Nonlinear Observer for Tightly Coupled Integration of Pseudorange and Inertial Measurements

Guide and Navigation Systems

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Outline

Introduction

This presentation shows the obtained results by the studying of the article written by Tor. A. Johansen and Thor I. Fossen.

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Nonlinear Observer for Tightly Coupled Integration of Pseudorange and Inertial Measurements

Tor A. Johansen and Thor I. Fossen

Introduction

The goal of navigation system is to estimate the vehicle position within a certain area. The most common approach is to implement a inertial navigation system (*INS*), based on *IMU* (Inertial Measurement Unit).

An *INS* is a navigation system that calculates the position, the orientation and the velocity of a moving vehicle without the need for external references, by using a computer, motion sensors (accelerometers), rotation sensors (gyroscopes) and occasionally magnetic sensors (magnetometers).

Introduction

In order to estimate position, velocity and orientation, inertial sensors measurements have to been integrated.

$$p = \int_0^t (v + \beta)dt = (\bar{v} + \beta)t$$

Therefore sensor bias and noise effects diverge during time. To solve this problem, *INS* is aid by other techniques, such as *GPS*, *range* − *range* measurements, etc..

Introduction

Here the goal is to support *INS* with *pseudorange* measurements, obtained by transponders, and *nonlinear observers*.

Introduction

There are two main approaches to design nonlinear observer:

- *Loosely Coupled*: range measurements are used to estimate position and velocity and given to the observer;
- *Tightly Coupled*: range measurements are directly used in a state observer together with inertial measurements.

Here the second approach is applied.

Outline

Vehicle Kinematics

The vehicle kinematic model is given by

$$\dot{p}^n = v^n$$

$$\dot{v}^n = R_b^n f^b + g^n$$

$$\dot{R}_b^n = R_b^n S(\omega_{ib}^b)$$

Where p^n, v^n, f^n are position, velocity and proper acceleration in NED (North-East-Down), respectively, while the attitude is described by a rotation matrix R_b^n that represents the rotation from *body* to NED; ω_{ib}^b represents the rotation rate of body with respect to ECI (Earth-Centered-Inertial) and g^n denotes the gravity vector. We also assume NED to be an inertial frame.

Inertial Sensor Models

The inertial sensor model is based on the strapdown assumption

$$f^b_{IMU} = f^b + \epsilon_f$$

$$\omega^b_{ib,IMU} = \omega^b_{ib} + b + \epsilon_\omega$$

$$\dot{b} = \epsilon_b$$

$$m^b_{mag} = m^b + \epsilon_m$$

where ϵ_f, ϵ_ω and ϵ_m account for noise, and b denotes the rate gyro bias that is driven by the noise ϵ_b and assumed to be bounded.
All sensors are 3-D.

Pseudorange Measurement Model

The range is generally measured indirectly by some receiver that measures signal time of arrival, phase difference or other variables. The geometric range

$$\rho_i = \|p^n - p_i^n\|_2$$

is a nonlinear function of the vehicle position p^n and the ith transponder position p_i^n, given by their Euclidean distance.

Pseudorange Measurement Model

The pseudorange measurement model is

$$y_i = \rho_i + \beta + \epsilon_{yi}$$

where $\beta \in \mathbb{R}$ is a bias parameter due to unknown clock synchronization errors or other unknown effects and ϵ_{yi} the noise. $i = 1, 2, ..., m$ where m is the number of transponders.

Pseudorange Measurement Model

The nonlinear model can be approximated with a linear one by an algebraic transformation

$$2C_{\delta x}x = \delta + \varepsilon$$

where the matrix $C_{\delta x} \in \mathbb{R}^{(m-1)\times 4}$ is

$$C_{\delta x} := \begin{pmatrix} (p_m^n - p_1^n)^T & y_1 - y_m \\ \vdots & \\ (p_m^n - p_{m-1}^n)^T & y_{m-1} - y_m \end{pmatrix}$$

$\varepsilon \in \mathbb{R}^{m-1}$ the noise and $\delta \in \mathbb{R}^{m-1}$ the vector of squared range measurements.
$x := (p_\Delta^n; \beta)$ where $p_\Delta^n = p^n - p_0^n$ (p_0^n is a reference point in NED).

