PROJECT PHASE 3

DESCRIPTION OF PROJECT :

Algorithm using Bracket Penalty Function Method

Step 1 :

Choose two termination parameters ϵ_1 ,ϵ_2 an initial solution $\boldsymbol{x}^{(0)}$, a penalty term Ω ,and an initial penalty parameter $\boldsymbol{R}^{(0)}$. Choose a parameter c to update R such that 0 <c< 1 is used for interior penalty terms and c ≥ 1 is used for exterior penalty terms. Set t = 0.

Step 2 :

Form P($\boldsymbol{x}^{(t)}$, $\boldsymbol{R}^{(t)}$) = f($\boldsymbol{x}^{(t)}$) + Ω($\boldsymbol{R}^{(t)}$,g($\boldsymbol{x}^{(t)}$), h($\boldsymbol{x}^{(t)}$).

Step 3 :

Use Newton Method. Starting with $\boldsymbol{x}^{(t)}$, find $\boldsymbol{x}^{(t+1)}$ such that P($\boldsymbol{x}^{(t)}$, $\boldsymbol{R}^{(t)}$) is minimum for a fixed value of $\boldsymbol{R}^{(t)}$.Use ϵ_1 to terminate the unconstrained search.

Step 3.1:

Choose a maximum number of iterations M to be performed , two termination parameters ϵ_1 ,ϵ_2 and set k = 0.

Step 3.2:

Calculate ∇f($\boldsymbol{x}^{(k)}$) ,the first derivative at the point $\boldsymbol{x}^{(k)}$**.**

Step 3.3 :

If |∇f($\boldsymbol{x}^{(k)}$)| ≤ ϵ_1 , **Terminate**,
Else if k ≥ M; **Terminate**;
Else go to Step 4

Step 3.4 :

Perform a unidirectional search to find $\boldsymbol{\alpha}^{(k)}$ using ϵ_2 such that f($\boldsymbol{x}^{(k+1)}$)
= f($\boldsymbol{x}^{(k)}$ - $\boldsymbol{\alpha}^{(k)}\nabla^2$f($\boldsymbol{x}^{(k)}$)$^{-1}$∇f($\boldsymbol{x}^{(k)}$)) is minimum.

Use Bounding Phase Method

Step 1 :

Choose an increment Δ. Set k = 0.

Step 2 :

If f($\boldsymbol{x}^{(\mathbf{0})}$)- | Δ |) ≥ $f(\boldsymbol{x}^{(\mathbf{0})})$ ≥ $f(\boldsymbol{x}^{(\mathbf{0})}$)+ | Δ |), then is positive;
Else if f($\boldsymbol{x}^{(\mathbf{0})}$)| Δ |) ≤ f($\boldsymbol{x}^{(\mathbf{0})}$) ≤ f($\boldsymbol{x}^{(\mathbf{0})}$+|Δ), then is negative; Else
go to Step 1.

Step 3 :

Set ($\boldsymbol{x}^{(\boldsymbol{k}+\mathbf{1})}$) = $\boldsymbol{x}^{(\boldsymbol{k})}$ + $\mathbf{2}^{(\boldsymbol{k})}$Δ. (other exponent can be used).

Step 4 :

If f($\boldsymbol{x}^{(\boldsymbol{k}+!)}$) < f($\boldsymbol{x}^{(\boldsymbol{k})}$), set k=k+1 and go to Step 3;
Else the minimum lies in the interval ($\boldsymbol{x}^{(\boldsymbol{k}-\mathbf{1})}$), $\boldsymbol{x}^{(\boldsymbol{k}+!)}$) and Terminate .

Use Interval Halving Method for $\boldsymbol{x}^{(\boldsymbol{k}-\mathbf{1})}$), $\boldsymbol{x}^{(\boldsymbol{k}+!)}$

Step 1 :

Choose a lower bound $\boldsymbol{x}^{(\boldsymbol{k}-\mathbf{1})}$ and an upper bound $\boldsymbol{x}^{(\boldsymbol{k}+!)}$. Choose also a small ϵ . Let x_m= (a+b)/2 , Lo = L = b-a. Compute f(x_m) .
% New equi-distance points

Step 2 :

Set x_1 = a + L/4, x_2 = b-L/4. Compute f(x_1) and f(x_2).
% Region elimination

Step 3 :

If f(x_1) < f(x_m) set b = x_m ; x_m = x_1 ; go to Step 5 ; Else
go to Step 4.

Step 4 :

If f(x_2) < f(x_m) set a = x_m ; x_m = x_2 ; go to Step 5 ; Else
set a = x_1 , b = x_2 ; go to Step 5.
% Termination condition

Step 5 :

Calculate L = b-a. If |L| < ϵ . Terminate . Else go to Step 2.

One criterion for termination is when $|\nabla f(\boldsymbol{x}^{(k+1)}) . \nabla f(\boldsymbol{x}^{(k)})| \leq \epsilon_2$.

Step 3.5 :

Is $\frac{||\boldsymbol{x}^{(k+1)} - \boldsymbol{x}^{(k)}||}{||\boldsymbol{x}^{(k)}||} \leq \epsilon_1$ .If yes,

Terminate; Else set k = k+1 and go to Step 2.

Step 4 :

Is $|P(\boldsymbol{x}^{(t+1)}, \boldsymbol{R}^{(t)}) - P(\boldsymbol{x}^{(t)}, \boldsymbol{R}^{(t-1)})| \leq \epsilon_2$?
If yes, set $\boldsymbol{x}^{(t)} = \boldsymbol{x}^{(t+1)}$ and **terminate** ; Else go to Step 5.

Step 5 :

Choose $\boldsymbol{R}^{(t+1)} = c\boldsymbol{R}^{(t)}$. Set t = t +1 and go to Step 2 .

