value in your data table.
8. Repeat Steps 5 – 7 two more times.
9. Measure the length of the incline,  $x$ , which is the distance between the two contact points of the ramp. See Figure 1.
10. Measure the height,  $h$ , the height of the book(s). These last two measurements will be used to determine the angle of the incline.
11. Raise the incline by placing a second book under the end. Adjust the books so that the distance,  $x$ , is the same as the previous reading.
12. Repeat Steps 5 – 10 for the new incline.

13. Repeat Steps 5 – 11 for 3, 4, and 5 books.
14. Repeat Steps 5 – 13 using a low-friction dynamics cart instead of the ball.

## DATA

Data using ball							
Number of books	Height of books, $h$ (m)	Length of incline, $x$ (m)	$\sin(\theta)$	Acceleration			Average acceleration (m/s <sup>2</sup> )
				trial 1 (m/s <sup>2</sup> )	trial 2 (m/s <sup>2</sup> )	trial 3 (m/s <sup>2</sup> )	
1							
2							
3							
4							
5							

Data using cart							
Number of books	Height of books, $h$ (m)	Length of incline, $x$ (m)	$\sin(\theta)$	Acceleration			Average acceleration (m/s <sup>2</sup> )
				trial 1 (m/s <sup>2</sup> )	trial 2 (m/s <sup>2</sup> )	trial 3 (m/s <sup>2</sup> )	
1							
2							
3							
4							
5							

## ANALYSIS

1. Calculate the average acceleration for each height.
2. Using trigonometry and your values of  $x$  and  $h$  in the data table, calculate the sine of the incline angle for each height. Note that  $x$  is the hypotenuse of a right triangle.
3. Plot a graph of the average acceleration ( $y$  axis) vs.  $\sin(\theta)$ . Use graph paper. Carry the  **$\sin(\theta)$  axis out to 1 (one) to leave room for extrapolation**.
4. Draw a **best-fit line by hand**, and determine the slope. The slope can be used to determine the acceleration of the ball on an incline of any angle.
5. On the graph, **carry the fitted line out to  $\sin(90^\circ) = 1$  on the horizontal axis**, and read the value of the acceleration.<sup>1</sup>
6. How well does the extrapolated value agree with the accepted value of free-fall acceleration ( $g = 9.8 \text{ m/s}^2$ )?

<sup>1</sup> Notice that extrapolating to the  $y$  value at the  $x = 1$  point is equivalent to using the slope of the fitted line.

7. Repeat the analysis, **including the extrapolation**, for the low-friction dynamics cart.
8. Why do you think the data for the dynamics cart resulted in an extrapolated value of  $g$  that was closer to the accepted value than the rolling ball data?
9. Discuss the validity of extrapolating the acceleration value to an angle of  $90^\circ$ .

## **EXTENSIONS**

1. Use the Motion Sensor to measure the actual free fall of a ball. Compare the results of your extrapolation with the measurement for free fall.
2. Compare your results in this experiment with other groups. Show all the class results on a new chart and graph.
3. Use a free-body diagram to analyze the forces on a rolling ball or cart. Predict the acceleration as a function of ramp angle, and compare your prediction to your experimental results.