Pseudorange Measurement Model

If $m \geqslant 5$ and if there is no measurement noise, the unique solution to

$$2C_{\delta x}x = \delta + \epsilon$$

is given by

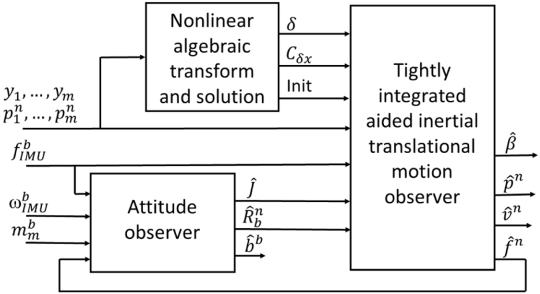
$$\hat{x} = \frac{C_{\delta x}^{+}\delta}{2}$$

where $\hat{x} := (\hat{p_{\Delta}^{n}}; \hat{\beta})$ is an estimation of the vehicle position and of the clock bias.

Outline

Nonlinear Observer

Two observer are designed: one for the attitude estimation and one for the translational motion estimation.



Nonlinear Observer

The Attitude Observer (*AO*) should provide an estimation of the bias gyroscope, in order to *clean* the angular velocity measurements, and of the rotation matrix from the body frame to the *NED* frame.

The pseudorange model provides an initialisation of the translational motion observer (*TMO*) which should estimate position, velocity and acceleration of the vehicle and the clock bias of the transponders.

Attitude Observer

The *AO* estimates the following variables

$$\dot{\hat{R}}_b^n = \hat{R}_b^n S(\omega_{ib,IMU}^b - \hat{b}) + \sigma K_p J(t, \hat{R}_b^n)$$

$$\dot{\hat{b}} = Proj(-k_I vex(\mathbb{P}_a(sat(\hat{R}_b^n)^T K_P J(t, \hat{R}_b^n))), M_{\hat{b}})$$

where $sat(\cdot)$ is an element-wise saturation, $vex(S(x)) = x$ and $k_I > 0$. $K_P > 0 \in \mathbb{R}^{3\times3}$ is a symmetric gain matrix, $\sigma \geq 1$. The second formula guarantees that the estimation of b is bounded:

$$\|\hat{b}\|_2 \leqslant M_{\hat{b}}$$

Every time $\hat{b}$ is available, this is subtracted to the gyroscope measurement.

Attitude Observer

The function $J(\cdot) \in \mathbb{R}^{3\times 3}$ is a stabilizing injection term that acts as an angular velocity when the discrepancy between measured vectors in body frame, here specific force and magnetic field from the accelerometer and the magnetometer, with the corresponding one in *NED* frame.

$$J(t, \hat{R}_b^n) = (E^n - \hat{R}_b^n E^b)(E^b)^T$$

Attitude Observer

$$E^b = (q_1^b, S(q_1^b)q_2^b, S^2(q_1^b)q_2^b)$$

$$E^n = (q_1^n, S(q_1^n)q_2^n, S^2(q_1^n)q_2^n)$$

$$q_1^b = m_{mag}^b / \|m_{mag}^b\|_2 \qquad q_2^b = f_{IMU}^b / \|g^n\|_2$$

$$q_1^n = m^n / \|m^n\|_2 \qquad q_2^n = \hat{f}^n / \|g^n\|_2$$

Translational Motion Observer

The *TMO* estimates p^n, v^n, f^n, β.

$$\dot{\hat{p}}^n_\Delta = \hat{v}^n + K_{pp}(\delta - \hat{\delta})$$

$$\dot{\hat{\beta}} = K_{\beta p}(\delta - \hat{\delta})$$

$$\dot{\hat{v}}^n = \hat{f}^n + g^n + K_{vp}(\delta - \hat{\delta})$$

$$\dot{\xi} = -\sigma K_P J(t, \hat{R}^n_b) f^b_{IMU} + K_{\xi p}(\delta - \hat{\delta})$$

$$\hat{f}^n = \hat{R}^n_b f^b_{IMU} + \xi$$

where $\hat{\delta} = 2C_{\delta x}\hat{x}$ and the gain matrix $K \in \mathbb{R}^{10\times(m-1)}$ is made of the matrices K_* and is in general time varying.

