Strong CP in the SFV/dSB Framework: A Bulk-Axion Route-II Module

Yukawa Overlap Phases, Axion Relaxation, and Bounce-Controlled PQ Quality

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Abstract

The SFV/dSB program has developed first-principles modules that (i) normalize UV gauge kinetic terms from brane-wall energetics and predict QCD dimensional transmutation, and (ii) generate fermion masses from localization/overlap integrals in a bounce-derived corridor geometry. A major remaining "QFT-native" target is the strong-CP sector: the observed smallness of the effective QCD vacuum angle $\bar{\theta}$. This note documents a minimal Route-II implementation in which an SFV phase mode (a bulk axion-like field) dynamically relaxes $\bar{\theta}$ to (nearly) zero even when complex Yukawa overlaps generate $\arg\det(M_q) \sim \mathcal{O}(1)$. The core SFV/dSB task becomes the *axion quality* problem: deriving an explicit PQ-breaking scale $\Lambda_{\rm PQ}$ from bounce microphysics and ensuring the residual $\theta_{\rm eff}$ lies below EDM bounds. Using the corridor bounce with $S_E \simeq 1078$ and geometric ratio $w/R \simeq 0.2985$, we show that wall-supported PQ breaking is naturally *exponentially* suppressed, while a naive "patch" support can be catastrophically large. A simple one-parameter family $S_{\rm break} = S_E(w/R)^p$ interpolates these extremes and selects $p \simeq 1.6$ as the borderline value compatible with $|\theta_{\rm eff}| \lesssim 10^{-10}$, thereby turning an EDM bound into a constraint on the effective support/geometry of brane–bulk induced PQ breaking.

Contents

1 Motivation and context within SFV/dSB

SFV/dSB models the observable Universe as the wall of a nucleated bubble (a moderate-thick brane shell) inside a superfluid false-vacuum bulk. Existing modules already provide:

1. **UV gauge normalization → QCD running:** the brane wall sets UV gauge kinetic normalization and, via RG flow, predicts $\alpha_s(M_Z)$ and $\Lambda_{\rm QCD}$ [9, 10].
2. **Fermion masses from overlaps:** quark and lepton Yukawas arise from localized wavefunctions whose overlap integrals are fixed by bounce-derived corridor geometry (the "string-site" picture).

The strong-CP sector introduces one additional QFT datum: the effective QCD vacuum angle $\bar\theta$. The goal is to incorporate $\bar\theta$ into the same first-principles chain (SFV microphysics → SM/QFT observables).

2 Strong CP basics: what is $\bar\theta$?

The CP-odd topological term in QCD is

$$\mathcal{L}_\theta = \theta\,\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu}\,, \tag{1}$$

where $\tilde{G}^{a\,\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}G^a_{\rho\sigma}$. After electroweak symmetry breaking, chiral rotations shift θ into quark-mass phases. The physical combination is

$$\bar\theta \equiv \theta + \arg\det(M_q)\,, \tag{2}$$

with M_q the complex quark mass matrix. Experimental limits on the neutron EDM imply $|\bar\theta| \lesssim 10^{-10}$ [6].

Key point. Determining $\alpha_s(\mu)$ and $\Lambda_{\rm QCD}$ through RG flow does *not* determine $\bar\theta$: the θ operator is topological/CP-odd and requires additional structure.

3 Route II: a bulk SFV phase mode as an axion-like relaxer

3.1 Minimal EFT

Route II introduces a pseudoscalar $a(x)$ (interpreted here as a bulk SFV phase excitation or a bulk mode with support overlapping the brane) with anomalous coupling to QCD:

$$\mathcal{L} \supset \frac{1}{2}(\partial a)^2 + \frac{a}{f_a}\frac{g_s^2}{32\pi^2}\,G^a_{\mu\nu}\tilde{G}^{a\,\mu\nu} + \cdots\,, \tag{3}$$

so that the effective angle is

$$\theta_{\text{eff}} = \bar{\theta} + \frac{a}{f_a} \,. \tag{4}$$

Non-perturbative QCD generates an axion potential

$$V_{\text{QCD}}(a) \simeq \chi_{\text{QCD}} \left(1 - \cos\theta_{\text{eff}}\right) , \tag{5}$$

where χ_{QCD} is the topological susceptibility. Minimization gives

$$\langle\theta_{\text{eff}}\rangle \to 0 \quad \Rightarrow \quad \frac{a_*}{f_a} = -\bar{\theta} \pmod{2\pi} \,, \tag{6}$$

thereby solving strong CP dynamically (Peccei–Quinn mechanism [2, 3]; axion [4, 5]).

