Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
A		2227089

Power Levels of a Cyclist: It's Over 9000!!

The problem given was to use various factors of cyclists and courses in order to determine the power profile and the optimal power output of the rider at each point during a course. We were also tasked with considering different perturbations that we could encounter in our model. For example, we had to take into account differing wind conditions and potential deviations that the rider could take. Finally, we had to discuss the application of our model to a team of cyclists, all working together to collectively achieve the fastest time.

Firstly, after declaring our assumptions and variables, we worked on creating a model to describe the power profile of a cyclist. By considering the limits of the body with respect to VO2 and P_i concentration, we determined a piece-wise function that would calculate the maximum power output of a general cyclist after cycling for a given amount of time. However, different types of cyclists have different power profiles, which we needed to consider. We therefore used empirically determined data to find out the specific power outputs of time trial specialists and sprinters.

Next, we made a model to determine the optimal power output of a cyclist given a course. In this step, we created different models to have two exponents: one regarding external conditions and the other considering the biochemistry of the situation. After developing the equations, we had to simulate the course in order to test them out. We tried different approaches, such as using the Runge-Kutta method for solving differential equations, but we ultimately decided to use a modified genetic algorithm to simulate many different cyclists with different power curves, eventually finding the trends that were prominent in the most successful cyclists.

Afterwards, we extended and analyzed our model. Specifically, we tested different perturbations, such as varying weather conditions and rider deviations, and used principles from chaos theory to determine whether or not those perturbations would be disastrous to our model. Through this analysis, we found that these perturbations did not affect our model by a significant amount. Additionally, we found how a team of riders could finish the course differently than an individual and determined the degree to which this occurred.

Finally, we created an article for a cyclist and their Directeur Sportif to analyze deficiencies in the rider's performance. We also gave suggestions to them on how the cyclist's performance could be improved, based on the trends displayed by the simulation of our model.

Contents

1 Introduction

1.1 Background

For any given activity that requires muscular output, the body exerts an output of energy over time. In general, a person's intensity during demanding physical activity decreases over time due to fatigue, leading to a reduction in their power [5]. The power output of an exercising individual (and its decay) can be displayed in what is called a power curve.

With cycling in particular, power curves are a common metric in the analysis of a rider's maximal power output at different points of time. There are many different types of races, and various cyclists have distinctive power exertion profiles. Thus, different cyclists should ride with unique power curves in order to achieve an optimal time during a race, and matching this distribution with a cyclist is essential. While training has been revolutionized with the use of mobile power meters, there is not yet a clear connection between the power profile of a cyclist and their specific stratagems for a given course.

1.2 Problem Restatement

In order to properly model the situation, we need to understand what the problem asks of us. This problem contains five elements that we need to tackle:

- Determine the power profiles of two different types of riders (one time trial specialist and one other), considering gender. We define a power profile to be a measure of the maximum power output of a cyclist over time.

- Create a model that finds any type of rider's optimal power output based on their position on any course. We need to consider the maximum total possible energy output of the rider, their previous exertions of energy, and exceeding of the power curve.

- Simulate the track conditions of various time trial courses and apply our model to it in order to determine the model's efficacy. We also need to create our own track and test our model against it.

- Analyze our model in the presence of different disturbances. These variances can come in the form of weather conditions or the rider's deviations from the optimal power curves.

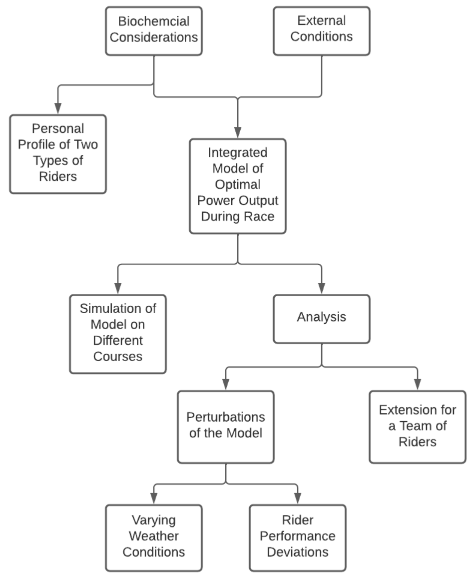


Figure 1.1: A broad outline of the work that we do for the model

- Extend our model and apply it to a team of racers, considering how the bikers might interact with each other.