FLOW CHART

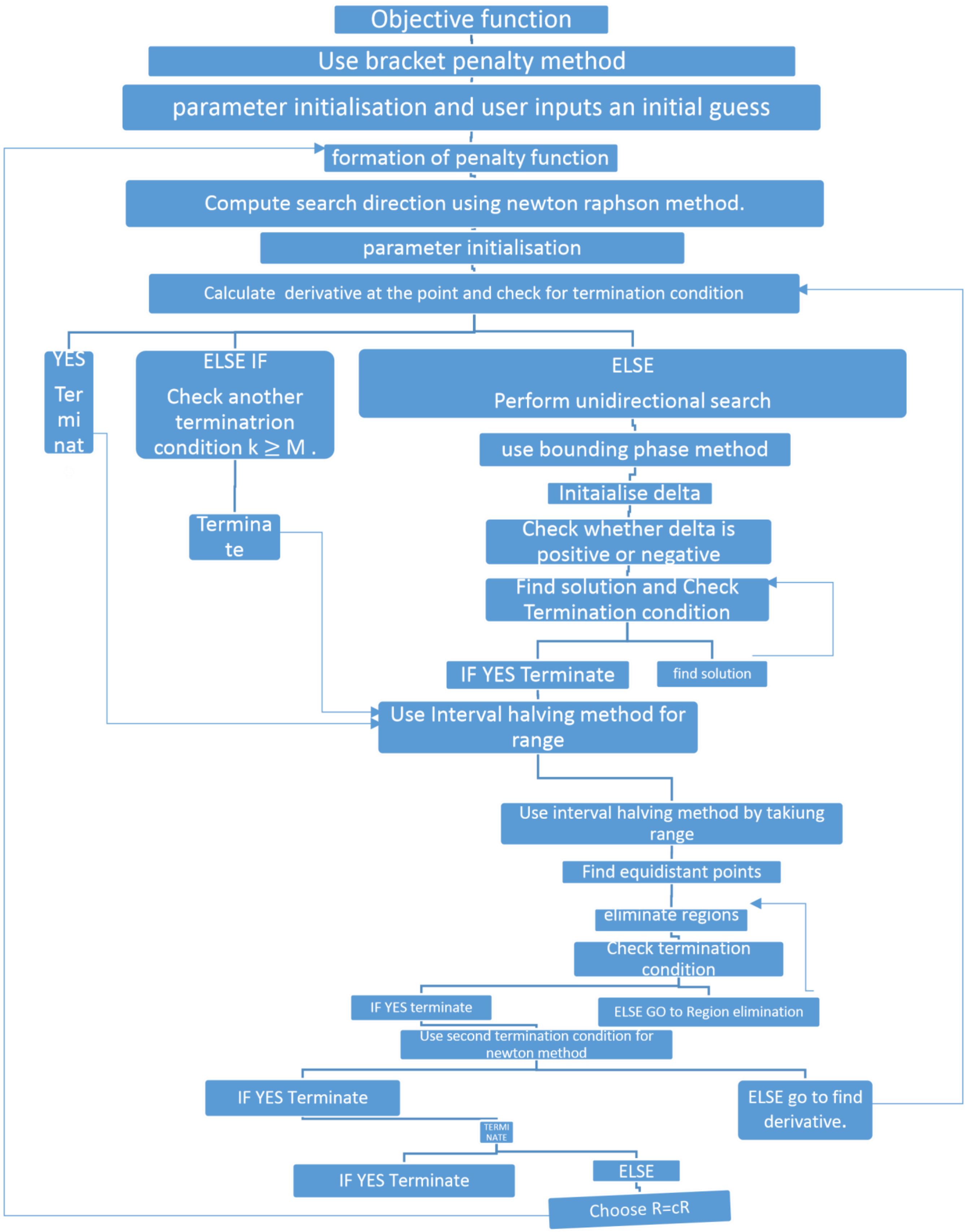
Objective function
Use bracket penalty method
parameter initialisation and user inputs an initial guess
formation of penalty function
Compute search direction using newton raphson method.
parameter initialisation
Calculate derivative at the point and check for termination condition
YES Ter mi nat
ELSE IF Check another terminatrion condition k ≥ M .
ELSE Perform unidirectional search
Termina te
use bounding phase method
Initaialise delta
Check whether delta is positive or negative
Find solution and Check Termination condition
IF YES Terminate
find solution
Use Interval halving method for range
Use interval halving method by takiung range
Find equidistant points
eliminate regions
Check termination condition
IF YES terminate
ELSE GO to Region elimination
Use second termination condition for newton method
IF YES Terminate
ELSE go to find derivative.
TERMI NATE
IF YES Terminate
ELSE
Choose R=cR

TABLE :

Q = 1, starting point = (15,2)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	5.697419	-6.340379	1894.949101
2	6.871726	-4.572813	1039.872858
3	11.212342	-2.666514	1506.974625
4	13.601778	-0.054307	439.455987
5	14.038107	0.726945	58.337188
6	14.089197	0.830944	6.080790
7	14.094419	0.841756	0.613523
8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (15,1)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	5.697419	-6.340379	1894.949101
2	6.871726	-4.572813	1039.872858
3	11.212342	-2.666514	1506.974625
4	13.601778	-0.054307	439.455987
5	14.038107	0.726945	58.337188
6	14.089197	0.830944	6.080790
7	14.094419	0.841756	0.613523
8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (16,3)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	5.697419	-6.340379	1894.949099
2	6.871726	-4.572813	1039.872858

3	11.212342	-2.666514	1506.974625
4	13.601778	-0.054307	439.455987
5	14.038107	0.726945	58.337188
6	14.089197	0.830944	6.080790
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8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (17,2)

Iteration	**X1**	**X2**	$\|P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\|$
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2	6.871726	-4.572813	1039.872858
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6	14.089197	0.830944	6.080790
7	14.094419	0.841756	0.613523
8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (18,3)

