

Executive Summary

Due to the fact that bees are responsible for the pollination, and thus production of many of our crops, the importance of bees are undeniable. [12] However, due to increases in diseases, temperature, and variation in the seasons, the average size of bee hives and amount of hives in general has started to decline rapidly.

In this paper, we use mathematics to model the amount of bees in a hive. Using differential equations and computational methods, any set of parameters can be entered for a specific bee hive and the amount of bees can then be predicted over a large period of time. This is an important tool as it allows us to model the growth and decline of hives due to certain factors such as diseases and loss of habitats. Once a differential equation was set and all factors that were significant were found, we used Python in order to create a program to plot the number of bees in a hive as time moves forward. In addition, this way of modeling bee hives allows for additional factors to be added in the future. Thus, making it a lot more appealing than other types of models.

For the second part of the problem, we were tasked with creating a model for the amount of pollination done in a plot of land. This is important as farmers and others in the agricultural industry must work with bees to increase their farm's productivity. In order to do so, a model and simulation using pollination dynamics and other factors such as the amount of nectar that is available in each flower. This model allows us to find the optimal amount and location of beehives in a square plot of land. This is an important tool to farmers as it allows for them to strategically place artificial hives in order to promote productivity and reduce the amount of wasted potential of the hives while maximizing the number of foraging bees and, therefore, increasing the amount of pollination occurring in that plot of land.

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1 Introduction

1.1 Background

Bees are responsible for about one-third of the food eaten by Americans. [5] They and other insects pollinate about three quarters of the world's crops that produce food for human consumption, including apples, broccoli, hay, nuts, and other plants. [4] [5] [10] Unfortunately, the bee population has recently been in decline. In fact, the number of bee colonies per hectare has dropped by approximately 90 percent since 1962, which has a critical impact on plant life and biodiversity. [6] In order to protect plant life and human agriculture, it is important to determine the causes of this decline and find potential mitigation strategies.

1.2 Problem Restatement

Our goal is to determine the number and placement of beehives in a 20-acre section of land that best optimizes crop yield. This problem is split essentially into two parts: determining the change in the honeybee population over time and determining the pollinating behavior of a set of honeybee colonies. This will result in two models: one that determines the population of a single honeybee colony over time and another that evaluates how the number of honeybee hives affects the pollination of a 20-acre parcel of land.

We will also perform a sensitivity analysis on the population model in order to determine the most important influences on bee population, which will help inform our analysis of the best ways to promote the bee population.

1.3 Our Work

We created models to study the change in honey bee population over time and to evaluate the optimal number of beehives necessary to fully pollinate a twenty-acre farm. For the population model, we used a computer model based on a generational approximation of a system of differential equations. For the pollination model, we used a distributional grid-based model to determine the amount that each hive pollinates the flowers in its vicinity.

2 Assumptions and Definitions

2.1 Variables

| Symbol | Definition |
|------------|---|
| p | The population of a colony of bees |
| P_0 | The total initial population of bees in the 20-acre field |
| P | The total population of bees in the 20-acre field (a function of t , time) |
| R | The number of resources demanded by the bees in the field (a function of P). Here, resources are defined as any food a bee needs |
| R_{\max} | The total number of resources available in the field |

2.2 Assumptions

Assumption 1: Individual honeybee colonies are not territorial over flowers.

Justification: Honeybees, unlike some types of stingless bees, are not territorial over flowers [2]. Therefore, it is not necessary to consider territorial interactions between colonies.

Assumption 2: Wind does not play a significant role in determining the foraging behaviors of honeybees.

Justification: Because wind is highly variable based on location, time of day, time of year, and random weather patterns, considering it would add a tremendous amount of complexity to the model.

Assumption 3: Bees are cooperative within their hives and are therefore optimal and consistent workers based on outlined factors.

Justification: Bees, like ants and other hive animals are not smart individually, but rather smart in collective which means that they usually cooperate and work very efficiently due to the chemical communication cues within them.

Assumption 4: Bees mature into workers and drones at a constant proportion.

Justification: While the proportion of drone to worker bee populations fluctuate throughout the year, the distribution of adult bees between these populations was not

significant in the model.

