Performance of Delta Hedging

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July 14, 2016

Delta

- Delta() is defined as the rate of change of the option price with respect of price of the underlying asset price.

$$\Delta = \frac{\theta C}{\theta S}$$

where C is the option price, S is the price of underlying asset.

Delta in Black-Scholes-Merton model

- For BSM model, the delta of European call option without dividend is:

$$\delta_c = N(d_1)$$

- For BSM model, the delta of European call option without dividend is:

$$\delta_p = N(d_1) - 1$$

where $d_1 = \frac{\ln \frac{S}{K} + r*\tau + \sigma^2 * \tau}{\sigma * \sqrt{\tau}}$, $N()$ is the cumulative distribution function for a standard normal distribution. And S, K, r, σ, τ denote the current stock price, strike price of option, the interest rate, the stock volatility, and the time to expiration of option, respectively.

Delta Hedging Strategy

- An investor who has sold call options to buy M shares of a stock. Then the position of underlying assets could be hedged by buying $M * \Delta$ shares.
- The gain (loss) on the underlying stock position would then tend to offset the loss (gain) on the option position.
- A position with a delta of zero is referred to as delta neutral.

Dynamic Aspects of Delta Hedging

- We assume where 100,000 call options are sold. The hedge is assumed to be adjusted or rebalanced weekly. Simulation parameters are showed as table 1.

Table 1 : Simulation parameters

NAME	VALUE
Current time t	6 weeks
Maturity T	26 weeks
Time to maturity τ	20 weeks=0.3846
Continuous annual interest rate r	0.05
Annualized stock volatility	0.20
Current stock price S_t	98
Exercise price K	100

Dynamic Aspects of Delta Hedging

- The simulation result of dynamic delta hedging is showed as table 2. Option closes in the money and cost of hedging is 281,976.00 dollars.

Table 2 : Simulation Results

Week	Stock Price	Delta	Purchased Shares	Cumulative cost
0	98.00	0.522	52,160.47	5,111,726.00
1	95.90	0.446	-7,524.66	4,390,124.00
2	99.64	0.570	12,316.13	5,617,286.00
3	100.36	0.592	2,210.97	5,839,184.00
4	102.31	0.656	6,390.28	6,492,996.00
5	104.35	0.720	6,488.05	7,170,001.00
6	106.23	0.778	5,723.04	7,777,976.00
7	106.74	0.796	1,843.13	7,974,720.00
8	105.89	0.777	-1,858.75	7,777,899.00
9	102.14	0.652	-12,552.01	6,495,899.00
10	98.59	0.497	-15,513.79	4,966,331.00
11	96.43	0.385	-11,134.85	3,892,582.00
12	100.26	0.567	18,198.86	5,717,135.00
13	100.40	0.573	528.69	5,770,217.00
14	102.75	0.698	12,518.89	7,056,558.00
15	104.27	0.783	8,508.04	7,943,674.00
16	103.90	0.784	134.12	7,957,610.00
17	106.65	0.923	13,841.30	9,433,765.00
18	109.67	0.992	6,950.34	10,196,009.00
19	111.39	1.000	767.65	10,281,519.00
20	111.47	1.000	4.10	10,281,976.00

Dynamic Aspects of Delta Hedging

- Table 3 illustrates an alternative sequence of events such that the option closes out of the money. As it becomes clear that the option will not be exercised, delta approaches zero. In Week 20, the investor has a naked position and has suffer from total costs 176,701.20 dollars.

Performance of delta hedging

Table 3 : Simulation Results

Time(weeks)	Stock Price	Delta	Purchased Shares	Cumulative cost
0	98.00	0.522	52,160.47	5,111,725.68
1	100.07	0.586	6,452.37	5,757,433.60
2	101.60	0.633	4,729.30	6,237,936.50
3	108.37	0.817	18,323.77	8,223,609.19
4	109.93	0.853	3,584.25	8,617,612.69
5	113.37	0.912	5,996.16	9,297,388.68
6	115.57	0.942	3,002.69	9,644,409.67
7	116.75	0.958	1,513.35	9,821,094.63
8	117.57	0.968	1,041.13	9,943,498.03
9	117.49	0.972	408.59	9,991,501.57
10	112.16	0.928	-4,404.26	9,497,532.51
11	113.58	0.953	2,505.71	9,782,131.75
12	111.63	0.938	-1,494.97	9,615,248.90
13	106.01	0.822	-11,609.88	8,384,523.88
14	104.25	0.768	-5,422.68	7,819,198.71
15	102.40	0.689	-7,931.93	7,006,930.19
16	103.75	0.776	8,793.25	7,919,209.65
17	102.13	0.699	-7,704.38	7,132,362.93
18	99.23	0.449	-25,033.39	4,648,252.85
19	100.05	0.526	7,724.82	5,421,110.64
20	99.64	0.000	-52,634.37	176,701.20

Performance of delta hedging

- For BSM model, the price of this call option is 4.80 dollars.
- The performance measure is the ratio of the standard deviation of the cost of hedging the option to the Black–Scholes–Merton price of the option. Table 4 shows statistics on the performance of delta hedging obtained from one million random stock price paths in our example.

Table 4 : Performance of Delta Hedging

Time between hedge rebalancing(weeks)	5	4	2	1	1/2	1/4
Performance measure	0.42	0.38	0.28	0.21	0.16	0.13

- The limit yields the riskless BS portfolio strategy as $\delta t \to 0$.

THANKS !