MIE479 - MSF Capstone

Crypto Portfolio Optimization

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Overview

This paper examines the crypto currency asset class and discusses an app to evaluate crypto portfolios using multiple optimization techniques while also integrating SPY and a risk free asset (USDT) into the portfolio.

The intended users of the app are portfolio managers, hedge fund managers, or investors looking to manage crypto currency. There are very limited tools available to portfolio managers or ETF creators to test portfolio optimization tools in the crypto space. The purpose of the app is to be able to customise and evaluate unique optimization techniques, adjust factors, and see backtested results in real time. Since crypto has unique characteristics compared to equities, such as high correlation making diversification harder, this tool can help identify unique optimization combinations that work well for cryptocurrencies. The crypto space as a whole continues to evolve. While crypto volatility remains high, crypto will not be an ideal asset to dominate a long term retirement portfolio. As a result, the app is largely meant for accredited investors looking to take on risk and generate alpha, not generate long term wealth.

Mean Variance Optimization (MVO) and Value at Risk (VaR) are the two main approaches used to find an optimal portfolio which is standard with industry norms. Both approaches have their use cases and can be ideal for different situations thus the app allows for easy evaluation using customizable MVO and VaR optimization methodologies.

Literature Review

Factor models are a well established method of forecasting asset returns utilised by asset managers around the world. Fama and French famously extended the 1-factor CAPM model to 3 factors, and now there exists a large body of literature on new factors and advanced techniques for forecasting returns based on factors.

However, to our knowledge there is a limited body of literature on the application of factor models to cryptocurrencies. Urquhart and Wang discuss how cryptocurrencies have been shown to not follow the Efficient Market Hypothesis, and thus propose a 3-factor model that vastly outperforms CAPM for cryptocurrencies.Their three factors are beta (correlation with the market), big vs. small cryptocurrencies and down vs. up return cryptocurrencies. Further, Barrera and Minovitsky at MSCI explored a 7-factor model which expands on this model to include age, volatility, liquidity and momentum. They found that these 7 factors help explain 45% of the variance in cryptocurrency returns, as explained by an R^2 of 0.45. This is a comparable result to factor models in equities. Lastly, Yukun, Aleh and Wu present a comprehensive review of 25 factors that are applicable to the cryptocurrency market. They find that market returns, size and momentum factors are most statistically significant in explaining cryptocurrency returns and useful in creating long-short strategies that result in excess weekly returns over SPY. Kao and Roy propose a regularised PCA algorithm for creating factor models that we can adapt to our data to efficiently create factor models for cryptocurrencies.

Factor models are widely used to reduce estimation error in return and covariances due to their computational efficiency and explainability. Regression and its regularised formulations are quick to train and use to forecast once the coefficients have been generated. This means that factor models can be extended to forecast returns over a shorter period (for example, predicting minutely returns) if we decide to go in that direction. Further, factor models are very explainable as the coefficients in a regression are easy to interpret. However, these models have traditionally been linear and fail to capture non-linear relationships. These models also do not have strong predictive powers, with most R^2 values being within 0.4-0.5.

In dealing with the risk profiles of any asset, it becomes necessary to understand the distributions of the underlying returns to assess your relative probabilities of attaining different outcomes. Understanding the underlying statistical properties of your asset return can inform your investment decisions and allocations to properly understand exposure to tail, idiosyncratic, and market risk when selecting portfolios for investment.

It is common in industry to use a Value at Risk (VaR) in high volatility environments, as VaR assesses the amount of money lost by a portfolio allotment in the worst case scenarios based on estimated distribution of asset returns. For any base application of Value at Risk (VaR) approaches to asset allocation, it is necessary to understand the return distribution to properly evaluate your true values at risk with the true level of confidence to protect against larger tail risk than allotted for by the portfolio manager. Traditional approaches to portfolio optimization and market correlated risks as suggested by Markowitz (1952), operate under the approximation that stock prices can be modelled by geometric brownian motion, and thus the returns would be roughly normally distributed. However these assumptions when applied to crypto assets rapidly fall apart as returns do not remotely appear normal. A statistical analysis on the asset returns of the largest crypto-assets showed a p-value near zero using the Shapiro-Wilk test for normality (Mendez, Carniero 2020). Other important features to note are high variance, large and strictly positive kurtosis, an inability to reject a mean return of zero at 99% confidence and mutual heteroskedasticity amongst the asset class, leading to a complicated distribution to cohesively model (ibid). Mendez and Carniero suggested a mixture model from McLachlan and Peel 2000, to approximate individual features of the empirical distribution and verified using goodness of fit analysis. The cohesive methodology and consideration for use in VaR and Conditional Value at Risk (CVaR) models, makes this methodology a desirable approach for this project. Yet, the analysis only encompasses up to 2020, so inclusion of the data from years 2020-2022 is necessary to ensure model validity.

Many other methodologies around estimating the return distribution of crypto assets have been proposed at one time or another, and each would require consideration for their suitability in a portfolio generation context. A time series approach is proposed in Hu et al. 2021, which generates asset returns using an ARIMA-GARCH (autoregressive moving average - general autoregressive conditional heteroscedastic) model to define the underlying stochastic process. Though this method will not yield a probability density function, a curve can be generated empirically via a monte carlo simulation for evaluation. In the end, a combination of the relevant literature would likely yield the best return modelling for our purposes and the desires of our investors, yet can only serve as one component of a cohesive portfolio generating strategy.

Return Predictive Models

We chose our main return prediction methodology as linear regression factor-based models (we will call these factor models). Note that we are using factor models not in a traditional factor modelling approach for both return and covariance prediction, but rather in a predictive sense of estimating our next day returns. As such, the covariance matrix is always calculated as the covariance between the last 30 days of return for each asset. Fama and French have famous factor models for equities that rely on 3 or 5 factors that they have developed. However, it does not make sense to use the same factors as used in the equity models for cryptocurrencies. We must create new factors on which we can train the factor model. To that end, we have come up with the following factors:

1. **Crypto Market** - Market Cap weighted average daily return of all 22 cryptocurrency/USD pairs
2. **Big vs. Small** - (Average daily return of Top 11 cryptocurrencies by market cap) - (Average daily return of Bottom 11 cryptocurrencies by market cap)
3. **Volume Big vs. Small** - (Average daily return of Top 11 cryptocurrencies by daily volume) - (Average daily return of Bottom 11 cryptocurrencies by daily volume)
4. **Daily Variation High vs. Low** - (Average daily return of Top 11 cryptocurrencies by daily variation) - (Average daily return of Bottom 11 cryptocurrencies by daily variation)
   a. Daily variation = (High Price - Low Price)/Low Price
5. **Momentum -** 7 day rolling average of price return for each cryptocurrency.
   a. Note that this is the only symbol-specific factor.

Please note that for SPY (S&P ETF) we use the Fama-French 3 factor model, and that these aforementioned calculations are only for cryptocurrency assets.

Factor Model Calculation Methodology

Crypto Market Factor

The crypto factor is calculated using the following methodology:

1. Calculate cryptocurrency/USD daily percentage returns
2. Calculate cryptocurrency market cap weighted average return by date

Momentum Factor

The momentum factor is calculated using the following methodology:

1. For each symbol,
   a. For every date, get the average of the returns of the last 7 days

Other Factors

The remaining factors have a consistent methodology, which is:

1. Calculate cryptocurrency/USD daily percentage returns
2. Calculate average value of factor (one of the above 4) over time period by cryptocurrency
3. Sort cryptocurrencies by factor, from highest to lowest
4. Split above list in half, creating a **Top** portfolio and a **Bottom** portfolio
5. Get average daily return for **Top** and **Bottom** portfolios
6. Get Factor Daily Return as **Top** Daily Return - **Bottom** Daily Return

We want to compare the factor model to a baseline model to determine whether it is an improvement over a baseline model. For our baseline model, we use the average return 7 days prior to the date for which we would like to predict the return. Then, we fit a linear regression with an intercept term. The model looks like:

$$R_{t+1} = \beta_0 + \beta_1 * (R_{(t-7,t)})$$

where $R_{(t-7,t)}$ is the previous 7 day average return

For our predictive model, we fit a linear regression on a 1-day lagged version of the four factors described above, plus an intercept term. The model for this looks like:

$$R_{t+1} = \beta_0 + \beta_1 * (CryptoMkt) + \beta_2 * (BigVsSmall) + \beta_3 * (VolumeHighVsLow) + \beta_4 * (VariationHighVsLow) + \beta_5 * (Momentum)$$

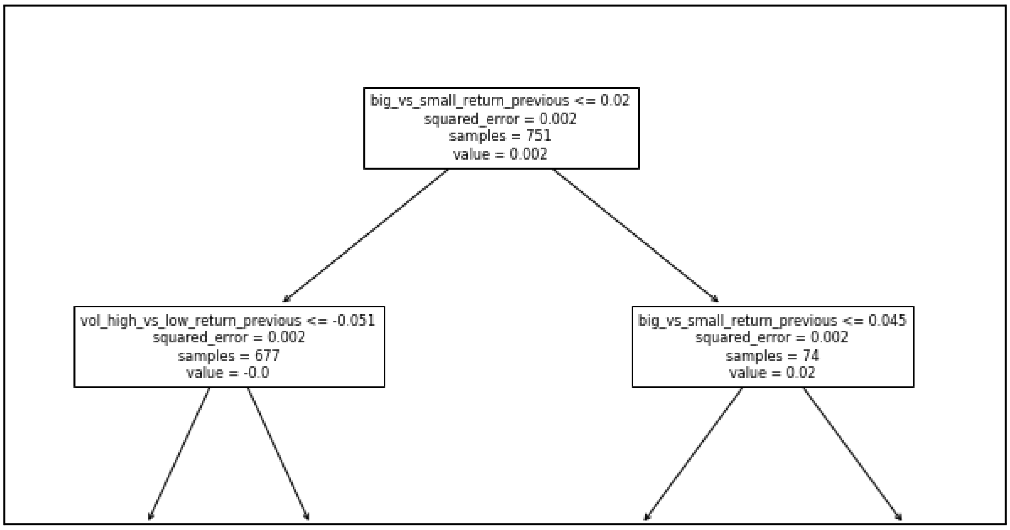
The table below summarises the R^2 values of the baseline model and the factor model. We can clearly see that the previous 30 day model has very little predictive power. A lot of R^2 values for validation dataset are negative, which implies that a horizontal data line holds more predictive power. The new proposed factor model has a fairly high R^2, for both training and validation sets. Overall, we can conclude that our factor model has already shown to have significant predictive power over the baseline model.

