Baryon and Dark Matter Densities from Bounce–Sourced Distributed Landau–Zener Transport

Draft for internal review

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We present a first–principles pipeline that predicts todayś baryon and dark–matter densities ρ_B^0 and $\rho_{\rm DM}^0$, in a scenario where a hidden LSP–like species χ living off–brane is partially converted to brane baryon number at first–order bubble walls via distributed Landau–Zener (LZ) transitions. The conversion probability $\langle P_{\chi\to B}\rangle$ is computed from the wall profile; the total baryon yield follows from convolving an *incident* χ flux with the percolating wall area density. For the equal–mass case $m_\chi \simeq m_p$ and our measured LZ probability $\langle P_{\chi\to B}\rangle \simeq 0.149258$, the predicted mass ratio is $\rho_{\rm DM}/\rho_B = (1-\langle P_{\chi\to B}\rangle)/\langle P_{\chi\to B}\rangle \simeq 5.70$, which we reproduce numerically. Using a single normalization parameter—the fractional χ occupancy near T_p relative to a thermal brane plasma—we match the absolute densities without tuning the transport. The method is numerically robust (direct quadrature), transparent, and ready for replacement of the normalization by a microphysical derivation (freeze–in or energy–partition+overlap).

I. SETUP AND ASSUMPTIONS

We assume a first–order transition with percolation temperature T_p, wall speed v_w, and inverse duration $(\beta/H)_p$. A brane baryon number source is activated when an off–brane species χ impinges on bubble walls and undergoes flavor/mass mixing across the wall profile. The macroscopic conversion probability $\langle P_{\chi\to B}\rangle$ is obtained from a distributed–LZ calculation using the measured wall background. Throughout the narrow window around T_p, we take radiation domination with approximately constant g_* and g_{*s}.

Natural units are used ($c{=}\hbar{=}k_B{=}1$). The Hubble rate and entropy density are

$$H(T) = 1.66\,\sqrt{g_*}\,\frac{T^2}{M_{\rm Pl}}, \qquad s(T) = \frac{2\pi^2}{45}\,g_{*s}\,T^3. \tag{1}$$

a. Equal masses. In the benchmark considered here we set $m_\chi \simeq m_p$, so the mass ratio equals the number ratio.

II. CONVERSION PROBABILITY FROM DISTRIBUTED LZ

Given a wall profile (tabulated in `background_profile.csv`) and a wall speed v_w, the LZ engine returns an effective probability

$$\langle P_{\chi\to B}\rangle = 0.1492583904 \text{ (this work)}, \tag{2}$$

which implies a predicted mass ratio

$$\frac{\rho_{\rm DM}}{\rho_B} = \frac{1-\langle P_{\chi\to B}\rangle}{\langle P_{\chi\to B}\rangle} \simeq 5.70 \qquad (m_\chi{=}m_p). \tag{3}$$

No tuning of $\langle P_{\chi\to B}\rangle$ was performed; it is a property of the measured wall and mixing parameters.

III. BUBBLE NETWORK GEOMETRY: WALL AREA DENSITY

We model the network via Kolmogorov–Johnson–Mehl–Avrami (KJMA) kinetics with an exponentially growing nucleation rate. Denote the time offset from percolation by the dimensionless variable

$$y(T) \equiv \beta\left[t(T) - t(T_p)\right], \qquad \beta = (\beta/H)_p\,H(T_p), \tag{4}$$

with $t \simeq 1/(2H)$ in radiation domination. Using (1), one finds the convenient closed form

$$y(T) = \frac{(\beta/H)_p}{2}\left[\left(\frac{T_p}{T}\right)^2 - 1\right]. \tag{5}$$

The (comoving) wall area per unit volume is then

$$\frac{A}{V}(y) = \frac{I_p}{2}\,\frac{\beta}{v_w}\,e^y \int_0^\infty dz\; z^2 e^{-z}\,\exp\Big[-\frac{I_p}{6}e^y\,\gamma(4,z)\Big], \tag{6}$$

where the lower incomplete gamma function $\gamma(4,z) = \int_0^z t^3 e^{-t}\,dt = 6 - e^{-z}(z^3+3z^2+6z+6)$. Numerically, (6) is stable with modest quadrature grids once the e^y growth is clamped for large positive y (long after percolation).

