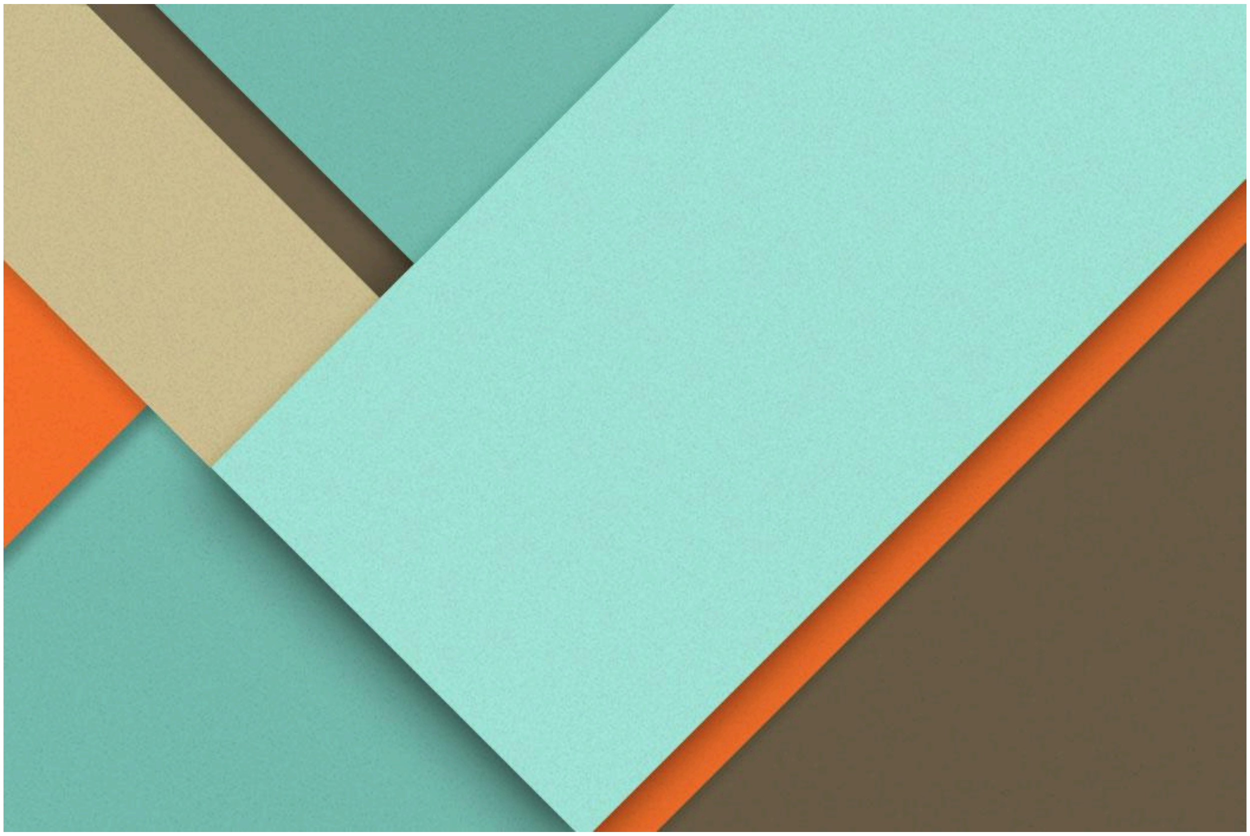


Communication Assignment

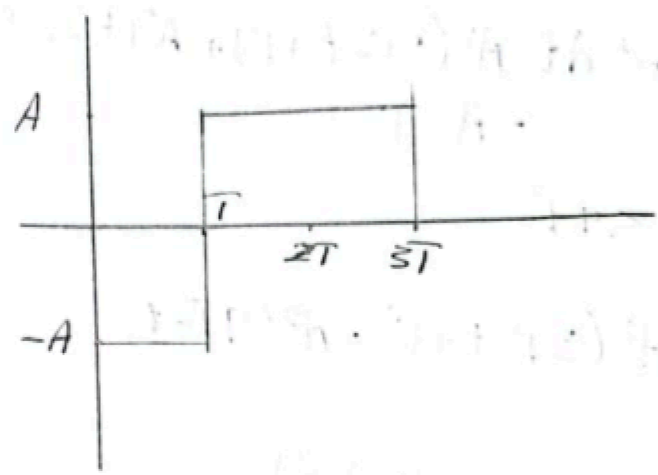
Name	SEC	BN
Sara Bisheer Fikry	1	18
Menna Mohammed AbdelBaset	2	26

Part1

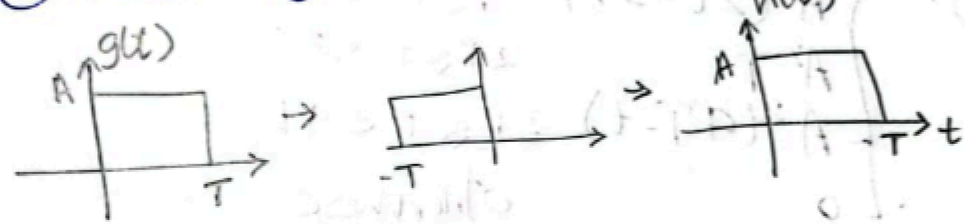
Hand Analysis:

1. Part 1:-

(a)



