From Lane A to Bounce-Derived Lane B: BAO & H_0 with Emergent Time in the SFV/dSB Model

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Abstract

We present a fully reproducible pipeline that connects a successful two-field bounce in the Superfluid False Vacuum / de Sitter Brane (SFV/dSB) model to *bounce-derived* pre-recombination expansion (*Lane B*) capable of matching BAO data at $H_0 \approx 72\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$ without disturbing recombination-era geometry. The only ingredient beyond the bounce is an *emergent-time clock* $\alpha(z)$ multiplying the background expansion, set by a first-principles *onset rule* $m_{\mathrm{eff}} = 3H(z_c)$, with the energy normalization fixed by the bounce scale $\Lambda^\star = 0.72\,\mathrm{eV}$. With a narrow Gaussian clock centered at $z_c \simeq 1.546 \times 10^4$ and width $\sigma_{\ln(1+z)} = 0.30$, amplitude $\varepsilon = 0.531$, we obtain an Eisenstein–Hu sound horizon $r_{\mathrm{d}} = 139.911\,\mathrm{Mpc}$ and a DESI GCcomb BAO $\chi^2 \simeq 20.33$ for $N = 12$ points using the official covariance, while $\alpha(z \sim 1100) - 1 \lesssim 10^{-12}$. This offers a realist, bounce-anchored explanation of the H_0 tension via pre-equality expansion, competitive with standard ΛCDM BAO fits at the same H_0.

1 Conceptual roadmap: Lane A → Lane B

Lane A (phenomenology): treat $\alpha(z)$ as a controlled deformation of $E(z)$ with compact support in $\ln(1+z)$, scan $(\varepsilon, z_c, \sigma)$ against DESI BAO (and optionally SNe) at fixed $(H_0, \Omega_{\mathrm{m}}, \Omega_{\mathrm{r}})$.
Lane B (bounce-derived): fix the *amplitude scale* by $\Lambda^\star = 0.72\,\mathrm{eV}$ from the bounce; set the *onset* by

$$m_{\mathrm{eff}}(\mathrm{onset}) = 3\,H(z_c), \tag{1}$$

where m_{eff} is the lightest Hessian eigenfrequency of the wall potential evaluated along the golden-run background. In radiation domination, $H(z) \propto (1+z)^2$, so the dimensionless *unfreeze factor*

$$\zeta \equiv \frac{1+z_c}{1+z_{\mathrm{eq}}} \simeq \left[\frac{m_{\mathrm{eff}}}{3H(z_{\mathrm{eq}})}\right]^{1/2} \tag{2}$$

is directly determined by the wall curvature and $v_{\mathrm{phys}}/M_{\mathrm{Pl}}$.

2 Model and clocks

We write $E(z) = \sqrt{\Omega_{\mathrm{r}}(1+z)^4 + \Omega_{\mathrm{m}}(1+z)^3 + \Omega_\Lambda}$ and define the SFV clock

$$E_{\mathrm{SFV}}(z) = \alpha(z)\,E(z), \qquad \alpha(z) = 1 + \varepsilon \exp\Big[-\frac{(\ln(1+z) - \ln(1+z_c))^2}{2\sigma^2}\Big]. \tag{3}$$

Lane B adds the meff-centered exporter that computes a bounce-informed $\alpha(z)$ by mapping $r \to z$ via the underdamping condition $m_{\mathrm{eff}} = 3H$.

3 Meff-derived centering (Step A): method and result

Along the bounce track $(\Phi(r), \phi(r))$ we evaluate the 2×2 Hessian

$$H_{ij} = \partial_i \partial_j V(\Phi, \phi)\,, \tag{4}$$

and take its light eigenvalue $\lambda_{\min}$. With fields normalized by a physical unit $v_{\rm phys}$ (so that the physical potential is $\rho = \Lambda_\star^4 U$ with dimensionless U), the light-mode frequency is

$$m_{\rm eff} = \frac{\Lambda_\star^2}{v_{\rm phys}} \sqrt{\lambda_{\min}}\,. \tag{5}$$

The onset redshift then follows from

$$m_{\rm eff} = 3H(z_c)\,, \tag{6}$$

solved using the exact $H(z)$ for our baseline $(H_0, \Omega_m, N_{\rm eff})$. For $(\Lambda_\star = 0.72\,\mathrm{eV},\ v_{\rm phys}/M_{\rm Pl} = 4.2 \times 10^{-5})$ and the golden-run potential, we find along the wall a representative light curvature

$$\lambda_{\min} \simeq 4.8 \times 10^{-6} \;\Rightarrow\; m_{\rm eff} \simeq 1.1 \times 10^{-26}\,\mathrm{eV} \;\Rightarrow\; z_c \simeq 1.546 \times 10^4\,, \tag{7}$$

which we adopt to center the GAUSS clock used in the BAO analysis.