Translational Motion Observer

The injection term is provided by

$$\delta - \hat{\delta}$$

where

- δ is computed by the pseudorange model;
- $\hat{\delta}$ by the TMO.

For each variable to estimate, the injection term is weighted by a gain K_{ij}.

Translational Motion Observer

The gain matrix K is time varying and calculated as

$$K := PC^T R^{-1}$$

where P is solution of the *Riccati* equation

$$\dot{P} = PA + A^T P - PC^T R^{-1} CP + Q$$

Translational Motion Observer

The matrices $A \in \mathbb{R}^{10\times 10}$,$B \in \mathbb{R}^{10\times 3}$,$C \in \mathbb{R}^{(m-1)\times 10}$ and $K \in \mathbb{R}^{10\times(m-1)}$ are described as follows

$$A := \begin{pmatrix} 0 & 0 & I_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad B := \begin{pmatrix} 0 \\ 0 \\ 0 \\ I_3 \end{pmatrix}$$

$$K := \begin{pmatrix} K_{pp} \\ K_{\beta p} \\ K_{vp} \\ K_{\xi p} \end{pmatrix} \qquad C := \begin{pmatrix} 2C_{\delta x} & 0 & 0 \end{pmatrix}$$

Outline

Implementation

The whole model has been implemented with MATLAB/Simulink (R2016b).

In the following each observer is presented in terms of structure and results, i.e. the difference between the true value of the vehicle position and of the bias parameter and their estimation.

Implementation

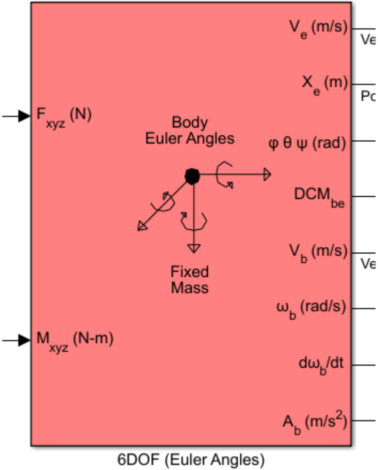
Data used by the authors are

- pseudorange measurements obtained by radio beacons;
- accelerations and angular velocities by IMU.

The inertial sensor model is based on the strapdown assumption, i.e. the inertial measurement unit is fixed to the body frame.

Data Generation

In order to generate data the 6*DOF* Simulink block has been used



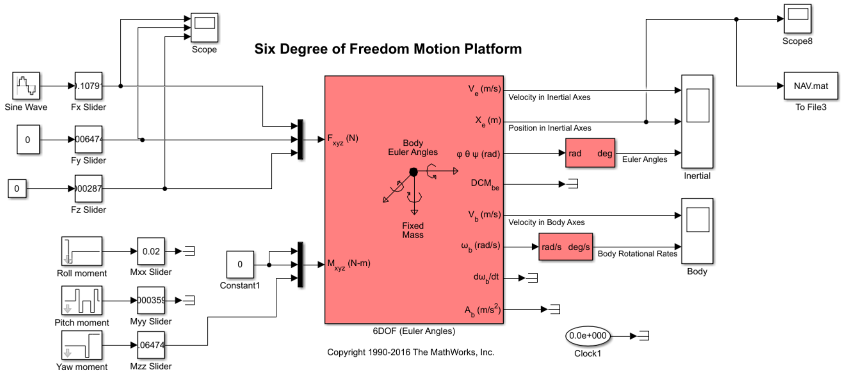
Data Generation

- The real position of the vehicle is given from $X_e(m)$;
- The range measurements always by $X_e(m)$, sampled at $0.1Hz$ and corrupted by noise;
- Specific forces from $A_b(m/s^2)$, sampled at $0.01Hz$;
- Angular velocities from $\omega_b(rad/s)$, sampled at $0.01Hz$.

The range measurements and *IMU* data have been generated with two different simulink model (*generator_of_pseudo_measures.slx*, *generator_of_imu.slx*),because of the system complexity and the non-possibility to incorporate everything in one model.