(b) $h(t) = kg(T-t)$



(c) $y(t) = h(t) * r(t)$

$= h(t) * g(t) + \cancel{h(t) * w(t)}$ ignore noise

• Case I: $0 \leqslant t \leqslant T$

$$\int_0^t A(-A)\,dt = -A^2(t)$$

• Case II: $T \leqslant t \leqslant 2T$

$$\int_{t-T}^{T} -A^2\,dt + \int_T^t A^2\,dt = -A^2(2T-t) + A^2(t-T)$$

$$= A^2(-3T+2t)$$

2

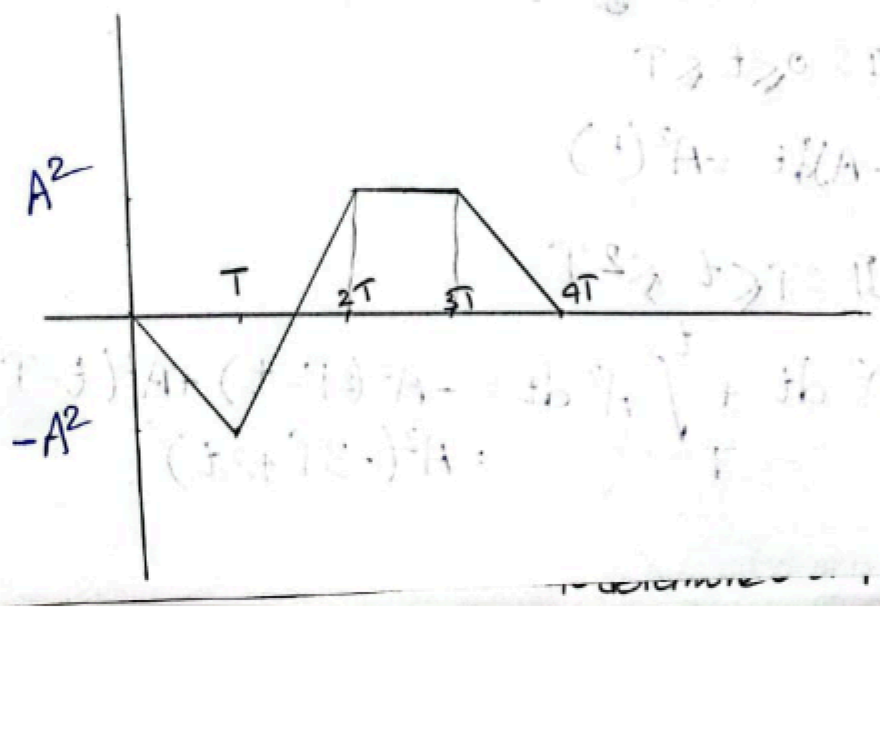
Case III : $2T \leq t < 3T$

$$\int_{t-T}^{2T} A^2\,dt + \int_{2T}^{t} A^2\,dt = A^2(2T - t + T) + A^2(t - 2T)$$
$$= A^2 T$$

Case IV : $3T \leq t < 4T$

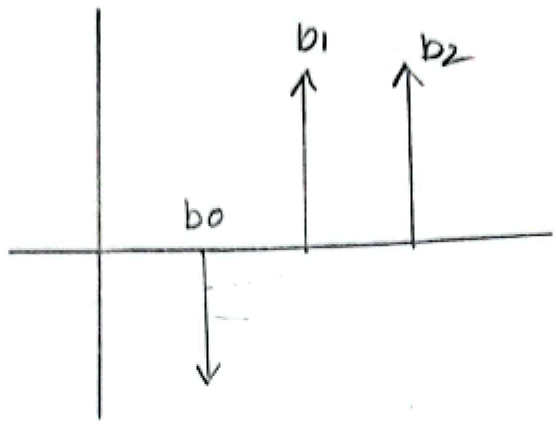
$$\int_{t-T}^{3T} A^2\,dt = A^2(3T - t + T) = A^2(4T - t)$$

$$y(t) = \begin{cases} -A^2 t & 0 \leq t < T \\ A^2(2t - 3T) & T \leq t < 2T \\ A^2 T & 2T \leq t < 3T \\ A^2(4T - t) & 3T \leq t < 4T \\ 0 & \text{otherwise} \end{cases}$$

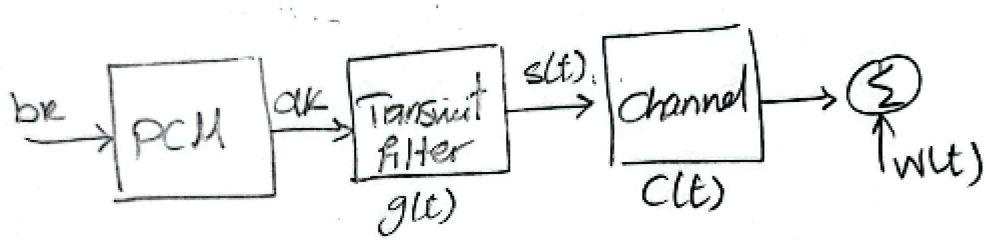


3

(c)

