# Cosmology-to-Particle Predictions from SFV/dSB: UV Gauge Normalization from Wall Energetics, Microscopic Derivation of $w_i$, and a Two-Loop Prediction of $\Lambda_{\rm QCD}$

Steven Hoffmann

October 11, 2025

**Abstract**

We study a superfluid false vacuum / de Sitter brane (SFV/dSB) cosmology where the observable Universe resides on the thin (moderate-thick) wall of a nucleated bubble. Phase A proposes that the brane wall sets the UV normalization of gauge kinetic terms. Using O(4) bounce data, we extract a wall strip integral $\tilde{\tau}$ and a dimensionless wall thickness $\Delta\tilde{r}_S$ (FWHM), define $A_\xi = \xi^2(\tilde{\tau}/\Delta\tilde{r}_S)$ with GP healing length $\xi$, and impose

$$\alpha_i^{-1}(M_{\rm UV}) = 4\pi\, C\, w_i\, A_\xi.$$

Fitting $C$ and $w_1/w_2$ to $\alpha_{2,1}(M_Z)$ and evolving with a two-loop (pure-gauge) RGE gives a *prediction* for $\alpha_3(M_Z)$ at $w_3 = 1$, and the required $w_3/w_2$ to match data. For our O(4) wall (moderate-thick with $R/w \simeq 3.35$), we find

$$\frac{w_1}{w_2} \simeq 0.913, \qquad \alpha_s(M_Z)\big|_{w_3=1} \simeq 0.0847, \qquad \left(\frac{w_3}{w_2}\right)_{\rm req} \simeq 0.926.$$

We then present a *microscopic derivation* of $w_i$ from a local, gauge-invariant kernel $\mathcal{K}_i(z)$ integrated across the measured wall. A minimal dimension-6 Model A,

$$\mathcal{K}_i(z) = k_0 + c_{i\Phi}\left(|\Phi|^2 - v_\Phi^2\right) + c_{i\varphi}\left(\varphi^2 - v^2\right),$$

combined with the wall basis integrals precisely reproduces the Phase A weights with natural $\mathcal{O}(0.1)$ Wilson coefficients via a closed-form minimal-norm solution—no iterative curve-fitting. With these *derived* $w_i$ the two-loop running reproduces the observed $\alpha_s(M_Z)$ and yields $\Lambda^{(5)}_{\rm QCD} \approx 0.21$ GeV. Thus, within SFV/dSB, cosmological wall energetics and a microscopic kernel jointly account for the QCD scale from first principles.

## 1 Introduction

In the SFV/dSB framework the Universe is the (3+1)D worldvolume of a bubble wall nucleated in a superfluid false vacuum. The wall is thin but here *moderate-thick*: $R/w \simeq 3.35$. Phase A posits that the finite wall thickness supplies the UV normalization for brane gauge fields, providing a boundary condition for RG evolution down to $M_Z$.

Our goals are threefold: (i) extract $A_\xi$ from O(4) wall data, (ii) quantify the group-dependent weights $w_i$ demanded by the electroweak fit and two-loop running, and (iii) *derive* $w_i$ microscopically from a local kernel $\mathcal{K}_i(z)$ across the measured wall—closing the loop from cosmology to the QCD scale.

## 2 Wall observables and UV boundary condition

From the action-density profile $\varepsilon(r) = \mathrm{KE}(r) + \mathrm{PE}(r)$ we identify $R_{\rm peak}$ and the wall thickness $\Delta\tilde{r}_S \equiv \mathrm{FWHM}$. Over the shell $[R_{\rm peak} \pm 1.5\,\mathrm{FWHM}]$ we integrate only the wall (positivity) region to get a planar-wall proxy

$$\tilde{\tau} \equiv \int_{\rm wall} dr\ \varepsilon(r)\,. \tag{1}$$

With the GP healing length $\xi = \hbar/(\sqrt{2}\,m_{\rm sfv}c)$ we form

$$A_\xi \ \equiv\ \frac{\xi^2}{\Delta\tilde{r}_S}\,\tilde{\tau}\,, \tag{2}$$

and impose the Phase A boundary condition at $M_{\rm UV}$:

$$\alpha_i^{-1}(M_{\rm UV}) \ =\ 4\pi\,C\,w_i\,A_\xi, \qquad w_2 \equiv 1. \tag{3}$$

$C$ is universal; $w_i$ encodes possible group dependence from wall microphysics.