1.3 Our Work

A flow chart of the work that we do to construct our model is shown in Figure 1.1. As displayed, we created models to determine the power of any given cyclist and the power needed for any given course. Then, we analyze our models in various ways, through simulations and perturbations, and provide an extension that applies to a team. Finally, we write a guidance for a cyclist and their Directeur Sportif to analyze the results of the biker's performance and suggest improvements.

2 Assumptions and Definitions

2.1 Variables

Symbol	Definition
x	The position of the cyclist on the course
$\dot{x}$	The speed of the cyclist
$\vec{\dot{x}}$	The velocity of the cyclist
$\ddot{x}$	The magnitude of the acceleration of the cyclist
P_i	The ith component of power needed to overcome external conditions (specified in their respective discussions)

2.2 Assumptions

ASSUMPTION 1: The effect of calories when cycling is negligible.
JUSTIFICATION: Since we are only considering the elite cyclists, we can assert that their diets are sufficient such that they will not run out of calories during a time trial.

ASSUMPTION 2: The gradient of any given cycling course will generally not exceed 10%.
JUSTIFICATION: Most courses follow this rule, and the cycle highway manual for North-west Europe sets a limit for a gradient 10% and only for very short sections [6]. While some time trial courses may have gradients of over 10%, these will only occur for very short distances and are therefore negligible.

ASSUMPTION 3: The change in O2 saturation of the air does not change.
JUSTIFICATION: Through out the course, the greatest amount of altitude chance of 200m. In this space, the body barley notices a change of O2 concentration or pressure meaning that the O2 saturation would not change significantly.

ASSUMPTION 4: The amount of power and consequent fatigue produced from pushing off/ going from stationary to kinetic movement at the beginning of the race is negligible.
JUSTIFICATION: While the power that is exerted is very large when kicking off, it only occurs for a very short amount of time, therefore the amount of fatigue does not increase significantly (see Fatigue equation).

2.3 Glossary

Power Profile: A set of numbers that define how well a cyclist is at certain situations. These sets of numbers usually include a 5 sec maximum power per kg, 1 min maximum power per kg, 5 min maximum power per kg, and 20 min maximum power per kg. Where under each of these conditions, the maximum power is the maximum power that can be sustained during the time period specified. Expressed in watts per kilogram

3 Cyclists' Profiles and Power Curve

3.1 Biochemistry of a Cyclist

There are two variables that must be considered:

- Cyclist's VO2 Max
- P_i concentration

The VO2 Max determines how much oxygen the body can utilize. As one approaches their VO2 max, which is determined by how much they are exerting in a period of time, more and more of the body starts fermenting lactic acid. In general, people start lactic acid fermentation at various VO2 levels. However, most trained professionals start uncompensated lactic acid concentration at around 80% of their VO2 Max. This is directly related with the P_i concentration. The P_i concentration is the next important factor. Until recently, lactic acid levels in the muscle and blood were believed to be the contributing factor to fatigue and loss of force when exercising [5]. However, new research has indicated that fatigue is related to the concentration of P_i, which are phosphate groups that form ATP.

When a muscle contracts, it uses ATP to phosphorlate proteins which creates a phosphorylated protein and ADP. Afterwards some proteins shed the phosphate group in order to recycle it with ADP to make ATP. However, the recombination of ADP and a phosphate group into ATP requires energy, which is usually supplied by aerobic respiration. Aerobic respiration which produces upwards of 36 ATP per glucose consumed. On the other hand, anaerobic respiration can only produce 2 ATP per glucose consumed. This dramatically decreases the rate of which ADP and P_i is being recycled into ATP. This increases the amount of P_i and ADP. While ADP is relatively benign, P_i is a charged particle which can cause interference in cell signalling or nervous signalling [8]. With the increase of P_i, the efficiency of muscles decrease which causes a reduction of up to 70% of force given by the muscle [1].

