# Nonlinear Observer for Tightly Coupled Integration of Pseudorange and Inertial Measurements

## Guide and Navigation Systems

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# Outline

# Introduction

This presentation shows the obtained results by the studying of the article written by Tor. A. Johansen and Thor I. Fossen.

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Nonlinear Observer for Tightly Coupled Integration of Pseudorange and Inertial Measurements

Tor A. Johansen and Thor I. Fossen

# Introduction

The goal of navigation system is to estimate the vehicle position within a certain area. The most common approach is to implement a inertial navigation system, based on *IMU* (Inertial Measurement Unit).

An inertial navigation system (*INS*) is a navigation system that calculates the position, the orientation and the velocity of a moving vehicle without the need for external references, by using a computer, motion sensors (accelerometers), rotation sensors (gyroscopes) and occasionally magnetic sensors (magnetometers).

# Introduction

In order to estimate position, velocity and orientation, inertial sensors measurements have to been integrated.

$$p = \int_0^t v dt = \bar{v} t$$

Therefore sensor bias and noise effects diverge during time. To

solve this problem, *INS* is aid by other techniques, such as *GPS*, $range - range$ measurements, etc..

# Introduction

Here the goal is to support *INS* with *pseudorange* measurements, obtained by transponders, and *nonlinear observers*.

# Outline

# Vehicle Kinematics

The vehicle kinematic model is given by

$$\dot{p}^n = v^n$$

$$\dot{v}^n = R_b^n f^b + g^n$$

$$\dot{R}_b^n = R_b^n S(\omega_{ib}^b)$$

Where $p^n$, $v^n$, $f^n$ are position, velocity and proper acceleration in NED (North-East-Down), respectively, while the attitude is described by a rotation matrix $R_b^n$ that represents the rotation from *body* to NED; $\omega_{ib}^b$ represents the rotation rate of body with respect to ECI (Earth-Centered-Inertial) and $g^n$ denotes the gravity vector. We also assume NED to be an inertial frame.

# Inertial Sensor Models

The inertial sensor model is based on the strapdown assumption

$$f^b_{IMU} = f^b + \epsilon_f$$

$$\omega^b_{ib,IMU} = \omega^b_{ib} + b + \epsilon_\omega$$

$$\dot{b} = \epsilon_b$$

$$m^b_{mag} = m^b + \epsilon_m$$

where $\epsilon_f$, $\epsilon_\omega$ and $\epsilon_m$ account for noise, and $b$ denotes the rate gyro bias that is driven by the noise $\epsilon_b$ and assumed to be bounded.
All sensors are 3-D.

# Pseudorange Measurement Model

The range is generally measured indirectly by some receiver that measures signal time of arrival, phase difference or other variables. The geometric range

$$\rho_i = \|p^n - p_i^n\|_2$$

is a nonlinear function of the vehicle position $p^n$ and the $i$th transponder position $p_i^n$, given by their Euclidean distance.

# Pseudorange Measurement Model

The pseudorange measurement model is

$$y_i = \rho_i + \beta + \epsilon_{yi}$$

where $\beta \in \mathbb{R}$ is a bias parameter due to unknown clock synchronization errors or other unknown effects and $\epsilon_{yi}$ the noise. $i = 1, 2, ..., m$ where $m$ is the number of transponders.

# Pseudorange Measurement Model

The nonlinear model can be approximated with a linear one by an algebraic transformation

$$2C_{\delta x}x = \delta + \varepsilon$$

where the matrix $C_{\delta x} \in \mathbb{R}^{(m-1)\times 4}$ is

$$C_{\delta x} := \begin{pmatrix} (p_m^n - p_1^n)^T & y_1 - y_m \\ \vdots & \\ (p_m^n - p_{m-1}^n)^T & y_{m-1} - y_m \end{pmatrix}$$

$\varepsilon \in \mathbb{R}^{m-1}$ the noise and $\delta \in \mathbb{R}^{m-1}$ the vector of squared range measurements.
$x := (p_\Delta^n; \beta)$ where $p_\Delta^n = p^n - p_0^n$ ($p_0^n$ is a reference point in NED).

