H

National
Qualifications
2025

2025 Mathematics

Higher - Paper 1

Question Paper Finalised Marking Instructions

Marking Instructions for each question

Question			Generic scheme	Illustrative scheme	Max mark
1.			•[1] find y-coordinate	•[1] 5	**4**
			•[2] differentiate expression	•[2] $3x^2-4x$	
			•[3] evaluate $\frac{dy}{dx}$ when $x=2$	•[3] 4	
			•[4] determine equation of tangent	•[4] $y=4x-3$	

Notes:

1. Where candidates integrate, only •[1] is available. For example, see Candidate F.
2. •[3] is not available for $y=4$. However, where $y=4$ comes from substituting into $3x^2-4x$, or 4 is then used as the gradient of the straight line, •[3] may be awarded. See Candidates A, B and C.
3. •[4] is only available where candidates attempt to find the gradient by substituting $x=2$ into their derivative.
4. Where $x=2$ has not been used to determine the y-coordinate, •[4] is not available.

Commonly Observed Responses:

Candidate A - incorrect labels and no evidence of substitution

$y=5$	•[1] ✓ - BoD
$y=3x^2-4x$	•[2] ✓
$y=4$	•[3] ✗

Evidence for •[4] would need to appear in the equation of the line for •[3] to be awarded

Candidate B - incorrect labels and no evidence of substitution

$y=5$	•[1] ✓ - BoD
$y=3x^2-4x$	•[2] ✓
$y=4$	
$y=4x-3$	•[4] ✓ •[3] ✓ - BoD

Candidate C - incorrect labels with substitution

$y=5$	•[1] ✓
$y=3x^2-4x$	•[2] ✓
$y=3(2)^2-4(2)=4$	•[3] ✓

Candidate D - no evidence of substitution

$y=5$	•[1] ✓
$\frac{dy}{dx}=3x^2-4x$	•[2] ✓
$m=4$	•[3] ✓

Candidate E - equating to zero

$y=5$	•[1] ✓
$\frac{dy}{dx}=3x^2-4x=0$	•[2] ✓
$3(2)^2-4(2)=0$	
$m=4$	•[3] ✗
$y=4x-3$	•[4] ✓$_1$

Candidate F - appearance of '$+c$'

$y=5$	•[1] ✓
$\frac{dy}{dx}=3x^2-4x+c$	•[2] ✗ •[3] ✗ •[4] ✗

Question			Generic scheme	Illustrative scheme	Max mark
2.			•[1] find the midpoint of AB •[2] calculate gradient of AB •[3] state perpendicular gradient •[4] determine equation of perpendicular bisector	•[1] $(5,7)$ •[2] $\frac{3}{4}$ •[3] $-\frac{4}{3}$ stated or implied by •[4] •[4] $3y=-4x+41$	**4**

Notes:

1. •[2] may be awarded for $\frac{-6}{-8}$ or equivalent.
2. The gradient of the perpendicular bisector must appear in fully simplified form at the •[3] or •[4] stage for •[4] to be awarded. See Candidate A.
3. •[4] is only available as a consequence of using a perpendicular gradient and a midpoint.
4. At •[4] accept $3y+4x=41$, $3y+4x-41=0$, $y=-\frac{4}{3}x+\frac{41}{3}$ or any other rearrangement of the equation where the constant terms have been simplified.

Commonly Observed Responses:

Candidate A		Candidate B - $\boldsymbol{m} = \boldsymbol{m}_{\text{perp}}$	
$(5,7)$	•[1] ✓	$(5,7)$	•[1] ✓
$m=\frac{6}{8}$	•[2] ✓	$m=\frac{3}{4}=-\frac{4}{3}$	•[2] ✓ •[3] ✗
$m_{\perp}=-\frac{8}{6}$	•[3] ✓	$3y=-4x+41$	•[4] ✓$_1$ - BOD
$6y=-8x+82$	•[4] ^	However	
		$m=\frac{3}{4}$	•[2] ✓
		$=-\frac{4}{3}$	•[3] ✓ - BOD
		$3y=-4x+41$	•[4] ✓

Question			Generic scheme	Illustrative scheme	Max mark
3.			•¹ express first term in integrable form	•¹ $12x^{-2}\ldots$	4
			•² integrate first term	•² $\dfrac{12x^{-1}}{-1}$	
			•³ integrate second term	•³ $\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}$	
			•⁴ complete integration	•⁴ $-12x^{-1}+\frac{2}{3}x^{\frac{3}{2}}+c$	

Notes:

1. Do not penalise the appearance of '$+c$', missing integral signs or missing 'dx' at •¹.
2. •² is only available for integrating a term with a negative power.
3. Do not penalise the appearance of an integral sign and/or dx throughout.
4. Do not penalise the omission of '$+c$' at •² or •³.
5. The '$+c$' must appear in the first line of working where coefficients of both terms are fully simplified. See Candidates C and D.
6. All coefficients must be simplified at •⁴ stage for•⁴ to be awarded.
7. •⁴ is not available within an invalid strategy, such as differentiation.
8. Accept $\frac{12}{-x}+\frac{2}{3}x^{\frac{3}{2}}+c$ for •⁴. However, do not accept $\frac{12}{-1x}+\frac{2}{3}x^{\frac{3}{2}}+c$.

Commonly Observed Responses:

Candidate A - Integrating over two lines

$12x^{-2}+\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}$	•¹ ✓
$\dfrac{12x^{-1}}{-1}+\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}$	•² ✓ •³ ✗
$-12x^{-1}+\dfrac{2x^{\frac{3}{2}}}{3}+c$	•⁴ ✓1

Candidate B - error in integration

$\int\left(12x^{-2}+x^{\frac{1}{2}}\right)dx$	•¹ ✓
$\dfrac{12x^{-3}}{-3}+\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}+c$	•² ✗ •³ ✓
$-4x^{-3}+\frac{2}{3}x^{\frac{3}{2}}+c$	•⁴ ✓1

Candidate C - integration incomplete at •⁴ stage

$-12x^{-1}+\dfrac{2x^{\frac{3}{2}}}{3}$	•² ✓ •³ ✓ •⁴ ^
$-\dfrac{12}{x}+\dfrac{2}{3}\sqrt{x^3}+c$	

Candidate D

$\dfrac{12x^{-1}}{-1}+\dfrac{x^{\frac{3}{2}}}{\frac{3}{2}}$	•² ✓ •³ ✓
$\boxed{\dfrac{12x^{-1}}{-1}}+\dfrac{2x^{\frac{3}{2}}}{3}+c$	
$-\dfrac{12}{x}+\boxed{\dfrac{2\sqrt[3]{x^2}}{3}}+c$	•⁴ ✗

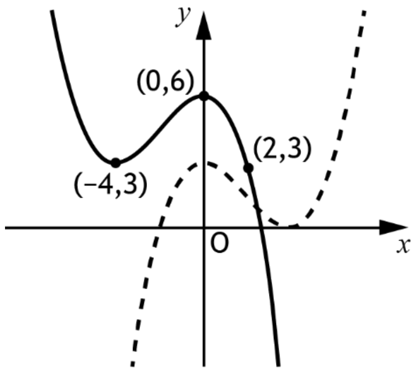
Question			Generic scheme	Illustrative scheme	Max mark
4.			$\bullet^1$ apply $m\log_3 x = \log_3 x^m$ $\bullet^2$ apply $\log_3 x + \log_3 y = \log_3 xy$ $\bullet^3$ evaluate expression	$\bullet^1$ $\log_3 2^3 \ldots$ $\bullet^2$ $\log_3\left(2^3 \times \frac{1}{24}\right)$ $\bullet^3$ -1	3

Notes:

1. Do not penalise the omission of the base of the logarithm at $\bullet^1$ or $\bullet^2$.
2. Do not penalise the omission of brackets at $\bullet^2$.
3. Correct answer with no working, award 0/3.
4. $\bullet^3$ is only available for evaluating the logarithm of a unitary fraction.
5. Each line of working must be equivalent to the line above within a valid strategy. However, see Candidates B and C.

Commonly Observed Responses:

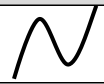
Candidate A - introducing a variable	Candidate B
$\log_3 \frac{1}{3}$ $\bullet^1$ ✓ $\bullet^2$ ✓ $3^x = \frac{1}{3}$ $x = -1$ $\bullet^3$ ✓	$3\log_3\left(2 \times \frac{1}{24}\right)$ $\bullet^2$ ✗ $\log_3\left(2 \times \frac{1}{24}\right)^3$ $\bullet^1$ $✓_1$ $\bullet^3$ ^
Candidate C $\log_3\left(9 \times \frac{1}{24}\right)$ with no supporting working $\bullet^1$ ✗ $\bullet^2$ $✓_1$	

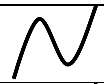
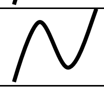
Question			Generic scheme	Illustrative scheme	Max mark
5.			•[1] reflection in y-axis identifiable from graph •[2] vertical translation of '$+3$' identifiable from graph	•[1,2] 	**2**

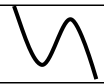
Notes:

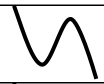
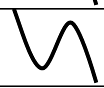
1. Where candidates do not sketch a cubic function, award 0/2.
2. Where only two of the three points are correctly transformed, award 0/2. However, see the table for possible exceptions.
3. Note that the position of the root, where shown, is not being assessed.
4. Ignore intersections (or lack of intersections) with the original graph.

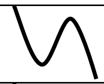
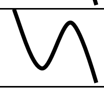
Commonly Observed Responses:

Function	Transformation of (−2,0) & (4,0)	Transformation of (0,3)	Shape	Award
Incorrect orientation	(2,3) & (−4,3)	(0,6)		0/2

$y = f(x) + 3$	(−2,3) & (4,3)	(0,6)		1/2
$y = f(x) - 3$	(−2,−3) & (4,−3)	(0,0)		0/2

$y = f(-x) - 3$	(2,−3) & (−4,−3)	(0,0)		1/2

$y = -f(x) + 3$	(−2,3) & (4,3)	(0,0)		1/2
$y = -f(x) - 3$	(−2,−3) & (4,−3)	(0,−6)		0/2

Horizontal translation	(5,0) & (−1,0)	(3,3)		1/2
Horizontal translation	(−1,0) & (−7,0)	(−3,3)		1/2

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)	(i)	$\bullet^1$ determine $\sin q$ or $\cos q$	$\bullet^1$ $\sin q = \dfrac{1}{\sqrt{26}}$ OR $\cos q = \dfrac{5}{\sqrt{26}}$ stated or implied by $\bullet^2$	**3**
			$\bullet^2$ substitute into formula for $\sin 2q$	$\bullet^2$ $2 \times \dfrac{1}{\sqrt{26}} \times \dfrac{5}{\sqrt{26}}$	
			$\bullet^3$ calculate $\sin 2q$	$\bullet^3$ $\dfrac{5}{13}$	
		(ii)	$\bullet^4$ calculate $\cos 2q$	$\bullet^4$ $\dfrac{12}{13}$	**1**

Notes:

1. Evidence for $\bullet^1$ may appear in (a)(ii).
2. Accept $q = \sin \triangleleft \left(\dfrac{1}{\sqrt{26}}\right)$, $q = \cos^{-1}\left(\dfrac{5}{\sqrt{26}}\right)$, $\sin\left(\dfrac{1}{\sqrt{26}}\right)$ or $\cos\left(\dfrac{5}{\sqrt{26}}\right)$ for $\bullet^1$.
3. Where candidates substitute an incorrect value for $\sin q$ or $\cos q$, $\bullet^2$ may be awarded if this value has previously been stated or it can be implied by a diagram or Pythagoras calculation. See Candidates C and D.
4. Where candidates explicitly state a value for $\sin q$ or $\cos q$, subsequent working must follow from that value for $\bullet^2$ to be awarded.
5. $\bullet^3$ is only available as a consequence of substituting into a valid formula at $\bullet^2$.
6. Do not penalise trigonometric ratios which are less than −1 or greater than 1 throughout this question.
7. For $\sin 2q = \dfrac{1}{\sqrt{26}}$ and $\cos 2q = \dfrac{5}{\sqrt{26}}$ $\bullet^3$ and $\bullet^4$ are not available.