Recall that professional elites start uncompensated P_i increase when they exceed 80% of their VO2 Max. Therefore we can make the connection that if

$$\frac{VO2}{VO2_{Max}} > 0.80,$$

then the build up of P_i will start. We refer to the time at which this occurs as the critical point. Once the concentration of P_i starts increasing, the force that is applied by the muscle will start decreasing, where, eventually, the reduced muscle output will become 70% of maximum muscle force which generally occurs at concentrations of 30 mM of P_i. This yields [1]:

$$R = \begin{cases} R_t + \frac{0.70}{1+e^{-k_1(t-10)}} & : \frac{VO2}{VO2_{Max}} > 0.80 \\ R_t + \frac{2*R_t}{1+e^{k_2 t}} & : \frac{VO2}{VO2_{Max}} < 0.80 \\ R_t & : \frac{VO2}{VO2_{Max}} = 0.80 \end{cases}.$$

where R is the percentage reduction in force as a function of time, k_1 is the rate of P_i concentration increase, and t is time passed. R_0 is the R value from before the function crossed the critical point of 0.8. [1]. k_1 is also required for this problem which can be represented as a linear equation with VO2 max and VO2. The relation between k and $\frac{VO2}{VO2_{Max}}$ is linear in normal humans[2]. Therefore, the it can be said that:

$$k_1 = \frac{VO2}{VO2_{Max}} Q_1 \text{ when } \frac{VO2}{VO2_{Max}} > 0.80,$$

where Q is a constant that is different for every person and is determined by experimentation.

Let intended force be F_0 and leg velocity v_{leg}. Therefore the power generated over time for a cyclist can be expressed as:

$$P = (RF_0)\, v_{\text{leg}}$$

Previously we looked at the power output of a cyclist when riding and how the power output in is related to the amount of time spent over a proportion of VO2 over VO2 max. This is actually related to how much power the cyclist has used previously. As power usage increases, the body automatically adjusts VO2 to compensate for the increase amount of energy needed to get to that power level. This occurs in two ways that increase in respiratory rate (RR) and Tidal Volume. This results in:

$$VO2 = \frac{V_{\text{in}} O_{2,\,\text{air}}}{V_{\text{out}} O_{2,\,\text{exhalation}}}$$

as the cyclist exerts themselves more and more, the amount volume of air inhaled and exhaled increases and so does the amount of O2 that is removed from the air inhaled. This means that as the cyclist exerts more energy, VO2 increases. Recall that the P_i will only increase once VO2 increases above a certain range so that $\frac{VO2}{VO2_{max}} > 0.80$. Therefore we can conclude that $P = (RF_0)\, v_{\text{leg}}$ does in fact rely on amount of previous exertion.

So far, these equations have looked at how fast fatigue goes up and how power goes down; however, it must be noted that if a biker was to go below 0.8, the build up of P_i would be reversed as the amount of oxygen that is needed to preform aerobic respiration is being met making the concentration of P_i go down. This can be modeled by the equation for k_2. Since $\frac{VO2}{VO2_{max}}$ is a proportion of how much oxygen is being used/supplied, it also is linear with how fast P_i is removed from the system. This gives us:

$$k_2 = \frac{VO2}{VO2_{max}} Q_2 \text{ when } \frac{VO2}{VO2_{Max}} < 0.80,$$

Putting it all together, we get:

$$P = \begin{cases} F_0 \left(R_t + \frac{0.70}{1+e^{-\frac{VO2}{VO2_{Max}} Q_1 (t-10)}} \right) v_{leg} & : \frac{VO2}{VO2_{Max}} > 0.80 \\ F_0 \left(R_t + \frac{2R_t}{1+e^{\frac{VO2}{VO2_{max}} Q_2 t}} \right) v_{leg} & : \frac{VO2}{VO2_{Max}} < 0.80 \\ F_0 R_t v_{leg} & : \frac{VO2}{VO2_{Max}} = 0.80 \end{cases}.$$