Assumption 5: Survival rates are dictated by the classification of the bee.

Justification: Fairly low variance was found amongst bees of the set class named.

Assumption 6: The growth in honeybee population is primarily dictated by the size of the population.

Justification: While environmental resources may be abundant, honeybee populations cannot grow exponentially due to limitations in food resources, nursing space, and other factors. [9]

3 Honeybee Population Model

3.1 Factors of Honeybee Population

The population of honey bees can be attributed to two main factors: the egg laying rate of the queen and the death rate in the colony. These two factors are then influenced by factors including the maturation rate of the bees, environmental hazards, pheromones, food, and the composition of the colony population.

To model the problem, the bee population was split into different classes in which the bees will spend a certain amount of time. These classes were egg, larva, pupa, worker, and drone. Additionally, each of these classes were found to have different survivability rates, in which the percentage of the class of bees that would live each day was a different proportion. As such, the following weights were given for the development time and the rate of survival:

| | Egg | Larva | Pupa | Worker | Drone |
|-----------------------------------|------|-------|-------|--------|-------|
| Average Development Period (days) | 3 | 5 | 12 | 21 | 21 |
| Survival Rate (% each day) | 0.97 | 0.99 | 0.999 | 0.985 | 0.985 |

The model start with the number of eggs produced per day. In previous research, an equation used to model the egg production coefficient relative to the seasonal changes in bee fertility was used in our model. [11] Using the equation, the coefficient was multiplied by a maximum number of 1600 bees produced each day:

$$\text{season}(t) = \max \left(-\frac{1}{1 + x_1 e^{\frac{-2t}{x_2}}}, \frac{1}{1 + x_3 e^{\frac{-2(t-x_4)}{x_5}}} \right)$$

Where the x coefficients are determined to be 385, 30, 36, 155, and 30 in chronological order, while t is the number of days starting at the beginning of the year.

Over the period of each developmental phase, each class of bee was placed in a numpy array, with the survival rates and the growth into the next stage of development both accounted for through code. At times when the population exceeded 80,000 honeybees, the larva population was reduced by 2.3%, 3%, 5.8%, and 6%, where each percentage correlates with a different day of development.

Finally, each springtime, new queens are birthed, and migrate out of the beehive, bringing half the population of worker and drone bees. [9]. As such, the model also accounts for this behavior, when it detects that a certain period of the year has been reached.

3.2 Development of Model

We developed this model based on the idea of a system of differential equations. However, we used a discrete generation-based approach. Due to the complexity of the model, we created a Python program to execute the model.

We chose to base our model on the idea of differential equations because they model change. Trends in population are ultimately dependent on the rates of input (i.e. births) and output (i.e. deaths). Differential equations allowed us to dynamically model those changes. Rather than use a simpler model with only one differential equation, we used a system because it allowed us to model the heterogeneous composition of the bee population.

4 Analysis of Population Model

The model, which gives the graph seen in Figure 4.1, shows seasonal trends within the bee population, but the colony does not fully die out during the winter or fall which shows that the climate that we chose to model is relatively mild has some warm winters/falls.

It can be seen that in this model, there is a subtle increase in the bee population, however, if you were to continue to run the model up to about 16 years, the curve begins to flatten and oscillate around 100,000 bees in the hive which is consistent with actual sizes of very large bee hives. This may seem like a lot, but it is worth considering that diseases such as parasites and other deaths were not included in the bee hive and, therefore, the beehive is expected to be very large.

It was found that variables such as the initial population of bees and the survival

rates affected the model very little, but the days of development along with the seasonal equations for the bee population altered the entire model by 25% if removed or altered. Additionally, the dip in the graphs found at the peaks annually show the bee migration due to new queen bees. This change also affects the model very little, with the population only affecting long term growth by less than 5%.

4.1 Sensitivity Analysis

4.1.1 Cross Checking with Professional Models

There are already many different models that model bee populations, the most prominent being found in David J. Torres, et al. [3]. In this article, they have already developed Matlab code in order to model the amount of individuals in a given hive. We modified this code to address this problem. After adjustments, certain features were added such as being able to change the starting ratios of the different castes as well as the ability to add prints to find exact number of bees in each caste. A rounding system was also implemented in order to stop the occurrences of fractional bees. In addition, a total bees and a final print out was added just to make the reading of the graph easier. The resulting graph is shown in Figure 4.2.

