Nonlinear Observer for Tightly Coupled Integration of Pseudorange and Inertial Measurements

Guide and Navigation Systems

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Outline

Introduction

This presentation shows the obtained results by the implementation of the article written by Tor. A. Johansen and Thor I. Fossen.

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Tor A. Johansen and Thor I. Fossen

Introduction

The goal of navigation systems is to estimate the vehicle position within a certain area. This is let generally by a inertial navigation system, based on *IMU* (Inertial Measurement Unit).
Using an interconnection of a nonlinear attitude observer and a translational motion observer based on pseudorange and range-range measurements, a tightly coupled integrated aided inertial navigation system is designed!?!?!?
Due to the problem of measurements integration, this method is supported by other techniques. In this work, the inertial navigation system is aid by pseudorange measurements, obtained by transponders.

Vehicle Kinematics

The vehicle kinematic model is given by

$$\dot{p}^n = v^n$$

$$\dot{v}^n = R_b^n f^b + g^n$$

$$\dot{R}_b^n = R_b^n S(\omega_{ib}^b)$$

Where p^n, v^n, f^n are position, velocity and proper acceleration in NED (North-East-Down), respectively, while the attitude is described by a rotation matrix R_b^n that represents the rotation from *body* to NED; ω_{ib}^b represents the rotation rate of body with respect to ECI (Earth-Centered-Inertial) and g^n denotes the gravity vector. We also assume NED to be an inertial frame.

Inertial Sensor Models

The inertial sensor model is based on the strapdown assumption

$$f^b_{IMU} = f^b + \epsilon_f$$

$$\omega^b_{ib,IMU} = \omega^b_{ib} + b + \epsilon_\omega$$

$$\dot{b} = \epsilon_b$$

$$m^b_{mag} = m^b + \epsilon_m$$

where ϵ_f, ϵ_ω and ϵ_m account for noise, and b denotes the rate gyro bias that is driven by the noise ϵ_b and assumed to be bounded.
All sensors are 3-D.

Pseudorange Measurement Model

The geometric range

$$\rho_i = \|p^n - p_i^n\|_2$$

is a nonlinear function of the vehicle position p^n and the ith transponder position p_i^n, given by their Euclidean distance.
The pseudorange measurement model is

$$y_i = \rho_i + \beta + \epsilon_{yi}$$

where $\beta \in \mathbb{R}$ is a bias parameter due to unknown clock synchronization errors or other unknown effects and ϵ_{yi} the noise.
$i = 1, 2, ..., m$ where m is the number of measurements.

Pseudorange Measurement Model

The nonlinear model can be approximated with a linear one by an algebraic transformation

$$2C_{\delta x}x = \delta + \varepsilon$$

where the matrix $C_{\delta x} \in \mathbb{R}^{(m-1)\times 4}$ is

$$C_{\delta x} := \begin{pmatrix} (p_m^n - p_1^n)^T & y_1 - y_m \\ \vdots & \\ (p_m^n - p_{m-1}^n)^T & y_{m-1} - y_m \end{pmatrix}$$

$\varepsilon \in \mathbb{R}^{m-1}$ the noise and $\delta \in \mathbb{R}^{m-1}$ the vector of squared range measurements.
$x := (p_\Delta^n; \beta)$ where $p_\Delta^n = p^n - p_0^n$ (p_Δ^n is a reference point in NED).

Observer Design

Two observer are designed: one for the attitude estimation and one for the translational motion estimation.

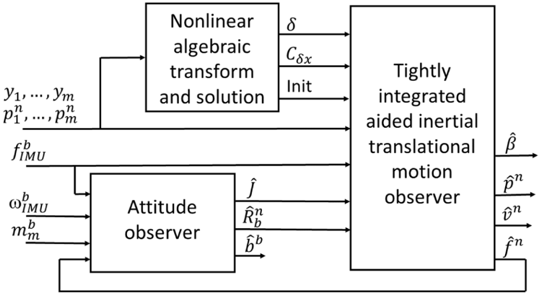


Figure : Overall block diagram for tightly integrated observer.

Attiture Observer

The attitude variables to estimate are R_b^n and b.

$$\dot{\hat{R}}_b^n = \hat{R}_b^n S(\omega_{ib,IMU}^b - \hat{b}) + \sigma K_p J(t, \hat{R}_b^n)$$

$$\dot{\hat{b}} = Proj(-k_I vex(\mathbb{P}_a(sat(\hat{R}_b^n)^T K_P J(t, \hat{R}_b^n))), M_{\hat{b}})$$

where $K_P > 0 \in \mathbb{R}^{3\times 3}$ is a symmetric gain matrix, $K_I > 0$ is a scalar gain and $\sigma \geq 1$.
The function $sat(\cdot)$ is an element-wise saturation, while $Proj(\cdot)$ is a parameter projection which ensures that $\|\hat{b}\|_2$ is bounded.

Attiture Observer

The function $J(\cdot) \in \mathbb{R}^{3\times 3}$ is a stabilizing injection term

$$J(t, \hat{R}_b^n) = (E^n - \hat{R}_b^n E^b)(E^b)^T$$

based on the vector measurements m_{mag}^b and f_{IMU}^b and their NED reference vetors m^n and $\hat{f}^n$ used to define vectors scaled by nonzero terms

$$q_1^b = m_{mag}^b / \|m_{mag}^b\|_2 \qquad q_2^b = f_{IMU}^b / \|g^n\|_2$$

$$q_1^n = m^n / \|m^n\|_2 \qquad q_2^n = \hat{f}^n / \|g^n\|_2$$

and the 3×3 matrices

$$E^b = (q_1^b, S(q_1^b)q_2^b, S^2(q_1^b)q_2^b)$$

$$E^n = (q_1^n, S(q_1^n)q_2^n, S^2(q_1^n)q_2^n)$$

Translational Motion Observer

The variables to estimate are p^n,v^n,f^n,β.