3.2 Power Profiles of Time Trial Specialists

The power profile of a time trial specialist would likely be a low 5sec, low 1 min, average 5 min, and high 20 min maximum power per kilogram, when compared to other world class cyclists of different specialties. A time trial specialist focuses on getting the fastest time for a given course. However, since they only race against the clock and not against each other, the psychological component of trying to overcome opponents is taken away. This means that the best way to win a time trial is to maintain a steady speed. Shorter bursts of 1 sec and 1 min of maximum speed will only serve to slow a time trialist down in the end. Therefore a time trialist would likely have lower 1 sec and 1 min maximum sustained power. Meanwhile, 5 min bursts of higher energy may be more useful towards the end of the race if the trialist deems that he or she have enough energy. The most useful number for a time trial specialist and the number that should be the most helpful is the 20 min maximum sustained power output as it determines how fast they can sustain a certain pace during the time trial

[3]. Using previously determined empirical data from this source, we can find the values for a power profile of a male world class time trialist named "Boe Jiden" and a female world class time trialist named "Alice":

Boe Jiden's Power Profile

5 sec	1 min	5 min	20 min
23.00	11.16	7.39	6.40

Top World Class Performance for All Categories of Power Profile (Males)

5 sec	1 min	5 min	20 min
25.18	11.5	7.60	6.40

Alice's Power Profile

5 sec	1 min	5 min	20 min
18.56	9.02	6.55	5.69

Top World Class Performance for All Categories of Power Profile (Females)

5 sec	1 min	5 min	20 min
19.42	9.29	6.74	5.69

3.3 Power Profiles of Sprinters

Unlike time trial specialists, sprinters do not specialize in sustaining a speed over time, in fact, they are the opposite. Sprinters need to go as fast as possible to reach their finish line in the shortest amount of time. Generally this is a small distance to cover meaning that sprinters will have high 1 sec and 1 min powers but lower 5 min and 20 min powers. This is evident through the fact that the two races that sprinters and time trialists take are very different in nature and thus require a vastly different set of skills [3]. Using data from the website, we can create a power profile for a male world class sprinter named "Gon Soku" and a female world class sprinter named "Feyene":

Gon Soku's Power Profile

5 sec	1 min	5 min	20 min
25.18	11.39	7.19	6.13

Top World Class Performance for All Categories of Power Profile (Males)

5 sec	1 min	5 min	20 min
25.18	11.5	7.60	6.40

Feyene's Power Profile

5 sec	1 min	5 min	20 min
19.42	9.20	6.36	5.36

Top World Class Performance for All Categories of Power Profile (Females)

5 sec	1 min	5 min	20 min
19.42	9.29	6.74	5.69

4 Race Modeling

Our race model focuses on two main components in order to determine the optimal power outputs of a rider for the course. Namely, we discuss the effect of external conditions on the power curve. Then, we determine the influence of a person's biochemistry on the power they will be able to output. Finally, we combine the two aspects in order to form an accurate model.

4.1 The External Conditions Component

The effect of the external conditions of the course on power output can be split into five different elements [7]:

- Aerodynamic drag
- Rolling friction
- Turns
- Changes in (gravitational) potential energy

Aerodynamic drag. The force produced by aerodynamic drag F_D can be determined by:

$$F_D = \frac{1}{2}\rho C_D A v_{\text{rel}}^2,$$

where ρ is the density of air, C_D is the drag coefficient, A is the cross-sectional area of the cyclist facing the direction of travel, and v_{rel} is the speed of the cyclist relative to air.

At any given point in time, power is equal to the product of force and velocity. Since power is measured relative to the ground, we use the velocity of the biker with respect to the ground when determining the power needed to overcome aerodynamic resistance. Assuming that wind speed is negligible,

$$P_D = F_D\dot{x} = \frac{1}{2}\rho C_D A\dot{x}^3.$$

Rolling friction and turns. Both of these elements contribute to friction against the bicycle's movement. To analyze their effects, we move into the frame of the cyclist. They experience a gravitational force downwards and a (fictitious) centrifugal force perpendicular to the motion of the bike. Since these two forces are perpendicular to each other, the magnitude of the normal force on the bike N is given by:

$$N = \sqrt{(mg\cos\theta)^2 + \left(m\kappa\dot{x}^2\right)^2},$$

where m is the total mass of the rider and bike, g is the gravitational acceleration, $\theta = \tan^{-1}\left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)$ is the angle of the slope, and κ is the curvature of the track at any given point.