However, it can be seen that there are some problems with this model. The model seems to provide too low of an estimate for bees and that is because of some blatantly wrong factors that they use. Once putting in more accurate numbers, it can be seen that the peaks should be much closer to 100,000 bees during peak season. The main contributor to this problem was the low ratio of foragers to drones which decreased the amount of food that can go into the hive and therefore the max population.

Comparing it to our model, we see multiple key differences. One of which is the fact that the amount of bees that died during winter is vastly different and therefore, there is some discrepancies in the models. However, because the periods of the lapses and everything else seems to be somewhat similar, it can be concluded that the models have one major difference which is how harsh the winters are.

4.2 Strengths

The strengths of approaching the model like this is the fact that additional variables can be added in the future which makes this model very flexible and adaptable compared to the professional model in which everything is a complex series of sums [3].

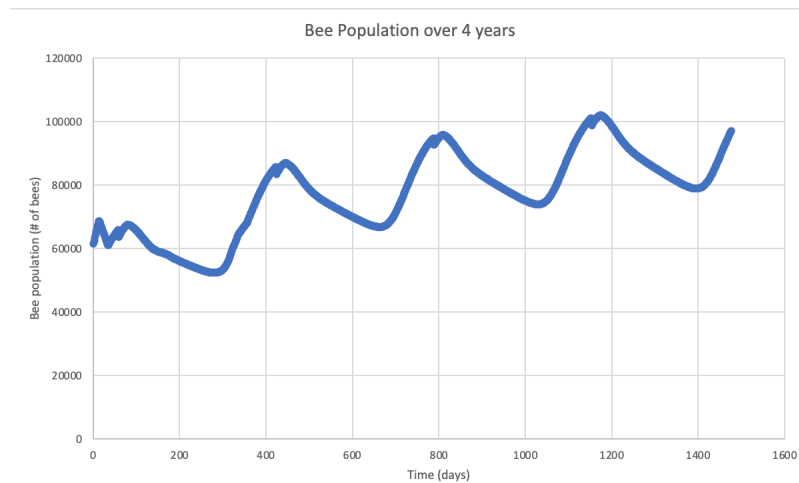


Figure 4.1: Number of bees in a colony vs time (our model)

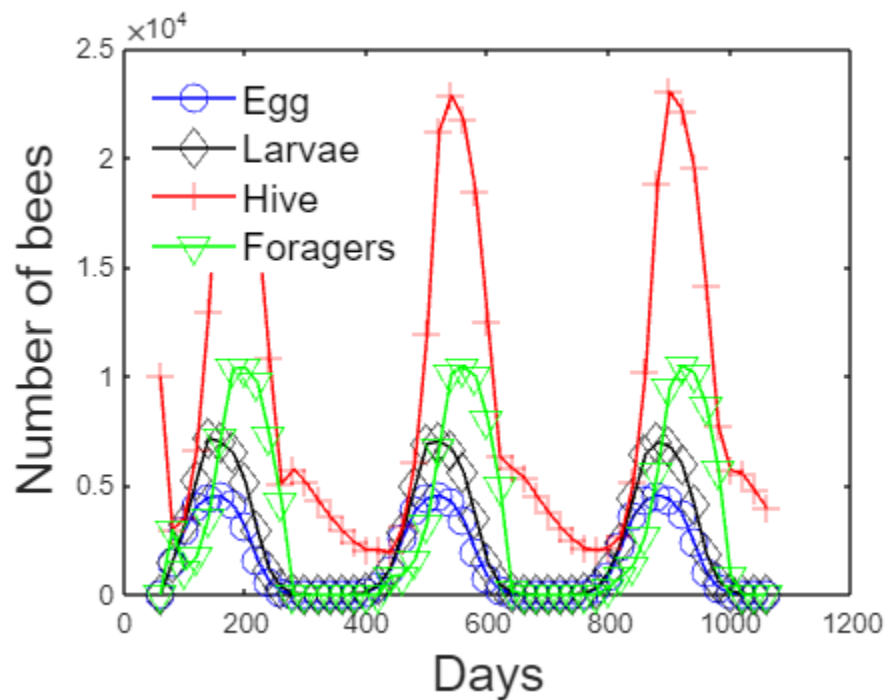


Figure 4.2: Number of bees in a colony vs time (Modified Professional Model)