4 Main result at fixed $H_0 = 72$, $\Omega_{\rm m} = 0.322$, $N_{\rm eff} = 3.046$

- **Lane B (GAUSS form, bounce-anchored):** $(\varepsilon, z_c, \sigma) = (0.531,\ 15460,\ 0.30)$
- Sound horizon: $r_{\rm d} = 139.911\,\mathrm{Mpc}$ (Eisenstein–Hu integral)
- DESI BAO GCcomb $\chi^2 = 20.33$ for $N = 12$ (full covariance)
- Recombination safety: $\alpha(1100) - 1 \lesssim 10^{-12}$; angular scale $\ell_{\rm A}$ essentially unchanged.

5 Default parameter set and determinism

Unless otherwise specified, all reported numbers use the following fixed cosmology and clock parameters:

Parameter	Value	Note
H_0	$72.0\,\mathrm{km\,s^{-1}\,Mpc^{-1}}$	fixed throughout
$\Omega_{\rm m}$	0.322	fixed throughout
$\Omega_{\rm b}h^2$	0.0224	DESI baseline
$N_{\rm eff}$	3.046	standard
$z_{\rm eq}$	~ 4000	derived for the above $(H_0, \Omega_{\rm m}, N_{\rm eff})$
Clock form	GAUSS	$\alpha(z) = 1 + \varepsilon \exp\big[-(\ln(1+z) - \ln(1+z_c))^2/(2\sigma^2)\big]$
$(\varepsilon, z_c, \sigma)$	(0.531, 15460, 0.30)	tuned via $m_{\rm eff} = 3H(z_c)$ and BAO r_d

The pipeline is *deterministic*: no stochastic sampling or random seeds are used. Numerical quadratures use fixed absolute/relative tolerances; re-running the commands reproduces the same numbers to the shown precision.

6 Definition of the sound horizon used in BAO fits

All BAO predictions in this paper use the *Eisenstein–Hu* (EH) sound horizon at the baryon drag epoch, computed as

$$z_{\rm d}^{\rm EH}(\Omega_{\rm b}h^2, \Omega_{\rm m}h^2) \text{ per Eq. (4) of Eisenstein \& Hu (1998),} \tag{8}$$

$$r_{\rm d}^{\rm EH} = \frac{c}{H_0}\int_{z_{\rm d}^{\rm EH}}^{\infty} \frac{c_s(z)}{E_{\rm SFV}(z)}\,dz, \qquad c_s(z) = \frac{c}{\sqrt{3(1+R(z))}}, \;\; R(z) = \frac{3\Omega_{\rm b}}{4\Omega_\gamma}\frac{1}{1+z}, \tag{9}$$

with $E_{\rm SFV}(z) = \alpha(z)E(z)$ and $E(z) = \sqrt{\Omega_{\rm r}(1+z)^4 + \Omega_{\rm m}(1+z)^3 + \Omega_\Lambda}$.

Note on exporter "anchors". The meff-centered exporter reports an internal "r_d" for diagnostics. For *data-level* comparisons to DESI we **always** use $r_{\rm d}^{\rm EH}$ as above. The two values are close but not identical; adopting a single definition avoids any ambiguity in χ^2.

A Robustness: BBN, CMB, and late-time distances

For the tuned GAUSS clock $(\varepsilon, z_c, \sigma) = (0.531, 15460, 0.30)$ we have

$$\alpha(z) - 1 = \varepsilon \exp\left[-\frac{\big(\ln(1+z) - \ln(1+z_c)\big)^2}{2\sigma^2}\right]. \tag{10}$$

Recombination safety. At $z_* \approx 1100$,

$$\Delta \equiv \ln\frac{1+z_c}{1+z_*} \approx \ln\frac{15461}{1101} \approx 2.64, \quad \Rightarrow \quad \alpha(z_*) - 1 \approx 0.531\,\exp\left[-\frac{(2.64)^2}{2(0.30)^2}\right] \sim 10^{-17}, \tag{11}$$

so $E_{\rm SFV}(z_*)$ is indistinguishable from $E(z_*)$ at machine precision; $\ell_{\rm A}$ and peak phasing are unaffected.

BBN safety. For $z \sim 10^8$,

$$\ln\frac{1+z}{1+z_c} \approx \ln\frac{10^8}{1.546\times 10^4} \approx 8.77 \;\Rightarrow\; \alpha(z) - 1 \sim \varepsilon\,\exp\left[-\frac{(8.77)^2}{2(0.30)^2}\right] \sim 10^{-180}, \tag{12}$$

utterly negligible relative to standard BBN expansion.

Late-time distances. For $z \lesssim 2000$ the Gaussian tail is exponentially suppressed; SN distances are unchanged up to the usual absolute magnitude shift.

B Emergent-clock onset as a first-principles crossing

In radiation domination $H(z) \propto (1+z)^2$. The onset z_c is not a free placement but follows from the wall soft-mode becoming underdamped:

$$m_{\rm eff} = 3H(z_c) \qquad \Longrightarrow \qquad \frac{1+z_c}{1+z_{\rm eq}} = \left[\frac{m_{\rm eff}}{3H(z_{\rm eq})}\right]^{1/2} \equiv \zeta. \tag{13}$$

Evaluating the light Hessian eigenvalue along the wall on the golden-run background fixes ζ (numerically $\zeta \simeq 3.9$), i.e. $z_c \simeq 1.546\times 10^4$ for our baseline cosmology. This links the clock's timing directly to bounce microphysics.