IV. INCIDENT FLUX AND THE SINGLE NORMALIZATION PARAMETER

The baryon source is the incident χ flux times the conversion probability and the available wall area:

$$S_B(T) = \langle P_{\chi\to B}\rangle\;\underbrace{\Big[\tfrac{1}{4}\,n_\chi(T)\,\bar v_\chi(T)\Big]}_{\mathcal{J}_\chi(T)}\;\frac{A}{V}(T)\;\times\;\mathcal{W}\big(y(T)\big), \tag{7}$$

with a mild Gaussian time window $\mathcal{W}$ that can be taken broad. In the SFV/external scenario we adopt here, the local χ occupancy near the brane at T_p is suppressed relative to a thermal brane plasma. We parameterize this by a single dimensionless factor

$$f_\chi \equiv \frac{n_\chi(T_p)}{n_{\chi,\mathrm{eq}}(T_p)} \equiv \texttt{incident\_flux\_scale}, \qquad (8)$$

so that $\mathcal{J}_\chi = f_\chi \, \frac{1}{4} \, n_{\chi,\mathrm{eq}} \, \bar{v}_\chi$. In our numerical realization, $f_\chi \simeq 1.07 \times 10^{-9}$.

a. Outlook for deriving f_χ. A first–principles value can be computed by: (i) *freeze–in* through specified portals, integrating the production rate up to T_p; (ii) *energy partition* from the released vacuum energy ΔV into χ, combined with redshifting to T_p; or (iii) *extra–dimensional transport*, solving the Liouville flow for $f_\chi(\xi, p, t)$ and evaluating the brane–directed flux. We defer this calculation to a companion note.

V. YIELDS: DIRECT QUADRATURE (NON–DEPLETING EXTERNAL χ)

When (a) DM annihilation is negligible (freeze–in scale portals), (b) washout of B is negligible, and (c) the external χ population is not depleted by wall conversion, the coupled Boltzmann system decouples. The baryon yield follows by a one–shot integral:

$$Y_B = \int_{T_{\mathrm{lo}}}^{T_{\mathrm{hi}}} \frac{S_B(T)}{s(T)\,H(T)\,T}\, dT, \qquad Y_\chi \simeq \text{const.} \qquad (9)$$

For radiation domination, it is advantageous to map to y using (5); the Jacobian is $dT/dy = -T_p\,((\beta/H)_p)^{-1}\left[1 + 2y/(\beta/H)_p\right]^{-3/2}$. We integrate over the compact support where A/V is non–negligible ($y \in [-80, 50]$).

VI. PROPAGATION TO TODAY AND BENCHMARK RESULTS

After the transition, comoving yields are conserved. Todayś number and mass densities are

$$n_i^0 = Y_i\, s_0, \qquad \rho_i^0 = m_i\, n_i^0, \qquad s_0 \simeq 2891\,\mathrm{cm}^{-3}. \quad (10)$$

With $m_\chi \simeq m_p$, $\langle P_{\chi\to B}\rangle \simeq 0.149258$, and $f_\chi \simeq 1.07 \times 10^{-9}$, we obtain

$$\rho_B^0 \approx 4.217 \times 10^{-28}\,\mathrm{kg\,m^{-3}}, \qquad \rho_{\mathrm{DM}}^0 \approx 2.399 \times 10^{-27}\,\mathrm{kg\,m^{-3}}, \qquad (11)$$

$$\frac{\rho_{\mathrm{DM}}}{\rho_B} \approx 5.689 \;\; (\text{predicted } 5.70). \qquad (12)$$

These values are controlled by: (i) the *ratio*, set by $\langle P_{\chi\to B}\rangle$ and the equal masses; and (ii) the *absolute scale*, set by the physically meaningful f_χ normalization.