Commonly Observed Responses:

Candidate A

$2 \times \sin \dfrac{1}{\sqrt{26}} \times \cos \dfrac{5}{\sqrt{26}}$ $\bullet^1$ ✓ $\bullet^2$ ✗

$\dfrac{5}{13}$ $\bullet^3$ ✗

Candidate B - unsimplified final answers

$\sin q = \dfrac{1}{\sqrt{26}}$ $\bullet^1$ ✓

$\sin 2q = 2 \times \dfrac{1}{\sqrt{26}} \times \dfrac{5}{\sqrt{26}}$ $\bullet^2$ ✓

$\sin 2q = \dfrac{10}{26}$ $\bullet^3$ ^

$\cos 2q = \dfrac{24}{26}$ $\bullet^4$ $✓_1$

$\sin(2q - r) = \dfrac{10}{26} \times \dfrac{4}{\sqrt{17}} - \dfrac{24}{26} \times \dfrac{1}{\sqrt{17}}$ $\bullet^5$ ✓ $\bullet^6$ ✓

$\sin(2q - r) = \dfrac{16}{26\sqrt{17}}$ $\bullet^7$ $✓_1$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)	(ii)	**(continued)**		

Candidate C - incorrect use of Pythagoras hypotenuse $=\sqrt{24}$ •¹ ✘ $\sin 2q = 2\times\frac{1}{\sqrt{24}}\times\frac{5}{\sqrt{24}}$ •² ✓₁ $\sin 2q = \frac{5}{12}$ •³ ✓₁	**Candidate D - no evidence of Pythagoras** $\sin 2q = 2\times\frac{1}{\sqrt{24}}\times\frac{5}{\sqrt{24}}$ •¹ ^ •² ✘ $\sin 2q = \frac{5}{12}$ •³ ✓₁

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		•5 expand $\sin(2q-r)$	•5 $\sin 2q\cos r-\cos 2q\sin r$	**3**
			•6 substitute into addition formula	•6 $\frac{5}{13}\times\frac{4}{\sqrt{17}}-\frac{12}{13}\times\frac{1}{\sqrt{17}}$	
			•7 evaluate $\sin(2q-r)$	•7 $\frac{8}{13\sqrt{17}}$	

Notes:

8. Where candidates write $\sin\frac{5}{13}\times\cos\frac{4}{\sqrt{17}}-\cos\frac{12}{13}\times\sin\frac{1}{\sqrt{17}}$, award •5. However, •6 and •7 are unavailable.
9. •6 and •7 are only available as a consequence of substituting into a formula involving $\sin 2q$, $\cos 2q$, $\sin r$ and $\cos r$ from •5.
10. For any attempt to use $\sin(2q-r)=\sin 2q-\sin r$, •6 and •7 are unavailable.
11. •7 is only available for an answer expressed as a single fraction.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
7.	(a)		•¹ use '−3' in synthetic division or in evaluation of a cubic •² complete division/evaluation and interpret result	•¹ $\begin{array}{r\|rrrr} -3 & 5 & 16 & -1 & -12 \\ \hline & 5 & & & \end{array}$ **OR** $5\times(-3)^3+16(-3)^2-(-3)-12$ •² $\begin{array}{r\|rrrr} -3 & 5 & 16 & -1 & -12 \\ & & -15 & -3 & 12 \\ \hline & 5 & 1 & -4 & 0 \end{array}$ Remainder $= 0$ $\therefore (x+3)$ is a factor **OR** $f(-3)=0 \therefore (x+3)$ is a factor	2

Notes:

1. Communication at •² must be consistent with working at that stage – ie a candidate's working must arrive legitimately at 0 before •² can be awarded.
2. Accept any of the following for •².
   - '$f(-3)=0$ so $(x+3)$ is a factor'.
   - 'since remainder = 0, it is a factor'.
   - the '0' from any method linked to the word 'factor' by 'so', 'hence', $\therefore$, $\rightarrow, \Rightarrow$ etc.
3. Do not accept any of the following for •²:
   - double underlining the '0' or boxing the '0' without a comment.
   - '$x=-3$ is a factor', '…is a root'.
   - the word 'factor' only with no link.

Commonly Observed Responses:

Candidate A - grid method

$$\begin{array}{c|c|c|c|} & 5x^2 & & \\ \hline x & 5x^3 & x^2 & \\ \hline 3 & 15x^2 & & \\ \hline \end{array}$$

•¹ ✓

$$\begin{array}{c|c|c|c|} & 5x^2 & x & -4 \\ \hline x & 5x^3 & x^2 & -4x \\ \hline 3 & 15x^2 & 3x & -12 \\ \hline \end{array}$$

'with no remainder'

$\therefore (x+3)$ is a factor •² ✓

Candidate B - grid method

$$\begin{array}{c|c|c|c|} & 5x^2 & & \\ \hline x & 5x^3 & x^2 & \\ \hline 3 & 15x^2 & & \\ \hline \end{array}$$

•¹ ✓

$$\begin{array}{c|c|c|c|} & 5x^2 & x & -4 \\ \hline x & 5x^3 & x^2 & -4x \\ \hline 3 & 15x^2 & 3x & -12 \\ \hline \end{array}$$

$\therefore (x+3)(5x^2+x-4)=5x^3+16x^2-x-12$

$\therefore (x+3)$ is a factor •² ✓

Question			Generic scheme	Illustrative scheme	Max mark
7.	(b)		$\bullet^3$ state quadratic factor $\bullet^4$ factorise quadratic $\bullet^5$ state solutions	$\bullet^3$ $5x^2+x-4$ stated or implied by $\bullet^4$ $\bullet^4$ $(5x-4)(x+1)$ $\bullet^5$ $-3,\ \frac{4}{5},\ -1$	**3**

Notes:

4. The appearance of '$=0$' is not required for $\bullet^5$ to be awarded.
5. Candidates who identify a different initial factor and subsequent quadratic factor can gain $\bullet^3$, $\bullet^4$ and $\bullet^5$. See Candidate C.
6. $\bullet^5$ is only available as a result of a valid strategy at $\bullet^3$ and $\bullet^4$.
7. Accept $(-3,0)$, $(-1,0)$, $\left(\frac{4}{5},0\right)$ for $\bullet^5$.
8. $x=-3$ may appear in the working for part (a).

Commonly Observed Responses:

Candidate C - starting again

$(x+1)(5x^2+11x-12)$ $\bullet^3$ ✓

$(x+1)(5x-4)(x+3)$ $\bullet^4$ ✓

$x=-1,\ \frac{4}{5},\ -3$ $\bullet^5$ ✓

Candidate D - alternative valid strategy

$$\begin{array}{r|rrr}
-1 & 5 & 1 & -4 \\
 & & -5 & 4 \\
\hline
 & 5 & -4 & 0
\end{array}$$

$\bullet^3$ ✓ $\bullet^4$ ✓

$x=-1,\ \frac{4}{5},\ -3$ $\bullet^5$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
8.			**Method 1**	**Method 1**	3
			$\bullet^1$ rearrange and apply $\log_a x - \log_a y = \log_a \frac{x}{y}$	$\bullet^1$ $\log_a \frac{75}{3} = 2$	
			$\bullet^2$ write in exponential form	$\bullet^2$ $\frac{75}{3} = a^2$ stated or implied by $\bullet^3$	
			$\bullet^3$ find a	$\bullet^3$ 5	
			Method 2	**Method 2**	
			$\bullet^1$ write 2 as $2\log_a a$ and apply $n\log_a x = \log_a x^n$	$\bullet^1$ $\log_a 75 = \log_a a^2 + \log_a 3$	
			$\bullet^2$ apply $\log_a x + \log_a y = \log_a xy$	$\bullet^2$ $\log_a 75 = \log_a 3a^2$	
			$\bullet^3$ find a	$\bullet^3$ 5	

Notes:

1. Where candidates state expressions such as $\sqrt{25} = a^2$ or $a = 25^2$, $\bullet^3$ is not available.
2. Each line of working must be equivalent to the line above within a valid strategy.
3. Where candidates state $a = -5$ without discarding that 'solution', $\bullet^3$ is not available.
4. Do not penalise candidates who score out 'log' from equations of the form $\log_a x = \log_a y$. See Candidate H.
5. For the correct answer without working, award 0/3.

Commonly Observed Responses:

Candidate A - invalid strategy $\log_a 75 = 2 + \log_a 3$ $75 = a^2 + 3$ $\quad\bullet^1 \times \ \bullet^2 \times \ \bullet^3 \times$ $a = \sqrt{72}$	**Candidate B - invalid strategy** $\log_a 75 = 2 + \log_a 3$ $\log_a 75 = \log_a 3^2$ $\quad\bullet^1 \times \ \bullet^2 \times \ \bullet^3 \times$
Candidate C - valid strategy incorrectly applied $\log_a 75 = 2 + \log_a 3$ $\log_a (225) = 2$ $\quad\bullet^1 \times$ $225 = a^2$ $\quad\bullet^2 \checkmark_1$ $a = 15$ $\quad\bullet^3 \checkmark_1$	**Candidate D - valid strategy incorrectly applied** $\log_a 75 = 2 + \log_a 3$ $\log_a \left(\frac{75}{3}\right) = 2$ $\quad\bullet^1 \checkmark$ $\log_a 15 = 2$ $15 = a^2$ $\quad\bullet^2 \times$ $a = \sqrt{15}$ $\quad\bullet^3 \checkmark_1$
Candidate E $\log_a 75 = 2 + \log_a 3$ $\log_a \left(\frac{75}{3}\right) - 2$ $\quad\bullet^1 \wedge \ \bullet^2 \wedge \ \bullet^3 \wedge$	**Candidate F - BEWARE** $\log_a 25 = 2$ $\quad\bullet^1 \checkmark$ $\log_a \sqrt{25}$ $\quad\bullet^2 \times \ \bullet^3 \times$ $\log_a 5$ $a = 5$ $\quad$ $\boxed{\text{SEEN}}$

Question	Generic scheme	Illustrative scheme	Max mark
8. (continued)			

Candidate G $\log_a 75 = 2 + \log_a 3$ $\dfrac{\log_a 75}{\log_a 3} = 2$ $\quad\bullet^1$ ✗ $\bullet^2$ ✗ $\bullet^3$ ✗ $25 = a^2$ $a = 5$	**Candidate H - scoring out** $\cancel{\log_a} 25 = \cancel{\log_a} a^2$ $a = 5$ $\quad\bullet^3$ ✓ **However** $\dfrac{\cancel{\log_a} 25}{\cancel{\log_a}} = \dfrac{\cancel{\log_a} a^2}{\cancel{\log_a}}$ $a = 5$ $\quad\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
9.			•1 substitute •2 write in standard quadratic form •3 find x-coordinates •4 find y-coordinates	•1 $x^2+(x+1)^2-2x+6(x+1)-15(=0)$ •2 $2x^2+6x-8=0$ •3 •4 •3 $-4,\quad 1$ •4 $-3,\quad 2$	4

Notes:

1. •2 is only available if '$=0$' appears at the •1 or •2 stage.
2. Accept $x^2+3x-4=0$ for •2.
3. The quadratic at •2 must lead to two distinct real roots for •3 and •4 to be available.
4. Where candidates arrive at an equation which is not a quadratic at •2 stage, then •3 and •4 are not available.
5. Where candidates identify **both** solutions by inspection, full marks may be awarded provided they justify that their points lie on **both** the line and the circle. Where candidates identify both solutions, but justify in only one equation, award 2/4.