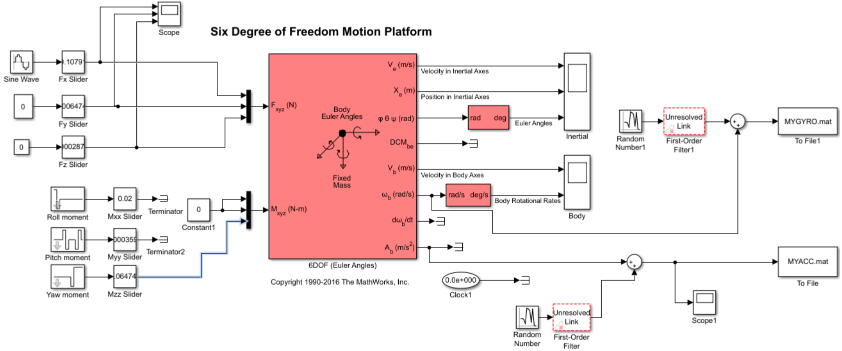
Data Generation

Pseudo Measures generation

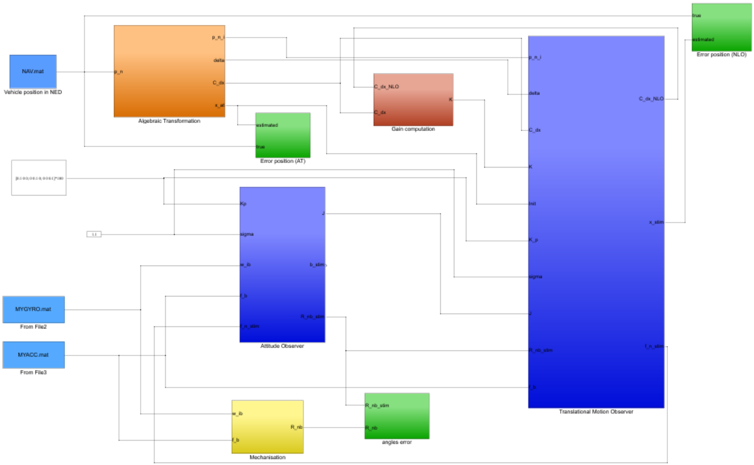


Data Generation

IMU generation



Simulink Model



Outline

Results

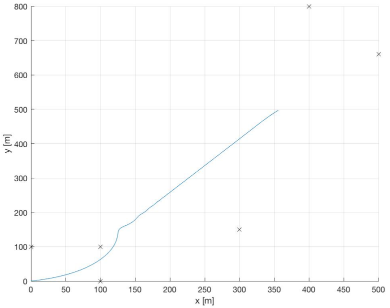
In this section, the results of the Nonlinear Observer are presented in terms of committed error between the true and estimated state. The transponder positions and the observer tuning have been decided by a *trial and error* approach.

Results

The following picture shows the transponder positions and the real trajectory to be estimated.

Transponder positions p_i^n, $i = 1, 2, ..6$.

$$
\begin{aligned}
p_1^n &= [100; 0; 0.1] \\
p_2^n &= [0; 100; 10] \\
p_3^n &= [100; 100; 20] \\
p_4^n &= [300; 150; 50] \\
p_5^n &= [500; 660; 60] \\
p_6^n &= [400; 800; 100]
\end{aligned}
$$

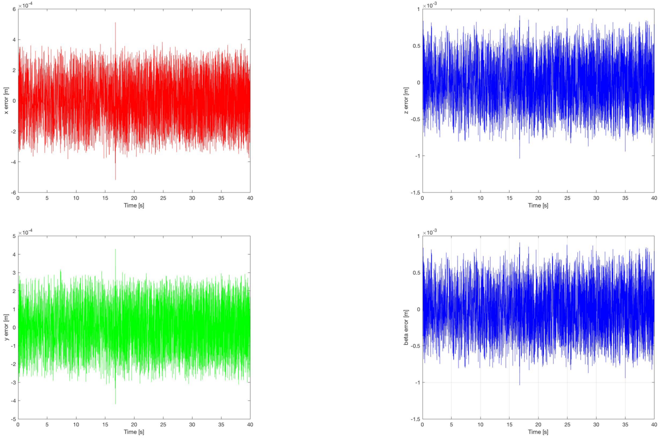


Pseudorange Model Design

The Pseudorange Model has been implemented with positions obtained directly from the previously described equations, without modelling the receiver on the vehicle that receives signals from transponders