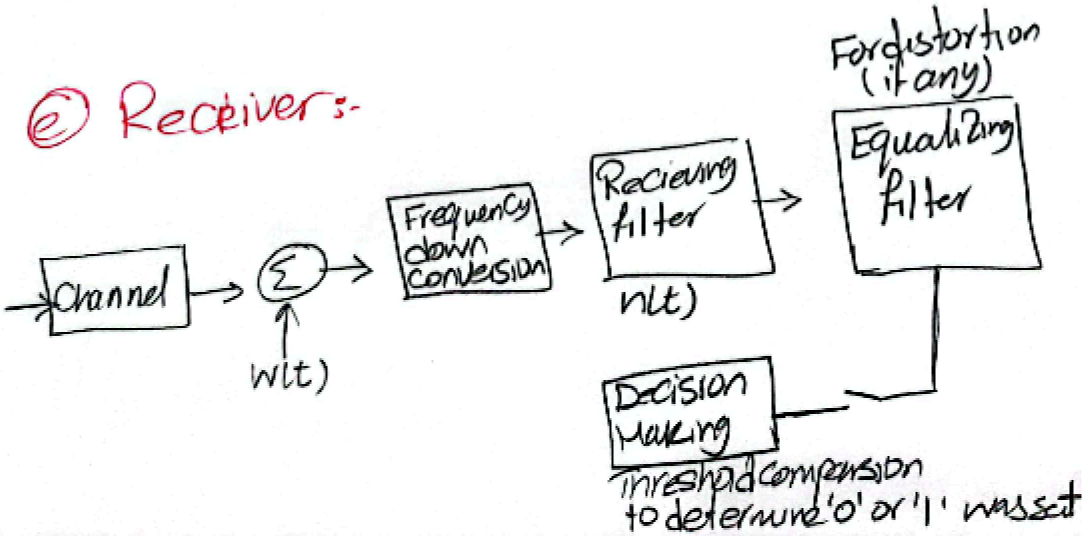


(d) Transmitter



$s(t) = \sum a_k\, g(t - kT_b)$

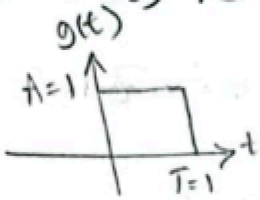
(e) Receiver:-

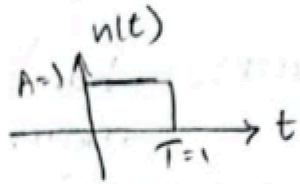


Part 2

* Analysis for part II

ⓐ h(t) is a matched filter with unit energy





$$h(t) = kg(T-t)$$

$$y(t) = h(t) * r(t) = h(t) * (g(t) + w(t))$$

$$= \underbrace{g(t) * h(t)}_{g_o(t)} + \underbrace{w(t) * h(t)}_{n(t)}$$

$$g_o(t) = g(t) * h(t) = \int g(t)\, \underbrace{h(t-z)}_{kg(T-t+z)}\, dz = k\int g(t) g(T-t+z)\, dz$$

$$g_o(T_p) = k\int_0^{T_p} A g(T_p)\, g(T_p)\, dz = k\int_0^{T_p} A \cdot -A\, dt = -kA^2 T_p$$

$$|g_o(T_p)| = kA^2 T_p$$

$$y(T_p) = \begin{cases} -kA^2 T_p + n(t_p) & \text{when '0' is sent} \\ kA^2 T_p + n(t_p) & \text{when '1' is sent} \end{cases}$$

* We can make $k = \dfrac{1}{T_p A}$

$$y(T_p) = \begin{cases} -A + n(t_p) & \rightarrow \text{when '0' is sent} \\ A + n(t_p) & \rightarrow \text{when '1' is sent} \end{cases}$$

2

$n(t) = h(t) * w(t)$

$n(t)$ is Gaussian Random noise with zero mean

$$Var(n(T_p)) = E[n^2[T_p]] - E^2[\underbrace{\cancel{n(T_p)}}_{0}]$$

$$E[n^2[T_p]] = \int G_n(f)\,df = \int |H(f)|^2 \underbrace{\cancel{G_w(f)}}_{\frac{N_0}{2}}\,df$$

'power'
average of energy

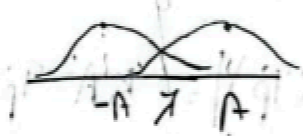
$$= \frac{N_0}{2}\int \underbrace{|H(f)|^2\,df}_{\frac{1}{T_p}\ (as\ E = A^2T)} = \frac{N_0}{2T_p}$$

To clarify:
$$= \frac{N_0}{2}k^2 \int |G(f)|^2\,df \doteq \frac{N_0}{2T_p}$$

$$\cancel{y(T_p)} = n(T_p) \sim N\left(0, \frac{N_0}{2T_p}\right)$$

$$y(T_p) \sim N\left(\pm A, \frac{N_0}{2T_p}\right)$$

$$P(error) = P(y<\lambda|'1')\,P('1') + P(y>\lambda|'0')\,P('0')$$



$$P(y<\lambda|'1') = Q\left(\frac{\lambda+A}{\sqrt{N_0/2T_p}}\right)$$

$$P(y>\lambda|'0') = 1 - Q\left(\frac{\lambda-A}{\sqrt{N_0/2T_p}}\right) = Q\left(\frac{A-\lambda}{\sqrt{N_0/2T_p}}\right)$$

Suppose $P('1') = P('0') = 0.5$

8

To Choose λ we can get $\frac{\partial P_{error}}{\partial \lambda} = 0$

then $\lambda_{opt} = \frac{N_0}{4AT_p} \ln\left(\frac{P('0')}{P('1')}\right)$

then

$$\lambda_{opt} = 0$$

$$P(error) = \frac{1}{2} Q\left(\frac{A}{\sqrt{N_0/2T_p}}\right) + \frac{1}{2} Q\left(\frac{A}{\sqrt{N_0/2T_p}}\right)$$

$\boxed{x}$ from Given $T = 1$, $A = 1$

$$P(error) = Q\left(\frac{1}{\sqrt{N_0/2}}\right)$$

$$erfc(x) = 2Q(\sqrt{2}x)$$

$$Q\left(\sqrt{2}\frac{1}{\sqrt{N_0}}\right) = \frac{1}{2} erfc\left(\frac{1}{\sqrt{N_0}}\right)$$

(b) $h(t)$ is not existent (ie $h(t) = \delta(t)$)