Symbol	Return MA Model	Factor Model	Decision Tree
SPY	0.29%	2.36%	5.69%
BTC	0.00%	3.70%	12.39%
LTC	0.02%	3.47%	13.61%
XRP	0.00%	3.84%	7.64%
DOGE	0.06%	1.69%	10.69%
XMR	0.12%	6.14%	19.02%
XLM	0.10%	5.46%	8.40%
USDT	4.37%	1.38%	5.92%
XEM	0.07%	4.32%	10.82%
ETH	0.21%	3.25%	16.24%
MIOTA	0.16%	5.10%	17.23%
EOS	0.01%	5.18%	14.56%
BNB	0.19%	5.49%	15.06%
TRX	0.02%	2.96%	14.82%
LINK	0.01%	2.79%	20.42%
ADA	0.55%	3.92%	19.46%
CRO	1.93%	0.95%	9.40%
WBTC	0.04%	1.38%	23.11%
ATOM	0.02%	2.64%	21.97%

Accuracy Results for Predictive Methods

While we see that the factor model presents a higher R^2 than the naive moving average model, the value is still not very high with magnitude around 1-6%. Hence, only a small percentage of the variation in returns is explained by these factors. As such, there may be a non-linear relationship which we may need a more sophisticated model to fit.

In order to tackle this non-linear relationship, we decided to also use decision tree regression. Decision trees break up the input space into smaller chunks which can be answered by “if-then” questions. There is no linear relationship between the input and the output as it follows this tree down to make the decision. An example of how this regression may make a decision is seen below. It creates these greater, large, or equal questions for each of our input factors, and based on the truthiness of that question it separates the data into the different values of the next day's return that it saw. We can see that the non-linearities introduced by this system are beneficial in increasing the accuracy of our model. Please note that we set the maximum depth of the Decision Tree to be 4, to prevent overfitting on the test dataset.



Starting Decision Criteria for Decision Tree Model for BTC

Optimization

MVO & Robust MVO

The mean variance approach is great for maximising the quadratic utility (or minimising risk) of a portfolio.

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \boldsymbol{x}^T \boldsymbol{Q} \boldsymbol{x} - \lambda \boldsymbol{\mu}^T \boldsymbol{x} \\ \text{s.t.} \quad & \mathbf{1}^T \boldsymbol{x} = 1 \\ & \quad (\boldsymbol{x} \geq \mathbf{0}) \end{aligned}$$

In the equation above, both the covariance matrix and the estimated returns are within the minimization function. Either term can be added to the constraints to focus the optimization around minimising variance or maximising returns.

One limitation of mean-variance optimization is that small changes in the input parameters can lead to large changes in the optimal solution. As the input parameters (returns - μ and the covariance matrix - Q) have a degree of uncertainty, small errors in the input parameters can cause large fluctuations in the optimal solution or render the problem infeasible (Costa, Robust MVO, 2021).

As a result, being able to quantify and incorporate uncertainties rather than taking the risk & return estimates as absolute within the input parameters will allow the model to be less sensitive to errors in risk & return.

Robust MVO

Robust MVO quantifies uncertainty within the input parameters and incorporates this into the optimization (Costa, Robust MVO, 2021) which reduces the sensitivity limitation of a standard MVO.

$$\begin{aligned} \min_{\boldsymbol{x}} \quad & \boldsymbol{x}^T \boldsymbol{Q} \ \boldsymbol{x} \\ \text{s.t.} \quad & \boldsymbol{\mu}^T \boldsymbol{x} - \varepsilon_2 \|\boldsymbol{\Theta}^{1/2} \boldsymbol{x}\|_2 \geq R \\ & \mathbf{1}^T \boldsymbol{x} = 1 \\ & (x_i \geq 0, \quad i = 1, \ \ldots, \ n) \end{aligned}$$

The main difference between the standard MVO and the robust MVO is the error term in the constraints (which could also be a part of the minimization). In the first constraint, ε2 and the function within the absolute value measures the ellipsoidal interval in which the estimated returns could be in. This adds a penalty to returns proportional to the uncertainty of the estimate.

The main advantage of adding a robust element to the MVO is to promote diversification. As can be seen in Yuan Hu's 2021 paper '*Modelling Crypto Asset Price Dynamics, Optimal Crypto Portfolio, and Crypto Option Valuation*', bitcoin accounts for 100% of the MVO portfolio during a large part of the testing period (Hu, 2021). Using a robust MVO along with concentration limits should help combat the over concentration issues seen in crypto min variance optimization.

CVaR

Conditional Value at Risk (CVaR) is a common optimization system deployed mainly for extremely volatile assets with large tail behaviour. Even to the casual observer, cryptos exhibit extreme volatility and consistently make headlines for either creating or erasing trillions of dollars of wealth in the span of a few months to a few years. In dealing with this extreme volatility, academic approaches would suggest CVaR as a viable optimization strategy such that tail risk exposure is minimised to prevent massive losses. In practice, CVaR takes the total returns of each asset in the worst n% of scenarios weighted by the probability of that outcome occurring, returning a portfolio with weights such that losses are minimised under those scenarios. The approach aims to have a high sharpe ratio, even if returns may be slightly lower on average.

In order to utilise CVaR, a distribution of possible returns must be generated for their tail behaviour to be analysed. In this application, we support a few different ways to simulate future returns, generate a distribution, and then perform CVaR optimization to find an investment portfolio for the basket of crypto assets. In order to mitigate the difficulty of modelling all features of long term crypto returns, predictions were fit on short lookback periods and rebalanced rapidly. Future returns were generated in three different ways: Historical Simulation, Short Term Normal Simulation, and Generation through a fitted ARIMA-GARCH time series simulation. These were chosen because despite being approximate measures, they still can capture short term variance for tail risk mitigation on our daily rebalance period. The desired result of the optimization is to avoid extreme tail risk while having passive positive exposure to upward trends.

A limitation to CVaR is that it is agnostic to the upside of the return distribution while also being extremely sensitive to estimation error of the lower tail. All the simulation methods have dense lower tails and should in theory insulate against asset assignment resulting in massive losses.

Simulation Methodologies

Historical: A distribution of future returns is assumed to be identically distributed as the past returns in a short term lookback period.

Normal: The short term returns of the asset are assumed to be distributed normally along the same observed parameters of the lookback period.

ARIMA-GARCH: (AutoRegressive Integrated Moving Average - Generalised AutoRegressive Conditional Heteroskedastic). The asset returns in the look back period are fit to an ARIMA(p, q, d) time series model where the model parameters are determined via a stepwise fitting procedure. The residuals of this fit are fit to a GARCH(1,1) model. Simulated means and volatilities are generated from the expected mean and variance of the sum of the ARIMA and GARCH terms to generate an empirical return distribution.

The parameters of these simulation methods are lookback period, look forward period and number of simulations. The lookback period is the number of previous days considered in determining the parameters of the simulation. The preferred window in our testing was 30 days, in order to try and minimise estimation error of the return variance. Look forward period is the amount of days in the future in which the returns are simulated. In this case, with daily rebalancing we simulated only 1 day in advance to again try and reduce parameter estimation error. For the number of simulations, 1000 was selected in order to give the CVaR optimizer enough observations within the tails to produce better results while also not ballooning run time of the program.

CVaR Optimization and Objective Function

The following is from Dr. Costa's course notes prepared for MIE377 at the University of Toronto:

The formal definition of CVaR is

$$\mathrm{CVaR}_\alpha(\boldsymbol{x}) = \frac{1}{1-\alpha}\int_{f(\boldsymbol{x},\boldsymbol{r})\geq \mathrm{VaR}_\alpha(\boldsymbol{x})} f(\boldsymbol{x},\boldsymbol{r})p(\boldsymbol{r})\ d\boldsymbol{r}$$

Where VaR is the amount of money lost in the worst $\boldsymbol{\alpha}$% of outcomes, and the function $f(x,r)$ being the loss incurred by portfolio x, given returns r and $p(r)$ being the probability of observing returns r. Alpha is a risk value that is selected as a hyper parameter for the optimization

$$\begin{aligned} \min_{x} \quad & \mathrm{CVaR}_\alpha(\boldsymbol{x}) \\ \text{s.t.} \quad & \boldsymbol{x} \in \mathcal{X} \end{aligned}$$

Where χ is the universe of acceptable portfolios (ie budget, holding allowances, diversification).