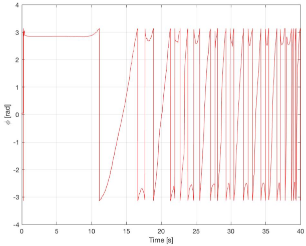
Pseudorange Model Results

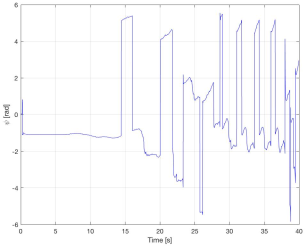
- Position and bias errors

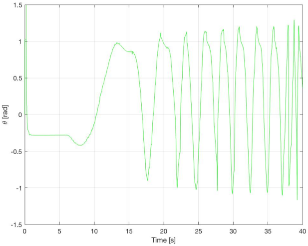


AO Results

- Angles errors

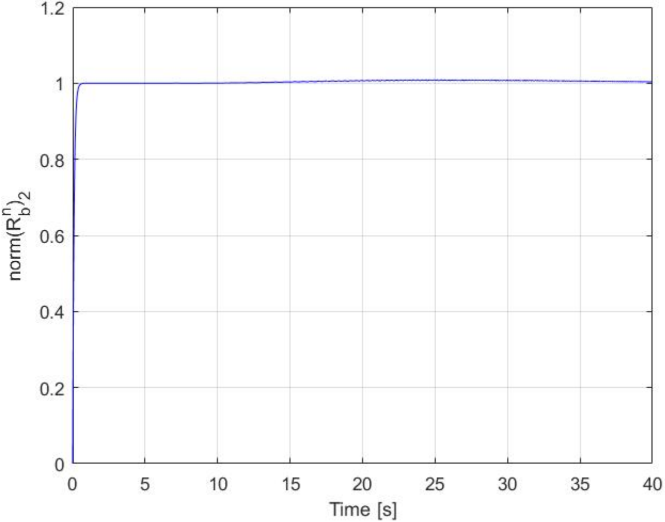






Whatever tuning does not allow to have a good estimation of the angles, in terms of error respect to the true ones.

AO Results



TMO Results

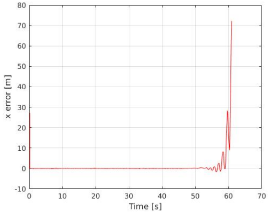
Despite of the non-estimation of *AO*, the *TMO* provides a good position estimation only for a simulation time of about 50s.

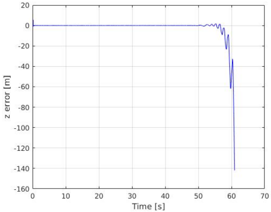
$$\dot{\hat{p}}^n_\Delta = \hat{v}^n + K_{pp}(\delta - \hat{\delta})$$

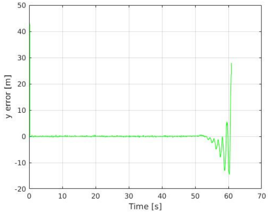
At the beginning, the weighted injection term $K_{pp}(\delta - \hat{\delta})$ prevails on $\hat{v}^n$. Then $\hat{v}^n$ prevails and brings a bad estimation due to the fact that depends on *AO*.

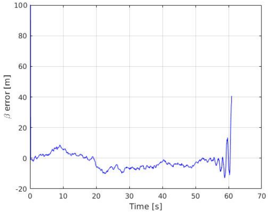
TMO Results

- This situation is shown in the following figures.