Commonly Observed Responses:

Candidate A - substituting for x	Candidate B - not squaring
$(y-1)^2+y^2-2(y-1)+6y-15=0$ •1 ✓ $2y^2+2y-12=0$ •2 ✓ $y=-3$ or $y=2$ •3 ✓ $x=-4$ or $x=1$ •4 ✓	$x^2+(x+1)-2x+6(x+1)-15=0$ •1 ✗ •2 ✗ $x^2+5x-8=0$ $x=\dfrac{-5+\sqrt{57}}{2}$ and $\dfrac{-5-\sqrt{57}}{2}$ •3 ✓1 $y=\dfrac{-3+\sqrt{57}}{2}$ and $\dfrac{-3-\sqrt{57}}{2}$ •4 ✓1 The working for •2 is not of equivalent difficulty

Question			Generic scheme	Illustrative scheme	Max mark
10.			•[1] calculate $\lvert \mathbf{u} \rvert$	•[1] $\sqrt{2}$	5
			•[2] find expression for $\lvert \mathbf{v} \rvert$	•[2] $\sqrt{10+k^2}$	
			•[3] evaluate $\mathbf{u} \cdot \mathbf{v}$	•[3] 4	
			•[4] substitute into formula for scalar product	•[4] $4 = \sqrt{2} \times \sqrt{10+k^2} \times \cos 45°$ # **OR** $\cos 45° = \dfrac{4}{\sqrt{2} \times \sqrt{10+k^2}}$	
			•[5] find k	•[5] $\sqrt{6}$	

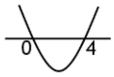
Notes:

1. Do not penalise the use of $\mathbf{a}$ and $\mathbf{b}$ in place of $\mathbf{u}$ and $\mathbf{v}$.
2. •[4] should not be awarded to candidates who simply state the formula $\cos 45° = \dfrac{\mathbf{u} \cdot \mathbf{v}}{\lvert \mathbf{u} \rvert \lvert \mathbf{v} \rvert}$.
3. Do not penalise the omission or incorrect use of units.
4. Where candidates find $\lvert \mathbf{v} \rvert = 4$ by substitution and equate that to $\sqrt{10+k^2}$, award •[3] and •[4].
5. •[5] is only available for a positive solution using a valid formula. However, do not penalise the appearance of $-\sqrt{6}$.

Commonly Observed Responses:

Candidate A - no reference to $\cos 45°$		Candidate B - substitution followed by equating	
$4 = \sqrt{2} \times \sqrt{10+k^2} \times \dfrac{1}{\sqrt{2}}$	•[1] ✓ •[2] ✓ •[3] ✓ •[4] ✓	$\mathbf{u} \cdot \mathbf{v} = \sqrt{2} \times \sqrt{10+k^2} \times \cos 45°$	•[1] ✓ •[2] ✓
$k = \sqrt{6}$	•[5] ✓	$\mathbf{u} \cdot \mathbf{v} = \sqrt{10+k^2}$	
		$\mathbf{u}.\mathbf{v} = 4$	•[3] ✓
		$4 = \sqrt{10+k^2}$	•[4] ✓

Candidate C - all working contained in formula		Candidate D - invalid notation	
$4 = \sqrt{1+1} \times \sqrt{1+3^2+k^2} \times \cos 45°$	•[3] ✓ •[4] ✓	$\mathbf{u} \cdot \mathbf{v} = \begin{pmatrix} 1 \\ 3 \\ 0 \end{pmatrix}$	•[3] ✗
$4 = \sqrt{2} \times \sqrt{10+k^2} \times \cos 45°$	•[1] ✓ •[2] ✓	$\mathbf{u}.\mathbf{v} = 4$	

Question			Generic scheme	Illustrative scheme	Max mark
11.			•¹ use the discriminant	•¹ $(3k)^2 - 4(9)(k)$	4
			•² apply condition	•² $(3k)^2 - 4(9)(k) > 0$	
			•³ identify roots of quadratic expression	•³ 0, 4	
			•⁴ state range with justification	•⁴ $k < 0$ and $k > 4$ with eg labelled sketch: 	

Notes:

1. At •¹, treat the inconsistent use of brackets, for example $3k^2 - 4 \times 9 \times k$, as bad form only if the unbracketed terms are dealt with correctly in the next line of working. See also Candidate F.
2. Where candidates do not state an inequality in terms of k but state expressions for a, b and c, and $b^2 - 4ac > 0$, award •². Where no expressions for a, b and c are stated, •² is only available where •⁴ is awarded. See Candidates A, B and C.
3. If candidates have the condition 'discriminant < 0', 'discriminant ≤ 0' or 'discriminant ≥ 0', then •² is lost but •³ and •⁴ are available.
4. Ignore the appearance of $b^2 - 4ac = 0$ where the correct condition has subsequently been applied. See Candidate D.
5. If candidates only work with the condition 'discriminant = 0', then •² and •⁴ are unavailable. See Candidate E.
6. Accept the appearance of 0 and 4 within inequalities for •³.
7. For the appearance of x in any expression of the discriminant, no further marks are available.

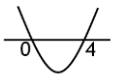
Commonly Observed Responses:

Candidate A - no expressions for *a*, *b* and *c*

Two real and distinct roots $b^2 - 4ac > 0$

$9k^2 - 36k = 0$ •¹ ✓

$k = 0,\ k = 4$ •³ ✓



$k < 0,\ k > 4$ •² ✓ •⁴ ✓

In this case •² is only available where •⁴ is awarded

Candidate B - expressions for *a*, *b* and *c*

$a = 9,\ b = 3k,\ c = k$

$b^2 - 4ac > 0$ •² ✓

Question	Generic scheme	Illustrative scheme	Max mark

11.(continued)

<table>
<tr>
<td>

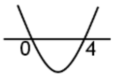
Candidate C - working with ='0'

$(3k)^2 - 4\times 9\times k$ $\bullet^1$ ✓

$9k^2 - 36k = 0$

$k = 0,\ k = 4$ $\bullet^3$ ✓

$b^2 - 4ac > 0$



$k < 0,\ k > 4$ $\bullet^2$ ✓ $\bullet^4$ ✓

In this case $\bullet^2$ is only available where $\bullet^4$ is awarded

</td>
<td>

Candidate D

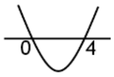
$(3k)^2 - 4\times 9\times k$ $\bullet^1$ ✓

$b^2 - 4ac = 0$

$9k^2 - 36k = 0$

$k = 0,\ k = 4$ $\bullet^3$ ✓

$9k^2 - 36k > 0$ $\bullet^2$ ✓



$k < 0,\ k > 4$ $\bullet^4$ ✓

</td>
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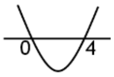
<tr>
<td>

Candidate E - no inequality stated

$(3k)^2 - 4\times 9\times k$ $\bullet^1$ ✓

$9k^2 - 36k = 0$ $\bullet^2$ ✗

$k = 0,\ k = 4$ $\bullet^3$ ✓



$k < 0,\ k > 4$ $\bullet^4$ ✗

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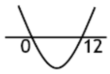
Candidate F - incorrect squaring

$3k^2 - 4\times 9\times k > 0$ $\bullet^1$ ✗ $\bullet^2$ $✓_1$

$k = 0,\ k = 12$ $\bullet^3$ $✓_1$



$k < 0,\ k > 12$ $\bullet^4$ $✓_1$

</td>
</tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
12.			•1 integrate one term	•1 $6\sin x\ldots$ or $\ldots-\frac{8}{2}\cos 2x$	4
			•2 complete integration	•2 $6\sin x-\frac{8}{2}\cos 2x+c$	
			•3 substitute for x and y	•3 $4=6\sin\frac{\pi}{6}-\frac{8}{2}\cos\left(2\times\frac{\pi}{6}\right)+c$	
			•4 state equation	•4 $y=6\sin x-4\cos 2x+3$ stated explicitly	

Notes:

1. Where candidates omit $+c$, only •1 is available.
2. Where candidates differentiate either term, •3 and •4 are not available.
3. Where candidates substitute into the original expression, •3 and •4 are not available.
4. Where candidates use an invalid approach, for example $6\cos x^2$, •3 and •4 are unavailable.
5. Do not penalise the appearance of an integral sign and/or 'dx' at •1, •2 or •3.
6. Do not penalise candidates who substitute 30° after integrating.
7. •4 is only available where candidates work with a double angle after integrating.
8. Where candidates work with an expansion of $\cos 2x$, an explicit statement of $y=\ldots$ must still be stated. See Candidate D.

Candidate A - incomplete substitution

$y=6\sin x-4\cos 2x+c$ •1 ✓ •2 ✓

$y=6\sin\frac{\pi}{6}-4\cos\frac{2\pi}{6}+c$

$c=3$ •3 ^

$y=6\sin x-4\cos 2x+3$ •4 $✓_1$

Candidate B - partial integration

$y=6\sin x+8\sin 2x+c$ •1 ✓ •2 ✗

$4=6\sin\frac{\pi}{6}+8\sin\frac{2\pi}{6}+c$ •3 $✓_1$

$c=1-4\sqrt{3}$

$y=6\sin x+8\sin 2x+1-4\sqrt{3}$ •4 $✓_1$

Candidate C - integration incomplete at •2 stage

$y=6\sin x-4\cos 2x$ •1 ✓ •2 ^

$y=6\sin x-4\cos 2x+c$

$4=6\sin\frac{\pi}{6}-4\cos\frac{2\pi}{6}+c$ •3 $✓_1$

$y=6\sin x-4\cos 2x+3$ •4 $✓_1$

Candidate D - use of double angles

$y=6\sin x-4\cos 2x+c$ •1 ✓ •2 ✓

$y=8\sin^2 x+6\sin x-4+c$

$4=8\sin^2\left(\frac{\pi}{6}\right)+6\sin\left(\frac{\pi}{6}\right)-4+c$ •3 ✓

$c=3$

$y=8\sin^2 x+6\sin x-1$ •4 ✓

All three expansions of $\cos 2x$ are valid.