3.2 Why "bulk" is natural in SFV/dSB

In the SFV/dSB transport picture, dark matter is associated with a bulk channel (LSP-like excitations of the SFV), while baryons are reheated/transported into the brane via a distinct channel. It is therefore natural to interpret the axion-like field as a bulk SFV mode whose anomalous coupling is *induced* on the brane.

4 Complex Yukawas from overlap integrals with SFV phases

4.1 Gaussian localization model with phase gradients

The overlap module represents each chiral fermion and the Higgs as localized wavefunctions along an extra coordinate y. A convenient minimal ansatz is Gaussian localization with an SFV-induced phase gradient:

$$\psi_{L_i}(y) = \mathcal{N}_{L_i} \exp\left[-\frac{(y - y_{L_i})^2}{2\sigma_L^2}\right] \exp(ik_{L_i} y) \,, \tag{7}$$

$$\psi_{R_j}(y) = \mathcal{N}_{R_j} \exp\left[-\frac{(y - y_{R_j})^2}{2\sigma_R^2}\right] \exp\Big(ik_{R_j} y\Big) \,, \tag{8}$$

$$h(y) = \mathcal{N}_H \exp\left[-\frac{(y - y_H)^2}{2\sigma_H^2}\right] \exp(ik_H y) \,. \tag{9}$$

The dimensionless k-parameters capture a minimal set of SFV phase gradients (or geometric phase factors) and were exposed as CLI controls in the diagnostic scripts:

$$k_L = (k_{L_1}, k_{L_2}, k_{L_3}), \quad k_{Ru} = (k_{Ru_1}, k_{Ru_2}, k_{Ru_3}), \quad k_{Rd} = (k_{Rd_1}, k_{Rd_2}, k_{Rd_3}), \quad k_H.$$

4.2 Yukawa matrices and $\arg\det(Y_u Y_d)$

With a 5D Yukawa kernel y_0, the induced 4D Yukawa matrices are modeled as

$$(Y_u)_{ij} = y_0^{(u)} \int dy\, \psi_{L_i}(y)\, \psi_{Ru_j}(y)\, h(y), \qquad (Y_d)_{ij} = y_0^{(d)} \int dy\, \psi_{L_i}(y)\, \psi_{Rd_j}(y)\, h(y). \tag{10}$$

Phases from the k-terms generate complex $Y_{u,d}$ and therefore contribute

$$\arg\det(M_q) \;\equiv\; \arg\det(Y_u Y_d) \,, \tag{11}$$

in the simplified diagnostic where Higgs-VEV phases are absorbed into Y. The strong-CP diagnostic computes

$$\bar{\theta} = \theta_{\text{UV}} + \arg\det(Y_u Y_d) \,, \tag{12}$$

and wraps to the principal branch $\bar{\theta} \in (-\pi, \pi]$ for reporting:

$$\bar{\theta}_{\text{wrap}} = \left((\bar{\theta} + \pi) \bmod 2\pi\right) - \pi.$$

4.3 CKM and CP violation

Diagonalizing Y_u and Y_d yields a CKM matrix V and a Jarlskog invariant J. This provides a basic sanity check that overlap phases can generate flavor CP violation while strong CP is removed by axion relaxation.

5 Bulk axion scale estimates from SFV microphysics

The Route-II module requires an estimate of the axion decay constant f_a. Because the axion is taken as a bulk SFV mode, f_a should be expressible in terms of bulk scales and a characteristic length L_a of the axion wavefunction along the transverse direction.

In the diagnostic implementation, three *candidate* scaling estimates were used:

$$f_a^{(1)} \sim \kappa_a \, \ell_s^{-1} \,, \tag{13}$$

$$f_a^{(2)} \sim \kappa_a \, M_* \sqrt{M_* \, L_a} \,, \tag{14}$$

$$f_a^{(\phi)} \sim \kappa_a \, v_{\rm bulk} \, M_{\rm Pl} \,, \tag{15}$$

where ℓ_s and M_* are SFV/dSB scales already present in the transport module, $v_{\rm bulk}$ is the bounce-derived bulk VEV in Planck units, and κ_a is an $\mathcal{O}(1)$ normalization factor to be fixed by the microscopic axion origin.

In the reported corridor run we used $M_* = M_{\rm brane}$ and L_a taken from the bounce profile FWHM:

$$L_a = w_{\rm FWHM} \, \ell_s \,. \tag{16}$$

Table 1: Key numerical inputs used in the example runs (from calibration JSON and bounce profile).

