Addendum: Two-Loop Validation Status for the Kernel-Derived w_i Pipeline

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December 22, 2025

Context

The main Phase A/Phase B note presents (i) a wall-energetics UV boundary condition, (ii) a microscopic kernel derivation of the non-universality weights w_i, and (iii) a two-loop running check that reproduces the observed strong coupling $\alpha_s(M_Z)$ and corresponding Λ_{QCD}.

Addendum (Correction / Clarification)

At the time of writing the main note, the *internal* two-loop RGE "sanity check" implemented inside the kernel-weight script `derive_wi_from_kernel_v2.py` (i.e. the `--two-loop-rge` option) was not reliably constrained to the *physical/perturbative* solution branch. In practice, the solver could drift to an unphysical branch in which one or more gauge couplings become non-perturbative during integration (blow-up), preventing stable reproduction of $(\alpha_1, \alpha_2, \alpha_3)$ at M_Z directly within that script.

Therefore:

- The microscopic derivation of the ratios w_1/w_2 and w_3/w_2 from the wall basis integrals and the kernel model (Model A minimal-norm solution) *remains valid* and is the intended output of `derive_wi_from_kernel_v2.py`.

- The two-loop running and Λ_{QCD} reporting quoted in the note should be understood as produced by the dedicated fitter `gauge_normalizer_from_wall.py` (using the extracted wall observables and two-loop evolution), rather than by the kernel script's internal two-loop validation pathway.

Planned resolution

Future updates will either (a) modify the kernel-script two-loop solver to explicitly select the perturbative branch (e.g. via bracketing and/or bounds enforcing $\alpha_i(\mu) \ll 1$ over $\mu \in [M_Z, M_{\mathrm{UV}}]$), or (b) remove the internal two-loop validation step and treat `gauge_normalizer_from_wall.py` as the sole reference implementation for two-loop evolution and Λ_{QCD} extraction.

Current recommended reproducible workflow

For reproducibility of results in this repository:

1. Use `kernel_integrals.py` and `derive_wi_from_kernel_v2.py` to compute the wall basis integrals and derive (w_1, w_3) from the microscopic kernel model.

2. Use `gauge_normalizer_from_wall.py` (two-loop option) to perform the electroweak fit, predict $\alpha_s(M_Z)$ for a chosen w_3, solve for the required w_3/w_2, and report Λ_{QCD}.

Note: This addendum is intended to clarify implementation status only; it does not alter the underlying Phase A/Phase B physical hypothesis or the kernel-based derivation of w_i.