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•¹ equate derivative to zero and solve for x	•¹ $(x+5)(2-x)=0$ **OR** $f'(x)=0$ **OR** $\frac{dy}{dx}=0$ **stated explicitly** leading to -5 and 2	**3**
			•² construct nature table(s)	•² (see table below)	
			•³ interpret table and state conclusions	•³ min at $x=-5$; max at $x=2$	

x	…	-5	…	2	…
$f'(x)$	$-$	0	$+$	0	$-$
(shape)	\	_	/	–	\

(•² for the $f'(x)$ row; •³ for the shape row)

Notes:

1. •² and •³ may be awarded vertically.
2. •³ is still available in cases where a candidates table of signs does not lead legitimately to a maximum/minimum shape.
3. Candidates may use the second derivative. See Candidates A and B.
4. Ignore any y-coordinates.
5. •² is not available where any errors are made in calculating values of $f'(x)$.

Commonly Observed Responses:

Candidate A - second derivative			**Candidate B - second derivative**		
$f''(x)=-2x-3$			$f''(x)=-2x-3$		
•² ✓	•³ ✓		•² ✓	•³ ✓	
$f''(-5)>0$	$f''(2)<0$	•² ✓	$f''(-5)=7>0$	$f''(2)=-7<0$	•² ✓
So min at $x=-5$	so max at $x=2$	•³ ✓	so min at $x=-5$	so max at $x=2$	•³ ✓

Question	Generic scheme	Illustrative scheme	Max mark

13.(a) (continued)

For the table of signs for a derivative, **accept**:

x	-5^-	-5	-5^+
$f'(x)$	$-$	0	$+$
Slope or shape	$\diagdown$	—	$\diagup$

x	$\rightarrow$	-5	$\rightarrow$
$f'(x)$	$-$	0	$+$
Slope or shape	$\diagdown$	—	$\diagup$

Arrows are taken to mean 'in the neighbourhood of'

x	a	-5	b
$f'(x)$	$-$	0	$+$
Slope or shape	$\diagdown$	—	$\diagup$

where $a<-5$ and $-5<b<2$

AND

x	2^-	2	2^+
$f'(x)$	$+$	0	$-$
Slope or shape	$\diagup$	—	$\diagdown$

AND

x	$\rightarrow$	2	$\rightarrow$
$f'(x)$	$+$	0	$-$
Slope or shape	$\diagup$	—	$\diagdown$

Arrows are taken to mean 'in the neighbourhood of'

AND

x	c	2	d
$f'(x)$	$+$	0	$-$
Slope or shape	$\diagup$	—	$\diagdown$

where $-5<c<2$ and $d>2$

For the table of signs for a derivative, **accept**:

x	$\rightarrow$	-5	$\rightarrow$	2	$\rightarrow$
$f'(x)$	$-$	0	$+$	0	$-$
Slope or shape	$\diagdown$	—	$\diagup$	—	$\diagdown$

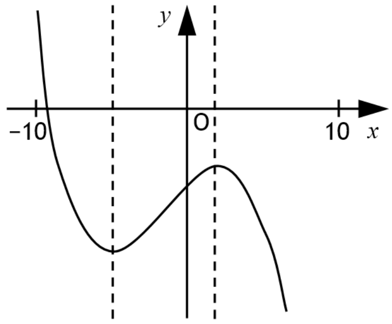
Since the function is continuous $-5 \rightarrow 2$ is acceptable

x	a	-5	b	2	c
$f'(x)$	$-$	0	$+$	0	$-$
Slope or shape	$\diagdown$	—	$\diagup$	—	$\diagdown$

Where $a<-5$, $-5<b<2$ and $c>2$
Since the function is continuous
$-5<b<2$ is acceptable

- For this question do not penalise the omission of 'x' or the word 'shape'/'slope'.
- Stating values of $f'(x)$ is an acceptable alternative to writing '+' or '–' signs.
- Acceptable variations of $f'(x)$ are: f', $\frac{df}{dx}$, $\frac{dy}{dx}$, $(x+5)(2-x)$ and $-x^2-3x+10$

Question			Generic scheme	Illustrative scheme	Max mark
13.	(b)		•[4] interpret result of (a) and first bullet point	•[4] cubic drawn with stationary points consistent with result of (a)	**3**
			•[5] interpret second bullet point within a cubic function	•[5] cubic drawn with y-intercept below the origin	
			•[6] interpret third bullet point within a cubic function	•[6] cubic drawn with exactly one root; $-10 < \text{root} < 10$	

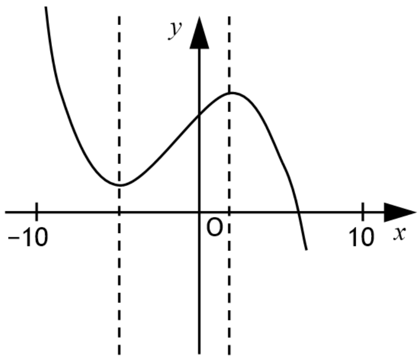


Notes:

6. •[4] is awarded for turning points in a cubic graph consistent with the natures stated in part (a), or where no natures have been stated, the shape of the graph from the nature table.
7. Do not penalise the appearance of y-coordinates.
8. For a cubic graph which does not have two stationary points, award 0/3.
9. •[6] is not available where an extension of the graph of $y = f(x)$ will cross the x-axis.

Commonly Observed Responses:

Candidate A - positive y-intercept

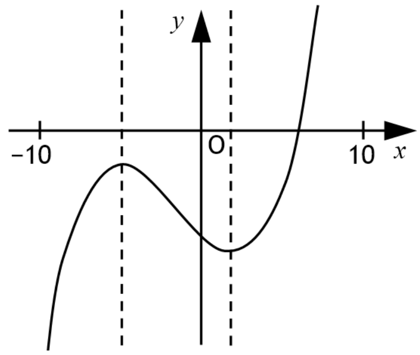


Award 2/3

Candidate B - follow through from (a)

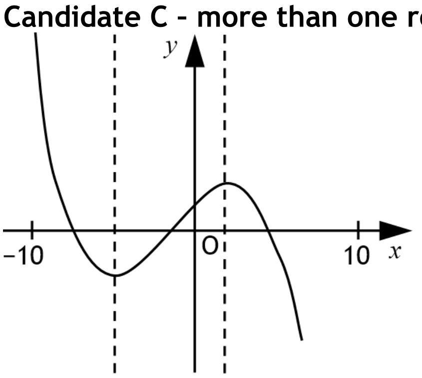
(a) max when $x = -5$ & min when $x = 2$

(b)



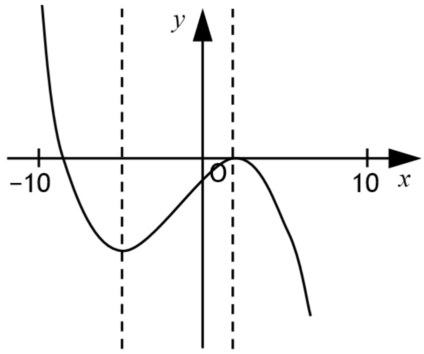
Award 3/3

Candidate C - more than one root



Award 1/3

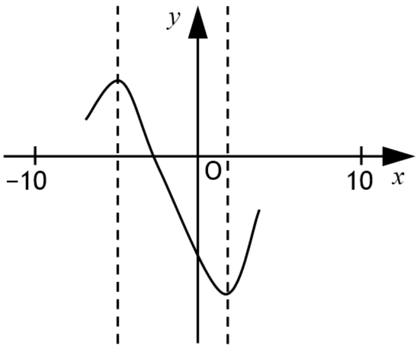
Candidate D - more than one root



Award 2/3

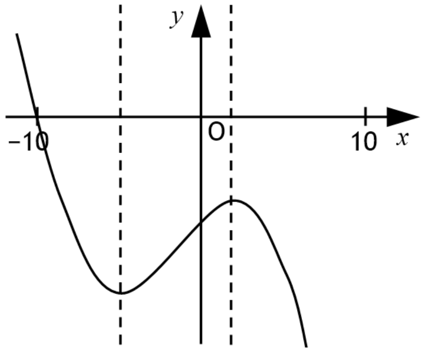
Question	Generic scheme	Illustrative scheme	Max mark
13.(b) (continued)			

Candidate E - more than one root implied



Award 1/3

Candidate F - root outwith −10 < root < 10



Award 2/3

[END OF MARKING INSTRUCTIONS]

National
Qualifications
2025

2025 Mathematics

Higher - Paper 2

Question Paper Finalised Marking Instructions

Marking instructions for each question

<table>
<tr><th colspan="3">Question</th><th>Generic scheme</th><th>Illustrative scheme</th><th>Max mark</th></tr>
<tr><td>1.</td><td>(a)</td><td></td><td>•1 find gradient of AC

•2 use property of perpendicular lines

•3 determine equation of altitude</td><td>•1 $-\frac{1}{3}$
OR
$-\frac{10}{30}$
•2 3
OR
$\frac{30}{10}$
•3 $y=3x-7$</td><td>3</td></tr>
<tr><td colspan="6">Notes:</td></tr>
<tr><td colspan="6">1. •3 is only available to candidates who find and use a perpendicular gradient.
2. The gradient of the altitude must appear in fully simplified form at the •2 or •3 stage for •3 to be awarded. See Candidate A.
3. At •3, accept any arrangement of a candidate’s equation where constant terms have been simplified.</td></tr>
<tr><td colspan="6">Commonly Observed Responses:</td></tr>
<tr><td colspan="4">Candidate A - not simplifying the gradient
$10y=30x-70$ •3 ✗</td><td colspan="2">Candidate B - m = m perp
$m=-\frac{1}{3}=3$ •1 ✓ •2 ✗
$y=3x-7$ •3 ✓1 - BOD

However

$m=-\frac{1}{3}$ •1 ✓
$=3$ •2 ✓ - BOD
$y=3x-7$ •3 ✓</td></tr>
</table>

Question			Generic scheme	Illustrative scheme	Max mark
1.	(b)		•[4] determine midpoint of BC	•[4] $(15,-2)$	**3**
			•[5] determine gradient of median	•[5] $\frac{1}{2}$ **OR** $\frac{12}{24}$	
			•[6] determine equation of median	•[6] $2y = x - 19$	

Notes:

4. •[5] is only available to candidates who use a midpoint to find a gradient.
5. •[6] is only available as a consequence of using a 'midpoint' of BC and the point A. See Candidates B to E.
6. The gradient of the median must appear in fully simplified form at the •[5] or •[6] stage for •[6] to be awarded.
7. At •[6], accept any arrangement of a candidate's equation where constant terms have been simplified.
8. •[6] is not available as a consequence of using a perpendicular gradient.