4.2.1 Consideration of Bee Heterogeneity

This model considers the composition of the bee population. Rather than simply model changes in the population as a function of population size, it models the population change as a function of the number of foragers, eggs, larvae, etc.

4.3 Weaknesses

4.3.1 Computational Limitations

This model was limited by the need for timely computation using personal computers. We were forced to make simplifying assumptions to reduce computational complexity. For example, we had to assume that survival rates were constant between each stage. When we tried to make a model with variable survival rates, it took too long to run.

5 20-Acre Parcel Pollination Model

5.1 Overview of Pollination Foraging Dynamics

Honeybee pollination starts when a bee lands on a flower. Pollen is attracted to the hair on the bee's legs by electrostatic forces. The stiff hairs allow the bee to carry pollen back to its hive. In a day, a bee can take pollen from around 100 flowers every foraging trip for 10 trips a day, for a total of 1,000 flowers. [8] When the bee lands on a different flower at a later point in time, there is a chance that some of the pollen stored on the bee's legs from previous flowers will be deposited on the new flower. When pollen is transferred from one flower to another of the same species, self-pollination occurs. [10] Self-pollination is critical for the survival of plant species. For our model, it is important to know what proportion of collected pollen will be self-pollinated before the bee makes it back to the hive.

The volume of pollen foraged by an individual bee depends on environmental factors and factors within the hive. For example, when pollen is removed from the hive or the hive population increases, bees tend to increase the intensity of their foraging. Additionally, it has been determined that bees can determine the volume of reward (like sucrose solution or nectar) from the feeding sources and tend to maximize their reward. [1] It has been previously shown that bees also maximize their energy efficiency (that is, the energy gained per unit of energy spent) rather than their energy gain per unit of time.

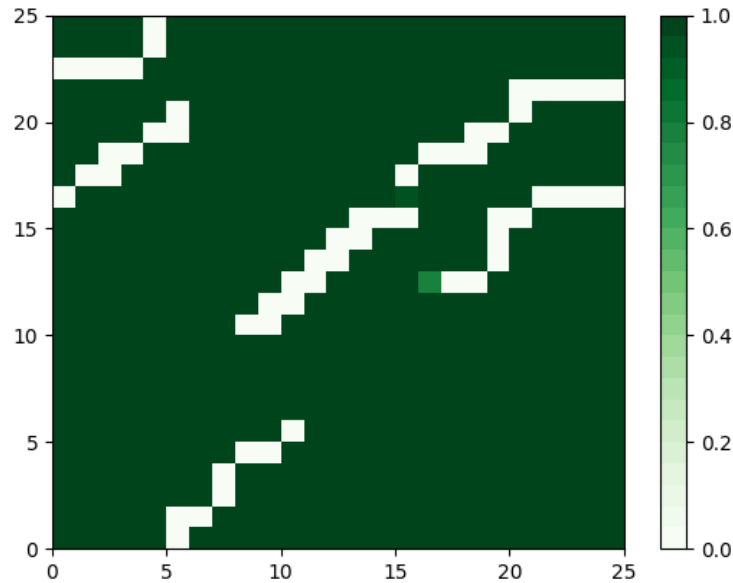


Figure 5.1: Resources after the first trip

5.2 Optimizing the Number of Bees

The main idea governing the pollination model is resource scarcity. There must be an upper bound to the number of bees because bees and beehives require a certain amount of resources to survive. If there are too many hives, then the bee population will collapse due to the lack of resources. [7] We created a simulation using the Python library Matplotlib in order to analyze the process of bees foraging for resources. In this simulation, we placed three hives at different locations around the field with the conservative estimate that each hive had less than 1000 bees. The results are shown in Figures 5.1, 5.2, and 5.3. A greener grid square indicates more resources are present in that location.

