Phase II Note: Microphysical PQ-Breaking Proxy in SFV/dSB Route-II Strong-CP

Motivation

In Phase I of the Route-II strong-CP module, explicit PQ breaking was modeled with a heuristic scaling

$$S_{\mathrm{break}} \equiv S_E \left(\frac{w}{R}\right)^p, \tag{1}$$

where S_E is the $O(4)$ bounce action and w/R characterizes the bubble wall thickness. The exponent p was selected by hand ("wall", "patch", or a chosen power), encoding an assumed geometric participation fraction for the PQ-violating event on the bubble wall.

Phase II replaces the free exponent p with an SFV/dSB *microphysical* estimate of the operator coherence scale on the wall, so that the effective geometric suppression becomes derived.

Phase II Replacement: Microphysical Patch Fraction

We interpret the geometric factor as the fraction of the S^3 wall that participates coherently in the PQ-violating process. Let the PQ-violating operator be coherent over tangential correlation lengths ℓ_1, ℓ_2, ℓ_3 on the wall. Then the participating fraction is

$$f_{\mathrm{geom}} \;=\; \frac{\ell_1 \ell_2 \ell_3}{R^3}, \qquad \ell_i \le R. \tag{2}$$

For an approximately isotropic coherence length ℓ_{micro}, we use

$$f_{\mathrm{geom}} \;=\; \left(\frac{\ell_{\mathrm{micro}}}{R}\right)^{n_{\tan}}, \tag{3}$$

where $n_{\tan}$ encodes the tangential dimensionality of the localized event on the S^3 wall (default $n_{\tan} = 3$ for a localized operator/defect patch on the 3-sphere).

The PQ-breaking action proxy is then

$$S_{\mathrm{break}} \;=\; S_E \, f_{\mathrm{geom}}. \tag{4}$$

This replaces the heuristic $(w/R)^p$ by a derived length ratio.

Conversion to Bounce-Dimensionless Units

The bounce solver provides R in its natural dimensionless radial coordinate. To compare with a microphysical length expressed in physical units, we convert the microphysical length to the same dimensionless coordinate. In Phase II v8 we take $\ell_{\mathrm{micro}} = \ell_s$ from the SFV$\to$EW matching (string-length proxy) and convert via

$$\tilde{\ell}_s \;=\; \ell_s \, v_{\mathrm{brane}}^{(\mathrm{GeV})}, \qquad v_{\mathrm{brane}}^{(\mathrm{GeV})} \;=\; v_{\mathrm{brane}} \, M_{\mathrm{Pl,red}}, \tag{5}$$

so that $\tilde{\ell}_s$ and R are in the same bounce-dimensionless units. We then clamp $\tilde{\ell}_s \le R$.

Derived Effective Exponent (Diagnostic Only)

For comparison to Phase I, we define a derived effective exponent $p_{\rm eff}$ by

$$f_{\rm geom} \equiv \left(\frac{w}{R}\right)^{p_{\rm eff}} \quad \Rightarrow \quad p_{\rm eff} = \frac{\ln f_{\rm geom}}{\ln(w/R)}. \tag{6}$$

In Phase II, $p_{\rm eff}$ is an *output diagnostic*, not an input parameter.

PQ-Breaking Scale Proxy

As in Phase I, we parameterize the explicit breaking scale with a semiclassical suppression

$$\Lambda_{\rm PQ} \;\approx\; M_* \exp\left(-\frac{S_{\rm break}}{4}\right), \tag{7}$$

where M_* is a UV amplitude scale (in v8 defaulting to $M_{\rm brane}$ from the SFV scale pack). This yields an explicit breaking term added to the axion potential, allowing a quantitative "PQ quality" diagnostic.

Numerical Illustration (Corridor Run)

For the benchmark corridor run with $S_E = 1078$ and wall thickness ratio $w/R \simeq 0.298$, Phase II uses $\ell_{\rm micro} = \ell_s$ with $n_{\rm tan} = 3$, giving

$$f_{\rm geom} \simeq 0.445, \qquad S_{\rm break} \simeq 480, \qquad p_{\rm eff} \simeq 0.67. \tag{8}$$

The corresponding proxy scale is extremely suppressed:

$$\Lambda_{\rm PQ} \sim M_* \exp(-S_{\rm break}/4) \;\ll\; 10^{-30}\ \text{GeV}, \tag{9}$$

implying a negligible explicit-breaking induced residual in $\theta_{\rm eff}$.

Scope and Next Steps

Phase II removes the free exponent p by tying the geometric suppression to derived microphysical length scales. However, the mapping $S_{\rm break} = S_E f_{\rm geom}$ remains a controlled proxy. A complete microscopic derivation requires specifying the dominant PQ-violating mechanism (instanton/defect/heavy operator) and computing its Euclidean action and prefactor directly in SFV/dSB microphysics, recovering the present scaling as an appropriate limit.