# Assignment AE2-002P

## Numerical calculation of the balance field length

## Introduction

The takeoff for aircraft, as illustrated in Figure 1, can be defined as the maneuver by which the aircraft in accelerated from rest, up to screen height, which is usually at an altitude of 10.7 meters.

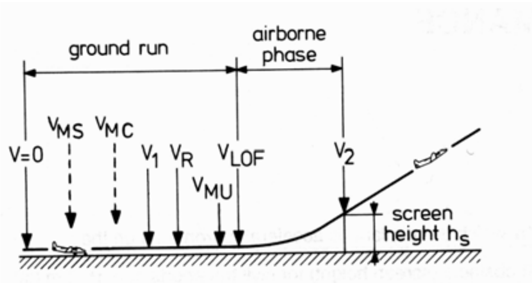



**Figure 1: Overview of speeds relevant to a take off**

At one point during the take-off procedure of a multi-engine aircraft $V_1$, the decision speed, will be called. After this point the aircraft must continue the take off in case of, most notably, an engine failure. The basic value of $V_1$ is given by the balanced field length, as illustrated in Figure 2. If a single engine failure occurs at this speed, the stopping and the take off distance are equal. The total traveled distance, plus a margin, specifies the minimum amount of runway which needs to be available[1].

The aim of this assignment is to calculate the balance field length, and the accompanying $V_1$, of a specified aircraft with a numerical simulation using the Euler integration method.

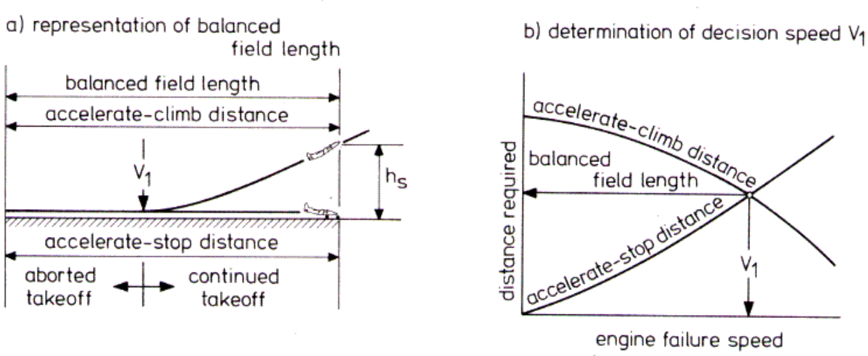



**Figure 2: Graphical representation of the balanced field length**

[1] When the available amount of runway is longer than this distance, the decision speed may be chosen more freely.

## Assumptions:

- The aircraft has a parabolic drag coefficient.
- For jet powered aircraft the thrust is independent of speed.
- For propeller powered aircraft the available power is independent of speed.
- The ground resistance and brake resistance coefficient are independent of speed.

## Equations

### *Aircraft parameters*

Here the formulas to be used are given. Please note that equations (8) and (10) are only calculated under specified conditions.  Please use a new routine for each formula.

| | | |
|---|---|---|
| Minimum (stall) speed | $V_{\min} = \sqrt{\dfrac{2W}{\rho S C_{l_{\max}}}}$ | (1) |
| Rotation speed | $V_{rot} = 1.2 V_{\min}$ | (2) |
| Angle of attack | $\alpha = \theta - \gamma$ | (3) |
| Lift coefficient | $C_l = \frac{dC_l}{d\alpha}\left[\alpha - \alpha_0\right]$ | (4) |
| Lift | $L = C_l \frac{1}{2} \rho V^2 S$ | (5) |
| Drag coefficient | $C_d = C_{d_0} + \dfrac{C_l^2}{\pi A e}$ | (6) |
| Ground force | $N = W - L, L < W \cap h < 0.01$ | (8) |
| Drag | $D = C_d \frac{1}{2} \rho V^2 S + \mu_{rol/brake} N$ | (7) |
| Speed acceleration | $\dfrac{dV}{dt} = \dfrac{g_0}{W}\left[T - D - W\sin(\gamma)\right]$ | (9) |
| Climb angle acceleration | $\dfrac{d\gamma}{dt} = \dfrac{g_0}{WV}\left[L - W + N\right], V > 1$ | (10) |

For propeller aircraft the available thrust can be gained by dividing the available power by the airspeed. If the airspeed is lower than 1 [m/s], divide the available power by one.