These figures show that if we increase the number of hives in an attempt to increase pollination, then the bees will not be able to support themselves, so pollination will actually *decrease*. In order to find the optimal number of bees, we must first determine the optimization formula that accurately describes our goal, given by

$$\begin{aligned} \max_t \quad & P_0 \\ \text{s.t.} \quad & R(P(t)) < R_{\max}, \\ & P(t) > 0. \end{aligned}$$

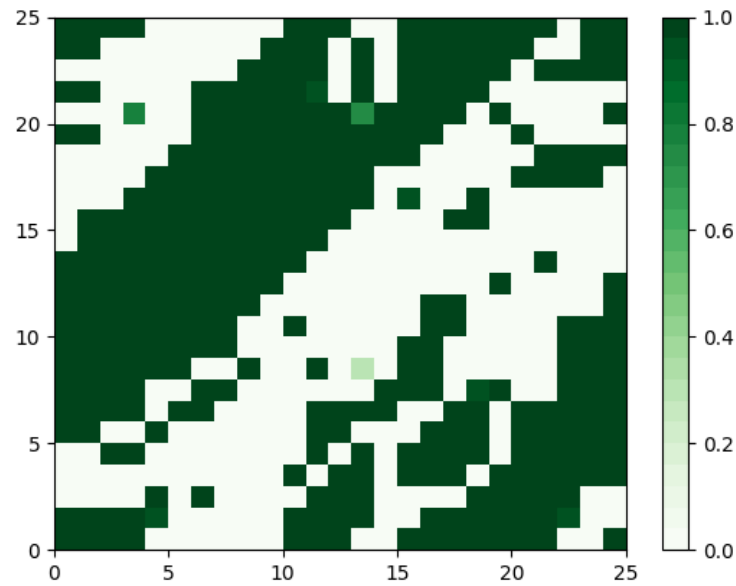


Figure 5.2: Resources after the fifth trip

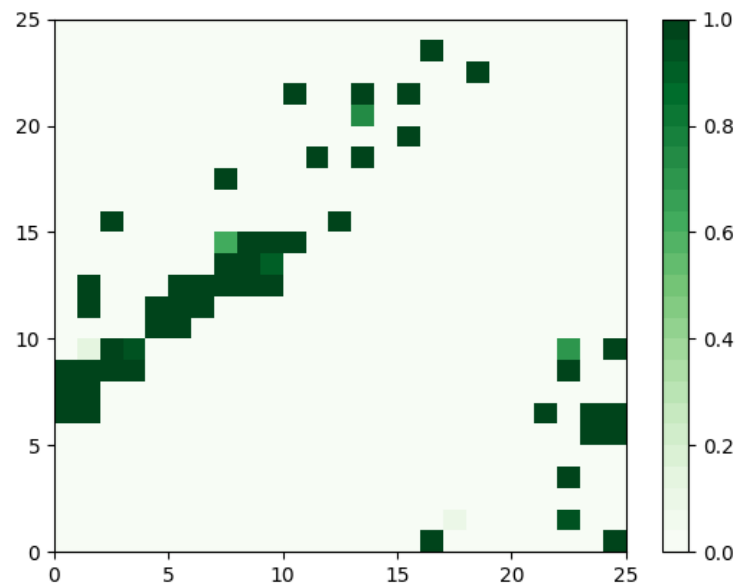


Figure 5.3: Resources after the tenth trip

In words, we wish to maximize the population of bees whose demand for resources is constrained above by the maximum number of resources available in the field. Solving this formula will give us the maximum initial population of the field that is sustainable. From there, we can get the number of hives that we should place in the field. When we solve this formula, we get that the number of hives that should be placed in the field is approximately 4. That is, if we put more than 4 hives in the field, then it is possible for the bees in at least one hive to die out from a lack of resources.

5.3 Model Analysis

Since there is only one degree of freedom in this model (that is, the initial population of the field), a sensitivity analysis for this model will not prove to be useful. However, it is still helpful to discuss the strengths and weaknesses of this model in order to evaluate its applicability to the real world.