Iteration	**X1**	**X2**	$\|P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\|$
1	5.697419	-6.340379	1894.949099
2	6.871726	-4.572813	1039.872858
3	11.212342	-2.666514	1506.974625
4	13.601778	-0.054307	439.455987
5	14.038107	0.726945	58.337188
6	14.089197	0.830944	6.080790
7	14.094419	0.841756	0.613523
8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (19,2)

Iteration	X1	X2	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	5.697419	-6.340379	1894.949100
2	6.871726	-4.572813	1039.872858
3	11.212342	-2.666514	1506.974625
4	13.601778	-0.054307	439.455987
5	14.038107	0.726945	58.337188
6	14.089197	0.830944	6.080790
7	14.094419	0.841756	0.613523
8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (16,1)

Iteration	X1	X2	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	5.697419	-6.340379	1894.949099
2	6.871726	-4.572813	1039.872858
3	11.212342	-2.666514	1506.974625
4	13.601778	-0.054307	439.455987
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8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (17,3)

Iteration	X1	X2	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	5.697419	-6.340379	1894.949101
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Q = 1, starting point = (18,1)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)}) \rvert$
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8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 1, starting point = (15,3)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)}) \rvert$
1	5.697419	-6.340379	1894.949100
2	6.871726	-4.572813	1039.872858
3	11.212342	-2.666514	1506.974625
4	13.601778	-0.054307	439.455987
5	14.038107	0.726945	58.337188
6	14.089197	0.830944	6.080790
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8	14.093891	0.840667	10.632593
9	14.094252	0.841410	24.475525
10	14.094289	0.841483	189.032353

Q = 2, starting point = (1,3)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	1.227969	4.245365	0.000001

Q = 2, starting point = (4,1)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	1.259465	3.350811	0.023610
2	1.674674	3.801553	0.000080
3	1.674063	3.802188	0.000008

Q = 2, starting point = (4,5)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	-0.005023	4.243568	1.019534
2	-0.001741	4.242824	10.125741
3	0.095941	4.203143	80.424199
4	1.674002	3.802254	0.000001

Q = 2, starting point = (6,2)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	-0.001916	5.238438	5.786347
2	0.023065	5.183122	50.835912
3	1.001879	3.956472	0.000000

Q = 2, starting point = (3,3)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	-0.005982	4.243964	1.021763
2	0.000235	4.239619	10.058352
3	0.095943	4.203140	80.424238
4	1.674002	3.802254	0.000001

Q = 2, starting point = (3,7)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	1.734139	4.746083	0.000000

Q = 2, starting point = (5,1)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	-0.005618	4.243925	1.021021
2	0.000835	4.239595	10.047068
3	0.095943	4.203141	80.424423
4	1.674002	3.802254	0.000001

Q = 2, starting point = (8,4)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	-0.002682	5.238445	5.789676
2	0.023070	5.183154	50.840996
3	1.001879	3.956476	0.000000

Q = 2, starting point = (4,6)

Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	-0.009397	2.245149	15.047243
2	0.008364	2.302722	134.958140
3	0.095950	4.203143	80.426303
4	1.674002	3.802254	0.000001

Q = 2, starting point = (1,1)

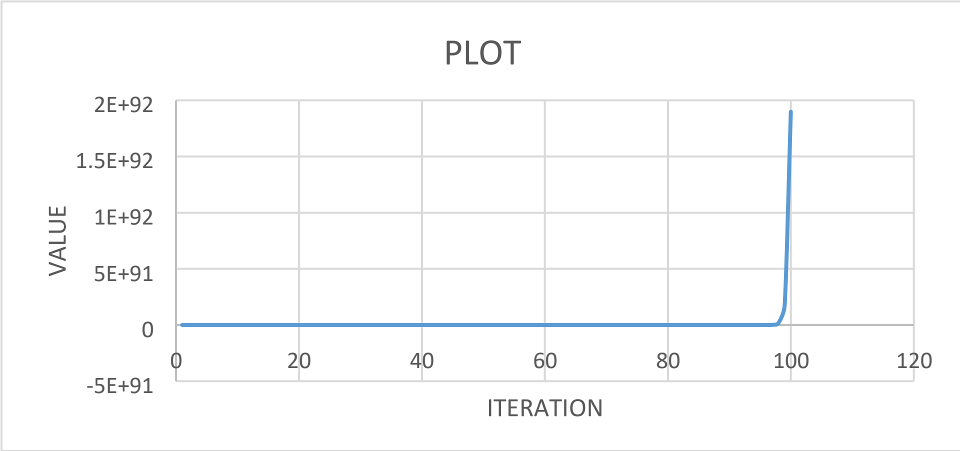
Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	0.723367	3.758853	0.098889
2	1.734132	4.746097	0.000000

Q = 2, starting point = (10,10)

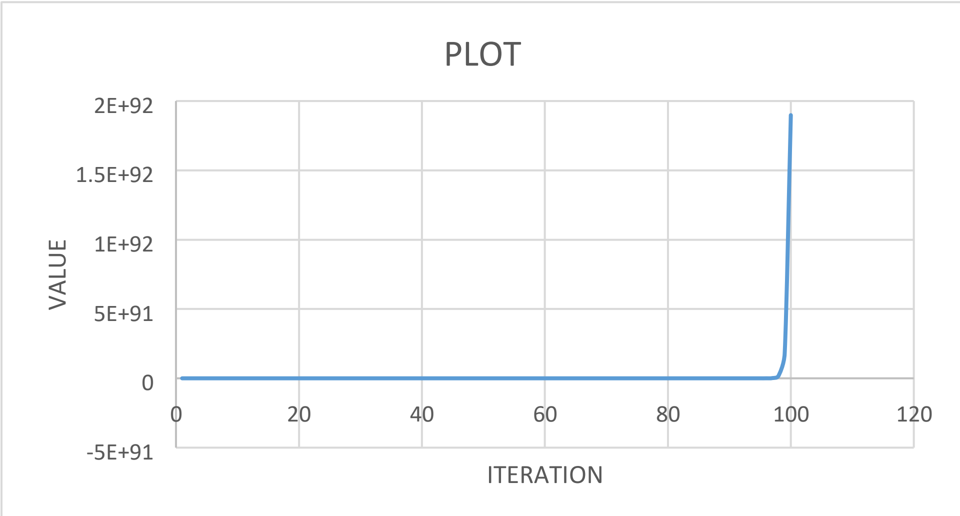
Iteration	**X1**	**X2**	$\lvert P(\boldsymbol{x}_{(t+1)}, \boldsymbol{R}_{(t)}) - P(\boldsymbol{x}_{(t)}, \boldsymbol{R}_{(t-1)})\rvert$
1	1.734138	4.746081	0.000000