This optimization was performed using the objective function transformation proposed by Dr. Costa in his MIE377 lecture notes on CVaR and the cvxpy open source solver library. This transformation makes the optimization convex with respect to γ, and since the distributions are empirically generated, we can assume that each observation is equally likely to occur.

$$F_\alpha(\boldsymbol{x},\gamma) = \gamma + \frac{1}{1-\alpha}\int \Big(f(\boldsymbol{x},\boldsymbol{r}) - \gamma\Big)^+ p(\boldsymbol{r})\ d\boldsymbol{r}$$

Important Properties of $F_\alpha(x,\gamma)$:

1. $F_\alpha(\boldsymbol{x},\gamma)$ is a convex function of γ.
2. $\mathrm{VaR}_\alpha(\boldsymbol{x})$ is the γ that minimizes $F_\alpha(\boldsymbol{x},\gamma)$.
3. The minimum value over γ of the function $F_\alpha(\boldsymbol{x},\gamma)$ is $\mathrm{CVaR}_\alpha(\boldsymbol{x})$,

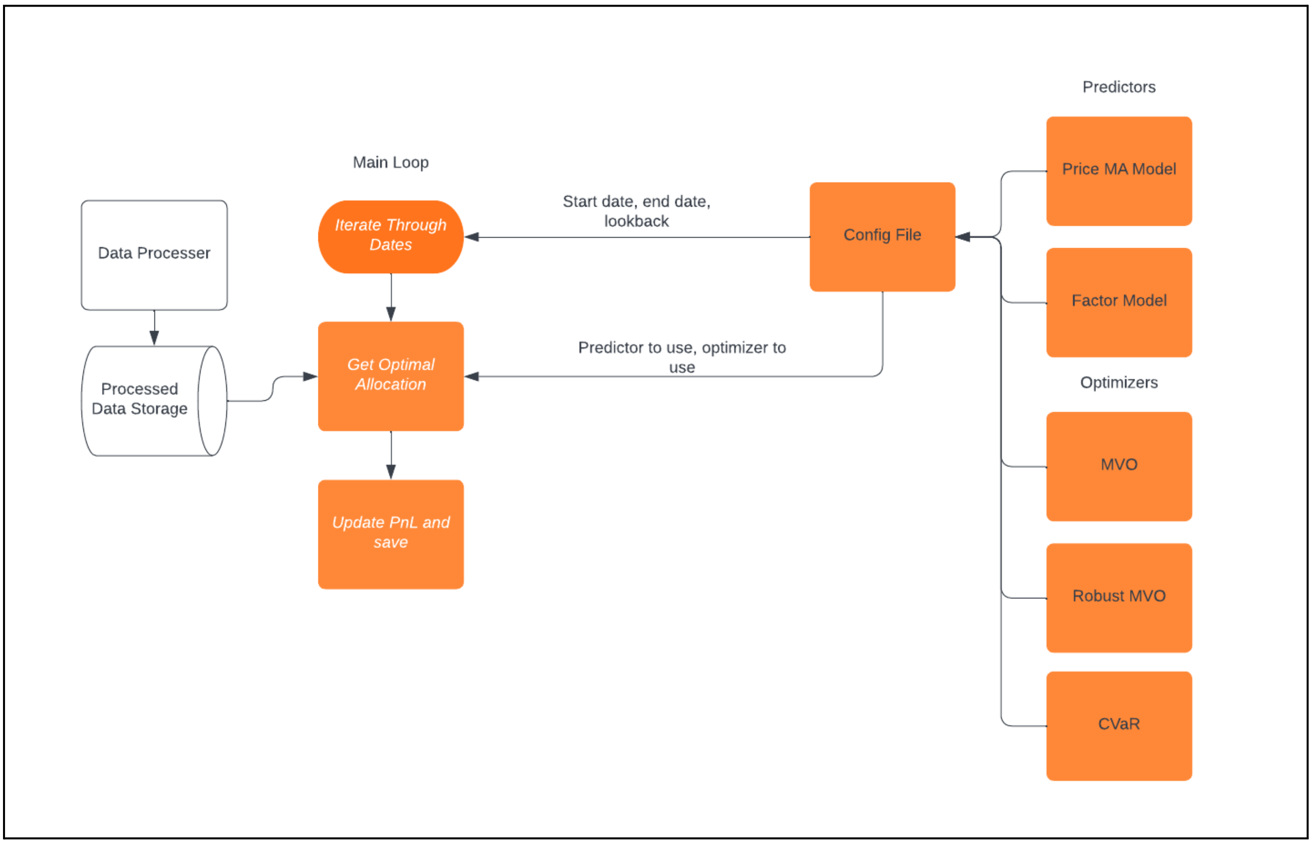
Transformed Objective Function of CVaR

Backtesting

Our backtesting system is detailed in the UML diagram below. Our main criterion when designing this system were:

1. Robustness against a variety of inputs
2. Speed at which it runs
3. Smooth error handling

To make sure that our system was robust and scalable for a variety of inputs and combinations, we made our backtesting system as modular as possible. As the diagram details, the Main Loop which gets the optimal holdings everyday is fed to the predictor to use and the optimizer to use from an external config file which is easy to update both manually and programatically. Further, in efforts to speed up our system, we also process the data and calculate the results of all factors and returns prior to any calls to the system being made. This way, we do not have to run the data processor every time there is a call to re-run a model. Lastly, we have wrapped our entire code in an error handling class. This allows the system to continue running if there is an error on one day, and does not stop the entire system.



UML Diagram of Backtesting System

Data Description

The dataset we have has daily price observations for 20 crypto assets and SPY over 2020 and 2021. After removing cryptocurrencies with messy data, we were left with 14 cryptocurrencies and SPY, for a total of 15 assets. SPY data was obtained from Yahoo Finance, cryptocurrency data was obtained from a past Kaggle competition: (https://www.kaggle.com/datasets/sudalairajkumar/cryptocurrencypricehistory).

Results

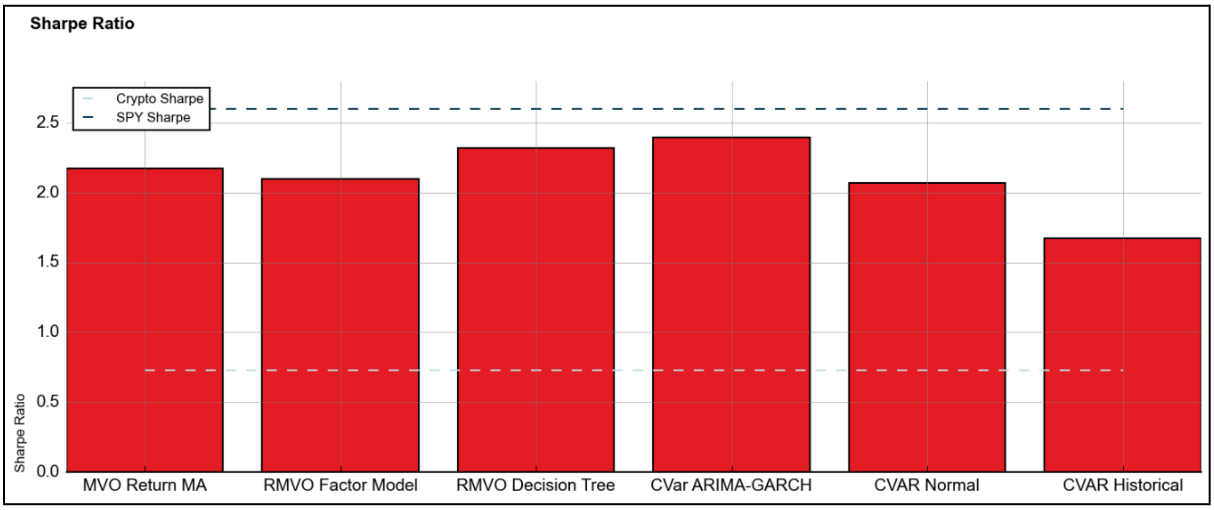
The main evaluation metric used for portfolio performance is Sharpe ratio (returns/standard deviation of returns). However, for our final model selection we also consider diversification as a key requirement for an ideal portfolio. Overall, the CVaR models perform better than the MVO models in Sharpe and diversification which is consistent with the current literature. Additionally, CVaR with an ARIMA-GARCH simulation methodology has the highest sharpe ratio, and we can see that it is the best model overall. However, we found that our regression and decision tree prediction models result in a higher sharpe ratio than other CVaR models. This is new when compared to current literature review, where all of the CVaR models had a higher sharpe ratio than all MVO approaches.

The table below shows the sharpe ratio of each model type. A model type is defined by the predictor and the optimizer used (for example, "Robust MVO with a Factor Model" is one model type). Note that the model whose data is showcased below is the model which had the highest sharpe per model type. Further details on model selection are available in the Final Model Selection section below.

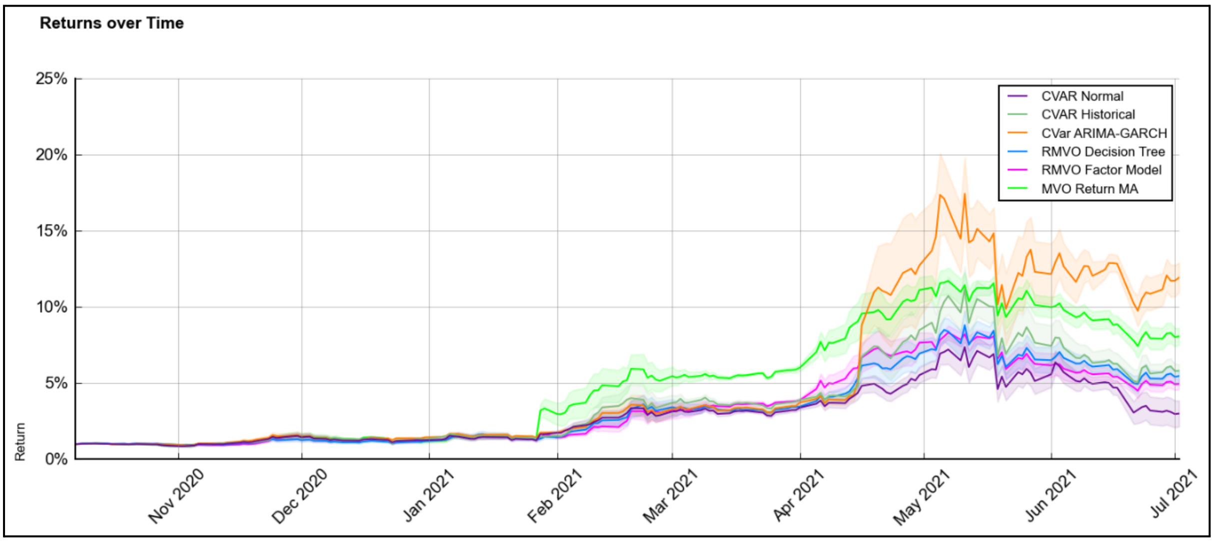
Optimization	Model	α	λ	Max Weight	Sharpe	Return	Std
RMVO	Price MA	0	100	0.4	2.175	8.17%	3.76%
RMVO	Factor Model	0	50	0.4	2.099	5.01%	2.39%
RMVO	Decision Tree	0	0.5	1	2.32	5.53%	2.39%
CVaR	ARIMA-GARCH	0.01	0	1	2.396	12.07%	5.04%
CVaR	Normal	0.01	0	1	2.068	5.88%	2.84%
CVaR	Historical	0.01	0	1	1.673	3.04%	1.82%
SPY	/	/	/	/	2.3		
Crypto Market	Cap Weighted	/	/	/	0.73		

Sharpe Ratio Through Whole Backtest Period by Model

The first thing to note is that the sharpe ratio of all models is higher than that of the crypto market. However, we see that direct investment in the S&P-500 index results in a higher sharpe ratio than all portfolios. This is important as it suggests that all of our models provide some improvement over a portfolio investing a market cap-weighted average of cryptocurrencies or the equities model. We can also see that the CVaR models give the highest sharpe ratio. This makes sense as cryptocurrencies are very volatile and CVaR aims to minimise exposure to these extreme volatility investments. Below is a graph highlighting the top sharpe ratios for each model (all evaluated hyper-parameters for each model is in appendix A).



