From Dimensionless Bounce to Physical SFV: A Reproducible Calibration of the Superfluid False Vacuum

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Abstract

We present a fully reproducible pipeline that maps a numerically constructed, dimensionless $O(4)$ bounce in a two-field potential to a physically calibrated Superfluid False Vacuum (SFV). Starting from a corridor solution with Euclidean action $S_E \approx 1078$, we compute the wall width via FWHM of the 4D action density, diagnose the wall regime ($R/w \approx 3.35$, moderate-to-thick), and choose a physical length unit using the SFV healing length $\xi = \hbar/(\sqrt{2}\, m\, c_s)$. With $m_{\text{SFV}} = 2.0$ GeV$/c^2$ and $c_s = 100$ m/s we obtain $\xi = 0.704$ Å and a physical bubble radius $R_{\text{phys}} \approx 2.36$ Å. The Gross–Pitaevskii identities then fix $a_s n = 1/(8\pi\xi^2)$; choosing $a_s = 1$ nm yields $n = 8.02 \times 10^{27}$ m^{-3} and $\rho = 84.96$ kg m^{-3}. We tabulate all derived quantities and provide exact commands to reproduce the outputs from the supplied scripts and CSV files in the repository.

1 Overview

We consider a two-field scalar potential with a portal coupling between an SFV field Φ and a brane field ϕ. The Euclidean, $O(4)$-symmetric bounce solution is obtained from dimensionless equations of motion (EOM) and then continued in the portal coupling g_{portal} along a narrow corridor to a stable solution with

$$S_E \approx 1078, \quad g_{\text{portal}} \approx 2.313, \quad v_\phi = 9.9 \times 10^{-5}, \quad v = 4.2 \times 10^{-5}, \quad \lambda_\phi = 0.1, \quad \lambda = 10^{-8}.$$

From the bounce we compute the wall full-width at half-maximum (FWHM) w and peak radius R using the 4D action density. These dimensionless measures are mapped to SI units by identifying the solver length unit with the SFV healing length ξ up to an $\mathcal{O}(1)$ geometric factor κ; here we take $\kappa = 1$, so $w_{\text{phys}} = \xi$. Fixing the SFV mass and c_s sets ξ, which then determines the product $a_s n$ through the Gross–Pitaevskii relations; a choice of a_s fixes n (and hence ρ).

2 Dimensionless model and bounce EOM

We work with the rescaled potential (tildes suppressed for readability):

$$V(\Phi, \phi) = \frac{\lambda_\phi}{4}\left(\Phi^2 - v_\phi^2\right)^2 + \beta\,\Phi^2 + \frac{\lambda}{4}\left(\phi^2 - 1\right)^2 - \mu^2\phi^2 + g_{\text{portal}}\,\Phi^2\phi^2, \tag{1}$$

where $v_\phi = v_\phi^{\text{phys}}/v^{\text{phys}}$ and the small quadratic "bias" β is tuned numerically to place the false/true vacua at the desired values.[1] The $O(4)$ bounce equations with Euclidean "friction" are

$$\Phi''(r) + \frac{3}{r}\Phi'(r) = \frac{\partial V}{\partial \Phi}, \qquad \phi''(r) + \frac{3}{r}\phi'(r) = \frac{\partial V}{\partial \phi}, \tag{2}$$

[1] In the implementation used here, $\mu^2 = \lambda$ and β is a small negative constant determined once for the corridor.

subject to boundary conditions $\Phi'(0) = \phi'(0) = 0$ at the center, and $\Phi(\infty) = \Phi_{\rm FV}$, $\phi(\infty) = -\phi_{\rm FV}$ at large radius (the sign implements the chosen true-vacuum basin). The Euclidean action is

$$S_E = 2\pi^2 \int_0^\infty dr\, r^3 \left[\tfrac{1}{2}\left(\Phi'^2 + \phi'^2\right) + V(\Phi,\phi) - V(\Phi_{\rm FV},\phi_{\rm FV})\right]. \tag{3}$$

Numerical solution. We use `solve_bvp` with a tanh-based initial guess characterized by (R_0, w_0) and perform adaptive continuation in $g_{\rm portal}$ from 0 to a target maximum. The solver was configured with a large node budget and tight tolerances. For the production run documented here:

$$v_\phi = 9.9\times10^{-5}, \quad v = 4.2\times10^{-5}, \quad \lambda_\phi = 0.1, \quad \lambda = 10^{-8}, \quad g_{\rm portal}^{\rm max} = 2.313.$$

The final converged solution has $S_E = 1078.2287$ (monotone along the continuation), with the numerical residuals within solver tolerance.