### *Integration*

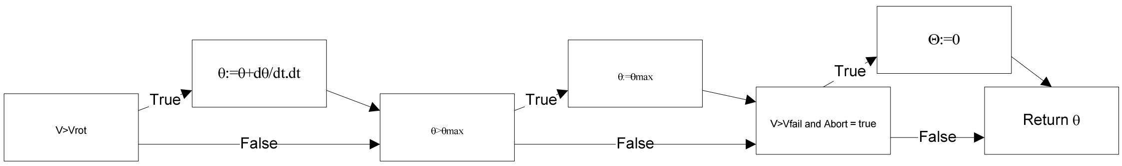
The following formulas can be used to calculate the values for the next step:

| | |
|---|---|
| Climb angle | $\gamma_{i+1} = \gamma_i + \frac{d\gamma}{dt} dt$ |
| True air speed | $V_{i+1} = V_i + \frac{dV}{dt} dt$ |
| Traveled distance | $S_{i+1} = S_i + V_{i+1} dt$ |
| Altitude | $H_{i+1} = H_i + V_{i+1} \sin(\gamma_{i+1}) dt$ |

# Controls

## *Rotation*

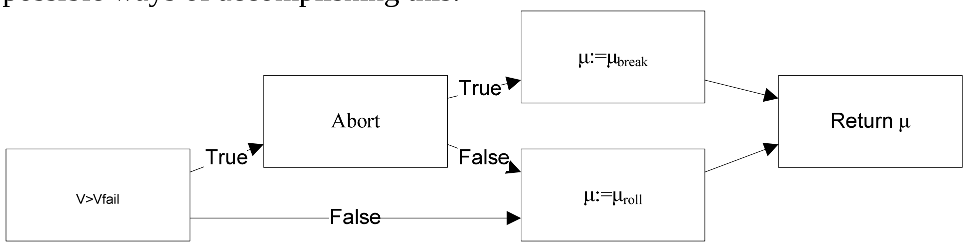
During the start procedure, at $V_{rot}$, the aircraft has to rotate. One of the ways to simulate this is to do as follows:




If the speed is higher than the rotation speed, the pitch angle is increase with a fixed amount per second up to a set maximum. When the failure speed is exceeded and the aircraft is set to abort, the pitch angle is set to zero.

## *After failure speed*

For the thrust and the braking and wheel resistance the following scheme is one of the possible ways of accomplishing this:




If the failure speed is exceeded and the aircraft is set to abort, the wheel resistance is set to the brake value. In all other cases the resistance is set to the rolling value. If the aircraft is off the ground, the ground force will be zero, so the value of the wheel resistance will not have any influence anyway. In a similar way, the thrust will be set to zero when the aircraft aborts, and will be set to $T = T_{\max}\left[1-\frac{1}{N_{engines}}\right]$ when the aircraft continues after the failure speed has been reached.

# Results of each simulation

As the aircraft has reached the screen height or, in case of an abort, slowed to a speed below zero meters per second, the simulation is stopped. The failure speed and the traveled distance are then returned.

## Calculating the balanced field length

For calculating the balance field length, it is best to run the simulation for a series of failure speeds both with and without abort. For this, it saves a lot of work if you can run the actual simulation by calling it with just two parameters; Failure speed and Abort (True/False). It is then possible to create a loop to calculate all the results automatically and return all the values of the failure speed and the traveled distance required for creating a graph and calculating the balanced field length.

## Assignment

The assignment has to be done in subgroups of two to three students, with each subgroup using a different aircraft. Each subgroup has to hand in its own report before the deadline.