Sharpe Ratio Through Whole Backtest Period by Model



Portfolio Returns Through Whole Backtest Period by Model

Looking at the return graph above, all optimization models follow a similar trend. One of the main dates to note is April 2021. In April 2021, many crypto currencies started to take off and they gained a lot of traction resulting in a short period of high growth. This was then followed by a market correction starting in May 2021.

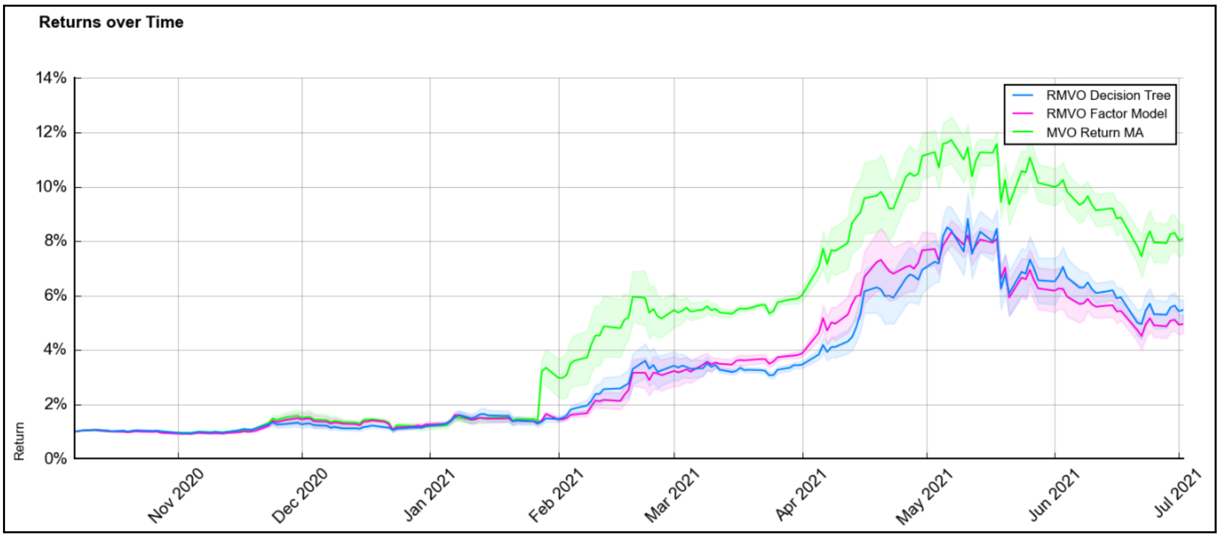
Minimum Variance Optimization Results

Prediction Models

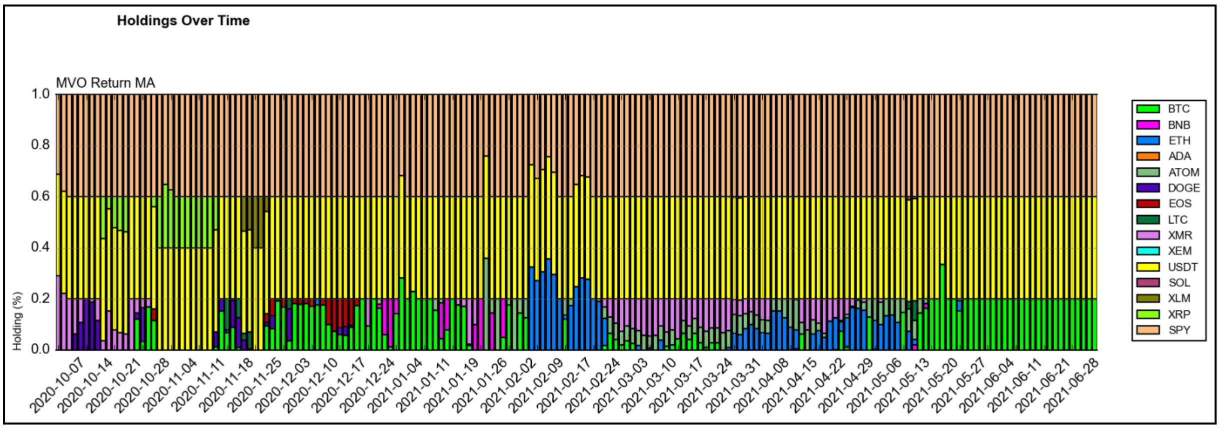
The graphs below show the holdings of Robust MVO with each of our prediction methodologies: Average Return Model, Factor Model and the Decision Tree Model. We can see in the Sharpe Ratio table above that the Decision Tree model has the best sharpe ratio and that the Moving Average Return model and factor model seems to underperform.

Regarding holdings, we can see that the decision tree model chooses to make a single asset pick everyday, and there is very little overall diversification. This is similar to the factor model and return MA holdings as well, where we see limited diversification across all models.

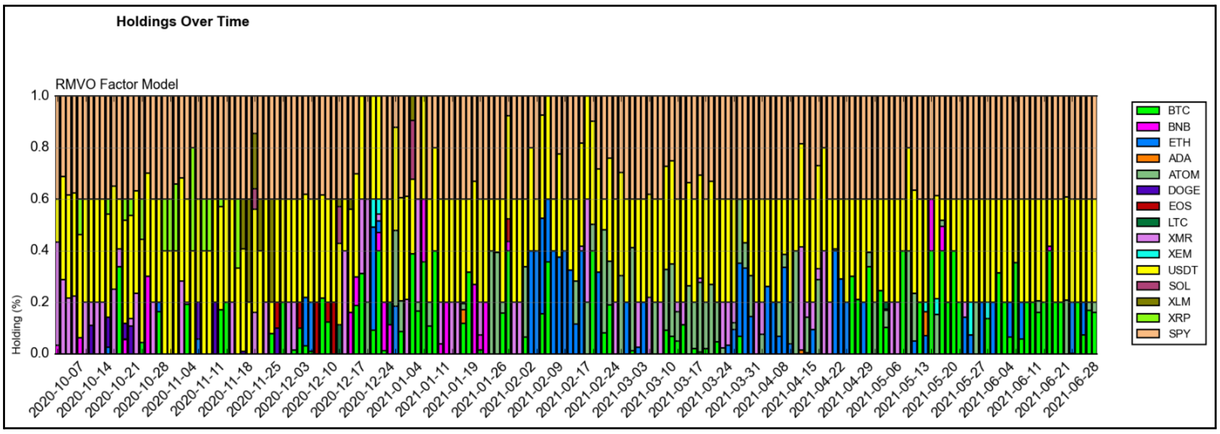
Although there is a lot of over concentration in the MVO portfolios, the main overconcentration is either in SPY and USDT (crypto risk free) as these assets are not very correlated with the rest of the crypto universe (see appendix for correlation matrix). To add, the MVO seemed to assess that the risk-reward tradeoff of most cryptos was not great thus concentrating most of the positions to SPY or USDT.



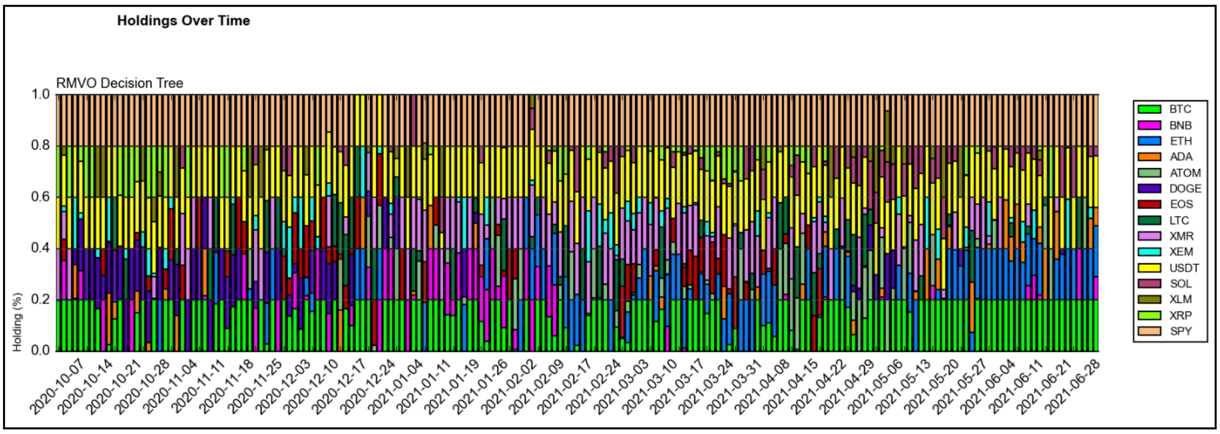
Returns of (R)MVO Optimised Portfolios with 15 Day Rolling Volatility Bands