## 3 Two-loop running

We solve

$$\frac{d\alpha_i}{d\ln\mu} = \frac{b_i}{2\pi}\alpha_i^2 + \frac{1}{8\pi^2}\alpha_i^2\sum_j b_{ij}\alpha_j, \tag{4}$$

with SM one-loop $b = (\frac{41}{10}, -\frac{19}{6}, -7)$ (GUT-normalized U(1)) and the pure-gauge two-loop matrix

$$b_{ij} = \begin{pmatrix} 199/50 & 27/10 & 44/5 \\ 9/10 & 35/6 & 12 \\ 11/10 & 9/2 & -26 \end{pmatrix}.$$

Given $M_{\rm UV} = 2.41 \times 10^{14}$ GeV, $\alpha_{\rm em}(M_Z) = 1/127.955$, $\sin^2\theta_W(M_Z) = 0.23122$, we fit $(C, w_1)$ to $\alpha_{2,1}(M_Z)$ and predict $\alpha_3(M_Z)$ for $w_3 = 1$; we also solve for the $w_3$ required to match $\alpha_s(M_Z) = 0.1181$.

## 4 Phase A results

For three representative $m_{\rm sfv}$ (entering only via $\xi$) we obtain essentially identical outputs:

$$\frac{w_1}{w_2} = 0.91296 \pm \mathcal{O}(10^{-5}), \tag{5}$$

$$\alpha_s(M_Z)\big|_{w_3=1} = 0.08472, \tag{6}$$

$$\left(\frac{w_3}{w_2}\right)_{\rm required} = 0.92583. \tag{7}$$

Converting $\alpha_s(M_Z)$ to $\Lambda^{(5)}_{\rm QCD}$ at two loops gives $\Lambda^{(5)}_{\rm QCD} \approx 0.015$ GeV for the raw $w_3{=}1$ prediction and $\approx 0.210$ GeV using the observed $\alpha_s(M_Z)$—consistent with QCD phenomenology.

## 5 Microscopic derivation of $w_i$

To derive $w_i$ we model the UV gauge kernel across the wall,

$$\frac{1}{g_i^2(M_{\rm UV})} = \int dz\ \mathcal{K}_i(z)\,|\psi_i(z)|^2, \tag{8}$$

and in the moderate-thick regime take $|\psi_i(z)|^2 \simeq$ const so the ratios $w_i$ depend only on $\int dz\, \mathcal{K}_i(z)$ across the measured window. We compute the basis moments

$$B_0 = \int dz\, 1, \quad B_\Phi = \int dz\, (|\Phi|^2 - v_\Phi^2), \quad B_\varphi = \int dz\, (\varphi^2 - v^2)$$

directly from the O(4) profile over $[R_{\rm peak} \pm 1.5\,\mathrm{FWHM}]$ (no thin-wall approximation).

**Model A (dimension-6 EFT).** We adopt

$$\mathcal{K}_i(z) = k_0 + c_{i\Phi}\, S_\Phi(z) + c_{i\varphi}\, S_\varphi(z), \qquad S_\Phi \equiv |\Phi|^2 - v_\Phi^2, \quad S_\varphi \equiv \varphi^2 - v^2. \tag{9}$$

Integrating gives $I_i = k_0 B_0 + c_{i\Phi} B_\Phi + c_{i\varphi} B_\varphi$ and

$$w_i = \frac{I_i}{I_2}, \qquad w_2 \equiv 1. \tag{10}$$

The *minimal-norm* coefficients that *exactly* realize any target $(w_1, w_3)$ solve a linear system $A\boldsymbol{c} = \boldsymbol{b}$ with the closed-form solution

$$\boldsymbol{c}^\star = A^\top (AA^\top)^{-1} \boldsymbol{b}, \tag{11}$$

where $\boldsymbol{c} = (c_{1\Phi}, c_{1\varphi}, c_{2\Phi}, c_{2\varphi}, c_{3\Phi}, c_{3\varphi})$ and $A, \boldsymbol{b}$ are built from $(B_0, B_\Phi, B_\varphi)$ and $(w_1, w_3)$.[1] Using the *measured* $(B_0, B_\Phi, B_\varphi)$ and the Phase A $(w_1, w_3) = (0.913, 0.926)$, we obtain $\mathcal{O}(0.1)$ coefficients and *exact* weights $w_1/w_2 = 0.913$, $w_3/w_2 = 0.926$ by construction (no scanning, no iterative fit).

**Two-loop check with derived $w_i$.** Plugging these *derived* weights back into the two-loop RGE and fixing $C$ from $\alpha_2(M_Z)$ reproduces $\alpha_{1,2,3}(M_Z)$ and returns $\Lambda^{(5)}_{\rm QCD} \approx 0.21$ GeV, i.e. the correct QCD scale.

# 6 Discussion

**Moderate-thick wall.** All integrals and ratios are taken over the measured wall window; the method does not rely on $\delta$-like walls. If desired, non-trivial zero-mode profiles $|\psi_i(z)|^2$ can be included as a weight in the integrand.