Since the friction force F_f is proportional to the normal force, we have

$$F_f = \mu m\sqrt{\left(g\cos\left(\tan^{-1}\left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)\right)\right)^2 + \left(\kappa\dot{x}^2\right)^2}.$$

We use the approximation that, for small angles x, $\cos x \approx 1$. The power needed to overcome frictional forces is therefore:

$$P_f = \mu m\dot{x}\sqrt{g^2 + \left(\kappa\dot{x}^2\right)^2}$$

Changes in (gravitational) potential energy. The increase in gravitational potential energy $\mathrm{d}U_g$ for a small change in position along the course $\mathrm{d}x$ is:

$$\mathrm{d}U_g = mg\mathrm{d}x\sin\theta,$$

where $\theta = \tan^{-1}\left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)$. The work done by the cyclist to overcome this is equal to the change in potential energy, so dividing by $\mathrm{d}t$ gives:

$$P_U = mg\dot{x}\sin\left(\tan^{-1}\left(\frac{\mathrm{d}h}{\mathrm{d}x}\right)\right).$$

Using again our second assumption that the gradient is small, we can use the approximation that $\sin x \approx \tan x \approx x$ and simplify so that

$$P_U = mg\dot{x}\frac{\mathrm{d}h}{\mathrm{d}x}.$$

In total, the external condition component of the power function is

$$P_{EC} = P_D + P_f + P_U = \dot{x}\left(\frac{1}{2}\rho C_D A\dot{x}^2 + \mu m\sqrt{g^2 + \left(\kappa\dot{x}^2\right)^2} + mg\frac{\mathrm{d}h}{\mathrm{d}x}\right).$$

4.2 The Biochemical Component

In addition to the external factors that are affecting the cyclist, there are biological and biochemical aspects at play. The groundwork for this component has been laid in Section 3, but we still must consider how previous power output effects current power output. As outlined in section 3, there is fatigue that is caused by over exertion by the rider. In that section we explored the effect of over exertion, but one question still remains, how does the previous power outputs of the race effect the fatigue of the rider? In section 3, the expression $\frac{VO2}{VO2_{Max}}$ showed up. This is the proportion of exertion to maximum possible exertion possible. This is calculated as

$$VO2 = \frac{V_{\text{in}}O_{2,\,\text{air}}}{V_{\text{out}}O_{2,\,\text{exhalation}}}.$$

This is a differential equation that is specific to every single rider.

When a rider exerts themselves, their rate of breathing in a time period increases which increases both their V_{in} and V_{out}. These numbers are usually they same. However, the change in the proportion of O2 concentration has a dramatic change as higher blood flow and higher demand for oxygen causes oxygen to be taken from the air and into the blood faster. This in turn increases the VO2 capacity. This O2 concentration is the differential equation portion of the fatigue model. Let p_O be the proportion of oxygen from breath in to breath out. It is said that the increase in oxygen uptake ($O_{2,\,\text{air}} - O_{2,\,\text{exhalation}}$) is exponential with respect to power previously [4]. Since $p_O = \frac{O_{2,\,\text{air}}}{O_{2,\,\text{exhalation}}}$ and $O_{2,\,\text{air}} - O_{2,\,\text{exhalation}} = ab^{P_p} + c$ where the exponential equation varies from person to person due to factors such as metabolism, muscle strength and efficiency, cardiac output, respiratory efficiency and P_p is the power output in the past over the amount of time that the power output occurred [4].

$$p_O = \frac{ab^{P_p} + c}{O_{2,\,\text{exhalation}}}$$

$$\frac{dp_O}{dP_p} = \frac{(ab^{P_p})(O_{2,\,\text{exhalation}}) - \left(\frac{dO_{2,\,\text{exhalation}}}{dP_p}\right)(ab^{P_p} + c)}{{O_{2,\,\text{exhalation}}}^2}$$

As seen in these two equations, the proportionality of oxygen uptake and the power that one has exerted over time during the workout/track and how it changes with increases or decreases of this number.