5.3.1 Strengths

One of the biggest strengths of this model is that we considered the effect of the number of available resources on the sustainability of the bee population. In order for a field to be consistently pollinated, there has to be pollinators. If the amount of hives leads to an equilibrium population that is above the carrying capacity of the field, then the pollinators will die out, and the field will not be supported.

5.3.2 Weaknesses

A weakness in this model is that we do not discuss the speed of the pollination. It is likely that one bee depositing pollen from one flower to another is not enough for the second flower to be fully pollinated. In order to ensure that a flower is properly pollinated and to sustain the pollination of the field as a whole, bees need to make multiple visits to the flowers of a plant. Additionally, the flight energy of the bee was not considered. Since the bee could be taking a convoluted path to get resources from each flower, it is likely that the bee expends a lot of energy carrying the heavy nectar from place to place. This could be considered to expand the model in the future.

6 Conclusion

Our bee population model has shown some important facets of bees. Firstly, the egg laying rate of the queen is the main way that the population increases, while

the mortality rate is how the population decreases. By taking these into account in different ways, we can form a comprehensive model. For example, in the summer, worker bees exert more energy and so have shorter lifespans, increasing the mortality rate. On the other hand, the bees have a greater access to pollen and nectar, two important food sources for bees, meaning that the queen has more energy to lay eggs. This interplay is captured in our model. We also realize that there are multiple types of bees. The ways that the population changes itself changes based on whether one looks at the larvae, pupi, workers, or scouts.

By using our population model, we can also model what would happen if we placed bee hives in a 20-acre parcel of land. Crucially, we realized that resources were not infinite for the bees in this parcel, which we showed by generating a simulation for a system of three arbitrary colonies. This means that we had to constrain our model from above in order to account for the maximum number of resources available on a given day.

All in all, our models are fairly accurate, especially that for the honeybee population. We used factors that are considered in research done in academia, and we were able to find patterns that are reproduced by real bee populations.

6.1 Implications on the Real World

The most important part of our model is how it interacts the world and what it says about bees. To maximize the usefulness of bees, we need to consider when they are most available. Specifically, in order to take advantage of the superior pollination skills of honeybees, a farmer should only use them in the summer. In the winter, the farmer should find another pollinator that is better suited to the harsh and cold conditions of that season.

Additionally, a farmer or beekeeper that wants to maximize crop yield should consider calculating the suitable amount of bee hives that will effectively pollinate and thrive in their field. Introducing too many hives into a field on paper looks like it would ensure complete pollination. However, this overintroduction would have the adverse effect of causing some of the hives to die out, resulting in the equilibrium we found of 5 hives. A farmer can conserve their monetary resources by only buying the infrastructure for the maximum number of sustainable hives and no more.

Buzz! Optimize Your Farm With the Power of Bees

Pollination is the agricultural management practice with the greatest contribution to yields. [4] Pollinators improve the quality and quantity of crops: in one study, bees were found to increase yield by up to 62% on average. [12]

There are a few important considerations to make when adding bees to your farm. Other than the season (make sure not to introduce bees to your farm in the winter!), the health of your queen bee and availability of food for the bees are most important to promoting bee populations. So, you should make sure to plant lots of flower-bearing crops to give bees the pollen and nectar they need to give you the most yield.

In addition, the type of plants that you have in your farm help to optimize the number of flowers that each bee can pollinate. For example, almond trees are some of the most nutritious plants for bees in terms of pollen. The pollen given by almond flowers actually has all of the proteins that your little bee larvae need to thrive.

Okay, you've been convinced: bees are a worthwhile and valuable addition to your farm. What's the best way to add them? We studied the number of beehives necessary to optimally pollinate a twenty-acre parcel of land, and found that 5 produces optimal results.

Notably, increasing the number of beehives does not necessarily result in increasing pollination. Each beehive requires resources to survive. If more beehives are present than the land is able to support, then there will be a dramatic decline in the bee population, resulting in worse pollination outcomes.