# Pseudorange Measurement Model

If $m \geqslant 5$ and if there is no measurement noise, the unique solution to

$$2C_{\delta x}x = \delta + \epsilon$$

is

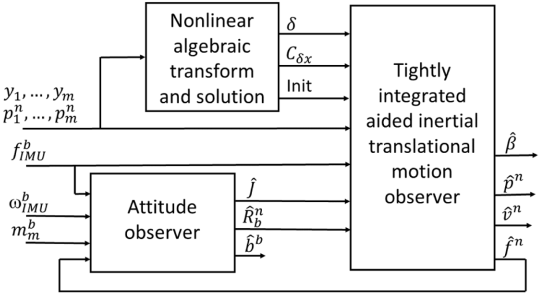
$$\hat{x} = \frac{C_{\delta x}^{+}\delta}{2}$$

where $\hat{x} := (\hat{p_{\Delta}^{n}}; \hat{\beta})$ is an estimation of the vehicle position and of the clock bias.

# Outline

# Nonlinear Observer

Two observer are designed: one for the attitude estimation and one for the translational motion estimation.



# Nonlinear Observer

The Attitude Observer (*AO*) should provide an estimation of the bias gyroscope, in order to *clean* the angular velocity measurements, and of the rotation matrix from the body frame to the *NED* frame.

The pseudorange model provides an initialisation of the translational motion observer (*TMO*) which should estimate position, velocity and acceleration of the vehicle and the clock bias of the transponders.

# Attiture Observer

One of the variables to estimate is the gyroscope bias

$$\dot{\hat{b}} = Proj(-k_I vex(\mathbb{P}_a(sat(\hat{R}_b^n)^T K_P J(t, \hat{R}_b^n))), M_{\hat{b}})$$

where $sat(\cdot)$ is an element-wise saturation, $vex(S(x)) = x$ and $k_I > 0$.

The formula guarantees that the estimation is bounded:

$$\|\hat{b}\|_2 \leqslant M_{\hat{b}}$$

# Attiture Observer

The second variable to estimate is the rotation matrix $R_b^n$.

$$\dot{\hat{R}}_b^n = \hat{R}_b^n S(\omega_{ib,IMU}^b - \hat{b}) + \sigma K_p J(t, \hat{R}_b^n)$$

where $K_P > 0 \in \mathbb{R}^{3\times 3}$ is a symmetric gain matrix, $\sigma \geq 1$.

Every time $\hat{b}$ is available, this is subtracted to the gyroscope measurement.

# Attitude Observer

The function $J(\cdot) \in \mathbb{R}^{3\times3}$ is a stabilizing injection term that acts as an angular velocity when the discrepancy between measured vectors in body frame, here specific force and magnetic field from the accelerometer and the magnetometer, with the corresponding one in *NED* frame.

$$J(t, \hat{R}_b^n) = (E^n - \hat{R}_b^n E^b)(E^b)^T$$

# Attitude Observer

$$E^b = (q_1^b, S(q_1^b)q_2^b, S^2(q_1^b)q_2^b)$$

$$E^n = (q_1^n, S(q_1^n)q_2^n, S^2(q_1^n)q_2^n)$$

$$q_1^b = m_{mag}^b / \|m_{mag}^b\|_2 \qquad q_2^b = f_{IMU}^b / \|g^n\|_2$$

$$q_1^n = m^n / \|m^n\|_2 \qquad q_2^n = \hat{f}^n / \|g^n\|_2$$

# Translational Motion Observer

The *TMO* estimates $p^n$, $v^n$, $f^n$, $\beta$.

$$\dot{\hat{p}}^n_\Delta = \hat{v}^n + K_{pp}(\delta - \hat{\delta})$$

$$\dot{\hat{\beta}} = K_{\beta p}(\delta - \hat{\delta})$$

$$\dot{\hat{v}}^n = \hat{f}^n + g^n + K_{vp}(\delta - \hat{\delta})$$

$$\dot{\xi} = -\sigma K_P J(t, \hat{R}^n_b) f^b_{IMU} + K_{\xi p}(\delta - \hat{\delta})$$

$$\hat{f}^n = \hat{R}^n_b f^b_{IMU} + \xi$$

where $\hat{\delta} = 2C_{\delta x}\hat{x}$ and the gain matrix $K \in \mathbb{R}^{10\times(m-1)}$ is made of the matrices $K_*$ and is in general time varying.