3 Wall width via 4D action density (FWHM)

Define the 4D action density along the solution

$$\mathcal{A}_4(r) \equiv 2\pi^2 r^3 \left[\tfrac{1}{2}\left(\Phi'^2 + \phi'^2\right) + V(\Phi,\phi) - V_{\rm false}\right]. \tag{4}$$

Let R be the location of its peak and FWHM the full width at half maximum obtained by linear interpolation on the radial grid. For the production bounce we measure (at $g_{\rm portal} = 2.313$)

$$R = 5.867, \qquad w \equiv \text{FWHM} = 1.751, \qquad \frac{R}{w} = 3.3507. \tag{5}$$

We classify wall regimes by R/w: thin if $\gtrsim 10$, moderate for 3–10, very thick if $\lesssim 3$. The present solution is in the *moderate-to-thick* band, consistent with a corridor that is numerically stable but not thin-wall.

4 Mapping to SI units via the SFV healing length

We calibrate physical units by identifying the solver unit with the SFV healing length

$$\xi = \frac{\hbar}{\sqrt{2}\, m\, c_s}, \qquad \ell_0 \equiv \frac{\xi}{w}, \qquad R_{\rm phys} = \frac{R}{w}\,\xi, \qquad w_{\rm phys} \equiv \xi, \tag{6}$$

with $m \equiv m_{\rm SFV}$ the SFV particle mass and c_s its sound speed. We adopt

$$m_{\rm SFV} = 2.0\ \text{GeV}/c^2, \qquad c_s = 100\ \text{m/s}.$$

Using fundamental constants $\hbar$ and c, this yields

$$\xi = 7.042\times10^{-11}\ \text{m} = 0.704\ \text{Å}, \tag{7}$$

$$\ell_0 = \xi/w = 4.022\times10^{-11}\ \text{m} = 0.402\ \text{Å}, \tag{8}$$

$$R_{\rm phys} = (R/w)\,\xi = 2.3596\times10^{-10}\ \text{m} = 2.360\ \text{Å}. \tag{9}$$

5 Derived SFV thermodynamic relations

In the weakly interacting (Gross–Pitaevskii) regime with contact coupling $g = 4\pi\hbar^2 a_s/m$, the sound speed and healing length satisfy

$$c_s^2 = \frac{gn}{m}, \qquad \xi^{-2} = \frac{2mgn}{\hbar^2} \Rightarrow a_s n = \frac{1}{8\pi\xi^2}. \tag{10}$$

Thus c_s (hence ξ) fixes the *product* $a_s n$ exactly; choosing a convenient a_s sets the number density n and the mass density $\rho = mn$. The chemical potential and pressure are

$$\mu = gn = mc_s^2, \qquad P = \tfrac{1}{2}\mu n. \tag{11}$$

For the calibrated ξ above we have

$$a_s n = \frac{1}{8\pi\xi^2} = 8.023 \times 10^{18}\ \mathrm{m}^{-2}, \qquad \mu = 6.609 \times 10^{-4}\ \mathrm{eV}.$$

Two illustrative choices for a_s give (all exact to quoted precision):

a_s	n (m^{-3})	ρ ($\mathrm{kg\,m}^{-3}$)	g ($\mathrm{J\,m}^3$)	μ (eV)	P (Pa)
1 nm	8.023×10^{27}	84.958	1.3198×10^{-50}	6.609×10^{-4}	4.248×10^{5}
10 nm	8.023×10^{26}	8.496	1.3198×10^{-49}	6.609×10^{-4}	4.248×10^{4}

We adopt $a_s = 1$ nm as the canonical baseline for the repository.