Commonly Observed Responses:

Candidate B - Perpendicular bisector of BC		**Candidate C - Altitude through A**	
$(15,-2)$	•[4] ✓		•[4] ^
$m_{BC} = -\frac{11}{3}$, $m_{\perp} = \frac{3}{11}$	•[5] ✗	$m_{BC} = -\frac{11}{3}$, $m_{\perp} = \frac{3}{11}$	•[5] ✗
$11y = 3x - 67$	•[6] ✗	$11y = 3x - 127$	•[6] ✗
For other perpendicular bisectors award 0/3			

Candidate D - Median through B		**Candidate E - Median through C**	
$\text{Midpoint}_{AC} = (6,-19)$	•[4] ✗	$\text{Midpoint}_{AB} = (0,3)$	•[4] ✗
$m_{BM} = 13$	•[5] $✓_1$	$m_{CM} = -\frac{9}{7}$	•[5] $✓_1$
$y = 13x - 97$	•[6] ✗	$7y = -9x + 21$	•[6] ✗

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		•[7] determine x-coordinate	•[7] -1	**2**
			•[8] determine y-coordinate	•[8] -10	

Notes:

9. For $(-1,-10)$ without working, award 2/2.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
2.			**Method 1** •1 identify common factor •2 complete the square •3 process for r and write in required form	**Method 1** •1 $2(x^2+8x\ldots$ stated or implied by •2 •2 $2(x+4)^2\ldots$ •3 $2(x+4)^2-27$	3
			Method 2 •1 expand completed square form •2 equate coefficients •3 process for q and r and write in required form	**Method 2** •1 $px^2+2pqx+pq^2+r$ stated or implied by •2 •2 $p=2$, $2pq=16$, $pq^2+r=5$ •3 $2(x+4)^2-27$	

Notes:

1. $2(x+4)^2-27$ with no working gains •1 and •2 only. However, see Candidate E.
2. Do not penalise candidates who do not work with p, q and r.

Commonly Observed Responses:

Candidate A $2(x^2+8)+5$ $2((x+4)^2-16)+5$ •1 ✓ •2 ✓ $2(x+4)^2-27$ •3 ✓ See exception to general marking principle (h)	**Candidate B - not using required form** $px^2+2pqx+pq^2+r$ •1 ✓ $p=2,\ 2pq=16,\ pq^2+r=5$ •2 ✓ $q=4,\ r=-27$ •3 ^ •3 is lost as answer is not in completed square form
Candidate C $2(x^2+16x)+5$ •1 ✗ $2((x+8)^2-64)+5$ •2 ✓1 $2(x+8)^2-123$ •3 ✓1	**Candidate D** $2((x+8)^2-64)+5$ •1 ✗ •2 ✗ $2(x+8)^2-123$ •3 ✓1
Candidate E $2(x+4)^2-27$ •1 ✓ •2 ✓ Check: $=2(x^2+8x+16)-27$ $=2x^2+16x+32-27$ $=2x^2+16x+5$ •3 ✓	**Candidate F** $2(x+4)^2=2x^2+16x+32$ •1 ✓ •2 ✓ $\therefore 2(x+4)^2-27=2x^2+16x+5$ •3 ✓

Question			Generic scheme	Illustrative scheme	Max mark
3.			•[1] state appropriate definite integral	•[1] $\int_2^4 \left(x^2 - 2x + 3\right) dx$	**4**
			•[2] integrate	•[2] $\frac{x^3}{3} - \frac{2x^2}{2} + 3x$	
			•[3] substitute limits	•[3] $\left(\frac{4^3}{3} - \frac{2(4)^2}{2} + 3(4)\right) - \left(\frac{2^3}{3} - \frac{2(2)^2}{2} + 3(2)\right)$	
			•[4] calculate area	•[4] $\frac{38}{3}$	

Notes:

1. •[1] is not available to candidates who omit 'dx'.
2. Limits must appear at the •[1] stage for •[1] to be awarded.
3. Where candidates differentiate one or more terms at •[2], then •[3] and •[4] are unavailable.
4. Candidates who substitute limits without integrating do not gain •[3] or •[4].
5. Do not penalise the inclusion of '$+c$' at •[2] or •[3].
6. Do not penalise the continued appearance of the integral sign or 'dx' after •[1].
7. Do not penalise rounded or truncated answers with at least one decimal place.

Commonly Observed Responses:

Candidate A - missing working		Candidate B - evidence of substitution using a calculator	
$\int_2^4 x^2 - 2x + 3$	•[1] ^		•[1] ^
$= \frac{x^3}{3} - \frac{2x^2}{2} + 3x$	•[2] ✓	$= \frac{x^3}{3} - \frac{2x^2}{2} + 3x$	•[2] ✓
	•[3] ^	$= \frac{52}{3} - \frac{14}{3}$	•[3] ✓
$= \frac{38}{3}$	•[4] ✓1	$= \frac{38}{3}$	•[4] ✓

Candidate C - limits not stated at •[1]		Candidate D - reversed limits	
	•[1] ^	$\int_4^2 \left(x^2 - 2x + 3\right) dx$	
$= \frac{x^3}{3} - \frac{2x^2}{2} + 3x$	•[2] ✓	...	•[2] ✓ •[3] ✓
$= \left(\frac{4^3}{3} - \frac{2(4)^2}{2} + 3(4)\right) - \left(\frac{2^3}{3} - \frac{2(2)^2}{2} + 3(2)\right)$	•[3] ✓	$= -\frac{38}{3}$, hence area is $\frac{38}{3}$	•[1] ✓ •[4] ✓
$= \frac{38}{3}$	•[4] ✓	However, $\int_4^2 \left(x^2 - 2x + 3\right) dx$ ...	•[2] ✓ •[3] ✓
		$-\frac{38}{3}$	
		$= \frac{38}{3}$ units^2	•[1] ✗ •[4] ✗

Question			Generic scheme	Illustrative scheme	Max mark
4.			**Method 1** •[1] equate composite function to x •[2] write $g\left(g^{-1}(x)\right)$ in terms of $g^{-1}(x)$ •[3] state inverse function	**Method 1** •[1] $g\left(g^{-1}(x)\right)=x$ •[2] $\left(g^{-1}(x)-4\right)^3=x$ •[3] $g^{-1}(x)=\sqrt[3]{x}+4$	3
			Method 2 •[1] write as $y=(x-4)^3$ and start to rearrange •[2] complete rearrangement •[3] state inverse function	**Method 2** •[1] $y=g(x)\Rightarrow x=g^{-1}(y)$ $\sqrt[3]{y}=x-4$ •[2] $x=4+\sqrt[3]{y}$ •[3] $g^{-1}(y)=\sqrt[3]{y}+4$ $\Rightarrow g^{-1}(x)=\sqrt[3]{x}+4$	

Notes:

1. In method 1, accept $x=\left(g^{-1}(x)-4\right)^3$ for •[1] and •[2].
2. In method 2, accept '$\sqrt[3]{y}=x-4$' without reference to $y=g(x)\Rightarrow x=g^{-1}(y)$ at •[1].
3. In method 2, accept $g^{-1}(x)=\sqrt[3]{x}+4$ without reference to $g^{-1}(y)$ at •[3].
4. In method 2, beware of candidates with working where each line is not mathematically equivalent. See Candidates A, B and C.
5. At •[3] stage, accept g^{-1} written in terms of any dummy variable. For example $g^{-1}(y)=\sqrt[3]{y}+4$.
6. $y=\sqrt[3]{x}+4$ does not gain •[3].
7. $g^{-1}(x)=\sqrt[3]{x}+4$ with no working gains 3/3.
8. In method 2, where candidates make multiple algebraic errors at the •[2] stage, •[3] is still available.
9. Marks should only be awarded for using a valid strategy to find the inverse of $g(x)$.

Commonly Observed Responses:

Candidate A		Candidate B	
$g(x)=(x-4)^3$		$g(x)=(x-4)^3$	
$y=(x-4)^3$		$y=(x-4)^3$	
$x=\sqrt[3]{y}+4$	•[1] ✓ •[2] ✓	$x=(y-4)^3$	•[1] ✗
$y=\sqrt[3]{x}+4$	•[3] ✗	$y=\sqrt[3]{x}+4$	•[2] $\checkmark_1$
$g^{-1}(x)=\sqrt[3]{x}+4$		$g^{-1}(x)=\sqrt[3]{x}+4$	•[3] $\checkmark_1$

Question	Generic scheme	Illustrative scheme	Max mark
4.	(continued)		

Candidate C $x=\left(g\left(x\right)-4\right)^3$ •1 ✗ $g\left(x\right)=\sqrt[3]{x}+4$ •2 ✓1 $g^{-1}\left(x\right)=\sqrt[3]{x}+4$ •3 ✓1	**Candidate D - Method 1** $g\left(g^{-1}\left(x\right)\right)=\left(g^{-1}\left(x\right)-4\right)^3$ •2 ✓ $x=\left(g^{-1}\left(x\right)-4\right)^3$ •1 ✓ $g^{-1}\left(x\right)=\sqrt[3]{x}+4$ •3 ✓
Candidate E - BEWARE of incorrect notation $g'\left(x\right)=\ldots$ •3 ✗ $f^{-1}\left(x\right)=\ldots$ •3 ✗	**Candidate F** $x\rightarrow x-4\rightarrow\left(x-4\right)^3=g\left(x\right)$ $-4\rightarrow\ ^3$ $\therefore\sqrt[3]{\ }\rightarrow\ +4$ •1 ✓ $\sqrt[3]{x}+4$ •2 ✓ $g^{-1}\left(x\right)=\sqrt[3]{x}+4$ •3 ✓

Question			Generic scheme	Illustrative scheme	Max mark
5.	(a)		$\bullet^1$ find an appropriate vector eg $\overrightarrow{AB}$ $\bullet^2$ find a second vector eg $\overrightarrow{BC}$ **AND** compare $\bullet^3$ appropriate conclusion	$\bullet^1$ eg $\overrightarrow{AB}=\begin{pmatrix}9\\-3\\6\end{pmatrix}$ $\bullet^2$ eg $\overrightarrow{BC}=\begin{pmatrix}6\\-2\\4\end{pmatrix}$ $\therefore \overrightarrow{AB}=\frac{3}{2}\overrightarrow{BC}$ $\bullet^3$ $\ldots \Rightarrow$ AB is parallel to BC (common direction) **AND** B is a common point $\Rightarrow$ A,B and C are collinear.	3

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. If no comparison of vectors or the trivial comparison '$\overrightarrow{AB}=\overrightarrow{BC}$' is made at $\bullet^2$, then $\bullet^3$ is not available.
3. $\bullet^3$ can only be awarded if a candidate has stated 'parallel', 'common point' and 'collinear'.
4. Candidates who state that 'points are parallel' or 'vectors are collinear' or 'parallel and share a common point $\Rightarrow$ collinear' do not gain $\bullet^3$. There must be a reference to the points.
5. Do not accept 'a, b and c are collinear' at $\bullet^3$.