CONVERGENCE PLOTS :

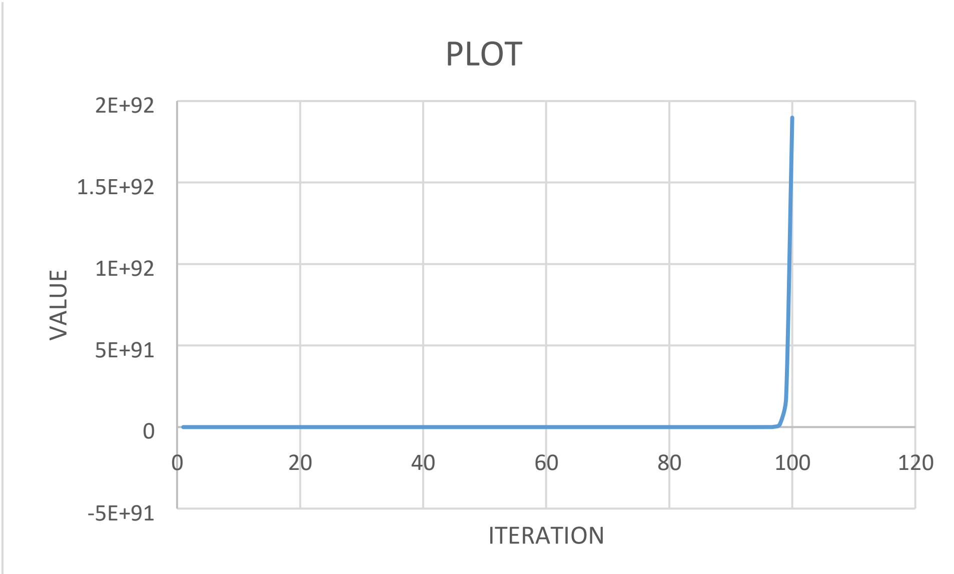
Q = 1, starting point = (15,2)



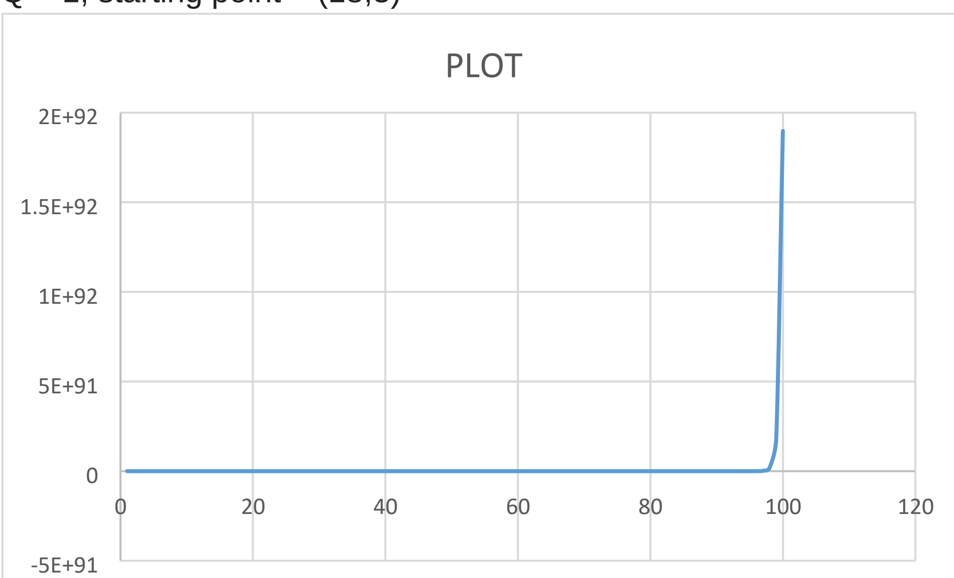
Q = 1, starting point = (15,1)



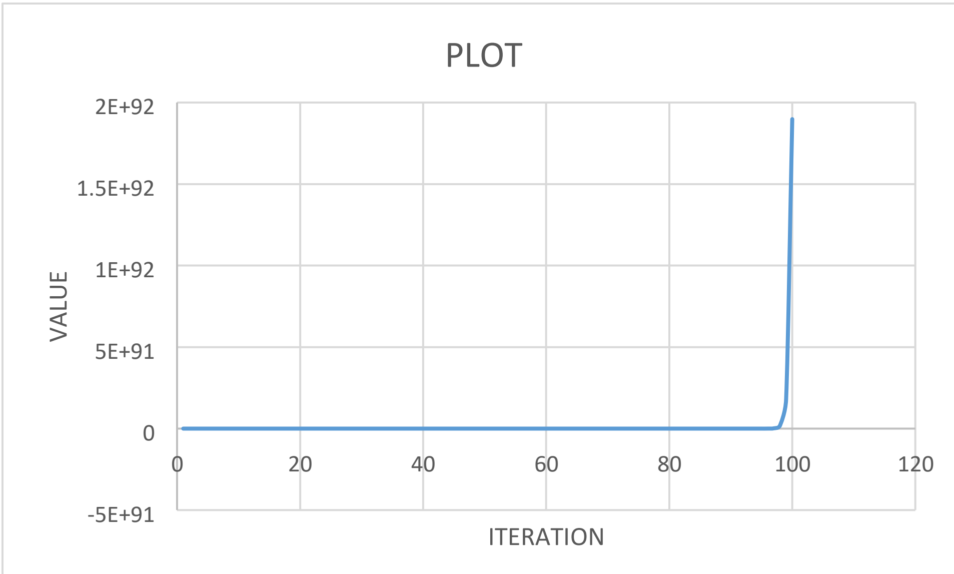
Q = 1, starting point = (17,2)