Quantity	Value	Notes
O(4) bounce action S_E	1078	corridor two-field bounce
Wall FWHM $w_{\rm FWHM}$	1.75113	dimensionless (from profile CSV)
Peak radius $R_{\rm peak}$	5.86660	dimensionless (from profile CSV)
w/R	0.298492	$= w_{\rm FWHM}/R_{\rm peak}$
ℓ_s	4.38×10^{-14} GeV^{-1}	SFV length scale
$M_* = M_{\rm brane}$	1.628×10^{13} GeV	transport/portal scale
L_a (bounce FWHM)	7.67×10^{-14} GeV^{-1}	$w_{\rm FWHM}\ell_s$
$\chi_{\rm QCD}$	3.249×10^{-5} GeV^4	$(75\,{\rm MeV})^4$ benchmark [7, 8]

6 Axion quality: explicit PQ breaking and the residual $\theta_{\rm eff}$

6.1 Adding explicit PQ breaking

Generic UV physics can break the PQ shift symmetry and spoil the $\theta_{\rm eff} \to 0$ minimum. A minimal parametrization adds a second cosine:

$$V(a) = \chi_{\rm QCD} \left[1 - \cos\!\left(\bar\theta + \frac{a}{f_a} \right) \right] + \Lambda_{\rm PQ}^4 \left[1 - \cos\!\left(\frac{a}{f_a} + \delta \right) \right] , \tag{17}$$

where δ is a relative phase.

For $\Lambda_{\rm PQ}^4 \ll \chi_{\rm QCD}$, the residual strong-CP angle after minimization is approximately

$$\theta_{\rm eff} \simeq -\frac{\Lambda_{\rm PQ}^4}{\chi_{\rm QCD}} \sin\delta \quad \Rightarrow \quad |\theta_{\rm eff}| \lesssim \theta_{\rm tol} \quad \Longrightarrow \quad \Lambda_{\rm PQ} \lesssim \left(\chi_{\rm QCD} \, \theta_{\rm tol} \right)^{1/4} \equiv \Lambda_{\rm PQ}^{(\rm max)} . \tag{18}$$

Using $\theta_{\rm tol} = 10^{-10}$ and the benchmark $\chi_{\rm QCD}$ above gives $\Lambda_{\rm PQ}^{(\rm max)} \simeq 2.39 \times 10^{-4}\,{\rm GeV}$ (for $\sin\delta \sim \mathcal{O}(1)$).

6.2 A bounce-derived proxy for $\Lambda_{\rm PQ}$

In SFV/dSB it is natural to model explicit PQ breaking as a brane–bulk induced nonperturbative effect suppressed by an action $S_{\rm break}$:

$$\Lambda_{\rm PQ} \sim M_*\, e^{-S_{\rm break}/4}. \tag{19}$$

The diagnostic then asks: can $S_{\rm break}$ be *derived* from the same bounce geometry that already controls other SFV/dSB modules?

We introduce a simple geometry-controlled proxy:

$$S_{\rm break} = S_E \left(\frac{w}{R}\right)^p, \tag{20}$$

where S_E is the O(4) bounce action and w/R is the wall thickness-to-radius ratio. Different "support" assumptions correspond to different p:

- **Wall-supported breaking:** $p = 1$ (breaking distributed across the shell thickness).
- **Patch-supported breaking:** $p = 2$ (breaking localized in a smaller subregion / patch).
- **Intermediate (effective) support:** p free, capturing partial localization/fractal support.

The EDM requirement translates to a minimum action

$$S_{\rm req} = 4 \ln\left(\frac{M_*}{\Lambda_{\rm PQ}^{(\rm max)}}\right). \tag{21}$$

With the values in Table 1, $S_{\rm req} \simeq 155.0$.

Table 2: PQ-breaking proxy from the bounce and strong-CP outcome.

Model	p	$S_{\rm break}$	$\Lambda_{\rm PQ}$ proxy [GeV]	Outcome
wall	1	321.77	1.89×10^{-22}	extremely safe
patch	2	96.05	6.07×10^{2}	fails badly
power	1.6	155.78	1.99×10^{-4}	borderline safe

Solving $S_{\rm break} = S_{\rm req}$ for p yields the critical exponent

$$p_{\rm crit} = \frac{\ln(S_{\rm req}/S_E)}{\ln(w/R)} \simeq 1.60, \tag{22}$$

so the nEDM bound maps directly into an SFV/dSB constraint on the effective geometry/support of explicit PQ breaking.

7 Worked example: the corridor run and script outputs

7.1 Overlap-phase input choice

A representative set of phase-gradient parameters used in the runs was

$$k_L = (0, 0.04, 0.09), \quad k_{Ru} = (0.01, 0.03, 0), \quad k_{Rd} = (-0.02, 0.02, 0), \quad k_H = 0.015,$$

with $\theta_{\rm UV} = 0$.

The overlap diagnostic produced

$$\arg\det(Y_u Y_d) = -2.675403\ {\rm rad}, \qquad \bar\theta = -2.675403\ {\rm rad}, \tag{23}$$

demonstrating that $\arg\det(M_q)$ is generically $\mathcal{O}(1)$ once complex overlap phases are allowed.