Holdings of Robust MVO Optimised Portfolio with Moving Average Predictors



Holdings of Robust MVO Optimised Portfolio with Factor Model Predictors



Holdings of Robust MVO Optimised Portfolio with Decision Tree Model Predictors

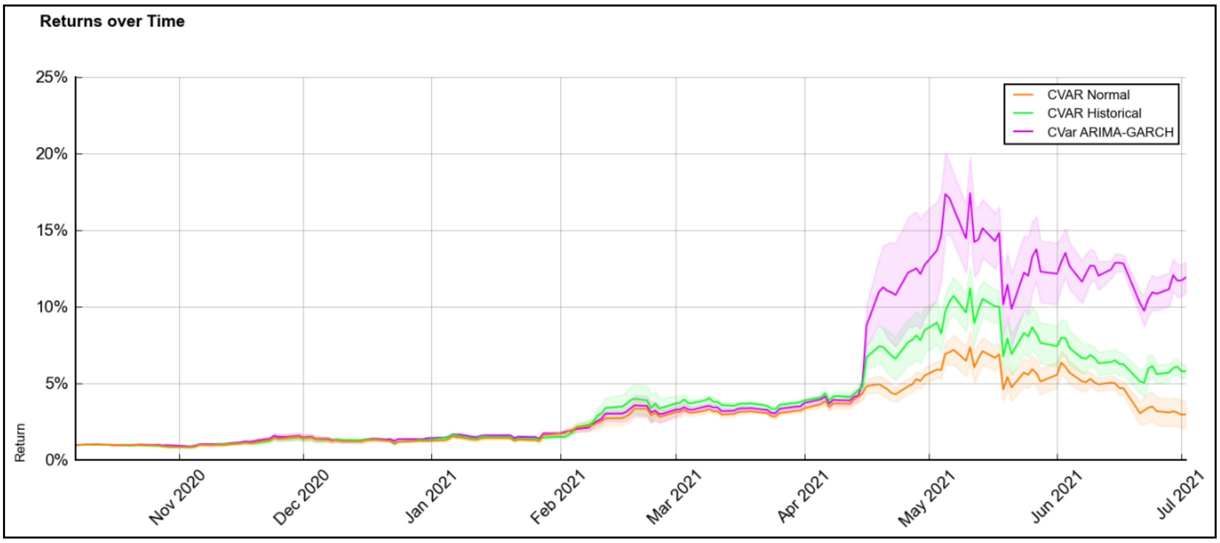
Robust vs regular MVO

Adding a robust element to the min variance optimization slightly improved the results. One of the difficulties with crypto is that there are zero fundamental evaluations for crypto prices. To gauge the future price, all projections are based off of past price movements. If one assumes any form of weak market efficiency or higher, these factors will not be good predictors of asset returns. Therefore, there is a lot of noise within the inputs leading to errors in the optimization using the MVO. The robust MVOs account for some of this uncertainty which reduces noise and sensitivity of the inputs. This in turn improved the results and is highlighted by the fact that all of the top minimum variance optimizers had a robust element.

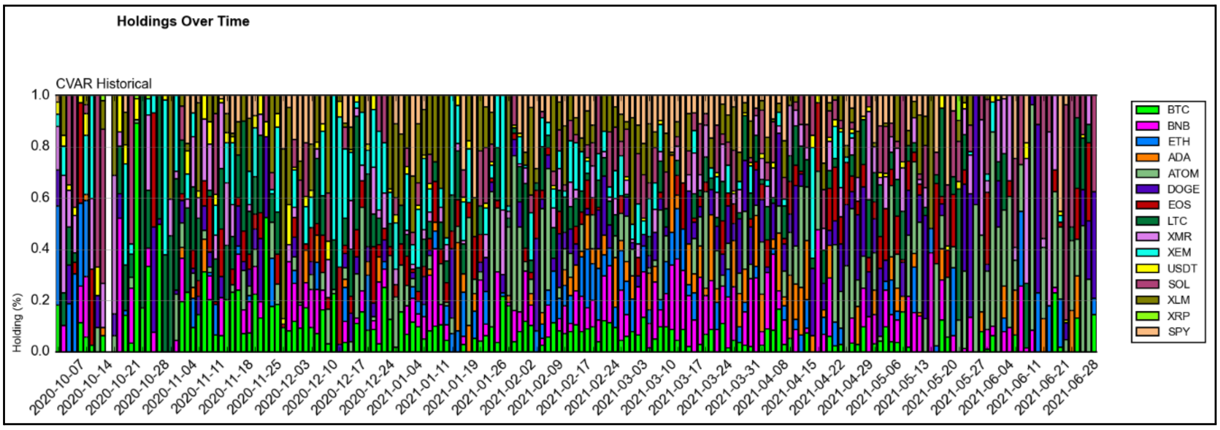
Another downfall of MVO is over concentration. Although the robust element did not substantially help promote diversification, the weight capping did. This in turn improved the sharpe ratios of all min variance models (which is highlighted by the fact that the top minimum variance models have weight caps).

CVaR

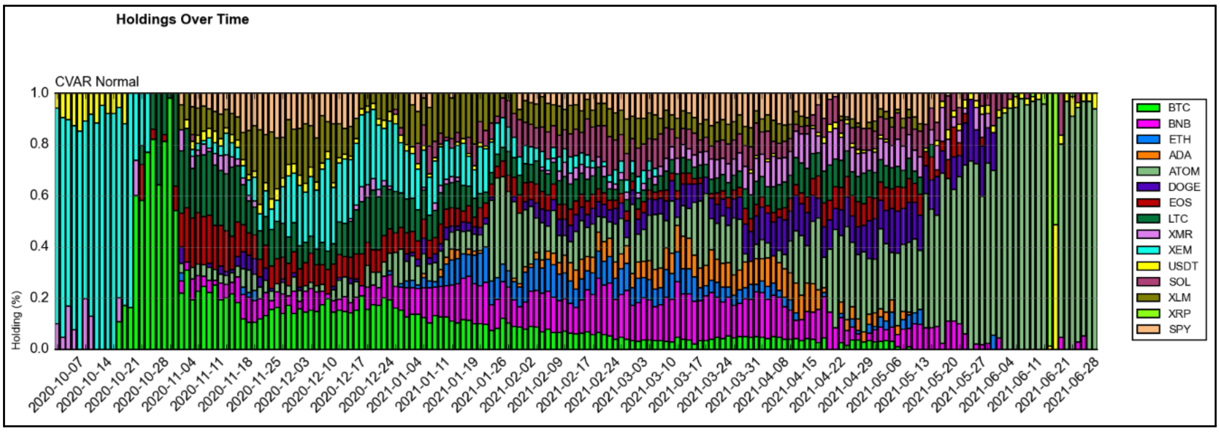
Overall CVaR performed in line with what the framework is designed to do, having decent sharpe ratios, but lower returns due to the optimization process being agnostic to the upper tail of the return distributions. There was significant variation in the performance of each of the best CVaR models. Historical and Normal simulations but showed low return variance and low returns, resulting in only moderate sharpe ratios by comparison. The ARIMA-GARCH methodology had the lowest return volatility and the highest returns amongst the CVaR methodologies. One common issue with CVaR, is that the objective function does not reward diversification as MVO does, resulting in over concentration in certain assets. Looking over the graphs of asset holding in the following section, this did not appear to be an issue in this case, as all approaches had largely diversified portfolios and moved between most assets throughout the backtest period. The one exception to the diversification issue can be observed at the end of the Normal and ARIMA simulations, where the portfolio was almost 100% concentrated on a singular asset.



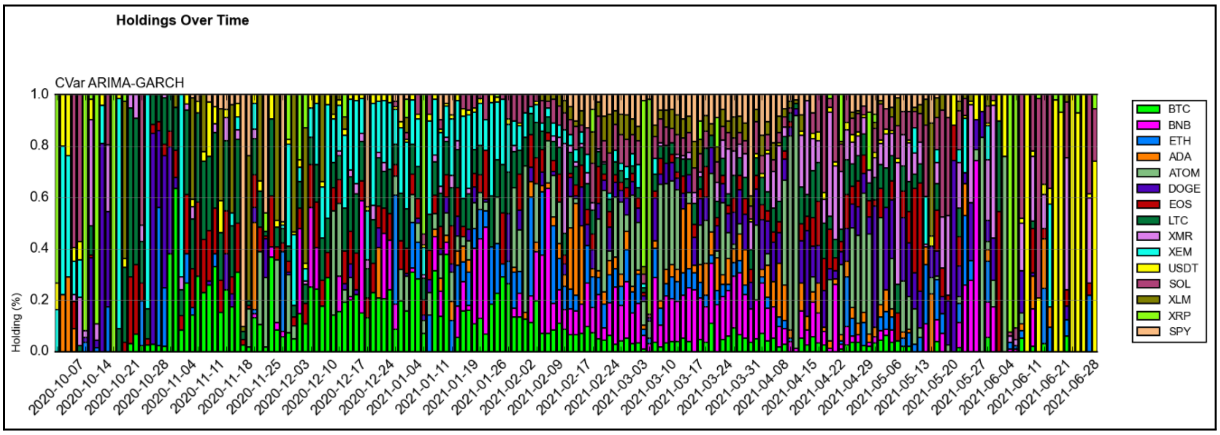
Returns of CVaR Optimised Portfolios with 15 Day Rolling Volatility Bands



Holdings of CVaR Optimised Portfolio with Historical Simulation



Holdings of CVaR Optimised Portfolio with Normal Simulation



Holdings of CVaR Optimised Portfolio with ARIMA-GARCH Simulation

Final Model Selection

The final model was obtained through a combination of programmatic Cross Validation Testing with a variety of hyperparameter combinations, and a holistic manual analysis of the top candidates' holdings graphs to get an idea of its diversification. For the cross validation testing, we tested the following parameters in these ranges:

Parameter	Ranges Tested
λ	[0.5, 1, 5, 10, 50, 100]
α	[0.01, 0.05, 0.1, 0.5]
Max Weight Constraint	(0,1] in steps of 0.2
Optimization Methodology	[CVaR, Robust MVO, MVO]
Simulation Methodology	[Normal, Historical, ARIMA-GARCH]
Prediction Methodology	[Return MA, Factor Model, Decision Tree]

We already saw that the CVaR model with ARIMA-GARCH and Robust MVO models with Decision Trees for prediction had the highest sharpe ratios, but as discussed earlier there was lower diversification in the MVO model as compared to the CVaR model. However, this again highlights the importance of the application we have developed as the final model depends on the requirements of the portfolio manager generating them. For the purposes of this project, we will assume that we want a good sharpe but also value diversification, for which CVaR presents the optimal portfolios.