$y(t) = h(t) * r(t) = h(t) * (g(t) + w(t))$

$= h(t) * g(t) + h(t) * w(t) = \delta(t) * g(t) + \delta(t) * w(t)$

$= g(t) + w(t)$

$$g(T_p) = \begin{cases} A & \text{when '1' is Sent} \\ -A & \text{when '0' is Sent} \end{cases}$$

$$y(T_p) = \begin{cases} A + w(t) & \text{when '1' is Sent} \\ -A + w(t) & \text{when '0' is Sent} \end{cases}$$

4

$w(t) \sim N(0, \frac{N_0}{2})$

$y(t) \sim (\pm A, \frac{N_0}{2})$

$P(error) = p(y<\lambda|'1')\, p('1') + p(y>\lambda|'0')\, p('0')$

$P(y<\lambda|'1') = Q\left(\frac{\lambda+A}{\sqrt{N_0/2}}\right)$

$P(y>\lambda|'0') = 1 - Q\left(\frac{\lambda-A}{\sqrt{N_0/2}}\right) = Q\left(\frac{A-\lambda}{\sqrt{N_0/2}}\right)$

taking $P('0') = P('1') = 0.5$

& $\lambda = 0$ as derived before in (a)

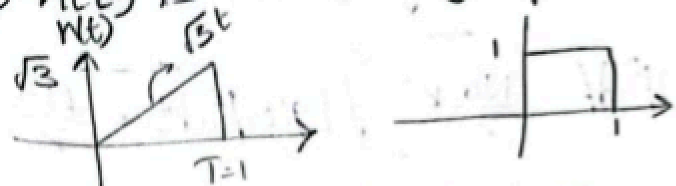
$P(error) = Q\left(\frac{\overset{1\ (\text{given})}{A}}{\sqrt{N_0/2}}\right)$

$erfc(x) = 2Q(\sqrt{2}x)$

$Q\left(\sqrt{2}\frac{1}{\sqrt{N_0}}\right) = \frac{1}{2} erfc\left(\frac{1}{\sqrt{N_0}}\right)$

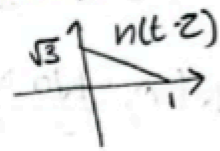
$P(error) = \frac{1}{2} erfc\left(\frac{1}{\sqrt{N_0}}\right)$

(c) $h(t)$ is the following impulse response



$$y(t) = \underbrace{h(t) * g(t)}_{g_o(t)} + h(t) * w(t)$$

$$g_o(t) = \int_0^1 h(t-\tau)\, g(\tau)\, d\tau$$



$$= \int_0^1 (-\sqrt{3}\,\tau + \sqrt{3})\, d\tau = \frac{\sqrt{3}}{2}$$

or we can make $g_o(t) = \int_0^1 h(t)\, g(t-\tau)\, d\tau)$

$$g_o(t) = \int_0^1 \sqrt{3}\, t\, dt = \frac{\sqrt{3}\, t^2}{2}\Big|_0^1 = \frac{\sqrt{3}}{2}$$

$$g_o(T_p) = \begin{cases} \frac{\sqrt{3}}{2} & \text{when '1' is sent} \\ -\frac{\sqrt{3}}{2} & \text{when '0' is sent} \end{cases}$$

$$y(T_p) = \begin{cases} \frac{\sqrt{3}}{2} + n(T_p) & \text{when '1' is sent} \\ -\frac{\sqrt{3}}{2} + n(T_p) & \text{when '0' is sent} \end{cases}$$

$$n(t) = h(t) * w(t)$$

$$w(t) \sim N(0, \tfrac{N_0}{2})$$

$$E[n[T_p]] = E\left[\int h(t-\tau)\, w(\tau)\, d\tau\right]$$
$$= \int h(t-\tau)\, \underbrace{E[w(\tau)]}_{0}\, d\tau = 0$$

6

$$E[n^2(t)] = \int G_n(f)\,df = \int_{-\infty}^{\infty} \frac{N_0}{2}|H(f)|^2\,df$$

$$= \frac{N_0}{2}\int_{-\infty}^{\infty} k^2 |G(f)|^2\,df = \frac{N_0}{2}k^2\int_{-\infty}^{\infty}|g(t)|^2\,dt = \frac{N_0}{2}$$

$$\mathrm{Var}(n(t)) = E[n^2(t)] - E^2\underbrace{(n(t))}_{0}$$

$$n(t) \sim N\left(0, \frac{N_0}{2}\right)$$

$$y(t) \sim N\left(\pm\frac{\sqrt{3}}{2}, \frac{N_0}{2}\right)$$

$$P(\text{error}) = P(y<\lambda\,|\,'1')P('1') + P(y>\lambda\,|\,'0')P('0')$$

$$P(y<\lambda\,|\,'1') = Q\left(\frac{\lambda+\frac{\sqrt{3}}{2}}{\sqrt{N_0/2}}\right)$$

$$P(y>\lambda\,|\,'0') = 1 - P(y>\lambda\,|\,'0') = 1 - Q\left(\frac{\lambda-\frac{\sqrt{3}}{2}}{\sqrt{N_0/2}}\right)$$

$$= Q\left(\frac{\frac{\sqrt{3}}{2}-\lambda}{\sqrt{N_0/2}}\right)$$

$$P(\text{error}) = Q\left(\frac{\frac{\sqrt{3}}{2}}{\sqrt{N_0/2}}\right) \quad \lambda=0,\quad P('1') = P('0') = \frac{1}{2}$$

$$Q\left(\sqrt{2}\,\frac{\frac{\sqrt{3}}{2}}{\sqrt{N_0}}\right) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{3}/2}{\sqrt{N_0}}\right)$$

$$P(\text{error}) = \frac{1}{2}\,\mathrm{erfc}\left(\frac{\sqrt{3}/2}{\sqrt{N_0}}\right)$$

5) Is the BER an increasing or a decreasing function of E/No? Why?