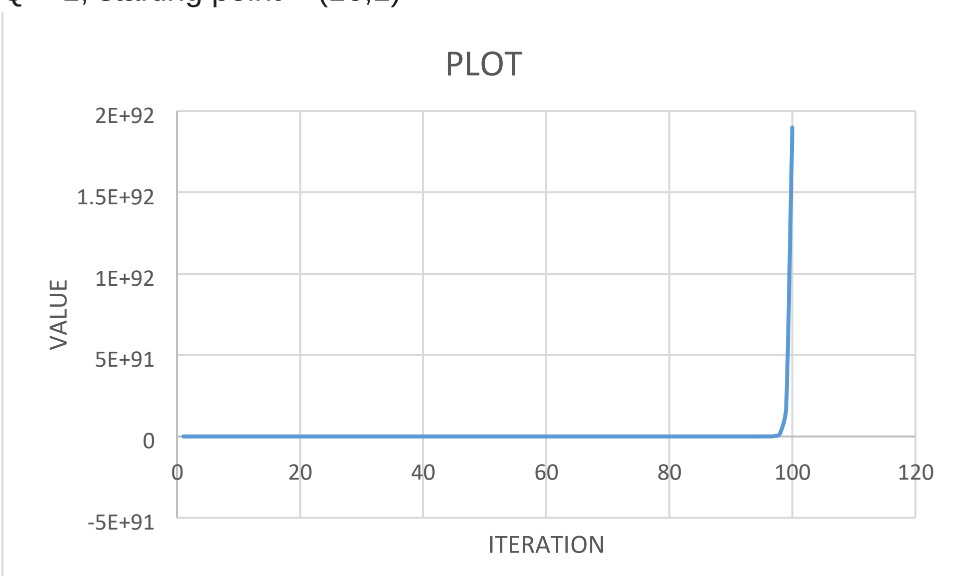
Q = 1, starting point = (18,3)



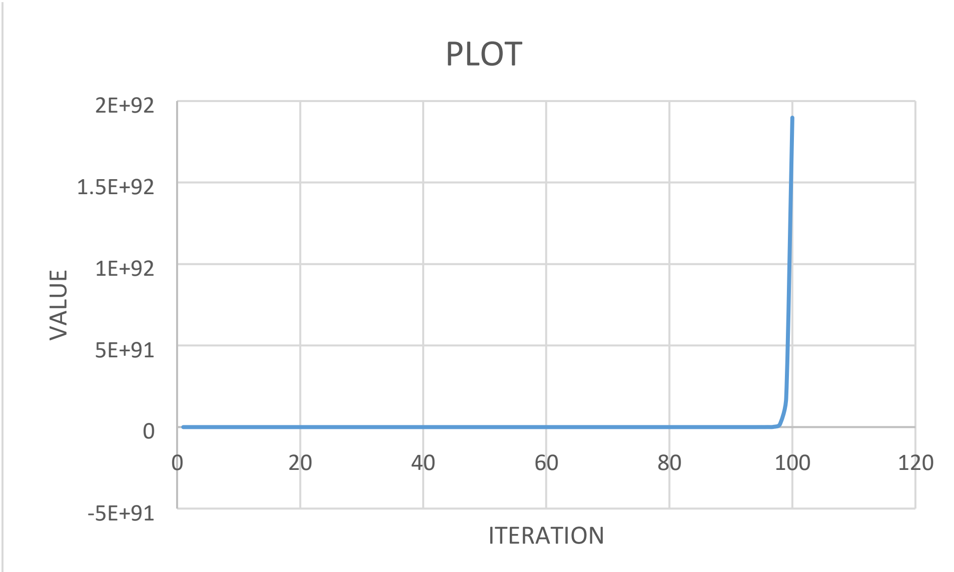
Q = 1, starting point = (19,2)



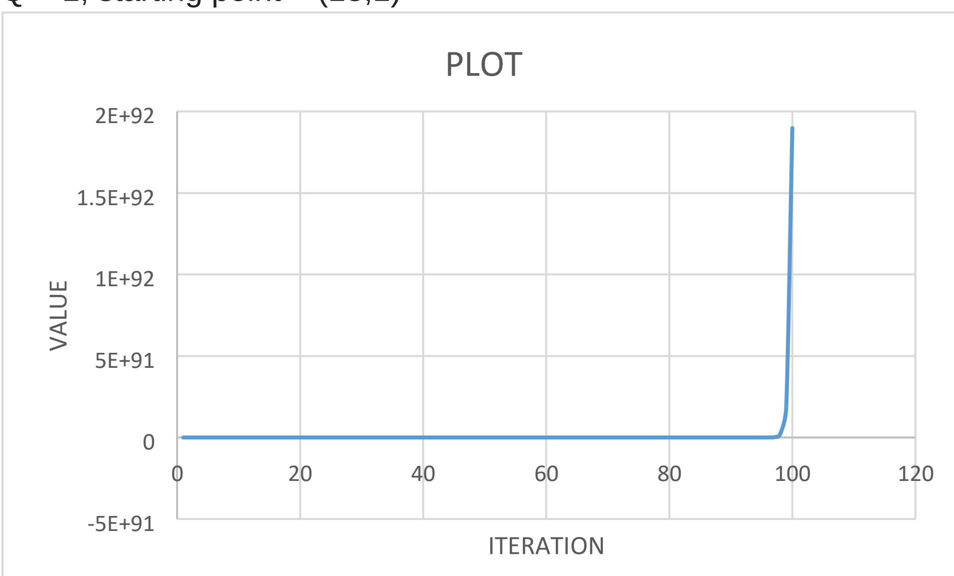
Q = 1, starting point = (16,1)