Takeaways:

- Bees are a highly effective way to boost agricultural yields.
- For best results, approximately 5 beehives for 20 acres of land are necessary.
- Bees' ability to pollinate is dependent on the strength of their colony, which depends on their food sources. For optimal results, plant lots of crops, especially almond trees!

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A Code for Population of Bee Hives

This code was made to calculate and solve by computation the differential equation and specific instances.

```
1  from math import exp
2  import numpy as np
3  np.set_printoptions(threshold=np.inf)
4
5  dE_list = np.array([])
6  dL_list = np.array([])
7  dP_list = np.array([])
8  dW_list = np.array([])
9  dD_list = np.array([])
10
11  # dE_list = []
12  # dLP_list = []
13  # dW_list = []
14  # dD_list = []
15
16  dE = 0
17  num_eggs = []
18  num_larva = []
19  num_pups = []
20  num_work = [300000]
21  num_drone = [500]
22  percent_work = 0.985
23  percent_drone = 0.015
24  total_bees = [num_work[0] + num_drone[0]]
25  cannibalism_larva = [0.23, 0.3, 0.58, 0.06, 0]
26
27  s_E = 0.97
28  s_L = 0.99
29  s_P = 0.999
30  s_W = 0.985
31  s_D = 0.985
32
33  days_E = 3
34  days_L = 5
35  days_P = 12
36  days_W = 21
37  days_D = 21
38
39  season1 = 0
40  season2 = 0
41
42  x1 = 385
43  x2 = 30
44  x3 = 36
45  x4 = 155
46  x5 = 30
47  sum = 0
48
```

```

49     egg_max = 1600
50
51     for t in range(4000):
52         season1 = 1 - (1/(1 + (x1 * exp(-2 * (t%365)/x2))))
53         season2 = 1/(1 + x3 * exp(-2 * (((t%365) - x4)/x5)))
54
55         dE = egg_max * (1 - np.maximum(season1, season2))
56         dE_list = np.append(dE_list, dE)
57         dL_list = np.append(dL_list, dE_list[t - days_E] if t - days_E >= 0 else
0)
58         dP_list = np.append(dP_list, dL_list[t - days_L] if t - days_L >= 0 else
0)
59         dW_list = np.append(dW_list, percent_work * dP_list[t - days_P] if t -
days_P >= 0 else 0)
60         dD_list = np.append(dD_list, percent_drone * dP_list[t - days_P] if t -
days_P >= 0 else 0)
61
62         if(t == 0):
63             dW_list[0] = num_work[0]
64             dD_list[0] = num_drone[0]
65
66
67         if t - days_E >= 0:
68             dE_list[t - days_E] = 0
69         if t - days_L >= 0:
70             dL_list[t - days_L] = 0
71         if t - days_P >= 0:
72             dL_list[t - days_P] = 0
73         if t - days_W >= 0:
74             dW_list[t - days_W] = 0
75         if t - days_D >= 0:
76             dD_list[t - days_D] = 0
77
78         dE_list = dE_list * s_E
79         dL_list = dL_list * s_L
80         dP_list = dP_list * s_P
81         dW_list = dW_list * s_W
82         dD_list = dD_list * s_D
83
84         num_eggs.append(np.sum(dE_list))
85         num_larva.append(np.sum(dL_list))
86         num_pups.append(np.sum(dP_list))
87         num_work.append(np.sum(dW_list))
88         num_drone.append(np.sum(dD_list))
89         sum = np.sum(dE_list) + np.sum(dL_list) + np.sum(dP_list) + np.sum(
dW_list) + np.sum(dD_list)
90         total_bees.append(sum)
91
92         if(sum > 50000):
93             dL_list[t - 1] = dL_list[t - 1] * (1 - 0.23)
94             dL_list[t - 2] = dL_list[t - 2] * (1 - 0.3)
95             dL_list[t - 3] = dL_list[t - 3] * (1 - 0.58)
96             dL_list[t - 4] = dL_list[t - 4] * (1 - 0.06)
97

