

CptS 223 Homework #3 - Heaps, Hashing, Sorting

Due Date: Nov 20th 2020

Please complete the homework problems and upload a pdf of the solutions to blackboard assignment and upload the PDF to Git.

1. [6] Starting with an empty hash table with a fixed size of 11, insert the following keys in order into three distinct hash tables (one for each collision mechanism): {12, 9, 1, 0, 42, 98, 70, 3}. You are only required to show the final result of each hash table. In the very likely event that a collision resolution mechanism is unable to successfully resolve, simply record the state of the last successful insert and note that collision resolution failed. For each hashtable type, compute the hash as follows:

$$\text{hashkey}(\text{key}) = (\text{key} * \text{key} + 3) \% 11$$

Separate Chaining (buckets)

	3		0 98	12 1			9 42			70
0	1	2	3	4	5	6	7	8	9	10

To probe on a collision, start at $\text{hashkey}(\text{key})$ and add the current $\text{probe}(i')$ offset. If that bucket is full, increment i until you find an empty bucket.

Linear Probing: $\text{probe}(i') = (i + 1) \% \text{TableSize}$

	3		0	12	1	98	9	42	70	
0	1	2	3	4	5	6	7	8	9	10

Quadratic Probing: $\text{probe}(i') = (i * i + 5) \% \text{TableSize}$

			0	12		42	9			1
0	1	2	3	4	5	6	7	8	9	10

Collision resolution failed when inserting 98

2. [3] For implementing a hash table. Which of these would probably be the best initial table size to pick?

Table Sizes:

1 100 101 15 500

Why did you choose that one?

It is the only prime number.

3. [4] For our running hash table, you'll need to decide if you need to rehash. You just inserted a new item into the table, bringing your data count up to 53491 entries. The table's vector is currently sized at 106963 buckets.

- Calculate the load factor (λ):

$$53491/106963 = .500$$

- Given a linear probing collision function should we rehash? Why?

It should not be rehashed. as the load factor would not be considered high enough to need it. A general rule of thumb is to rehash at a load factor of .75 or higher.

- Given a separate chaining collision function should we rehash? Why?

See above?

4. [4] What is the Big-O of these actions for a well designed and properly loaded hash table with N elements?

Function	Big-O complexity
Insert(x)	$O(1)$
Rehash()	$O(n)$
Remove(x)	$O(1)$
Contains(x)	$O(1)$

7. [3] I grabbed some code from the Internet for my linear probing based hash table at work because the Internet's always right (totally!). The hash table works, but once I put more than a few thousand entries, the whole thing starts to slow down. Searches, inserts, and contains calls start taking *much* longer than $O(1)$ time and my boss is pissed because it's slowing down the whole application services backend I'm in charge of. I think the bug is in my rehash code, but I'm not sure where. Any ideas why my hash table starts to suck as it grows bigger?

```
/**
 * Rehashing for linear probing hash table.
 */
void rehash( )
{
    ArrayList<HashItem<T>> oldArray = array;

    array = new ArrayList<HashItem<T>>( 2 * oldArray.size() );

    for( int i = 0; i < array.size(); i++ )
        array.get(i).info = EMPTY;
    // Copy old table over to new larger array
    for( int i = 0; i < oldArray.size(); i++ ) {
        if( oldArray.get(i).info == FULL )
        {
            addElement(oldArray.get(i).getKey(),
                        oldArray.get(i).getValue());
        }
    }
}
```

It looks like the rehashing sucks because the second for loop is nested in the first due to a lack of brackets, causing a big O of n^2 .

8. [4] Time for some heaping fun! What's the time complexity for these functions in a Java Library priority queue (binary heap) of size N ?

Function	Big-O complexity
push(x)	$O(\log n)$
top()	$O(1)$
pop()	$O(\log n)$
PriorityQueue(Collection<? extends E> c) // BuildHeap	$O(n)$

9. [4] What would a good application be for a priority queue (a binary heap)? Describe it in at least a paragraph of why it's a good choice for your example situation.

A priority queue is good for any list where you want to in general be "popping" the most important (higher priority) items first, in the order that they were received. Such as a printer queue where print jobs can be given different priorities. Print jobs with a higher priority will be placed further up in the queue to be printed even if other jobs were added before them.

10. [4] For an entry in our heap (root @ index 1) located at position i , where are its parent and children?

Parent: *the parent would be located at the position of the integer value $i/2$*

Children: *children would be located at positions $i*2$ and $(i*2) + 1$*

11. [6] Show the result of inserting 10, 12, 1, 14, 6, 5, 15, 3, and 11, one at a time, into an initially empty binary heap. Use a 1-based array like the book does. After insert(10):

10										
----	--	--	--	--	--	--	--	--	--	--

After insert (12):

10	12									
----	----	--	--	--	--	--	--	--	--	--

etc:

1	12	10								
---	----	----	--	--	--	--	--	--	--	--

1	12	10	14							
---	----	----	----	--	--	--	--	--	--	--

1	6	10	14	12						
---	---	----	----	----	--	--	--	--	--	--

1	6	5	14	12	10					
---	---	---	----	----	----	--	--	--	--	--

1	6	5	14	12	10	15				
---	---	---	----	----	----	----	--	--	--	--

1	3	5	6	12	10	15	14			
---	---	---	---	----	----	----	----	--	--	--

1	3	5	6	12	10	15	14	11		
---	---	---	---	----	----	----	----	----	--	--

12. [4] Show the same result (only the final result) of calling buildHeap() on the same vector of values: {10, 12, 1, 14, 6, 5, 15, 3, 11}

1	3	5	11	6	10	15	14	12		
---	---	---	----	---	----	----	----	----	--	--

13. [4] Now show the result of three successive deleteMin / pop operations from the prior heap:

3	6	5	11	12	10	15	14			
---	---	---	----	----	----	----	----	--	--	--

5	6	10	11	12	14	15				
---	---	----	----	----	----	----	--	--	--	--

6	11	10	15	12	14					
---	----	----	----	----	----	--	--	--	--	--

14. [4] What are the average complexities and the stability of these sorting algorithms:

Algorithm	Average complexity	Stable (yes/no)?
Bubble Sort	$O(n^2)$	yes
Insertion Sort	$O(n^2)$	yes
Heap sort	$O(n \log n)$	no
Merge Sort	$O(n \log n)$	yes
Radix sort	$O(kn)$	yes
Quicksort	$O(n \log n)$	no

15. [3] What are the key differences between Mergesort and Quicksort? How does this influence why languages choose one over the other?

merge sort places items in the index in order (is stable),

16. [4] Draw out how Mergesort would sort this list:

24	16	9	10	8	7	20
----	----	---	----	---	---	----

24	16	9	10
----	----	---	----

8	7	20
---	---	----

24	16
----	----

9	10
---	----

8	7
---	---

20

24

16

9

10

8

7

16	24
----	----

9	10
---	----

7	8
---	---

20

9	10	16	24
---	----	----	----

7	8	20
---	---	----

7	8	9	10	16	20	24
---	---	---	----	----	----	----

17. [4] Draw how Quicksort would sort this list:

24	16	9	10	8	7	20
----	----	---	----	---	---	----

24	16	9	20	8	7	10
----	----	---	----	---	---	----

7	16	9	20	8	24	10
---	----	---	----	---	----	----

7	8	9	20	16	24	10
---	---	---	----	----	----	----

7	8	9	10	16	24	20
---	---	---	----	----	----	----

7	8	9	10	16	20	24
---	---	---	----	----	----	----

Let me know what your pivot picking algorithm is (if it's not obvious):

Move the middle index to the end of the sublist.