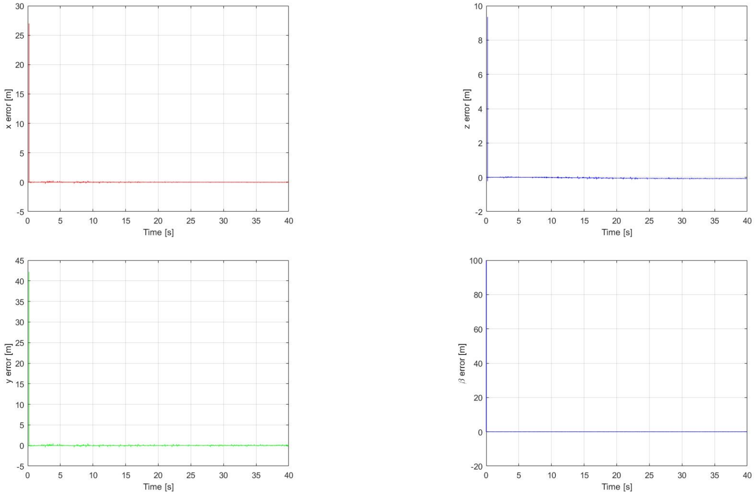






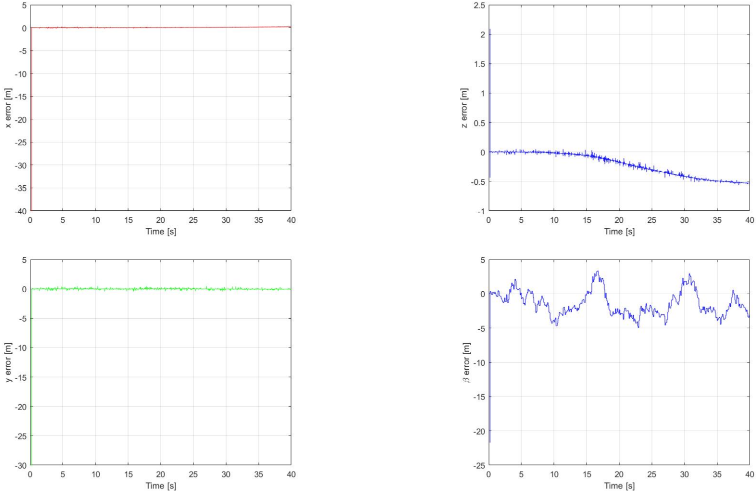
TMO Results

- No initial error.



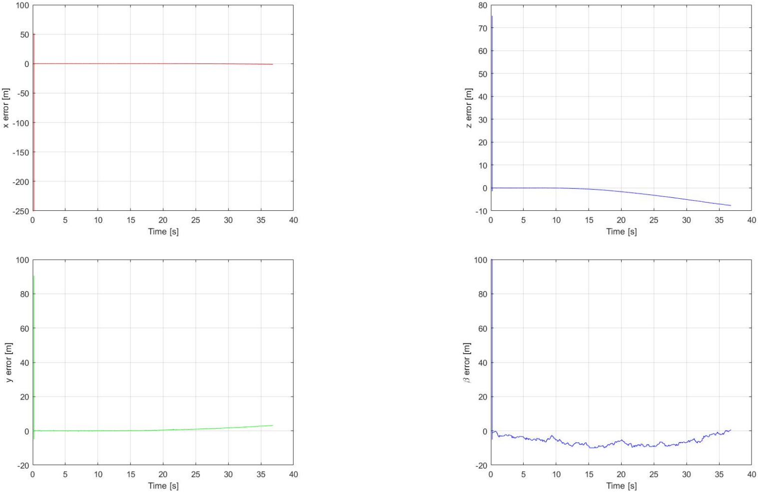
TMO Results

- Initial error [40;30;0;100].



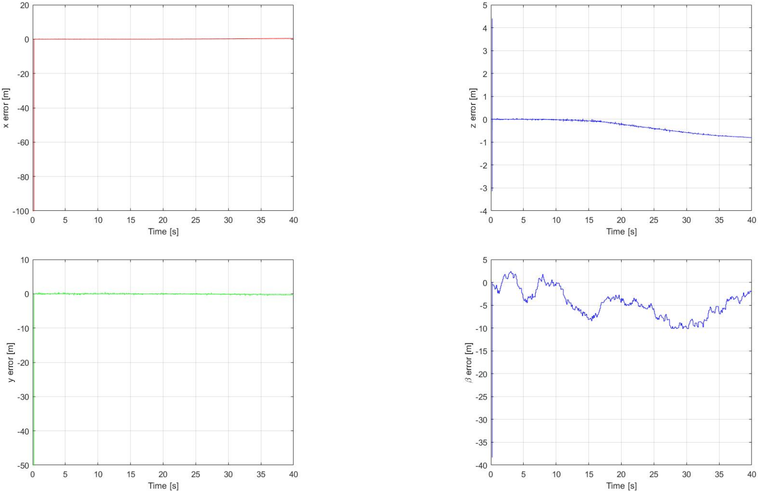
TMO Results

- Initial error [250;0;0;0].