```

```

98     if((t % 365) == 160 and sum > 50000):
99         dW_list = dW_list * 0.5
100        dD_list = dD_list * 0.5
101
102    if((t % 365) == 200 and sum > 50000):
103        dW_list = dW_list * 0.5
104        dD_list = dD_list * 0.5
105
106    # np.multiply(dE_list, s_E)
107    # np.multiply(dLP_list, s_LP)
108    # np.multiply(dW_list, s_W)
109    # np.multiply(dD_list, s_D)
110
111    # dE_list.append(1500)
112    # dLP_list.append(dE_list(t - 2) if t - 2 >= 0 else 0)
113    # dW_list.append(percent_work * dLP_list(t - 11) if t - 11 >= 0 else 0)
114    # dD_list.append(percent_drone * dLP_list(t - 11) if t - 11 >= 0 else 0)
115    # dE_list = dE_list * 2
116    print("Day")
117    # print(dE_list)
118    # print(dL_list)
119    # print(dP_list)
120    # print(dW_list)
121    # print(dD_list)
122    print(total_bees)

```

B Code for Pollination Foraging Simulation

This code was developed to display the foraging patterns of bees. Since this simulation only showed one day, the population model was not used. This code sample is rigorously commented in order to explain exactly what each section is doing.

```

1  import numpy as np
2  import matplotlib as mpl
3  import matplotlib.pyplot as plt
4  import imageio
5
6  # Necessary constants
7  GRID_DIM = 25
8  FLOWERS_PER_PLANTS = 1000
9  PLANTS_PER_ACRE = 36
10 TOTAL_PLANTS = PLANTS_PER_ACRE * 20
11 PLANTS_PER_SQUARE = TOTAL_PLANTS / (GRID_DIM ** 2)
12 FLOWERS_PER_SQUARE = FLOWERS_PER_PLANTS * PLANTS_PER_SQUARE
13 DELTA_T = 1/10
14
15 # Class that holds information about a given colony
16 class Colony():
17     def __init__(self, num_bees, coord):

```

```

18         self.num_bees = num_bees
19         self.worker_bees = 0.3 * self.num_bees # The number of worker bees
tends to be 30% of the population
20         self.x, self.y = coord # The location of a colony
21
22         greens = mpl.colormaps['Greens'].resampled(GRID_DIM)
23
24         # The code used to simulate the gathering of resources
25         def gather_resources(Pmat, x, y, needed):
26             # If a square has less than the flowers needed to be visited, then move
to a random adjacent square
27             if Pmat[x,y] < needed:
28                 remain = needed - Pmat[x,y] # The remainder of the flowers that need
to be visited
29                 Pmat[x,y] = 0
30                 # Recursively repeat the process at a square that is adjacent
31                 Pmat = gather_resources(Pmat, (x + np.random.randint(-1,1)) %
GRID_DIM, (y + np.random.randint(-1,1)) % GRID_DIM, remain)
32             else:
33                 Pmat[x,y] -= needed
34
35             return Pmat
36
37         def update(Pmat, Bset):
38             for b in Bset:
39                 Pmat = gather_resources(Pmat, b.x, b.y, (b.worker_bees * 100) / (
FLOWERS_PER_SQUARE)) # Each worker bee visits 100 flowers during one trip
40
41             return Pmat
42
43         # Function used to plot the heatmaps representing the field and save them in
a folder
44         def plot(Pmat, Bset):
45             files = []
46
47             for i in range(int(1/DELTA_T)):
48                 name = f'./plots/{i}.png'
49                 files.append(name)
50
51                 Pmat = update(Pmat, Bset)
52
53                 plt.cla()
54                 fig, ax = plt.figure(), plt.axes()
55                 psm = ax.pcolormesh(Pmat, cmap=greens, rasterized=True, vmin=0, vmax
=1)
56                 fig.colorbar(psm, ax=ax)
57                 plt.savefig(name)
58
59             return files
60
61         P = np.ones((GRID_DIM, GRID_DIM)) # The initial field (a 25x25 grid that has
all of its resources)
62         B = {Colony(1000, (5, 10)), Colony(500, (15, 15)), Colony(700, (20, 5))} # A
set containing three colonies placed at arbitrary locations

```

```
63
64     if __name__ == '__main__':
65         filenames = plot(P, B)
```