Commonly Observed Responses:

Candidate A - missing labels

$\begin{pmatrix}9\\-3\\6\end{pmatrix}$ $\bullet^1$ ^

$\begin{pmatrix}6\\-2\\4\end{pmatrix}$ $\therefore \overrightarrow{AB}=\frac{3}{2}\overrightarrow{BC}$ $\bullet^2$ ✓$_1$

Missing labels at $\bullet^2$ is a repeated error

$\Rightarrow$ AB is parallel to BC
and B is a common point
$\Rightarrow$ A, B and C are collinear $\bullet^3$ ✓$_1$

Candidate B

$\overrightarrow{AB}=\begin{pmatrix}9\\-3\\6\end{pmatrix}$ $\bullet^1$ ✓

$\overrightarrow{BC}=\begin{pmatrix}6\\-2\\4\end{pmatrix}$

$\begin{pmatrix}9\\-3\\6\end{pmatrix}=3\begin{pmatrix}3\\-1\\2\end{pmatrix}$ **AND** $\begin{pmatrix}6\\-2\\4\end{pmatrix}=2\begin{pmatrix}3\\-1\\2\end{pmatrix}$ $\bullet^2$ ✓

$\therefore \overrightarrow{AB}=\frac{2}{3}\overrightarrow{BC}$

Ignore working subsequent to correct statement made on previous line

$\Rightarrow$ AB is parallel to BC
and B is a common point
$\Rightarrow$ A, B and C are collinear $\bullet^3$ ✓

Question			Generic scheme	Illustrative scheme	Max mark
5.	(b)		•4 state ratio	•4 3:2	1

Notes:

6. Answers in (b) must be consistent with the components of the vectors in (a) or the comparison of the vectors in (a). See Candidates C and D. However, award •4 for '3:2' with no working.
7. In this question, the answer for •4 must be stated explicitly in part (b).
8. The only acceptable variations for •4 must be related explicitly to AB and BC. For $\frac{BC}{AB}=\frac{2}{3}$, $\frac{AB}{BC}=\frac{3}{2}$ or $BC:AB=2:3$ stated in part (b) award •4. See Candidate E.
9. Accept unitary ratios for •4, for example $\frac{3}{2}:1$ or $1:\frac{2}{3}$.
10. Where candidates state multiple ratios which are not equivalent, award 0/1.

Commonly Observed Responses:

Candidate C - using components of vectors

(a) $\overrightarrow{AB}=\begin{pmatrix}9\\-3\\6\end{pmatrix}$ •1 ✓

$\overrightarrow{BC}=\begin{pmatrix}6\\-2\\4\end{pmatrix}$

$\overrightarrow{BC}=\frac{3}{2}\overrightarrow{AB}$ •2 ✗

(b) $3:2$ •4 ✓

Candidate D - using comparison of vectors

(a) $\overrightarrow{AB}=\begin{pmatrix}9\\-3\\6\end{pmatrix}$ •1 ✓

$\overrightarrow{BC}=\begin{pmatrix}6\\-2\\4\end{pmatrix}$

$\overrightarrow{BC}=\frac{3}{2}\overrightarrow{AB}$ •2 ✗

(b) $2:3$ •4 ✓1

Candidate E - acceptable variation

$\frac{AB}{BC}=\frac{3}{2}$ •4 ✓

Ratio $=2:3$

Ignore working subsequent to correct statement made on previous line

Candidate F - trivial ratio

Ratio $=1:1$ •4 ✗

Question			Generic scheme	Illustrative scheme	Max mark
6.	(a)		•[1] use compound angle formula •[2] compare coefficients •[3] process for k •[4] process for a and express in required form	•[1] $k\cos x\cos a - k\sin x\sin a$ **stated explicitly** •[2] $k\cos a = 5$ and $k\sin a = 9$ **stated explicitly** •[3] $\sqrt{106}$ •[4] $\sqrt{106}\cos(x + 1.06\ldots)$	4

Notes:

1. Accept $k(\cos x\cos a - \sin x\sin a)$ for •[1].
2. Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the •[2] stage both contain k.
3. $\sqrt{106}\cos x\cos a - \sqrt{106}\sin x\sin a$ **or** $\sqrt{106}(\cos x\cos a - \sin x\sin a)$ are acceptable for •[1] and •[3].
4. •[2] is not available for $k\cos x = 5$ and $k\sin x = 9$, however •[4] may still be gained. See Candidate E.
5. Accept $-k\sin a = -9$ and $k\cos a = 5$ for •[2].
6. •[4] is not available for a value of a given in degrees.
7. Accept values of a which round to 1.1.
8. Candidates may state and use any form of the wave function for •[1], •[2] and •[3]. However, •[4] is only available if the wave is interpreted in the form $k\cos(x + a)$.
9. Evidence for •[4] may not appear until part (b) and must appear by the •[5] stage.

Commonly Observed Responses:

Candidate A

•[1] ^

$\sqrt{106}\cos a = 5$

$\sqrt{106}\sin a = 9$ •[2] ✓ •[3] ✓

$\tan a = \frac{9}{5}$

$a = 1.06\ldots$

$\sqrt{106}\cos(x + 1.06\ldots)$ •[4] ✓

Candidate B

$k\cos x\cos a - k\sin x\sin a$ •[1] ✓

$\cos a = 5$

$\sin a = 9$ •[2] ✗

$\tan a = \frac{9}{5}$

$a = 1.06\ldots$ (Not consistent with equations at •[2])

$\sqrt{106}\cos(x + 1.06\ldots)$ •[3] ✓ •[4] ✗

Candidate C

$\cos x\cos a - \sin x\sin a$ •[1] ✗

$\cos a = 5$

$\sin a = 9$ •[2] ✗

$k = \sqrt{106}$ •[3] ✓

$\tan a = \frac{9}{5}$

$a = 1.06\ldots$ (Not consistent with equations at •[2]

$\sqrt{106}\cos(x + 1.06\ldots)$ •[4] ✗

Candidate D - errors at •[2]

$k\cos x\cos a - k\sin x\sin a$ •[1] ✓

$k\cos a = 9$

$k\sin a = 5$ •[2] ✗

$\tan a = \frac{5}{9}$

$a = 0.507\ldots$

$\sqrt{106}\cos(x + 0.507\ldots)$ •[3] ✓ •[4] ✓1

Question	Generic scheme	Illustrative scheme	Max mark

6.(a) (continued)

Candidate E - use of x at $\bullet^2$		Candidate F	
$k\cos x\cos a - k\sin x\sin a$	$\bullet^1$ ✓	$k\cos \mathrm{A}\cos \mathrm{B} - k\sin \mathrm{A}\sin \mathrm{B}$	$\bullet^1$ ✗
$k\cos x = 5$ $k\sin x = 9$	$\bullet^2$ ✗	$k\cos \mathrm{A} = 5$ $k\sin \mathrm{A} = 9$	$\bullet^2$ ✗
$\tan a = \frac{9}{5}$ $a = 1.06\ldots$ $\sqrt{106}\cos(x + 1.06\ldots)$	$\bullet^3$ ✓ $\bullet^4$ $✓_1$	$\tan \mathrm{A} = \frac{9}{5}$ $a = 1.06\ldots$ $\sqrt{106}\cos(x + 1.06\ldots)$	$\bullet^3$ ✓ $\bullet^4$ $✓_1$

Question			Generic scheme	Illustrative scheme	Max mark
6.	(b)		$\bullet^5$ link to (a) $\bullet^6$ solve for $(x+a)$ $\bullet^7$ solve for x	$\bullet^5$ $\sqrt{106}\cos(x+1.06\ldots)=7$ $\bullet^6$ $\qquad\qquad\qquad\bullet^7$ $\bullet^6$ $0.82\ldots$ $(7.106\ldots)$, $5.46\ldots$ $\bullet^7$ $6.04\ldots$, $\qquad 4.396\ldots$	3

Notes:

10. In part (b), where candidates work in degrees throughout, the maximum mark available is 2/3.
11. $\bullet^7$ is only available for two solutions within the stated range. Ignore 'solutions' outwith the range.
12. At $\bullet^7$ accept values of x which round to 6.0, 6.05 or 4.4.

Commonly Observed Responses:

Candidate G - converting to radians

$\vdots$	$\bullet^1\checkmark$ $\bullet^2\checkmark$ $\bullet^3\checkmark$
$\sqrt{106}\cos(x+60.9\ldots)$	$\bullet^4$ ✗
$\sqrt{106}\cos(x+60.9\ldots)=7$	$\bullet^5$ $\checkmark_1$
$x+60\otimes 9\ldots=312.8\ldots,\ 407.1\ldots$	
$x=251.8\ldots,\ 346.2\ldots$	$\bullet^6$ $\checkmark_1$
$x=\dfrac{251.9\pi}{180},\ \dfrac{346.2.\pi}{180}$	$\bullet^7$ $\checkmark_1$

Candidate H - working in degrees

$\vdots$	$\bullet^1\checkmark$ $\bullet^2\checkmark$ $\bullet^3\checkmark$
$\sqrt{106}\cos(x+60.9\ldots)$	$\bullet^4$ ✗
$\sqrt{106}\cos(x+60.9\ldots)=7$	$\bullet^5$ $\checkmark_1$
$x+60.9\ldots=312.8\ldots,\ 407.1\ldots$	
$x=251.8\ldots,\ 346.2\ldots$	$\bullet^6$ $\checkmark_1$ $\bullet^7$ ^

Candidate I - working in degrees

$\vdots$	$\bullet^1\checkmark$ $\bullet^2\checkmark$ $\bullet^3\checkmark$
$\sqrt{106}\cos(x+60.9\ldots)$	$\bullet^4$ ✗
$\sqrt{106}\cos(x+60.9\ldots)=7$	$\bullet^5$ $\checkmark_1$
$x+60.9\ldots=312.8\ldots,$	
$x=251.8\ldots$	$\bullet^6$ ^ $\bullet^7$ ^

Candidate J - working in degrees

$\vdots$	$\bullet^1\checkmark$ $\bullet^2\checkmark$ $\bullet^3\checkmark$
$\sqrt{106}\cos(x+60.9\ldots)$	$\bullet^4$ ✗
$\sqrt{106}\cos(x+60.9\ldots)=7$	$\bullet^5$ $\checkmark_1$
$x+60.9\ldots=312.8\ldots,\ 407.1\ldots$	$\bullet^6$ ^ $\bullet^7$ ^

Question			Generic scheme	Illustrative scheme	Max mark
7.			•¹ start to integrate •² complete integration	•¹ $\frac{(3x+2)^8}{8}$ •² $\cdots\times\frac{1}{3}+c$	2

Notes:

1. Award •¹ for any appearance of $\frac{(3x+2)^8}{8}$ regardless of any constant multiplier.
2. Where candidates **differentiate throughout** or make **no attempt to integrate**, award 0/2.
3. Where candidates start to integrate individual terms within the bracket or use another invalid approach, award 0/2.
4. Do not penalise the continued appearance of the integral sign or 'dx'.

Commonly Observed Responses:

Candidate A $\frac{(3x+2)^8}{8}+c$ •¹ ✓ •² ^	**Candidate B - NOT Differentiating throughout** $7(3x+2)^6\times\frac{1}{3}+c$ •¹ ✗ •² ✓₁
Candidate C - 'Integrating' over two lines $\frac{(3x+2)^8}{8}$ •¹ ✓ •² ^ $\frac{(3x+2)^8}{8}\times\frac{1}{3}+c$ SEEN	**Candidate D - integration incomplete at •² stage** $\frac{(3x+2)^8}{8}\times\frac{1}{3}$ •¹ ✓ •² ^ $\frac{(3x+2)^8}{8}\times\frac{1}{3}+c$ SEEN
Candidate E - integration by substitution $\int u^7\times\frac{1}{3}du$ where $u=3x+2$ and $du=3dx$ •¹ ✓ $\frac{u^8}{24}+c$ $\frac{(3x+2)^8}{24}+c$ •² ✓ However, for $\int u^7du$ or $\int u^7dx$ leading to $\frac{(3x+2)^8}{24}+c$ award 0/2.	