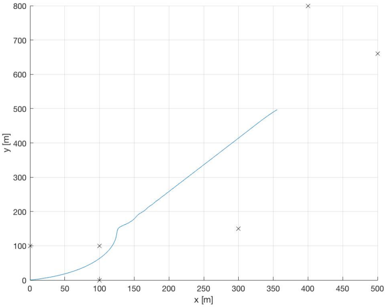
TMO Results

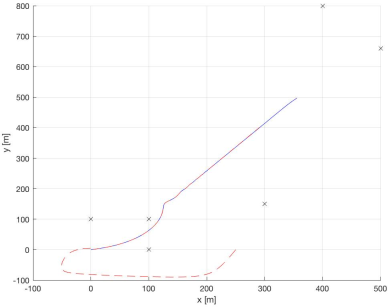
- Initial error [100;50;0;100].



TMO Results

Comparison between real and estimated trajectory.

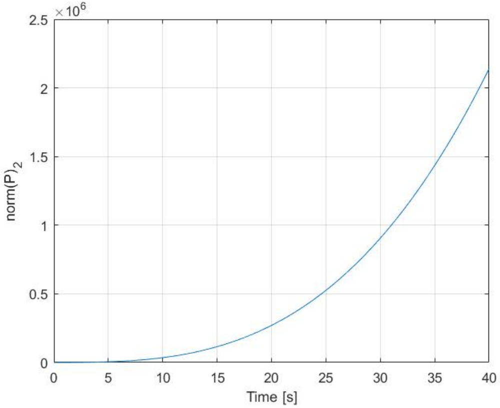




Outline

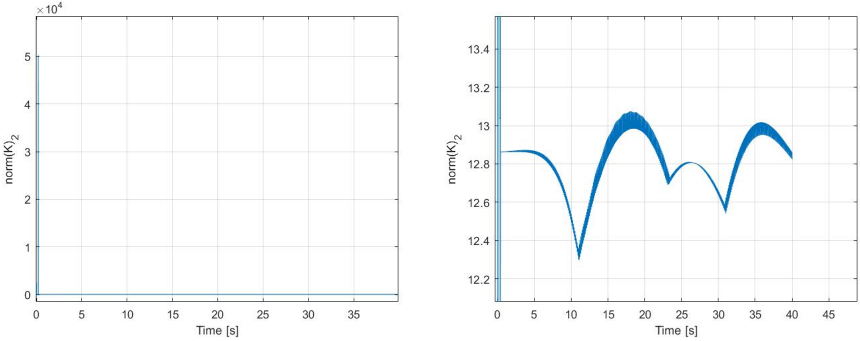
Unbounded P

Due to the time varying system and/or to the non-well-posedness, the covariance matrix P is unbounded.



Bounded K

Despite that, the gain matrix K results bounded.



Outline

Conclusions

This presentation has shown the results obtained by studying and implementation of a Nonlinear Observer for Tightly Coupled Integration of Pseudorange and Inertial Measurements designed by T.A. Johansen and T.I. Fossen.
Since the model does not work as well as described in the article, we texted the authors. Dr. Fossen told us that there are some tricks for the implementation of Nonlinear Observers and to refer to the article:

- **Bryne, T. H., J.M. Hansen, R.H. Rogne, N. Sokolova, T. A. Johansen and T. I. Fossen.** Nonlinear Observers for Integrated INS/GNSS Navigation - Implementation Aspects. *IEEE Control Systems Magazine*, Volume: 37, Issue 3, June 2017, pp. 59-86.

Conclusions

However the utilized approach is different respect to the ours, so it was not very useful.

In order to achieve better performance in terms of state estimation, we tried to modify the gain matrix K (computed by *Riccati* equation) with a pure gain, but it did not work since the only result has been a delayed error divergence.