Answer:

It is a decreasing function as if the energy of the signal compared the energy of noise is high then the noise will not affect the signal so much and BER will decrease

so the more the E/No is the less the BER

6) Which case has the lowest BER? Why?

Answer:

The first one with the matched filter.

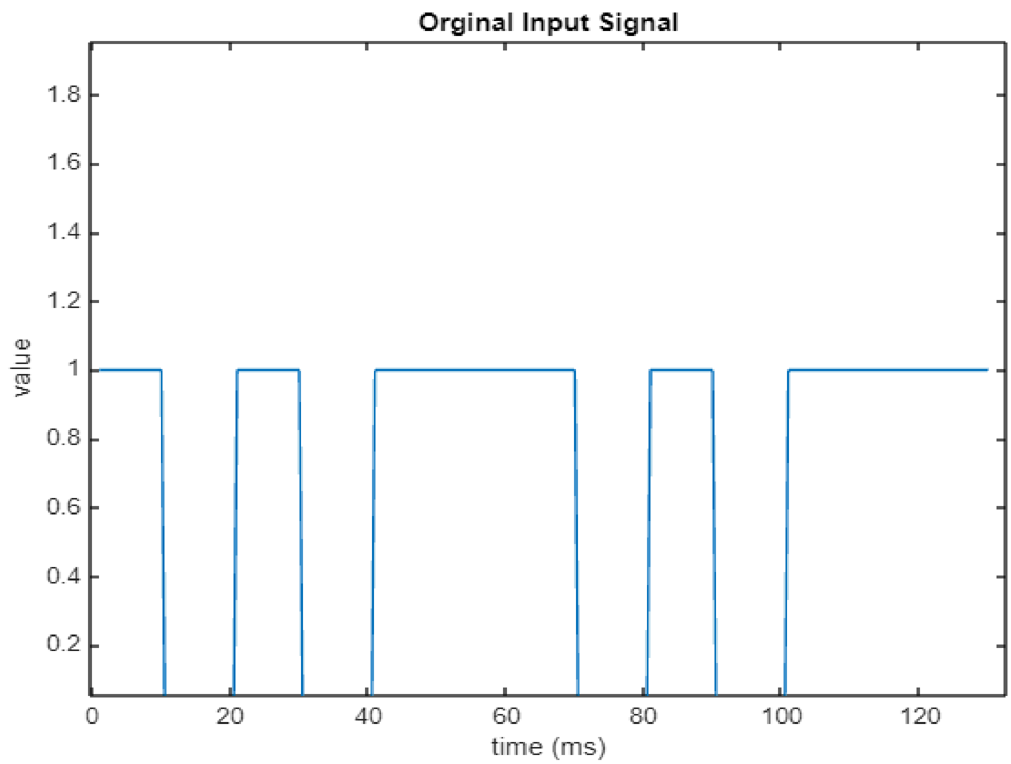
The scenario with the matched filter achieves the lowest BER. This is because the matched filter is specifically designed to maximize the peak SNR of the signal.

Consequently, this minimizes the probability of errors, leading to a lower BER.