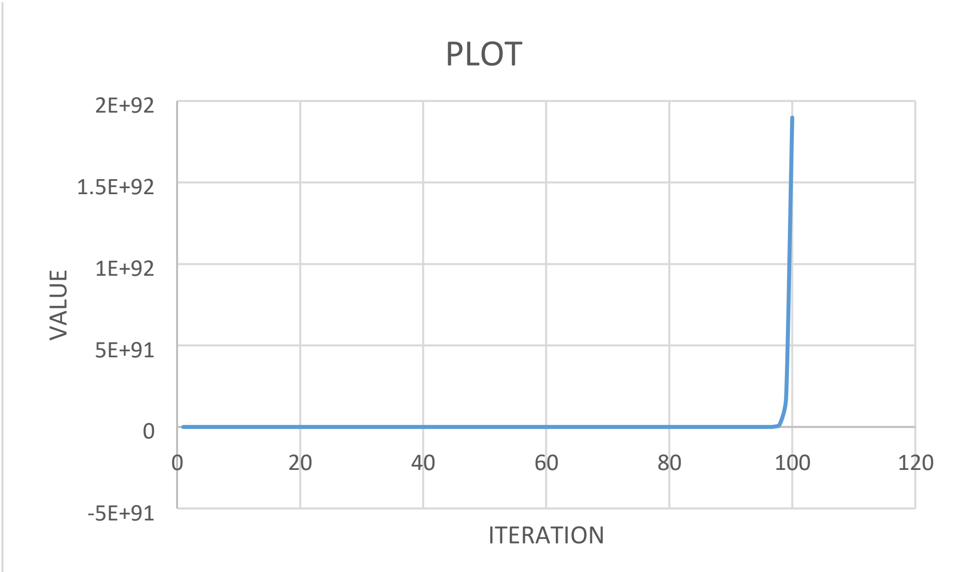
Q = 1, starting point = (17,3)



Q = 1, starting point = (18,1)

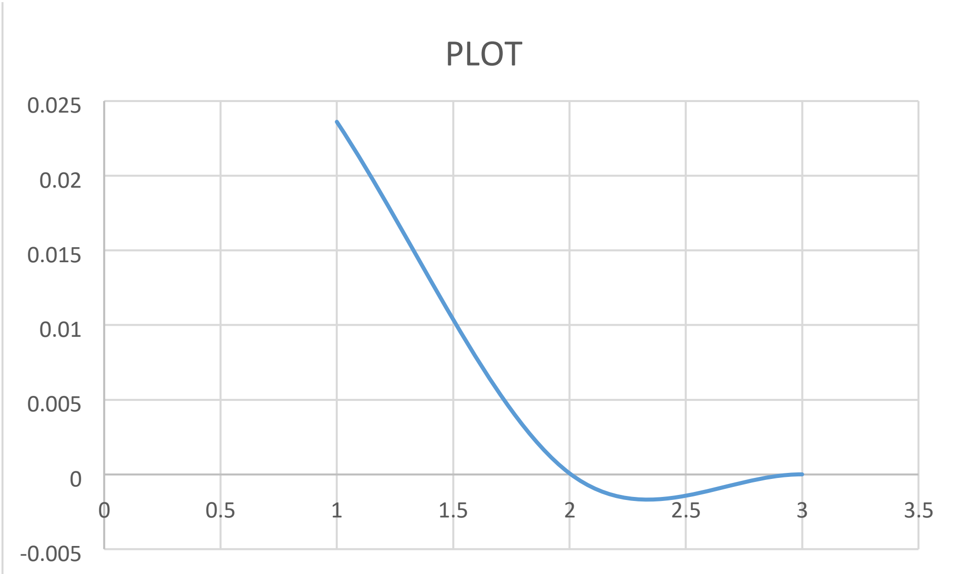


Q = 1, starting point = (15,3)

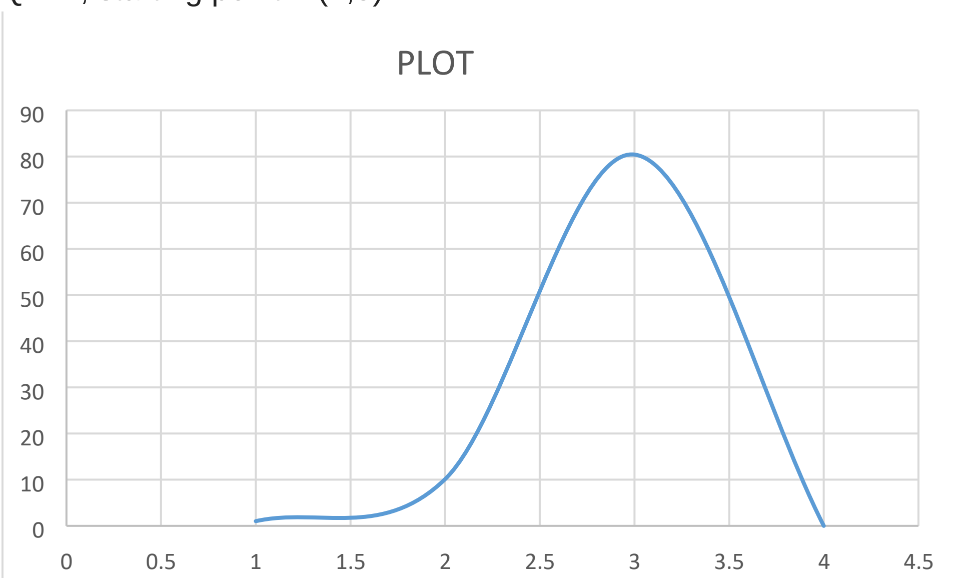


Q = 2, starting point = (1,3)