4.3 Final Model

We can now use both of the components of our model to determine the best course of action. Firstly, we determine the effect of power exerted and velocity on the acceleration of the cyclist. By using the idea that power is equivalent to the product of the force and velocity, the power needed to accelerate by a given amount at a certain velocity is:

$$\Delta P = m\ddot{x}\dot{x},$$

where m is the total mass of the rider and the bicycle. We also say that $P = \Delta P + P_{EC}$ because the rider deals with the external conditions. Using that, the acceleration of the cyclist is:

$$\ddot{x} = \frac{P - P_{EC}}{m\dot{x}}.$$

5 Simulation and Optimization of Model

In order to determine the most efficient strategy in traversing a course with varying gradients and turns, a simulation of each course was created and a genetic algorithm was applied to each case in order to determine the best approach in obtaining the fastest time. The 3 sections of our simulation were obtaining data, simulating time trials, and creating a genetic algorithm

5.1 Obtaining Data

Obtaining data on these courses requires first, the course layout as determined by the Union Cycliste Internationale (UCI) and secondly, the processing of the different factors as a result of the track's layout on the Earth.

5.1.1 Course Layout

The course layout was determined by locating the official images online that were provided by the Union Cycliste Internationale (UCI). Vision processing was implemented via the Scikit Image library of Python. the outlined track was obtained in this method, and GPS Geolocation helped obtain the coordinates and elevations needed for all the processing.

5.1.2 Location Processing

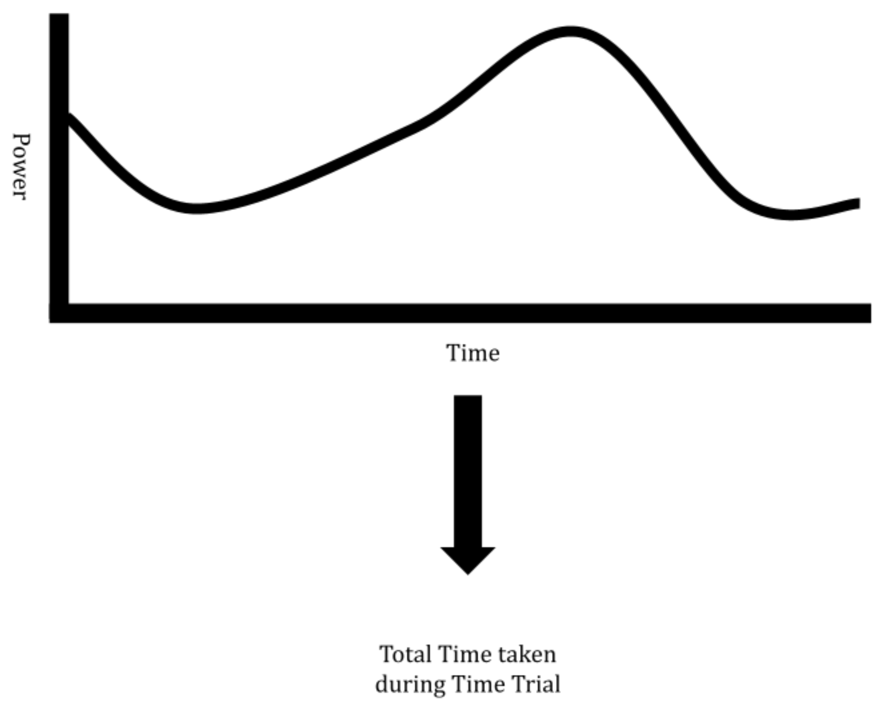
Additionally, the GPS coordinates were used to calculate both the change in elevation (gradient) along with the curvature of the path at each location. The dataset's structure can be seen below:

2021 Tokyo Olympic's Cycling Time Trial Coordinates

Longitude	Latitude	Elevation	Curvature	Elevation Change
138.9286	35.37317	591	0.009304	-0.09501