Parameter	Value
Rebalance Period	Daily (Business Days)
λ	N/A
Lookback Period	3 Months
Look Forward Period	5 days
α	0.01
Max Weight Constraint	1
Optimization Methodology	CVaR
Simulation Methodology	ARIMA-GARCH
Prediction Methodology	N/A

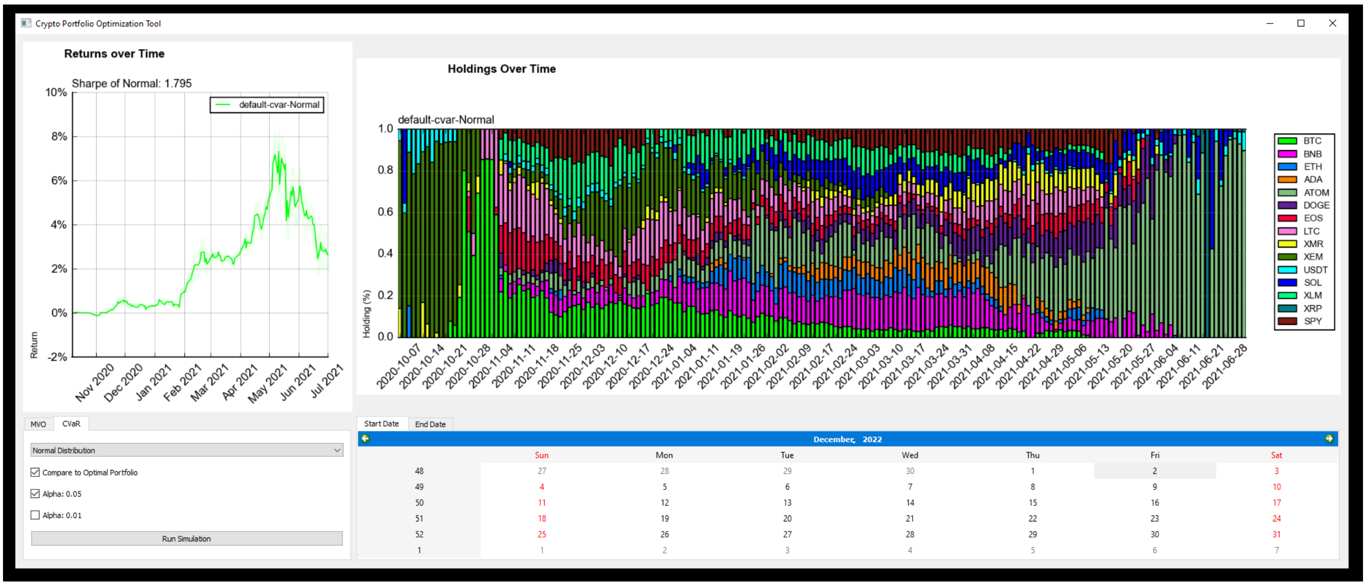
Table Showcasing Parameters of Final Model Selected

Application

The front end of the application is built using the GUI application PyQt5 in python and the backend is built with python as well. The backend pulls from local json files as nothing is required to be saved in a large/secure database.

The front end allows the user to quickly backtest crypto portfolios and will display important characteristics of the portfolio such as AUM over time, sharpe ratio, and holdings. The user can get those portfolio characteristics for any MVO (including Robust MVO) or CVaR portfolio with many customizable features over any choice of time period within the domain of the data set.

Below is a snapshot of the application where the user is testing a historical CVaR model with an alpha of 0.05. The application will run the backtest and show the returns over the selected time, sharpe, and the holdings over the time period.



GUI of Final Application

Alternative Design Choices

Machine Learning Approaches vs Regression/Decision Tree

To address non-linearities in return prediction, we had the choice between using a traditional statistical learning algorithm, or a deep learning algorithm. We had the following constraints within which to make a decision:

1. Speed of prototyping
2. Speed at which it runs for the eventual user

Neural networks have a lot of hyperparameters for training, such as the architecture of the system, activation functions, hyperparameters in training and so on. Further, developing an optimal design for time series data is a hotly researched topic, and some approaches still tend to underperform, and so there is no set conclusion on what works well. This would increase the time it takes to prototype a solution. Further, neural networks also take a long time to run and to make a decision as well. Thus, we decided that training a neural network for this problem may be out of the scope of this project, and went with a traditional statistical learning algorithm instead, namely decision trees.

CVaR Simulation Methodologies

The space of possible simulation methods for future returns is approximately infinite. In this specific approach, the lookback, look forward and number of simulations were fixed. These values were selected to attempt to mitigate the estimation error stemming from only using approximate modelling methodologies. As crypto returns are observed to be heteroskedastic, the longer the time period modelled by simulation, the larger the parameter estimation error of the variance. This was mitigated by having a short lookback period and short look forward period in testing. For simplicity, having dynamic lookback periods or another fitting methodology for these parameters was not considered.

The ARIMA-GARCH model and fitting methodology and model design was also chosen explicitly. A simpler ARIMA model was not used to fit the time series because it resulted in large residuals and did not appear to be a good fit. A stepwise fitting methodology using the Akaike information criterion was used for determining the ARIMA parameters was selected due to its simplicity and moderate computational efficiency to ensure run times did not balloon. We also decided against fractionally integrating the time series before fitting the ARIMA model to it. The aim of fractional integration is to augment the time series to make sure it is stationary and thus improve the predictive ability of the ARIMA model on time series with constantly fluctuating higher order moments. This was determined to not be necessary on the short term time series we were using, as in practice it is applied to years of return data to ensure goodness of fit of an ARIMA model aiming to capture all features of the series. We aimed to mitigate estimation error via constant refitting and short look back periods, instead of introducing the complication and extra computation time associated with determining integration parameters and transforming return series every time a new simulation is performed.

Website vs GUI

For the application, the final two design choices were a full website and a GUI. One of the original ideas was to make a website where users log-in and see their crypto portfolios with the selected metrics. However, the intended user of the application is a trader. Traders are busy and do not want to waste their time logging into a website and clicking through tabs. To add, most applications traders use on a day to day are quick desktop applications (Bloomberg, Fidessa, etc.) and not websites. We wanted the process of backtesting portfolios and seeing results to be as quick and user friendly. As a result, we decided to build our crypto backtesting model on a GUI rather than a website. This will make the process of using the application easier and save the user time while also conforming to industry standards.

Conclusion

We started this project with the goal of expanding the existing equity portfolio optimization techniques to cryptocurrencies and to create an application for portfolio and hedge fund managers to create and backtest their own cryptocurrency portfolios. We have determined that a CVaR approach with an ARIMA-GARCH methodology for returns estimation provides us with the highest sharpe ratio and the best model. This methodology beats the sharpe ratio of a naive portfolio of evenly-weighted cryptos. Finally, our application allows potential portfolio managers to select a combination of optimization and prediction approaches to generate portfolios and determine the best one for their requirements. In the future, the availability of more crypto data will allow for more in-depth analysis of the optimization techniques.

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Appendix

A - Sharpe, Returns, STD of portfolios

All data comes from /graphs/results-big_test.json and is cleaned in /graphs/Graphs.ipynb

Weight	Lambda	Prediction	Optimization	Alpha	CVaR	Sharpe	Return	Std
1	0	na	cvar	0.5	ARIMA-GARCH	0	0	0.002645
0.2	0	na	cvar	0.05	ARIMA-GARCH	1.890049	0.100018	0.052918
0.8	0	na	cvar	0.01	ARIMA-GARCH	1.934836	0.142218	0.073504
0.8	0	na	cvar	0.5	ARIMA-GARCH	2.026233	0.156695	0.077333
0.2	0	na	cvar	0.01	ARIMA-GARCH	2.027228	0.106932	0.052748
0.8	0	na	cvar	0.1	ARIMA-GARCH	2.035582	0.111905	0.054974
0.4	0	na	cvar	0.05	ARIMA-GARCH	2.073363	0.125211	0.06039
0.4	0	na	cvar	0.01	ARIMA-GARCH	2.107152	0.087215	0.04139
0.6	0	na	cvar	0.01	ARIMA-GARCH	2.115399	0.094017	0.044444
0.2	0	na	cvar	0.1	ARIMA-GARCH	2.120097	0.131356	0.061957
0.8	0	na	cvar	0.05	ARIMA-GARCH	2.124298	0.134146	0.063148
0.6	0	na	cvar	0.1	ARIMA-GARCH	2.18046	0.130718	0.05995

0.6	0	na	cvar	0.05	ARIMA-GARCH	2.192425	0.122932	0.056071
0.4	0	na	cvar	0.1	ARIMA-GARCH	2.199668	0.106569	0.048448
1	0	na	cvar	0.05	ARIMA-GARCH	2.251409	0.14388	0.063907
1	0	na	cvar	0.1	ARIMA-GARCH	2.304936	0.166241	0.072124
0.6	0	na	cvar	0.5	ARIMA-GARCH	2.361662	0.197662	0.083696
0.4	0	na	cvar	0.5	ARIMA-GARCH	2.36879	0.197226	0.08326
1	0	na	cvar	0.01	ARIMA-GARCH	2.396225	0.120663	0.050355
0.2	0	na	cvar	0.5	ARIMA-GARCH	2.49544	0.198974	0.079735
0.2	0	na	cvar	0.01	Historical	0	0	0.002384
0.2	0	na	cvar	0.05	Historical	0	0	0.00195
0.2	0	na	cvar	0.1	Historical	0	0	0.00241
0.2	0	na	cvar	0.5	Historical	0	0	0.000743
0.4	0	na	cvar	0.05	Historical	0	0	0.00208
0.4	0	na	cvar	0.1	Historical	0	0	0.00225
0.4	0	na	cvar	0.5	Historical	0	0	0.000743
0.6	0	na	cvar	0.1	Historical	0	0	0.001053
0.6	0	na	cvar	0.5	Historical	0	0	0.000743