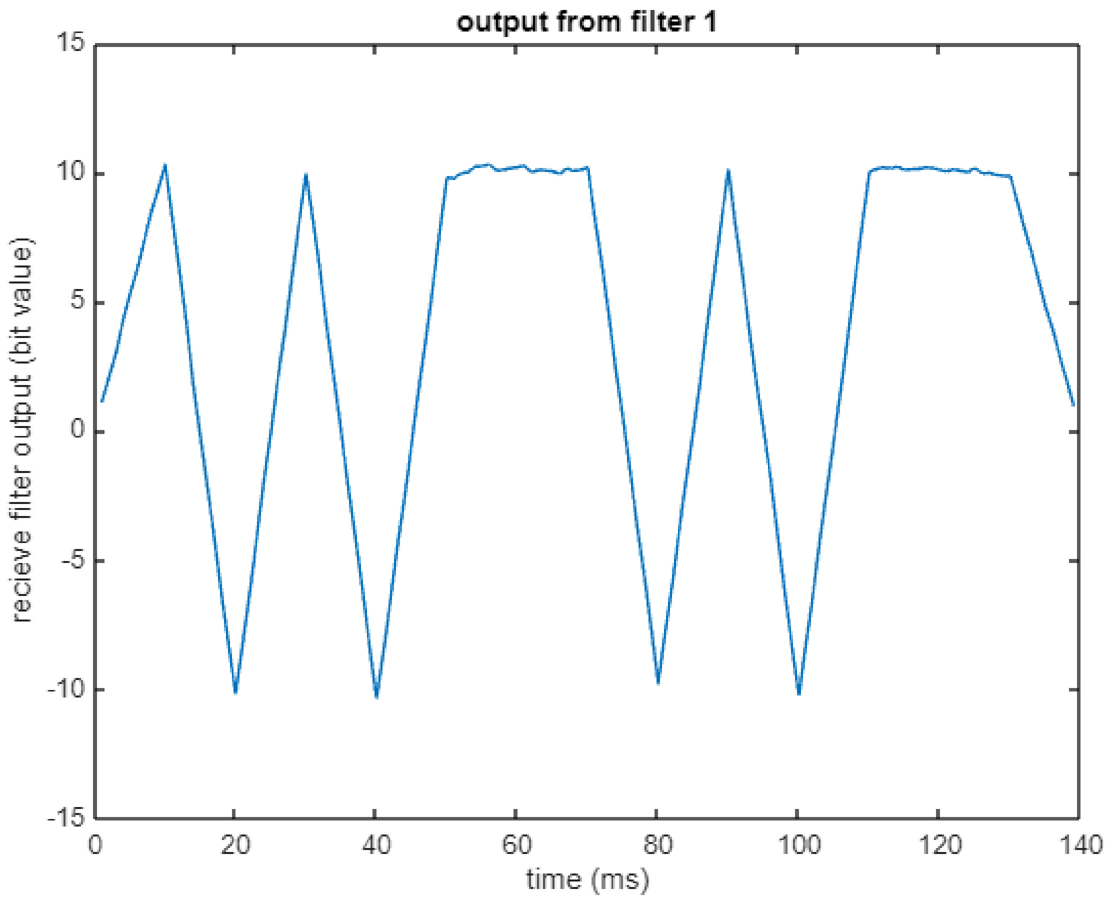
Simulation Of Part 2

The bits = `[1,0,1,0,1,1,1,0,1,0,1,1,1]`

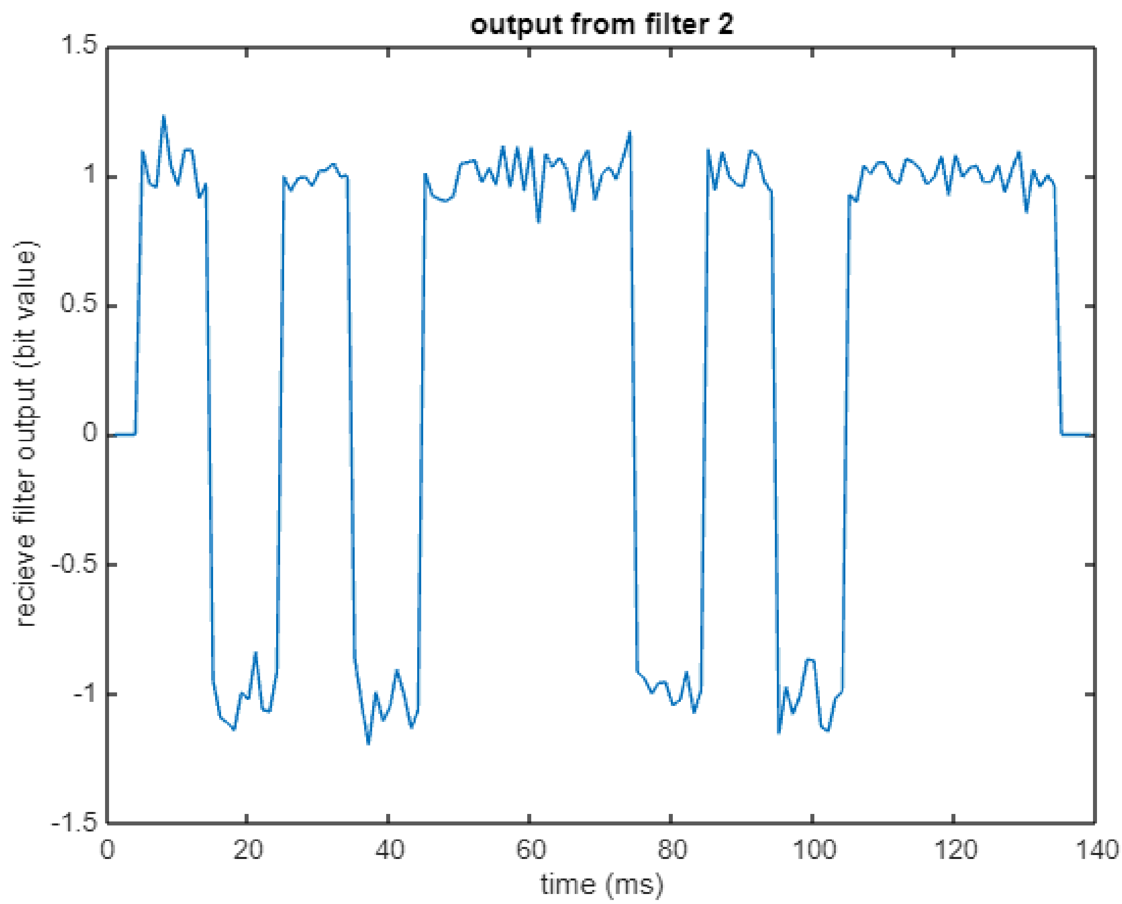
SNR = 20



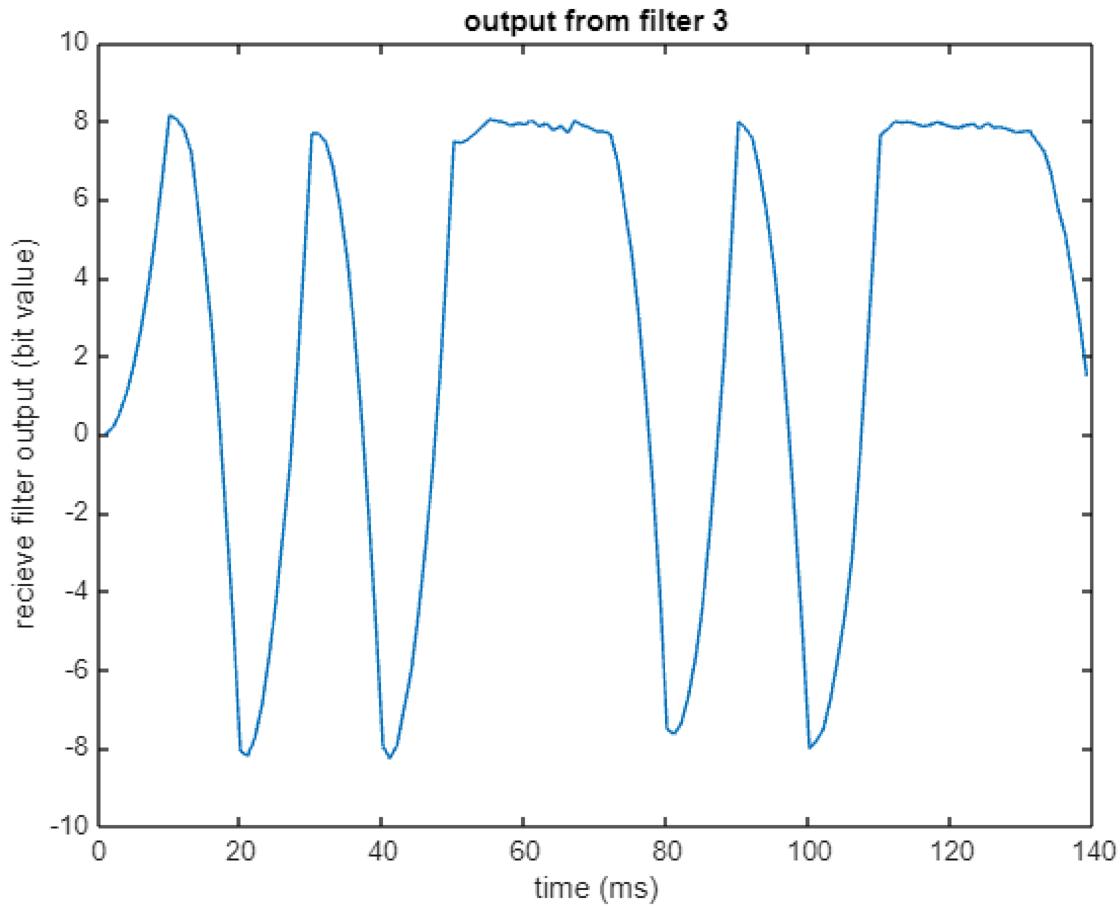
Matched Filter