Question			Generic scheme	Illustrative scheme	Max mark
8.			•[1] identify an appropriate pathway •[2] find $\overrightarrow{BE}$	•[1] eg $\overrightarrow{BC}+\overrightarrow{CD}+\overrightarrow{DE}$ stated or implied by •[2] •[2] $(0\mathbf{i}+)\,5\mathbf{j}+4\mathbf{k}$	2

Notes:

1. Do not penalise inconsistent vector notation (for example lack of arrows or brackets).
2. •[1] is not available for $\overrightarrow{BD}+\overrightarrow{DE}$ or $\overrightarrow{BC}+\overrightarrow{CE}$ or similar. However, see Candidate C.
3. •[2] is only available where a valid pathway has been stated.
4. Do not accept $\begin{pmatrix}0\mathbf{i}\\5\mathbf{j}\\4\mathbf{k}\end{pmatrix}$ or $\begin{pmatrix}0\\5\\4\end{pmatrix}$ for •[2].
5. Where an invalid pathway 'leads' to $5\mathbf{j}+4\mathbf{k}$ award 0/2.

Commonly Observed Responses:

Candidate A - using given vectors	Candidate B - using given vectors
$\overrightarrow{BE}=-\overrightarrow{DC}+\overrightarrow{AD}+\overrightarrow{DE}$ •[1] ✓ $5\mathbf{j}+4\mathbf{k}$ •[2] ✓	$\overrightarrow{BE}=(6\mathbf{i}+4\mathbf{j}+2\mathbf{k})-(2\mathbf{i}-4\mathbf{j}+2\mathbf{k})+(-4\mathbf{i}-3\mathbf{j}+4\mathbf{k})$ •[1] ✓ $\overrightarrow{BE}=\begin{pmatrix}6\\4\\2\end{pmatrix}-\begin{pmatrix}2\\-4\\2\end{pmatrix}+\begin{pmatrix}-4\\-3\\4\end{pmatrix}$ •[1] ✓
Candidate C $\overrightarrow{BD}+\overrightarrow{DE}$ $\overrightarrow{BD}=4\mathbf{i}+8\mathbf{j}$ •[1] ✓ $\overrightarrow{BE}=5\mathbf{j}+4\mathbf{k}$ •[2] ✓	

Question			Generic scheme	Illustrative scheme	Max mark
9.	(a)		•[1] substitute •[2] process	•[1] $10 = 10m + 4$ **OR** $10 = \dfrac{4}{1-m}$ •[2] $m = \dfrac{3}{5}$	2

Notes:

1. Correct answer with no working, award 1/2.
2. Where candidates state '$m = \dfrac{3}{5}$' or equivalent, and then verify the result from the given recurrence relationship, award 1/2.
3. Where candidates work in terms of a, a link to m must be made for •[1] to be awarded. See Candidates A, B and C.

Commonly Observed Responses:

Candidate A - working not in terms of *m*	**Candidate B - working not in terms of *m***
(a) $10 = \dfrac{4}{1-a}$ •[1] ^ $a = \dfrac{3}{5}$ •[2] ✓[1]	(a) $10 = \dfrac{4}{1-a}$ $a = \dfrac{3}{5}$ •[2] ✓ $m = \dfrac{3}{5}$ •[1] ✓
Candidate C - working not in terms of *m* (a) $10 = \dfrac{4}{1-a}$ $a = \dfrac{3}{5}$ •[2] ✓ (b) $19 = \dfrac{3}{5}u_0 + 4$ •[1] ✓ **•[1] is awarded when the calculated value of *a* is used in place of *m*.**	

Question			Generic scheme	Illustrative scheme	Max mark
9.	(b)		$\bullet^3$ calculate u_0	$\bullet^3$ 52	**1**

Notes:

4. Where candidates use an incorrect value of m without supporting working in part (a) or which is inconsistent with their answer in part (a), $\bullet^3$ is not available.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
10.	(a)		•1 express P in terms of x and y	•1 $P=12x+2y$	3
			•2 express y in terms of x	•2 $y=\dfrac{150-6x^2}{5x}$	
			•3 substitute for y and complete proof	•3 $P=12x+2\left(\dfrac{150-6x^2}{5x}\right)$ leading to $P=9.6x+\dfrac{60}{x}$	

Notes:

1. Accept $P=4x+3x+y+5x+y$ or equivalent for •1.
2. The substitution for y at •3 must be clearly shown for •3 to be available.
3. Do not penalise the omission of brackets at •3 leading to the correct solution. See Candidate A.

Commonly Observed Responses:

Candidate A - missing brackets ⋮ $P=12x+2\times\dfrac{150-6x^2}{5x}$ $P=9.6x+\dfrac{60}{x}$ •3 ✓	

Question			Generic scheme	Illustrative scheme	Max mark
10.	(b)		•⁴ express P in differentiable form	•⁴ $9.6x+60x^{-1}$ stated or implied by •⁵	**6**
			•⁵ differentiate	•⁵ $9.6-60x^{-2}$	
			•⁶ equate expression for derivative to 0	•⁶ $9.6-60x^{-2}=0$	
			•⁷ process for x	•⁷ 2.5 or $\frac{5}{2}$	
			•⁸ verify nature	•⁸ table of signs for a derivative $\therefore$ minimum **OR** $P''(x)=120x^{-3}$ and $P''(2.5)=7.68\ldots>0 \;\therefore$ minimum	
			•⁹ find minimum value of P	•⁹ 48(cm)	

Notes:

4. For a numerical approach, award 0/6.
5. •⁶ can be awarded for $60x^{-2}=9.6$.
6. Where candidates integrate any term at the •⁵ stage, only •⁶ is available on follow through for setting their 'derivative' to 0.
7. •⁷, •⁸ and •⁹ are only available for working with a derivative which contains an index ≤ -2.
8. $\left(\dfrac{60}{9.6}\right)^{0.5}$ or $\sqrt[-2]{\dfrac{9.6}{60}}$ must be simplified at •⁷ or •⁸ for •⁷ to be awarded.
9. Ignore the appearance of −2.5 at •⁷.
10. Notation for the derivative is only assessed at •⁸.
11. •⁸ is not available to candidates who consider a value of $x \leq 0$ in the neighbourhood of 2.5.
12. •⁹ is still available in cases where a candidate's table of signs does not lead legitimately to a minimum at •⁸.
13. •⁸ and •⁹ are not available to candidates who state that the minimum exists at a value of x where $x \leq 0$.

Commonly Observed Responses:

Candidate B - differentiating over multiple lines		**Candidate C - differentiating over multiple lines**	
$P'(x)=9.6+60x^{-1}$	•⁴ ✗	$P'(x)=9.6x+60x^{-1}$	•⁴ ✓
$P'(x)=9.6-60x^{-2}$	•⁵ ✗	$P'(x)=9.6-60x^{-2}$	•⁵ ✗
$9.6-60x^{-2}=0$	•⁶ $\checkmark_1$	$9.6-60x^{-2}=0$	•⁶ $\checkmark_1$
Candidate D			
Stationary points when $P'(x)=0$			
$P'(x)=9.6-60x^{-2}$	•⁴ ✓ •⁵ ✓ •⁶ ✓		

Question	Generic scheme	Illustrative scheme	Max mark

10.(b) (continued)

For the table of signs for a derivative, **accept**:

x	2.5^-	2.5	2.5^+
$P'(x)$	$-$	0	$+$
Slope or shape	\	—	/

x	$\rightarrow$	2.5	$\rightarrow$
$P'(x)$	$-$	0	$+$
Slope or shape	\	—	/

Arrows are taken to mean 'in the neighbourhood of'

x	a	2.5	b
$P'(x)$	$-$	0	$+$
Slope or shape	\	—	/

Where $0 < a < 2.5$ and $b > 2.5$

For the table of signs for a derivative, do **NOT accept**:

x	$\rightarrow$	-2.5	$\rightarrow$	2.5	$\rightarrow$
$P'(x)$	$+$	0	$-$	0	$+$
Slope or shape	/	—	\	—	/

Since the function is discontinuous $-2.5 \rightarrow 2.5$ is NOT acceptable

x	a	-2.5	b	2.5	c
$P'(x)$	$+$	0	$-$	0	$+$
Slope or shape	/	—	\	—	/

Since the function is discontinuous $-2.5 < b < 2.5$ is NOT acceptable

- For this question, do not penalise the omission of 'x' or the word 'shape'/'slope'
- Stating values of $P'(x)$ is an acceptable alternative to writing '+' or '–' signs
- Acceptable variations of $P'(x)$ are: P', $\frac{dP}{dx}$, and $9.6 - 60x^{-2}$. Accept $f'(x)$ only where candidates have previously used $f(x) = 9.6x + 60x^{-1}$ in their working
- Do not accept $\frac{dy}{dx}$ or $\frac{d^2y}{dx^2}$

Question			Generic scheme	Illustrative scheme	Max mark
11.			•1 substitute double angle formula for $\sin 2x°$ •2 factorise •3 solve for $\cos x°$ and $\sin x°$ •4 solve for x	•1 $3(2\sin x°\cos x°)+4\cos x°(=0)$ •2 eg $2\cos x°(3\sin x°+2)=0$ •3 $\cos x°=0$, $\sin x°=-\frac{2}{3}$ •4 90, 270, 221.8…, 318.1…	4

Notes:

1. Substituting $2\sin A\cos A$ for $\sin 2x°$ at the •1 stage should be treated as bad form provided the equation is written in terms of x at the •2 stage. Otherwise, •1 is not available.
2. '$=0$' must appear by the •2 stage for •2 to be awarded.
3. Do not penalise the absence of '2' as a common factor at •2.
4. Award •3 for $x=\cos^{-1}(0)$ **AND** $x=\sin^{-1}\left(-\frac{2}{3}\right)$.
5. Do not penalise the omission of degree signs.
6. Candidates who leave their answer in radians do not gain •4 (if marking horizontally) or •3 (if marking vertically).
7. Where equations for $\sin x°$ and/or $\cos x°$ do not have solutions, marks are unavailable for stating 'no solutions'.

Commonly Observed Responses:

Candidate A - dividing by cos x

$6\sin x°\cos x°=-4\cos x°$ •1 ✓
$6\sin x°=-4$ •2 ^ •3 ^
$x=221.8\ldots,\ 318.1\ldots$ •4 ✓1

Candidate B - insufficient evidence for •3

$6\sin x°\cos x°+4\cos x°=0$ •1 ✓
$2\cos x°(3\sin x°+2)=0$ •2 ✓
$2\cos x°=0,\ \sin x°=-\frac{2}{3}$ •3 ^ •4 ^
However,
$x=90,\ 270,\ 221.8\ldots,\ 318.1\ldots$ •3 ✓ •4 ✓

Candidate C

$\cos x°=0$, $\sin x°=-\frac{2}{3}$ •3 ✓
$x=90,\ 270$ $x=41$
$x=221.8\ldots,\ 318.1\ldots$ •4 ✗

However, where the final solution(s) are clearly identified by the candidate award •4.