0.8	0	na	cvar	0.1	Historical	0	0	0.000743
0.8	0	na	cvar	0.5	Historical	0	0	0.000743
1	0	na	cvar	0.1	Historical	0	0	0.001057
1	0	na	cvar	0.5	Historical	0	0	0.000743
0.6	0	na	cvar	0.01	Historical	2.050353	0.065647	0.032017
1	0	na	cvar	0.05	Historical	2.064468	0.061458	0.029769
1	0	na	cvar	0.01	Historical	2.068033	0.058829	0.028447
0.8	0	na	cvar	0.05	Historical	2.091077	0.027413	0.01311
0.4	0	na	cvar	0.01	Historical	2.283229	0.021451	0.009395
0.8	0	na	cvar	0.01	Historical	2.398444	0.075631	0.031533
0.6	0	na	cvar	0.05	Historical	2.576463	0.049386	0.019168
0.6	0	na	cvar	0.05	Normal	1.49482	0.032209	0.021547
0.2	0	na	cvar	0.05	Normal	1.546611	0.039221	0.025359
0.8	0	na	cvar	0.01	Normal	1.639237	0.036504	0.022269
0.4	0	na	cvar	0.01	Normal	1.639863	0.034964	0.021321
1	0	na	cvar	0.01	Normal	1.673086	0.030411	0.018176

0.2	0	na	cvar	0.01	Normal	1.803132	0.040182	0.022285
0.6	0	na	cvar	0.01	Normal	1.901067	0.039781	0.020926
0.4	0	na	cvar	0.05	Normal	1.992258	0.035488	0.017813
1	0.5	Decision Tree	MVO	0	na	1.589474	0.061666	0.038796
0.6	0.5	Decision Tree	MVO	0	na	1.591245	0.092706	0.05826
0.8	0.5	Decision Tree	MVO	0	na	1.658811	0.09408	0.056715
0.8	1	Decision Tree	MVO	0	na	1.745485	0.080964	0.046385
1	1	Decision Tree	MVO	0	na	1.745902	0.054137	0.031008
0.6	1	Decision Tree	MVO	0	na	1.815136	0.08335	0.045919
0.4	0.5	Decision Tree	MVO	0	na	1.867206	0.077041	0.04126
0.4	1	Decision Tree	MVO	0	na	2.119229	0.094932	0.044796
0.2	0.5	Decision Tree	MVO	0	na	2.207108	0.052822	0.023933
0.2	1	Decision Tree	MVO	0	na	2.254238	0.051339	0.022775
0.2	5	Decision Tree	MVO	0	na	2.274801	0.048979	0.021531
0.2	10	Decision Tree	MVO	0	na	2.353558	0.040242	0.017098
0.2	50	Decision Tree	MVO	0	na	2.429143	0.032423	0.013347

0.2	100	Decision Tree	MVO	0	na	2.466022	0.030215	0.012253
0.4	5	Decision Tree	MVO	0	na	2.467106	0.042792	0.017345
1	5	Decision Tree	MVO	0	na	2.589644	0.015121	0.005839
0.8	5	Decision Tree	MVO	0	na	2.661775	0.028803	0.010821
0.6	5	Decision Tree	MVO	0	na	2.752177	0.03364	0.012223
0.4	10	Decision Tree	MVO	0	na	2.752706	0.031016	0.011267
0.4	50	Decision Tree	MVO	0	na	3.084421	0.024079	0.007807
0.6	10	Decision Tree	MVO	0	na	3.096829	0.025541	0.008248
0.4	100	Decision Tree	MVO	0	na	3.244683	0.021968	0.00677
0.8	10	Decision Tree	MVO	0	na	3.498565	0.01807	0.005165
1	10	Decision Tree	MVO	0	na	4.080537	0.012915	0.003165
0.6	50	Decision Tree	MVO	0	na	4.559824	0.016717	0.003666
0.6	100	Decision Tree	MVO	0	na	4.73668	0.01599	0.003376
0.8	50	Decision Tree	MVO	0	na	7.178262	0.014791	0.00206
0.8	100	Decision Tree	MVO	0	na	8.442699	0.013353	0.001582
1	50	Decision Tree	MVO	0	na	15.912	0.011424	0.000718

1	100	Decision Tree	MVO	0	na	34.7822	0.01083	0.000311
1	0.5	Factor Model	MVO	0	na	1.399374	0.030552	0.021832
0.8	0.5	Factor Model	MVO	0	na	1.571167	0.045955	0.029249
1	1	Factor Model	MVO	0	na	1.682738	0.015183	0.009023
0.6	0.5	Factor Model	MVO	0	na	1.804704	0.057898	0.032082
0.8	1	Factor Model	MVO	0	na	1.883994	0.022997	0.012206
0.2	0.5	Factor Model	MVO	0	na	1.942221	0.05772	0.029718
0.4	0.5	Factor Model	MVO	0	na	2.013693	0.058946	0.029273
0.2	1	Factor Model	MVO	0	na	2.017888	0.053233	0.026381
0.6	1	Factor Model	MVO	0	na	2.1716	0.03203	0.01475
0.2	5	Factor Model	MVO	0	na	2.23605	0.041193	0.018422
0.4	1	Factor Model	MVO	0	na	2.293127	0.038639	0.01685
0.2	10	Factor Model	MVO	0	na	2.3549	0.034417	0.014615
1	5	Factor Model	MVO	0	na	2.486306	0.006675	0.002685
0.2	50	Factor Model	MVO	0	na	2.489995	0.029044	0.011664
0.2	100	Factor Model	MVO	0	na	2.507329	0.028548	0.011386

0.4	5	Factor Model	MVO	0	na	2.724085	0.031022	0.011388
0.4	10	Factor Model	MVO	0	na	2.93876	0.026093	0.008879
0.4	50	Factor Model	MVO	0	na	3.31662	0.021129	0.006371
0.8	5	Factor Model	MVO	0	na	3.341472	0.010095	0.003021
0.4	100	Factor Model	MVO	0	na	3.369171	0.020615	0.006119
0.6	5	Factor Model	MVO	0	na	3.564578	0.015681	0.004399
1	10	Factor Model	MVO	0	na	4.012958	0.008561	0.002133
0.6	10	Factor Model	MVO	0	na	4.094268	0.014516	0.003545
0.8	10	Factor Model	MVO	0	na	4.257772	0.010614	0.002493
0.6	100	Factor Model	MVO	0	na	4.898827	0.015483	0.003161
0.6	50	Factor Model	MVO	0	na	4.943665	0.015542	0.003144
0.8	50	Factor Model	MVO	0	na	8.924437	0.013204	0.001479
0.8	100	Factor Model	MVO	0	na	9.002327	0.013002	0.001444
1	50	Factor Model	MVO	0	na	17.43171	0.010188	0.000584
1	100	Factor Model	MVO	0	na	32.56062	0.01022	0.000314
1	0.5	Price MA	MVO	0	na	1.654579	0.246027	0.148694

1	1	Price MA	MVO	0	na	1.679356	0.074958	0.044635
0.8	0.5	Price MA	MVO	0	na	1.835951	0.2774	0.151093
0.8	1	Price MA	MVO	0	na	1.874908	0.09318	0.049699
0.2	0.5	Price MA	MVO	0	na	1.980395	0.109731	0.055409
0.2	1	Price MA	MVO	0	na	1.980803	0.096255	0.048594
0.6	0.5	Price MA	MVO	0	na	2.017968	0.225294	0.111644
0.6	1	Price MA	MVO	0	na	2.076942	0.093398	0.044969
0.4	0.5	Price MA	MVO	0	na	2.083834	0.148023	0.071034
0.2	5	Price MA	MVO	0	na	2.119282	0.051159	0.02414
0.4	1	Price MA	MVO	0	na	2.14038	0.096848	0.045248
0.2	10	Price MA	MVO	0	na	2.279789	0.038222	0.016766
0.2	50	Price MA	MVO	0	na	2.484508	0.02923	0.011765
0.2	100	Price MA	MVO	0	na	2.514557	0.028373	0.011284
0.4	5	Price MA	MVO	0	na	2.573306	0.034466	0.013394
0.4	10	Price MA	MVO	0	na	2.873112	0.026598	0.009257
0.6	5	Price MA	MVO	0	na	3.191707	0.024013	0.007524

0.4	50	Price MA	MVO	0	na	3.354971	0.020694	0.006168
0.4	100	Price MA	MVO	0	na	3.387638	0.020416	0.006027
0.8	5	Price MA	MVO	0	na	3.395783	0.021908	0.006452
1	5	Price MA	MVO	0	na	3.474801	0.02179	0.006271
0.6	10	Price MA	MVO	0	na	3.828348	0.020174	0.00527
0.8	10	Price MA	MVO	0	na	4.670016	0.018986	0.004065
0.6	50	Price MA	MVO	0	na	4.71713	0.016025	0.003397
0.6	100	Price MA	MVO	0	na	4.846581	0.015634	0.003226
1	10	Price MA	MVO	0	na	5.372348	0.016935	0.003152
0.8	50	Price MA	MVO	0	na	8.122776	0.013438	0.001654
0.8	100	Price MA	MVO	0	na	8.598248	0.013102	0.001524
1	50	Price MA	MVO	0	na	20.25878	0.011364	0.000561
1	100	Price MA	MVO	0	na	36.46252	0.01076	0.000295
1	10	Decision Tree	RMVO	0	na	1.350338	0.092376	0.06841
0.8	1	Decision Tree	RMVO	0	na	1.372682	0.062963	0.045868
1	5	Decision Tree	RMVO	0	na	1.388292	0.073478	0.052927

1	0.5	Decision Tree	RMVO	0	na	1.438818	0.100392	0.069774
0.8	10	Decision Tree	RMVO	0	na	1.543308	0.072138	0.046743
0.8	0.5	Decision Tree	RMVO	0	na	1.544824	0.109644	0.070975
0.6	1	Decision Tree	RMVO	0	na	1.57552	0.081991	0.052041
0.6	10	Decision Tree	RMVO	0	na	1.609197	0.097241	0.060428
1	50	Decision Tree	RMVO	0	na	1.621387	0.195267	0.120432
0.8	5	Decision Tree	RMVO	0	na	1.639251	0.128796	0.07857
1	100	Decision Tree	RMVO	0	na	1.673223	0.077619	0.046389
0.4	1	Decision Tree	RMVO	0	na	1.724613	0.069555	0.040331
0.6	50	Decision Tree	RMVO	0	na	1.735093	0.108243	0.062384
0.4	0.5	Decision Tree	RMVO	0	na	1.736638	0.084667	0.048753
1	1	Decision Tree	RMVO	0	na	1.740601	0.096042	0.055178
0.6	5	Decision Tree	RMVO	0	na	1.771909	0.120676	0.068105
0.6	0.5	Decision Tree	RMVO	0	na	1.781645	0.123199	0.069149
0.4	10	Decision Tree	RMVO	0	na	1.862516	0.105541	0.056666
0.8	100	Decision Tree	RMVO	0	na	1.875512	0.065344	0.034841