5.2 Simulation of Time Trials

The time trials were simulated with the idea the final model in mind. A program was written that calculated a time-step differential equation that used a power curve as an input and would output the time taken to finish the race. A visual is shown below:



5.3 Genetic Algorithm

Lastly, in order to optimize the time taken on the various courses, a genetic algorithm was created in order to generate a best power curve for each situation. This genetic algorithm first created generation one of a randomly generated a power curve, with 1000 children. The simulation was then run with the different power curves all resulting in a time output. The lowest 10 time values were kept, and each unique first step of the values produced 100 new children in the next generation, where the second step of the time value was randomly generated. This process continued until 100 different individual children finished the Time Trial, and the results were analyzed. The difficulties of creating this model were mainly in the random generation of the power curves. If too much energy were utilized, then the simulation should not be feasible, so energy calculations based on the biochemistry mentioned above were used to create a model where, if the power generated exceeded the W' requirement, then the rider would have to significantly reduce their power until it were below their critical power. At this point, energy reserves would be allowed to replenish through the aforementioned biochemical equations, and the rider would then be able to continue exerting power above the curve. If the external power calculations resulted in higher values than the power, the acceleration would decrease, but if the velocity became less than 0, the program simulated a resting state, where the rider would not continue to ride, but would instead stay in one place while recovering their energy threshold.

5.4 Results

The simulation results were quite interesting. For both individuals for time trialists, significantly more power was recommended during uphill portions compared to the individuals of the sprinter class. Additionally, the time trialists finished with faster times than the sprinters on all the tracks.

6 Perturbation of Model

In this section, we discuss how different perturbations could affect our model. We look at the effect of varying weather conditions, namely wind speed and direction, as well as the rider's deviations from optimal power output. In order to determine how disastrous perturbations are to our model, we use elements of chaos theory in order to determine whether or not our model is chaotic given these variations in initial conditions.

6.1 Varying Weather Conditions

There are two main considerations for weather:

- Wind speed
- Wind direction

These two components only affect the air resistance component of our model, so we do a deeper analysis of that aspect.

From earlier, we know that the force of air resistance without any wind is:

$$F_D = \frac{1}{2}\rho C_D A\dot{x}^2.$$

When wind speed is not negligible, we will consider it as a vector $\mathbf{v}_{\text{wind}}$. Combining this with $\vec{\dot{x}}$, we get a relative velocity of

$$\mathbf{v}_{\text{rel}} = \vec{\dot{x}} - \mathbf{v}_{\text{wind}}.$$

From this, we get that the power needed to overcome air resistance is:

$$P_D = \frac{1}{2}\rho C_D A v_{\text{rel}} \left(\vec{\dot{x}} \cdot \mathbf{v}_{\text{rel}}\right).$$

If the air speed is small relative to the cyclist's velocity, which it likely will be, then this new model for air resistance will not be a large enough difference from the old one to cause substantial chaos in the system.

6.2 Rider Deviations

Since our model depends on a set of differential equations, rider deviations from the optimal power output, as long as they are sufficiently small, will not disturb the model by a noticeable amount. Our simulations show general trends that are dependent on the course, so if the cyclist follows those trends, then the perturbations caused by the deviations will not affect the time taken to complete the time trial by a significant amount.

7 Extension of Model

For an extension of the model, we will consider a team of cyclists, where each team contains six cyclists. Since the time is only counted when the fourth rider finishes, we will consider the effects of how the team as a whole can contribute to a faster time than simply an individual.

The main way in which a team of riders can improve their collective time through a process called drafting. Drafting occurs when a cyclist is behind another cyclist, causing the cyclist in the back to face less air resistance than the rider in the front, decreasing the power needed for the biker in the back. Specifically, the team can all achieve an optimal slipstream by utilizing a Belgian tourniquet formation, which can effectively be modeled as a group of six cyclists rotating at a constant angular velocity ω while the system as a whole has a translational velocity.