7.2 Ideal axion relaxation

With only V_{QCD}, the required axion VEV is (Eq. 6)

$$\frac{a_*}{f_a} = +2.675403 \quad \Rightarrow \quad a_* \simeq 2.675403\, f_a, \tag{24}$$

which cancels $\bar{\theta}$ exactly on the principal branch.

7.3 Including explicit PQ breaking

For a fixed explicit-breaking scale Λ_{PQ} and phase $\delta = 0.7$, the minimizer finds a_*/f_a and reports the residual $\theta_{\text{eff}} = \bar{\theta} + a_*/f_a$. Two illustrative choices (both from the diagnostic runs) were:

- $\Lambda_{\text{PQ}} = 10^{-4}\,\text{GeV}$ gives $\theta_{\text{eff}} \simeq -2.83 \times 10^{-12}$ (safe).
- $\Lambda_{\text{PQ}} = 3 \times 10^{-4}\,\text{GeV}$ gives $\theta_{\text{eff}} \simeq -2.29 \times 10^{-10}$ (too large).

Thus the code-level "PQ quality diagnostic" reproduces the analytic bound Eq. (18).

When Λ_{PQ} is *derived* from the bounce proxy:

- wall model ($p = 1$) yields an exponentially tiny Λ_{PQ} and negligible residual θ_{eff}.
- patch model ($p = 2$) yields $\Lambda_{\text{PQ}} \sim 10^3\,\text{GeV}$ and completely destroys the PQ solution.
- power model ($p = 1.6$) yields $\Lambda_{\text{PQ}} \simeq 2.0 \times 10^{-4}\,\text{GeV}$ and $\theta_{\text{eff}} \simeq -4.41 \times 10^{-11}$, consistent with the 10^{-10} target.

8 Interpretation and SFV/dSB-specific discoveries

Discovery 1: Complex overlap phases easily generate $\arg\det(Y_u Y_d) \sim \mathcal{O}(1)$, so Route I (phase alignment without a new field) would be highly constrained.

Discovery 2: Route II trivially cancels $\bar{\theta}$ *in the absence of explicit PQ breaking.* Therefore the physically meaningful SFV/dSB derivation is the axion-quality sector: Λ_{PQ} must be derived and shown to be below $\Lambda_{\text{PQ}}^{(\text{max})}$.

Discovery 3: Using bounce-derived S_E and w/R provides a natural exponential suppression mechanism, but the *support model* matters qualitatively. A naive patch/localized model catastrophically over-breaks PQ, while wall-supported breaking is automatically safe. A continuous effective-support parameter p maps the EDM bound into a geometric constraint $p \lesssim 1.60$ for the corridor bounce.

Discovery 4 (bulk consistency): The PQ scale estimates based on bulk lengths and SFV scales naturally fall in the high-scale axion regime, $f_a \sim 10^{13}\,\text{GeV}$ for the simplest choices. This is compatible with a bulk axion interpretation, but demands a future cosmology module (misalignment/isocurvature) to check abundance constraints.

9 Next steps (Phase II of the strong-CP module)

1. **Derive κ_a and the correct f_a:** connect the axion mode normalization to the SFV order-parameter EFT and the bounce profile (beyond scaling estimates).

2. **Derive the support exponent p:** compute the actual brane–bulk induced PQ-breaking operator in the SFV/dSB microphysics, and show whether its effective support behaves like $p \simeq 1$ (safe) or yields an emergent $p \approx 1.6$ corridor.
3. **Axion phenomenology:** compute m_a and couplings (photons/nucleons) using standard QCD inputs [7], then check astrophysical/cosmological bounds.
4. **Coupling to transport:** if the axion is a bulk mode, relate its dynamics (and potential reheating-era evolution) to the dual-channel LZ transport framework without upsetting the successful B/DM corridor results.

A Implementation notes and script evolution

This module was developed iteratively as a diagnostic pipeline:

- **v1–v2:** introduced complex overlap phases and computed $\arg\det(Y_u Y_d)$; verified that a pure axion potential forces $\theta_{\text{eff}} \to 0$ by minimization.
- **v3–v4:** added explicit PQ-breaking cosine and the analytic "quality" bound; implemented Newton minimization and printed the axion VEV a_*.
- **v5–v7:** connected Λ_{PQ} to bounce information and added support models (wall, patch, power) via Eq. (20); fixed CLI reading of S_E by permitting a direct `--bounce_S_E` override, then validated the proxy against the quality bound.

For Windows `cmd.exe`, note that negative CSV arguments are easiest to pass using `--kRd=-0.02,0.02,0.0` to avoid option parsing ambiguity.

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