0.4	5	Decision Tree	RMVO	0	na	1.901144	0.070811	0.037247
0.8	50	Decision Tree	RMVO	0	na	1.902355	0.079542	0.041813
0.4	50	Decision Tree	RMVO	0	na	1.982141	0.081584	0.04116
0.2	0.5	Decision Tree	RMVO	0	na	1.989479	0.053586	0.026935
0.6	100	Decision Tree	RMVO	0	na	2.023258	0.089466	0.044219
0.2	1	Decision Tree	RMVO	0	na	2.026747	0.049254	0.024302
0.2	5	Decision Tree	RMVO	0	na	2.134302	0.04453	0.020864
0.2	50	Decision Tree	RMVO	0	na	2.158098	0.049302	0.022845
0.2	10	Decision Tree	RMVO	0	na	2.203202	0.052813	0.023971
0.4	100	Decision Tree	RMVO	0	na	2.225761	0.079838	0.03587
0.2	100	Decision Tree	RMVO	0	na	2.319864	0.055331	0.023851
1	0.5	Factor Model	RMVO	0	na	0.855711	0.141786	0.165694
1	1	Factor Model	RMVO	0	na	0.857319	0.137446	0.16032
0.8	0.5	Factor Model	RMVO	0	na	0.981201	0.139853	0.142532
0.8	1	Factor Model	RMVO	0	na	0.987864	0.132365	0.133991
1	5	Factor Model	RMVO	0	na	1.002318	0.104607	0.104365

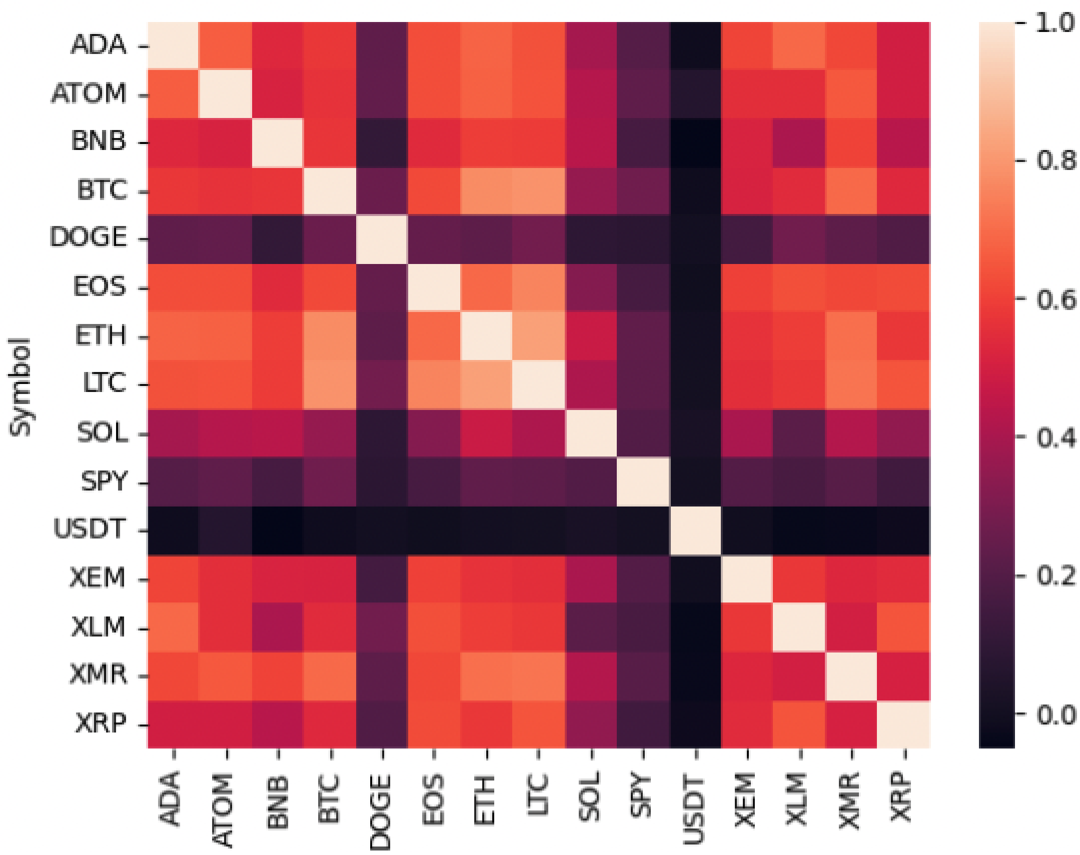
0.8	5	Factor Model	RMVO	0	na	1.107866	0.104715	0.09452
0.6	0.5	Factor Model	RMVO	0	na	1.114848	0.119375	0.107078
0.6	1	Factor Model	RMVO	0	na	1.147362	0.117005	0.101977
1	10	Factor Model	RMVO	0	na	1.162561	0.087871	0.075584
0.6	5	Factor Model	RMVO	0	na	1.277519	0.096324	0.075399
0.8	10	Factor Model	RMVO	0	na	1.302717	0.087034	0.066809
0.4	1	Factor Model	RMVO	0	na	1.380525	0.083878	0.060758
0.4	0.5	Factor Model	RMVO	0	na	1.392042	0.083241	0.059798
0.6	10	Factor Model	RMVO	0	na	1.436253	0.092872	0.064663
1	50	Factor Model	RMVO	0	na	1.477515	0.023216	0.015713
0.4	5	Factor Model	RMVO	0	na	1.490932	0.081751	0.054832
0.4	10	Factor Model	RMVO	0	na	1.554029	0.091978	0.059187
0.8	50	Factor Model	RMVO	0	na	1.65685	0.035397	0.021364
1	100	Factor Model	RMVO	0	na	1.805954	0.012995	0.007195
0.2	10	Factor Model	RMVO	0	na	1.868439	0.051405	0.027512
0.2	5	Factor Model	RMVO	0	na	1.9026	0.051633	0.027138

0.6	50	Factor Model	RMVO	0	na	1.905903	0.047173	0.024751
0.2	1	Factor Model	RMVO	0	na	1.949125	0.057581	0.029542
0.2	0.5	Factor Model	RMVO	0	na	1.955634	0.057625	0.029466
0.2	50	Factor Model	RMVO	0	na	1.965664	0.058885	0.029957
0.8	100	Factor Model	RMVO	0	na	2.032351	0.019085	0.009391
0.2	100	Factor Model	RMVO	0	na	2.068954	0.050538	0.024427
0.4	50	Factor Model	RMVO	0	na	2.099218	0.050093	0.023863
0.6	100	Factor Model	RMVO	0	na	2.313974	0.028239	0.012204
0.4	100	Factor Model	RMVO	0	na	2.378671	0.038133	0.016031
1	50	Price MA	RMVO	0	na	1.640338	0.16618	0.101308
1	0.5	Price MA	RMVO	0	na	1.673433	1.79944	1.075299
1	1	Price MA	RMVO	0	na	1.700496	2.002566	1.177636
1	10	Price MA	RMVO	0	na	1.703565	1.334581	0.783404
1	5	Price MA	RMVO	0	na	1.711134	2.310814	1.350458
1	100	Price MA	RMVO	0	na	1.73011	0.051975	0.030041
0.8	0.5	Price MA	RMVO	0	na	1.741107	1.30412	0.749018

0.8	1	Price MA	RMVO	0	na	1.773995	1.349435	0.760676
0.8	10	Price MA	RMVO	0	na	1.791439	1.008387	0.562892
0.8	5	Price MA	RMVO	0	na	1.808924	1.458276	0.806156
0.6	0.5	Price MA	RMVO	0	na	1.820641	0.789121	0.43343
0.8	50	Price MA	RMVO	0	na	1.829243	0.203742	0.111381
0.6	5	Price MA	RMVO	0	na	1.84699	0.755007	0.408777
0.6	1	Price MA	RMVO	0	na	1.847282	0.786389	0.4257
0.4	1	Price MA	RMVO	0	na	1.864898	0.420978	0.225738
0.4	5	Price MA	RMVO	0	na	1.866458	0.320861	0.171909
0.4	0.5	Price MA	RMVO	0	na	1.896133	0.440808	0.232477
0.6	10	Price MA	RMVO	0	na	1.901508	0.629978	0.331305
0.8	100	Price MA	RMVO	0	na	1.92835	0.063701	0.033034
0.4	10	Price MA	RMVO	0	na	1.931753	0.272863	0.141252
0.2	100	Price MA	RMVO	0	na	1.982441	0.088624	0.044705
0.2	50	Price MA	RMVO	0	na	1.992798	0.110105	0.055252
0.2	10	Price MA	RMVO	0	na	2.000847	0.131039	0.065492

0.2	0.5	Price MA	RMVO	0	na	2.022997	0.15143	0.074854
0.2	1	Price MA	RMVO	0	na	2.023438	0.151121	0.074685
0.2	5	Price MA	RMVO	0	na	2.028539	0.140697	0.069359
0.6	50	Price MA	RMVO	0	na	2.030261	0.174289	0.085846
0.4	50	Price MA	RMVO	0	na	2.106502	0.125269	0.059468
0.6	100	Price MA	RMVO	0	na	2.156735	0.067373	0.031238
0.4	100	Price MA	RMVO	0	na	2.175076	0.081708	0.037565

B - Correlation Matrix of Assets



This correlation matrix is created by looking at the correlation between the 2020 and 2021 daily returns of each asset.