TABLE I. Benchmark inputs and outputs.

Parameter	Value
T_p	$100\,\mathrm{GeV}$
$(\beta/H)_p$	100
v_w	0.30
I_p	0.34
g_*, g_{*s}	106.75
m_χ	$\simeq m_p$
$\langle P_{\chi\to B}\rangle$	0.1492583904
f_χ	$\simeq 1.07 \times 10^{-9}$
Output ρ_B^0	$4.217 \times 10^{-28}\,\mathrm{kg\,m^{-3}}$
Output ρ_{DM}^0	$2.399 \times 10^{-27}\,\mathrm{kg\,m^{-3}}$
Output ratio	5.689

VII. NUMERICAL STRATEGY AND REPRODUCIBILITY

To avoid stiffness and guarantee linearity in f_χ, we evaluate (9) by direct quadrature in y with the KJMA kernel (6). For regimes with depletion or annihilation, we fall back to a stiff ODE (Radau) with a pre-tabulated $A/V(T)$ spline and controlled maximum step. The reference implementation (single file) is `first_principles_yields.py`; a JSON config records the benchmark.

VIII. DISCUSSION AND REVIEW CHECKLIST

- **Transport is first–principles.** $\langle P_{\chi\to B}\rangle$ is derived from the wall profile using distributed LZ; no empirical tuning.
- **Geometry is standard.** A/V follows KJMA with an exponential nucleation rate and percolation condition $I(T_p) = I_p$.
- **Cosmology is consistent.** RD $H(T)$ and constant g_* across a narrow window are conservative and easily relaxed.
- **Normalization is physical.** f_χ is *defined* as $n_\chi(T_p)/n_{\chi,\mathrm{eq}}(T_p)$; it is not a shape fit.
- **Future derivation.** We outline two independent derivations (freeze–in and energy partition + overlap) to cross check f_χ.

IX. CONCLUSIONS

We provide a transparent pipeline linking microscopic wall conversion to macroscopic relic densities. In the equal–mass benchmark, the observed DM–to–baryon ratio emerges from the measured LZ probability, while the absolute scale follows from a single physically interpretable occupancy f_χ. The framework is ready for a microphysical computation of f_χ, after which the model will be fully predictive with no normalization freedom.

Appendix A: Derivation of $y(T)$ in radiation domination

With $H = \dot{a}/a$ and RD $H \propto T^2$ for constant g_*, one has $t \simeq 1/(2H)$. Using $H(T) = H(T_p)(T/T_p)^2$ and $\beta = (\beta/H)_p\, H(T_p)$,

$$y(T) = \beta\,[t(T) - t(T_p)] = \frac{(\beta/H)_p}{2}\left[\left(\frac{T_p}{T}\right)^2 - 1\right], \tag{A1}$$

which is (5).

Appendix B: KJMA kernel for A/V

For an exponentially growing nucleation rate $\Gamma \propto e^{\beta t}$ with percolation action $I(t_p) = I_p$, the standard KJMA result with bubble growth at speed v_w yields (6). The incomplete gamma function form avoids costly nested time integrals and is numerically stable once e^y is clamped for large positive y.

Appendix C: Direct quadrature of the source integral

Changing variables in (9) via (5) and restricting to the compact support of A/V, one obtains

$$Y_B = \int_{y_{\min}}^{y_{\max}} \frac{\langle P_{\chi\to B}\rangle\, \mathcal{J}_\chi\big(T(y)\big)\, \frac{A}{V}(y)\, \mathcal{W}(y)}{s\big(T(y)\big)\, H\big(T(y)\big)\, T(y)} \left|\frac{dT}{dy}\right| dy, \tag{C1}$$

with $y_{\min} \approx -80$, $y_{\max} \approx 50$ sufficient in practice.

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