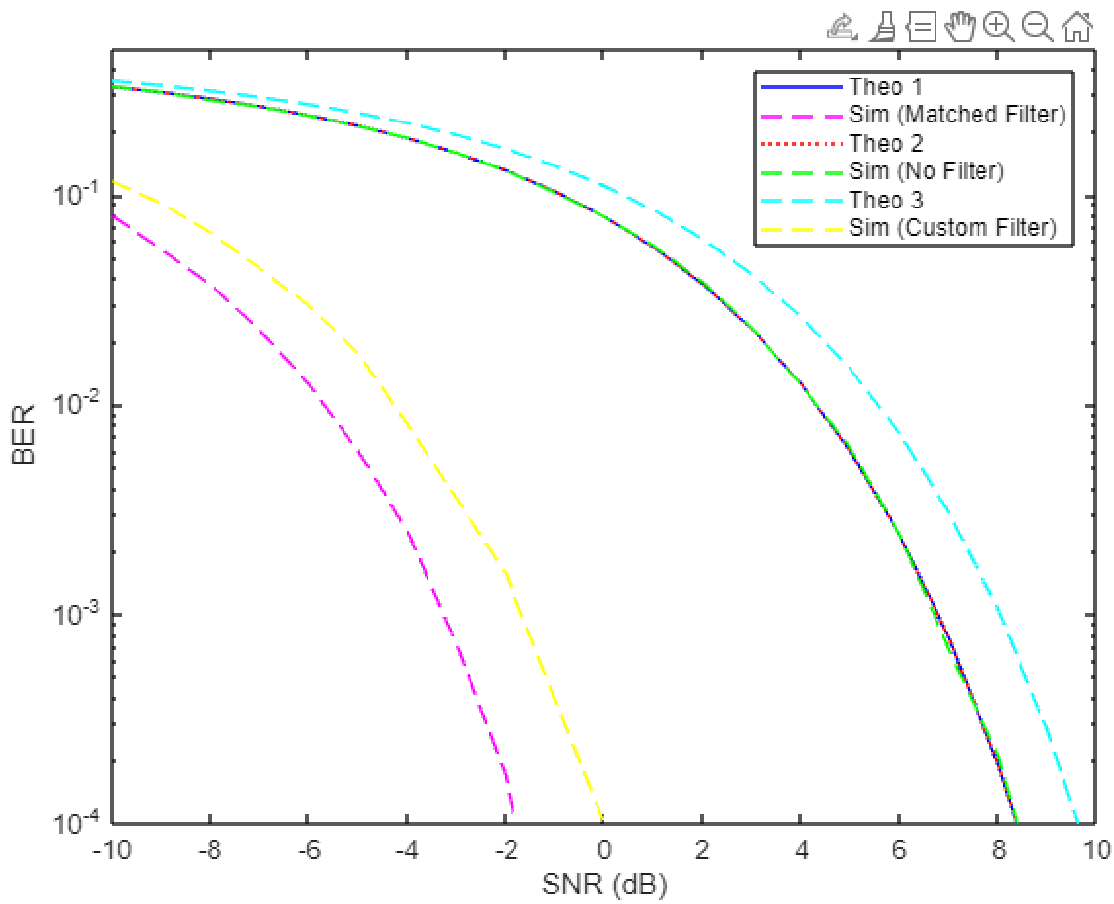
No Filter


output from filter 2
1.5
1
0.5
0
-0.5
-1
-1.5
recieve filter output (bit value)
0
20
40
60
80
100
120
140
time (ms)

Custom Filter (Triangle Filter)