$$\dot{\hat{p}}^n_\Delta = \hat{v}^n + K_{pp}(\delta - \hat{\delta})$$

$$\dot{\hat{\beta}} = K_{\beta p}(\delta - \hat{\delta})$$

$$\dot{\hat{v}}^n = \hat{f}^n + g^n + K_{vp}(\delta - \hat{\delta})$$

$$\dot{\xi} = -\sigma K_P J(t, \hat{R}^n_b) f^b_{IMU} + K_{\xi p}(\delta - \hat{\delta})$$

$$\hat{f}^n = \hat{R}^n_b f^b_{IMU} + \xi$$

where $\hat{\delta} = 2C_{\delta x}\hat{x}$ and the gain matrix $K \in \mathbb{R}^{10\times(m-1)}$ is made of the matrices K_* and is in general time varying.

Translational Motion Observer

Then it is possible to build the *estimated state* vector $\dot{\tilde{\chi}} = (\tilde{p}^n_\Delta; \tilde{\beta}; \tilde{\hat{v}}^n; \tilde{\hat{f}}^n) \in \mathbb{R}^{10}$ and the relative LTV error system

$$\dot{\tilde{\chi}} = (A - KC)\tilde{\chi} + Bu + B\epsilon_u + K\varepsilon$$

$$u = \tilde{R}^n_b \dot{f}^b + \tilde{R}^n_b S(\omega^b_{ib}) f^b - \hat{R}^n_b S(\tilde{b}) f^b$$

Translational Motion Observer

The matrices $A \in \mathbb{R}^{10\times10}$,$B \in \mathbb{R}^{10\times3}$,$C \in \mathbb{R}^{(m-1)\times10}$ and $K \in \mathbb{R}^{10\times(m-1)}$ are described as follows

$$A := \begin{pmatrix} 0 & 0 & I_3 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_3 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad B := \begin{pmatrix} 0 \\ 0 \\ 0 \\ I_3 \end{pmatrix}$$

$$K := \begin{pmatrix} K_{pp} \\ K_{\beta p} \\ K_{vp} \\ K_{\xi p} \end{pmatrix} \qquad C := \begin{pmatrix} 2C_{\delta x} & 0 & 0 \end{pmatrix}$$

Translational Motion Observer

The gain matrix K is time varying and calculated as

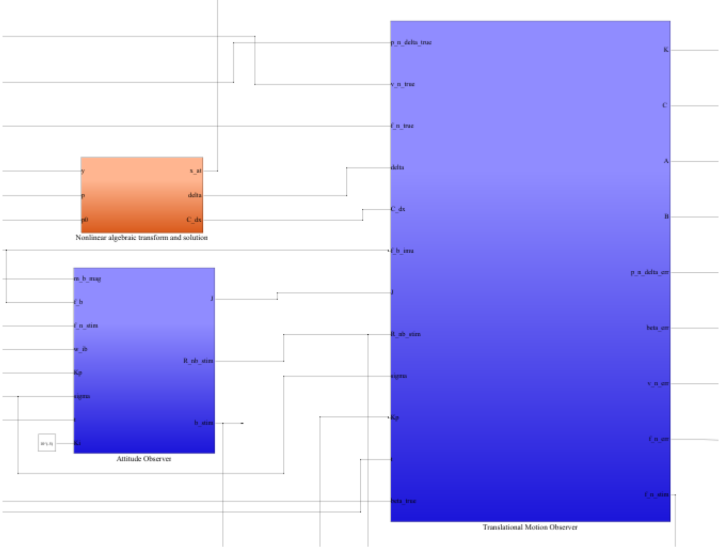
$$K := PC^T R^{-1}$$

where P is solution of the *Riccati* equation

$$\dot{P} = PA + A^T P - PC^T R^{-1} CP + Q$$

Implementation Design

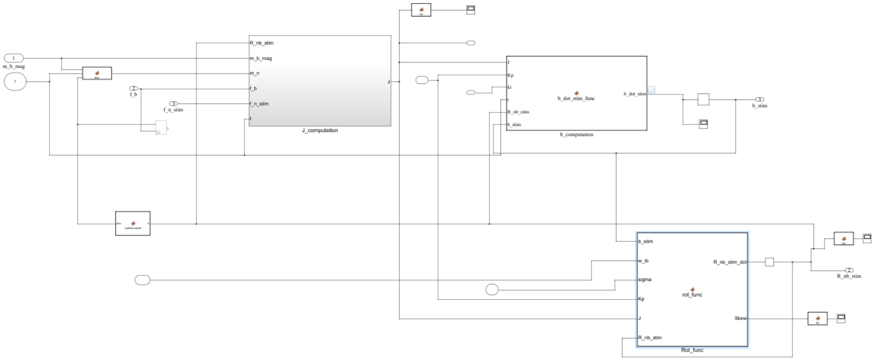
The Simulink implementation is similar to the one offered by the article:



Implementation Design

The Attitude Observer is made of three main function blocks: The first one implements the function $J(\cdot)$ as previously described. The function *b_computation* computes the dinamics of the bias by means of the $Proj(\cdot)$ function. and the function *Rot_func* computes an estimate of the rotation matrix. The following picture shows the Simulink scheme:

Implementation Design



Implementation Design

The *NonlinearAlgebraicTransform* block computes the $C_{\delta x}$ matrix and the following variable:

$$\hat{x} = \frac{C_{\delta x}^{+}\delta}{2}$$

that is the unique solution to

$$2C_{\delta x}x = \delta + \epsilon$$

in the case of $m \geqslant 5$ transponders.

Implementation Design