## Report requirements

For your report the following items are required, separate from the requirements stated by written reporting:

- A flow diagram showing the program structure.
- A graph with the required distance against the engine failure speed, with and without abort, for a relevant range of failure speeds.
- The calculated balanced field length and the accompanying value of the decision speed, the stall speed and the rotation speed.
- Graphs with the time vs. speed, traveled distance and altitude with an engine failure at the decision speed, with and without abort.
- Explanation of the results

## References


- **Elements of Airplane performance**, G.J.J. Ruijgrok, *Delft University Press*
- **AE2-201: Flight Path Simulations**, Prof.dr.ir. Th. Van Holten, ir. J.A.Krijnen

## Two engine powered jet aircraft

| **Parameter** | **Description** | **Value** | **Dimension** |
|---|---|---|---|
| $N_{engines}$ | Number of engines | **2** | - |
| A | Aspect ratio | 15 | - |
| S | Wing area | 100 | $m^2$ |
| W | Take off weight | 500 | kN |
| $\alpha_0$ | Zero lift angle of attack | -3 | Deg |
| $C_{L_\alpha}$ | Lift coefficient gradient | 4.85 | $Rad^{-1}$ |
| $C_{L_{\max}}$ | Maximum lift coefficient | 1.60 | - |
| $T_{\max}$ | Maximum available thrust (total) | 150 | kN |
| $\frac{d\theta}{dt}$ | Pitch angle gradient during rotation | 6 | Deg/sec |
| $\theta_{\max}$ | Maximum pitch angle during rotation | 16 | Deg |
| $\mu_{roll}$ | Rolling resistance coefficient | 0.02 | - |
| $\mu_{brake}$ | Braking resistance coefficient | 0.2 | - |
| $C_{D_0}$ | Rest drag coefficient, without engine failure | 0.021 | - |
| $C_{D_{0,fail}}$ | Rest drag coefficient, with single engine failure | 0.026 | - |
| e | Oswald Factor | 0.85 | - |
| $h_s$ | Screen height | 10.7 | m |

After the engine failure, available thrust will be reduced to 50% of the total amount. Also the rest drag coefficient ($C_{D_0}$) will increase, to the 'fail' value in the table above.

## Three engine powered jet aircraft

| Parameter | Description | Value | Dimension |
|---|---|---|---|
| $N_{engines}$ | Number of engines | **3** | - |
| A | Aspect ratio | 14 | - |
| S | Wing area | 200 | $m^2$ |
| W | Take off weight | 1200 | kN |
| $\alpha_0$ | Zero lift angle of attack | -4 | Deg |
| $C_{L_\alpha}$ | Lift coefficient gradient | 4.32 | $Rad^{-1}$ |
| $C_{L_{\max}}$ | Maximum lift coefficient | 1.45 | - |
| $T_{\max}$ | Maximum available thrust (total) | 360 | kN |
| $\frac{d\theta}{dt}$ | Pitch angle gradient during rotation | 5 | Deg/sec |
| $\theta_{\max}$ | Maximum pitch angle during rotation | 15 | Deg |
| $\mu_{roll}$ | Rolling resistance coefficient | 0.02 | - |
| $\mu_{brake}$ | Braking resistance coefficient | 0.2 | - |
| $C_{D_0}$ | Rest drag coefficient, without engine failure | 0.024 | - |
| $C_{D_{0,fail}}$ | Rest drag coefficient, with single engine failure | 0.028 | - |
| e | Oswald Factor | 0.80 | - |
| $h_s$ | Screen height | 10.7 | m |

After the engine failure, available thrust will be reduced to 67% of the given amount. Also the rest drag coefficient ($C_{D_0}$) will increase, to the ‘fail’ value in the table above.