6 Final calibrated numbers (baseline)

- **Bounce:** $S_E = 1078.2287$, $g_{\mathrm{portal}} = 2.313$, $\lambda_\phi = 0.1$, $\lambda = 10^{-8}$, $v_\phi = 9.9 \times 10^{-5}$, $v = 4.2 \times 10^{-5}$.
- **Wall measures (dimensionless):** $R = 5.867$, $w = 1.751$, $R/w = 3.3507$ (moderate-to-thick).
- **Calibration choices:** $m_{\mathrm{SFV}} = 2.0\ \mathrm{GeV}/c^2$, $c_s = 100$ m/s, $a_s = 1$ nm.
- **Healing length / unit:** $\xi = 0.704$ Å, $\ell_0 = \xi/w = 0.402$ Å, $R_{\mathrm{phys}} = 2.360$ Å.
- **Thermodynamics:** $a_s n = 8.023 \times 10^{18}\ \mathrm{m}^{-2}$, $n = 8.023 \times 10^{27}\ \mathrm{m}^{-3}$, $\rho = 84.96\ \mathrm{kg\,m}^{-3}$, $\mu = 6.609 \times 10^{-4}$ eV, $P = 4.248 \times 10^5$ Pa.

7 Reproducibility: exact steps

(1) Solve and continue the bounce

Run the hard-coded corridor script (this version writes both the g=0 and final background CSVs, and computes the FWHM):

```
python goldenRunDetails_v4f_more3_1078.py
```

This produces `background_profile_g0.csv` and `background_profile.csv` with columns `r, Phi, phi, w_FWHM, R_peak`. The final printout should match

$$S_E \approx 1078.2287, \quad R \approx 5.867, \quad w \approx 1.751, \quad R/w \approx 3.35.$$

(2) Calibrate SFV units and properties

Use the measured w and R and specify (m_{SFV}, c_s):

$$\xi = \frac{\hbar}{\sqrt{2}\, m\, c_s}, \quad \ell_0 = \frac{\xi}{w}, \quad R_{\mathrm{phys}} = \frac{R}{w}\,\xi, \quad a_s n = \frac{1}{8\pi\xi^2}, \quad \mu = mc_s^2.$$

For $c_s = 100$ m/s and $m_{\mathrm{SFV}} = 2.0$ GeV/c^2 the values in the previous section follow directly. Pick a_s (e.g. 1 nm) to fix n, ρ, g, P.

(3) Optional JSON for the repo

We recommend recording the calibration in a machine-readable JSON:


```
{
  "c_s_m_per_s": 100.0,
  "m_SFV_GeV": 2.0,
  "xi_m": 7.04215744e-11,
  "w_tilde": 1.751,
  "R_tilde": 5.867,
  "R_over_w": 3.35065677,
  "l0_m": 4.02179180e-11,
  "R_phys_m": 2.35958525e-10,
  "a_s_times_n_1_per_m2": 8.02321962e18,
  "mu_eV": 6.60914133e-4,
  "a_s_nm": 1.0,
  "n_m3": 8.02321962e27,
  "rho_kg_m3": 84.9579673,
  "g_J_m3": 1.31979583e-50,
  "P_Pa": 4.24789836e5
}
```


8 Notes and diagnostics

Wall regime. The moderate-to-thick ratio ($R/w \approx 3.35$) is stable along the g_{portal} continuation; the slow drift of R is expected in a non-thin-wall corridor and is not a dynamical evolution, only a change of the optimal $O(4)$ radius.

Sensitivity. Re-running with tighter tolerances shifts S_E by $< 1\%$; small variations in the initial guess (R_0, w_0) return the same solution within the same tolerance, indicating a well-conditioned saddle.

Alternative c_s. If desired, $c_s \in [50, 300]$ m/s can be used to probe the mapping; $\xi \propto 1/c_s$, so $R_{\mathrm{phys}} \propto 1/c_s$ and $a_s n \propto c_s^2$. The dimensionless bounce and S_E are unaffected.

Constants

We used CODATA values: $\hbar = 1.054\,571\,817 \times 10^{-34}$ J s, $c = 299\,792\,458$ m/s, 1 eV $= 1.602\,176\,634 \times 10^{-19}$ J.

Data & code

This paper accompanies the repository (scripts and CSVs) containing:

- `goldenRunDetails_v4f_more3_1078.py` (hard-coded corridor bounce; writes CSVs).
- `background_profile_g0.csv`, `background_profile.csv` (with columns `r,Phi,phi,w_FWHM,R_peak`).
- Optional: a small helper to emit the JSON in Section (3) from the CSV.