BER Theoretical and Simulation For The Three Filters



Code

```
clear ;
close all;
samples_number = 10;
bits = [1,0,1,0,1,1,1,0,1,0,1,1,1];
bits_number = length(bits);
% Pulse Shape
[input] = pulse_shape(bits_number,samples_number,bits);
orginal_input = reshape(input.', [], 1);
figure;
plot(orginal_input);
title("Original Inp");
xlabel('time (ms)');
ylabel('receive filter output (bit value)');
% Channel AWGN
% Generate Noise To Add
E = 1;
snr_range = -10:1:20;
snr = 10 ^(snr_range(30)/10);
[input_with_noise] = add_noise(bits_number,samples_number,input,E,snr);
plot(input_with_noise);
hold on ;
% filters definitions
delta_filter = zeros(1,samples_number);
delta_filter(samples_number/2)=1;
t = 0 : 1 : samples_number -1;
tri_filter = (sqrt(3)/samples_number)*t;
matched_filter = ones(1,samples_number);
filter ={matched_filter,delta_filter,tri_filter};
output = {0,0,0};
for k=1 : 3
   output{k} = conv(input_with_noise,filter{k});
```

```
end
% show output of each filter
for k=1 : 3
   % show the output
   figure;
   plot(output{k});
   title(sprintf('output from filter %d', k));
   xlabel('time (ms)');
   ylabel('receive filter output (bit value)');
   hold on ;
end
% sample the output to get stream of bits
for i=0:bits_number-1
   output_1_samples = sample(output{1},bits_number,samples_number);
   output_2_samples = sample(output{2},bits_number,samples_number);
   output_3_samples = sample(output{3},bits_number,samples_number);
end
% disp(output_1_samples)
% disp(output_2_samples)
% disp(output_3_samples)
% calculate accuracy of each filter
err_prob_1 = sum(output_1_samples ~= bits);
BER_1 = err_prob_1/bits_number;
err_prob_2 = sum(output_2_samples ~= bits);
BER_2 = err_prob_2/bits_number;
err_prob_3 = sum(output_3_samples ~= bits);
BER_3 = err_prob_3/bits_number;
disp(BER_1);
disp(BER_2);
disp(BER_3);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
```

```
% calculate BER for different SNR
bits_number = 100000;
samples_number = 10;
indices = randperm(bits_number, 1);
bits= ones(bits_number,1);
bits(indices)=0;
input =pulse_shape(bits_number,samples_number,bits);
snr_range = -10:1:20;
% Preallocate arrays to store BER simulations
BER_sim_1 = zeros (length(snr_range),1);
BER_sim_2 = zeros (length(snr_range),1);
BER_sim_3 = zeros (length(snr_range),1);
BER_theo_1 = zeros(length(snr_range),1);
BER_theo_2 = zeros(length(snr_range),1);
BER_theo_3 = zeros(length(snr_range),1);
for i = 1:length(snr_range)
   snr = 10 ^(snr_range(i)/10);
   input_with_noise = add_noise(bits_number,samples_number,input,E,snr);
   for k = 1:3
       output{k} = conv(input_with_noise, filter{k});  % Consider using
'same' to maintain dimensionality
   end
   % Extracting the middle point for each bit period after convolution
   output_1_samples = sample(output{1},bits_number,samples_number);
   output_2_samples = sample(output{2},bits_number,samples_number);
   output_3_samples = sample(output{3},bits_number,samples_number);
   % Calculate errors and BER for each filter
   err_prob_1 = sum(output_1_samples.' ~= bits);
   BER_sim_1(i) = err_prob_1 / bits_number;
   err_prob_2 = sum(output_2_samples.' ~= bits);
   BER_sim_2(i) = err_prob_2 / bits_number;
   err_prob_3 = sum(output_3_samples.' ~= bits);
```

```
    BER_sim_3(i) = err_prob_3 / bits_number;
    BER_theo_1(i)=0.5*erfc(sqrt(snr));
    BER_theo_2(i)=0.5*erfc(sqrt(snr));
    BER_theo_3(i)=0.5*erfc((sqrt(3)/(2)*sqrt(snr)));
end
% Update plot commands to reflect all data
figure;
semilogy(snr_range, BER_theo_1, 'b-');
hold on;
semilogy(snr_range, BER_sim_1, 'm--');
semilogy(snr_range, BER_theo_2, 'r:');
semilogy(snr_range, BER_sim_2, 'g--');
semilogy(snr_range, BER_theo_3, 'c--');
semilogy(snr_range, BER_sim_3, 'y--');
hold off;
ylim([10^-4 0.5]);
xlabel('SNR (dB)');
ylabel('BER');
legend('Theo 1', 'Sim (Matched Filter)', 'Theo 2', 'Sim (No Filter)',
'Theo 3', 'Sim (Custom Filter)');
function [input] = pulse_shape(bits_number,samples_number,bits)
  input = ones(bits_number,samples_number);
    for i=1 : bits_number
        if bits(i) == 0
        input(i,:) = -input(i,:);
        end
    end
end
function [input_with_noise] =
add_noise(bits_number,samples_number,input,E,snr)
    sigma = sqrt(E/(2.0*snr));
    noise = normrnd(0,sigma,[1,bits_number*samples_number]);
```

```
    input_with_noise = input;
    % add noise to input
    for i=1 : bits_number
        input_with_noise(i,:) = input_with_noise(i,:) +
noise((samples_number)*(i-1)+1:(samples_number)*(i));
    end
    input_with_noise = reshape(input_with_noise.', [], 1);
end
function [samples]=sample(output,bits_number,samples_number)
    samples = ones(1,bits_number);
    for i=0:bits_number-1
        samples(i+1) = (output((samples_number - 1) + samples_number *
i+1)) > 0;
    end
end
```