# Translational Motion Observer

The injection term is provided by

$$\delta - \hat{\delta}$$

where

- $\delta$ is computed by the pseudorange model;
- $\hat{\delta}$ by the TMO.

For each variable to estimate, the injection term is weighted by a gain $K_{ij}$.

# Translational Motion Observer

The gain matrix $K$ is time varying and calculated as

$$K := PC^T R^{-1}$$

where $P$ is solution of the *Riccati* equation

$$\dot{P} = PA + A^T P - PC^T R^{-1} CP + Q$$

# Translational Motion Observer

The matrices $A \in \mathbb{R}^{10\times 10}, B \in \mathbb{R}^{10\times 3}, C \in \mathbb{R}^{(m-1)\times 10}$ and $K \in \mathbb{R}^{10\times (m-1)}$ are described as follows

$$A := \begin{pmatrix} 0 & 0 & I_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad B := \begin{pmatrix} 0 \\ 0 \\ 0 \\ I_3 \end{pmatrix}$$

$$K := \begin{pmatrix} K_{pp} \\ K_{\beta p} \\ K_{vp} \\ K_{\xi p} \end{pmatrix} \qquad C := \begin{pmatrix} 2C_{\delta x} & 0 & 0 \end{pmatrix}$$

# Outline

# Implementation

The whole model has been implemented with MATLAB/Simulink (R2016b).

In the following each observer is presented in terms of structure and results, i.e. the difference between the true value of the vehicle position and of the bias parameter and their estimation.

# Implementation

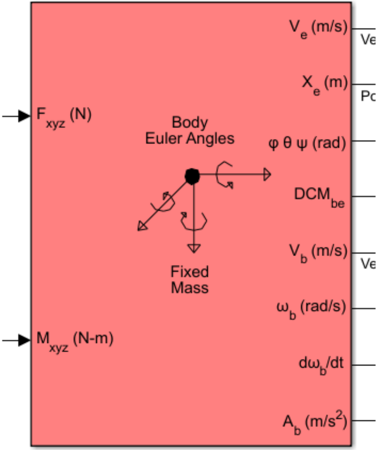
Data used by the authors are

- pseudorange measurements obtained by radio beacons;
- accelerations and angular velocities by IMU.

The inertial sensor model is based on the strapdown assumption, i.e. the inertial measurement unit is fixed to the body frame.

# Data Generation

In order to generate data the 6*DOF* Simulink block has been used




6DOF (Euler Angles)

# Data Generation

- The real position of the vehicle is given from $X_e(m)$;
- The range measurements always by $X_e(m)$ but corrupted by noise;
- Specific forces from $A_b(m/s^2)$;
- Angular velocities from $\omega_b(rad/s)$.

# Outline

# Pseudorange Model Results




# *AO* Results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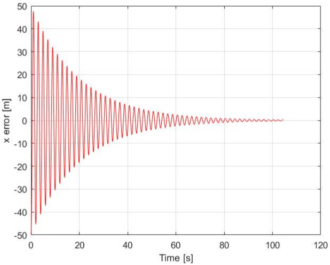
# Results

Whatever tuning does not allow to have a good estimation of the angles, in terms of error respect to the true ones.

# *TMO* Results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x error [m]
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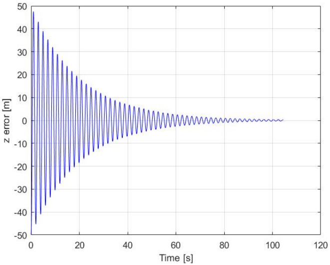
Time [s]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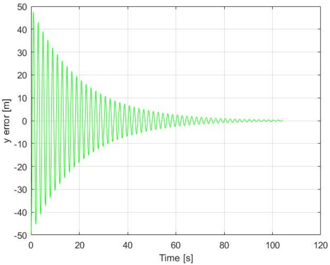
Time [s]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