Question			Generic scheme	Illustrative scheme	Max mark
12.	(a)		•¹ interpret notation •² state expression for $f(g(x))$	•¹ $f(1-x^3)$ **OR** $(g(x))^5+3$ •² $(1-x^3)^5+3$	**2**

Notes:

1. For $(1-x^3)^5+3$ without working, award 2/2.

Commonly Observed Responses:

Candidate A $f(g(x))=(1-x^3)^5+3$ •¹ ✓ •² ✓ $h(x)=4-x^{15}$	**Candidate B - two 'attempts'** $f(g(x))=x^5+3$ •¹ ✘ •² ✘ $f(g(x))=(1-x^3)^5+3$
Candidate C $f(g(x))=1-(x^5+3)^3$ •¹ ✘ •² ✓₁	

Question			Generic scheme	Illustrative scheme	Max mark
12.	(b)		•³ start to differentiate •⁴ complete differentiation	•³ $5\left(1-x^3\right)^4 \ldots$ •⁴ $\ldots\times\left(-3x^2\right)$	2

Notes:

2. For $-15x^2\left(1-x^3\right)^4$, award 2/2.
3. •³ and •⁴ are not available for working with $4-x^{15}$.
4. Accept '$5u^4$ where $u=1-x^3$' for •³.
5. Do not award •⁴ where the answer includes '$+c$'.

Commonly Observed Responses:

Candidate D - differentiating over two lines	Candidate E - poor notation
$5\left(1-x^3\right)^4$ •³ ✓ $5\left(1-x^3\right)^4\times\left(-3x^2\right)$ •⁴ ^	$y=\left(1-x^3\right)^5$ $y=1-x^3$ $\frac{dy}{dx}=-3x^2$ $\frac{dy}{dx}=5\left(1-x^3\right)^4\times\left(-3x\right)^2$ •³ ✓ •⁴ ✓
Candidate F - poor communication	**Candidate G - insufficient evidence for •³**
$y=\left(1-x^3\right)^5+3$ $y=5\left(1-x^3\right)^4\times\left(-3x^2\right)$ •³ ✓ •⁴ ✓	$-15\left(1-x^3\right)^4$ •³ ✘ •⁴ ✘

Question			Generic scheme	Illustrative scheme	Max mark
13.	(a)		•[1] identify initial mass	•[1] 150 (micrograms)	**1**

Notes:

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		•[2] interpret information	•[2] $120 = 150e^{-0.0054t}$ stated or implied by •[3]	**4**
			•[3] process equation	•[3] $e^{-0.0054t} = \dfrac{120}{150}$	
			•[4] write in logarithmic form	•[4] $\log_e\left(\dfrac{120}{150}\right) = -0.0054t$	
			•[5] process for t	•[5] 41.32… (years)	

Notes:

1. Where values other than 120 are used in the substitution, •[3], •[4] and •[5] are still available.
2. •[3] may be implied by •[4].
3. Evidence for •[4] must be stated explicitly. See Candidate B.
4. At •[4] all exponentials must be processed.
5. Any base may be used at •[4] stage. See Candidate A.
6. Accept $\ln 0.8 = -0.0054t \ln e$ and $-0.223\ldots = -0.0054t$ for •[4].
7. •[5] is unavailable where candidates round the value of $\ln 0.8$ to fewer than 2 decimal places.
8. Accept answers where $40.7 \le t \le 42$ at •[5].
9. The calculation at •[5] must follow from the valid use of exponentials and logarithms at •[3] and •[4].
10. Where candidates show no working or take an iterative approach to arrive at $t = 41$ or $t = 42$, award 1/4. However, if, in any iterations M is evaluated for $t = 41$ and $t = 42$ leading to a final answer of $t = 42$ (years), then award 4/4.

Commonly Observed Responses:

Candidate A - using other bases

$120 = 150e^{-0.0054t}$ •[2] ✓

$0.8 = e^{-0.0054t}$ •[3] ✓

$\log_{10} 0.8 = -0.0054t \log_{10} e$ •[4] ✓

$t = 41.32\ldots$ (years) •[5] ✓

Candidate B - missing working

$120 = 150e^{-0.0054t}$ •[2] ✓

$0.8 = e^{-0.0054t}$ •[3] ✓

•[4] ^

$t = 41$ •[5] ✓

Candidate C - iterative approach

$t = 40 \Rightarrow M = 120.86\ldots$

$t = 41 \Rightarrow M = 120.20\ldots$

$t = 42 \Rightarrow M = 119.56\ldots$

So $t = 42$ (years) **Award 4/4**

Candidate D - taking logarithms of both sides

$120 = 150e^{-0.0054t}$ •[2] ✓

$\log_e 120 = \log_e\left(150e^{-0.0054t}\right)$

$\log_e 120 = \log_e 150 + \log_e e^{-0.0054t}$ •[3] ✓

$\log_e 120 - \log_e 150 = -0.0054t$ •[4] ✓

$t = 41.32\ldots$ (years) •[5] ✓

Question			Generic scheme	Illustrative scheme	Max mark
14.	(a)		$\bullet^1$ state coordinates of centre $\bullet^2$ state radius	$\bullet^1$ $(-5,6)$ $\bullet^2$ 3	**2**

Notes:

1. Accept $x=-5$, $y=6$ for $\bullet^1$.
2. Do not accept '$a=\ldots, b=\ldots$' or '−5,6' for $\bullet^1$.

Commonly Observed Responses:

Question			Generic scheme	Illustrative scheme	Max mark
	(b)		$\bullet^3$ find coordinates of centre $\bullet^4$ find radius	$\bullet^3$ $(7,-3)$ $\bullet^4$ 2	**2**

Notes:

3. Accept $x=7$, $y=-3$ for $\bullet^3$.
4. Do not accept '$g=\ldots$, $f=\ldots$' or '7,−3' for $\bullet^3$.
5. Do not penalise candidates who treat negatives with a lack of rigour when calculating the radius.
   For example, accept $\sqrt{7^2+3^2-54}=2$ or $\sqrt{7^2+-3^2-54}=2$ or $\sqrt{-7^2+3^2-54}=2$ for $\bullet^4$.
   However, do not accept $\sqrt{7^2-3^2-54}=2$ for $\bullet^4$.

Commonly Observed Responses:

Candidate A - repeated error within a question			**Candidate B - two errors**		
(a)	−5,6	$\bullet^1$ ✗	(a)	5,−6	$\bullet^1$ ✗
(b)	7,−3	$\bullet^3$ ✓$_1$	(b)	7,−3	$\bullet^3$ ✗

Question			Generic scheme	Illustrative scheme	Max mark
	(c)		$\bullet^5$ find distance between centres of C_1 and C_2 $\bullet^6$ calculate radius of C_3 $\bullet^7$ find centre of C_3 and state equation of C_3	$\bullet^5$ 15 $\bullet^6$ 8 $\bullet^7$ $(-1,3)$ **stated explicitly** and $(x+1)^2+(y-3)^2=64$	**3**

Notes:

6. $\bullet^5$ may be awarded for $\sqrt{(-5-7)^2+(6+3)^2}$ within a valid calculation for the radius of C_3.
7. $\bullet^6$ is only available where a valid approach to finding the distance between the centres of C_1 and C_2 has been used.
8. Where candidates use a radius without valid supporting working, $\bullet^7$ is not available.
9. Accept the centre of C_3 written as a position vector.

Question	Generic scheme	Illustrative scheme	Max mark
14.(c) (continued)			

Commonly Observed Responses:

Valid approaches for finding *r*

$r = \dfrac{15+3-2}{2} \therefore r = 8$ $\bullet^6$ ✓

$r = \dfrac{\sqrt{(7+5)^2+(6+3)^2}+3-2}{2} \therefore r = 8$ $\bullet^5$ ✓ $\bullet^6$ ✓

$r = 3+3+\dfrac{(13-9)}{2} \therefore r = 8$ $\bullet^6$ ✓

Let $x = r_3 - r_1$. Since $r_3 = 13 - x$,
$x = (13-x)-3 \Rightarrow x = 5 \therefore r = 8$ $\bullet^6$ ✓
(may also use $y + 3 = r_3 - r_1$)

This list is not exhaustive

Invalid approaches and/or insufficient communication for finding *r*

$r = 3+3+2 \therefore r = 8$ $\bullet^6$ ^

gap $= 2 \therefore r = 10-2 \therefore r = 8$ $\bullet^6$ ^

$r = \dfrac{15}{3}+3 \therefore r = 8$ $\bullet^6$ ^

Distance = 15 $\therefore r = 8$ $\bullet^6$ ^
(with no supporting working)

This list is not exhaustive

Valid approaches for finding centre of C_3

Ratio $= 1:2 \therefore C_3(-1,3)$

$C_3\left(-5+12\times\dfrac{1}{3}, 6-9\times\dfrac{1}{3}\right) \therefore C_3(-1,3)$

Ratio $= 15:10$ & stepping out $\therefore C_3(-1,3)$
(may be drawn as similar triangles)

Let $x = r_3 - r_1$. Since $r_3 = 13 - x$,
$x = (13-x)-3 \Rightarrow x = 5$
$\Rightarrow$ Ratio $= 5:10 \therefore C_3(-1,3)$
(may also use $y + 3 = r_3 - r_1$)

$2\overrightarrow{C_1C_3} = \overrightarrow{C_3C_2} \therefore C_3 = \begin{pmatrix}-1\\3\end{pmatrix}$

This list is not exhaustive

Invalid approaches and/or insufficient communication for finding centre of C_3

$m_{C_1C_2} = -\dfrac{3}{4} \therefore C_1(-5,6) \rightarrow C_3(-1,3)$ $\bullet^7$ ✗

$\dfrac{y-6}{x+5} = \dfrac{-3-y}{7-x} = -\dfrac{3}{4} \therefore C_3(-1,3)$ $\bullet^7$ ✗

$\{3,4,5\}$ triangle with no evidence
of a ratio $\therefore C_3(-1,3)$ $\bullet^7$ ✗

This list is not exhaustive

[END OF MARKING INSTRUCTIONS]

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If you are uncertain how to assess a specific candidate response because it is not covered by the general marking principles or the detailed marking instructions, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6\times6=12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$ $x-4x+3=0$ $(x-3)(x-1)=0$ $x=1$ or 3

(i) Horizontal/vertical marking

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x=2$	$x=-4$
$\bullet^6$	$y=5$	$y=-7$

Horizontal: $\bullet^5$ $x=2$ and $x=-4$ Vertical: $\bullet^5$ $x=2$ and $y=5$

$\bullet^6$ $y=5$ and $y=-7$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0.3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*****

*The square root of perfect squares up to and including 144 must be known.

(k) Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide when marking similar non-routine candidate responses.

(l) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example
  $\left(x^3+2x^2+3x+2\right)\left(2x+1\right)$ written as
  $\left(x^3+2x^2+3x+2\right)\times 2x+1$
  $=2x^4+5x^3+8x^2+7x+2$ gains full credit
- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

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Horizontal: $\bullet^5$ $x=2$ and $x=-4$ Vertical: $\bullet^5$ $x=2$ and $y=5$
$\bullet^6$ $y=5$ and $y=-7$ $\bullet^6$ $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

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- repeated error within a question, but not between questions or papers

(m) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(n) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(o) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(p) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

<u>Note: Marking from Image (MFI) annotation change from 2025</u>

A double cross-tick is used to indicate correct working which is irrelevant or insufficient to score any marks. In MFI marking instructions prior to 2025 this was shown as **ü**$_2$ or ü2.

From 2025, the double cross-tick will no longer be used in MFI. A cross or omission symbol will be used instead.