For the sake of simplicity, we will assume that every rider except the rider in the front, due to drafting, experiences no air resistance. This means that the additional power exerted by the rider in the front when compared to the other rider in the team is:

$$P_D = \frac{1}{2}\rho C_D A\dot{x}^3.$$

However, by symmetry, each rider experiences $\frac{1}{6}$ of this additional power for every full revolution. Therefore, on average, when considering that the time for each revolution is small compared to the total time of the race, the average extra power gained over individually cycling is:

$$P_{\text{extra}} = P_D - \frac{1}{6}P_D = \frac{5}{12}\rho C_D A\dot{x}^3.$$

We can then subtract this value from the power needed to overcome external conditions (derived in Section 4.1) to get the average power needed to overcome external conditions in a team:

$$P_{EC,\,\text{team}} = P_{EC} - P_{\text{extra}} = \dot{x}\left(\frac{1}{12}\rho C_D A\dot{x}^2 + \mu m\sqrt{g^2 + \left(\kappa\dot{x}^2\right)^2} + mg\frac{\mathrm{d}h}{\mathrm{d}x}\right).$$

Now, we can solve the set of different equations numerically again (using the process suggested in Section 5) to get the time needed to complete a team time trial and the power curve associated with it. An interesting thing to note about this model is that the revolution rate of the tourniquet did not factor into determining the power.

8 Rider's Race Guidance

We have split this analysis into multiple sections. The first dictates time trialist recommendations, the second sprinters, the third climbers, and the fourth puncheurs.

I. The time trialists have a good all-round performance on all sections of the race. As such, we recommend that they use these strengths to exert less power over uphill climbs, while using significantly more power on level fields. Additionally, power exertion should stay low at the beginning, while it should increase on average as the time continues.

II. The sprinter has a very difficult time with longer courses, but to maximize the effort, sprinters should focus on using great spurts of power in short periods of time accompanied by times of recovery. A balance should be met, where power exerted should not exceed an amount where times of over 10 minutes would be needed to recover from these efforts.

III. The climbers are also relatively versatile in terms of their long term endurance. As such, less power should be utilized during periods of flat terrain, and more should be used when riding up hills. That way, more speed can be gathered, and downhill portions can be used in recovery efforts.

IV. The puncheur has a very similar profile to the sprinters. As such, they should focus on having great spurts of power over time, while also exerting high amounts of power during climbs. Similar to the climbers, the downhill portions can be used as recovery efforts along with the relatively flat areas, if no hills are present.

References

[1] Christopher W Sundberg et. al. "Effects of elevated H + and P i on the contractile mechanics of skeletal muscle fibres from young and old men: implications for muscle fatigue in humans". In: *The Journal of Physiology* 596 (2018).

[2] Donna M. Mancini et. al. "Detection of abnormal calf muscle metabolism in patients with heart failure using phosphorus-31 nuclear magnetic resonance". In: *American Journal of Cardiology* 62 (1988), pp. 1234–1240.

[3] Coach Damien. *Power Profile*. URL: %5Curl%20%7Bhttps://cyklopedia.cc/cycling-tips/power-profile/#:~:text=What%20is%20Power%20Profile%3F%20Power%20Profile%20is%20telling,capability%20and%2020%20min%20to%20describe%20our%20FTP.%7D.

[4] E C Rhodes F Xu. "Oxygen uptake kinetics during exercise". In: *Sports Medecine* 27 (1999), pp. 313–327.

[5] Asok Kumar Ghosh. "Anaerobic Threshold: Its Concept and Role in Endurance Sport". In: *The Malaysian Journal of Medical Sciences* 11.1 (2004), pp. 24–36.

[6] Interreg. *Slopes and Gradients*. 2022. URL: https://cyclehighways.eu/design-and-build/design-principles/slopes-and-gradients.html.

[7] James C. Martin et al. "Validation of a Mathematical Model for Road Cycling Power". In: *Journal of Applied Biomechanics* 14 (1998), pp. 276–291.

[8] Håkan Westerblad, David G. Allen, and Jan Lännergren. "Muscle Fatigue: Lactic Acid or Inorganic Phosphate the Major Cause?" In: *Physiology* 17 (2002), pp. 17–21.