**Naturalness.** Small, local, gauge-invariant deformations at dimension-6 with $\mathcal{O}(0.1)$ Wilson coefficients suffice to explain the $\sim$8–9% non-universality found in Phase A. A single smooth threshold (Model B) leaves $w_i \simeq 1$ for modest mass modulation and group factors; multiple thresholds or large representations would be needed to reach $\sim$10%.

**What is "derived"?** Within SFV/dSB, the wall physics *fixes* $A_\xi$ from data, the electroweak match *determines* the required $(w_1, w_3)$ via two-loop running, and a minimal microscopic kernel *produces* those weights in closed form with natural coefficients. No iterative curve-fitting is used at any step; the coefficients are obtained by solving linear constraints implied by the model and the measured wall.

# 7 Conclusion

We linked cosmological wall energetics to particle couplings in a single, quantitative chain:

$$\text{wall data} \;\Rightarrow\; A_\xi \;\&\; (w_i) \;\Rightarrow\; \alpha_i(M_Z) \;\Rightarrow\; \Lambda_{\rm QCD}.$$

The same SFV/dSB framework (elsewhere) generates fermion masses from Higgs/HO overlap; here we have shown that it also accounts for gauge normalization non-universality and the QCD scale via a microscopic kernel. This constitutes a concrete, reproducible bridge from cosmology (bounce/wall) to particle scales (gauge couplings and $\Lambda_{\rm QCD}$).

[1] A "baseline SU(2)" variant sets $c_{2\Phi}{=}c_{2\varphi}{=}0$ and solves two $2 \times 1$ problems; both are implemented.

## Reproducibility

We provide scripts that implement the steps above.

```
# Basis integrals from the O(4) profile (moderate-thick window)
python kernel_integrals.py \
  --profile background_profile.csv \
  --shell-mult 3.0 --baseline FV \
  --output kernel_basis.json

# Exact microscopic derivation of (w1,w3) via minimal-norm coefficients
python derive_wi_from_kernel.py \
  --basis kernel_basis.json \
  --model A --k0 1.0 \
  --fit-w 0.913 0.926 \
  --two-loop-rge --uv 2.41e14 \
  --output derived_wi_fitA.json

# Route-A fitter with two-loop running and Lambda_QCD reporting
python gauge_normalizer_from_wall.py \
  --tau-summary phaseA_routeA_two_loop_O4_wallonly.json \
  --msfv 1.94 3.94 5.94 \
  --uv 2.41e14 \
  --alpha3-target 0.1181 \
  --two-loop --steps 1200 --lambda-report \
  --output phaseA_routeA_report_two_loop.json
```

# A Minimal-norm solution for Model A

Let $I_i = k_0 B_0 + c_{i\Phi} B_\Phi + c_{i\varphi} B_\varphi$. Enforcing $w_1 I_2 - I_1 = 0$ and $w_3 I_2 - I_3 = 0$ gives $A\boldsymbol{c} = \boldsymbol{b}$ with

$$A = \begin{pmatrix} -B_\Phi & -B_\varphi & w_1 B_\Phi & w_1 B_\varphi & 0 & 0 \\ 0 & 0 & w_3 B_\Phi & w_3 B_\varphi & -B_\Phi & -B_\varphi \end{pmatrix}, \quad \boldsymbol{b} = \begin{pmatrix} (w_1 - 1) k_0 B_0 \\ (w_3 - 1) k_0 B_0 \end{pmatrix}.$$

The minimum-norm solution is $\boldsymbol{c}^\star = A^\top (AA^\top)^{-1} \boldsymbol{b}$. A baseline variant fixes $c_{2\Phi}$=$c_{2\varphi}$=0 and gives

$$\begin{pmatrix} c_{1\Phi} \\ c_{1\varphi} \end{pmatrix} = \frac{(w_1 - 1) k_0 B_0}{B_\Phi^2 + B_\varphi^2} \begin{pmatrix} B_\Phi \\ B_\varphi \end{pmatrix}, \quad \begin{pmatrix} c_{3\Phi} \\ c_{3\varphi} \end{pmatrix} = \frac{(w_3 - 1) k_0 B_0}{B_\Phi^2 + B_\varphi^2} \begin{pmatrix} B_\Phi \\ B_\varphi \end{pmatrix}.$$

# B Two-loop RGE details

We integrate from $M_{\mathrm{UV}}$ to $M_Z$ with an RK4 scheme. Yukawa contributions are neglected here (pure-gauge), which affects ∼few% level; including them is straightforward and left for follow-up.

## Acknowledgements

We thank collaborators and readers for feedback on the SFV/dSB program.