# Four engine powered jet aircraft

| Parameter | Description | Value | Dimension |
|---|---|---|---|
| $N_{engines}$ | Number of engines | **4** | - |
| A | Aspect ratio | 12 | - |
| S | Wing area | 500 | $m^2$ |
| W | Take off weight | 3500 | kN |
| $\alpha_0$ | Zero lift angle of attack | -5 | Deg |
| $C_{L_\alpha}$ | Lift coefficient gradient | 3.95 | $Rad^{-1}$ |
| $C_{L_{\max}}$ | Maximum lift coefficient | 1.40 | - |
| $T_{\max}$ | Maximum available thrust (total) | 1200 | kN |
| $\frac{d\theta}{dt}$ | Pitch angle gradient during rotation | 4 | Deg/sec |
| $\theta_{\max}$ | Maximum pitch angle during rotation | 14 | Deg |
| $\mu_{roll}$ | Rolling resistance coefficient | 0.02 | - |
| $\mu_{brake}$ | Braking resistance coefficient | 0.2 | - |
| $C_{D_0}$ | Rest drag coefficient, without engine failure | 0.026 | - |
| $C_{D_{0,fail}}$ | Rest drag coefficient, with single engine failure | 0.029 | - |
| e | Oswald Factor | 0.82 | - |
| $h_s$ | Screen height | 10.7 | m |

After the engine failure, available thrust will be reduced to 75% of the given amount. Also the rest drag coefficient ($C_{D_0}$) will increase, to the 'fail' value in the table above.

# Two engine powered propeller aircraft

| Parameter | Description | Value | Dimension |
|---|---|---|---|
| $N_{engines}$ | Number of engines | **2** | - |
| A | Aspect ratio | 8 | - |
| S | Wing area | 20 | $m^2$ |
| W | Take off weight | 60 | kN |
| $\alpha_0$ | Zero lift angle of attack | -2 | Deg |
| $C_{L_\alpha}$ | Lift coefficient gradient | 4.85 | $Rad^{-1}$ |
| $C_{L_{\max}}$ | Maximum lift coefficient | 1.30 | - |
| $T_{\max}$ | Maximum available power (total) | 1500 | kW |
| $\frac{d\theta}{dt}$ | Pitch angle gradient during rotation | 6 | Deg/sec |
| $\theta_{\max}$ | Maximum pitch angle during rotation | 14 | Deg |
| $\mu_{roll}$ | Rolling resistance coefficient | 0.02 | - |
| $\mu_{brake}$ | Braking resistance coefficient | 0.2 | - |
| $C_{D_0}$ | Rest drag coefficient, without engine failure | 0.030 | - |
| $C_{D_{0,fail}}$ | Rest drag coefficient, with single engine failure | 0.035 | - |
| e | Oswald Factor | 0.75 | - |
| $h_s$ | Screen height | 10.7 | m |

After the engine failure, available power will be reduced to 50% of the given amount. Also the rest drag coefficient ($C_{D_0}$) will increase, to the ‘fail’ value in the table above.

# Four engine powered propeller aircraft

| Parameter | Description | Value | Dimension |
|---|---|---|---|
| $N_{engines}$ | Number of engines | **4** | - |
| A | Aspect ratio | 10 | - |
| S | Wing area | 60 | $m^2$ |
| W | Take off weight | 240 | kN |
| $\alpha_0$ | Zero lift angle of attack | -3 | Deg |
| $C_{L_\alpha}$ | Lift coefficient gradient | 4.90 | $Rad^{-1}$ |
| $C_{L_{\max}}$ | Maximum lift coefficient | 1.40 | - |
| $T_{\max}$ | Maximum available power (total) | 4800 | kW |
| $\frac{d\theta}{dt}$ | Pitch angle gradient during rotation | 4 | Deg/sec |
| $\theta_{\max}$ | Maximum pitch angle during rotation | 13 | Deg |
| $\mu_{roll}$ | Rolling resistance coefficient | 0.02 | - |
| $\mu_{brake}$ | Braking resistance coefficient | 0.2 | - |
| $C_{D_0}$ | Rest drag coefficient, without engine failure | 0.028 | - |
| $C_{D_{0,fail}}$ | Rest drag coefficient, with single engine failure | 0.032 | - |
| e | Oswald Factor | 0.82 | - |
| $h_s$ | Screen height | 10.7 | m |

After the engine failure, available power will be reduced to 75% of the given amount. Also the rest drag coefficient ($C_{D_0}$) will increase, to the ‘fail’ value in the table above.