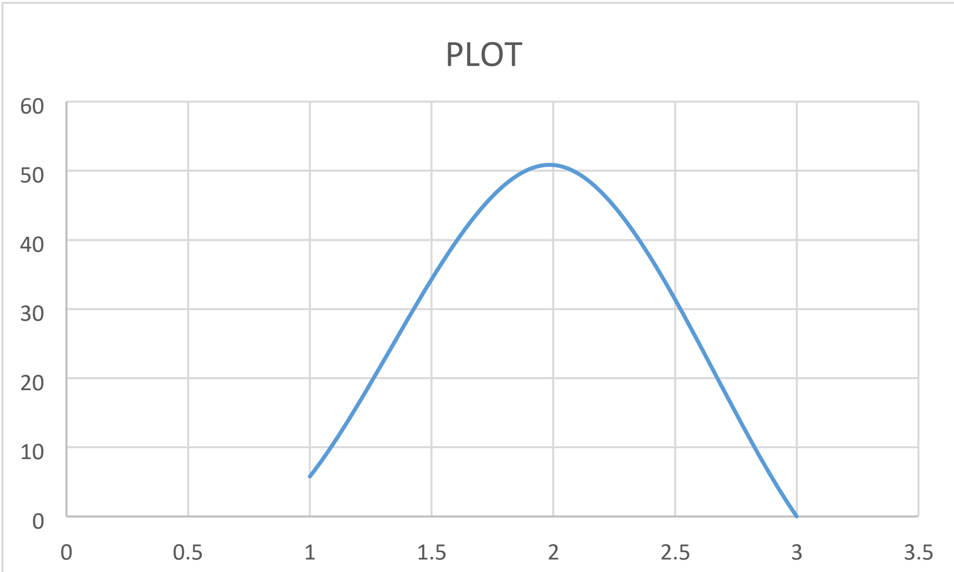
Q = 2, starting point = (4,1)



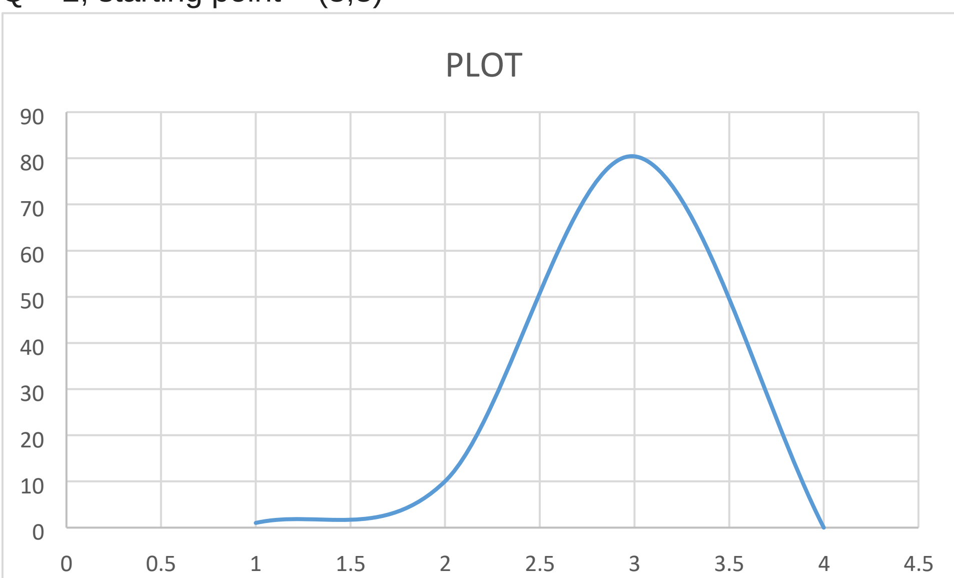
Q = 2, starting point = (4,5)



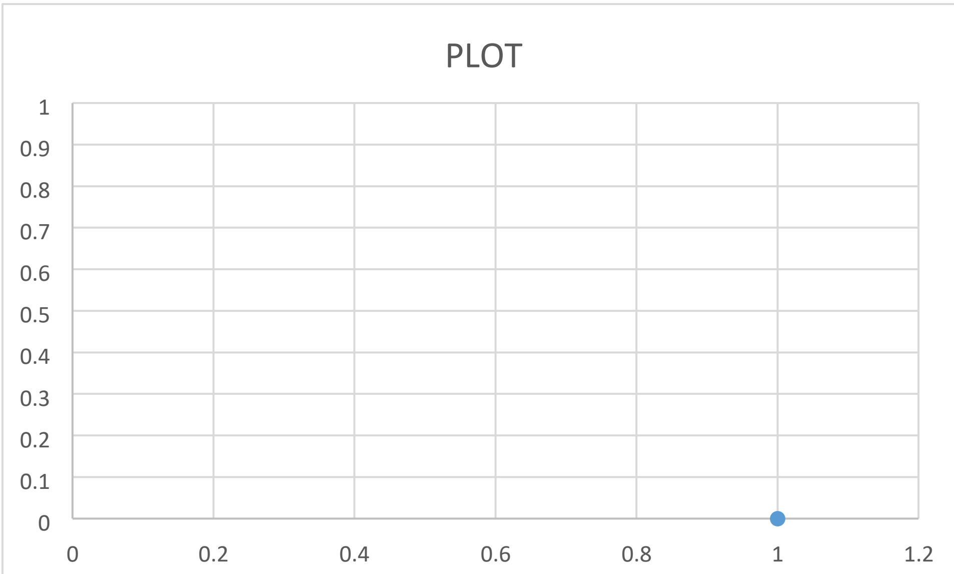
Q = 2, starting point = (6,2)



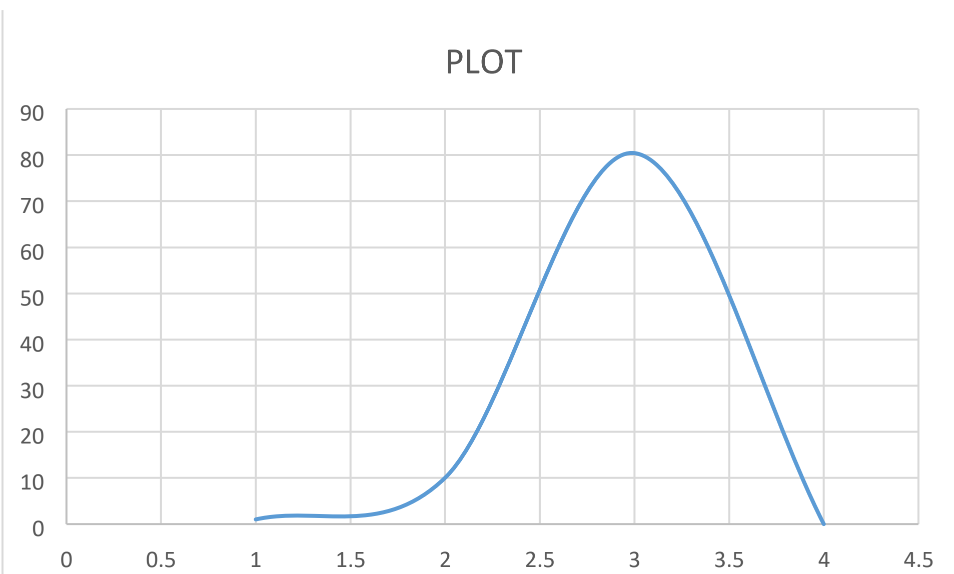
Q = 2, starting point = (3,3)



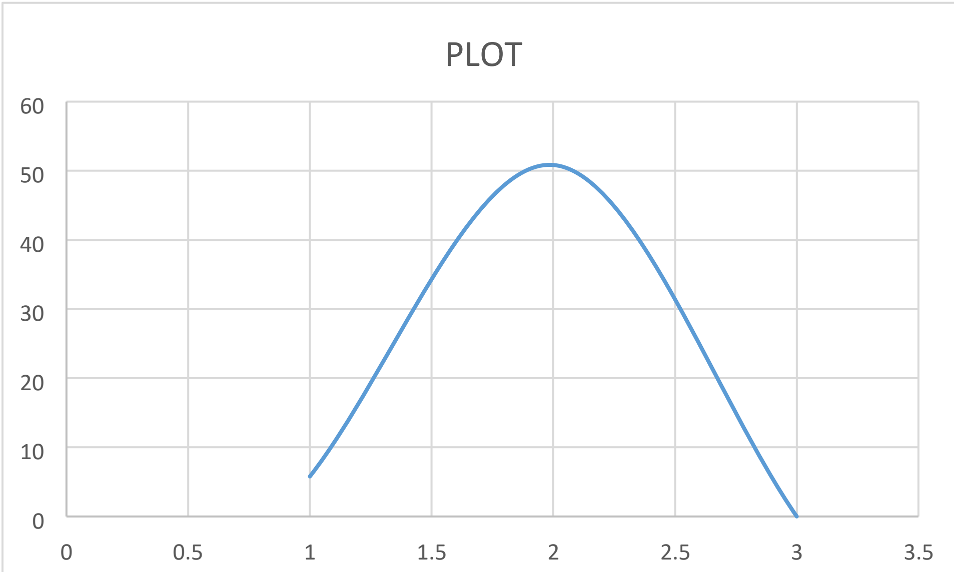
Q = 2, starting point = (3,7)



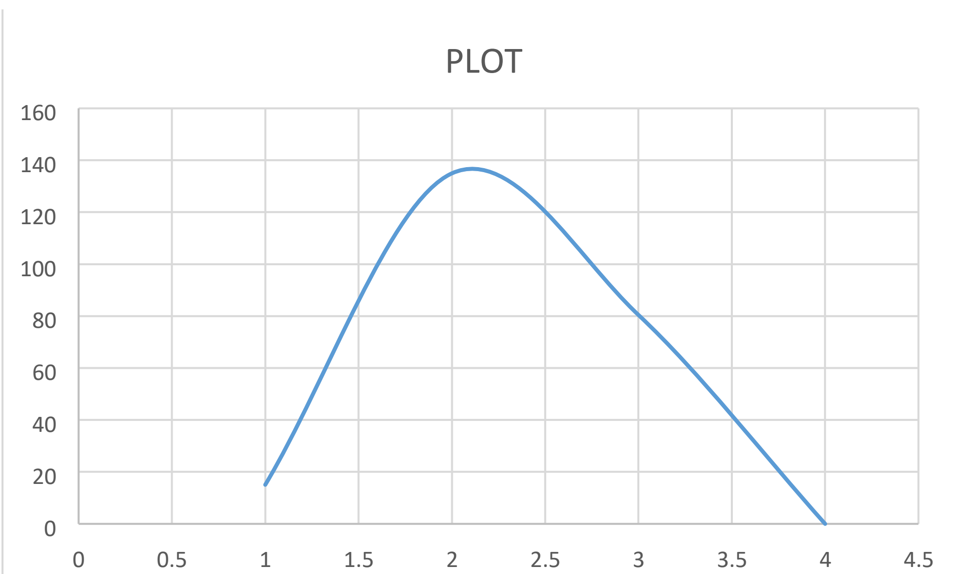
Q = 2, starting point = (5,1)



Q = 2, starting point = (8,4)



Q = 2, starting point = (4,6)



Q = 2, starting point = (1,1)